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COMPARISON OF LASER MODELS

By

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Comparison of Laser Models

Notation.

In this report the following notations will be used:

$$x_1 = [RI] \qquad x_4 = [I_2]$$
$$x_2 = [R] \qquad x_5 = [I^*]$$
$$x_3 = [R_2] \qquad x_6 = [I]$$
$$x_7 = [\rho]$$

where [ ] denotes concentration and R is a perflouride radical, I is iodine, I^* is an excited state of iodine, and ρ is the photon density.

Oscillatory Model.

The following is the unscaled version of the time varying chemical kinetics equations for the simulation of the iodine laser operation

$$\frac{dx_1}{dt} = k_1x_2x_5 + k_2x_2x_6 - \psi_1x_1 - k_4x_1x_2$$
$$\frac{dx_2}{dt} = -k_1x_2x_5 - k_2x_2x_6 - 2k_3x_2^2 + \psi_1x_1 - k_4x_1x_2$$
$$\frac{dx_3}{dt} = k_3x_2^2 + k_4x_1x_2$$
$$\frac{dx_4}{dt} = c_1x_1x_5x_6 + c_2x_1x_6^2 + c_3x_4x_5x_6 - \psi_2x_4 + c_4x_4x_6^2$$
$$\frac{dx_5}{dt} = -k_1x_2x_5 - c_1x_1x_5x_6 - c_3x_4x_5x_6 - Q_1x_1x_5 - Q_2x_4x_5 - Ax_5 + \psi_1x_1 + 0.51\psi_2x_4 - \Gamma_{max}$$
$$\frac{dx_6}{dt} = 1.49\psi_2x_4 + Q_1x_1x_5 + Q_2x_4x_5 + Ax_5 - c_1x_1x_5x_6 + \Gamma_{max} + k_4x_1x_2 - 2c_2x_1x_6^2 - c_3x_4x_5x_6 - k_2x_2x_6 - 2c_4x_4x_6^2$$
$$\frac{dx_7}{dt} = \sqrt{\frac{\pi}{2.77}}\left(\frac{L}{L_c}\right)\Gamma_{max} + \frac{\log(R_1R_2)}{2\left(\frac{L_c}{c}\right)}x_7 + 2\left(\frac{0.18}{L_c}\right)^2 Ax_5$$
$$\text{where} \quad \Gamma_{max} = \frac{cx_7(x_5 - \frac{1}{2}x_6)}{A_{00} + B_{00}x_1}$$

The above equations are solved subject to the initial conditions

$$x_1(0) = P * (3.5E16) \quad \text{and} \quad x_i(0) = 0, \quad i = 2, \ldots, 7$$

where P is the pressure in torr. The above system was solved using a variable step size Runge-Kutta-Fehlberg alogorithm. The figures 1.1 through 1.4 illustrated a typical set of solution curves to the above system of differential equations. Note the violent oscillations which occur during start-up of the laser. These oscillations require that small step sizes be taken during the numerical integration process in order to achieve some desired degree of accurracy. The numerical method becomes unstable and the results are meaningless if too large a step size is selected. This is not an uncommon situation in trying to numerical solve chemical kinetic systems of equations.

Nonoscillatory Model.

The inversion term $[I^*] - \frac{1}{2}[I] = x_5 - \frac{1}{2}x_6$ which occurs in the Γ_{max} term as well as the equations which define the rate of change of $[I^*] = x_5$ and $[I] = x_6$, is the term which causes the oscillations in the nonlinear system of coupled ordinary differential equations. Under quasi-steady state conditions we set the rate of change of this term equal to zero. That is we require that

$$\frac{d}{dt}\left([I^*] - \frac{1}{2}[I]\right) = 0$$

and then solve for Γ_{max} which is then relabled as $(\Gamma_{max})_{ss}$. We find that

$$\begin{aligned}(\Gamma_{max}) = &- Ax_5 - Q_2x_4x_5 + \{2\psi_1x_1 - 0.47\psi_2x_4 - 2k_1x_2x_5 + k_2x_2x_6 \\ &- c_1x_1x_5x_6 + 2c_2x_1x_6^2 - c_3x_4x_5x_6 + 2c_4x_4x_6^2 - k_4x_1x_2\}/3\end{aligned}$$

Initially the Γ_{max} term is removed from the above equations as it is assumed to be initially zero. As time increases, $[I^*]$ and $[I]$ build up and when the threshold condition is met we have

$$[I^*] - \frac{1}{2}[I] = -\frac{1}{2}\frac{\ln(R_1R_2)}{\sqrt{(\pi/2.77)}}(L\sigma)^{-1} = I_{th}.$$

Here it is assumed that steady state conditions have been achieved and gains balance with losses. The above steady state value is subsituted for Γ_{max}. In this way the ocillatory character of the solution is removed and the integration can be made to proceed at a much faster rate. The results from this second nonoscillatory model were generated by L.Stock and are presented in the figures 1.5, 1.6. Observe how the oscillatory character of the solutions has been removed. Also, as a check on the model, the values of the power output were compared for both models and both values were found to be in agreement. The figure 1.7 lists the values of the reaction rate coefficients used in both models.

