Chaos Motion in Robot Manipulators

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~~Abstract. This paper shows~~ It is shown that a simple two-link planar manipulator exhibits a phenomenon of global instability in a subspace of its configuration space. A numerical example, as well as results of a graphic simulation, is given.

I. Introduction.

The problem of unpredictability in deterministic mechanical systems without random disturbances was posed more than a century ago in connection with turbulence motion. A new interest in the problem was aroused only recently when it became clear that even low order deterministic dynamical systems can be unpredictable from any practical point of view. A classical example of a Henon-Heiles system (1) [2]

$$H = 0.5(p_1{}^2 + q^{12} + p_2{}^2 + q_2{}^2) + q_1{}^2q_2 - q_3{}^2/3 \tag{1}$$

represents a case of a well posed deterministic conservative system with only two degrees of freedom. While (1) cannot be solved analytically for arbitrary initial values, it can be integrated numerically. Henon and Heiles did it in 1964, and the results showed that for the system energy above E=1/6, a phase portrait looks seemingly random, while for the E=1/12 <for energy levels above E=1/8, while for E<1/8 system (1) exhibits traditional smooth curves (Fig. 1). During the last twenty years, existence of chaos in nonlinear dynamic systems became a well established fact.

Another type of chaos that is important for us can be well demonstrated by equation (2).

$$X(n+1) = 2*X(n) \text{ MOD}(1) \tag{2}$$

For a binary representation of X(n), it simply states that on each step we are shifting the decimal point in X one bit right and throwing out an integer part. It is easy to see that (2) has an analytical solution (3)

$$X(n) = 2^n*X(0) \text{ MOD}(1) \tag{3}$$

but to compute a result for a given N, one must know exactly N bits in the initial value of X(0).

This example demonstrates a case of orbital instability that was first studied in [1]. Equation (3) is a case in which an initial separation between two close solutions grows exponentially along the trajectories. Of course for any real dynamical system, orbital instability can exist only for the general coordinates that don't increase ths system's total energy. It is worth mentioning here that while an exponential "explosion" of solutions is very well known in the Linear System Theory, there it only means that a linear system description cannot be used, and the system moves toward its limited circle or breaks down as a result of too high stresses. Conversely, an orbital instability in nonlinear systems does not lead to an alternative stable equilibrium, and the system description is done, in the framework of Newtonian mechanics, without any simplification and linearization.

II. Geometric Approach.

For our future discussion, a geometrical representation of orbital instability may be useful. Let us start from an example of an inertial motion of a single particle M on a smooth surface S. In the absence of external forces, a point mass M would move along the geodesics line on this surface. It is shown in differential geometry [5] that the distance between two initially close geodesics is

$$d(t) = d_0*\exp(t\sqrt{-G}\ ) \tag{4}$$

where d_0 is an initial separation, G - Gaussian curvature, and t is a trajectory parameter not necessarily time. One can see from (4) that for a negative surface curvature the separation increases exponentially. Such a case is shown in Fig. 2. Alternatively, for the surfaces with a positive curvature, separation is bounded by its initial value.

This geometrical representation is very important for a question of system orbital stability. Indeed if it is possible to find a space where the system behavior can be described as an inertial motion of a single particle, one need examine only the sign of a space Gaussian curvature without solving the equations themselves [4].

For the case of conservative finite-degree-of-freedom systems, such a space is very well known. It is a configuration space with a metric tensor corresponding to the structure of the system kinetic energy [4]. Let q^i (i=1..N) denote generalized coordinates, and the kinetic energy is

$$E = a_{ij}\dot{q}^i\dot{q}^j \qquad (5)$$

then a_{ij} should be used as a new metric tensor for the configuration space. In (5) and thereafter, summation is assumed upon repeated indexes. In such constructed space, the solution for the free motion of the original system will correspond to an inertial motion of a single particle of a unit mass along the geodesic lines [5,6].

For this metrics triangle, equality is not true any more. Now an elementary arc is

$$ds^2 = a_{ij}dq^idq^j \qquad (6)$$

and ds^2 can be less, greater, or equal to the sum of dq_i^2. The sign of the resulting Gaussian curvature is connected to this relation. An illustration of a two dimensional case is shown in Fig. 3.

In the rest of the paper, we are going to show that a free frictionless motion of a simple mechanical system of a two-link planar manipulator, in the absence of gravity, can demonstrate orbital instability that can be characterized as a "weak chaos" [4].

