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## ABSTRACT

These proceedings report the results of a workshop on space telerobotics, which was held at the Jet Propulsion Laboratory, January 20-22, 1987. Sponsored by the NASA Office of Aeronautics and Space Technology (OAST), the wcrkshop reflected NASA's interest in developing new telerobotics technology for automating the space systems planned for the 1990 s and beyond. The workshop provided a window into NASA telerobotics research, allowing leading researchers in telerobotics to exchange ideas on manipulation, control, syatem architectures, artificial intelligence, and machine sensing. One of the objectives was to identify important unsolved problems of current interest. The workshop consisted of surveys, tutorials, and contributed papers of both theoretical and pratical interest. Several sessions were held with the themes of sensing and perception, control execution, operator interface, planning and reasoning, and system architecture. Discussion periods were also held in each of these major topics.

## ACKNOWLEDGNENT

Thanks are due to D.R. Meldrum, R.S. Doshi, K. Kreutz, B. Seraji, M.B. Milman, and S.K. Owen for significant contributions to the organization of the workshop.

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# PLANNING AND SCHEDULING: BASIC RESEARCH AND TOOLS 

# Planning and Scheduling: Is there a Difference? 

K.G. Kempf<br>FMC Corporation<br>Santa Clara, CA 95052

## 1. Alutract

It is proponed that the olitixy to danify grobient by oype and solation morniques by appliakility in advatergous for any problem aotver. Yet to clvirimation achemes are avilable in the importont arena of robotic plasaing and achenting. A standerd appouch to developing schemes band os wook reported is the literatare is actined bat in fouad so be rather unomiofying. An alternacive approweh in stetend besed upon the mocertinety invereat in dealine with the mel worid, and is fand to captere gore of the intritive differcoces betwuen plazing ated schenaling A trial explane fign for the diecisction berwen plataing and yalmeling in then offered The moid of three sumerch grompe on mbotic


## 2. Introdection

Arguments are introdeced in this mecticn in the conterat of providing perspective and morivation for the dimanion Thich follows The wopies include our appronches to the stand of arificial intelligence and competer scievce an well as robacios

In some fields of seady, it im poibic wasity after the expenditure of great effort to syachaies 7aws which decribe various phenomena of inceresc Duese laws an be unad to exphis the remals of pesteperimetes or wo predict the cuscome of futare triak Une to exphim and prodict an furthor confirm the accurnicy and gererality of the laves or hightight mecemary reviaion and refinement Eveatoalty the late an be utilized in the comerection of entities to dieplay spacix properties in the damais of intuter Entmpies of thim progremion can be drame from the manal miencen (rach os physica) and their appliantion disciptines (weh as mechanical engineeringl

Some researchers sabicribe to the belief that inquities inmo the nature of intrellizence will head to the diacomery of a (small) ser of socurate gevenl lavs of intelligence Dey believe that mon lives mefricienty refiond aod undersood, mighe eabble the conerucrion of eacioies which dinplay inest ligence and use homen a an erimence proof that gate ex-
 the ammption that a of lave underlize intelligent be bavior in a directly amiopone fantion to the phyrical hewrs which form the foumption of phyiot Thy argee mened that intellipence is inherenty beaed on bearnize and uning a lane and drvers ber of mighly epecializod tricks Consorverinf entities which dimplay insilligence can be socomplinhed by discovering a few of then ericks encoding them is acmputer progrtan and uing them to solve problemes fintet ligeoce ond ie incrementiily improval by ahtion more rixiks

 mechaing elet conid te end in te ronkionin of an inet ligent embicy which openows anywher in the poctur branded by thene erroune vipw be conpreteional serms is conares (cococepanilly) of two date scructases and three peocemer The firts dese atrecuse conbits a denription of a spere of problen sypa in erne of the fancrse vocibatory Which the gyte to ecocie problems suost which it
 the greot of getution matriphe with which ofe syuces in



 solecios tuchnige spere for the perpote of alecting the
 the problem The thise proces manap the inamaition and exacrion of the indicmeat methote to provide a zolution.

At the uricktend exurene the problem type and solerion trehoique demerpoing would be very specifics while at de
 the procelse at either erreme perform the man puing even though thry might be quite differemt in opetational deonil In any one thin motel mpgets that the implliquce of an entity rutide in being atie to eategorize probiems (the firt proces and die fure date orvetarel, minat the appropriate rolution approch (the mocod proceth and the moond den sorecturel and prodecr a solaxion (the kived procenal Far thermore the mplel puovites for varion focen of learaing by expanion refinemeat and ab-unction of the data sorreunes as will simprovement of the mapping procemex

The object of rainise chin argument is to point out the there suems be at eymmearic distointion of work reported in the lisernoure on planaing and setrateling in the
 the been mach work sitact at gyachaitiay powefol genal purpone phaning mechaomeng and wore rert doce to buld eqpecin parpon planaers and schoblers viciat ootve upecific probleme This wort corremponds to the mocod deta strucoure and the thind proces in ols madel, reppectuely. As mighe be expuctaf free the model, the epmafic ghnmers ave wery of teen specinlizations of the general phaners Unformmaniy. it is midice (if ever) the cine that a chear staternent of the charmatermoins of the pootiens for which any of the plamoers

 carried ont comarping the vocabelary of prodem typer Coriousily. wichoue the vocabulary. is abo vers difficult of make progres as the procense of clmonjriat problem topes

 work tumion on tive developpent of a problen type rocubliary to probere al indellipat encity.




 being able to ch-ify protiters which it emonaters and to
 ing omaple of the powe wich on be peneratiod by explicitry molyzing probiem and solution spoce can be found in two of ote mot importint coeceper of compecter riemot cocoppotetitity and compleaity. The firt aide wa in the caneporization of probleme into thow which are in principle comprative and thom wich are aot compartibe The soood sapplies the mexic for decition whether a solution approech to a concupatable peoblem will be relatively efficient (polypouinl ran cime) or rehtavely inefficient (exponential run timel

It is sagnated that this type of analysia and more is isdicaced for the problem tope and the solution rechnique speces in plonning and secheduling. It would be useful for remearchers to bave a problem dexcripcion vocabulary though which thy wald discuse the similaricies and differences among prodecon Furthermores to have a map of the keown portions of the solution appronch spece would help developers anen the remits of existing remearch and would belp focm new reverch effort on the unknown repions in addition. thete two speos are neowary for the construction of intelligent entitiet under the model dencribed earlier.

## 3. A Steniard Approech

In an effort to construct an initial dencription of these spaces for planing typer of problems, apartil literature review has been attempead. Planoing problems are usoally considered to the a inpers mate descriptiona action descriptioms and path conscraines, and to produce as outpat some ordered set of scaves and extices which obey the path coscroints One form of problem and solution cateforimation scheme an be bead an the eristing liternture in which this input/cutput riew of planning is taken.

The stare deatriptions taken as input consist of an initial state and a soll mace A problem caregocization scheme an une as oove mearore the sixe and complexity of the initial and goal stavel given The methodology used to encode the distioguished initial and gonl grates in also used by the planner mo construct tral intermediate sterse during plan syathesis and so partially deacribe the plas which the sysuem outpats it in ofter the case in robocic problems that scate deacripcion 000 cern, it some level of abucraction, objects, their propertien, and their relationships to ocher objects A solution ategorimition sheme could ses as soe mensure the rechoique used by the planner to reprevent staten

The action deacripoices atken as input for robotic probleme include a cataiog of agente and facilition involved along with their capebilitea is cerme of manipularica and percepoion Ooe useful memare for probiem atregorinion is the number and anture of the acente described in problems involvize robock the lawe of phyion mense always be considered as supplyint actices meh gravity which all agente must reocyaize bas which 20 agent can control Beyond emother nature". there can be one ageat or many apenth If there are many agentin they mose be defined in rerme of their relationship (cocperteve seatril. semperitive), cemmunications (nose occasional coctinuoual and ehared information (mones partini. compieteh to same but a fey of the relevnat chargcteristoc:

Eurly piansex umally condered ringle mate planning siontion [1, 2,3] More mieatiy, many rumarchers have foond on maltingent plaming problema and the chonely related arin co dincribated plaming [4, 5, 6, 7, 8, 9, 10, 11, 12, 13,14 15. 16, 17!

The action dencripelion methodalopy aned by the planner ceis alo be rised in a sohation categorimation scheme. In robet problema, ections ofter change the propertien of objece or the relmoions amons objects One ueful meare for solmion caregorimetion is whether or mor ection decasion is explicidy repremented The ourty plamars did aot explicitly comider durstion [1, 2, 3] but mors recently the representation and use of action daracion has been a fertile arta of remarch (18. 19. 20, 21. 22l Intesuing quentions involve the une of time in an absiove or relative way and the reptementation of time a points or intervale

The pach constraints taken sonplet are and to define the general type of solutice path detirod and often have the form of restriction an propertios of the initina and gool sutes Four catequries are anily dimingaishod a being mefal in problem clamirication Problems with properties whick ane prevent in the initial sace and which must also be present in the goal state are preservation problema Complementary prevention problems have propertien which are abment in the initimil sate and which must also be abwent in the gool sate Problems with propertise which are prement in the initil state but must be abnent in the goal sate are deveraction probleme The complementary coosoruction problems whict have been most stodied in the literntate have peoperies which are abeent in the initial state but must be proient in the goal state.

Further meanuremeats for the construction of the slution rechnique spece an be found by coumidering the may in which the planner toe about syntbetizing its outpat [23] Early plapsers unod weak methods ach an meemeach amalysis and simple beck-chaining to solve rather imple problems. As researchers began to attempe more and more complex problems, more powerful rechaique were used The iden of hierarchical abtraction was used to focus the pinmer on the hardest parts of the problem first [24, 25] Nert on be considered were coajunct ordering sibpoal internction, and the nonlinear nature of plans [26, 27, 28, 29, 30, 31, 32, 33, 34) Liter, preformed plan mequent were built into phannem to provide trem with a mechenien of avoitiat repentedly planning typical take from ecratech [35 36 ] Finally. plannem have been given mea-level control so tat they can plan aboat planning as well as planning about solvinf the problem ar haod [37. 38] Related, but alrearative views of planning also have been takea. Opponed to the view that the planner should strive to make perfort phass exclusively, one planner proponed making an imperfact phan and debugging it [39] Another planner mook the view thit an attempt should first be made to disprove that goals an be reached before attempting to reach thome goals [40]

Pursuit of this type ai "standard" approech will probebily produce a useful clenification scbeme for planning problam ogper and solution mehniquar Much mare work is rupuind and even then it remaina to be seen whether practionly tefal reprementations or data structure can be coossorexted in the contert of the intelligent entity model which scimatmed this inventigation It is aggested that a similar effor on cheduling problema, that is one besed on inpur/oapper properties and on representation/reasoning characteriatics woald yield similat remulte Unfortunately, this gives a set of scructures for plaming and another at of structuren for schoduling either avoiding the question of the difference bsween planaing and scheduling altogether, or answening that there is a difference beod on the narrow input/outpor definsscons uned to build the structurea. Given that the ens
"phaning" and "echoduling" cover a very broed aet of problems, such a narrow answer in mot very minfying. It seem intuicively obvious that a workable clanafication scheme should differentiate a et of problem which are purely planning problema, a met which are clearly acheduling problema, and ant which have characteristict of both, or it should show that there in a epectrum of problems none of Which are purely planaing problems or purely scheduling problema

It Is suggented that two other observations about planning and achoduling problems an belp to rectify this senve of dis satifaction. One concerns the gature of the uncertaintics which are encountered in robocic make The other concerna whye in which to manage the uncertainty. Taken together, there two idens provide a new perspective on the difference between planning and scheduling.

## 4. A New Perspective - Obecrvation 1

An intererting exercise is to consider what is known in the earliest stages of a plan/schedule/execute cyele and to contratt thin with what is known at the completion of such a cycle. In the early stagen, not much is known. In robotic traks, it may be as little as an abatract goal (the need to repair a broken aubryitem on a spece rtation), mome idea of the resources which might be called into play (any toole and sparea on or available to the station), the objects which may be involved (the rools and all parts in and around the aubsystem with the unknown fault) and the manipulatory and percepoual capebilities of the robot(s) in the area as well as ite currently amigned task In the final atagen, there is a plethors of informatico. Every importans intermediate state can be known to the realution of the system sensors including the apecific objects involved, the values of all of their relevant propertics, and the details of all relationchips between objecta Every important action can be known within sensor resolution including start times, durations end times facilities utilized, even robor apatial and force trajectorien Uncerainty remaing, to be aure, but hindsight is greatly superior to foresight in robotic planoing, scheduling and erecution.

From this perspective, the important queation is what information is required to revolve various uncertainties and when can this information be known with sufficient accurscy that commitments can be made. Planoing scheduling, and execution an be viewod am a continuous series of refinements in which the initial abutract concept is converted into the final concrete realization. An interesting characteristic of robotic tasks is that they are executed in the real world which in the domnin of Murphy's laws as well at Newton's laws Something will probably go wrong, but it cannot be determined in advance what it will be or when it will happen. Furthermore, actions executed in the real world are not easily retracted. In orher worda the penalty for commitment made too early will be extra expense, because, once executed, the wroog action will require effort to reverse, if it is reversible at all

The uncertainty which is encountered during planning, scheduling, and execution has many sources. A major source stems from the fact that the robot (and any aupporting computational device) bolds an incomplete ineract digital model of an anslog world The ircompletenes is due to the inberent finitenem of the computer relatively to the practically infinite richness of detril in the world. The ineractoes is due to the imperfect sature of the input and output channels which the robor (and any upporting computational device) han to its worldly environment Sersore es input chanoels and mechanimene a output chaooels are ooly accurate within bounde. Any form of intelligent entity dealing with robot takk muat recognize and deal with there facte even though it
belplem to chnnge them. This uncertainty in reflected in the structure of reprementations of statea and acticne in aysrems which must deal with the real world Sate deacriptions include tolerances an the valued amocinted with object propertica. Action descriptions include sccuracies on the values achievable by theis application. Recently, mechanizo. have been reported which can be uned to plan about anch uncertainty [41]

Since all robocic systems must handle uncertinity at the modeling aencor, and mechanimen leval, this type of uncertainty in obvioully Dot of murh uen for clemifiration (other than realizing that the level of uncertainty in the eyseam murt be matched to the leval of uncertainty in the terl). Another mource of uncervinty which cocurs at a higher level in uneful, however. Statem and state demeripoices involve a degree of uncertrinty. In micro-acalo, it is noc alway clant when a particular object which in required will be avalisbla, or, if it is available at aome point, whether it will exill be suitable when needed. In macro-scale, it is certioly pomible to imagine that the specifications of initial and goll stelten can change with tima. Similar comments and be mede ebout actions At the micro-male the availability of facilitias changen with time, sometime unerpectedly as a higher priority tank encounters difficultien or the facility thelf requirn adjustment or repair. At the mecro-acale, it is eveo pomible to imagine that the capability of $t$ facility chenge or thet facilitice are added to or cieleted from the pool of rescurces. These examples each point out that everything in the environment (except the laws of physics) in anceptibia to change over time sometimes gradually, somecime drametically. Of fundemental concern it the uncertaiaty about the availability and suitability of agents objecta and facilition, and when and to what degree this unewrainty cala be remolved.

It is certainly well understood in the robotica commanity that plans and schodules for robota are fragik in terma of executional integrity [42, 43] The carlieat planoing systems included execution monitore [44], and more recent work has been aimed at knowledge-based ayntema for error recovery (45, 46) Furthermore, methods have been proposed for incompletely specified or dynamic enviromments (47, 48, 49] However, it is not clear that an uncertainty model in a popular one for the clamification of planning and scheduling probleme. None the lem, it is proponed that it is procively the view of planaing, scheduling, and execution at atep by step resolution of uncertainty which begins 0 formalize the way in which we intuitively view the difference between planning and scheduling probleme

## 5. A New Perspective - Obeervation 2

The second observation complemente chis way of looking at problems by providing a perspective for looking at solution techniquea to manage the uncertainties involved in robotic rasks. The key here is the concept of leart commitment in the face of uncertainty, various options can be taken At one extreme. a guem can be mede. If the guem can be made in such a way that it can be eaxily retracted if it is wroag. then this is a uneful suraceg [37] At the other extreme, the decision can be deferred until enough information cen be gathered so that the uncertainty can be atmolutely reaolved. Of course, it is seldom the case in robotic tasks that eitber of these extremes is obtained. It is not often thas so litvle information is available that a blind gues tuar be made, or that so much information is available that no doubt remaine It is left to the implementor of planners and achedulers to provide sufficient knowledge to their syoteme to be able to decide what degree of commitment is warranted by given level of information. The poist is that it in a difficult decinon.

Another point in that it in the ordering of actions, the assignment of facilities to carry out sctions, and the amociation of searting and stopping times with actions which are being deferred. Thee iden are atro not new to the AI or robotice communities, but have been uned in different wrys than to clasify planning and scheduling molution rechniques As hat been pointed out, one of the main jobs of a planoing eyutem is to order atatee and actiona, so all syatema have a bedic ordering capability [23] In addition, the concept of abetraction coatains the idee of ondering tuke by importance to provide a focus for the reasoning syntem [23]. Furthermore, sdvanced planners ase based on the principle of least commitment in the ordering of plan fragments to avoid adverse aubgoal interactions [23, 30, 31) But in keeping with the spirit of this document, it has been suggented only afer time that the output of a planner should at times be limited to a partinlly ordered set of actions, the final order to be reoolved by the erecution environment [42, 50] in the extreme, syrtems have been implemented that plan onily incrementally, either based on the immediate need to do something other than an obvious atomic action [51] or on the opportunity to enally achieve ane of a list of goals [52] both driven by the enviroament.

## 6. Is There a Difference?

On one hand, planning and scheduling problems need to be clamified by the form and content of the input socepted and the output delivered. Planaing and acheduling techniques need to be clamified by the reprementation and reasoning mechaniams which tranaform the inputs into the outputs On the other hand, plapoing and achoduling types of problems can be clavified by the time at which rufficient information is available to define the initial and goal statea, the facilities catalog, the identity and relationahip among agenta, the desired actions, the ordering of the actionk, the asmignmeat of agenas and facilities to the actions, and the amociation of atart and stop times to the actions. Planaing and cheduling solution tochniques can be clamified by the manner is which each of thene decisions is delayed until sufficient information is available to justify commitment.

The planner is mainly concerned with what should be done and is invo..ed with using the initial and goal statem along with the capabilities of the facilitice in the catalog to select the denired actions. The scheduler is mainly interested in when the desired actions should be accomplished and une the availabilities of the facilitia in the catalog to amign facilitica and times to actions. Sometimes the planner must order the actions based on precondition and poarcondition arguments, and sometimen the scheduler must order actions besed on environmental conditions Sometimea the environment in very atable and enough information is available that the planner and seheduler can work together to make mont decisions prior to the start of execution. Other times the environment is very volatile and so little information is available that the planner and the scheduler must delay almort every decision until an instant before execution.

Confusion between planning and scheduling of ten occurs because the planner over-commits, taking on the tak of ordering actions when there is no justification to prefer one order over another, assigning facilities before the details of availability can be known, and even asociating start and stop times before environmental conditions warrant such deciniona Such overcommitment by the planner prematurely removea all decision proceses from the scheduler and makes the execution environment unnecesearily susceptible to failures. It in the responsibility of the planser to know when it is abmolurely necesary to make thene decisoons because of unavoidable requirements based on physical laws of facility capabilitien, and When it is possibie, or even desmatie, in leave these dericices for the sheduler.

To underscors thene coscepta, it should be noted that thrme different research group have beome interested (appareniliy independently and simultapocualy) in what can be comidered a paradigmatic problem in the division of labor berween a "planner" and a "rebeduler". The domain is robocic anembly, evpecially in the casan where the amembly an to ao complished by a number of different pethways. A particular problem might cocur when a robot is amembling from a cole plete but random stack of parts in an envirommeat in which all the required facilitien are alway available. The planner can commit to one particular amembly mequence, veuping the job of the scheduler, but oalj at the cont of requiring the execution rystesn to aiwaye reorder the parte so that thay ase available in a preacribed requence. If instend the planser in willing to supply the required eet of actions to the achoduler with only the minimum of of osdering constraints the scheduler can efficiently roect to the ordering uncertainty in the avaliability of parts to guarantee efficient erecution.

Notice that the same inuca arive if all the parta are available all of the time (for example, bins of ench individeal part are available in the work celll, but the apecinlized toois to perform the amembly are only avaliable at random times (for example, because thay are used in a number of cellial An even more complex can is encountered if availability of both the parts and the tools is randomized. In all of thase circumstances, it is clear that the planner thould produce a minimally ordered plan to that the scheduler can capitalize on the uncertainty in the environment rather than being hampered by it.

The groupe involved have been interested in the amembly problem besed on its own merit and have not been overily concerned with the impect of their work on the categorimtion of planning and acheduling problem typee of molution techniques. They have been more interented in the quextion of devising the beat way of representing schaduling options gained from the planner and reasoning over these options during executions. The representations ued have included seta of partiai orders $\{53,54,55,56,57\}$ AND/OR graphs [58] and Petri nets $[59,60]$ It is proposed that the robot arsembly problem an formulated by these researchers is an outstanding example of a problem which clearly distinguishem between planning and echeduling, and which clearly indicatea the penalties incurred for ignoring the dirtinctions It deserves further study in the effort to develop clamificacion schemes for planning and acheduling problems it is further proposed that the three methods which the researchers have developed deserve comparative study in the effort to develop clemification achemes for molution techniques for planning and scheduling probleme

## 7. Conclusions

It is concluded that there in a distinct difference between planning and scheduling. Until an adequate cincrification scheme can be developed for planning and schoduling problem typea, the case of robotic amembly under various typer of uncertainty about availability is offered as a paradigmatic example. It is hoped that this erample will atimulate work on the clamification of problem typea as well a solution rechniques.

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# The Mechanisms of Temporal Inference* 

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#### Abstract

Ah format the properties of a temporal language are determined by its constituent elements: the temporal objects which it can represent. the attributes of those objects, the relationships between then, the axioms which define the default relationships, and the rules which define the statements that can be formulated. The methods of inference which can be applied to a temporal language are derived in part from a small number of axioms which define the meaning of equality and order and how those relationships can be properated. More complex inferences involve detailed analysis of the stated relationships. Perhaps the most challenging area of temporal inference is reasoning over disjunctive temporal constraints. Simple forms of disjunction do not sufficiently increase the expressive power of a language while unrestricted use of disjunction makes the analysis NP-herd. In many cases a set of disjunctive constraints can be converted to disjunctive normal form and familiar methods of inference can be applied to the conjunctive sub-expressions. This process itself is NP-hard but it is made more tractable by careful expansion of a tree-structured search space.


## 1. Introduction

An intelligent autonomous system operating in a resole, unstructured environment must have three capabilities. First, it must be able to create a plan or course of action according to an initial state of the world, al state of the world, and some knowledge of its own abilities. Second, it must be able to determine a sequence of actions, according to the constraints on the steps of the plan and the evolving state of the world. Finally, it must be able to produce the desired effect of those actions according to its abilities and the present state of the world.

The performance of the planner, the sequencer, and the executor components of such a system can be very much affected by the language used to represent plans. The concepts of action must be suitable for the planner, which must reason about coals and effects, but at the same time be tractable for the executor, which must produce the desired effects. The concepts of order must be sufficient for the planner, which must control undesired interactions between operations, but at the same time they must not impose unnecessary constraint on the sequencer, which must adapt the sequence of actions to the dynamically chancing state of the world. The methods of formulation must enable the planner to produce the most central plans possible, yet at the same time it must be feasible for the sequencer to derive a sequence of actions from those plans. The language must be terse. The size of the plan must be proportional only to its complexity.

[^0]It has been repeatedir succested that reasoming over time is an essential element of planaiag and specific temporal representations have beea proposed to facilitate the planning proceas. including a linear prograning model of Milik and Binford[1]. the spece-time maps of Miller[2], the interval alebra of Allen[3]. the point algebra of Vilain and Reuta[4], and the end-point represeatation of Choeseman[5]. Al though not concerped with pleming but instead with the problems of sequencing the activities of robots. Fax and Eempf[8] propose a language of temporal constraints as the tarcet ropresentation for planera.

With this abundance of temporal languages for planaing and sequencing. it is iaportant to establish the propertios of the proposed languages and to understand the inference mothods which can be applied to them. Althouch a complete surver of temporal reasonine is beyond the scope of this paper, an examination of the basic elements of temporal representation and the rothods of temporal inference will establish the primary criteria for comparing these lancuages.

## 2. Elementa of Temporal Depresentation

The properties of a language are mbodied in its syntactic form and its semantic interpretation. The concern here is with the seanatic elements of a lancuage rather than with its syntactic details. Novartheless, in order to discuss the variety of possible temporal languaces, t- is necessary to introduce some simple syntactic structures which represent abstract seanntic entities.

Temporal languaces are concerned primarily with teaporal objects: instants and intervals of time. Some authors maintain that instants of time present some semantic difficulties and therefore propose that tiee intervals should be the priaitive element of temporal reasoning[3]. Others maintain that intervals of time can be defined by their endpoints and propose that instants shoald be treated as the basic olement of temporal reasoning. Some sequencing problems involve activities, which in reality occur over some interval of time. but for purposes of analysis can be treated as atomic and indivisible. In the following discussion, instants of time will be treated as prinitive objects, denoted by alphanumeric symbols such as X. Y, and mipe-o'clock. Likewise, intervals will be denoted by alphanueeric symbols, such as $Z$. $W$, install-clip, and drill-hole. but when useful or necessary. the initial and final endpoints of an interval will be denoted by a suffix letter $i$ or $f$ attached to the interval name, such as $Z i$ and $2 f$.

Temporal languages are concerned with the attributes of temporal objects. Some languages may allow the specification of the absolute time of some instant or it may be possible to specify the duration of an interval. Specialized systems may associate the properties of physical processes with intervals, such as rates, loads, or volumes. Planning systems may associate propositional variables and their values with temporal objects. Oltimately, each temporal object is associated with some event, activity, or proposition. For instance, it is possible to refer to the instant which begins an occultation, or the interval of time when the action install-clip is performed, or the interval of time over which the proposition chanael-is-avallable is true.

Temporal languaces are concernet with the relationships between temporal objects. The most primitive involve the relationships between instants of time. Two instants may be equal, denoted by the operator $=$, they may te inequal, denoted by the operator <>, or they may be ordered, as denoted by the operator <. In order to avoid any syntactic anbiguity, such relationships are written in fully parenthesized infix notation, as in the expression ( $X, Y$ ). The relationships between two intervals of time, as defined by Allen, are shown schematically in Figure 1. The relationships between instants and intervals of time can be defined in a similar fashion. All of these relationships can be specified by their respective endpoint relationships as indicated in the right hand column of Figure 1.

In addition to the facilitios for explicitly stating the relationships between temporal objects, a temporal lansuage must include some axions which define the relationships between objects that are not otherwise constrained. Commonly, it is assumed that, in the absence of other explicit constraints, two instants of time, $X$ and $Y$, are ordered as either ( $X<Y$ ) or ( $Y<X$ ). or they are equal. ( $\mathbf{X}=\mathbf{Y}$ ). Likewise, unless otherwise constrained, two intervals can be related in any of the 13 possible ways shown in Figure 1. In some applications involving the serial execution of a set of operations, there is no opportunity for any of the operations to be done concurrently. In such cases an axiom which defines the default relationship between intervals states that, unless otherwise constrained, two intervals, $X$ and $Y$ are disjoint and ordered as either ( $X$ before $Y$ ) or ( $Y$ before $X$ ). Such axioms play a significant role in the treatment of negation and the processes of inference.

Temporal languaces are subject to certain rules of formulation. The simplest rule is to assume that a siven set of prinitive constraints is to be treated as a conjunction and that they mast all be satisfied simultaneously. In contrast, the lagguge defined by Allen allows a restricted form of disjunction. The relationship betwean a civen pair of intervals can be specified as a disjunction of any of the 13 possible primitive relationships. This ankes it possible to circumscribe indefinite relationships or to prescribe some relationships which cannot be expressed as one of the 13. For instance, suppose that two intervals, $X$ and $Y$, must becin at the same tim but that there is no constraint on thoir termination. It would artificially constrain the intervals to state that (X bedina Y) because thia primitive relationship requires that $X$ terminate before T. Likewise it would be an artificial constraint to require that ( 7 bedias X ). Osing this vocabulary of 13 primitive relationships, the relationship between $I$ and $Y$ can only be stated as a disjunction, ( (I begins Y) or (Y bedins I)). In Allens's languace, disjunction is restricted to phrases that define the relationship between a single pair of intervals and cannot be used to pose constraints such as, ( (X before Y) or (Z before W)).

The properties of a temporal language are deternined by the combination of these elements: the objects, attributes, relationships, default axioms, and rules of formulation. Tocether these elements determine the set of probless that can be represented. For instance, the language of strict partial orders is composed of symbols which denote instants of time, the priaitive ordering relationship \&, the default axion that states for all $X$ and $Y$ either ( $X<Y$ ) or ( $Y$ < I), and a rule of formulation that allows only conjunctions of primitive ordering constraints. A given constraint expression in this language defines a set of admissible total orderings over a set of instants of time. For example. The conjunction $((X<Z)$ and $(Y<Z))$ defines 2 admissible orderings of $X, Y$. and 2: [ $X, Y, Z]$ and $[Y, X, Z]$. The limited rule of formulation in the language of strict partial orders makes it impossible to state the constraints for a problea which admits the 4 linear orderings [X,Y,Z]. [Y,X,Z]. [Y,Z,X], and [Z,Y,X]. There is no conjunction of primitive ordering constraints which defines exactly this set of linear orderings! The limited forms of disjunction included in Allen's interval algebra or the point algebra defined by Vilain and Kautz encompass some sense of indefiniteness in the relationship between temporal objects but these forms of disjunction are not sufficient to represent the full range of possible ordering problems.

A number of common temporal representations can be quickly distinguished by their constituent elements. For instance, the language of equivalence classes is composed of symbols which denote atomic temporal objects, the equality and inequality relationships, = and <>, an axiom which states that for all $X$ and $Y$. $(X=Y$ ) or ( $X$ < $Y$ ), and a rule of formulation which allows only conjunctions of equality constraints. In contrast, the language of graph coloring problems has a similar structure but the rule of formulation allows only conjunctions of inequality constraints. The language of temporal constraints proposed by Fox and Kempf[6] is composed of symbols which denote atomic temporal objects, the ordering relation-ship $<$, an axiom of serial processes which states that for all $X$ and $Y$. ( $X<Y$ ) or ( $Y<X$ ), and a rule of formulation which allows arbitrary use of conjunction, disjunction, and negation. This axion limits the scope of this language to problems that involve activities that must be done one at a time, such as a robot performing an assembly task. Bowever, the unrestricted use of disjunction guarantees that this language can represent any problem within that domain. Portrait, a temporal language under development by Fox and Green allows arbitrary use of equality, ordering. conjunction, disjunction, and negation.