Currently there are four models for the simulation of an iodine laser. These models can be described by:

1. A time dependent model which is given by the above set of coupled nonlinear differential equations which describe the chemical kinetics at a fixed point as a function of time t.
2. A quasi-steady state model whereby the Γ_{max} term is replaced by its steady state value. This model is also a time dependent model with much faster integration times.
3. A noncompressible model which solves the above system of differential equations as a function of distance z along the axis of the laser tube. This model is characterized by substituting a material derivative in place of the time derivatives in the above differential equations. For an assumed steady flow, we make the substitutions:

$$\frac{d}{dt}[\,] = \frac{\partial[\,]}{\partial t} + \frac{\partial[\,]}{\partial z}\frac{dz}{dt}$$

   and assume that $\frac{dz}{dt} = \omega$ is a constant. The time dependence is removed by assuming steady state values. Problems of stiffness arise in the numerical integration of this system.
4. A compressible flow laser model which solves the above system of differential equations with the material derivative replacing the time derivatives. In addition to these equations there is added: (i) a continuity equation (ii) an equation of state (iii) a momentum equation and (iv) an energy equation.

The figures 1.8 illustrates power vs beam radius which was obtained from the model 4, above assuming a parabolic velocity profile. The figure 1.9 illustrated power vs reflectivity for various pumping distances along a 45cm laser tube.

Current research is being directed toward:

1. Parameter studies using the compressible flow laser model.
2. Development of a two pass amplifier model.
3. Solve system of equations describing operation of high powerhh iodine MOPA (Master Oscillator Power Amplifier). These equations are derived from the Frantz-Nodvik Amplifier theory.

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1. L.V. Stock, J.W.Wilson, R.J.DeYoung, A Model for the Kinetics of a Solar-pumped Long Path Laser Experiment., NASA Technical Memorandum 87668, May 1986.
2. J.W.Wilson, S.Raju, Y.J. Shiu, Solar-Simulator-Pumped Atomic Iodine Laser Kinetics, NASA Technical Paper 2182, August 1983.

