Differential Surface Models for Tactile Perception of Shape and On-Line Tracking of Features

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1. Abstract

Tactile perception of shape involves an on-line controller and a shape perceptor. The purpose of the on-line controller is to maintain gliding or rolling contact with the surface, and collect information, or track specific features of the surface such as edges of a certain sharpness. The shape perceptor uses the information to perceive, estimate the parameters of, or recognize the shape. The differential surface model depends on the information collected and on the a-priori information known about the robot and its physical parameters. These differential models are certain functionals that are projections of the dynamics of the robot onto the surface gradient or onto the tangent plane. They involve the states of the robot (i.e., angles and angular velocities), input torques or forces to the robot, the coefficient of friction μ, and some of the differential properties of the surface such as the units of tangent and normal to the surface, gradient, Hessian, and the radius of curvature and its projections onto planes. A number of these differential properties may be directly measured from present day tactile sensors. Others may have to be indirectly computed from measurements. Others may constitute design objectives for distributed tactile sensors of the future. A parameterization of the surface leads to linear and nonlinear sequential parameter estimation techniques for identification of the surface. Many interesting compromises between measurement and computation are possible.

2. Introduction

Tactile perception of shape by natural systems has been the subject of many recent studies [1]. Tactile perception in robotic systems requires maintenance of gliding and/or rolling contact with the unknown object and infering information about the shape. A major component of this kind of probing is the controller. The controller needs on-line construction of the kinematics [2], force feedback [3], and inverse dynamics [4] to generate the needed input torques to the robot joints. The available tactile sensors to date, however, are not adequate for fast and efficient execution of rolling and gliding manipulations [5,6]. Once gliding or rolling is maintained, the perception of shapes involves using kinematic, and dynamic information to gather information about the manipulated object [7]. The process of determining the shape involves availability of a-priori computational and symbolic models of shape [8].

For smooth surfaces, that are linear in an unknown parameter vector, linear sequential estimation algorithms can be used to arrive at these parameters [9,10,11]. Alternatively, solution of partial differential equations or nonlinear estimation algorithms are needed [10].

When the object or surface is known, the trajectory of the robot end effector can be a-priori determined the control of both gliding [12,13] and rolling [14] on known surfaces has been studied before. Brown [15] has considered the control problem for gliding on unknown objects. This paper deals with the kinematics and dynamics of gliding and rolling contact of a known end effector gliding and/or rolling on an unknown surface. Two differential surface models for perception are derived. For parametric surfaces a linear sequential estimation algorithm is sketched.

3. The Kinematic Problem

The on-line kinematic problem for purposes of gliding and rolling on an unknown surface is discussed here by a simple two rigid body problem. A planar rigid body end effector is considered that maintains contact with an unknown rigid body by gliding or rolling on it (Fig. 1). The internal coordinate system of the end effector is the y_1y_2 axes centered at the center of gravity of the end effector A and parallel with the principal axes of the end effector. The smooth surface of the end effector is assumed to be known implicitly or parametrically in its own coordinate system.

$$C(Y) = 0 \tag{1.a}$$

$$Y = Y(\alpha) = [y_1(\alpha),\ y_2(\alpha)]^T \tag{1.b}$$

and the point of contact B (in gliding) is specified by α_1:

$$Y_B = Y(\alpha_1) \tag{2}$$

Similarly the smooth unknown surface may be characterized parameterically or implicity. An implicit representation is assumed here:

$$D(X) = D(x_1, x_2) = 0 \tag{3}$$

Let the coordinates of A be given by the two-vector X_A. The coordinates of the contact point B in the inertial coordinate system are

$$X_B = X_A + \begin{bmatrix} \cos\theta_3 & -\sin\theta_3 \\ \sin\theta_3 & \cos\theta_3 \end{bmatrix} Y_B \tag{4}$$

The control requirements for gliding contact are:

1. existence of the normal contact force to making sure that the contact is maintained,
2. knowledge about the point of contact,
3. guiding the motion of the end effector, along the unknown surface,

and finally, 4. knowledge of the radius of curvature of the unknown surface.

along the unknown surface. A control input to this guidance is the tangential velocity of contact $v_B(t)$. The latter guidance requires sensing of the unit tangent vector T at B in the end effector coordinate system and transforming it to the inertial coordinate system

$$\dot{X}_B(t) = v(t)T \tag{5}$$

Differentiating Eq. (4) with respect to time and substituting in Eq. (5) gives

$$\dot{X}_A(t) = v(t)T + \dot{\theta}_3 \begin{bmatrix} \sin\theta_3 & \cos\theta_3 \\ -\cos\theta_3 & +\sin\theta_3 \end{bmatrix} Y_B \tag{6}$$

Eq. (6) relates the local translational velocities $\dot{X}_A$ to the angular velocity of the end effector $\dot{\theta}_3$.