## 3. Methods of Temporal Inference

Temporal reasoning is a frocess of deriving the properties of temporal objects and the relationships between temporal objects that are implied but may not be explicitly stated in a given set of temporal constraints. The most familiar form of temporal reasoning is constraint propogation. In the language of equivalence classes constraint propogation is based upon two axioms. The first defines the symmetry of equivalence: for all $X$ and $Y,(X=Y$ ) implies that $(Y=X)$. The second defines the method for propogating equivalence: for all $X$, $Y$, and $Z,(X=Y)$ and $(Y=Z)$ implies that $(X=Z)$. There is no symmetry in the language of strict partial orders, only an axiom which defines the method for propagating order: for all $X, Y$, and $Z,(X<Y)$ and ( $Y<Z$ ) implies that ( $X<2$ ). The language of partially ordered sets includes an axiom which defines how a disjunction of order and equality can be propogated: for all $Z, Y$, and 2 , $(X<=Y)$ and $(Y<=Z)$ implies that $(X<=Z)$. Coupled with this is an axiom which defines how constraints over a given pair of temporal objects can be resolved: for all $X$ and $Y,(X<=Y$ ) and $(Y \ll X)$ implies that $(X=Y)$. If, after the complete propogation of constraints, only one of the constraints ( $X<=Y$ ) or ( $Y<=X$ ) has been imposed then it can be assumed that the two objects are not
equal. Vilain and Kautz dofine a language over instants of time which, for a given pair of instants, allows an arbitrary disjunction of the 3 possible relationships between.that pair. In this context, the propogation of constraints can be best defined by a matrix as shown in Fizure 2. Conjunctions of constraints over a single pair of instants can be resolved by a rule of intersection as shown in the matrix of Figure 4. Vilain has demonstrated that constraint propogation within this language is both complete and correct. Allen's interval algebra relies upon similar, tabular rules of inference, but because of the added complexity of this language, constraint propagation is not cuaranteed to be complete.

In most circumstances these constraint propagation axioms can be applied in reverse in order to identify the essential constraints in a problem and to elininate any implied constraints. Givon the complete set of implied and easential constraints it is a simple matter to identify the equivalence class of some teaporal object along with all of its predecessors, direct predecessors, siblings, successors, and direct successors. For instance, the axiom which dofines the predecessors of a temporal object $Z$ states that $X$ is a predecessor of 2 if ( $\overline{2}<2$ ). The direct predecessors of $Z$ include all those temporal objects $\boldsymbol{X}$ such that $(X<Z$ ) but there does not exist $Y$ such that ( $X<Y$ ) and ( $Y<Z$ ). These can be easily identified by scanning the set of essential constraints.

Reasoning about the admissible ordering of temporal objects is directly related to an analysis of precedessors and successors. For instance, in the language of strict partial orders, the controlling axiom specifies that a temporal object $X$ can occur only after all of the predecessors of X. Of course, those objects which have no predecessors can occur at any time. This axion can be used to incrementally build sequences of activities. At aach step of the process simply choose one of those activities which can occur next.

In most sequencing problems the combined ordering constraints limit the admissible sequences of activities but do not remove every sequencing option. In most problems there are many admissible sequences. The number of admissible sequences can serve as a useful indicator of the available sequencing options. In some problems this may provide an estimate of the effort required to find the best sequence of activities. In other problems it may provide an estimate of the inherent flexibility that can be exploited in sequencing those activities. The naive apprcach to computing this number would be to explicitly enumerate all of the feasitle sequences by exhaustive application of the sequencing axion or other more sophisticated algorithms[7]. Unfortunately, the simplest of problems will prove the most intractable. Consider a serial task of 15 steps with no sequencing constraints. There exists $15!=1,307,674,368,000$ sequences. Even if one sequence could be generated each microsecond it would still requie 15 days to enumerate the entire set. Fortunately, general methods are available which can determine the number of feasible sequences over a strict partial order without explicit enumeration. These methods are first roported in a textbook by Wells[8] but soveral refinements of these methods were doveloped at MDRL by the authors. Generally, this computation can be accomplished by recursive application of 3 simple rules:
(1) if a set of activities can be divided into two subsets such that all of the activities in the first set must precede all of the activities in the second set, then the total number of feasible sequences equals the number of feasible sequences for performing the activities in the first set times the number of feasible sequences for performing the activities in the second set.
(2) if a set of activities can be divided into two subsets such that all of the activities in the first set can be performed independently of the acitvities in the second set, then the total number of feasible sequences equals the total number of feasible sequences for performing the activites in the first set times the number of feasible sequences for performing the activities in the second set times the number of ways that one sequence from the first set can be interleaved with one sequence from the second set.
3) if a set of activites cannot be divided into two subsets accordine to rules (1) or (2) then that sot of activites can be partitioned into two strategies for performing those activies which have no feasible sequences in common, and the the total number of feasible sequences will be the number of feasible sequences under the first strategy plus the number of feasible sequences under the second strategy. The partition is generated by identify a pair of unconstrained activitios, $X$ and $Y$. The first strategy is defined by the orginal set of constraints plus the constraint that $X$ must precede $Y$, ( $X<v$ ), and the second strategy adds the constraint that $\mathbf{Y}$ must precede $\mathbf{X}$,
(I $<X$ ). (Repeated mpplication of these 3 rules is muaranteed to work recardless of the I and I chosen when unins rule 3, but the number of partition cenerated is sipalifenatly affeoted by the choice. By carefully selectine the steps $X$ and $Y$, it is possible to control the number of partitions ultimately cenerated.)

By recursive application of these rules it is possible to determine the number of feasible sequences for performing a set of activiites from start to finish, or it can be used to deternine the number of ways of completinc the task from any siven state. In most circunstances the number of feasible sequences corresponds closely to the decree of flexibility inherent in the sequencing of the activites and it can be used as a valuable metric for comparing difierent plans or strategies. As a side-effect, application of the 3 rules stated above results in the decomposition of a siven task into sets of dependent activities, sets of independent activities, and into disjoint sub-strategies. This decomposition can be used by human analysts to better understand the structure of the tasks that they must plan and coordinate.

Dnfortunately, inference over a disjunctive languace, such as that developed by Fox and Kempf, is much more difficult. One way of resolving the constraints in a disjunctive constraint expression is to convert a eiven set constraints into disjunctive normal form, i.e. a disjunction of conjunctions of the primitive ordering constraints, keeping only the satisfiable and nonredundant subexpressions. In that form, the methods of inference sketched above can be applied separately to each conjunction of constraints and the results combined under an appropriate interpretation of disjunction. The production of this reduced disjunctive normal form is very difficult, in fact it is NP-hard, but it is an essential part of more general temporal reasoning

For instance, the constraint expression shown in Figure 4 is typical of the constraints imposed on small assembly problems. Production of the disjunction normal form of that constraint expression, using the distributive law of boolean algebra, ( $X$ and ( $Y$ or $Z$ )) --> ( $X$ and $Y$ ) or ( $X$ and $Z$ ), results in a set of 1024 conjunctions. In general, the size of the disjunctive normal form grows exponentially with the number of applications of the distributive law. Some of the resulting conjunctions are inconsistent and should never be considered, others are specific cases of more ceneral sub-expressions in the result and can safely be removed. Other simple methods for producinc the disjunctive normal form have the same result. However, all of the admissible sequences for performing the task defined by these constraints are embodied in only 22 conjunctions.

An officient method for deriving that set of 22 conjunctions is closely related to methods for determining the satisfiabliity of boolean expression and is based on the expansion of a tree structured search space. Each node in the search space consists of 2 parts. The first is a partially formed conjunction, and the second is a constraint expression which remains to be satisiled. The root node consists of an empty conjunction coupled with the initial constraint expression. Successor nodes are formed by propagating primitive constraints from the constraint expression into the conjunction being constructed. The target leaf nodes consist of a completed conjunction which satisfies the original constraint expression and an empty set of constraints remaining to be satisfied. Specific heuristics have been developed which make it possible to prune redundant or inconsistent solutions early in the tree expansion. Using these methods the constraint expression shown in Figure $\&$ produced 28 consistent conjunctions, 6 of which were subsequently identifed as redundant. Subtree expansion was terminated 58 times because inconsistencies were detected and 12 times because redundancies were detected. This is considerably more efficient than producing 1024 conjunctions and then attempting to prune the inconconsistent and redundant sub-expressions.

## 4. Conclusion

The properties of a temporal languace are determined by its constituent elements: the temporal objects which it can represent, the attributes of those objects, the relationships between those objects, the axioms which define the default relationships, and the rules which define the statements that can be formulated. The methods of inforence which can be applied to a temporal language are dorived in part from a small number of axions which define the maning of equality and order and how those relationships can be propageted. More complex inferences involve detailed analysis of the stated relationships. Perhaps the most challencing area of temporal inference is reasoning over disjunctive temporal constraints. Simple forms of disjunction do not sufficiently increase the expressive power of a language while unrestricted use of disjunction makes the analysis NP-hard. In many cases a set of disjunctive constraints can be converted to disjunctive normal form and familiar methods of inference can be applied to the conjunctive sub-expressions. This process itself is NP-hard but it is made more tractable by careful expansion of a tree-structured search space.

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Figure-1.
Thirteen possible interval relationships.


Fisure 2.
Matrix of constraint proparation in the point algebra．

|  | $x R y$ | ＜ | $=$ | $>$ | $<$ | ＞$=$ | ＜＞ | ＜${ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | ＜ | x | $x$ | ＜ | $x$ | ＜ | $<$ |
| $=$ |  | x | $=$ | $x$ | ＝ | ＝ | x | $=$ |
| ＞ |  | $x$ | x | $>$ | x | ＞ | ） | $\rangle$ |
| く |  | $\leqslant$ | $=$ | $x$ | ＜ | $=$ | $<$ | く |
| ＞ |  | $x$ | ＝ | $>$ | ＝ | ＞$=$ | ＞ | $\rangle=$ |
| ＜＞ |  | $<$ | $x$ | ＞ | $<$ | ＞ | ＜＞ | 〈＞ |
| ＜ |  | ＜ | ＝ | ＞ | ＜ | $\rangle=$ | －＞ | く |

Fisure 3.
Matrix of constraint resolution in the point algebra．
（co before cl）
（ba before cl）or（co before dr））and
（（co before st）or
（（dr before co）or（dr before ba））and
（（ba before dr）or（ba before ca））and
（（ra before co）or（ra before ba））and
（（mi before ra）or（mi before ms））and
（（mi before co）or（mi before ba））and
（（sm before mi）or（smbefore ba））and
（（sm before co）or（smbefore ba））and
（（ri before co）or（ri before ba））

Figure 4.
Typical disjunctive constraints on the steps of an assembly problem．

# Contingent Plan Structures for Spacecraft 

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## 1. Abstract.

Most current AI planners build partially ordered plan structures which delay decisions on action ordering. Such , / structures cannot easily represent contingent actions. This -paper present a representation which can The representation has some other useful features: it provides a good account of the causal structure of a plan, can be used to describe disjunctive actions, and it offers a planner the opportunity of even less commitment than the classical partial order on actions. The use of this representation is demonstrated in an on-board spacecraft activity sequencing problem. Contingent plan execution in a spacecraft context highlights the requirements for a fully disjunctive representation, since communication delays often prohibit extensive ground-based accounting for remotely sensed information and replanning on execution failure.

## 2. Introduction.

Plan generation isn't problem solving. Planning problems are physical realities which require physical solutions. Planning can only be construed as problem solving when it's part of a larger system which also addresses plan execution; only execution can realize the solution a plan specifies. We use this theme of plan execution to bring together some important issues in AI planning. We consider least commitment plan construction, the representation of teleological information, disjunctive plans, and contingent plan execution in realistically complex domains.

We begin in the next section by briefly discussing the way that most Al planners operate. Commonly used techniques include least commitment action ordering and object selection; we discuss both. Following this, in section 4, we describe an actual planner called O-Plan [1] which uses these techniques to good effect. We cover the essentials of O-Plan's search for an acceptable plan, leaving aside low level details. This discussion is used to show how O-Plan relegates the responsibility for reasoning about disjunctive actions to its search space management component. We argue that what a planner needs is a plan structure which is able to describe the disjunction of action implied by the choices encountered during plan construction. In section 5 we present a solution to the problem. A representation is given which has the properties we seek: it can be used to do least commitment plan construction; it explicitly represents teleological information; and it can describe disjunctive actions. Together these abilities allow our plans to be used for plan execution in realistically complex domains. To motivate this, section 6 places the ideas in the context of a spacecraft activity sequencing problem: planetary observation. This example causes us to reflect on the basic principle of least commitment problem solving in general, since it supports a form of least commitment reasoning which commits even less than current techniques.

The primary result of this paper is a representation we call C-Plans. We claim that the representation is suitable for use in sequencing the activities of automated spacecraft. Further applications-oriented research is required to substantiate this claim.

## 3. Current Al planners: least commitment plan construction.

An AI planner is given the responsibility of constructing a plan of action. Such a planner is given an initial state description, a set of goals, and a set of action schemas. The schemas are parameterized plans, suitable for solving limited problems. A plan produced by the system is an artifact built "om individual operators, appropriately instantiated and ordered. This plan must be sanctioned by the system as a feasible means of achieving the given goals. In this section we examine briefly two of the main operations required to produce this plan: action ordering and variable instantiation.

### 3.1. Action ordering

Early planners built mocally ordered structures: a plan was a sequence of actions. This not oaly applied to the final plans produced by the system. bet also to the parial plans tax were beitt during search. Win Sacerdoti's Nown [3] system this all changed. NONH baik plans as partial orders on actioas. This meare that it was possible for any two given zetions to be mordered with respect to each ocher. The interition betiod this idea is that a partial order on actions characterizes very many sotal orders. Toray such plans are often called nontinear. It mighe seem thax a system which builds nonlinear plans would be exponentially more efficient than one which builds linear, or toolly ordered plans. Unfortunately this has never been proven. Onty the inmicion exists that nonlinear is betwer than linear, but this inuition is betrer than nothing. See Chapman [4] and Dramanood [5] for more on this.

## 31. Object selection

There is another sort of least commitment found in some planners which relates to the way tha the objects referred to in plans are selected. Variable instantiation is the process of selecting constants to bind to variables. Exh variable can be bound to a specific constant, or unbound, meaning that mo constant has yet been selected as appropriate. But it is possible to operate in a more sophisticated way. we can post constraints on the permissible constants for any given variable. In this way, we constrain the possible bindings for a variable, rather than sck... one as correct ourright Information can be gathered during the process of plan construction which leads to the deletion of particular constants from the set of possibilities. The bope is that eventually the set of possibilities will be narrowed to one altemative, or reduced to the empty set indicating that the consuraints posted on the variable are so strict that there is no sacisfactory object. This method of associating constants with variables is known as least commitment object selection. Its genesis was in Molgen [2].

## 4. A framework for doing this: O-Plan.

O-plan is a modern planning system which owes many of is ideas to Noalin [6]. NonLin derives from Noall, and extends it in many ways. For our purposes the essential contribution of Nonlin is is completion of the search space of parial plans: Nonlin could find plans that NOAH could nor. This is because Nonlin had plan modification operations available to it which defined its search space of partial plans so as to include plans that Nonif would never consider. O-Plan inherits its definition of the search space from NonLin. In the next section we consider the mechanisms used by O-Plan to search the space of possibilities. We explain how it keeps track of alternatives, and how it searches through the space of partial plans. We thea go on to consider bow this mechanism can be extended through a more flexible representation for plans.

### 4.1. Agenda-based partial plan search.

The planning components in the 0 -Plan framework employ various techniques to lessen the amount of porential search in any paricular appication. The tectriques include least commitment variable binding. constraint cut-off. temporal coherence and various beuristic functions. O-Plan searches through a space of partial plan states. guided by these techniques, where each partial plan state is derived from the application of a plan modification operator to some current partial plan. This is essentially a search space of plan modifications, or operations whose application results in a new (partial) plan state.

An O-Plan Plan State is a structure of some detail and it includes the partial order of activities that is curreruly being built (essentially the plan so far), a log of effects and conditions asserted or required in the plan, the teleological information ased during plan generation (and available thereafur), variables used during planning and. tinally, information on outstandiag tasks generated during the planning process. These pending tasks are colkected together into agenda lists from which each task can subsequenly be scheduled in some opportunistic fashion. This mechanism provides for a dynamic approach similar to that provided by "blackboard" based systems.
in practice there are two main agenda list opes, cre for ask specier ations which are fully instantiated and one for tasix specifications where cerain information has yet to te determined. A chird "alkermatives" agenda list is currenty employed which should evenouslly disappear, but uhich has been used in the absemeie of a comf: $:$ e method for dependency-based plan repair.

Task selection is done under the control of a scheduler, which provides the opportunism for the overall process. This schedaler can be regarded as a "plug-in" module in the O-Plan system and therefore it can reflect various schedaling strategies. The scheduling of a task from the agendas causes a handler (or knowledge source) to process that paricular task. The relevant handier is invoked by the type of the task scheduled bence the systern is data driven by the tasks themselves. As well as changes to the current Plan State, the processing of a task generally results in the creation of new tasks or the amendmeal of existing tasks on the agendas.

When choices are made, the task handiers have the option to either post dependency information in the Plan State, or to simply spawn almenative Plan Stales via the alternatives agenda mentioned earier. The former method has the advantage that it offers the porential for proper plan repair where only the affected parts of the current (partial) plan are stripped off after a failure, while oseful parts are saved and the work dooc in producing them protected. We are researching how this can be done by using partial plans angmented with teleology information, although there are many outstanding problems.

Processing proceeds in cycles and finishes when all tasks have been processed or when there is reason for a particular task to terminate planning. In theory the handers are independent of one another but they do have the ability to "poison" the carrent Plan State if they detect inconsistency or constraint violations. This is the time to backtrack, plan repair or simply give up. Search through the space of partial plans is therefore controlled by the scheduler which chooses the next best thing to do, using information provided by the tasks generated during planning. More detail of the O-Plan control structure can be found in [1].

### 4.2. The requirement for truly disjunctive structures.

O-Plan searches through a space of partial plans. When there's a choice that cannot be delayed, the current O-Plan task scheduler pursues one of the available options by incorporating it into the current Plan State. On failure. OPlan may reconsider all previous Plan States on the altematives agenda and pursue a previously ignored plan modification operation. In this way it follows a "one-then-best" search strategy as in Noalin.

An alternative approach is demonstrated by the following scenario. Consider that at some point during its search for an acceptable plan, the system identifies an outstanding goal, G. Assume that there are two action schemas which after analysis appear suitable for achieving $G$. The urditional approach says that this choice induces a bifurcation in the search space, each path considering one of the two possible actions. However if our developing plan is able to represent disjunction, such a bifurcation is unnecessary. Both possible actions (resulting from instantiating the schemas' variables) can be installed in the plan. The only requirement is that the plan record the fact that these two actions stand in a disjunctive relationsthip.

By the above discussion we aren't suggessing that a pianner consider all possible options at each point in its search; such behavior is doomed to failure, since the number of options open will inevitably be huge. Much of the information rueded for later planning also becomes uncertain in a plan with too much disjunction. However if the plan represemation is able to describe disjunction, then the system will have the option of including action disjunction as appropriate.

Conaingent plans are also necessary for doing realistic plan execution monitoring. When a plan is generated, it's unlikely that the generation component can guarantee what the world will be like when plan execution begins. To properly handle this we need disjunctive plans. The planner can produce plans which contain actions to deal with whatever conuingencies it deems worth considering. Such a contingen' plan must specify the conditions under which each of the planned actions is appropriate, to allow the execution component to correctly select which action to execute.

So: we would like to formalize a plan structure able to represent disjunction of action. But in doing this there's a trap to avoid. We could easily over-simplify the data structures used by a system such as O-Plan. It would appear possible to formalize a nonlinear plan as a partially ordered set Mathematically all one requires is a set of actions and an ordering relation over that set. (See [4] for an example.) The ordering relation is required to be irreflexive and unnsitive, therefore asymmerric. The problem with such a simple formalization is that is fails to capture much of the information that O-Plan exploits during plan generation. In particular, it does not capture the goal structure of a pian [7]; that is, the causal structure that exists among the planned actions.

There are other requirements on the formalization that we won't consider in this paper. In particular, we won't address formalizing least commitment object selection. Data structures to support such operations are simple to formalize, but for ease of exposition, we won't do it here. It is straighforward to add this to the formalism ue present

## 5. Formalizing contingent plans

We can borrow some notions and notation from Net Theory [8]. Not all the constructions that we need are part of net theory, so we'll have to add a few bits onto the basic framework. We won't motivate our additions; for a brief discussion, see [5], and for more extensive motivation [9]. Essentially, we use Condition/Event systems augmented with event occurrence preference orderings; we also idenify the conditions and events of the system with predicates of a simple language. In this section, we'll proceed by informally defining the constructs of our plan language, building up the overall structure we require. The eventual goal is to define $C$-plans, or Contingent Plans, following on the arguments above. It is possible to be quite formal in defining these C-plans, but this paper simply explains and mocivates them.

### 5.1. Basic C-plan structure.

A proposition is a functor applied to arguments. A functor is written in lower case, followed by its arguments in parentheses. Arguments are variables or constants; we allow infinitely many of each. Variables are written in upper case, constanss are wriwen in lower case. For example, both on(a,X), and skew-plafform(15.lefi) are propositions.

Propositions are identified with what we call b-elements and e-elements. A b-element is intended to denote a condition in the world, and can be oue or false. For instance, the b-etement clear(c) under a blocks world interpretation is true if and only if the block denoted by $c$ has nothing on its upper surface. Propositions are also identified with e-elements. An e-element is intended to denote an action, the occurrence of which changes the holding of certain conditions. ${ }^{1}$ For instance the e-element move( $a, b, c$ ) in a blocks world context might denote the action of moving the block denoted by a from the block denoted by $b$ to the block denoted by $c$. Certain conditions must hold if this action is to occur; furthermore, when the action does occur, certain conditions in the world will no longer hold, and centain others which did net hold will begin to do so. For example, in the case of the block movement we might expect that $a$ can only be moved from $b$ to $c$ if $a$ is initially on $b$. Following the movernent, $a$ will be on $c$. We need to capaure chese condition-action relationships in our plan representation.

To do this we introduce the notion of a flow relation. A flow relation is a set of ordered pairs, each pair in the set ordering either a b-element and e-element, or e-lement and $b$-element. The ordeing of a $b$-element and $e$ element is interpreted as an crable relation. Thus, the holding of certain conditions is understood to enable the occurrence of certain actions. The ordering of an e-element and b-element is interpreted as a cause relation: actions can cause the holding of certain conditions. The flow relation describes the relationship between any given event and that event's enabling conditions and effects. It captures what O-Plan and Nonlin call Goal Structure; the best dictionary word for this concept is probably teleology. We use the word to refer to the reasons for some event or condition being included in a plan. The flow relation of a net allows a formal analysis of which actions can be used to enable which other actions; this is essentially the reasoning that O-Plan performs to generate a plan. Other modern planners, such as SIPE [10] also include such information in their plan data structures.

We will refer to the b-elements which are ordered immediately before an e-element as that eelement's preconditions; similarly, we will refer to the b-elements ordered immediately after it as its postconditions.

Graphically we present b-elements as circles and e-elements as squares. Each circle is labeled with the proposition which is the b-element, and each square is labeled with the proposition which is the e-element. The flow relation is drawn as arcs from circles to squares and from squares to circles. If an arrow is to go from a circle to a square, and another from the same square to the same circle, we draw only one line, and use an arrow-head on each end of the line to indicate the two arcs.

One other ordering relation is needed to complete the basic C-plan structure. This is the before relation, used to constrain the way that a net can execute. Intuitively, the before order is a specification of which events must occur before which other events if a plan is to run to its intended completion. We often refer to the before relaticis as execution advice. Consider: the cause and enable orderings in a C-plan's llow relation describe what is causally possible. But in planning we are often interested in only one of generally many causally permitted execution sequences. Causal orderings will not always uniquely constrain a set of actions to describe just those behaviors

[^1]which achieve a planner's overall goals.
A classic example of this occurs in blocks-world tower construction problems. For example: given the problem of creating a tower with block $C$ on the boteom, block $B$ in the middle, and block $A$ on top, the plan construction reasoning must order the two required stack actions to refect its overall goals. To see this, sasume that all blocks are initially clear and on the table. If a plan cals for stacking $A$ on $B$, and $B$ on $C$, then both stack actions are enabled in the initial state. His not an ordering enforced by causation that requires the stacking of $B$ on C before $A$ on $B$. Rather, it is the agent's intention regarding overall plan execution outcome that directs the sequencing of the two actions.

So a C-plan is defined by specifying a set of b-elements (which denote the conditions of interest in the domain being modelied), a set of e-elements (which denote the relevant actions), and an ordering relation oa the members of these two sets (lechnically, the relation is bipartite, since it orders members of two different sets). The C-plan is augmented by giving some execution advice for causally underconstrained actions. This advice takes the form of an ordering relation on C-plan e-elements. To keep the graphical presentation of C-plans simple, we do not draw arcs between e-elements ordered in the execution advice. Instead, the ordered pairs are simply listed beside the nel.

A simple blocks world plan basically compatible with what we have defined here can be found in [11].

### 5.2. C-plan projection.

We now have to say something about the projection of a C-plan. A projection is a structure which supports reasoning about the behaviors that a C-plan describes. First we must say something about the conditions under which events can occur and what changes they realize by occurring. Second we must build up the projection structure which describes the overall behavior of a C-plan, using the definition of individual event occurrence as a building block. E-element occurrence can be used as a "state generator" to create a state-space account of the behaviors permitted by a plan.

We will call an ar'jitrary set of b-element propositions a case. We interpret such a set of propositions 25 a partial description of a state of the word. If a proposition is in a case, then it is true; if it is not in the case, then it is false. Graphically, we present cases only in terms of C-plans -- when doing so, we place a dot (a token) inside each and only those circles labelled with propositions in the plan which are also in the given case.

We can use this idea of a case as a partial world state description to define when an individual e-element is enabled; that is, when the action it denotes is allowed to occur. To model this, we can say that an e-element is enabled in a case if and only if its preconditions are a subset of the case, i.e., if the enabling condivions of the event are true. We also require that none of the e-element's postconditions are already in the case, unless they are also preconditions. Further, we can specify how the world is changed under the occurrence of the action, by defining how an e-element's enabling case is modified to gain a successor. We can generate a new case through the occurrence of an e-element the new case is defined to be the old one, minus all the e-element's preconditions, plus all the e-element's postconditions. The effects of an event are made true in the successor case, and the enabling conditions are made false. If a precondition is not made false by the occurrence of an action, one need only make the relevant b-element a postcondition of the e-element as well.

This definition of e-eiement occurrence can be used to build up a state-space graph structure which tells a story about the possible behaviors of a C-plan. Given an initial case and a C-plan, we can build up a projection graph as follows. The initial case is used as the starting node of the projection graph. E-elements of the given Cplan are repeatedly applied in non-terminal projection graph cases until there are no more cases in which any of the C-plan's e-elements have concession. Arcs leading from node to node in this graph are labelled with e-elements. An arc directed from one node $\alpha$ to another node $\beta$ indicates that the e-ele;nent labelling the arc has concession in the case contained in $\alpha$ and under occurrence, produces the case contained in $\beta$.

The idea is that the graph structure defined in this way contains a given initial case as its starting node, and that each node in the graph contains a case reachable under e-element occurrence. With the interpretation of a case as a partial description of the world, the projection gives us a prediction of what a C-plan can do in terms of the possible world states it might give rise to. The initial case describes the "current" state of the world, and cases in the graph reachable from the initial case describe future possible world states. The arcs in the graph denote transitions from one world state to another, and these transitions can be realized through the actual execution of the actions that correspond to the e-elements labelling the arcs.

So the nodes of our projection graph contain cases, and the arcs are labelled with steps. We can map this structure onto the classical AI picture of planning as follows. The first node of our projection is the initial state given in the problem specification. In order to represent a solution to the problem, the plan's projection must give rise to a node which contains the required goals. We can say that a C-plan is a pocential solution to a planning problem if it is applicable in the initial case of the problem, and under projection gives rise to a case which contains the given goals. Also, a particular case reachable under e-element occurrence in a partially developed C. Plan can be used for "Question Answering" operations in the planner during plan generation.

Using the idea of projection we can now say something more precise about a C-plan's execution advice. Recall the basic idea. Execution advice must contain the information required to remove harmful residual nondeterminism. The advice should not restrict legitimately causally independent actions from occurring concurrently, but it should prevent planned actions from occurring in an order permitued by the causal structure of the plan but unintended by the planner. We can explain the meaning of a plan's execution advice by isterpreting it as a guide to navigation through the projection structure. Basically, we say that a C-plan's execution advice is sound (with respect to a given problem specification) if and only if for all choice points in the projection, if there is any hope for success at the choice point, then either all choices lead to success, or for each choice point that could lead to failure, there is advice about another possible altemative, such that the suggested altemative can lead to success. In essence, when there is still hope for success the advice prevents the wrong sequencing choice from being made. To achieve this the advice must prescribe an order on e-elements which prevents certain paths through the projection from being considered at execution time.

It is possible to generalize the projection we have defined to deal with least commitment reasoning about action ordering. To do this, one need only say when a set of e-elements are causally independent, and use this definition to specify when sets of e-elements can be applied to a case, in bulk, to derive a successor. If this is done the arcs of the projection graph are labelled with sets of e-elements which describe the parallel occurrence of the denoted actions. This means that if some events are causally independent the e-elements which describe them can be applied as a set, and reasoning can continue from the resulting case.

## 6. A spacecraft activity sequencing example.

This section presents an example problem and its representation using $C$-plans. This problem would be difficult if not impossible to represent using the classic partially ordered structures found in systems like NONH and NonLin.

The basic scenario for the example is as follows. While on a deep spaue mission, a spacecraft is to pass very close to the planet Jinx. Earth-based observation has determined that two weather systems oblain on Jinx: crystal clear skies and turbulent sand storms. While it isn't known exaclly what conditions will hold when the spacecraft arrives, it is certain to be one of these two. So useful observations can be made regardless of the atmospheric conditions. If the atmosphere is unclouded, then visible light pictures should be uliken. If a sand storm is in progress, then infrared pictures will be most effective.