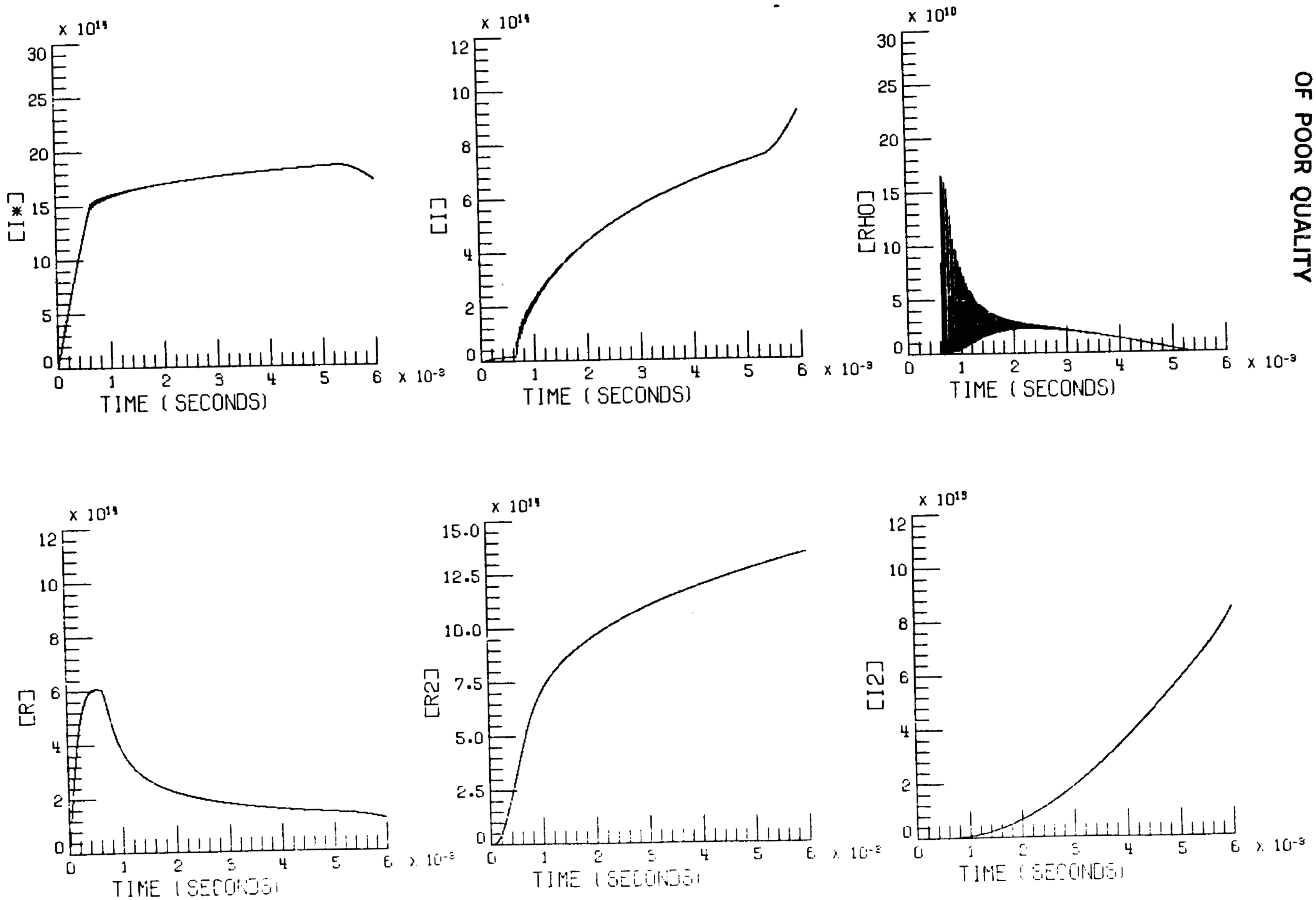

Fig. 1.1. Nominal Solution to Iodine Laser Equations.