III. Solution for a Two-link Arm.

A model for a two-link planar manipulator is shown in Fig.4. Angles f_1 and f_2 can be chosen as generalized coordinates q_1, q_2. System kinetic energy is

$$E = a_{11}*\dot{f}_1^2 + a_{12}*\dot{f}_1*\dot{f}_2 + a_{22}*\dot{f}_2^2 \qquad (6)$$

$$a_{11} = (I_1 + m*L^2)$$
$$a_{12} = m*l*lg*\cos(f_2-f_1) \qquad (7)$$
$$a_{22} = I_2$$

where I_1 and I_2 are moments of inertia, m - mass of the link 2, L - length of link 1, and lg is the distance from B to the center of inertia of the link 2.

The curvature of the resulting two dimensional space can be computed explicitly

$$G*a^2 = m*L*lg*(a^2+[m*L*lg*\sin(f_2-f_1)]^2)^2*\cos(f_2-f_1) \qquad (8)$$

and differential equations for f_1 and f_2 can be solved numerically.

One can see from (8) that the sign of the curvature G depends only on the $\cos(f_2-f_1)$. Fig.5 gives the region on (f_1,f_2) plane where G is positive and therefore the system is orbitally unstable. In other words folded arm configurations are orbitally unstable, and extended arm configurations are orbitally stable.

IV. Numerical Simulation.

For a numerical simulation, we chose parameter values that had been used for a real manipulator [7].

$$I_1 = 0.126,\ I_2 = 0.075,\ m = 4.978$$
$$L = 0.27,\ lg = 0.0485 \qquad (9)$$

The system was exercised in the following way. For various initial conditions, two arms were run with a slight difference in their start points. The time history, configuration plane trajectories, as well as graphical animation of the arms themselves had been displayed.

Also we computed and graphically displayed a running estimation of a Lyapunov exponent for the f_2-f_1

$$L(t) = [Ln(d(t)/d_0)]/t \qquad (10)$$

where d(t) is the difference between the arms in the value of an internal angle ($f_2(t)$-$f_1(t)$). It had been shown [8] that a motion is chaotic if

$$L(t) \rightarrow c > 0; \text{ when } t \rightarrow \inf \qquad (11)$$

Integration was done by using Runge-Kutta with adaptive size steps. To avoid the influence of numerical errors, integration was repeated with a tolerance parameter varying more than an order of magnitude. All runs gave the same results within desired tolerance.

V. Results.

While the fact of positive but not constant curvature over the whole space is a necessary but not sufficient condition for the orbital stability, the opposite is true. A system is orbitally unstable if the curvature of the space defined by (5),(6) is negative in all points.

Since there are "good" and "bad" regions on the f_1 vs f_2 plane (as shown on Fig.5), to get definite results it would be desirable to find an initial condition that would keep the system only in the region with positive or negative curvature. However it is clear, that if one can hope to find a trajectory that stays completely in the "good" region (G>0), there is no trajectory that would stay in the region with orbital instability. Indeed, any solution for the f_2-f_1 is of MOD(2PI), and the separation can not grow indefinitely without forcing an arm to "unfold".

In our simulation we found a case when an arm starting from unfolded position would stay there. In that case the original difference between the arm almost did not grow as can be seen from Fig.6a. This case will be further referred to as Case I. Its initial conditions were :

arm1 : $f_1 = 80$, $f_2 = 95$ [dg], $\dot{f}_1 = 300$, $\dot{f}_2 = 0$ [dg/sec]
arm2 : $f_1 = 80$, $f_2 = 97$ [dg], $\dot{f}_1 = 300$, $\dot{f}_2 = 0$ [dg/sec]

For the arm started from a folded position a small initial difference grew very fast as could be seen from Fig.6b. Fig.6c shows arms in one of the intermediate positions. It is clear how far apart they become. The initial conditions for the Case II were :

arm1 : $f_1 = 80$, $f_2 = 200$ [dg], $\dot{f}_1 = 300$, $\dot{f}_2 = 0$ [dg/sec]
arm2 : $f_1 = 80$, $f_2 = 202$ [dg], $\dot{f}_1 = 300$, $\dot{f}_2 = 0$ [dg/sec]

The estimate of the Lyapunov exponent for the Case I clearly goes to zero, while Case II stayed positive at least during the time of observation - Fig.7a,b. It is not clear nevertheless that it will never go to zero, due to the reasons described above (crossing both good as well as bad regions).

VI. Discussion.

A phenomenon of chaotic motion has been theoretically found and numerically illustrated for a simple mechanical system of a two-link manipulator. It has been shown that a folded arm is orbitally unstable, opposite to an extended one.