The camera used for visible light and infrared pictures is the same, $\mathbf{0} 0$ it is impossible to take 2 visible light and infrared picture in parallel. An inicialization step is required in order to prepare the camera for visible light or infrared work. Regardless of the sort of picture uken, a digital image is stored in a frame buffer on board. The frame buffer is only large enough to store one picture. Each time a picture is written to the frame buffer by the camera, a transfer operation must free the buffer by copying the information to an on-board tape storage medium. For this simple example, we do not address the problem of transferring the stored images back to Earth.
it would be nice to avoid specifying an utscration pregramure igicily in advance. Sirce linx is an far frome Earth to permit the up-loading of an appropriate command sequence (using information gathered closer to the encounter) it is preferable to be opportunistic, and exploit the atmospheric conditions which obtain when the spacecraft arrives. During the period of contact, conditions may change, and the pictures being taken should reflect current opportunity.

From an AI planning perspective, the problem is to have a plan which represents the disjunctive observational requirement simply and economically. Notice that it is not a problem to have an on-board computer which runs a contingent program during the Jinx encounter phase. In principle, the program could be written in any language whatever, compiled, and up-loaded to the spacecraft well in advance. But for an AI planner the problem is one of representing the disjunction in a way that permits reasoning about a plan, since the plan will form part of a lager scenario with unexpected events and changing requirements. We give a C-plan which does this. It specifies what
each of the individual observation operations are and the conditions under which they are to be carried out.
The plan of Gigure 1 is projected in Ggure 2. The projection describes the behaviors that are possible for the plan. Each arc in the projection is labelled with an integer as used for each event in igure 1. Notice that for this example no execution advice is required. See [11] for an example of how this ordering relation is used.

The plan describes the following behaviors. While it doesn't matter what conditions obtain when the spacecraft arrives at Jinx, assume for the sake of agument that: the spacecraft camera is initialized for infrared work; that the weather on Jinx is clear, and that the frame buffer is empty. A case which describes these conditions is contained in the projection node SI. Two events are possible, as described by the C-plan's e-elements clowding (2) and setup(vis) (4). Clouding (2) denotes the event of the atmosphere becoming clouded by a storm. The setup(vis) (4) e-element denotes the action of configuring the camera to take visible lighe pictures. Similarly, the eelement setup(ir) (3) denotes the action of configuring the camera so tike infrared pictures.

There are two tight cycles in the projection, one between 52 and S3, and one between S4 and S5. These cycles model the normal behavior of the plan during a period when the atmosphere is in a stable state. A transition from $\Omega \Omega$ to $S 3$ models the action of taking a visible light picture, and a transition from $S 3$ to $S 2$ models the action of transferring the picture information from the frame buffer to tape. Lilsewise, a transition from 55 to $S 4$ models the action of taking an infrared picture, and a transition from $S 4$ to $S 5$ models clearing the frame buffer to tape. All other transitions in the projection can be eacily read as setup actions in response to changes in the planet's atmosphere.

## 7. Conclusions.

There is a relationship between the choices of action schema to achieve goals at plan generation time, and contingent plans which support fiexible plan execution. Plan generation is reasoning about goals and the means to achieve them. Plan execution is about actually realizing these promised goals. If fash efficient and texible Al planning systems are to ever exist, they must strike a balance between reasoning about disjunction in advance, and


Figure 1: A contingent plan for taking visible light or infrared pictures.


Figure 2: The projection of the picture-taki. 8 plan.
reasoning about it only when necessitated by plan execution failures. This paper goes some way towards the construction of such a planner by defining a flexible and expressive plan representation which has the ability to represent disjunctive plans. It does this without losing information such as Goal Structure, used by systems like NonLin [6], O-Plan [1] and supe [10].

It is importans to realize that what we have defined is a representation able to describe contingent actions which is useful from $2 n \mathrm{AI}$ perspective. It is not hard to write contingent computer programs. But the eventual goal is to automate spacecraft command generation. It is likely that Al bechniques will be used so perform this task. A startat this has been made with the Deviser planner for the Voyager spacecraft [12]. What this means is that Al representations must be used, and where inadequate, must be improved. Since disjunctive situations will often arise. any planner automatically generating spacecraft commands must be able to reason about disjunction.

We are now working on adapting O-Plan to generate C-plans. Simple disjunctive plans can already be generated; more interesting examples will require more complex generation algorithms. We are currently working on an algorithm to achieve a specified marking in a Petri Net to belp produce a robust and efficient C-plan generation algorithm.

## 8. Acknowledgements.

This work was supported by the U.K. Alvey programme and the Science and Engineering Research Council on Grants GR/D/58987 (An Architecture for Knowiedge Based Planning and Control) and GR/E/O5421 (T-SAT: AI Applied to a Spacecraft). The support of System Designers ple for the work of the AIAI Knowledge-Based Planning Group is gratefully acknowledged.

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# Reasoning and Planning in Dynamic Domains: An Experiment With a Mobile Robot 

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## Abstract



Wederewbe progress made toward having an autonomous mobile robot reason and plan complex tasks in real world environment/ To cope with the dynamic and uncertain nature of the world we use a highly reactive system to which is attributed attitudes of belief, deane, and intention. Because theme attitudes as explicitly represented, they can be manipulated and reasoned about, resulting in complex goal-directed and reflective behaviors. Unlike mont planning byatema, the plans or intentions formed by the system need only be party elaborated before it decides to act. Thin allow the ayatem to avoid overly strong expectations about the environment. overly constrained plant of action, and other forms of over-commitment common to previous planners. In addition, the system is continuously reactive and has the ability to change its goals and intentions as situations warrant. Thus, while thecyntem architecture allows for reasoning about means and end in much the same way as traditional planners, it also pomesea the reartivity required for marvival in complex real-morld domains.

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Whaveteatedne syatemnaing SR's autonomow robot (Flakey) in a scenario involving navigation and the performance of an emergency task in apter station scenario.

## 1 Introduction

One feature that is critical to the survival of all living creatures is their ability to act appropriately in dynamic environments. For lower life forms, it seems that sufficient capability is provided by stimuhisreaponse and feedback mechanisms. Higher life forms, however, require more complex abilities, such as reasoning about the future and forming plans of action to achieve their goals. The dean of reasoning and planning systems that are situated in environments with real time constraints can thus be seen a fundamental to the development of intelligent autonomous machines.
In this paper, we describe a system for reasoning about ant performing complex tack a in dynamic environments and demonstrate how the system can be applied to the control of an autonomous mobile robot. The syttem, called a procedural reasoning system (PRS), is endowed with the paychological attitudes of belief, desire, and intention. At any instant, the actions that the system considers performing depend not only on the current detiget or goals of the system, but also on its beliefs and previously formed intentions. The system also hat the ability to reason about its own internal state - that is, to reflect on its own beliefs, desires, and intentions and to modify these as it chooses. This architecture allows the system to reason about means and ends in much the same way as traditiutial planners, but provides the reactivity required for survival in complex real-world domains.
As the task domain, we envisage a robot in a space station acting as any astronaut's assistant. When asked to get a wrench, for example, the robot work a out where the wrench is kept, plans a route to get it, and goo there. If the wrench is not where expected, the robot may ream further about bow to obtain information on its whereabouts. It then finally return e to the astronaut with the wrench or explains why it could not be retrieved.
In another scenario, the robot may be midway through the tank of retrieving be wrench when it notices a malfunction light for one of the jets in the reactant control system of the space station. It reasons that handling this malfunction jo of higher priority than retrieving the wrench and seth about diagnoming the fault and correcting it. Having done this, it continues with its original teak. finally telling the astronaut what has happened.
To accomplish these tasks, the robot must not only be able to crease and execute plans, but must be willing to interrupt or abandon a plan when circumstances demand it. Moreover, because the world in which the robot is situcted is continuously changing and because other agents and processes can issue demands at arbitrary times. performance of these tanks requires an architecture that is highly reactive at well as goal-directed.
We have used PRS with the new SRI robot. Flakey, to accomplish much of the two scenarios deactihed above, including both :he navigation and malfunctionhandling take. In the next section, we discuss some of the ;problems with traditional planning systems. The architecture and operation of PRS is then described, and Flakey's primitive capabilities are delineated. We then give a more detaked analyan of the problems pooed by this application and our progress to date. We concentrate on the navigation task; the knowledge base used for the jet malfunction handling ia described elsewhere [15,17].

## 2 Previous approaches

Most architectures for intelligent autonomous systems consist of a plan constructor and a plan executor. Typically, the plan constructor plans an entire course of action before commencing execution of the plan ( $1,11,25.28,30,32.33,34$ ). The plan itself is usually composed of primitive actions - that is, actions

[^2]that ase directly performable by the aystem. The motivation for thin approseh, of course, in to ensure that the planned sequence of actions setually achievea the preceribed goal. An the plan is executed, the aystem performa the primitive ections in the plan by calling various low-level routines. Usually, execution is monitored to ensure that thece routines achieve the desired effecta; if they do not, the aystem may return control to the plan constructor to modify the existing plan appropriately. There are, however, a number of nerious drawbacks with this architecture as the bacia for the design of autonomoun agents.
Firat, this kind of planaing is very time coneuming, requiring exponential nearch through potentially enormous problem apaces. It is thus usual for clasical Al planners to spend conaiderable time thinking before performing any effector setions. While this may be acceptable in some situationa, it is not euited to domains where replanning is frequently necemary and where symem viability depende on readinem to act. In real-world domains, unanticipated events are the norm rather than the exception, necesaitating frequent replanning. Furthermore, the real-time conatrainte of the domain often require almont immediate reaction to changed circumstances, allowing inauficient time for this kind of planning.
Second, in real-world domains, much of the information about how beat to achieve a given goal is acquired during plan execution. For expemple, in planning to get from home to the airport, the particular sequence of actions performed depends on information acquired on the way -auch at which turnof to take, which lane to get into, when to slow down and apeed up, and won. Traditional plannern can only cope with this uncertainty in two ways: (1) by building highly conditional plans, most of whove branches will never be used, or (2) by leaving low-leval tacks to be accomplished by fixed primitive operatora that are themelves highly conditional (e.g., the intermediate level actiona (ILAs) uned by SHAKEY [23]). The former case is combinatorially exploaive or simply canot be done - the world around us is simply too dycamic to anticipate all circumatances. The latter, as ually implemented, aerioualy reatricts flexibility and remoning capabilities. Of course, in situations where we can paint ourseives into a corner, some preplanning is necesary. But even this need aot involve expandiag plans down to the level of primitive operalors; indeed, we may do the planaing in quile a different abatraction apeos than that ueed to guide our setions in the real worid (see, for example, the representations in the mimionaries and cannibale problem diseused by Amarel (3)).
A thisd drambech of traditional planning aystema is that they usually provide no mechaniams for reacting to new situations or goals during plan exeeution, let alone during plan formation. For example, many robots (e.g., SHAKEY [23]) effectively shut off their abilitien to react to new situations and goula while moving from one location to another. Only low-level feedback mechaniama and emergeney sensors auch as colliaion detectors remaia eaabled. Such dieregard for senwory isput is particularly undesirable in realiatic eavironmente in which unpredictable evente may occur or other agente may be active because of inaaccurate information about the actual atate of the world, actions may be chosen that are inappropriate to achieving the goals of the aystern. By remaining continuowly aware of the environment, an agent can modify ita actiona and goale an the situation warranta.
Indeed, the very survival of an autonomous syatem may depend on ite ability to react quickly to new situationa and to modify its goals and intentions accordingly. For example, in the scenario described above, the robot muat be capable of deferring the task of fetehing a wrench when it notices something more critical that needs attention (such an a jet failure). The robot thus needs to be able to reason about ite current intentions, changing and modifying these in the light of ite possibly changing beliefa and goala. While many existing planners have replanning capabilitiea, none have accomodated modifications to the system's underlying set of goal priorities.
Finally, current planners are overcommitted to the planning strategy iteelf - no matter what the situation, or how urgent the need for action, these syatems sifays spend as much time as necessary to plan and reason about achieving a given goal before performing any external actions whatsoever. They do not have the ability to decide when to stop planning, nor to reacon about the trade-offa between further planning and longer available execution time. Furthermore, they are committed to one particular planning strategy, and cannot opt for different methods in different situations. This clearly mitigates againat survival in the real world.
In sum, the central problem with traditional planning systems may be viewed an one of overcommitment. These syatens have atrong expectations about the behavior of the environment and make strong assumptions about the future succese of their own actions. They are strongly committed to their goals and intentions and, except in certain simple ways, cannot modify them as circumstances demand. This would be fine if it were possible to build plans that accommodate all the complexities to which an agent must be responsive; unfortunately, in most real-world domains, the construction of such plans is infeasible.
Of course, we are not suggesting that preplanning, followed by later replanning, can be completely avoided: because of unanticipated changea in the environment, an agent will often have to reconaider its goals or its intended means of achieving these. This is a property of the environment that an agent can do little about. If the agent did not make some assumptions about the behavior of the environment, there is little chance it would ever be able to act. On the other hand an agent should not make too many asoumptions about the environment - to the extent poaibie, deciaions should be deferred until they have to be made. The reason for deferring decisions is that an agent can only acquire more information as time passes; thus, the quality of its decisions can only be expected to improve. Of course, there are limitations resulting from the need to coordinate activities in advance and the difficulty of manipulating exceasive amounts of information, but some degree of deferred decision-making ia clearly desirable.
There has been some work on developing planning systern that interleave plan formation and execution [10,21,29]. While these aystems can cope far better with uncertain worlds than traditional planners, they are atill strongly committed to achieving the goals that were initially set them. They have no mechanisms for changing focus, adopting different goals, or reacting to sudden and unexpected changes in their environment. The reactive ayaterns used in robotics aleo handle changes in astuation better than traditional planning ayatema [2,7,18]. Even SHAKEY [23] utilized reactive procedures (ILAs) to realize the primitive actions of the high-level planner (STRIPS), and this idea is pursued further in some recent work by Nilseon [24). However, there is no indication of how these sytems could reason rationally about their future behaviora, auch at to weigh the prom and cons of taking one course of action over snother.

## 3 Knowledge Representation

The system we used for controlling and carrying out the high-level reasoning of the robot is called a Procedural Reasoning System (PRS) [15]. The system consiats of a data base containing current behefs or facts about the world, a set of current gosls or desires to be realized, a set of procederes (which, for historical reasons, are called knowledge areas or KAs) deacribing how certain sequences of actions and tests may be performed to achieve given goals or to react to particular situations, and an interpreter (or inference mechanism) for manipulating these components. At any moment, the system will also have a process stack (containing all currently active KAs) which can be viewed as the system's current intentions for achieving its goals or reacting to some observed situation.
The basic structure of PRS is shown in Figure 1. A brief description of each component and its usage is given below. ${ }^{2}$ Later sections will give examples of PRS use in the the robot scenario.
${ }^{1}$ Fhakey in being wed in a variety of experimente as SRI, and PRS in juat one of various aybtema being employed for contralling Flakey.
${ }^{2}$ A more formal dencription of PRS may be found in [17].


Figure 1: Syatem Structure

### 3.1 The System Data Base

The contents of the PRS data base may be viewed as reprementing the curreat beliefs of the syatem. Some of these beliefa may be provided initially by the syatem user. Typically, theee will include facte about atatic propertien of the application domain - for example, the atructure of some subayatem, or the phyical lawe that come mechanical componente must obey. Other beliefe are derived by PRS itsalf as it executes its KAe. These will typically be current obmervations about the world or conclusions derived by the system from these obeervation. Conevquently, at some times the aystem may believe that it is in a particular hallway, and at other times, in another. Updates to the data baee therefore necesaitate the use of consiatency maintenance techniques.
The data base itself conaiste of a set of atate descriptions deacribing what ia fbelieved to bej true at the curreat inatant of time. We use firm-order prediente calculus for the atate deacription language. Froe variables, represented by aymbols prefixed with $\$$, are asumed to be univeraally quantified. The atatement

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for example, represents states of the worldin which every object on the table is red. Data bave querieu are done using unification over the set of data base predicates.
State descriptions that describe internal system atates are called metalevel expremions. The bacic metalevel predicates and functions are predefined by the ayntem. For example, the metalevel expreasion (goal $g$ ) is true if $g$ is a current goal of the syatem.

### 3.2 Goals

Goals appear both on the syatem goal stack and in the representation of KAa. Unlike moat AI planning systems, PRS goals represent desired behaviors of the syatem, rather than static world statea that are to be (eventually) achieved. Hence goals are expremed as conditions on some interval of time (i.e., some sequence of world states).
Goal behaviors may be deacribed in two waya. One is to apply a temporal predicate to an $n$-luple of terms. Each temporal predicate denoten an ection type or a set of state sequences. That is, an exprestion like "(walk a b)" can be considered to denote the ret of state sequences which embody walking actiona from point a to $b$.
A behavior deacription can also be formed by applying a temporal operator to a state deacription. Three temporal operatora are currently uned. The expremion ( $1 p$ ), where $p$ is some atate description (posibly involving logical connoctives), is true of a sequence of atates if $p$ is true of the late atate in the sequence; that is, it denotea those behaviore that achieve $p$. Thus we might use the behavior deacription (l (vaiked a b)) rather than (raak a b). Similaply, (?p) is true if $p$ is true of the first state in the sequence - that is, it can be conaidered to denote those behaviors that reault from a succesaful teat for $p$. Finally, ( $a_{p}$ ) is true if $p$ is preserved (maintained invariant) throughout the sequence.
Beffavior dencriptions can be combined using the logical operatora $\wedge$ and $V$. Them denote, reapectively, the intersection and union of the composite behaviors.
As with state descriptions, behavior descriptions are not reatrictad to deecribing the externai environment, but can also be used to describe the internal behavior of the system. Such behavior specifications are called metaleval behavior apecifications. One important metalevel behavior is deacribed by 25 expression of the form ( $s$ ) $p$ ). This specifies a behavior that places the atate description $p$ in the aystem data base. Another way of describing this bethavior might be (! (belief p)).

### 3.3 Knowledge Areas

Knowledge about how to accompliah given goals or react to certain situations is represented in PRS by declarative procedure specifications called knowledge areas (KAs). Each KA consiste of a body, deacribing the atepe of the procedure, and an invocation condition that specifies under what aitualions the KA ia uneful.

The body of a KA is represented as a graphical network and can be viewed as a plan or plan chema. However, it differa in a very important way from the plane produced by most Al planners: it doem not comast of poesible cequences of primitive actions, but, zather, of powible sequences of sulyode to be echieved. Thus, the bodies of KAs are much more like the high-level "operators" ueod in planning aysteme areh as NOAB (28] and SIPE [34] They differ in that (1) the subgoals appearing in the body can be described by complex temporal exprealona (l.e., the gool expremions deacribed in the precediag rection) and (2) the allowed control conatructs are much richer, and include conditionale, loopa, and recursion. One important advantage of using abatract anbgoule rather than fixed ealle to actions is that the knowledge expreaped in any given KA is largely independent of other KAs, thereby providing a very high degree of modularity. It is thus pomible to build domain knowledge incrementally, with each component KA having a well-defined and eaily underntood cemantics.
The invocation part of a KA containg an arbitrarily complex lopical expremion describing under what conditiona the KA ia uteful. Uaually, thin consiste of some conditions on current syatem goals (in which eace, the KA is invoked in a goaldirected fachion) or current ayatem beliefs (rexulting in dets-directed or reaetive invocation), and may involve both. Together, the invocation condition and body of a KA exprem a declarative fact about the effects of performing certain sequences of actione under certain conditions.
The aet of KAs in a PRS application aystem consiats not only of procedural knowledge about a specific domain, but also includes metalevel RAs - that is, information about the manipulation of the beliefs. desires, and intentiona of PRS iteelf. For example, a typical metalevel KA would supply a method for choosing between multiple relevant KAs , or how to achieve a conjunction of goals, or how much further planning or reasoning can be undertaken given the real-time conatraints of the problem domain. These metalevel KAs may, of course, utilize domain-apecific knowledge as well. In addition to aser-aupplied KAs, each PRS application contains a set of ayatem-defined default KAs. These are typically domain-independent metalevel KAs.

### 3.4 The System Interpreter

The syatem interpreter runa the entire ayatem. From a conceptual viewpoint, it operaten in a relatively simple way. At any particular time, certain goals are active in the system, and certain beliefin are held in the system data base. Given these extant goals and beliefs, a subset of KAs in the system will be relovant (applicable). One of these KAs will then be chosen for execution.
In the course of traveraing the body of the chomen KA, new subgoals will be posted and new beliefs will be derived. When new goals are peabed onto the soal stack, the interpreter checks to see if any new KAs are relevant, and exeeutes them. Likewise, whenever a new belief is added to the data base, the interpreter will perform appropriate consistency-maintenance procedures and posibly activate new apphcable KAs. During this procese, various metalevel KAa may also be called to make choices between alternative patha of execution, to choose between multiple applicable KAs, to decompose composite goals into achievable components, and to make other decisions.

This reaults in an interleaving of plan selection, formation, and execution. In essence, the system forms a partial overall plan (chooses a KA), figures out near term means (tries to find out how to achieve the first subgoal), executes them, further expands the near-term plan of action, executes further, and wo on. At any time, the plans the system is intending to execute (i.e., the selected KAs) are both partial and hierarchical - that is, while certain general goals have been decided upon, apecific questions about the meana to achieve these ends are left open to future deliberation.
This approach has many advantages. First, aystems generally lack sufficient knowledge to expand a plan of action to the lowest levels of detail -at lead if the plan is expected to operate effectively in a real-world aituation. The world around un is simply too dynamic to anticipate all circumstances. By finding and executing relevant proceduret only when needed and only when sufficient information is available to make wise decisions, the system standa a better chance of achieving its goals under real-time conatraints.

Because the syatem is repeatedly assessing its current set of goala, beliefs, and the applicability of KAs, the aystem also exhibits a very reactive form of behavior, rather than being merely goal-driven. By reactive, we mean more than a capability of modifying current plans in order to accompliah given goals; a reactive system should also be able to completely change its focus and pursue new goals when the stuation warrants it. This is easential for domaina in which emergencies can occur and is an integral component of human practical reasoning
Because PRS expands plana dynamically and incrementally and also allows for new reactive KAs to respond when they are relevant, there are frequent opportunities for it to react to new situations and to change goals. The aystem is therefore able to modify ita intentiona rapidly on the bame of what it currantly perceives as well as upon what it already believes, intends, and desires. It can even change its intentions regarding its own reasomiag procesoes for example, the system may fecide that, given the current situation, it has no time for further reasoning and must act immediately.

### 3.5 Multiple Asynchronous PRSs

In some applications, it is neceasary to monitor and procese many sources of information at the same time. PRS was therefore designed to allow several instantiations of the bseic syatem to run in parallel. Each PRS instantiation has its own data base, goals, and KAs, and operates asynchronowely with ocher PRS instantiations, communicating with them by sending mesages. The mesgagea are written into the data base of the receiving PRS, wivich must then decide what to do with the new information, if anything

Typically, this decision is made by a fact-invoked KA (in the receiving PRS), which reaponds upon reeeipt of the external mesaage. On the basia of such factors as the reliability of the sender, the type of the message, and the beliefa, goale, and current intentiona of the receiver, it is determised what to do about the measage - for example, to acquire a new belief, eatabliah a new goal, or modify intentiona.
We have found the ability to perform multiple activities concurrently to be crucial in the robot domain. Although some syatems do generate pians, portions of which can be executed in parallel (e.g., NOAH [28] and SIPE [34]), our motivations for parallelism are quite different. In our case, the paralleliam is essential for processing the constant stream of sensory information and for controlling devices continuously. That is, parallelism is required for the system's proper operation. In NOAH and SIPE, however, the parallelism is simply fortuitous and does not result from any demands on proceasing speed or distributed functionality.

## 4 Flakey the Robot

Flakey was designed and built within SRI's Artificial Intelligence Center, and is being used by several research teams to test software-organization ideas. It contains two onboard computers, a SUN II workstation (with 12 Mb disk) and a $\mathbf{Z 8 0}$ microprocessor. The $\mathbf{Z 8 0}$ is the low-level controiler, receiving instructions from, and returning current information to, the SUN. The SUN, in turn, can be connected to an ethernet cable, allowing the robot to operate in either stand-alone or remote-control modes. The SUN can also be accessed from a small console on Flakey itself.

The $\mathbf{Z 8 0}$ manages 12 sonars, 16 bumper contacts, and 2 stepper motors for left and right wheels. Voice output and video input are managed by the SUN. A robot arm will be added in the future. The application described here uses only the sonars, voice. and wheels.


Figure 2: Top-level Strategy

The 12 sonars are located approximately 5 inches of the ground, 4 facing forward, 4 backward, and 2 on each side. To obtain a sonar reading, the SUN must issue a request to the $\mathbf{Z 8 0}$ and then wait until the result has been returned. While waiting, the SUN can continue with other processing. At present, the SUN can obtain no more than a few sonar readinga per second.
The motors for the left and right wheels can be controlled independently, again by having the SUN send a request to the Z80. For each wheel one ean specify a desired distance, a maximum forward speed, and a desired acceleration. The $\mathbf{Z 8 0}$ uses the given acceleration to achieve the maximum speed compatible with the desired distance.

Changing direction is done by requesting different speeds for the two wheels. When the robot is stationary, this can be reduced to a simple rotation; when the robot is moving, more complex algorithms are required. Direction changes are much more difficult when they must be negotiated during a forward acceleration.
As well as receiving the desired values of distance, speed, and acceleration from the SUN, the $\mathbf{Z 8 0}$ transmits current actual values to the SUN. This is done using interrupts that occur at a rate of approximately fifty times per second. The $\mathbf{Z 8 0}$ aleo runs a position iutegrator, thus making available the robot's position and orientation relative to particular reference axes. In line with our wish to avoid reliance on dead reckoning, however, we did not use the position integrator for the top-level navigation task; it was used, however, for such low-level tasks as estimating the robot's alignment within a hallway.
There is significant noise in every measurement available to the SUN. The sonars, while generally accurate to about 5 millimeters, can occasioally return invalid readings and can also fail to see an object if the angle of incidence is high enough. Furthermore, Flakey's sonars sense the closest object within a 30 -degree cone, so that open doorways are aot seen until the sonars are well past the doorpoat. Similariy, Flakey will atop within about 5 millimeters of the requested distance and will travel at speeds which fluetuate up to 10 millimeters/second above and below the requeated maximum speed.

## 5 The Domain Knowledge

The scenario described in the introduction includea problema of route planning, navigation to keep on route, and various general tasks such as malfunction handling and requests for information. In this paper, we will concentrate on the route planning and navigation tasks. However, it is importamt to realise that the knowledge representation provided by PRS is used for reaconing about all task that the aystem performs.
The way the robot (that is, Flaxey under the control of PRS) solves the taske of the space atation acenario is roughly as follows. To reach a particular destination, it knows that it must first plan a route and then navigate that route to the desired location (see the KA depicted in Figure 2). In planning the route, the robot uses knowledge of the topology of the station to work out a route to the target location, as is typically done in navigation tarks for autonomous robots $[6,7,22$ ]. The topological knowledge is of a very high-level form, stating which rooms are in which corridors and how corridors are connected. A plan formed by the robot is also high-level, typically having the following kind of form: Travel to the end of the corridor, turn right, then go to the third room on the lef." The robot's knowledge of the topology of the problem domain is stored in its data base, and its knowledge of how to plan a route is represented in various route-planning KAs (see Figures 4. 5, and 6). This is quite different from the approach adopted by traditional Al planners, whirh would find a route by symbolically executing the actual operators speeifying possible movements through rooms and down hallways.
A different set of KAs is used for navigating the route mapped out by the route-planning KAs (see Figures 7, 8, and 9). The navigation Kds perform such tasks as sensing the environment, determining when to turn, adjusting bearinga where necessary, and counting doors and other openings.
Yet other KAs perform the various other tasks required of the robot. Many of these are described by us elsewhere [17]. Metalevel Kis choose between different means to realize any given goal. and determine the priority of tasks when mutually inconsistent goals (such as diagnosing a jer failure and fetching a wrench) arise. If the robot's route plan fails, the route-planning KAs can again take over and replan a route to the target destination. In the implementation described herein, however, we have not provided any KAs for reestablishing location once the robot has left its room of departure, and so it does not currently exhibit any replanning capsbility.

### 5.1 The Planning Space

As stated above, the robot's route planaing is done in a very abatract space containing only tupological information about how the roons ad hallway connect. It is, in fact, the kind of map found in street or buildiag directorica, atripped of precier distances and anglea. Thin is quite antarst ahen one thinks of going home from the office, one considere primarily the copology of the hallways, footpathe, and roede to be followed, not precively hor loag ench in, not the consequences of drifling from side to side - that is too low a level of defail to be considered before setting out along the choven ronte.
The three primary KAs uned to plan paths are ahown in Figures 4, 5, and 6. Given knowledge of the start- and end-points, they first select some intermediate point. They then repeat the process for the two resulting subpathe until all pathe are reduced to straightive trajectories aloag single hallway Althoudh it is not the planner we would advocate for more general route planning, it in quite aufficient for our purpores. Indeed, the top-level route planning is probably the simpleat aspect of the navigation task.
The topological information needed for route planning is stored in the syatem data bace sa a act of facts (beliefis) about how wings, hallwayn and roocrs
 indirectly via yet further connections), (in-wing j1 joiag) (hallway jl is in wing jwing), and (in-hal2 oj225 j1 east 14 ) (room aj225 is in hall $j 1$ on the eant mide of the hall, fourteen rooms from the end). A typical plan constructed by the pach-planning KAs in ahown in Figure 3. Thip plen ras formed to satiafy the goal of reaching a target room (ej270 in wing j2) from the robot's present location (ej233 in wing j1) and was produced in leat than a second. No further predictive planaing is required for the robot to negotiate the path.
(forlow-pech hall ci23s j1)
(follow-path hall j 1 j 4 )
(follow-pech ball $j 4 j 2$ )
(follom-pech hall j2 ej270)

## Figure 3: Route from ej233 to ej270

It is important to emphasise that, even during this relatively abort planning stage, the robot remains continuously reactive. Thua, for example, thould che robor notice indication of a jet failure on the apace atation, it may well decide to interrupt its route planning and attend instead to the tant $\alpha$ remedying the jet problem.