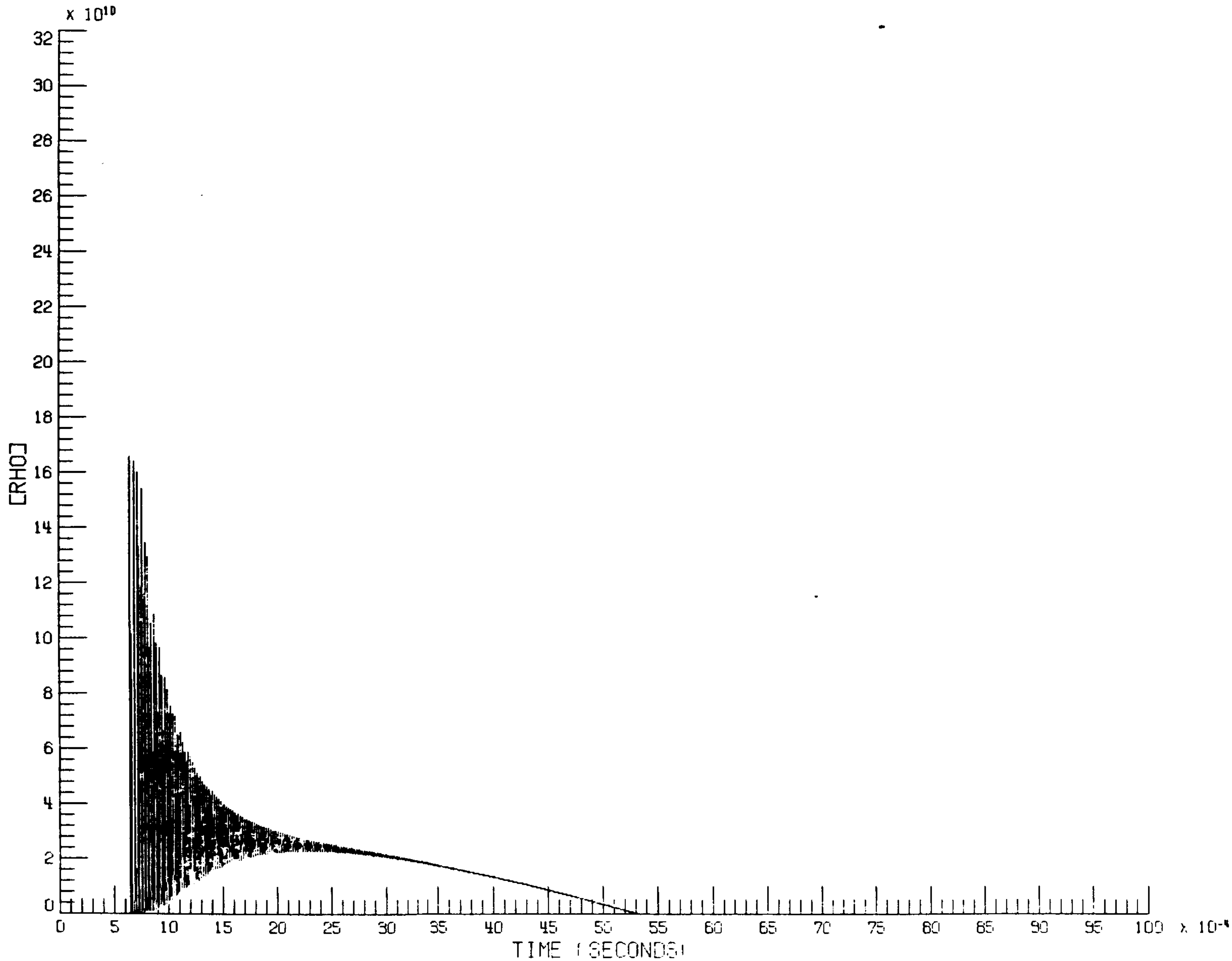

Fig. 1.2 Photon Density vs. Time

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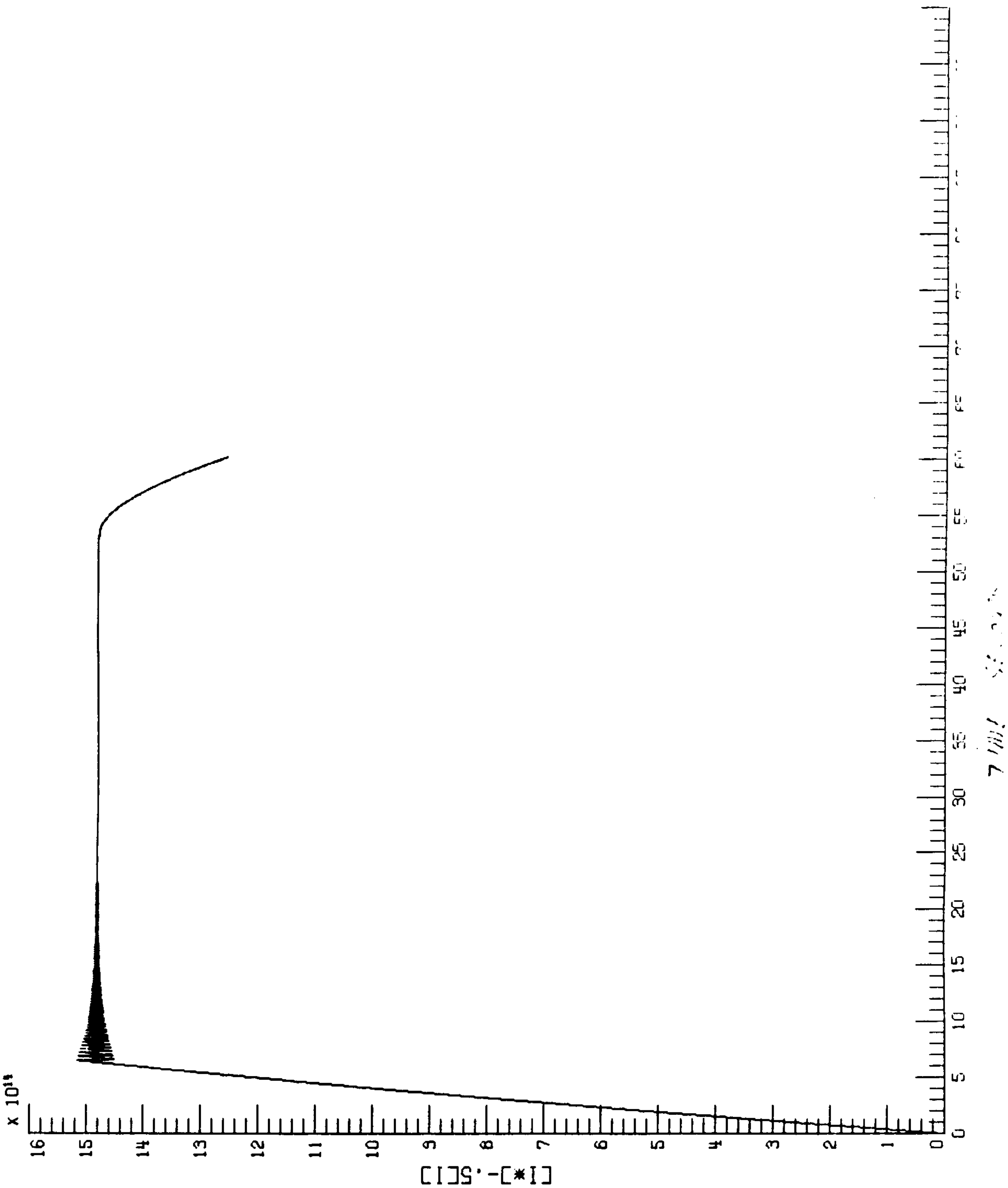