Another interpretation of Eq. (6) is that the terms on the right side of Eq. (6) are respectively a small translation of point A and a small rotation of point A about point B. The angular velocity $\dot{\theta}_3$ itself is a function of the local curvature of the unknown surface at the point of contact. Let ds be the traversed distance on the unknown surface. By definition, the radius of curvature is given by

$$\rho_u = \frac{1}{d\theta_3/ds} = \frac{ds}{d\theta_3} \tag{7}$$

$$\dot{\theta}_3 = \frac{d\theta_3}{dt} = \frac{1}{\rho_u}\frac{ds}{dt} = \frac{v(t)}{\rho_u} \tag{8}$$

Therefore, Eqs (6) and (8) together define the instantaneous kinematics of the gliding motion.

In the rolling motion the contact point moves on the end effector as well as on the unknown surface so that the incremental distances traversed on both surfaces are equal. In addition to the four requirements of gliding motion as before, the end effector should have knowledge of its own local radius of curvature at the point of contact. Assume $v(t) = ds/dt$ is the specified control input, and assume a convex surface, and a convex end effector surface as in Fig. 1. It is not difficult to show that

$$\dot{\theta}_3 = v(t)\left(\frac{1}{\rho_e} + \frac{1}{\rho_u}\right) \tag{9}$$

where ρ_e and ρ_u are respectively the local radii of curvature of the end effector and the unknown surface at the point of contact.

Similarly from the incremental form of Eq. (4) and the definition of rolling, it follows that

$$dX_A = \begin{bmatrix} +\sin\theta_3 & \cos\theta_3 \\ -\cos\theta_3 & \sin\theta_3 \end{bmatrix} d\theta_3 \, Y_B \quad (10) \qquad \text{or} \qquad \dot{X}_A = \dot{\theta}_3 \begin{bmatrix} \sin\theta_3 & \cos\theta_3 \\ -\cos\theta_3 & \sin\theta_3 \end{bmatrix} Y_B \quad (11)$$

To summarize, Eqs (9) and (11) are the instantaneous kinematics of the rolling motion.

If the unknown surface is convex, Eq. (9) is replaced by

$$\dot{\theta}_3 = v(t)\left(\frac{1}{\rho_e} - \frac{1}{\rho_u}\right) \tag{12}$$

Suppose the center of gravity of the end effector is connected to a two link robot (Fig. 2).

$$X_A = \begin{bmatrix} \ell_1\cos\theta_1 + \ell_2\cos\theta_2 \\ \ell_1\sin\theta_1 + \ell_2\sin\theta_2 \end{bmatrix} \tag{13}$$

Differentiating Eq. (13) with respect to time gives

$$\dot{X}_A = \begin{bmatrix} -\ell_1\sin\theta_1 & -\ell_2\sin\theta_2 \\ \ell_1\cos\theta_1 & + \ell_2\cos\theta_2 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix} \tag{14}$$

The latter equation provides the instantaneous kinematic of the three-link system.

$$\dot{\theta} = [\dot{\theta}_1 \;\; \dot{\theta}_2 \;\; \dot{\theta}_3]^\tau \tag{15}$$

for either of the gliding or rolling motion.

From the above discussion it can be stated that measuring or estimating the local radius of curvature ρ_u of the unknown object and determining convexity or concavity are two important parameters for the kinematics of rolling or gliding. The local radius of curvature has two more uses. 1) It is needed for an ideal inverse dynamics systems where the accelerations are needed to construct the input torques [11]. It is needed for detecting snarp edges (small ρ_u) and consequent tracking of such sharp edges on three dimensional surfaces.

4. The Dynamics Problem for Point Contact

Consider the three-link planar robot of Fig. 2 with no contact with any object or surface, the equations of motion for this system [12,13] are

$$J(\theta)\ddot{\theta} + B(\theta)\dot{\theta}^2 + E(\theta) = CU \tag{16}$$

where U is the vector of torque actuators at the joints. Suppose the gliding is on a frictionless surface. The contact force is along the unit normal vector N to the surface, and assume its magnitude is γ. The incremental motion of the contact point on the robot is

$$dX_B = dX_A + \begin{bmatrix} -\sin\theta_3 & -\cos\theta_3 \\ \cos\theta_3 & -\sin\theta_3 \end{bmatrix} Y_B \, d\theta_3$$