### 5.2 Reactive, Goal-Directed Behavior

The KAs used to anvigate the route fall into three clames: those that interpret the path plan and establish intermediate target locations, thove that are used to follow the path, and those that handle critical taks such a obatacle avoidance and reacting wemergencies. Each KA manifests a m-contained behavior, poesibly including both sensory and effector componente. Moreover, the set of KAs is naturally partitioned according to level of fuacticanlity (df. (7]): low-level functions (emergency reactions, obstacle avoidance, etc.), middle-level functions (following already eatabliahed paths and trajectories), and higher-level functions (figuring out how to execute a topological route). All of these KAs are simultaneously sctive, performing their function whenever they may be applicable. Thus, while trying to follow a path down a hallway, an obetacle avoidance procedure may simultancoualy canse the robot to veer slightly from its original path.
Once a plan ia formed by the route-planning KAs, thal plan most be converted into some useable form. Ideally, the plan shown in Figure 3 should be represented as a procedural KA containing the goale "leave room ej233 and go into hall j1," "go to the jl-j4 junction," etc. Since it is not currely poesible for KAs to create or modify other KAs, we have, instead. defined a group of KAs that react to the presence of a plan (in the data bace) by tranalating it into the appropriate sequence of subgoals. Each leg of the original plan generates subgoals such asturning a corner, travelling the hallway, aad opdatipe the data base to indicate progress. The second group of navigation KAs reacts to these gonle by actually doing the wort of reading the soass, imerpretimg the readings, counting doorways, aligning the robot within the hallway, and watching for obetaclea anead.
For example, consider the KAs in Figures 7 and 8. After having planned out a path as directed by the KA in Figure 2, the robot is given a goal of the forma (! (roon-laft sfroom)) (the variable siroon will be bound to some particular constant representing the room that the robot in trying to kave). The KA in Figure 8 will reapood and actually perform steps for leaviag the given room. The lact atep in this KA will insert a fact into the systern database of the form (origin sfroon sthall) (again, the variables will be bound $\omega$ apecific conatanta). This fact alerta a path interpretation KA (depicted in Figure 7) that the robot is now ready to execute a leg of it path, and sapplies the KA with the robot's starting position (i.e., the room adjacent to the robot, sfroon, and the hall in which it stands. \$5hall). Assuming that the facts deacribing a path have been placed in the database (for example, the met of facts in Figure 3), the fact-invoked FIID-IEXT KA in Figure 7 will rempoed and begin to interpret the path. It will then proceed and travel down the hallway $=$ instructed. This mill in turn establish a new origin position, thereby allowing for the next step of the path to be executed.
A third group of KAs reacts to contingencies obeerved by the robol as it interprets and execules its path. For example. these will include Khs ihat respond to the presence of an obotacle ahead (see Figure 9) or the fact than an emergency light has been seen. These reactive KAs are invoked solety oathe bais of certan facts becoming known to the robot. Implicit in their iavocation, however, is an underlying goal to "avoid obstacles" or "remain safe."
Since a fact-invoked KA can be executed as soon as ita triggering facta are known, the KAs invoked by these contingencies can interrupt whatever elee is happening. Of course, this may not always be desirable. Ideally, domain-specific metalevel KAs should deterinine whether and when preemption is desirable, but, at this stage of the project, we have not used metalevel KAs betides those provided as PRS defaults (which give immediate preemption). An alternative to preemption is to send a contingency mearge to anotber PRS instantiation that can process that measage in paralle.

### 5.3 Parallelism and Mediation

Because of the reat-ime constraints and the need for performing several takk concurrently, it is desirable to use multiple instances of PRS ranning in parallel. In particuiar, paralbelism can be used for handling contiagencies without interrupting other oagoing tasks. Multiple PRS ingtantiation can aboo be used as inforration filters to protect other instantistions from a barrage of unintereating reanory information. (The need for such filters arian in maay problem domaing - for example, in monitoring sensors on the space shutlle (4).) The strongest reasons, bowever, have to do with the inheresaly paraliel and largely independent nature of the various computations that mast be performed in dynamic envirouments.
For example, as the robot rolls down a hallway, it fires ita soane to determine bow far it is frocn the walk, and also to count doors. Suppose it decides that the walls are too close and a change in course in warranted. Becume apeed changen cannot be accomplished instantaneously, changing courre may take as long as two seconds. This is long enough for the robor to roll pana doorway. If the procedure that monitors sonar readinga is interrupted to effect the

## PLAN-PATH



Figure 4: Pach Planning KA


Figure 5: Pach Planning KA


Figure 6: Patin Planning KA

FIND-NEXT



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Figure 7: Plan Interpretation KA


Figure 8: Route Navigation KA

HALL-BLOCKED



Figure 9: Reactive KA
courre change, the robot might completely misa a door reading. Conversely, delaying course changes for the sake of sonar monitoring could make ecolliaion with a wall inevitable. Of course, travelling at lower speede would solve the problem, but would also render the robot too slow to be uecful.
The most effective way to handle this problem is to allow multiple PRS instantiations to execute concurrently. Running reveral inatantiationa aynchronously has its own problema, however. For example, it is desirable to have one PRS instantiation devoted to the tank of keeping the robot in the center of the halway, with another driving the robot to the target location and adjusting speed appropriately (e.g., alowing down when approaching the target). Changes in course are effected by changing the relative velocities of the two wheela, depending on their current velocity, and changea in apeed by changing the accelerations of the wheels. The problem in that, if both taske need to be performed at once, the required wheel operations may interfere with one another. Thia is an intereating example of a situation in which domain-independent decomposition operators will not work - because of the real-time constraints of the problem domain, it is not suitable to achieve one goal (asy, a change in direction) and subnequently achieve the other (change in apeed); neither can each goal be achieved independently, as the means for accompliahing theme goals interact with one another.
To mediate between interacting goals, we chove to implement a third PRS capable of accepting both apeed and direction change requeste mynchronoualy. This PRS could be viewed an a virtual controller. Because the virtual controller is in complete control of the wheels, it can incue instructiona that achieve both kinds of requeste at once. In this reapect, it serves as a special-purpose solution to a particular kind of conjunctive goal; goals to chagge both speed and direction are decomposed into independent goale to change the lef and right wheel apeeda.
Related to the problem of interacting goals is that of goal conflict: just as one may have poavibly conficting beliefs sbout a aituation that need to be reaolved (the problem of situation memesment), one may also have conflicting goals (or deaires) that need mediation [18]. For example, the virtual controller discussed above often gets conflicting apeed requeats from KAs: the hallway traversal KA might requent that a certain velocity be maintained, the KA that detecta approach of the target location may request a decrease in velocity, and the KA that detects obatacles could requeat that the robot atop altogether. At the same time, other KAs might request changes in direction to stay in the center of the hall or to pase around mall obstacles.
To resolve theme conflicting goals, the virtual controller has to be able to reason about their urgency and criticality. Thin, in turn, may involve further communication with the systems requesting these goals. Our present solution is to define domain-dependent mediators where necessary, but, at present, io general approach to this problem has been attempted.

### 5.4 Coping with Reality

Our initial implementation of the robot application used multiple PRS instances interacting with a robot simulator, all running on the Symbolica $\mathbf{3 6 n 0}$ This worked well, and demonatrated the suitability of the system for controlling complex autonomous devices. That done, we began work on driving the real robot. This transfer took considerably longer than estimated. Two major problems caused this divergence between expectations and reality.
First, because PRS was implemented on a Lisp machine, interaction with Flakey was confined to occur via an ethernet cable. Software for remote procedure calls over the ethernet limited communication to 15 function calls per second - too slow for timely response to sensor input. Consequently, we were foreed to transfer much of the functionality of PRS to Flakry': SUN. This required translating the functionality of the lower-level KAs into C code, at well a explicit coding for measage and clock-signal handling. Unfortunately Flakey's operating system also did not supf ort interprocess communication at the bandwidth and efficiency we wanted. This forced us to implement communication through shared memory, with all the concommitant synchroniantion code needed. After these efforts, the information fiowing over the ethernet was at the level of "move $\mathbf{N}$ doora" (PRS to Flakey) and "fim stopping for an obstacle" (Flakey to PRS). Obviously, the translated aystem is no longer solely constructed from instances of PRS. As a reault, our final implementation in considerably more constrained than the simulation version in its ability to resson about ita low-level actions and to react appropriately to changing goals.
The second obstacie to translating from our simulated application to the one that could function in the physical world is the nature of the real world iteell. A realistic environment is simply not controlled enough to foster efficient debugging. It is hard to repeat experimente (and get the aame bugs), time delays become critical, and the behaviors of real sensora and effectors can differ significantly from simulated ones.
The configuration of our current application system is shown is Figure 10. Three machines are involved, a Symbolica 3600, a SUN, and a 280, runniag six application processes. The wheels and sonara are also depicted, and may be regarded as physical processes. The rectangular box represente the SUN; shared memory area; arrows represent interprocess communication.


Figure 10: Processes Used in the Implementation

## 6 Discussion

The primary purpose of this research was to show that thr BDI architecture of PRS, the partial hierarchical planning strategy it supports, and its metalews (refective) eapabilities could be effertive in real-world dynamic domains. Furthermore. the design of PRS aeets some of the more importarit desiderats for autonomous systems: modularity, awarcness, and robustness [18]. In this section, we will briffly compare our appruach to other work in the areas $\boldsymbol{l}$ planning and robotics.

The partial hierarchical planning atrategy and the reflective reaconiag capabilities uned by PRS allow many of the difficulties amociated with traditional planning ayntems to be avoided, without denying the ability to plan ahead when neceseary. By finding and exceuting relevant procedurea only when sufficient information is available to make wise deciaiona, the syatern stande a better chance of achieving its goale under real-time constrainta.
For example, the speed and direction of the robot is determined during plan execution, and depends on auch things as proximity of obatacles and the actual course of the robot. Even the method for determining this course depends dynamically on the situation, auch as whether the robot is between awo haliway walls, adjacent to an open door, at a T-intersection, or passing an unknown obetacle. Similarly, the choice of how to normalize fuel or oxidant tank preaure while handling a jet failure depends on observations made during the diagnootic procem.
Because PRS expande plans dynamically and incrementally, there are aloo frequent opportunitiea to react to new situatione and changing goals. The ayatem is therefore able to modify ita intentions (plans of action) rapidly on the basia of what it currently perceiven as well an upon what it already believes, intenda, and desires. For example, when the aystem notices a jet-fail alarm while it is attempting to fetch a wrench, it has the ability to reamon about the prioritice of these taaks, and, if so decided, auspend the wrench-fetching task while it attends to the jet failure. Indeed, the aystem even continues to moaitor the world while it is route planning (in contrast to most robot aystems), and this activity too can be interrupted if the situation so demands.
The powerful control constructe used in PRS procedure bodies (auch at conditionals, loops, and recursion) are also advantageous. As a reault, the robor can display behaviora of the form "do $X$ until B becomea true." When $X$ is "maintain speed at $400 \mathrm{~mm} / \mathrm{cec}$ " and $B$ ia " $N$ doorways have been obeaved" we see why we could diapense with coordiaate grids and dead reckoning: we could defiwe the robot's behaviora in terms of conditiona that changed over time. In contrast, clasaical planning systems often have difficulty in reasoning about auch behavior and are thus reatricted to using unchanging features such as fixed locations or distances.

PRS is also very robust in that there may be many different KAs available for achieving some given goal. Each may vary in ita ability to accomplish the goal, and in its applicability and appropriatenem in particular aituations. Thus, if there is insufficient information about the current situation co allow one KA to be used, some other - perhaps lese reliable - KA may be available instend. For example, if a topological map of an area is unavailable for planaing purposes, the robot need not be rendered ineffective - there may, for example, be some other KA that seta the robot off in the general direction of the target. Paralleliom and reactivity aloo help in providing robustnese. For example, if one PRS inatantiation is busy planning a route, lower-level inatantiations can remain active, monitoring changes to the environment, keeping the robot in a stable configuration, and avoiding dangers.
The system we propcse alwo meets many of the criteria of rational agency advanced in the philowphical literature on practical reasoning (e.g., see the work of Bratman (5)). Driven by the demards of explaining reourceboundednesa and inter- and iu.cra-agent coordination, recent wort in the philosophy of action has moved beyond belief-desire architectures for rational agents and has provided insights into the nature of plans and intentiona, and especially the nature of intention formation.
In particuiar, plans are viewed as being subject to two kinds of constrainta: consostency constraints and requirements of means-ends coherence. That is, an agent's plans need to be both internally consiatent and consistent with ite beliefs and goals. It should be poasible for an agent's plane to be anceemfully executed (that is, to achieve the more important nuala of the aystem) in a world in which ita beliefs are true. Secondly, plana, though partial, meed to be filled in to a certain extent as time goea by, with sub-plans concerning means, preliminary ateps, and relatively apecific courses of action. These subplans must be at leat as extensive as the agent believes is required to succesfully execute the plan; otherwise they will suffer for means-enda incoherence.

These constraints on the beliefs, desires (goals), and intentions of an agent are realized by the aystem propoed herein, and as auch it can be viewed at an implementation of a rational agent. In addition, the notion of intention we use meets the major requirements put forward by Bratman (3), who considers intentions to have the following properties:

- They lead to action,
- They are parts of larger plans,
- They involve commitment.
- They constrain the adoption of other intentions.
- They are adopted relative to the beliefs, goals, and other intentions of the system.

Of course. out system is far from manifesting the behavioral complexity of real rational agents; however, it is a step in the direction of a better understanding of eational action.
In contrast to most Al planaing work, research in robotics has been very concerned with reactivity and feedback [2,18.23]. However, most of this work has not been concerned with general problem solving and commonsense reaconing - the work is almoat exclusively devoted to problems of navigation and execution of low-level actions. Furthermore, the reactivity is not of the general form we advocated above; although the syaterns can adjuat the measa for achieving given goals depending on ineoming sensory information, they do not exhibit the ability to completely change goal priorities, to modify, defer, or abandon current plans, or to reason about what is beat to do in light of the current situation.
Recently, Brooks [7] has advanced some intriguing ideas concerning the structure of autonomous systerms. Rather than the horizontal structure typical of most robot systems (where lower levels are restricted to performing basic sensory and effector processing, and the higher levela to planning and reasoning) Brooks advocates a vertical decomposition in which distinct beheviors of the robot are separately realized, each making use of the robot's sensory, effector, and reasoning capabilities as needed
Similarly, PRS provides a vertical, rather than horizontal, decomposition of the robot trak domain. Each KA defines a particular behavior of the systern, and can involve both processing of sensory information and the execution of effector actions. For example. there is a KA that manifeats a betavior to reman clear of obstacles, another lid whose behavior corresponds to keeping a straight course in a corridor. and jet another that chooser and traverseas routes from one room to another. All these KAs use both sensors and effectors to greater or lesser degrees - there is no single subsystem that preprocesees the sensory data before sending it to the reasoning system, and there is no post-procesoing of plan information that determines actual effector actions
In this sense, our system is very similar in structure to that propowed by Brooks - indeed, it can claim the same positive benefita [ $s$ ]

- There are many parallel paths of rontrol through the system [many different procedures can be used in a given stuation] Thus the performance of the system in a given situation is not dependent on the performance of the weakest link in that situation. Rather, it is dependent on the strongeat relevant behavior for the stuation
- Often more than one behavior is appropriate in a given situation. The fact that the behaviors are [can bej generated by parallel systems [multuple PRS mantances) provides redundary and robusturss to the overall system. There is no central bottleneck (through which all the processing or reasoning must occurl.

 oppodup; and (3) a acharally exteavible aycoms.

The main difference betweep our gyotem asd that advanced by Brooke in that we employ a much more general mochaniem for melecting betmeea appropriate behavions than he does: wherem Brooks use intibitory and axcitalory links to iategrate the aet of bebaviors defined by each of the aypern's fuactioal components, we use generla metaleval proceduren and commanieation protocole to perform the selection and integration. Of courm, mech generality will likely proclude meeting come of the reat time conetrainte of the enviromment, in which eace the metalevel proceduree might need to be compiled into a form closer to that enviaged by Brooks. Similarly, while our aytem aturally mape onto cotrre-grate parallel machinee, oophisticated compilation techniques would be required to map the lower-level function onto bighly parallel archilecturea.
Curseatly, PRS does not remon about other mubaytem (i.c., other PRS instantiationa) in any but the aimplear ways. However, the meamage-paeciag mechaniams we have amployed ahould allow un to integrate more complex resconing about interprocems communication, auch at described by Cohen aad Lovesque [9]. Remoniag about procem interference and aynchronisation is aloo important where concurrency in involved. The mecheaiman developed by us for receoning sbout these problems [12,13,14,19,20,31] could aloo potentially be integrated within PRS. Our future reaearch plans iaclucle both wort on communication and synchronisation within the PRS framework.
Finally, in giving a deseription of the PRS architecture, it is important to note that the actual implementation of PRS is not of primary concern. That is, while we believe that attributing beliefo. devires, and intentions to sutonomous aystame can aid in apecifying complex behaviors of these aysterna, and can ariat in communicating and interacting with them, we are not demanding that such aydema actually be atructured with diatinct dela atructures chat explicitly represent these paycbotogical attitudes (athough, indeed, that is the way we have chowen to implement our aystem). We can instend view the specification of the PRS syatem, cosether with the various metalevel and object-level KAb, aimply as a description of the deaired bethevior of the robor. Thia deacription, auitably formalised,' could be realised in (or compiled into) any suitable implementation we choose. In particular, the beliefs, desires, and intentions of the robot may no longer be explicitly reprecented within the syatem. Some inleresting work on this problem is being carried out currently by Rosenechein and Kaelbling (26].

## 7 Limitations

The primary thruat of this work has been to evaluate an architecture for autonomous aystems that provides a meana of achieving goal-directed, yet reactive. behaviof. We have made enough progrem to ahow that this approech morks. However, the research is only in its initial alages and there are on numer of limitations that atill need to be addremed.
First, there is a cles of facta our current ayutem must be told; for inatance, the robot's starting location. If the robot is initialised in sorse unknown pomition on the topological map, the planner will abort. It would be atraightforward to solve these problema by including KAs that ack for the anming information, but a completely autonomous recovery would be a much more chalienging problem. Poasible approsches might involve exploration of the terrain (including movernent around the neighboring area) and pattern matching onto known topological landmarks.
Second, there are many amumptions behind the procedurea (KAs) used. For example, we have amumed that hallways are straight and cornese rectangular; that all haliways are the same width and have that width for their entire length (except for doorways and intersecting halls); that there is only one layer of obriaclea in front of any wall (nowhere is there a garbage can in front of a cupboard); that all doors are open and unobatructed; and that other objects move much slower than the robot.
We have also made anumptiona that limit the robot's reactivity. For example, we amume that the robot does, in fact, arrive at the jubction it planned to reach. If the robot mimcounta doorways. it will ator in the wrong place, turn, and atart the next leg of ite journey without realising ite minake. The malt is generally that the robot will be found begging a wall to "plence make way." If the robot realized it was in the wrong poaition, it could replan to achieve ita goala. However, because we aseume that the door count is always right, the route plannes is never reinvoked.
In addition, some goals are not made as expiticit as one would like, but are implicit in the KAs ued by the robot. For example, the robot in deaigned to move an fast an pomible without miscounting doorwaye and to travel along the center of the hallway while accepting the fact that thia ideal will parely be achieved. Such goals are not represented explicitly within KAs. Handling the firat kind of goal ("move as fat an poasible") would be relalively atraightforward, requiring aimply that axiorne relating robot apeed and perceptive capabilitiea be provided to the ayatem. However, it in not sbviove how to explicitly represent the second kind of goal, in which the aystem attempta to maintain a particular condition but expecta at beat only to approximate it.
Finally, increased parallelism would have been preferable, allowing tbe robot to perform more take concurrently. For example, we could bave included many more law-level proceduree for, any, avoiding dangera and exploring the aurroundings. This would have provided a much more severe hat of the aymern's capability to coordinate varioua plana of action, to modify intentions appropriately, and to change ita focus of attention.

## 8 Acknowledgments

Joshua Singer and Mabry Tyson dehugged marly PRS code and extended the implementation as needed. Stan Reifel and Sandy Wella deagned Flakey and ite interfaces. and ansisted with the implementation deacribed hetrin. We have also benefited from our participation and interactions with members of CSLIIs Rational Agency (ifoup (RaTA(i), particulasly Michael Bratman. Phil Cohen, Kurt Konolige. David Israel, and Martha Pollack Lealie Pack-Kaelbling and Stan Homenachein alos provided helpful advice and intereating commenta.

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# Route Planning in a Four-Dimensional Environment 

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Despite the variety of the techniques mentioned above, all of the systems discussed share some basic limitations. None of the systems takes into account the quality of the surface upon which the robot travels, relying on the surface being either traversable or not. Such a restricted view is cominually contradicted by the way people move about. Pcople stray off the sidewalk or jay-walk across a street whenever it is convenient and safe, hence a realistic robot should be able to behave similarly. A further limitation is that all the aforementioned systems are designed to operate under static conditions, where the only aspect of the world that changes is the position of the robol. This is an unrealistic and unacceptable limitation for almost all applications.

In addition to being able to function in a dynamic world, a robot should be able to reason about dynamic processes and how they may alfect it. For example, if a robot knows the local train schedule and needs to get to the other side of the train Iracks, then it should use that information when planning to get to the other side. If the robor has information predicting that a long freight Irain will be coming just before it can reach the tracks, then given the choice between a short path that involves crossing the train tracks, and a slightly longer plan to so under the tracks, the robot should choose the latier plan. Similarly, if the robot's task is to rob a Iraitt, then tise ability to plot a path that will allow the rohol to jump onto the moving train is necessary.

Unpredictable dynamic processes must also be laken into account during route planning. A cavalry robot that "fears" an allack by a tribe of Indian robuts would be belter off planaing to get in the fort across the opell plain, rather than passing through the narrow passageway of Ambush Canyon. The primary reason for this is an allack in the canyon would more effeclively block the rubot from its destination. thereby mandating backiracking.

The single property that most distinguishes this work from previous systens is that it moilels not only space, but time as well. Rather than making a calculation aboul whether the robot can traverse a particular area independent of time, this system models the ability of the robot 10 traverse that area at different times. We lave accounted for icmporal as well as spatial changes in the enviromment. The message passing lechnique used allows time to be considered while allowing the system designer to mucici quaiitizs of the domain, such as the cost of moving from one position to another and the ease of traversing a particular area of space. The remainder of this paper describes a representation and route planning sysiem for use in unpredictable dynansic domains.

## 2 - The Algorithin

This section will describe an algorithm that finds the best path through predictable n-dimer:sional space using user-defined evaluation functions. The algorithm provides for an effective representation of space-time and the ability to functionally define predictabie static and dynamic objects that map into a subset of a given space-time.

## 2.1 - Spatial Representalion

A path-finding algorithm (hat is of any use must provide for: 1) an effective representation of space, 2) the ictationship between the elements makiug up the space, and 3) the ability to define the quality of that space with respect to a robot using the generated plans.

To ellectivly represent space, this model uses uniformly shaped, n-dimensional hyper-cubes called "nodes". Each node represents small chunk of space. Arbitrarily shaped n-dimensiona! areas can be defined through the spatial concatenation of nodes along common ( $n-1$ dimensional) surlaces. The collective area occupied by the nodes is called "space", while the remainder of cxistence is referred to as "void". For example. Figure I shows an arbitrary iwo-dimensional space conslnicted from the spatial concatenation of square shaped nodes. In gencral, the size of a node will be of at least sullicient size to subsume the size of the robot.


Figure 1

The relationship hetween adjoining nodes (such as the ability to move from one note to another) is represented by unidirectional links between each of the nodes (sce Figure 2b). The reflexive relationship a node has with itself is represented as a link that points back to the node and represents the ability $t 0$ remain at that node. Associated with each of the links is a cost. The cost represents such things as: whether the surface between the two nodes is continuous, a dowingrade, ill three-dimensions directly below, or if a node represents a safc place 10 stop. The function of the links is to provide a communication path over which messages can be sent. A node can have up to 2 n communication links with its neighboring nodes and one reflexive communication link. Nodes lliat lie on the edge between space and the void will have fewer communicalion connections.

The lopological features of the space are represented by the cost on links between indes, whereas the actual traversability of a particular region of space is specificd by lic node's traversability constant. The traversability constant represents the relative ability of the robot to move over given node. For example, for a robot will wheels, a concrete floor would have a traversability constant near 1.0 while that of quicksand might be on the order of 0.001 .

Consider the spatial representation of the iwo-dimensional space shown in Figure 2 . liart a of the Figure shows the physical layout of the example space. latt b shows how the nodes that define the space are interconnected with one another. The reason for the missing links between some of the noves is that the cost associated with the link is infinite (ilie cost on other links is not indicated). As a result, no patis are generated that make those transitions. While lhe sutiface is three-dimensional, only a iwo-dimensional represcutation is necessary (for this parlicular example) because the costs on the links between nodes allows for the representation of hills, absorbing the three-dimensional aspect of the space. For example, a link going down hill could have a lower cost than one going up hill. If the features described above are taken together, a robust represemation of llie salicut fealures of space can be created.


Figure 2

## 2.2 - Representing predictable objects

For a route planning system to be of value, not only does it have to represent the salient featurcs of space, bu: it must also provide an cffective means for representing predictable objects. Predictable ohjects are those objects, both stalic and dynamic, that have fully predictable behavior it both lime and space. An object is represented as a function having the defined space and time in its domain, and some subset of the nodes that make up space in its range. The set of nodes generated on any application of the function consists of those nodes in the given space that are occupied (fully or partially) by the ohject during the given time. For example. consider a model of a simple two ditucnsional revolving doer. This can be represented by defining a function that has four nodes forming a square in its range. Then, by setting the function to map onto two of the nodes that are diagomal during odd time units and onto the other two nodes during even times, a simple, predictable, revolving door catl be cicated. Such functions can be encoded into each of the nodes at selup time, thus refucing the amoum of outside communication dusing operation.

## 2.3 - An SIMD Algorilhm for Route Planning

The representation of the spatial features and predictable objects thus far described provides a basia for an algorithm that can be directily implemented on an SIMD machine, such as that in [llillis85]. This is accomplished by assigning each node to a processor. Each processor has message communication links to other processors that transcribe directly from the node links. A message represents the value of possible transition from one node to another and the quality of the entire path leading up to that transition. To perform the task of reclaiming. the generated paths from the processors, cach processor must maintain a stack. The stack represents a storage place lor logging the listory of the activity at the processor. For now, the simplifying assumption will be made that cach of the moves made by the robot being simulated will take one unit of time. For example, the time required for the robot to move ftom one node 10 an adjacent node, takes one unit of time regardless of the robots previous state. The removal of this assumption will be discussed later (see additional fealures section 2.5).

Using a synchronous, step-wise process of passing messages from processor to processor, all possible patis that the robot could take through lime and space in altaining the desired, static destination location can be considered. The process has two phases and a terminating condition.

The first plase sets up the intitial message set. Using the node in space that represents the current location of lic robor, a set of messages is created, one message for each communication link associated with the node. Liach message has assuciated with it an encrgy value that reflects the cost of moving the robot to the space represented by the adjoining node. The particular value of a message's energy is detemined by a user-defined evaluation function. The evaluation function considers such things as: the cuirent state of the robot, the cost on the lithk that the message is to be seltt over, the liaversability of the node currently being occupied by the robot, etc. The set of messages is then sent to its iespective destimations along the communications links of the node.

The second phase is the operational phase. It is defined by having each node in space that is not occupied by an object during the current simulation time perform the following process in a synchronous manner: form the base message by selling it to the incoming message with the minimum energy. All other messages can be throwil out because they represent more costly paths that altain the same location in space-time. In a manner similar to that described in the first phase, a new set of messages is created. Each message in the new set is assigned an energy that is a function of the base message and the link over which the message is intended to travel. The base message is then tagged with the time and a pointer indicating the node that created it. The base message is then added to the node's stack. Finally, the node sends the newly ireated list of messages out along their respective communication link. This process is repeated, until the lermination condition is mel, each repetition represcming a subsequent time unit.

The termitating condition is defined as the state of the system when the energy associated with each of the messages currently being processed in the system is greater than the global bound. The global bound is the minimum energy for all the messages that have reached the destination node (similar to zorch decay in (Charniak86)).

After the ending condition is met, the path through space-time that has the lowest energy associated with it can be retrieved from the destination node. This is done by locating the message on the destination node's stack with the lowest energy value. Once this is done, the path can be obtained by following the pointers back though space (other processors) and time (the stacks associated with the processors) until the robots original location in space-time is encountered. Tite stack allows interprocessor communications to be kept to a minimum.

## 2.4 - A Prediclable Example

This sectinn shows how predictable dynamic objects are represented and antripated. Shown helow in ligure 3 is a two-dimensional space made of ramps that are to be navigated by the rohot. A dynamic object emters the world at time $t=8$ and leaves the world after $t=15$, indicated by the blackened square in the second of the two Figutes. White the object is present, the node it occupies cannot be traversed by the robot. The nodes over which the robot must travel are, for the most part. casily traversable and thus have a high traversability constant of 0.9 . Two of the nodes, however, are made of loose sand and have the low traversability constant of 0.2 .


Figure 3
The routc planning objective is, starting at $t=0$, to move the robot from its current location (indicated by the $R$ ) to the static destination location (indicated by the $\mathbf{D}$ ) along the space-time path of minimumin energy without going over a cliff. By counting the number of node transitions that must be made in traveling from $\mathbf{R}$ to I), one can determine that the shortest path the robot could take would be 13 moves long and would use 13 lime units. Dut, when the 8 th move is being made, the dynamic object enters the space and disables the indicated node, keeping it from receiving or sending messages. The key is that after seven time units, the message passing process has not propagated to the node with the dynamic object in it. This causes all message activity to be confined to the back portion of the defined space until after $t=15$. The blockage is due to the fact that all paths from $R$ to D must pass through the node that is occupied by the object. After $t=15$ units, the occupied node is freed and resumes processing and passing messages, eventually allowing the system to mect its terminating condition.

A number of points can be made here that are based on the choice of the evaluation function used to determine the message energies. One is that a proper evaluation function will allow the best path to avoid the nodes that are made of sand by giving a high energy value to any message that represents a transition into such a node. A more important point is that the evaluation function determincs how and where the robot spends ths time while it waits for the object to leave. for exannple, the sotkit could remain at $R$, charging its batteries, wander along the path, or hurry to the object and wait there until the dynamic object goes away. So, depending on the requirements and the siluation (c.g., maximize charge time), an evaluation function can be written to detemine how and where the robot waits (c.g., add a cost for stopping and starting, or time spent not chaiging).

## 2.5 - Additional Features

Some of the system's most powerful features have been omilted thus far for clarity. Among these reatures ate: The ability to describe the destination as a function of both space and time. the ability to consider the opemess of a node with respect to its spatial location, and the ability to accurately cofnsider the movement capabilities of the robot using the generated plans.