While the assumption of a nonfriction, zero gravity environment is quite unrealistic, we believe that our finding warrants further and more detailed investigations of the described phenomena. It is quite possible that low-friction, many degrees of freedom redundant flexible arms of the future will exhibit more complicated behavior that could lead to orbital instability under more realistic conditions.

The importance of the geodesics in the configuration space with metric a_{ij} as minimum time trajectories has been recently shown by K.G. Shin and N.D. McKay [9]. Our result suggests that an open loop control should not be used in the region with negative curvature since trajectories there rapidly diverge.

A question of a closed loop control in the area of negative curvature also deserves more detailed investigation. While it had been shown that a simple PID control under the same conditions (no friction and no gravity) makes an arbitrary robot manipulator asymptotically stable [10] it is not clear how trajectory curvature affects an admissible sampling rate.

VII. Conclusion.

Using a geometric approach we have shown that a simple robot manipulator can be orbitally unstable depending on its configuration. A numerical simulation supported this finding. We feel that further efforts in this direction will help better understanding of the dynamical properties of such complicated nonlinear systems as robot manipulators.

Acknowledgement

This work was done at the Jet Propulsion Laboratory, California Institute of Technology under a contract with the National Aeronautics and Space Administration (NASA).

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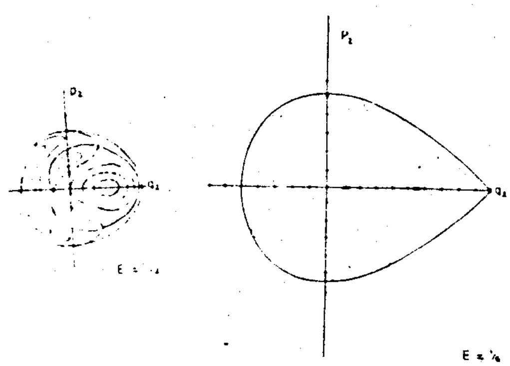


FIGURE 1

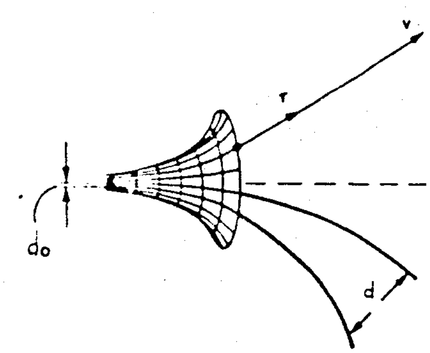
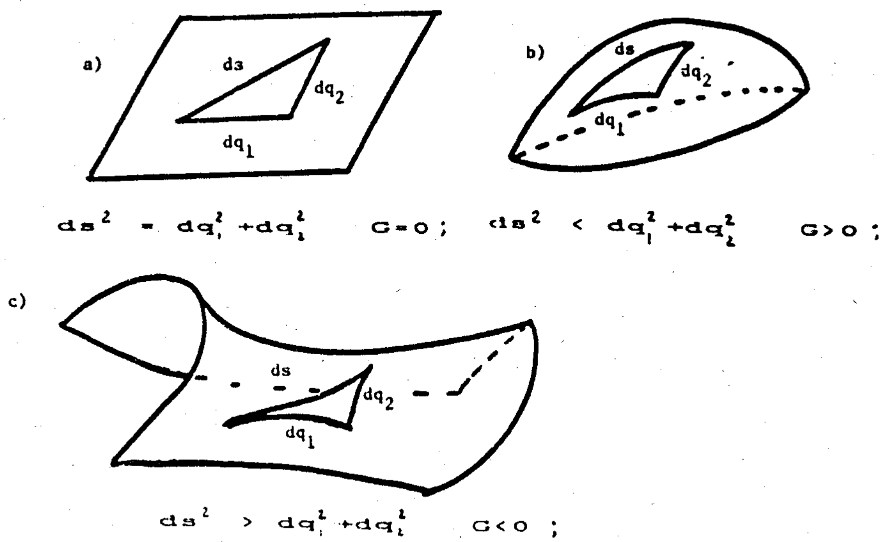


FIGURE 2



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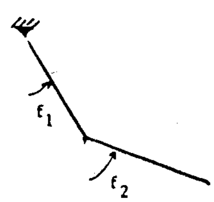