Fig. 1.3 Inversion Term vs. Time

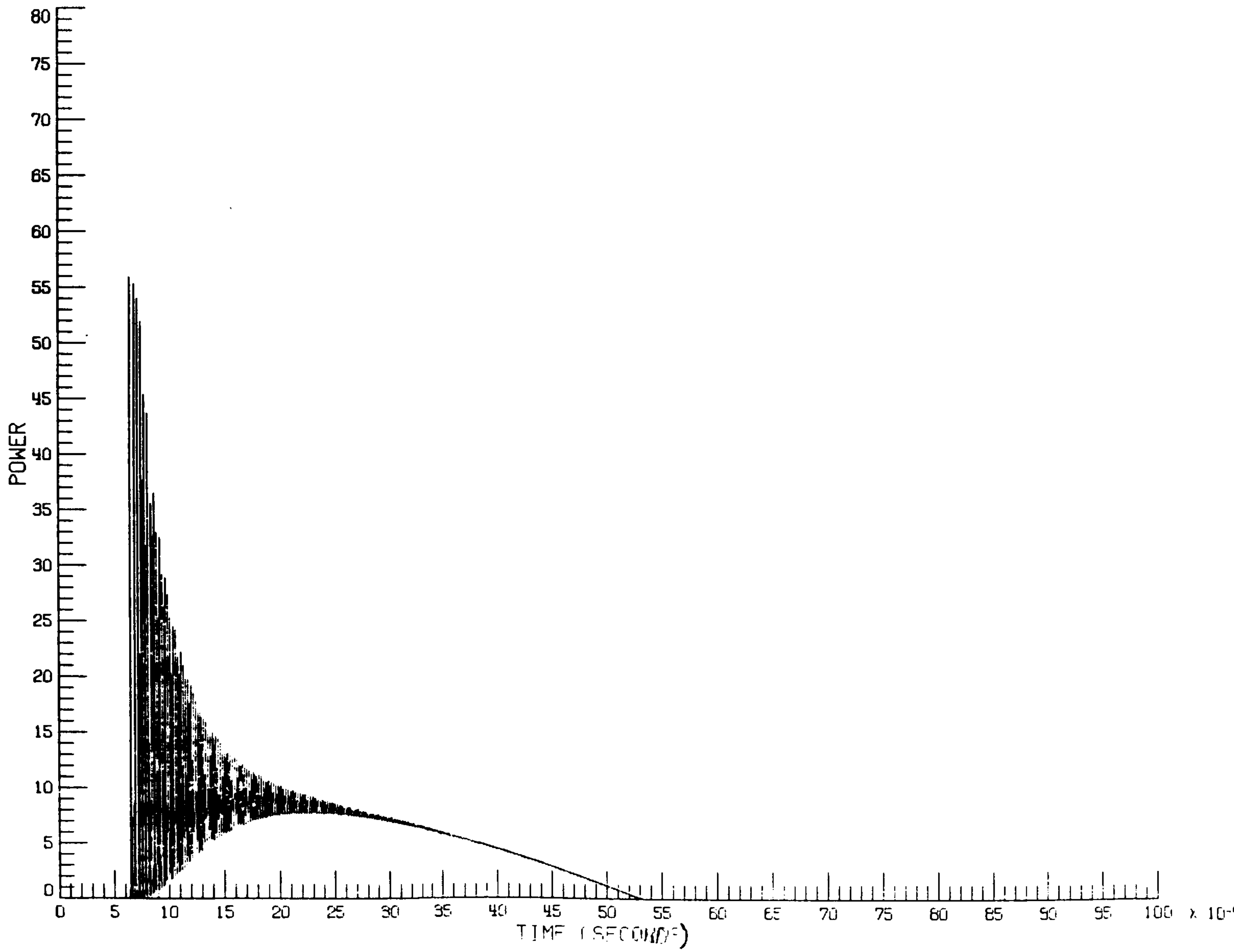

Fig. 1.4 Power vs. Time

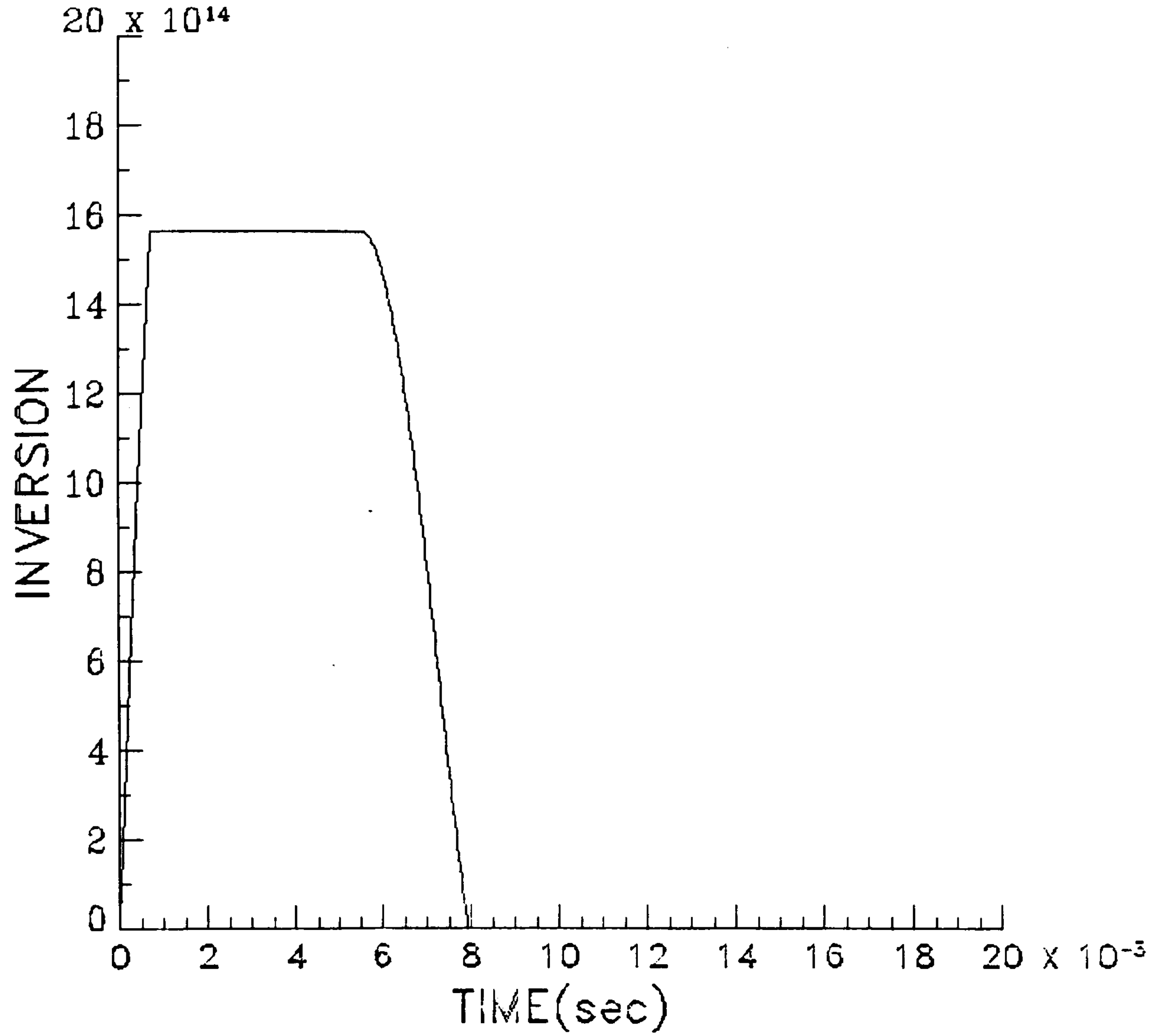

Fig. 1.5 Inversion vs. Time

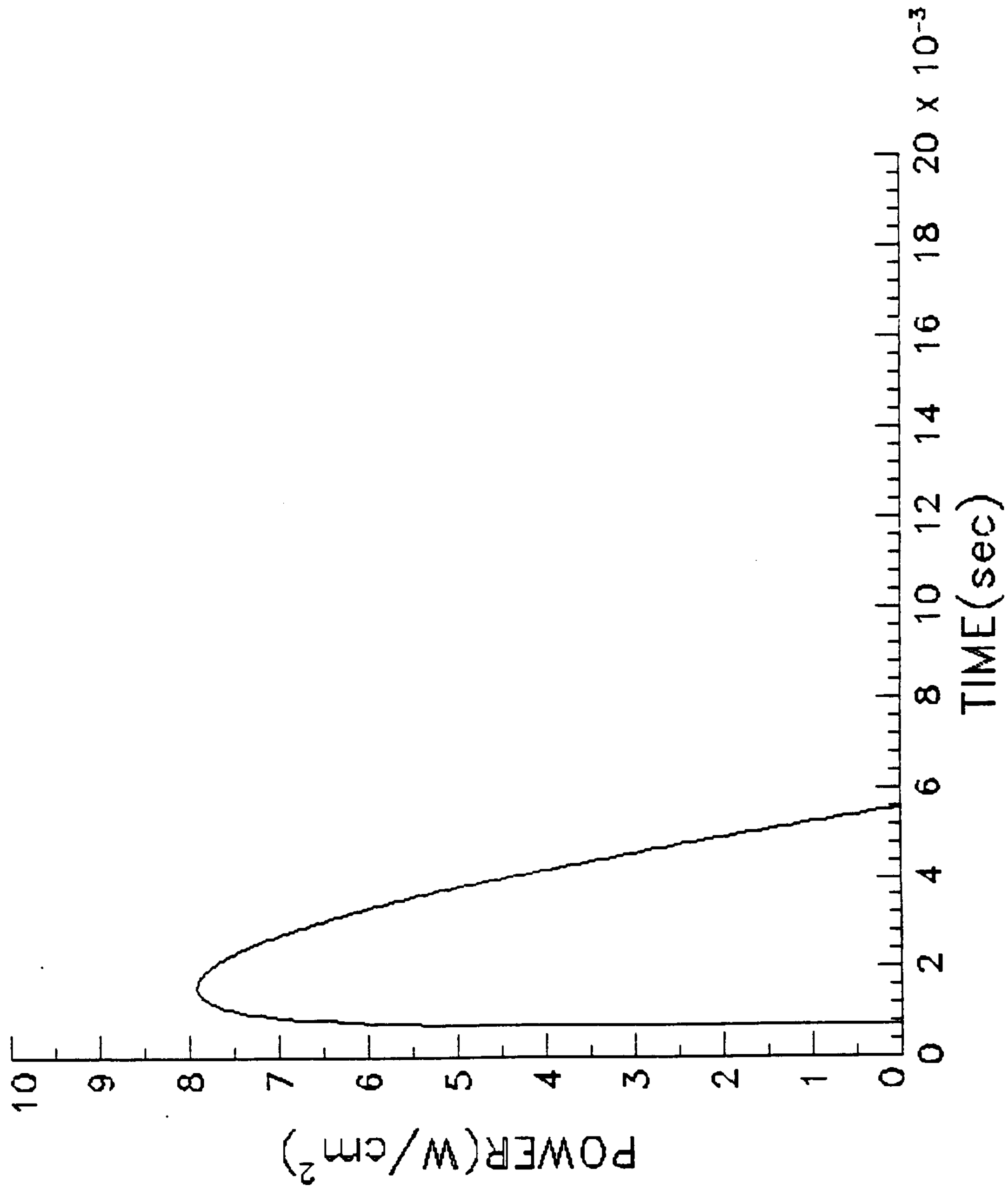

Fig. 1.6 Power vs. Time

TABLE I - Reaction rate coefficients and their associated reactions for the lasant n-C_3F_7I along with the parameters used to modify the optical time constant (ref. 2).

Lasant Reactions	n-C_3F_7I Symbol	Reaction Rate Coefficient $(cm^3)^n/sec$
$R + I^* \rightarrow RI$	K_1	$.5600 \times 10^{-12}$
$R + I \rightarrow RI$	K_2	$.2300 \times 10^{-10}$
$R + R \rightarrow R_2$	K_3	$.2600 \times 10^{-11}$
$R + RI \rightarrow R_2 + I$	K_4	$.3000 \times 10^{-15}$
$R + I_2 \rightarrow RI + I$	K_5	$.0000 \times 10^{+00}$
$I^* + I + RI \rightarrow I_2 + RI$	C_1	$.3200 \times 10^{-32}$
$I + I + RI \rightarrow I_2 + RI$	C_2	$.8500 \times 10^{-31}$
$I^* + I + I_2 \rightarrow I_2 + I_2$	C_3	$.8000 \times 10^{-31}$
$I + I + I_2 \rightarrow I_2 + I_2$	C_4	$.3800 \times 10^{-29}$
$I^* + RI \rightarrow I + RI$	Q_1	$.2000 \times 10^{-15}$
$I^* + I_2 \rightarrow I + I_2$	Q_2	$.1900 \times 10^{-10}$
	T_s	1.0000
	α	.0000