The ability to describe the destination as a function of both time and space allows the system to solve problems involving alternative planning (e.g. if you can't get to the post oflice by five, gn io the diug store for the stamps) and problems involving coordinating actions with dynamic objects (e.g. jumping on a moving train). This ability is incorpotated into the system, by modifying the termination condition of the algoritlon to consider a time ordered set of possible destimation nodes.

The ability to consider the openness of a node as a spatial relation between it and the nodes surrounding it can be used to generate paths that avoid narrow passages, if possibie. Generned plans avoid moving the robot though paths that would become blocked if an unpredictable object were to be enconntered during the execution of the plan.

The following gives an example of how an openness function might be defined iteratively for a two-dimensional space. First, assign each node in space a value of one if it is not occupied and a value of zero if it is occupied. Second, have each node send its value along all of its communication links. Third, cach node creates a temporary value by summing the values of all incoming information and then integer-dividing them by 1 the first itctation and $2,4,8,16$, ... in each subscquent iteration. Lastly, if the temporary value is greater than zero, a new node value is set by multiptying the original node value by the temporaty value. Using this scheme, the openness value associated with
the nodes will eventually converge to a stable node value pattern. The pattern will be such that the nodes that are in the biggest, most spacious areas will have the highest values, and the nodes in corners or alcoves will have the lowest values. The addition of openness to nodes allows evaluation functions in be written that considers the trade-offs between short path lengit and increased chance of backtracking due to the chosen route becoming blocked by an unpredictable object.

The ability to effectively represent the time required by a robot to make simulated moves allows plans to be generated that take full advantage of the robot's abilities. For exampic, moving from rest to another node should lake longer than moving from one node to another when the robot is already moving in the desired direction. This is significanily different from the scheme used up to this point, where all moves were considered to take one unit of time. The ability to consider the capabilities of the robot in the generated paths has been incorporated into the model by selting the model to operate in a more asynchronous manner. This asynchrony is accomplished by associating a real-lime with each message. The time value of created messages is set by adding to the time in the incoming message the amount of time that is required for the robot to make the move represented by each of the new messages. The ability to effectively predict the performance of the robot is bounded by the precision with which the real-time actions of the robot moving through space-time can be modeled.

## 3 - Dynamic vs. Unpredictable

Thus far, olly the generation of plans that involve predictable objects has been considered. To move autonomous robots in the real world, a route planning system nust be able to handle the unpredictability that the real world has to offer as in the case of a robot that must walk across a busy street. The process of incremental route planning has been identified to handle this problem.

Incremental route plaming can be viewed as the repeated use of a route planner that executes ill a predictable dynamic environmem. After each step, the state of the world is tested and updated with ally new information. for identification of any unpredictable objects. Incremental route planning is effectively handled by this system because it is structured to operate most efficiently in the incremental form. Unpredictability is handled by the system's ability to rapidly calculate the next best step after every puimitive move the robot makes.

By making a simple modification, an incremental version of the algorithm has been constructed frotil the framework of the previously defined algorithm for predictable domains. The stack is eliminated from cack of the processors by making an addition to the messages being passed around the system. The modification involves the addition of an initial direction header. This change is made because all that is needed is the next best move and not the entire path. The headers, of the messages created in phase olle of the algorithm, are set to a value representing the link along which that particular message is to be sent. The header, of the messages created during phase two, is copied from the header of the incoming nessage. To identify the next best move, the terminating condition is modified to keep track of the message representing the current global message energy bound. When the systen halts, the header of the global message energy bound indicates the direction of the next best move. This represents a significant simplification of the system, as it eliminates concerns involving the potentially unbounded growth of the node stacks.

## 4 - Implementation

The algorithm, when fully implemented on an SIMD machine, operates in $O$ ( $p$ ) time, where $p$ is the length of the longest path through space-time that is bounded by the global message energy bound.

A simulation version of the algorithm, written in NISP [McDermoti83], is currently up and rutning on a VAX $11-785$. It functions on the examples given plus others involving unpredictable dynamic environments. The implementation includes software for simulating the SIMD architecture.

## 5 - Further Research

There are several possible extensions to this model that would incresse its representational power. Among the most useful are:

- The granularity of the nodes: It becomes difficult to ensure that there is an adequate path for the robot to traverse, when the gramularity of the nodes used to represent space is smaller than the size of the robot or the exact size of the nodes is undetermined.
- Uncertainty in dynamic times: Special concern should be given to route planning where objects entering the space are predictable in their behavior but have uncertain arrival times. For example, the subway train that is running a few minutes behind schedule.
- Achieving maximal efficiency over a set of destinations: This is similar to the traveling salesman problem.
- Modeling unpredictable processes: The power of an incremental route planner can be increased for a particular domain with some model of the typical behavior of the unpredictable objects in that domain. For example, the route planner could give more useful advice to a robot crossing a street if the system had a model of the speed, maneuverability, and direction of travel for the autos traveling the road.
- Representation and coordination of multiple robols moving through space-time: For example, getting lluey, Duey and Luey to meet in the garden on the east end of the space ship at 3pm.

This list represents some of the extensions to our mode! that are currently under investigation. Exte:sions into more abstract domains, such as general problem solving using state transition graphs, are also under consideration.

## 6 - Summary and Conclusions

Planning robot movement in dynamic environments demands that the dynamic aspects of the environment be modeled in at least as much detail as the movements of the robot. We have created a representation system that allows dynamic aspects of the environment and performance aspects of the robet to be easily modeled. It also integrates this model with a high-performance route-planning algorithm. This system has been extended into an incremental route planner which can be used for real-time tactical planning in unpredictable domains. This system has been implemented in an SIMD simulator running on a VAX.

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# Prediction and Causal Reasoning in Planning 

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Nonlinear planners (erg., Nemenfied) are often touted as having an efficiency advantage over linear planners ( 0.8 , STRIPS) The reason usually given is that nonlinear planners, unlike their linear connterparts, are not forced to make arbitrary commitments to the order in which actions are to be performed. This ability to delay commitment enables nonlinear planners to solve certain problems with far less effort than would be required of linear planners. fm-this-paper_menegue that this advantage is bought with a significant reduction in the ability of a nonlinear planner to accurately predict the consequences of actions. Unfortunately, the general problem of predicting the consequences of a partially ordered set of actions is intractable. In gaining the predictive power of linear planners, nonlinear planners sacrifice their efficiency advantage. There are, however, other advantages to nonlinear planning (egg., the ability to reason about partial orders and incomplete information) that make it well worth the effort needed to extend nonlinear methods. In_this-paper,wesupply a framework for causal inference that supports reasoning about partially ordered events and actions whose effects depend upon the context in which they are executed. As an alternative to a complete but potentially exponential-time algorithm, we provide a provably sound polynomial-time algorithm for predicting the consequences of partially ordered events. If the events turn out to be totally ordered, then the algorithm is complete as well as sound.
Keywords: causal reasoning, planning, prediction, nonmonotonic inference, data base management.

## 1 Introduction

In this paper, we are concerned with the process of incrementally constructing nonlinear plans (ie., plans represented as sets of actions whose order is only partially specified). Nonlinear planning [2] has long been considered to have distinct advantages over linear planning systems such as STRIPS [8] and its descendents. One supposed advantage [10] has to do with the idea that, by delaying commitment to the order in which independent actions are to be performed, a planner can avoid unnecessary backtracking. Linear planners are often forced to make arbitrary commitments regarding the order in which actions are to be carried out. Such arbitrary orderings often fail to lead to a solution and have to be reversed. By ordering only actions known to interact with one another (i.e., actions whose outcomes depend upon the order in which they are executed) the expectation was that nonlinear planners would avoid a lot of unnecessary work.

The problem in getting delayed-commitment planning to work is that it is often difficult to determine if two actions actually are independent. In order to determine whether or not two actions are independent, it is necessary to determine what the effects of those actions are. Unfortunately, in order to determine the

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effects of an action $a$ it is necessary to determine what is true prior to a being executed, and this in turn requires that we know the effects of those actions that precede $a$. In general there is no way to determine whether or not two actions are independent without actually considering all of the possible total orderings involving those two actions.

Planning depends upon the ability to predict the consequences of acting. Past planning systems capable of reasoning about partial orders (i.e., nonlinear planners) have either employed weak (and often unsound) methods for performing predictive inference or they have sought to delay prediction until the conditions immediately preceding an action are known with certainty. Delaying predictive inference can serve to avoid inconsistency, but it can also result in extensive backtracking in those very situations that nonlinear planners were designed to handle efficiently.

It our contention that the initial success of Sacerdoti's NOAH [10] program and the promise of NOAH's style of least-commitment planning has caused researchers to ignore important issues in reasoning with incomplete information. The idea of least-commitment planning is not the only reason for building plannery capable of reasoning about partially ordered events. Most events will not be under a planner's control and more often than not it will impossible to determine the order of all events with absolute certainty. Planning systems for realistic applications will have to reason about partially ordered events.

In this paper, we consider the problem of reasoning about the effects of partially ordered actions. A theory for reasoning about the effects of actions (or, more generally, the consequences of events) is referred to as a causal theory. We will describe a language for constructing causal theories that is capable of representing indirect effects and the effects of actions that depend upon the situation in which they are applied. We will describe a series of algorithms for reasoning about such causal theories. All of these algorithms are polynomial-time, incremental, and insensitive to the order in which facts are added to or deleted from the data base. We show that a particular algorithm is complete for causal theories in which the events are totally ordered, but is potentially inconsistent in cases where the events are not totally ordered. In [6] we show that the general problem of reasoning about conditional actions is $N P$-complete, and, in this paper, we provide a partial decision procedure that, while not complete, is provably sound. What this means for a planner constructing a plan is that the procedure is guaranteed not to misslead the planner into committing to a plan that is provably impossible given what is currently known. If the decision procedure answers yes, then the conditions are guaranteed to hold in every totally ordered extension of the current partial order; if the decision procedure answers no, there is a chance that the conditions hold in every total order, but to determine this with certainty might require an exponential amount of time or space.

## 2 Temporal Data Base Management

In this section, we consider a particular type of inference system, referred to as a temporal data base management system (or TDBMS) [4], that is used to keep track of what is known about the order, duration, and time of occurrence of a set of events and their consequences. The user of a TDBMS is allowed to add two sorts of information: that which is unconditionally believed and that which is believed just in case certain conditions can be shown to hold. The former includes information concerning events that have
been observed or are assumed inevitable and information in the form of general rules that are believed to govern the physics of a particular domain.

In specifying conditional beliefs, the user explicitly states what the conditions are, and the TDBMS ensures that those beliefs (and their consequences) are present in the data base just in case the conditions are met. This is achieved through the use of data dependencies [7]. In a TDBMS, the primary forms of data dependency (in addition to those common in atatic situations) are concerned with some fact being true at a point in time or throughout an interval. In addition, there is a nonmonotonic form of temporal data dependency concerned with it being consiatent to believe that a fact is not true at a point in time or during any part of an interval. These forms of temporal data dependency are handled in the TDBMS uaing the mechanism of temporal reason maintenance [4]. Language constructs are supplied in the TDBMS that allow an application program to query the data base in order to establish certain antecedent conditions (including temporal conditions) and then, on the basis of these conditions, to assert consequent predictions. These predictions remain valid just in case the antecedent conditions continue to hold.

Perhaps one of the most important and most overlooked characteristics of a temporal reasoning aystem is the ability to handle incomplete information of the sort one invariably encounters in realistic applications. For example, we seldom know the exact duration or time of occurrence of most events. Moreover, for those durations and offsets we do know, they are seldom with respect to a global frame of reference such as a clock or calendar. In the TDBMS, every point is a frame of reference, and it is possible to constrain the distance between any two points simply by specifying bounds on the distance in time separating the two points. By allowing bounds to be both numeric and symbolic, the same framework supports both qualitative (i.e. ordering) and quantitative (distance) relationships.

Another important aspect of reasoning with incomplete information has to do with the default character of temporal inference. In general, it is difficult to predict in advance how long a fact made true will persist. It would be convenient to leave it up to the system to decide how long facts persist based upon the simple default rule [9] that a fact made true continues to be so until something serves to make it false. This is exactly what the TDBMS does. The term persistence is used to refer to an interval corresponding to a particular (type of) fact becoming true and remaining so for some length of time. A fact is determined to be true throughout an interval $I$ just in case there is a persistence that begins before the beginning of I and it can't be shown that the persistence ends before the end of $I$.

The TDBMS permits the specification of partial orders, but it imposes orderings in situations leading to potential incoherency. If the TDBMS encounters two persistence intervals of contradictory types that are not ordered with respect to one another, it prompts the calling program to resolve the possible contradiction by either imposing an order or explicitly introducing a disjunction. By introducing a disjunction, the calling program effectively splits the data base, producing two time lines. The answers returned by queries to the TDBMS indicate the disjuncts that must be true for a query to succeed (i.e., the particular time line that satisfies the query). There are also language constructs that allow a calling program to eliminate disjuncts (and hence time lines) that have been ruled out. Unfortunately, as we will see in Section 5, eliminating explicit contradictions is not sufficient to ensure consistency in a system capable of making conditional predictions from a set of partially ordered events [ 1 ;. Before we continue our discussion it will help to introduce some notation.

Relations. Let II be the set of points corresponding to the begin and end of ovents in a particular temporal data base. We define a function Dist to denote the beat known bounde on the distance in time separating two points. Given $\pi_{1}, \pi_{2} \in \Pi$ such that $\operatorname{DIST}\left(\pi_{1}, \pi_{2}\right)=\langle$ low, high $)$, we have:

```
- \(\pi_{1}<\pi_{3} \Leftrightarrow\) low \(\geq \epsilon^{1}\)
- \(\pi_{1}<_{M} \pi_{2} \Leftrightarrow\) high \(\geq 6\)
```

- $\pi_{1} \equiv \pi_{2} \Leftrightarrow\langle$ low, high $\rangle=\langle 0,0\rangle \quad-\pi_{1}$ is coincident with $\pi_{2}$
- $\pi_{1} \preceq \pi_{2} \Leftrightarrow\left(\pi_{1}<\pi_{2}\right) \vee\left(\pi_{1} \equiv \pi_{2}\right) \quad-\pi_{1}$ precedes or is coincident with $\pi_{2}$
- $\pi_{1} \underline{\Omega}_{M} \pi_{2} \Leftrightarrow$ high $\geq 0 \quad-\pi_{1}$ possibly precedes or is coincident with $\pi_{2}$

Tokens. We denote a set of time tokens $T=\left\{t_{0}, t_{1}, \ldots t_{n}\right\}$ for referring to intervals of time during which certain events occur or certain facts are known to become true and remain so for some period of time. The latter correspond to what we have been calling persistences. For a given token $t$ :

- $\operatorname{BEgin}(t), \operatorname{ENd}(t) \in \Pi$.
- $\operatorname{STATUS}(t) \in\{$ IN,OUT $\}$, determined by whether the token is warranted (IN) or not (OUT) by the current premises and causal theory.
- $\operatorname{TYPE}(t)=P$ where $P$ is an atomic predicate calculus formula with no variables
- $\operatorname{duration}(t)=\operatorname{dist}(\operatorname{begin}(t), \operatorname{End}(t))$

Types. As defined above, the type of an individual token is an atomic formula with no variables (e.g., (on block14 table42)). In general, any atomic formula, including those containing variables, can be used to specify a type. In describing the user interface, universally quantified variables are notated ?variable-name, the scope of the variable being the entire formula in which it is contained (e.g., (on ? x ? y )). In describing the behavior of the inference system, we will use variables of the form $t_{P}$ to quantify over tokens of type $P$ (e.g., $\forall t_{P} \in \operatorname{TYPE}\left(t_{P}\right)=P$ ).

## 3 Reasoning about Causality

In the TDBMS, a causal theory is simply a collection of rules, called projection rulea, that are used to specify the behavior of processes. In the following rule, $P_{1} \ldots P_{n}, Q_{1} \ldots Q_{m}, E$, and $R$ designate types, and delay and duration designate constraints (e.g., ( $\epsilon, \infty\rangle$ ). In:
(project (and $P_{1} \ldots P_{n}$
( $\mathrm{M}\left(\right.$ not $\left(\right.$ and $\left.Q_{1} \ldots Q_{m}\right)$ ) $)$
$E$ delay $R$ duration)
$P_{1} \ldots P_{n}$ and $Q_{1} \ldots Q_{m}$ are referred to as antecedent conditions, $E$ is the type of the triggering event, and $R$ refers to the type of the consequent prediction. The above projection rule states that, if an event

[^3]R1: (project (and $P_{\text {L }} \ldots P_{n}$ (M)(not (and $\left.\left.Q_{1} \ldots Q_{m}\right)\right)$ )
$\boldsymbol{E} \boldsymbol{R})$

R2: (project (and $\left.P_{1} \ldots P_{n}\right)$ $\boldsymbol{E} \boldsymbol{R}$ )

R3: (diasble (and $Q_{1} \ldots Q_{m}$ )
(ab R2))
R4: (disable (and $\boldsymbol{R}_{1} \ldots \boldsymbol{R}_{0}$ )
(ab R3))

Figure 1: Hierarchically arranged projection and disabling rules
of type $E$ occurs corresponding to the token $t_{E}$ and $P_{1} \ldots P_{n}$ are believed to be true at the outeet ${ }^{2}$ of $t_{E}$ and it is consistent to believe that the conjunction of $Q_{1} \ldots Q_{m}$ is not true at the outset of $t_{E}$, then, after an interval of time following the end of $t_{E}$ determined by delay, $R$ will become true and remain so for a pericd of time constrained by duration (if delay and duration are not apecified, they default to $\langle 0,0\rangle$ and $(\epsilon, \infty)$, respectively). In the following, we will be considering a restricted form of causal theory, called a type 1 theory, such that the delay always specifies a positive offset (causes always precede their effects).

We also allow the user to specify rules that serve to disable other rules [11]. Figure 1 shows a standard projection rule R1 and a pair of projection and disabling rules R2 and R3 that replace R1. The rule R3 is further conditioned by the rule R4. Assuming just the rules R2, R3, and R4, any application of R2 with respect to a particular token $t$ of type $E$ is said to be abnormal with regard to $t$ just in case $Q_{1} \ldots Q_{m}$ hold at the outset of $t$ and it is consistent to believe that $R 3$ is not abnormal with regard to $t$. The nonmonotonic behavior of type 1 causal theories is specified entirely in terms of disabling rules and the default rule of persistence (see Section 2). In addition to their usefulness for handling various forms of incomplete information, disabling rules make it possible to reason about the consequences of simultaneous actions. The reader interested in a more detailed treatment of causal theories may refer to one of [4], [5], or [11]. In parts of this paper, we will ignore disabling rules and speak of causal theories consisting solely of simple projection rules of the form (project (and $P_{1} \ldots P_{n}$ ) $E R$ ).

One of the most problematic aspects of designing a temporal inference system involves defining precisely what it means for a fact to be true at a point or throughout an interval (i.e., the conditions under which a query of the form $T T\left(P, \pi_{1}, \pi_{2}\right)$ will succeed). As a first approximation, we offer the following definition, which we will refer to as weak true throughout:

[^4]```
Vtg E T
    ((sTATUS(tg) = IN) ^
    (3\mp@subsup{t}{\mp@subsup{P}{1}{}}{\prime,.t\mp@subsup{t}{\mp@subsup{P}{0}{}}{}}\in\mathbb{T}
        ~1\leqi\leqn(sTATvs(tm) = IM) ^
```



```
                    (BEam(ts)\MEND(tmi)) )) ^
~(#t ma ...totam ET
        ~1\leqj\leqm(status(tom)=IN)^
                    (BEGIN(t\mp@subsup{O}{l}{})\BEGDN(tB)) ^
                    (BEGIN(ts)\MEEND(to,)) )))
# 3tr \inT
                (status(t/R)=IN) ^
                (DIST(END (ts), BEGIN(trg)) \subseteq delay)^
                (DIst(begin}(\mp@subsup{t}{R}{}),\operatorname{END}(\mp@subsup{t}{R}{\prime}))\subseteqduration
```

Figure 2: Weak projection

```
\(\forall x_{1} \pi_{2} \in \Pi\)
    \(\exists t_{p} \in T\)
        (STATUS \(\left.\left(t_{p}\right)=I N\right) \wedge\)
        \(\left(\operatorname{Begin}\left(t_{P}\right) \preceq \pi_{1}\right) \wedge\)
        ( \(\pi_{2}\) 亿M END \(\left(t_{P}\right)\) )
    \(\Leftrightarrow \operatorname{TT}\left(P, \pi_{1}, \pi_{2}\right)\)
```

In order to apecify the behavior of a temporal inference system such as the TDBMS, we also need to define the criterion for inferring consequent effects from antecedent conditions via causal rules. Our firat such criterion will be referred to as weak projection (Figure 2) and is defined with respect to the general form of a projection rule (1). Weak true throughout and weak projection correspond to the assumption that "what you don't know won't hurt you." Only those evtnta that can be shown to be ordered with respect to a particular point will have any effect at that point. As we will see in Section 5 , there are some problems with this formulation.

## 4 Transactions on the Data Base: the User Interface

Every inference system provides some means for the user to specify rules (referred to collectively as a causal theory) for inferring additional consequences of the data (referred to here as a set of basic facts). An application program interacts with an inference system by adding and removing items from the set of basic facts, which in the TDBMS corresponds to a set of tokens and a set of constraints. The state of a temporal data base is completely defined by a temporal constraint graph (TCG), consisting of the begin and end points of tokens and constraints between them, and a causal dependency graph (CDG), consisting of dependency structures corresponding to the application of causal rules in deriving new tokens. Each transaction performed on the temporal data base results in changes to these two data structures. The TDBMS is responsible for maintaining the temporal data base so that it captures exactly those consequences that follow from the causal theory and the current set of basic facts.

Generally, the causel theory remaine fixed for a particular applieation, and interaction with the TDBMs consists of a series of transactions and queries. A transection consiats of aither adding or removing some token or conatraint from the set of basic facte. A query consiste of a predicate calculus formula corresponding to a question of the form "Could come fact $P$ be true over a particular interval I?" An affirmative answer returned by the TDBMS in reaponce to such a query will include a eot of asumptions necessary for concluding that the fact is indeed true. Any aseertions made on the bacis of the answer to such a query are made to depend upon these assumptions. There is also a mechaniam that enables the TDBMs to detect and, with the asaistance of the calling program, resolve inconsistencies in the eet of conatrainte.

## 5 Completeness and Consistency

The primary source of ambiguity in the TDBMS arises from the fact that the eet of constraints eeldom determines a total ordering of the tokens in $T$. Given that moat inferences depend only upon what in true during intervals defined by points corresponding to the begin and ond of tokens in $T$, all that we are really interested in is what facts are true in what intervals in the different total orderings of time points consistent with the :nitial set of constrainta. For each total ordering we can identify a unique set of tokens that intuitively should be IN given a particular causal theory.

As far as we are concerned, an inference procedure is fully specified by a criterion for inforring consequent effects from antecedent causc via causal rulea (e.g. weak projection), a method for actually applying that criterion (an update algorithm), and a criterion for determining if a fact is true throughout some interval (e.g., weak true throughout). Wo will say that a particular inference procedure is complete for a clase of causal theories, if for any set of basic facts and causal theory in the clasa, the atatemente of the form TT( $P, \pi_{1}, \pi_{2}$ ) warranted by the inference procedure are exactly those that are true in all total orders consistent with the constraints in the TCG. Similarly, we will say that an inference procedure is eound for a class of causal theories, if for any set of basic facts and causal theory in the clase, each statement $T T\left(P, \pi_{1}, \pi_{2}\right)$ warranted by the inference procedure is true for any total order consistent with the eet of constraints. Given the preceding definitions, it is easy to show that the TDBMS is complete and sound for type 1 causal theories, assuming that the tokens in $T$ are totally ordered [6].

In situations where the set of basic facts does not determine a total order, it's easy to show that the TDBMS can end up in a state with IN tokens that allow one to conclude statements of the form $\operatorname{TT}\left(P, \pi_{1}, \pi_{2}\right)$ that are not true in any totally ordered extension. One thing we might do to improve the chances of the TDBMS warranting only valid statements of the form $\operatorname{TT}\left(P, \pi_{1}, \pi_{2}\right)$ in strengthen the criterion for belief in a given token. We can determine a class of tokens that are said to be atrongly protected, using the axioms in Figure 3. In these axioms and the rest of the paper, we use $T_{B}$ to denote the tokens in the set of basic facts. If the set of constraints determines a total ordering, then the set of strongiy protected tokens is identical to the set of tokens that are IN, but generally the former is a subset of the latter. Using this notion of strongly protected, we can define a stronger true throughout criterion that we will refer to as strong true throughout:

```
vi E TB
    STRONGLY-PROTECTIND(t)
Vig E T
    (STRONGLY-PROTECTED(ts) ^
        (#p,...tp ET
        N1\leqi\leqn STRONGLY-PROTDCTED(tp.) ^
```



```
                            (azam(ts) {m Emp(tpd)) ^
                    Nt-p, \inT
                            (STATUS(G-P若 = OUT) v
```



```
                            (END(ts)< &ROM(G-p,l))\))
# *r, 隹
    STRONGLY-PROTECTED(tR) ^
        (DIET(END(tg), BEOM(tg)) ¢ delay) ^
        (DIST(BECIN(tR),\operatorname{END}(\mp@subsup{t}{R}{\prime}))\subseteqduration)
```

Figure 3：Strong protection defined for simple projection rules

```
Vig e T
    ((status(ts) \(=\) IN) ^
        \(\left(\exists t_{p_{1}} \ldots t_{p_{n}} \in T\right.\)
        \(\left(\forall 1 \leq i \leq n\left(\operatorname{sTATUS}\left(t_{P_{1}}\right)=I M\right) \wedge\right.\)
                        \(\left(\operatorname{Becin}\left(t_{p_{i}}\right) \Omega_{M} \operatorname{BEGIN}\left(t_{S}\right)\right) \wedge\)
```




```
                            \(\neg S T R O N G L Y-P R O T E C T E D\left(t_{-} p_{i}\right) \vee\)
                            \(\left(\operatorname{BEGin}\left(t_{-} p_{1}\right)<_{M} \operatorname{BEGin}\left(t_{p_{f}}\right)\right) V\)
                            \(\left.\left(\operatorname{Begin}\left(t_{E}\right)<_{\mu} \operatorname{Begin}\left(t_{\rightarrow} p_{t}\right)\right)\right)\) ))
    \(\Rightarrow \exists t_{R} \in T\)
                            (sTATUS \(\left.\left(t_{R}\right)=I N\right) \wedge\)
                        \(\left(\operatorname{DIsT}\left(\operatorname{END}\left(t_{R}\right), \operatorname{BEGM}\left(t_{R}\right)\right) \subseteq\right.\) delay) \(\wedge\)
                        (DIST(BEGIN( \(\left.\left.t_{R}\right), \operatorname{END}\left(t_{R}\right)\right) \subseteq\) duration)
```

Figure 4：Improbably weak projection defined for simple projection rules

```
\forall\mp@subsup{\pi}{1}{}\mp@subsup{\pi}{2}{}\in|
    \existstp}\in
        STRONGLY-PROTECTED(tP)^
        (BEGIN(tP) \preceq }\mp@subsup{\pi}{1}{\prime})
        ( }\mp@subsup{\pi}{2}{}\mp@subsup{\⿴囗⿱一一心}{M}{END}(\mp@subsup{t}{P}{\prime})
    |TT(P, \mp@subsup{\pi}{1}{},\mp@subsup{\pi}{2}{})
```

As it turns out，weak projection and atrong true throughout atill do not constitute a sound inference procedure．We can show that，even when we restrict ourselves to strongly protected tokens，most interesting decision problems are intractable．In fact，we can prove that the problem of determining if $T T\left(P, \pi_{1}, \pi_{2}\right)$ is
true for a type 1 causal theory, with or without disabling rules, is NP-complete [6]. Although completeness is computationally infeasible, it is powable to devise an inference procedure that is both sound and capable of performing useful prediction. Firat, we provide e criterion for generating consequent predictions that takes into account every consequence that might be true in any total order, called improbably weak projection (Figure 4). Second, we provide a criterion for tres theo:shout thet zuecosis only if the correoponding formula will be true in all total ders consistent with the current set of constrainte. We define improbably strong true throughout:

```
\(\forall \pi_{1} \pi_{2} \in \Pi\)
    \(\exists t_{p} \in T\)
        STRONGLY-PROTECTED( \(\left.t_{P}\right) \wedge\)
        ( \(\left.\operatorname{BEGIN}\left(t_{P}\right) \preceq \pi_{1}\right) \wedge\)
        \(\left(\forall t_{-p} \in T\right.\)
            (status \(\left(t_{\sim} p\right)=\) OUT) \(V\)
            (BEGIN \(\left.\left(t_{-P}\right)<\operatorname{BEGIN}\left(t_{p}\right)\right) \vee\)
            \(\left.\left(\pi_{2} \prec \operatorname{BEGIN}\left(t_{\sim} P\right)\right)\right)\)
    \(\Leftrightarrow T T\left(P, \pi_{1}, \pi_{2}\right)\)
```

There is a simple decision procedure for generating all consequences and computing the eet of atrongly protected tokens. Let $T_{1}=T_{B}$, and initially assume that no tokens are strongly protected. Let $i=1$. To compute the consequences of $T_{i}$, set $T_{i+1}=T_{i}$, compute the consequent tokens of each token in $T_{i}$ using the criterion of improbably weak projection, and add any new tokens to $T_{i+1}$. Continue to compute new consequent tokens in this manner incrementing $i$ as needed until $T_{i}=T_{i+1}$. Set $T=T_{i}$. At this point, perform a sweep forward in time (relative to the current partial order) determining for each token in $T$ whether or not it is strongly protected and the status, IN or OUT, of each its consequents. In [6], we prove that this decision procedure is sound for a partially ordered set of tokens, and sound and complete for a totally ordered set.

In the same paper, we give two incremental update algorithms with polynomial-time worst case behavior, one for weak projection and weak true throughout, and one for improbably weak projection and improbably strong true throughout. The latter algorithm does not model the decision procedure given above-there is a more complicated procedure with the same behavior that is more efficient by a constant factor. We prove that these algorithms support exactly the conclusions licensed by their respective inference methods. Proving that the algorithms terminate is in one sense impossible. Using the TDBMS and a type 1 causal theory with arithmetic functions (e.g., (project (contents ?register ?n) (increment ?register) (contents ?register ( +1 ? $n$ )) ), we can easily simulate a Turing machine. There are a number of methods for either restricting what the user can encode in a causal theory or limiting the scope of the inferences computed by the TDBMS in such a way that we can guarantee that the update terminates. By limiting the scope of the inferences computed by the TDBMS, we potentially sacrifice completeness, but we have shown that to ensure completeness may require an exponential amount for time for other reasons.

## 6 Conclusions

This paper is concerned with computational approaches to reasoning about time and causality, particularly in domains invclving partial orders and incomplete information. We have described a class of causal theories
involving a carefully restrictod uee of nonmonotonic inference, capable of reprecenting conditional effecte and the effecta of simultaneoum actions. We showed in [6] that the decision problem for nontrivial inference systems involving conditional effecte and partially ordered events is NP-complete. As an alternative to e complete but potentially exponential-time inference procedure, we have described a decision procedure, for which there is an incremental polynomial-time algorithm, that gonerates a useful aubeot of those inferencea that will be true in all total orders consistent with the initially specified partial order. The deciaion procedure is provably sound and the resulting conclusions are guaranteed consistent if the underlying causal theory is consistent. If the events turn out to be totally ordered, the procedure is complete an well as sound.

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# The Generic Task Toolset: High Level Languages for the Construction of Planning and Problem Solving Systems 

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We begin hy/eriticising the current generation of languages for the construction of knowledg-hased systems as being at too low a level of abstraction andmarione the need for higher level languages for building problem solve. ing systems. We-iatraduce-our Potion of generic information processing tasks in knowledge-based problem solving, and-deweribe s toolset which can be used to build rxpet systems in way that enhances intelligibility and productivity in knowledge acquisition and system construelion. Whee illustrate the power of these ideas by paying special attention to a high level language called DSPL. and namibe how it was used in the construction of a mysterm called MPA, which assists with planning in the domain of offensive counter air missions.
$\rightarrow$ Scope and Intent $\quad$ indrodect.
 Scope and Intent
This paper is intended to be an introduction to the generic tasks approach to analyzing knowledge systems. descriptions of some of the particular generic tasks that have been dentified, and a description of the software tools that hive been build as a result. The approach is illustrated by discussing a particular mission-planning symterm, MPA, which was built using the DSPL language (or tool). The intent of the paper is primarily tutorial, much of the material is summary or repetition of material already presented elsewhere.1. 2

Level of Abstraction Problem in Characterizing Knowledge-Baced Systems

Much of Al can be said to be a search for the "Hols Grail" of a level of abstraction at which intelligence behavior qua intelligent behavior emerges. Above this level. presumably, are particular pieces of knowledge of the task domain, and below this level are specific details of how the intelligent level $\$$ implemented. For example, in a rule-based system such as Myrin. everyone would agree

[^5]that rules of medicine are not per se matters of Al research. That is, the content of the knowledge base itself is made up of domain particulars. At the same time the language in which the rule system is written, egg.. Lisp, is typically thought of as an implementation-level detail, of no particular interest as Al.
The Al interest of a given expert system is often a mat. ter of the level at which it is viewed. Clearly, taking Mycin as an example, all the following points of view are correct: (i) it suggests therapy for certain kinds of inferthous diseases, (ii) it is an embodiment of a diagnostic and therapeutic strategy, and (iii) it is a derision-maker which uses backward chaining to navigate a knowledge base of \} u l e s , ~ c o n n e c t i n g ~ p i e c e s ~ o f ~ e v i d e n c e ~ t o ~ c o n c l u s i o n s , ~ i n ~ o r d e r ~ oo arrive at a reliable conclusion from a given set of data. point of view (i) is of limited interest to AI, and point of flew (iii) has been the level at which the system as an Al system has been generally presented; this is the level at which the claim for generality of the underlying approach is made. All the excitement surrounding the "knowledge base/inference engine" paradigm, and the idea that knowledge acquisition and explanation can be keyed to phenomena at the rule level, emphasizes that the level corresponding (iii) has been the level of abstraction at which Al interest has been expressed, and claims of progress have been made.

What of the level corresponding to (ii)? We have for several years at Ohio State worked on the hypothesis that this is indeed an important level of abstraction for Al: and that knowledge representation, control regimes for problems solving, knowledge acquisition, and explanation all can be significantly advanced by looking at phenomena at this level. B. Chandrasekaran has put forth this view in ${ }^{1,3,4}$. In this paper we give a critical analysis of an important part of Al, viz., knowledge-based reasoning, and propose that a point of view based on a particular level of abstraction corresponding to generic information processing tasks has a number of advantages both for clarity of analysis and for systeth design. This is not pure theoretical speculation; researchers in our laboratory have built many systems and tools based on this framework. We wish to point to this level of abstraction as a productive level for concentration, and to indicate the conceptual advantages of it. Knowledge acquisition, systerm design, control of problem solving and generation of explanation all are facilitated at the same time, indicating that there is a naturalness to looking at phenomena at this level.

Intuitively one thinks that there must be some commonality of reasoning processes that characterize "diagnosis" as a generic activity, even acroas domains as different as medicine and mechanical aystems. There should be control strategies and ways of using knowledge that are common to diagnostic reasoning as such, or at least typical of diagnoatic reasoning. Similarly there should be common types of knowledge structures and control strategies for, say, design as a kind of reasoning artivity. Further, we expect that the structures and control regimes for diagnostic reasoning will be generally different from those for design reasoning. However, when one looks at the formalisms (or equivalently the languages or shells). that are cotmmonly used in expert system design, the knowledge representation and control regimes do not typically capture these distinctions. For example, in diagnontic reasoning, one might generically wish to speak in terms of malfunction hirfarchies, rule-out strategies, setting up a differential. etc., while for design, the generic terms might be device/component hierarchies, design plans, ordering of design subtasks, etc. Ideally one would like to represent diagnostic knowledge in a domain by using the vocabulary that is appropriate for the task. But typirally the languages in which the expert systems have been implemented have sought uniformity acrons lasks, and thus have lost clarity of representation at the task level. The computational universality of representation languages such as Emycin or OP'SS .- i.e.. the fact that any computer program can in principle the written in these languages -often ronfuses, the issue, vince after the system is finally built it is often unclear which portions of the systern represent domain expertise, and which are programming devices. In addition, the control regimes that these languages come with (such as forward or backward chaining) do not explicitly indicate the real control structure of the system at the task level. For example, the fact that iki 5 performs a linear sequence of subtasks .- an atypically simple stratigy for design probletn solving .- is not explicitly mented; the system desigumer son to speak "encrypted" this control in the pattern-matehing control of Op:

These comments need not be restrieted to the rule-based framework. One could represent knowledge as sentences in a togical calculus and use logiral inference merhanisms to solve problems: or one could represent it in a fratue hierarehy with procedural attachneents in the slots. (It is a relatively straightforward thing, e.g. to rewrite MICOX ${ }^{10}$ in this manner. wes.) In the former. the control isules would deal with choier of predirates and clansery, and in the latter. they will be at the level of whish links to pine sue for inheritance, "te. Vone of these have atm matural connertion with the control issues oprific to the task.

The other side of the coill. io to spaka, regarding control is the foltowing: bectuse of the relatively fow level of abstraction relative to the information processing task. there are control issues that are artifacts of the representation, but often in our opinion misinterproted as issues at the "knowledge-level." F.g.. rule-based approarhes ofton concern themselves with conlier remolution itrategies. If the knowledge were viewed at the level of abstration appropriate to the task, often there will he organizational elements which would result in only a small set of highly relevant pieces of knowledge or rules to being brought up for consideration, without any conflict remolution strategies, being needed. Of course, these organizational constructs
could be "programmed" in the rule language, but because of the status assigned to the rules and their control as knowledge-level phenomena (as opposed to the implementation level phenomena, which they often are), knowledge acquisition is often directed towards (typically syntactic) strategies for conflict resolution, wherean the really operational expert knowledge is at the organizational level.

This level problem with control structures is mirrored in the relative poverty of knowledge-level primitives for representation. For example the epistemology of rule systems is exhausted by data patterns (antecedents or subgoala) and partial decisions (consequents or gosh), that of logic is similarly exhausted by predicates, functions, inference rules, and related primitives. If one wishes to talk about types of goals or predicates in such a way that control behavior can be indexed over this topology, such a behavior can often be programmed into these systerns, but no explicit rendering of them is possible. E.g., Clanrey * found in his work using Mycin to teach students that for explanation he needed to attach to each rule in the Mycin knowledge base encodings of types of goals so that explanation of its behavior can be couched in terms of this encoding, rather than only in terms of "Kerause .. . was a subgnal of -..." This is not to argue that rule representations and backward or forward chaining rontrols are not "natural" for some situations. If all that a problem solver has for knowledge in a domain is in the form of a large collection of unorganized associative patterns. then data-directed or goal-direrted associations may be the best that the agent can do. But that is precisely the occasion for weak methods such as hypothesize and match (of which the above associations are variants), and. typically, successful solutions cannot be expected in complex problens without combinatorial srarches, Gencrally. however, expertise can be experted in ronsists of much more organized collections of knowledge. with rontrol behavior indexed by the kinds of organizations and forms of knowledge in them.

Thus, there is a need for understanding the generic information processing tasks that underlie knowledge-based reasoning. Knowledge ought to be directly encoded at the appropriate level by using primitives that naturally describe the domain knowledge for a given generir task. Problem solving behavior for the task ought to be controlled by regimes that are appropriate for the task. If done correctly, this would simultaneousiy farilitate knowledge representation, problem solving, and explanation.

At this point it will be useful to make further distinctions. Typically many tasks that we intuitively think of as generic tasks are really complex generic tasks. I. e.. they are further decomposable into components which are more elementary in the sense that each of them has a more homogeneous control regime and knowledge structures. For example, what one thinks of as the diagnostic task, while it may be generic in the sense that the task may be quite similar across domains, it is not a unitary task structure. Diagnosis may involve classificatory reasoning at a certain point, reasoning from one daturn to another datum at another point, and abductive assembly of multiple diagnostic hypotheses at another point. Hierarchical classification has a different form of knowledge and control behavior from those for data-to-data reasoning. which in turn is dissimilar in these dimensions from assembling hypotheses. Our research focuses on tasks at
both these levels, but the latter are viewed as somewhat "atomic" tasks into which more complex, but still generic, tasks such as diagnosis and design can often be decomposed.

To summarize the view presented so far: There is a need for understanding the generic information processing tasks that underlie knowledge-based reasoning. Knowledge ought to be directly encoded at the appropriate level by using primitives that naturally describe the domain knowledge for a given generic task. Problem solving behavior for the task ought to be controlled by regimes that are appropriate for the task. If done correctly, this would simultaneously facilitate knowledge representation, problem solving, and explanation.

Over the years, we have identified, and built systems using, six such generic tasks. Our work on MDX'. ${ }^{10}$. e.g., identified hierarchical classification. knowledge-directed information passing. and hypothesis matching as three generic tasks, and showed how certain illasses of diagnostic problems can be implemented as an integration of these generic tasks. Since then we have identified several others: object syuthesis by plan selection and refinement ${ }^{\text {" }}$, state abstraction ${ }^{4}$, and abductive assembly of hypotheses ${ }^{\prime 2}$. There is no claim that these six are exhaustive; in fact, our ongoing research objective is to identify other useful generic tasks and understand their knowledge representations and strategies for control of problem solving.

## Some Generic Tasks

## Characterization of Generic Tasks

Each generic task is characterized by: a task specification in the form of generic types of input and output information; specific forms of knowledge needed for the task. and specific organization of knowledge particular to the task; a family of control regimes that are appropriate for the task.

A task-specific control regime comes with certain characteristic types of strategic goals. These goal types will play a role in providing explanations of its problem solving behavior.

When a complex task is decomposed into a set of generic tasks, it will in general be necessary to provide for communication between the different structures sperializing in these diferent types of problem solving. Also there is not necessarily a unique decomposition. Depending upon the availability of particular pieces of knowledge, different architectures of generic tasks will typically be possible for a given complex task.

We will now give brief characterizations of the generic tasks that we have identified.

## Taxonomic ©lassification

Task specification: Classify a (possibly romplex) description of a situation as an elernent, as specific as possible. in a classification hierarchy. E.g. classify a medical rase description as an element of a disease hierarchy.

Forms of knowledge: one main form is <partial situation description> --> evidence, belief about confirmation or disconfirmation of classificatory hypotheses. E.g., in medicine, a piece of classificatory knowledse may be: certain pattern in X-ray \& bilirubin in blood $\rightarrow$ high evidence for cholestasis.

Organization of knowledge: knowledge of the form above distributed among concepts in a classificatory concept hierarchy. Each conceptual "specialist" ideally contains knowledge that helps it determine whether it (the concept it stands for) can be establiched or rejected.

Control Regime: Problem solving is top down, each concept when called upon tries to establish itself. If it succeeds, it lists the reasons for its success, and calls its successors, which repeat the process. If a specialist fails in its attempt to establish itself, it rejects itself, and all its successors are also automatically rejected. This control strategy can be called Establish-Refine, and results in a specific classification of the case. (The account is a simplified one. The reader is referred to ${ }^{10}$ for details and elaborations.)
Goal types: E.g., Establish .<concept>, Refine (subclassify) <concept>

Example Use: Medical diagnosis can often be viewed as a classification problem. In planning, it is often useful to classify a situation as of a certain type, which then might suggest an appropriate plan.

## Object Synthesis by Plan Selection and Refinement

Task Specification: Design an object satisfying specifications (object in an abstract sense: they can be plans, programs, etc.).
Forms of knowledge: Object strac'ure is known at some level of abstraction, and pre-compiled plans are available which can make choices of components, and have lists of concepts to call upon for refining the design at that level of abstraction.

Organization of Knowledge: Concepts corresponding to components organized in a hierarchy mirroring the object structure. Each concept has plans which can be used to make commitments for various "dimensions" of the component.

Control Regime: Top down in general. The following is done recursively until a complete design is worked out: A specialist corresponding to a component of the object is called, the specialist chooses a plan based on the sperifications and problem state, instantiates and executes the plan which suggests further sperialists to call to set details of the subromponents. Plan failures are passed up unti. appropriate changes are made by higher level specialists.

Goal Types: E.g., Choose plan, execute plan element refine <plan>, redesign (modify) partial design - o respond to failure of esubplan... select alternative plan. etc.

Example: Expert design tasks, routine synthesis of plans of action.

We will characterize the other generic tasks more succinctly. The reader is referred to ${ }^{1}$ for more details.

## Knowledge-Directed Information Passing

Tank: It is desired to obtain attributes of some datum. by deriving from some conceptually related datum. Some forms of knowledge are: <attribute> of <datum> is inherited from <attribute> $\psi$ pof parent of <datum>, <attribute> of <datum> is related as <relation> to <attribute> of <concept>. Organization: concepts are organized as a frame hierarchy, with IS-A and PART-OF links. Each frame is a specialist in knowledge-directed data inference for the concept. This is basically a hierarchical information-passing control regime.
Example usen: knowledge-based data retrieval tasks in wide variety of situations, as an intelligent data base in support of problem solvers of other types.

## Abductive Assembly of Explanatory Hypotheses

Task Specification: Given a situation (described by a set of data items) to be explained by the best explanatory account, an: oiven a number of h). theses, each associated with a degree of belief, and each of which offers to explain a portion of the data (possibly overlapping with data to be accounted for by other hypotheses), construct the best composite explanatory hypothesis. Some forms of knowledge are: causal or other relations (e.g. special case of, incompatibility, suggestiveness) between the hypotheses, relative significance of data items, and ways to group data items to be explained. Organization: one main, or a hierarchical rommunity of active abducers, each sperializing in explaining a certain portion of the data by composing and criticizing hypotheses. Control Regimp: (Sise $13^{3}$ for a fuller discussion.) A specialized means-ends regime is in control, driven by the goals of explaining all the significant lindings, with an economical hypothesis. which is consistent, and has been criticized for certain strengths and weaknesses. Some goal types are: accountfor <datum-: rheck-superfluousness-of - hypothesis>. choose the most significant unexplained finding. The Internist system ${ }^{14}$ and the Dendral system ${ }^{15}$ perform abdurtive assembly as part of their problem solving.

## State Abstraction

Task Specification: Given a change in some state of a system, provide an account of the changes that can be expected in the functioning of the system. (Useful for reasoning about consequences of changes on complex systems.) One knowledge form is • change in state of subsystem> ...-. change in functionality of subsystem $=$ change in state of the immediately larger system -. The knowledge is organized into conceptual specialists corresponding to systems and subsystems connected in a way mirroring the way the systems and subsystems are put together. The control is basically bottom up. following the architecture of the system, subsystem relationship. The changes in states are followed through. interpreted as changes in functionalities of subsystems, until the changes in the functionalities at the level of abstraction desired are obtained. This form of reasoning is useful for answering questions like: "What system functionalities will be compromised if this valve fails closed?".

## Hypothesis Matching

Given a concept and a set of data that describe the problem state. decide if the concept matches the situation. The idea here is to encode the routine knowledge for
verification and refutation that the concept applies to the situation. One way this can be done is by using a hierarchical representation of evidence abstractions, where the top node determines the overall degree of matching of the hypothesis to the data, and lower-level nodes represent components or features of evidence for the evidence abstraction at higher levels. Form of knowledge are auch as to enable evaluation of strength for each evidence abstraction, and to support mapping degrees of belief :r each of these evidence abstractions, to degree of belief in the higher abstractions. Strength for an evidence abstraction can be determined Each evidence abstraction can be determined by matching against prototypical patterna which have evidential significance. Samuel's signature tables can be thought of as performing this task.

## How Existing Expert Systems can be Analyzed in This Framework

Separating the implementation language and the intrinsic nature of the tasks has been argued as being salutary for a number of reasons. Let us look at some of the better known expert systems from the perspertive of the framework developed so $f_{i a}$ in this paper.

MYCIN's task is to (i) classify a number of observations describing a patient's infection as due to one or another organism, and (ii) once that is done, to instantiate a plat with parameters appropriate for the particular pasient situation. We have shown in ${ }^{16}$ how the diagnostic portion of MYCIN can be recast as a classification problem solver, with a more direct encoding of domain knowledge and a control structure that is directly appropriate to this form of problem solving.
Prospector ${ }^{17}$ classifies a geological description as one of a pre-enumerated set of formations. Internist ${ }^{14}$ generates candidate hypotheses by a form of enumeration (plausibility jcoring and keeping only the top few) and uses a form of abductive assembly to build a composite hypothesis that accounts for all of the data. Assembly and hypothesis enumeration alternate. Dendral ${ }^{13}$ generates candidate hypotheses by a form of hypothesis matching and uses a form of abductive assembly which puts together the best molecular hypothesis from the fragments produced by the matching process.
Note that in these analyses we have not mentioned rules (Mycin), networks (Prospector), graphs (Dendral). etc., which are the means of encoding and carrying out the tasks. This separation is an essential aspect of what we mean by the "right" level of abstraction in analysis.

## Encoding Knowledge at the Level of the Task: The Generic Task Toolset

for each ge:cric task, the form and organization of the knowledge directly suggests the appropriate representation in terms of which domain knowledge for that task can be encoded. Since there is a control regime associated with each task, the problern solver can be implicit in :tie representation language. That is, a shell for each generic task can be constructed such that, as soon as knowledge is represented in the shell, a problem solver which uses the control regime on the knowledge can be created by the interpreter. This is similar to what representation systems such as EMYCIN do, but note that we are
deliberately trading generality at a lower level to gain specificity, clarity, and richness of ontology and control at a higher level.

We have designed and implemented representation languages for versions of each of the six generic tasks we described. Here is a list of the generic task tools, each with a brief description of the task for which it is designed.

- HYPER for matching concept to situation to determine confidence or appropriateness.
- CSRL for taxonomir classification, typically a major component of diagnostic reasoning. ${ }^{18}$
- DSPL for object synthesis by plan selection and refinement, captures knowledge for certain routine design and planning tasks. ${ }^{19}$
- IDABLE for knowledge-directed information passing for intelligent data retrieval.
- PEIRCE for assembly and criticism of composite explanatory hypotheses, a form of abduction or best-explanation reasoning. ${ }^{20}$
- WWHI (What Would Happen If) for prediction by abstracting state changes.

We have described how this approach directly helps in providing intelligible explanations of problem soiving in expert systems. ${ }^{2}$ The approach has a number of other implications. E.g.. uncertainty handling in problem solving is usefully viewed as consisting of different types for each kind of problem solving, rather than as a uniform general method.?

In principle the tools can be used together to build composite problem solvers that integrate the different types of reasoning assoriated with the generic tasks. Systems have been built integrating more than one type of reasoning (the Red ${ }^{23}$ system for example relies on four of the types) but these systems predate the availability of the tools. At present the actual toolset consists of separately implemented tools in a variety of languages: each tool having an incarnation in Interlisp. LAIR has under development an integrated version of the toolset in Common Lisp.
The computational architerture of a problem-solving sys(em) (or system romponent) built with any of the tools is based on functionally distinct, highly modular elements. tightly organized. A generis lask problem solver is a community of agents, where each agent is of a specific type. each has its own embedded knowledge. The agents are organized so that they have specific lines of communication with each other; and. depending on the generic task. they pass control around in a well-defined way in order to cooperate and solve the problem. A HYPER-built system is made up of knowledge groups. hierarchically organized: a cerlobuit sytem consists of a hierarchiral community of rlassification sperialist, each specializing in a single elassificatory ronerpt. This sort of system architerture. besides making implementation in an object-oriented programming framework fairly pasy, makes for systems that are distributable and have predictable concurrencies. The high degree of modularity--modules having clear functions. and clear interactions with other modules--makes for good software engincering at the knowledge level. "structured programming of knowledge bases" if you will.

## DSPL for building a Mission Planning Assistant

We will describe the use of DSPL (Design Specialists and Plans Language) for the design and implementation of an expert system in the domain of tactiral air mission planning. After investigating KNOBS system from MITRE Corporation, an existing mission planning system. we developed the Mission Planning Assistant (MPA) using DSPL and our generic task approach to building expert systems. KNOBS was the primary source of domain knowledge for the MPA system. Our project had two main goals. First, we wanted to explore the use of DSPL for routine planning tasks. Initially, DSPL was developed as a result of studying a routine mechanical design task. ${ }^{11}$ It seemed to us, however, that routine planning shares many of the characteristics of routine design, suggesting that they might require some of the same kinds of problem-solving activities. Serondly, we wanted to investigate the explanation farilities that are necessary in planning systems. We wanted to demonstrate that our generir task architectures provide very natural and comprehensive frameworks for explanation.

Tartical mission planning in the Air Force essentially involves the assignment of resources to various tasks. The resources are primarily aircraft and their stores incated at airbases across the theater of operations. The tasks are specified by an "apportionment" order issued by the Joint Task Force Commander to a Tactical Air Control Cienter (TACC). This order deseribes the overall military ohjertives as determined by the Task Forre Commander. The TACC: is responsible for assigning aireraft and personnel from specific military units to meet the objertives of the apportionment order. The result of these assignments is an "Air Tasking Order" (ATO) which summarizes the responsibilities of earh unit with respert to the day: missions. Fach mission planned requiress attention to such details as the selection of aircraft type appropriate to the mission, selection of a base from which to fly the mission. and roxrdination with other misions.

The MPA system we developed currencly addressen onls a single type of mission. the Offensive Counter-Air (OC:N) mission. An OCA mission is an air strike directed sperifically against an enemy's airbase. Our selection of the OCA mission arose primarily because of the availability of the KNOBS system and its knowledge base of relevant domain facts. Our approach to tactical mission planning treats the Air Tasking Order (ATO) as an abstract device to be designed. The planning of the missions of the completed ATO involves a process similar to the process a designer undergoes when faced with a complex merhanical device to design. A view of design problem solving should illuminate this idea. For a more comprehensive description of design see ${ }^{11}$.

## Routine Design and DSPL

The general domain of design is vast: it involves creativity, many different problem-solving techniques, and many kinds of knowledge. Goals are often poorly specified, and may change during the course of problem solving. A spectrum of classes of design problems can be discerned, varying in complexity from those problems requiring significant amounts of "creativity". to the most routine design problems requiring no creativity at all. The complexity of a design problem will depend on what
pieces of knowledge are available to the problem soiver prior to the start of design, that is, the right pieces of knowledge can remove the need for creativity and turn a complex design task into a routine one.
What we have called "Class 3 Design" characterizes a form of routine design activity which postulates that several distinct types of knowledge are available prior to problem solving. First we assume that complete knowledge of the component breakdown of the to-bedesigned device is available to the problem solver, including knowledge of what component attributes need to be specified in order to specify a design. The final design will consist only of components known in advance, and no novel components need to be synthesized. Secondly, wa assume that knowledge is available in the form of plat fraximents deseribing the actions required to design earh component. A plan for designing a component will typically include the designing of subcomponents as steps in the plan. Thirdly, we assume that rerngnition knowledge is available that will allow the problem olver to select between the alternative plans for devigning a component. depending on the design recruirements and the state of the problem solving. The problom solving prosereds by following a top-down process of plan selection and refinement. with localized back up and selertion of alternative plans upon faiture of a denign plan at any level. While the chosiros at each point mat be cimple. the design process overall may be puite eomplex. and objects of significant complexity ran be designed. It apperars that a significant portion of the overyday artivity of practifing designers can be analyzed ax rlass 3 dessign.

In bsipl. a design problem solvet consists of a hierarchy of roxperating. conerpttal uperialists, with each specialist responsible for a particular portion of the design. sperialists higher up in the hierarchy deal with the more general aspects of the device being designed, while specialists lower in the hierarchy design sperific subportions of the devier. or address other design subtasks. Any perialist may arcess a design data-base (mediated by an intelligent data-base assistant). The organization of the sperialists and the sperific content of each one is intended to prerisely rapture the human designer's expertise in the problem domain. Each sperialist in the design hierarchy contains lecally the design knowledge necessary to accomplish that portion of the design for which it is responsible. There are several types of knowledge represented in each specialist, three of which are described here. First. explicit design plans in each specialist encode sequencess of possible actions to sucecesfully emmplete the sperialist's task. Different design plans within a sperialist may encode alternative action sequencess. but all of the plans within a particular specialist are always aimed at achieving the aperific design goals of that sperialist. A second type of knowledge encoded within "pecialists is encoded in design plan sponsors. Earh design plan has an assoriated sponsor to determine the appropriateness of the plan in the run-time context. The thied type of planning knowledge in a specialist is encoded in design plan selectors. The function of the selector knowledge is to examine the run-time judgments of the design plan sponsors and determine which of the design plans within the sperialist is most appropriate to the current probiem context.

Control in a DsPl. system procerds downwatds from the top-most aperialist in the design hierarchy. Begimening
with the top-most specialist, each specialist selects a deaign plan appropriate to the requirements of the problem and the current state of the molution. The selected plan is executed by performing the design actions specified by the plan. This may include computing and assigning specific values to attributes of the device, checking constraints to test the progress of the design, of invoking sub-specialists to accomplish sub-portions of the design. Thus a design plan which reffers in a subsperialist is refined by passing control to that subsperialist in its turn. DSPL also includen facilities for the handling of various types of plan failures. and for controft ing redesign suggested by such failures. ${ }^{24}$

## Mission Planning as Routine Design

We view tactical mission planning as predominantly 2 routine design task. The problem can be decomposed into the design of subcomponents of the mission plan, where each component can be designed in a fairly independent fashion. The Air Tasking Drder is decomposed into various missions or groups of missions of known types. Each mission or group of missions can be planned relatively independently of the others, modulo resource contention considerations. In both the mission planning and the mechanical design domains, each of the solutions to the subproblems must be appropriately combined into the solution for the problem which they decompose. Due to the well known limitations of human problem solving capacities, it is apparent that a hurnan problem solver can be successful in such a situation only to the extent that she can decompose the prohiem into a manageable number of somewhat independent sub-problems, which can be solved separately and combined into a final solution. The MPA system uses DSiPl. as a natural mechanism for representing the necessary knowledge.

## The MPA System

The MPA system contains six specialists. The topmost sperialist. OCA, accepts the mission requirements and uttimately produces the final mission plan. The OCA specialist divides its work between two subspecialists, base and aireraft. The base specialist is responsible for selecting an appropriate base, while the aircraft specialist selects an airreaft type. The aircraft vperialist has three subsperialists, one for each of throe aircraft types known to the MPA system. As needed, one of these sperialists will select an appropriate configuration for its aireraft type. Problem solving begins when the OCA sperialist is requested to plan a mission. Currently, the OCA specialist contains only a single design plan which first requests the base uperialist to determine a base. and then requests the aireraft specialist to determine (and configure) an appropriate aircraft for the mission. The current base sperialist simply selerts a base from a list of candidate bases geographically near the target. The airraft speciatist uses considerations of threat types and weather conditions at the target to select an appropriate airraft type and number for the mission. The aireraft specialist and its three configuration subsperialists represent the most elaborated domain knowledge in the MPA system.

Suppose the mission requirements call for a night raid. The plan sponsors for both the $A-10$ and $F-1$ would rule out the possibility of using these aircraft, sine (in our
domain model) neither of these airrraft have night nying capability. The F-11t plan aponsor, since it is an allweather firinter with night rapabilities, would not be excluded. The plan sponsor for the F-111, hased on this and other considerations (range, ability to carry appropriate ordinance. target characteristics, rte.) would find the F-111 suitable for the mission. The plan selector in the aircraft specialist, finding that two design plans have ruled out, would select the "suitable" F-1tI design plan. and return this information to the specialist. The specialist executes the F - 111 design plan, which includes setting the aircraft type in the mission template to "F-111", and invoking the F-lll configuration specialint which in turn decides an acceptable ordinance load for the F-llt for this mission. Once the configuration of the aircraft is known. the single aireraft probability of destruction in the mission rontext ran be computed. Finally. knowing the mission capabilities of each aircraft, the pequired number of airraft ran b're determined in order to achieve the required probability of destruction, and the neenssary number of aircraft ran be reverved from the proper unit.

## Summary

We have argued the nord for high-level task-upecific toonls for constructing knowledge-hayed problem-solving systems. We described our approach, based on the notion of generic information processing tasks, and described a toonset which can be used to efficiently build export systems. Expert systerns built acrording to this methedology have all of the advantages of control strategies and knowledge representations that are especially suited to their information processing tasks. Advantages include knowing what kinds of knowledge to collert, and where to put it away for use in the groblem oolver, cffirient provessing at run time, and "xplanations of whtem" behavior in terms of strategies and problem solving gods keyded to the type of reasoning. Wis have illostrated the powers of these ideas by paying speciat atcontion to a high lewe language called Dsple, and have shown how it was asod in the construc. tion of a yystem called MPI. for assisting with mission plaming in the domain of offonvise commer air missions.

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# Monitoring Robot Actions for Error Detection and Recovery 

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#### Abstract


Reliability is a serious problem in computer controlled robot systems. Although robots serve successfully in relatively simple applications such as painting and spot welding, their potential in areas such as automated assembly is hampered by programming problems. A program for assembling parts may be logically correct. execute correctly on a simulator, and even execute correctly on a robot most of the time, yet still fail unexpectedly in the lace of real world uncertainties. Recovery from such errors is far more complicated than recovery from simple controller errors, since even expected errors can often manifest themselves in unexpected ways. Here, is pliesety In this paper weapocorila novel approactifor improving robot reliability. Instead of anticipating errors, worse knowledge-based programming techniques so that the robot can autonomously exploit knowledge about its task and environment to detect and recover from failures. He describe preliminary experiment of a system that wo we designed and constructed in our Robotics Lab.

## then <br> 1. INTRODUCTION

We want to make robots more dependable so that they can be trusted when left unattended. This paper describes the design and development of a robot system that continues to operate satisfactorily even after it encounters a serious error [Gin83b]. [Gin85c].
Failures in achieving a task are the result of errors, but not every error produces an immediately detectable failure. Errors can occur at many levels, at the mechanical level (a joint becomes locked), at the hardware level (a sensor does not function properly so that the robot is driven to exceed its joint limits). at the controller level, in the computer controlling the robot (either at the hardware or the software level). and in the environment. We are mostly interested in errors in the environment because they tend to be more unpredictable and difficult to characterize with mathematical models. We are interested in errors in the component parts used for the assembly, and in errors in the work cell (loaders, feeders, conveyor belts. tools). Our goal is to automatically detect problems caused by collisions, jammed parts, gripper slip. misorientation, alignment errors, and missing parts.

In practice, robot systems that can recover from errors without human intervention do not exist today because robot control programs cannot handle the vast range of possible error conditions. It takes uncommon skill and experience to develop such a program, and the resulting program will then only apply to the specific robot task at hand. Moreover, the program may have to be largely rewritten to handle even a minor change in the robot's task [Car85]. [Loz83].

A difficult problem in automatic error detection and recovery is detecting that something significant has occurred. Many events are usually reported to the robot controller but not all of them are significant. The same event may be important in some circumstances and almost irrelevant in others. Deciding when something is important is the first step in the error detection process. The second step involves detecting the cause of the error and its effect on the robot environment. Errors might appear a long time after what caused them happened making it more difficult to detect them. Some errors do not affect the execution of the task so they could be left unrecovered. Only after the cause for the error has been identified or, at least. after alternative plausible causes have been found the recovery activity can start.

The robot system discussed here is geared towards industrial assembly tasks. The assembly to be performed is described in a robotics language. If an error occurs while the robot is performing the task, the robot detects the error, and dynamically plans the steps it must take to recover. To do this, the robot applies general knowledge about robots. and assembly tasks, plus specific information about the robot and the program in question.

Our approach simplifies robot programming by putting the burden for general error recovery on the robot system itself. The programmer can concentrate on the task at hand and minimize later maintenance if the robot recovers from most errors itself. This saves engineering tifne as well as robot downtime.

The system described here works in conjunction with an existing robot programming language. We show
 addition to developing the testbed with the IBM robot we have developed prototype components of a simulated system. The simulated version is currenty more complete than the testbed version since it is much easier to control a simulated robot than a real one.

## 2. INTELLIGENT ROBOT ERROR RECOVERY

Our system operates in two phases: oftline, and online. Figure 1 illustrates its structure. Each box represents a separate program that may run as an independent process. The Preprocessor prepares the assembly task for execution by the robot by generating an Augmented Program (AP). The AP Executive interprets the robot program and monitors the robol's operation. If a serious error occurs, it activates the Recoverer. The Recoverer examines information from the event trace and from the task knowledge base and devises a recovery plan. More details on the component of the system are provided later in the paper.

Since the Preprocessor and the Recoverer both rely on symbolic computational techniques and do not require real-time performance tiley both reside on the same processor, referred to here as the Manager. Robot control operations require real-time response and reside on other processors.

## Design Features of the Testbed

The design of the error recovery testbed incorporates four features of particular interest. First, the automated reasoning of the Manager and real-lime functions of the AP Executive operate independently and execute on separate processors. Second, sensors are activated and monitored selectively according to the robot's current action. Third, sonsor data are evaluated and assigned qualitative meanings at several levels


Figure 1: Eirror Recovery System Components
of the system. Fourth, if the robot's task fails repeatedly, the AP Executive will handle successive restarts and recoveries without ill effect.

The present configuration of the error recovery testbed manages an IBM 7565 manufacturing manipulator. The components of the error recovery Manager exist on an LMI Lambda processor. Motion control and sensor filtering for the IBM 7565 is implemented on an IBM Series 1 minicomputer using the AML programming system [Tay82]. The AP Executive resides on an MC68000 based personal computer system, an Apple Macintosh.

The IBM 7565 is well suited for these experiments. It is a cartesian robot moving on linear tracks over a rectangular work cell. The gripper has six degrees of freedom and its jaw contains six strain gauges. The 7565 is provided with the AML robot programming system which can be used to develop relatively sophisticated programs. AML provides facilities for monitoring the robot sensors and for performing robot motion subject to the presence of appropriate sensor readings.

## Separation of reasoning and real-time

The testbed utilizes separate processors for executing the reactive, or real-time, software components and for executing the reflective, or symbolic reasoning, components of the system. Providing separate processors for the real-time and the automated reasoning components of the system prevents time-critical soltware components from having to compete for computation time. The choice also allows us to choose a managing processor for its symbolic computation capabilities rather than its real-time capabilities. Since the AP Executive is the only component that interacts with the robot continuously, a large scale system could probably share a single Manager among several independent work celis, each with its own AP Executive.

The Manager initiates a task by transnitting the appropriate Augmented Progran (AP) to that cell's AP Executive. In return, the AP Executive transmits the event trace of the task's execution. The physical separation of functions makes this information exchange particularly important. The AP must contain all information the AP Executive requires to operate the robot in the work cell. The event trace must contain all information relating to the task's progress necessary to reconstruct activities that took place in the work cell. Whenever feasible and appropriate, the event trace contains specific numerical sensor readings from the work cell.

## 3. THE PREPROCESSOR

Our system uses a manipulator level robot programming language to specity the task the rohot is to execute. This description is given in the AL robot programming language [Gin85b]. [Muj79], though any other manipulator level language should work as well. Even though AL is r.Jt used in any commercial robot, it has many of features many languages have adopted. We have expanded AL to handle descriptions of objects so that it can be used to drive our graphic simulation system. We have chosen to use a "robot level language" to describe the task rather than a "task level language" because robots in real use are programmed with robot level languages. Starting from an existing and accepted level of language will allow us to grow to more sophisticated languages and yet to keep our ability to experiment with existing commercial robots.

We assume that the task description is accurate and correct in the sense that a robot simulator would execute it reliably. Thus the only errors the system should expect will be introduced by real world uncertainties.

We can't use the original AL program for online monitoring; we rely on an augmented version of the AL program to direct the job of monitoring the robot's activities. We call this program Augmented Program (or AP). The AP is structured as a finite state machine. The machine is represented with a directed graph in which the nodes are arbitrary states. Activities performed by the procedure are specified on the arcs connecting the states; you derive the sequence of actions in the procedure by traversing a series of arcs in the diagram. Each arc has 'events' or 'preconditions' attached to its activities. If a particular arc is leaving the state the system currently is in, then the activities on that arc are performed when the preconditions are satisfied. It two or more arcs leave a given state. then the AP Executive chooses the arc whose preconditions are satisfied first. This structure is especially useful in systems where several asynchronous events (sensor inputs) select and trigger subsequent actions.

This approach lends itself readily to the representation of AL programs. The sequencing of instructions in the original AL program is replaced by transitions in the AP. The AP represente!! Jn also relates explicit sensor
data to what the robot is supposed to be doing. Crucial sensor readings preceding some action in the AL program will correspond to the preconditions of the corresponding state transition in the AP. The preconditions on arcs leading out of a given state will correspond to the set of sensor readings to be monitored by the sensor handier. Thus, the set of significant sensor readings will change automatically as the robot proceeds through its task.

## Extracting high-lovel intentions from an AL. program

Functioning of the AP Executive is highly dependent upon the information derived from the off-line portion of the system. In order for the AP Executive to interpret sensory data, the off-line component must provide the intent of the AL instructions. For example, if the intent of an instruction "move." is to transport an object to a given location it is important to check for slippage by monitoring the touch sensors. If the intent is merely to move the arm. touch sensor data may be irrelevant. The off-line component must be able to extract the semantics of the program.

In our system methods from extracting intents of programs are based on syntactic matching and heuristics. By syntactic matching we mean identifying sequences of instructions that suggest operations such as grasping an object or releasing it. We have explored techniques of this type with excellent results. We can already handie difficult cases, such as, for instance, identifying when an object is grasped from inside a hole. Additional details can be found in [Gin85a], [Gin85c].

Figure 2 shows a state transition diagram used in conjunction with syntactic matching to identify intentions of AL instructions. Arcs shown in gray indicate transitions that have no clear interpretation and that require user intervention.


Higure 2: Sente Transition Diaeram

## Constructing the Expected World Model

The world model is another critical piece of information for the AP Executive. Suppose that a 'move object instruction is in execution when the proximity sensors of the hand are activated. Knowledge about the
placement of objects in the environment would help determine the cause of the impending collision. In addition to placement of all objects in the environment. geometric and non-geometric properties of objects will be important. If an object being transported is slippery, the chance of dropping it is increased. It it is quite large, chances of a collision may increase, and so on.

The Preprocessor simulates the execution of the AL program to build an Expected World Model. In our current implementation the Expected World Model contains properties of the objects in the erivironment such as their position after the execution of each instruction, when they are moved, wich robot moves them, etc.

A classical problem with creating a world model offline is caused by branches in the program that create alternative models. Our world model has a tree structure in which branching nodes are labelled by the condition that controls the branch. The existence of the Expected World Model reduces the amount of reasoning during the recovery because we can compare the current with the expected situation to get clues about problems.

## Translating an AL Program Into AP.

The Preprocessor module has to provide information on how errors can be detected and what errors are likely for each instruction. For example, a 'move object' instruction might lead to a slippage error with high probability. This indicates that finger separation or touch sensors should be checked during the movement. Using the intent of the AL instructions, the Preprocissor generates in the AP program the appropriate sensory conditions to be checked to guarantee the correct execution of the instruction.

We need also to know what sensor values are relevant to detect unexpected events. Some of this knowledge is obtained from the robot program (such as the position of the robot, or the opening of the fingers), some could be obtained from the CAD data base (such as the maximum pressure to exert when grasping an object). Some values cannot be known before executing the program. Sometimes the position of an object is identified only by sensors. Hence it is important to know that no value is known in advance and that sensor data will be used.

A simple AL program and the corresponding AP are illustrated in Figure 3a and 3b. The task described in the AL program is a simple pickup operation. In the AL program WOKH indicates the robot hand and WOKA the robot arm. The example shows that we have modified AL to allow for different names of robots and different