Let N be resolved in the inertial coordinate system. The incremental work of the contact force is

$$dW = \langle \gamma N, dX_B \rangle$$

where $\langle\ \rangle$ is the inner product, and

$$\frac{dW}{d\theta} = \langle \gamma N, \frac{dX_B}{d\theta} \rangle$$

where

$$\frac{dX^\tau_B}{d\theta} = \begin{bmatrix} -\ell_1\sin\theta_1 & \ell_1\cos\theta_1 \\ -\ell_2\sin\theta_2 & \ell_2\cos\theta_2 \\ -y_{1B}\sin\theta_3 - y_{2B}\cos\theta_3 & +y_{1B}\cos\theta_3 - y_{2B}\sin\theta_3 \end{bmatrix}$$

The equation of motion with the contact in effect are:

$$J(\theta)\ddot{\theta} + B(\theta)\dot{\theta}^2 + E(\theta) = CU + \left[\frac{dX^\tau_B}{d\theta}\right] \gamma N \tag{17}$$

The holonomic constraint governing the dynamics is

$$D(X_R) = 0 \tag{18}$$

Differentiating Eq. (18) with respect to time gives

$$\dot{\theta}^{\tau} \frac{dX_R{}^{\tau}}{d\theta} \frac{dD}{dX} = \frac{dD^{\tau}}{dX} \frac{dX_R}{d\theta} \dot{\theta} = 0 \tag{19}$$

A final relation that is important for the analysis to follow is the definition of the unit normal vector N. The gradient vector of the unknown surface is dD/dX and by definition

$$N = \frac{dD}{dX} \Big/ \left\| \frac{dD}{dX} \right\| \tag{20}$$

where $\| \ \|$ is the Euclidean norm.

Consider the rolling type of constrained motion. Let the magnitude of the tangential constraint force be λ. The contribution of these forces to the equations of motions is

$$\frac{dW}{d\theta} = \left\langle \frac{dX_R}{d\theta}, (N\gamma + T\lambda) \right\rangle \tag{21}$$

For the rolling motion, there are two constraints. The holonomic contact constraint of Eq. (18) and the non-holonomic roll constraint --no motion along T at the point of contact.

$$T^{\tau} \frac{dX_R}{dt} = T^{\tau} \frac{dX_R}{d\theta} \dot{\theta} = 0 \tag{22}$$

Eq. (19) implies no motion of the contact point along the unit normal vector. Eq. (22) means no motion of the contact point along the unit tangent vector. Consequently the only possible motion is a rotation about the contact point and hence a rolling motion.

In order for rolling to occur and no slippage or gliding to take place, the coefficient of friction μ must be different than zero and the forces of constraint must be governed by

$$0 < \lambda < \mu\gamma \tag{23}$$

5. Differential Surface Models

In this section constituent relations between the state θ, $\dot{\theta}$, the input U, the forces and the surface geometry are derived. If the surface is known, these equations can be used to solve for the forces of constraint γ and λ. If alternatively, the forces are known, these equations can be used as differential surface models, and used for estimating the shape of the surface. These constituent relations are arrived at by differentiating the constraint Eqs. (18) and (22) with respect to time and eliminating the acceleration $\ddot{\theta}$ between the latter second derivatives and the equations of motion.

The above procedure could be carried out simultaneously for both constraints. However, it is done for the individual constraints here in order to demonstrate two alternative formulations, one more analytical, one slightly more suitable for computational purposes.

5.1 First Formulation. The differentiation of Eq. (18) with respect to time gives:

$$\dot{\theta}^{\tau} \frac{dX_R{}^{\tau}}{d\theta} \frac{dD}{dX} = 0 \tag{24}$$

This equation implies no velocity component exists along the unit normal. An alternative form for Eq. (24) is

$$\dot{X}_R{}^{\tau}(t) N = 0 \tag{25}$$

The differentiation of Eq. (24) with respect to time gives

$$\ddot{\theta}^{\tau} \frac{dX_R{}^{\tau}}{d\theta} \frac{dD}{dX} + \dot{\theta}^{\tau} \frac{dX_R{}^{\tau}}{d\theta} \frac{d^2D}{dX^2} \frac{dX_R}{d\theta} \dot{\theta} + \dot{\theta}^{\tau} \frac{\partial}{\partial\theta} \left[\frac{dD^{\tau}}{dX} \frac{dX_R}{d\theta} \right] \dot{\theta} = 0 \tag{26}$$