FIGURE 4

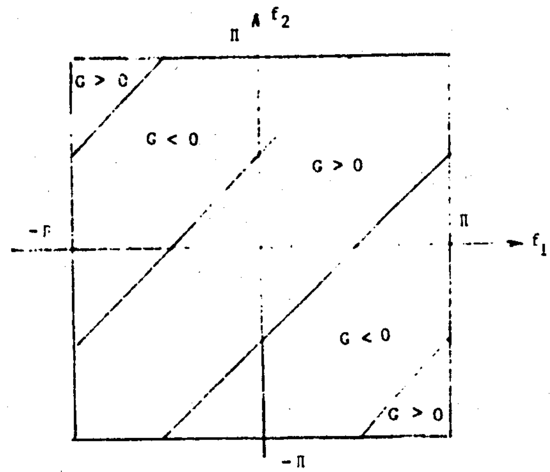
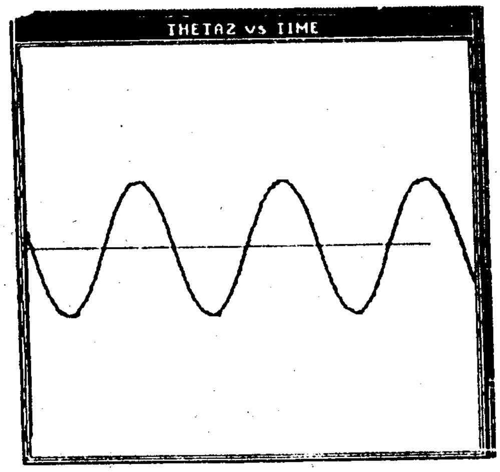
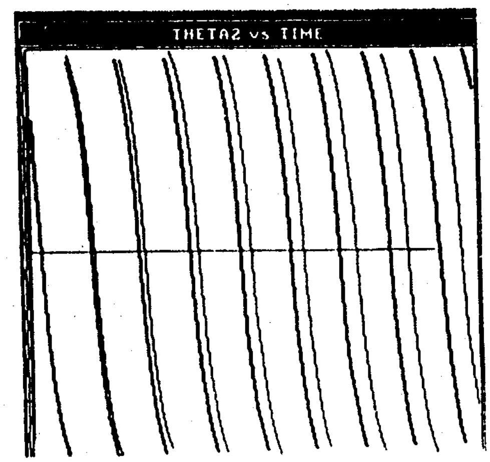


FIGURE 5

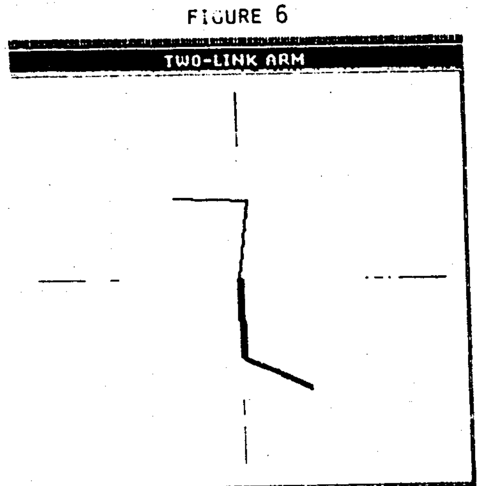


A



B

FIGURE 6



C

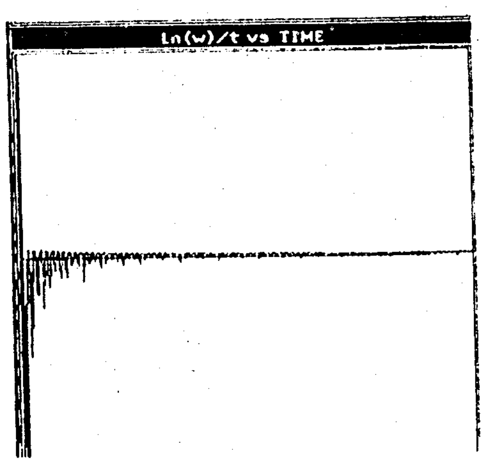


FIGURE 7A

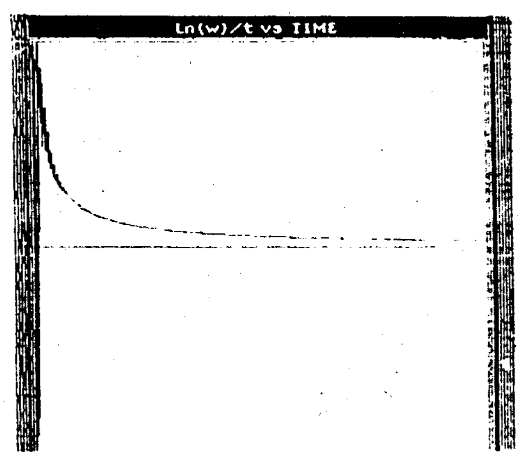


FIGURE 7B