```
CPEV NOKH TO 1.5:
MCVE \CKA TO FRAME (ROT (ROLL,-45),
    TMCR (-7.77, -14.41, 4.4));
    ZZER WOKH;
MME NCKA TO ERAME (ROT (ROLL,-45),
    GITOR (-7.77, -14.41, 7));
```

Figure 3a: Example AL Program

```
((1) (robot-do open wok 1.5))
    (lopen wok) 2
    ((hand-error wok) 12))
(2 ((robot-do move wok ( \(-7.77-14.414 .4-4500)\) )
    ( (reach wok) 3)
    ((hit wok) 12)
    ((joint-error wok) 12))
(3 ( (imply create abj-block ( -7.77 -14.41 4.4 -4500\()\) )
        (expect grasp wok obj-block)
        (robot-do center wok))
        ( (center wok) 4)
        ( (crush wok) 12)
        ((hand-error wok) 12))
(4 ((imply grasp wok obj-block)
        (expect carry wok obj-block)
        (robot-do move nuk ( \(-7.77-14.417-45001) 1\)
    ((reach wok) 5)
    ( (hit wok) 12)
    ( (untouch wok) 12)
    ((joint-error wok) 12))
                                    (other states in the program ... )
(12 ((irply error) (robot-do notice operator)))
```

Figure 3b: Augmented Program derived from Figure 3a
ways of expressing rotations. In the corresponding AP states are numbered. Each state contains a collection of entries. The first of them describes the action the robot has to do ("robot-do") and the meaning of the instruction in the physical world. In particular, "expect" shows what is expected to happen at the end of the state if all goes well, "imply" shows logical deductions about objects or the robot that can be made from the intentions extracted during the preprocessing phase. Each other entry specifies a condition to be checked using sensor data and the next state if the condition becomes true. Since the Preprocessor generates the conditions using knowledge about the intention of each robot action the same AL statement usually generates different conditions. As an example we can look at the states 2 and 4 in Figure 3b.

We should note that there is only one error state in the AP. When an error is detected a transition to the error state is generated. We do not use specitic error states because we need to check causes of the error to find the recovery procedure. We consider the sensor data obtained as symptoms of the error not as its diagnosis. Often software that handles failures does not make the distinction between symptoms and errors or it assumes that there is a deterministic mapping between symptoms and errors.

## 4. THE AP EXECUTIVE

The AP Executive is responsible for maintaining an accurate picture of what the robot does. The Recoverer needs to know the robot's situation but the potential complexity of the robot's activities make it hard to derive the necessary details from the program's state. It is easiest for the AP Executive to keep track of what the robot is doing and what objects it is manipulating. The AP Executive can then provide the robot's recent history and a catalog of objects in the workspace when an error occurs.

The AP Executive does more than simply observe and report on the robot's actions. It takes responsibility for issuing commands to move the robot. When the AP says that a robot action is to occur, the AP executive sends the command to the robot. The AP Executive tracks the robot's activities by monitoring data from the robot's sensors. The sequence of sensor data yields an event trace from which we get the robot's recent history.

## The Workspace Model

To recover from an error, the system needs to know what objects are in the workspace and where the objects are. At the time of error it should be easy to find out where the AL program failed and what the values of the program's variables are. Unfortunately, we can't deduce the state of the workspace from the state of the program. The program just doesn't keep the right kind of information. But it is possible to deduce when and how the AL program manipulates objects. To monitor objects in the workspace the AP Executive has to be told when an object is acquired, grasped, moved, and discarded. Typically the robot 'acquires' an object from a part dispenser, moves it somewhere, and maybe 'discards' a part by placing it on a conveyor. This is sufficient to keep track of what objects are in the workspace and where they are. The workspace model update process can then follow objects by monitoring such activities in the event trace.

The minimum workspace model is a catalog of objects and their locations. Along with the robot's most recent activities, this model gives enough information to determine what was going on at the time of an error. The Recoverer uses this information to key into more information about the object stored in the offline world model. More details about the Workspace modeller can be found in [Smi86a].

## Sensor management through filtration

In the error recovery testbed, sensor information is filtered several ways. Initially, the task's AP identifies specific sensor information that is significant to the execution of that task. This information is given in terms of sensory events specified symbolically that can cause state transitions in the AP. The sensory information is used both to identify potential state transition events and to filter sensory information for the event trace. This information is also passed to "sensor fitter" tasks that activate appropriate sensors and map sensor values into events signiticant to the progress of the robot's task.

Augmented Programs are instruction sequences structured as finite automatons. AP transitions are caused by discrete events, so a robot's progress at its task depends on the occurrence of events that cause
appropriate transitions. Significant sensory readings must be mapped into events that cause state transitions. This mapping provides one form of sensory filtration: sensory readings are reported only when the value is significant to the progress of the robot's task. In some cases the identity of the event is the only specitic sensory information returned and in other cases numerical data is included in the event trace as well.

Each AP state contains event predicates identifying sensor readings that would be significant to the successful execution of that state. The AP Executive passes the information in the event predicates to the appropriate sensor filters before initiating robot motion. The sensor filters activate app
ropriate procedures so that necessary sensor readings will take place. The AP Executive omits all sensor information from the event trace that is not identified in the AP as being significant.

Grip sensor filtering on the IBM 7565 is implemented using "monitor" facility of the AML language [Tay82]. Monitors are used to define ranges of sensor values that can activate user-defined procedures or terminate robot motions. When initializing the IBM 7565, the AP Executive delines a set of monitors for classifying gripping forces and associates each monitor with an AP event type. The numerical values used for classifying gripping force depend on the objects being used in the robot's task and the actions performed on them, so these values may be adjusted during initialization.

When the AP Executive gives the IBM 7565 a motion command, it also specilies a set of monitors to activate. The AML system collects the appropriate sensor readings for each active moritor and "trips" the appropriate monitor if its sensor enters the monitor's defined range. This terminates the motion in progress and generates a message to the AP Executive identifying the qualitative value of the sensor reading, as determined from the monitor that was tripped. If no monitor terminates the active motion, a similar message indicating uninterrupted completion is sent instead. The AP Executive then generates a sensor event and, if necessary, updates the event trace and performs an AP state transition.

Since the IBM 7565 is normally programmed in the AML language, the AP Executive translates AP commands and sensor specifications into an AML compatible form. Since AML is a manipulator level language, the mapping of robot actions from the AP form is straightforward. Mapping sensor operations is more complex since APS specity symbolic sensor readings. Figure 4 shows an example of the transformation of a "move" operatiori in an AP state into the corresponding AML commands executed by the robot. The desired destination and the desired AML monitoring sets to be activated (E_HIT and E_UNTOUCH) are passed to the APM procedure. This procedure, written in AML and executing on the IBM series 1, performs the MOVE operation and the related filtering for the 7565 's sensors. The procedure activates the appropriate monitors and performs the motion subject to the selected monitors.

## Qualitative sensor interpretation

Although the event trace often provides nurnerical sensor data. such information is not of primary importance when reasoning about the robot's activities. To meet this need, the error recovery system assigns symbolic meanings to numerical sensor values in a number of ways. Spatial locations and critical dimensions are assigned symbolic names. Gripping forces are assigned qualitative values according to the range in which a force value falls.

Qualitative classification of sensor data otten serves a second purpose as well. When executing an AP, the AP Executive responds to events in terms of symbolic classifications. Upon successful completion of a motion command the AP responds to a "reach" sensor event instead of examining and matching the robot's reported destination. If the gripper drops an object and the gripping force drops to a small value, the AP responds to an "untouch" sensor event instead of testing the specific force value. The classification of sensor values into different types of AP events is performed by a sensor filter procedure that operates on the behalf of the AP Executive, as described above.

Ideally, qualitative classification of sensor data should be performed by a component of the Manager and exploit its increased computational capabilities and knowledge bases. Symbolic classification of spatial information is an example of this. The robot's sensor filter identifies whether successful completion of a robot motion caused the robot to reach its sensed position, but the filter does not try to identity the location in terms of the robot task's overall goals. Identification of the particular location is handled by the Manager's work cell modeller.

Wher an error occurs the Manager produces a model of the robot work cell in terms of its probable state and its intended state. Locations visted by the robot or by objects in the work cell are assigned symbolic identifiers, and a history is produced of visits for each location, object, and robot gripper in the work cell. Error recovery planning consists of producing a sequence of robot actions to change the work cell from its erroneous state to its intended state.


Figure 5: Qualitative interpretation of graspling forces

Figure 4: Converting from AP statements to AML statements

Qualitative interpretation of gripper forces, on the other hand, must be performed by a sensor filter. Gripper forces, when they are significant, determine whether the gripper is touching an object and hoiding with an adequate force. Identification of appropriate touching and grasping forces must be communicated to the AP Executive so that appropriate AP state transitions occur depending on the gripping forces encountered. The sensor filter classifies gripping forces into specific ranges according to the robot's current action. Each range corresponds to a type of sensing event that can be produced by the gripping force sensor. Figure 5 shows an example of that.

## 5. THE RECOVERER

We are interested in discovering causes for errors and errors might appear a long time after what caused them happened [Smi86b]. Error interpretation becomes more difficult as the complexity of the task increases. For example, consider a task where a robot moves cubes from a feeder to a shipping pallet, twelve at a time. What might happen if the the cube falls from the gripper and lands on the pallet, knocking another object off? Most of the failure reason models available only apply to the objects and situations directly related to the sensor reading indicating the error. The robot thus only associates an error with a part if it uses its sensors on the part and finds an error. The lost part won't be missed until someone down the line tries to unload the pallet and finds it one part short.

The symjolic model of the work cell constructed before execution of the task and the trace of events are used here. When something unexpected happens we can trace the error back until we find critical measurements or assumptions made that were not supported by sensor data. For instance, in general we assume that if something is leit in a stable corifiguration it will remain there until a fact appears that shows the configuration has changed. If we discover later that the object moved it means that something happened to move it that was not explicitly noted.

We have developed methods for symbolic tracking of objects from event traces. Common sense heuristics are used for symbolic tracking. For instance, if the robot is holding an object and the robot moves, then we know that the object moves to the same place. Tracking an object in real time with sensors is too expensive, unless we know where to look for it, and how it looks like. Symbolic tracking is less expensive from a
computational point of view, and helps in reducing the number of possible causes for errors. After that number has been reduced we can get additional sensor data to guarantee that the correct cause for the error has been identified.

It is curlous to observe that most of the Al work in planning has concentrated on checking preconditions before executing every action to guarantee that they are satisfied in the current state of the world. Postconditions are not checked, but are used solely to update the world model. So an error can go unchecked for a while and can be detected only when it affects the preconditions of another action. We check selected conditions after the execution of each instruction to guarantee that failures are identified as soon as they appear. This still does not solve the problem of errors caused by the robot during the movement that require different sensors to be checked. For instance, it the arm bumps into objects during a transfer motion without losing the part it is carrying no error will be reported. This requires failure reason analysis when an error is found.

Once an error has been detected, the recovery process can start using a trace of relevant events and whatever information is available about the task to determine the causes and effects of the error. It is only after the cause for the error has been identified or, at least, after alternative plausible causes has been found the recovery process can start.

To be more specific, if an AP state transition leads to an error state, a message to that effect is appended to the event trace and the trace is passed to the Recoverer component of the Manager. The Recoverer generates a model of the current work cell's state and of its desired state. This model is used to produce a recovery plan in the form of AP states to be appended to the task's existing AP. The Manager passes these additional states back to the AP Executive where they are executed. If the new states each execute successfully, they will lead the task back to a state in the original AP.

To successfully effect recoveries in this manner, the Recoverer requires a copy of the task's AP and the information in the event trace. The Recoverer can also exploit knowledge about the robot's task, the parts involved, and the work cell to produce the recovery plan. To simplify experiments with error recovery as well as for improved performance in industrial situations, the testbed's AP Executive can handle repeated failures and subsequent recoveries by a robot task. The AP Executive can also display messages on the AP Executive's display screen for explaining error diagnoses or for instructing the robot's operator.

During normal execution, the AP Executive contains a copy of the robot task's AP. If an error recovery occurs, the Recoverer passes additional AP states to the AP Executive. These additional states do not replace existing states in the AP; they are appended to them. To recover, the AP Executive resumes task execution with the first of recovery states passed to it. Once the recovery execution begins, the AP Executive treats the recovery states identically to the states in the original AP. If another failure occurs, whether during the recovery or after completing the recovery, the AP Executive again reports the failure to the Recoverer and resumes executiun when it receives a set of recovery states.

For example, if the robot loses a part. it can attempt a recovery by opening the gripper, moving to the work cell surface, and trying to grab the part. If the part is there, the recovery can proceed. If the grasp fails, the AP Executive simply informs the Recoverer which can then produce another recovery plan and try again.

The ability to do multiple recoveries allows the Recoverer to profit from mistakes in a recovery plan. When faced with multiple recovery choices, the Recoverer can choose the one that is most likely to reduce uncertainty about the state of the work cell. The Recoverer can also produce recovery plans with the sole purpose of taking sensor readings in the robot work cell. If the Recoverer needs to probe a specific spatial location it can produce a recovery plan that performs the desired sensor reading and then immediately fails. The resulting event traces will increase the amount of information in the work cell model and the unsuccessful recovery will not prevent a subsequent recovery from being attempted.

Another useful feature during error recovery is the AP Executive's ability to display messages for the robot's operator. These messages are produced by statements in the AP and thus may be generated by the Recoverer. This facility allows the Recoverer to request specific operator intervention when necessary. This permits experiments with failures that tax the available sensory or reasoning facilities. In the Testbed it also allows experiments with primitive Recoverer software that simply diagnoses the problem and asks for operator intervention. This capability may also have worthwhile industrial applications: the Recoverer could produce messages to guide the robot's operator in manually correcting problems in a complex and unfamiliar assembly. An example is shown in Figure 6.


Figure 6: Operator display by AP Executive

## 6. RELATED WORK AND SUMMARY

Our approach differs from other approaches in significant aspects. The current state of the ant in industriat robots is that either the robot executes its task regardless of its success or it quits every time it encounters something unexpected [Luh83]. A better approach is to handle the situaticn by preprogramming error checking and error recovery procedures for every probable error [Bon82]. [Gin83a], [Gin85b], [Tay82] This is an expensive method both in engineering and in robot computational resources. It : also easy to forget some important checks.

Since it is difficult to consider all possible errors, many of which might never happen, another method is to generate from the task level description a program that is guaranteed to be correctly executed ever in the presence of uncertainties in the environment. This requires models of robot kinematics and dynamics, and models of physical properties of objects such as friction. This approach has been applied only to fine motions for specific tasks such as insertion operations (Loz84]. Modeling uncertainties [Bro82] and taking into account errors in the model [Don86] helps but the real world is so complex that it might not be worth developing sophisticated models of it.

Much previous research in Artificial Intelligence has centered on detection and correction of errors in simulated robot systems [Wil84]. These studies all make a number of assumptions: knowledge about events is correct, each action produces precisely defined postconditions, there are no uncertain data. correc: predicates are generated from sensor data every time they are needed, and sufficient know:edge is provided to take into account all the possible states of the environment. These assumptions are too stict to be realistic.

A few exceptions exist. The most notable is STRIPS [Fik71] the system used to control the mobile robot Shakey. Srinivas [Sri76] [Fri77] has designed a system for analyzing the causes of failures in robot programs and for replanning the robot activity. More recently, work has been done on monitoring the execution of programs with real robots [Lee83], [Lop86]. There is a growing interest in modeling sensors [Fox83], [Hen84]. and planning for their use [Doy86] that will provide needed background for work on error detection and recovery.

Our approach is more similar to the way people handle errors and unexpected events. By relating events to general knowledge human beings can identify unexpected situations and by applying common sense and domain specific knowledge they can find solutions to situations never seen before. The key to human performance is in the knowledge about the environment and about the specific task at hand. We want to do something similar for assembly robots. Since the domain is limited and reasonably constrained the amount of knowledge needed can be managed by using present technology [CAR84].

## ACKNOWLEDGEMENTS

Several individuals have contributed to the research presented in this paper. We would like to acknowledge the contribution of Rajkumar Doshi, Sharon Garber, Marc Gluch, Shideh Hojat, and Imran Zualkeman.
This work was funded in part by NSF grant DMC-8518735, b: the Microelectronic and Information Sciences Center and by the Productivity Center of the University of Minnesota.

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# Recovering From Execution Errors in SIPE 

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Note: Thin paper originally appeared in Computational Intelligence, Val. 1, No. 1, pp. 9s-45, Pabruary 1985.


#### Abstract

$p^{2}$ In real-world domains (es., a mobile robot environment), thing do not always proceed at planned, so it in important to develop better execution-monitoring techniques and replanning capabilities. Thie-pepowdeacribe thee e capabilities in the SIPE planning system. The motivation behind SIPE in to place enough limitations on the representation so that planning can be done efficiently, while retaining sufficient power to still be useful. This work assumes that now information given to the execution monitor is in the form of predicates, thus avoiding the difficult problem of how to generate these predicates from information provided by cancers. The replanning module presented here takes advantage of the rich structure of SIPE plank and is intimately connected with the planner, which can be called as a subroutine. This allows the uses of SIPE'a capabilities to determine efficiently how unexpected events affect the plan being executed and, in many cases, to retain moat of the original plan by making changes in it to avoid problems caused by these unexpected events. SIPE,is also capable of shortening the original plan when serendipitous events occur. A general set of replanning actions is presented along with a general replanting capability that hae been implemented by using these actions.


## 1 Introduction

A principal goal of our research in planning and plan execution is the development of a domain-independent, heuristic system that can plan an activity and then monitor the execution of that plan. Over the last two years we have deigned and implemented ouch a system, called SIPE (System for Interactive Planning and Execution Monitoring). ${ }^{1}$ The basic approach to planning ia to work within the hierarchical-planning paradigm, representing plans in procedural networks - a has been done in NOAH [0] and other ayoteme. Several extensions of previous planning systems have been implemented, including the development of a perspicuous formalism for describing operators and objects, the use of constraints for the partial description of objects, the creation of mechanisms that permit concurrent exploration of alternative plans, the incorporation of heuristics for reasoning about resource, and the creation of mechanisms that make it possible to perform deductions.
Given a description of the world and a set of operators that it can apply, SIPE can generate a plan to achieve a goal in the given world. (Operators are the system's description of actions that it may perform.) However, in reat-world domains, things do not always proceed an planned. Therefore, it is desirable to develop better execution-monitoring techniques and better capabilities to replan when things do not go as expected. In complex domains it becomes Increasingly important to use as much as possible of the old plan, rather than to start all over when thing go wrong.
This paper describes the execution-monitoring and replanning abilities that have recently been incorporated into the SIPE ayatem. The particular advantage that can be obtained by using the rich structure in our plan representation are shown, well as more general problems. The environment of a mobile robot has been used as a motivating domain in the development of some of the abilities here, though the implementation hat been carried out in a general, domain-independent manner. This document does not describe resources, constraints, plan generation, and other features of SIPE, nor does it attempt to justify the basic assumption e underlying the system. The interested reader is referred to (10 |for this.

The problem we are addressing is the following: given a plan, a world description, and some appropriate description of an unanticipated situation that occurs during execution of the plan, our talk is to transform the plan, retaining as much of the old plan as in reasonable, into one that will still accomplish the original goal from the current situation. This process can be divided into four steps: ( 1 ) discovering or imputing information about the current situation; (2) determining the problems this causes in the plan, if any, (similarly, determining shortcuts that could be taken in the plan after unexpected but helpful events); (3) creating "fixes" that change the old plan, possibly by deleting part of it and inserting some newly created aubplan; and (4) determiniag whether any changes effected by such fixes will conflict with remaining parts of the old plan. Steps 2 and 4 , and possibly 3 as well, involve determining which aspects of a situation later parts of the plan depend upon. Part of this problem is an instance of the standard truth maintenance problem, and SIPE's solution is described in Section 4. In SIPE, Step 4 becomes part of Step 3, 3 only those fixes that are guaranteed to work are produced. In addition, serendipitous effects are used to shorten the original plan in certain cases.
The major contributions of the replanning module in SIPE result from taking advantage of the system's rich plan representation and from imbedding it within the planning system itself, rather than implementing it as an independent module. This provides a number of benefits, of which the most important follow: (1) the replanning module can exploit the efficient frame reasoning mechanisms in SIPE to discover problems and potential fixes quickly; (2) the deductive capabilities of SIPE are used to provide a reasonable solution to the truth maintenance problem described above; and (3) the planner can be called as a subroutine to solve problems after the deplaning module has inserted new goals into the plan.
Another important contribution is the development of a general replanning capability (see Section 6) that has been implemented by using a general set of replanning actions. In general, recovery from an arbitrary error poses a difficult problem. Often very little of the existing plan can be reused. One can always fall back on solving the original problem in the new situation, ignoring the plan that was being executed. The re; anning part of SIPE, however,

[^6]

Figure 1: Control and Data Flow in SIPE Modulea
tries to change the old plan, while retaining a much of it at ponible. Since the problem is so dificult, one would not expect very impresive performance from a ganeral replanaer auch a SIPE's.
Bettor performance requires domain-apecific information for dealing with errors. In many domsins, the bypes of erron that are commonly ancountered eas be predicted (e.e., the robot arm droppiag something it was holding, or misting somathing it was trying to grasp). For this reason, the feneral replane is based on a number of general replannine actione (i.e., actions that modify a plan in ways that are uaful for handling unexpected situationt) that cat be referred to in a Janguage for providing domain-apecific error recovery instructiona. Section 6 gives the outline of anch alanguage.

### 1.1 Assumptions

SIPE aneumes that information provided about unexpected evente is correct and, to a certain extent, complete. This acoumption avoids many of the herdeat problems involved in getting a planner auch a SIPE to control a mobile robot. The challenging task of determining how to generate correct prodicalat from information provided by the sensort is not addressed. We expect the translation of the information from the robot's censort (e.f., the pixele from the camera or the range information from ultracouad) into the highes-level predicates used by the planaer to be crucial in applying a SLPE-like planner io 1 mobile robot. We hope to deal with this problem in the naar future.

In a mobile robot domain, it may often be important to expend conaiderable efort in checking for thiaga that might have gone wrong baides the ueaxpected occurrente already noticed. There is a aubstantial tradeof involved here, as interpreting the visual input of unanticipated acenes may be expetaive. The research described in this paper does not examine this problem either. It asumet that nothing has cone wrong besidea reported erron and effecta thet car be deduced from them. The problem of uncertain or unroliable sensors or information is also largely unaddressed, except thet eome predicates and rariable may be apecified as unknown. What is diecuseed here is what to do with new information in the form of predicalet (if we asaume that anch predicates haw somehow been discovered). Replanning appropriately with auch information is an eacential part of i.se overall solution.

### 1.2 Overview

Figure 1 shows the various modules ia the SIPE execution-monitoriag aystem. The solid arrowe show which modulea call which others. The broben arown ohow the flow of data and information through the ayatem an it replana for an anexpected situation. These arrows are labelod with a deceription of th data beiag paned.
The general replanner in given the list of problems found by the problem recogniser and trien certaia replanaing actione in various casea, but will aot atway find a colution. The general replanaer changes the plan so that it will look like an aneolved problem to the stasdard planaer in SIPE (e.E., by ineartiay new goalo). After the replanner has dealt with all the problems that were found, the planner ia called on the plan (which now incledee ancolved coath). I it produces a new plan, this new plan ahould solve correctly all the problema that were found.

Section 2 of thin paper describen the featuren of plaa represatation in SIPE that are relevant to its replanaing capabilitiea. To deacribe axexpectax nituationa, a user (at present a human, bat eventually this may be a program controlliag and iaterpreting the robot's wasors) can enter arbitrary prodicate at any point in the execution of can specify certain thinge me unknown. Section 3 deacribee the detailo of thin procese. Once the description of the umexpactox situation has been accumalatod, the execution monitor calls a problem recogniser described is Sectiom 4, which returns a lint of all the problems it doeecto in the plan. The replanning actions are described in Section 5 and the general replanner in Section 6 . Section 7 showi examplea of the general replanze in operation.