Elimination of $\ddot{\theta}$ between Eqs. (26) and the dynamics of the system

$$J\ddot{\theta} + B(\theta)\dot{\theta}^2 + E(\theta) = CU + \left\langle \frac{dX_R}{d\theta}, N\gamma + T\lambda \right\rangle \tag{27}$$

gives the first form of the constituent equation

$$-\dot{\theta}^{\tau} \frac{dX_R^{\tau}}{d\theta} \frac{d^2D}{dX^2} \frac{dX_R}{d\theta} \dot{\theta} - \dot{\theta}^{\tau} \frac{\partial}{\partial\theta} \left[\frac{dD^{\tau}}{dX} \frac{dX_R}{d\theta} \right] \dot{\theta} = \frac{dD^{\tau}}{dX} \frac{dX_R}{d\theta} J^{-1} \left[-R\dot{\theta}^2 - E\theta + CU + \frac{dX^{\tau}_R}{d\theta} (N\gamma + T\lambda) \right] \quad (28)$$

If all parameters and quantities in Eq. (28) are available, it is a constituent relation in the unknown gradient and Hessian dD/dX and d^2D/dX^2. If N and T are also not available, it is easy to include Eq. (20) for N and the following for T

$$T = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} N \quad (29)$$

The result is a more complex constituent relation. Because Eq. (28) involves the first and second partial derivatives of the surface, it is a differential model of the unknown surface. If λ and γ are negligibly small, and/or if $\dot{\theta}$ --the angular velocities --are relatively small, Eq. (28) simplifies.

5.2 Second Formulation. Eq. (28) establishes one relation among γ and λ. A second relation among λ and γ could be obtained in a similar fashion from the second constraint equation. However, an alternative formulation is given here.

If Eq. (22) is differentiated with respect to time, one obtains

$$T^{\tau} \frac{dX_R}{d\theta} \ddot{\theta} + \dot{\theta}^{\tau} \frac{d}{d\theta} \left(T^{\tau} \frac{dX_R}{d\theta} \right) \dot{\theta} = 0 \quad (30)$$

Between Eqs. (27) and (30) one can eliminate $\ddot{\theta}$ to obtain the second differential surface model

$$-\dot{\theta}^{\tau} \frac{d}{d\theta} \left(T^{\tau} \frac{dX_R}{d\theta} \right) \dot{\theta} + T^{\tau} \frac{dX_R}{d\theta} J^{-1} (R\dot{\theta}^2 + E) = T^{\tau} \frac{dX_R}{d\theta} J^{-1} \left[CU + \frac{dX^{\tau}_R}{d\theta} (N\gamma + \lambda T) \right] \quad (31)$$

The above two differential surface models, are two independent Eqs. in γ and λ. If everything else is known including the unknown surface, these equations can be solved for these constraint forces as functions of the state $[\theta^{\tau}, \dot{\theta}^{\tau}]^{\tau}$, input U and the surface $D(X) = 0$. They provide differential information about the surface, if everything else including γ and λ is known.

6. Shape Perception by Parameter Estimation

Suppose the unknown surface is representable by a vector of unknown parameters β.

$$D(X) = P^{\tau}(X)\beta = 1 \quad (32)$$

one may use coordinates of the many contact point X_R to arrive at a system of linear equations in β, under the above assumption, the gradient and the Hessian are also linear in β. As a result the constituent differential surface models, sampled at some interval T of time provide independent information about vector β. These systems of over specified linear equations can be solved for β. Let the overspecified system be

$$M\beta = N \quad (33)$$

where each row of the equation is one additional piece of information from sampling the constituent surface models or Eq. (32), etc. (an example is worked out in [10]). The best estimate for β is the mean square error sense [9] is

$$\beta = (M^{\tau}M)^{-1}M^{\tau}N \quad (34)$$

This equation is also robust with respect to a certain amount of independent random measurement noise.

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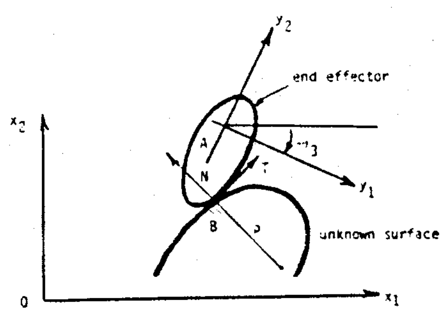

Figure 1: The planar two-body contact problem.

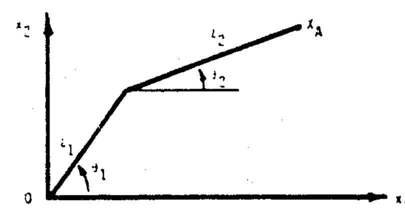
Figure 2: Gross motion of the end effector center of gravity by a two-link robot.