Fig. 1.7 Table I. Reaction Rate Coefficients

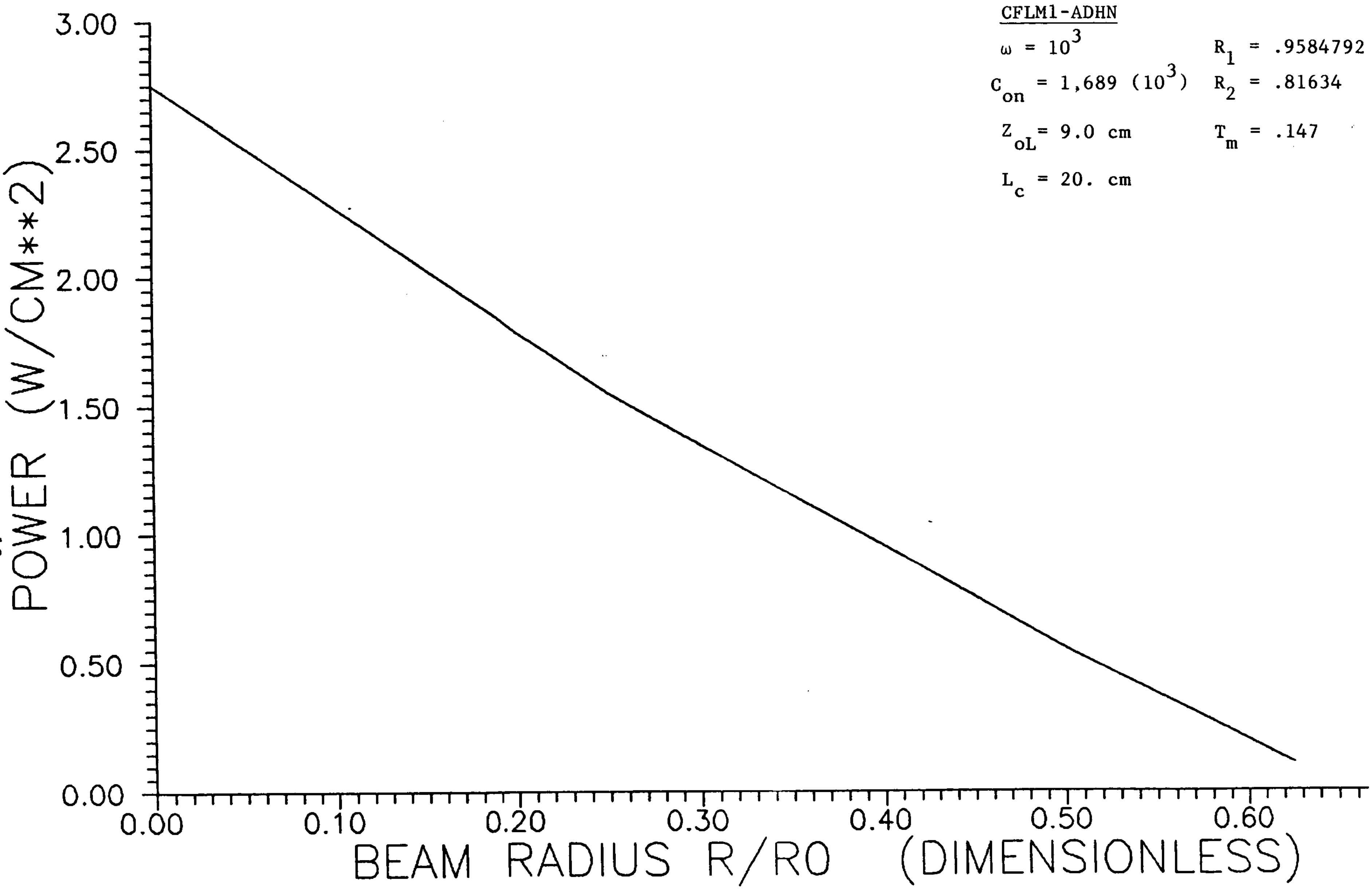

Fig. 1.8 Power vs. Beam Radius

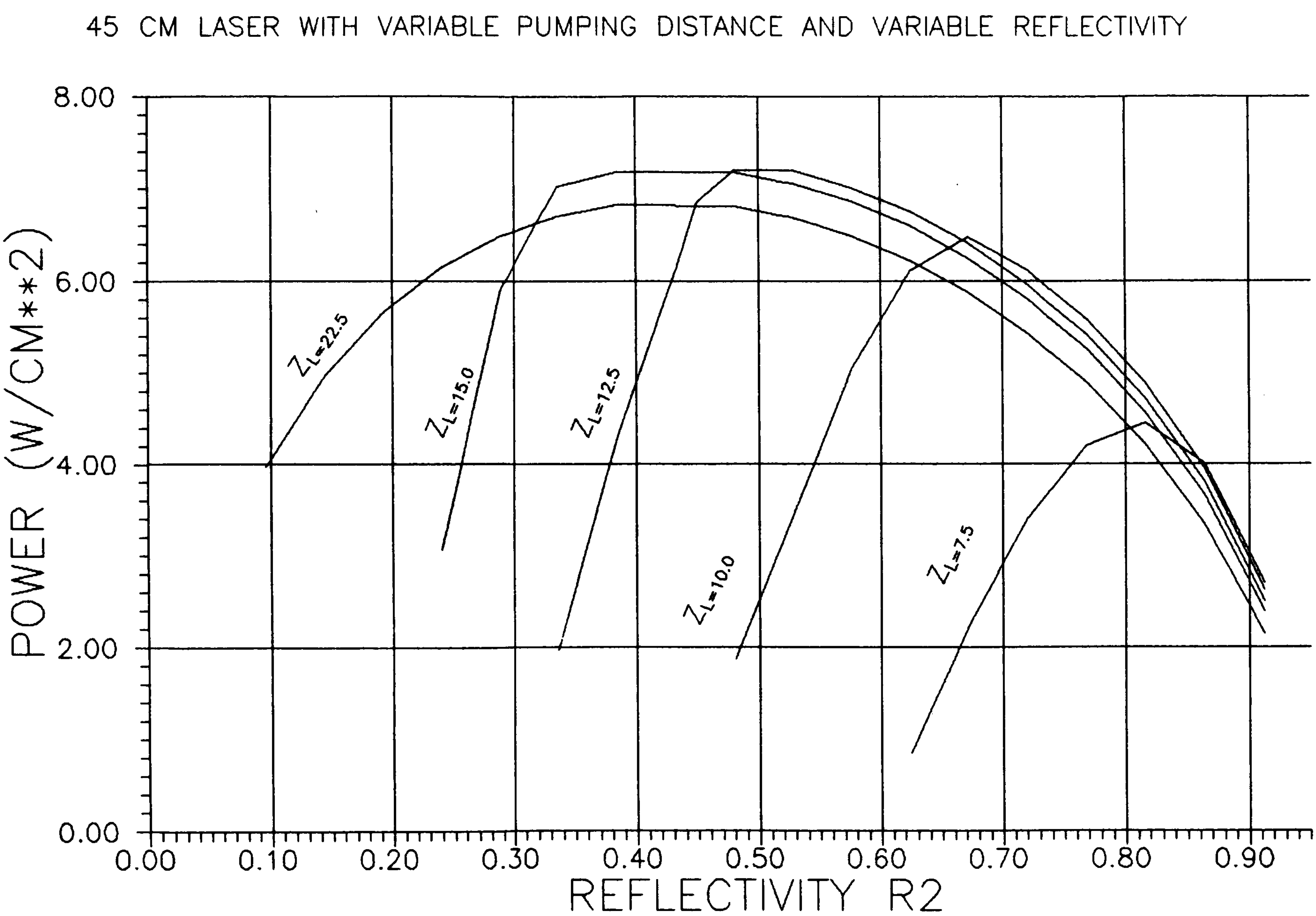

Fig. 1.9 Power vs. Reflectivity