## 2 Plans in SIPii

Plans in SıL $E$ are represented as procedural networka $|6|$, with temporal information encoded in the predecessor and successor links between nodet The plan rationale, of primary importance to the execution monitor, is encoded in the network by MAINSTEP links between nodes and by che ase o PRECONDITION nodes (dencribed below). MAINSTEP links describe how long each condition that has been achieved muat be maiatained. A consex
 in plane are deacribed below to the extent necesary for understanding the execution-monitoring capabilities.
SPLIT and JOIN nodes provide for parallel actions. SPLITs have multiple succeanors and JOINs have multiple predecenore so that partially orderad plan can be produced. JOIN nodes have a parallel-postcondition alot, which apecifiea the predicates that muat all be true in the situatica repreaented by th JOIN node. If a JOIN node originally hae $N$ predecesora, there will be $N$ conjunctiona of predicates that must all be true at the JOIN node. 'Som

(a) Plans at Different Lovels

(b) Wodgee Uned by the Execution Monitor

Figure 2: SIPE Plan Viewed from Differeat Penpoctivee
branches may have been linearised, wo there may be fewer than $N$ predecesoors after planning.) it is easiar to record this at the JOIN node (than by havint previoua nodes point to the JOIN as their purpoen, since a failed parallel poatcondition can more easily be retried during execution monitorine if thert in easy accest to all parallel postcondition. The parallel postcondition slot is flled only when the JOIN is firat introduced into the plan; it is not updated as more detailed levels of the hierarchy are expanded. As long at the higheat level predicates are as deaired, it is asoumed that the lowar-lovel predicatee at irrelevant.

COND, ENDCOND, and CONDPATTERN nodea implement conditional plane. COND and ENDCOND are aimilar to SPLIT and JOIN, but asch aucceacor of the COND begins with a CONDPATTERN node that determinea which succeasor will be axecuted.
CHOICE nodes denote branching pointe in the search apace. Thay have multiple succesers, but the context melecte one of thene at being in the curreat plan. Coastrainte on variables may be posted relative to thia choice point. Thus, if the part of a plan after a CHOICE aode is removed, the corrmpondiat choice point in the context should atoo be removed to that constrainte that are no longer valid will be ignored.

GOAL nodes do not occur in final plana, aince thoy represeat problems that have not yet beon oolved. A GOAL node apecifice a prodicate that muat be achieved, but which is not true in the aituation ropreenated by its location in the procedural notwork. Repleaaing actione will ineart COAL moden in the plan. Each GOAL aode has a MAINSTEP slot, which denotes a point later in the plan that depands on the GOAL. (This deecribee the rationale for havieg the GOAL in the plan.) Esch coal muat be maintained at true antil the node which is its MAINSTEP ie executed. A MAINSTEP slot can have the aloem PURPOSE as ita value, denoting that the given predicate in the maia perpoee of the plan, not proparation for some latar action.

PHANTOM nodes are similar to GOAL nodes except that they are afready true in the situation ropieseated by their lucation in tine procedural aetwork. They are part of the plan becaue their truth muet be monitored at the plan is being executed. They aleo contain MAINSTEP alots.
PROCESS nodes represent actions to be performed during execution of the plan; they aloo have MAINSTEP alota, a do PHANTOM and GOAL noden In a fnal plan, all PROCESS nodes will denote primitive actions. (There are alov CHOICEPROCESS nodes, which are like PROCESS nodes except that they have a list of actiona, one of which muat be performed.)
PRECONDITION nodes provide a list of predicatea that must be true in the aituation represented by their location in the procedural network. Operalors may apecify preconditiona that must obtain in the world atate before the operator can be applied. The concept of precondition hers differs from tas counterpart in come planners, since the system will make no effort to render the procondition true. A fabe precondition aimply meane that the operator in not appropriate. Conditions that the planner should make true (aed therefore backward-chais on) cas be expressed as goal or proceat aodee.

By diatinguishing between PRECONDITIONS, GOALS and PROCESSES, we effectively encode metaknowledge about how to achirve gral. SIPE will are any means to solve a goal node, only the operators listed to solve a procesa node, but no operators to solve a PRECONDITION node. Thus, a preconditios's becoming false does not mean that it should be made into a goal; rather it means that the part of the plan produced by the operator which initially insertad this precondition is invalid. PRECONDITION nodee sho help encode the rationale of a plan, since in effect they mean that the part of the plan aseocined with them (see below) was produced on the aseumption that the prodicates in the pr fadition ware true.
In addition to the "horisontal" MAINSTEP, prodecessor, and succesor linke within one level of a plan, there are "vertical" anka betwoen diffaront lovela of the hierarchy. Each node that is expanded by the application of an operator has decendant links to each node eo producel. The deccendant nodee in turn have ancestor links back to the original node one level higher is the hierarchy. Starting with a node that wat expanded 'v an operator application, a wedge of the plan is determined by following all ite deacendant linke (in the current context) repeatedly (i.e., includiag deacendants of deacendante, and so on) to the lowet level. (Thin definition of wedges is the same an that used by Sacerdoti (0|.) Figure 2 depicte thia graphically, with the large boxed in

Pa: : (b) representige wodsea. The node originally expanded by an operator application is called the top of the wedge. A wodge with its top at a high lovel in the hierarchy will generally contain many lowerbevel wodges wilhia iteoll. The only nodee that can be the tope of wodgee are GOAL, PROCESS, and CHOICEPROCESS.

Since PRECONDITION nodet are created only whea an operstor is applied, the part of a plan amociated with a PRECONDITION node can be found by acendiag along the accestor linkt to the point at which the procondition firat became part of the plan (once intartod, PRECONDITION aodes are copied dowa from level to level). The node that wae expanded by an oparator to create thie procondition ie one leval higher than whers the first PRECONDITION mode appeart and in the top of the wedge acociated with each of the PRECONDITION aodet that are copiod from the frot oxe.

## 3 The Input of Unexpected Situations

During execusion of a plan in SIPE, some perion or computer syatem monitoriag the axocution can apocify what actions have been performed and what changee have occurred in the domain boing modoled. SIPE change ite origiad world model permanantiy so an to show the effecte of actiona atready performed. At any point during execution, the syatom will secopt two typen of ialormation about the domaia: (1) an arbitrary prodicate whon argumente are ground inatancue, that is now true, falee or unkaown; and (2) a local variable name that is now uaknowa. SIPE first checka whether the troth-values for the new predicatem difer from ite expectations aad, ff they do, it applien lise deductive operators to dedy:e more changed predicates.
If is important to nove that the inputing of prodicates does not eolve the "pixele to prodicates" problem, which in the crucial iswe in using a planaer such
 information from ultrecound) iato the highor-lovel predicates ueed by the plapaer. The research deecribed here is concerned with what mast be doae a ith the prodicates once thay have been esteblished but doee not take up the quention of how to determine them automatically. We hope to addrese thin ta', $t$ task in the near future.

### 3.1 Unknowns

Unknowne are a new addition to SIPE, ae it previoualy aesumed complete knowledge of the world. Having unknown quantitios constitutes a fundamental modification because even the method of determining whather a predicate is true must be changed. If the truth-values of critical predicatea are unknown, the planner will quickly fail, aince none of the operatorn will be applicable. (Naither a negated nor an unnegated predicate in a procondition will match an unknown one.) Operators can require predicates to be unknown as part of their procondition in case there are appropriate actions to take when things are uncertain. Conditional plans have aloo been implemented at part of the execution-monitorine package in SIPE; thus, an operator might produce a plan with an action to perceive the unknown value, followed by a conditional plan that apecifes the correct course of action for eact poenible outcome of the perception action. The deductive capabilities have also been enhanced so that operators can deduce that romethiag ie unknown.
The ability to apecify variables as unknown is aimply a tool provided by the syatem that will presumably be useful in some domains, particularly in a mobile robot domain. The idea behind this tool is that the location of an object may become unknown during execution. Rathar than make predicates unknown, which may cause the application of operatore to fail, we simply say that the variable rapresentiag the location is inatantiated to the atom UNKNOWN, rather than to its original location. All prodicates with thie variable as an argument may then atill mateh as if they were true. Thua, the aydem can continue planning as if the location were known. The only reatriction is that no action can be executed that uses an unknown varisble as an argumeat. When such an action is to be executed (e.g., go to LOCATION1), then the actua inatantiation of the variable must be determined before the action is executed (posaibly through a perception action). Note that it would be incorrect to continue planning if the eruth-values of important predicates depended on the instantiation of the location variable. It is the responaibility of the uer not to une the unhnown variable if preaicatea dapend on the latter's value.

### 3.2 Interpreting the input

The user need not report all predicatee that have changed aince many of these may be deduced by SIPE's deductive oparatora. The ayotem's deductive power has been increased recently (see next section) so many effecte can be deduced from certain critical predicatea. SIPE does not check for additional unexpected predicates. Alternatively, we could decide on some basis (which would have to be provided as part of the dumain-apecific description) juat how much effort to expend on perception actions to find out whether more than the minimum hae gone wrone. For example, if we are told that (ON A B) is not true when we expected it to be, we might want to check to see if $B$ is where we thought it was. Ae it is, SIPE will cimply deduce that $B$ is clear (if no other block in on B] and will not try to execute actiona to make further checke with regard to the world. This latter procedure could be very expeanive for a mobile robot in the absence of good domain-apecific knowledge about what wa worth checking.
There is a problem with unexpected effecte in deciding how they interact with the effecte of the action that wat currently being executed (e.f., did they happen before, duriat, or after the expected effecta?). Our colution to this problem is to aseume that the action took place as expected and to simply ineert a "Mother Nature" action after it that ie preaumed to bring about the unexpected effects (and thinge deduced from them). The ayatem anaumes that any effecta of the action being executed that did not actually become true are either provided or can be deduced from the information provided. This solution interiaces cleanly and elegantly with the rest of the planner and avoids having to model the way in which the unexpected effecta might unteract with their expected counterparts.

## 4. Finding Problems in a Plan

Havin، jutt inserted a MOTHER-NATURE node (MN node) in a plan being executed, SIPE muat now determine how the effecte of thin node intuence the remainder of the plan. There are two aspecta to this: the first involves planning decisions that were baeed on the effecta of this node, and the second involvee deductions about the state of the worid that were based on those effects. Section 4.1 describes tha problem recogniser in SIPE, which finds all problerns in the remainder of the plan that might be caused by the effects of the $M N$ node. Becauce of the rich information content in the plan representalion (including the plan rationale), there are only six problems that muat be checked. As shown in Figure 1 , the problems found by the problem recogaiser are given to the general replanner. The problem recogniser aleo notices pousible serendipitous effects.
The second aspect mentioned sbove involves solving the traditional truth maintenance problem. Many effects deduced later in the plan may no longer be true if they depeaded on predicates that are negated by the MN node. The validity of such deductions muat be checked so that the remainder of the plan represents the state of the world accurately. Section 4.2 describes how SIPE solven this problem, correctly updating deductions later in the plan. Deductiona that ara changed may or may not cause problems that should be recognized by the problem recogniser. If such problema are generated, they will be found by the problem recogaiser deacribed in Section 4.1 , since the deductions are correctly updated before the problem recognirer is called.


Pigure 3: Blocke World Problom and Plan

### 4.1 Problems found by the problem recognizer

All occurrences of the six problems lised below are found by the problem recogaiser. These problems conatitute the only thinga that can go wroag with a plen in SIPE after addition of a MN node at the current execution poist. The blockeworld problem in Figure $\mathbf{3}$ will be used to show an example of ench type of problem.
1-Purpose not echicved. If the MN node negates any of the main effecte of tha setion juat executed, thers is a problem. The main effecte muet be reachiored. If during execution of the Arat PUTON node in the plan in Figure 3 , aither 7 (ON B C) or (ON B D) is given as an uaexpected effect, then the MN node iaserted after the PUTON node will negate the puspoee of the PUTON node - thereby reaulting in an inctance of this type of problem.

2 - Previous phantome not maintained. SIPE keepe a list of phantom nodes that occur before the current execution point (including thoee on parallel branches), and whose MAINSTEP slot specifes a poiat in the plan that has not yet been executed. These are phantoms that must be maintained. If the MN node negates any of these, then there is a problam. The phantome that are no louger true must be reachisved. Suppoes that duriag axecution of the frat PICKUP node in the plan in Figure 3, -(CLEAR C) is givea as an enexpected effect. This type of problem will then cecur, since the phantom node (CLEAR C) has a MAINSTEP slot (not shown in the fgure) pointing to the first PUTON node, bat hat been negated by the MN node alter the first PICKUP node.

3 - Process node using unknown seriable as argwment. If a variable hae been declared at unknown, then the firat action using it a an argument must be proceded by a perception action for determiaing the value of the variable (eee Suction 3). Wf the B in the plan were the inetantiation of the variable BLOCK1 (instead of being given as part of the problem), and UNKNOWN BLOCK1 were entered during execution of the firat PICKUP action, then this type of problem would occur with the immediacely following PUTON action, aince it would be appliod to an UNKNOWN argumert.

4 - Fuisre phantome no longer true. A phantom node aftor the current exceation point may no longer be true. It mant be chaged to a GOAL node so that the planner will try to schieve it. In the sample plan, suppose that (ON D B) were given an an effoct during execuition of the firat PUTON node. Thia type of problem would then occus with the lat (CLEAR B) phantom node in the plan, aince it would no longer be true when it in expected to be.
5 - Futere precondition no longer trua. A PRECONDITION node afler the current erecution point may mo lomger be tree. In thie cace, we do not want to reschiove it, but rather pop up the hierarehy and perform some alternative action to schieve the goal at that level of the hierarchy. Because the sample plan contains no PRECONDITION nodes, we consider an axample of this type in the travel planning domain. Suppoee there is an operstor for John's taking a taxi to the airport, which hat a precondition that Joha's car is inoperative. If, during execution of the first part of the plan, SIPE is told that John's car is not broken, this type of problem will occur. In this caee the reacon for taking a taxi to the airport has been invalidated, and the general replanner will pop up the hierarchy and apply a differeat operator to get John to the sirport (presumably driving his car).
6 - Parallet postcondition not true. All the parallel postconditions may no longer be troe at a JOIN node. (This could be handled by maintainimg phantome, tat is more convenient to handie separately.) ln this cace, we mast insert a set $\alpha$ parallel goale after the JOIN, one for each antrue paralbl poatcondition. The parallel poetconditions of the naw JOIN will be the same as thow on the old JOIN. In the sample plan, the last JOIN aode will have both (ON A B) and (ON B C) as parallel postconditions (since they were in parallel originally). Suppose that (ON B TABLE) were given at an effect during the execution of the lat PUTON node in the plan. This type of problem would then occur, since the parallel pootcondition of (ON B C) would ao loager be true.

Becance of :he way plans are encoded in SIPE, these are the only things that need to be checked when determining whether an MN node affecte the remainder of a plan. This illustrates how she rich atructure of plans in SIPE helpe produce efficient problem detection. It should be noted, however, that processes (actions) are aspumed to work whenever their precondition is true and when all phantoma whowe MAINSTEP slot points to the proceas are true. (All such necesaary conditions should be encoded as either preconditions or goals, in any case.) There is currently no check for loops cassed by the aame error happening repeatedly, with the ame fix being propoed by the general replanner each time. Various simple checks could easily be added if this were a problem.
Finally, there are two important points to note with regard to the problem recogniser. First, in addition to the Pove problems, possible serendipitous effecta are also noted and inciuded in the liat of problems. If the main effect of some action later in the plan is true bfore the action is executed, then that is noted as a possible place to shorten the plan (this is discused in more detail in the next section). Second, only the last three probleme above interact with the wolm:-a to the truth maintenance problem, since only they involve the truth-value of predicates in situations after the current execution point. The proble... . ciogniser takes into account any changed deductions (eee Section 4.2) while looking for the latter three problems.

### 4.2 Solution to the truth maintenance problem

 Whonever their procondition in troe and all phantoma whom MAINSTEP slot pointe to the proceno are true, only deduced aflocte nood to be checked for their dependence on mexpected effects. (The axecution monitor will solve problems having to do with proconditiont and phantome that ane not trae).
 it neverthelen keepe deduction under control by everely reatrictiag the deduction that can be made, an well as by haviag trigeres to control the application of deductive operators. All deductionathat can be made are performod at the time a mode in ineertod into the plan. siace deduction in sot axpensive, the truth maintesaaca problem in solvod simply by redoing the doductions at ecch node in the plea after an MN node. Even bhis cas be avoided in simple caces, becanse SIPE carriee a lint of changed predicales an it goon through the plan and, if they all become true later in the plan (without any deduced efocte changing in, ( iterim), then the execution monitor noed not book at the remainder of the plan (either for rodoing dedectione or for fiadiag probleme).

## 5 Replanning Actions

The eight mplanniag actione deacribed below, iEEINSTANTIATE, INSERT, INSERT-CONDITIONAL, RETRY, REDO, INSERT-PARALLEL, POP. REDO, and POP.REMOVE have all been implemeuted in SIPE. Theen actiona provide sufficient power to alter plaas in a way that often retaine much of the original plan. These are domain-independens setions, and thay form the benie of the general raplapaer. Ther should aloo prove aceful as a basis for domain-apecific error recovery operatore. Both of these uses are described in more detsil in Section 6 . The frat aeven actions can ath be used co solve problems found by the problem recogniser, while the last is used to take full advantage of wereadipitous effecta.
Four of the replanning actions change the plan co that it will contain uacolvod problems. The intention (see Figure 1) is that the plan will then later be given to the normal planning module of SIPE (pooujby after a number of thome replanaias uctione have changed the plan). The planaer will then ablempe to find a solution that rolves all the problems that have been corrected is the plan. The planner automatically checka to determine whetier nodee is aplicee into the middle of the plan cause problema later, so that any solution found will be correct. (tt does this when copyiag nodes down to the acxt lower level during planning.) In all actions described below, the context argument meroly apecifas the context of the current plan.

- REINSTANTIATE (predicate node context)

This action attempte to instantiate a variable difierently so as to make the givell predicate true in the situation apecified by the given aode. This appeare to be a commonly useful replanning action. For example, it might correspond to uaiag a difereat reeource if scmething hae gone wrong with the one originally employed in the plan, or deciding to return to the hopper for another acrew rather than trying to find the one that hat juat been dropped.
An attempted reinatantiation is done by looping through the arguments of the given prodicate. For each argument that is a planning variable (ae oppowed to an actual ground instance), SIPE checke to see if there is another inatantiation fur it that will make the predicate true. This is cheap and efliciont in SIPE, since it merely involvet removing the INSTAN conatraint on the variable from the current context (and also from all variablee coastrained wo be the same ae this one), and then calling the normal matcher (which will return possible instantiationt) to determine if the predicate is now tme. Note that all other conatraints that have beetu accumulated on this variable are left intact, so only inatantiations that meet all relevant requiremento are found.

If new instantiations are found, the REINSTANTIATE action checke the remainder of the plan to see if any parts of it might be affected by the new instantiation. This ia done by a routine similar to the problem detector described in Section 4 (in fact, the two thare much of their code). REINSTANTIATE currently accepte new inatantiations only if they cause no new problems (see discussion below on trado-offa). If all new instantiations are rejected, the old INSTAN conatraint is simply replaced. Note that replanning may be done later in the plan after the REINSTANTIATE action becaum of other probleme that were found; the only requirement here is that the REINSTANTIATE action itself not introduce new problema later in the plan.
One might use REINSTANTIATE to help with the above mentioned problem of dropping ascrew in the following way. Suppoee that SCREW 1 is a planning variable, while S1 and S2 are particular ecrewa. The plan baing executed could have SCREWI inatantiated to SI, a phantom to be maintaiaed with the goal of (KNOWN-LOCATION SCREWI), and a PROCESS node for moving SCREW1 to achieve (AT SCREW1 WORKBENCH). Duriag execution of the latter node, SIPE is told that the finger separation of the arm io zero. Prom this it could deduce (amons other thingi) - (KNOWN.LOCATION SCREW 1 ) and -(AT SCREW 1 WORKBENCH). The problem of not achievins the purpose of the PROCESS node will result ia an (AT SCREWI WORKBENCH) goal being inserted in the plan. Without REINSTANTIATE, this would involve finding the location of S1 and movine it to the workbench - which may be a very hard problem (as anyone who han ever dropped a acrew is aware). The problem of not maiataining the phantom node could triger REINSTA NTIATE on the predicate (KNOWN-LOCATION SCREW1), which woald reoult in SCREW 1 being reinatantiated to $S 2$ (whoee location in known). Thia woald introduce no new problems and SIPE could proced to colve the (AT SCREWI WORKBENCH) goal by getting S2 from the hopper.
To prevent the introduction of a large cearch apact, REINSTANTIATE is limited by the requirement that it not introduce aew problems. There are aleo tradeoffs in deciding when to apply REINSTANTIATE as it exists, but theee are diecuaced later in the paper. The implementution deacribed above opts for reinatantiation only when it is likely to be the correct solution. This is consistent with SIPE', runaing eficiently on the probieme it does solve. Alternatively, new instantiations could be accepted even though they caused problems - as long as the latter are lem severe than the probleme incurred hy keeping the ofl inatantsation. Since SIPE has to way of comparing the difficuley of two sets of problems, REINSTANTIATE does not do thia. Doing so would introduce another very large search apace into the replanning process. However, it would not be difficult to change SIPE to explore thia search space if a domain warranted it. There are aloo ways to partially lift this restriction at the cont of a moderately increased search space (though the tradeoffs involved probably depend on the domain).
One could alao expend more effort in thding new instantiations. As amplemented, this replanning action will find reinstantiationa when onl; one variablis changed. Some problems could be solved by reinstantiating a whole set of variables, but this would be more expensive and involve a search problem to decide which variables to include in the set. The decision to try only one variable was made because it is efficient while evidently powerful enough to be useful. If the ability to reinatantiate sets of variables appeared vaeful, implementing it would certainly be tractable.

- INSERT (node 1 node \&)

This action inserts the subplan beginning with nocel (which haa been constructed) into the current plan after node2. All links between the new subplan and the old plan are inserted correctly. This is used as a subroutine by many of the actiona below.

- INSERT.CONDITIONAL (variable node coniert)

This action is not very interesting, but complements the unknown variable feature, which may be useful. It simply inserts a conditional around the given node that testa whether the given variable is known. If it is, the given node is executed next; ocherwise a failure node is executed.

- RETRY (node)

Thim replanaing action in very uimple. The given node in acoumed to be a phantom node and it ie changed to a goal node so that the planaer will perceive if a uncolvod.

- REDO (pradicate nole costext)

This action crentes a GOAL sode whoee goal is the given prodicate. It then calle INSERT to place this new node after the given node in the plan. The planaer will see the aew sode as an unsolved sonl.

- INSERT-PARALLEL (node prodiceles context)

Thin action meantially does a REDO on asch prodicate in the liat PREDICATES and puts the resultiag COAL nodas in parallel botwoen anmly created SPL:T and JOIN. Thin subplan is ineerted after NODE in the plan. The planaer will see theee new nodes an unsolvod goala. This action in useful for reachlovist parallal poatconditions.

## - POP-REDO (node predicates contast)

This and POP-REMOVE are the most complicatod of the replanaiag actions; it is used to remove a hierarchical wedge from the plan and ruplace it with a node at the loweat loval. POP-REDO is ueed when a PRECONDITION node in no longer true and another action mato be appliod at a higher lovel. It could also be uned to find higher-laval goale from which to raplan when there are wideopresed probleme causiag the replaraiag to fail (thin in aot curreatly implameated).

Whan redoing a precondition failare, it is easy te determine the wedge to be romovad, since PRECONDITION noden are copiod down from ome lovel to another. The top of the wedse to be romoved is the node that wae expanded to initially pl" te the given PRECONDITION node (or oae of ite ancemore that is a PRECONDITION aode) in the plan. Actually, only the bottom of the wedge in apliced out of the plan, a planaing will coatiane oaly from the lowest lovel. The aubplan that is removed at the luwest level is repleced by a copy of the GOAL or CHOICEPROCESS node that wat at the top of the wedse. (The INSERT replanning action is used for this.) This is seen as an uneolved goal by the planner, which automatically checke to asertain whether expansiona of this node cause problems later in the plan.

Let us coneider the example mentioned earlier of John planning to take a taxi to the airport when hin car in broken. The operator for takiag the taxi could have a procondition -(HAS-CAR JOHN AUTO1) $\vee$ (BROKEN AUTO1). (This will match Joha't not having a car or his car being broken.) This operator is applied to solve the GOAL node (AT JOHN AIRPORT) at a high lovel in the plan, causing a PRECONDITION node for the above procoadition to be inserted intr ane plan and copied down to all lower levale of the plan. Suppose that, during execution, -(BROKEN AUTO1) is entered an an unexpected effect during execution of a procese before the PRECONDITION node. Thie node is a future PRECONDITION which becomes false, and the general replanaer will apply POP-REDO to the problem. The wedge that is deleted hae the GOAL node (AT JOHN AIRPORT) at the top. This may be e very large wodge if its lowest leval is as detailed os "find the phone book, look up taxi in the yollow pages, dial a taxi company," atc. At the lowest level, the whole plan of Gading a taxi and taking it to the airport is apliced out and replaced by an (AT JOHN AIRPORT) GOAL node. Whon SIPE's planaer is later called on this plan, thin GOAL node may be solved by John's driving his ear to the airport.

There is one potentially serious complication in the above deacription of POP-REDO. Namely, various conatrainta may have been poated on the planaine variables becaune of decisions made in the wedge of the plan that has been effectively removed. Fortunately, becauce of SIPE's use of alternative contexts, this is eseily colved. A coatext is a lint of choice pointa, and constraints are pooted relative to the choice point that forced them to be poated. Therefore, thie problem in solved by removing from the current context all the choice points thet occurred in the wedge of the plan that was effectively removed. This new contoxt in given an the context argameat to future planaing action, and no further action aced be taken. Thia resulta in ignoriag procisely thoee conatrainto that should be ignored.

## - POP-REMOVE (node predicates context)

SIPE takes advantage of cerendipitous effects to shortea a plan by using POP.REMOVE, which removes a wedge but doee not insert a mode. (It shares much of ite code with POP-REDO.) However, in this cane it is nontrivial to decide which wedge to remove. There may be various wedges that are candidates and, as with REINSTANTIATE, these candidates may cause problems later in the plan if they are removed. SIPE currently handlea thin case in the otame way it handles REINSTANTIATE. Namely, it removes a wedge, checke to see if this causes any problems, and, if there are any, replaces the wodge. Thue, serendipitoun effects are exploited only if doing $w$ does not change the rest of the plan. This is a trade-off like the one discuseed previounly. SIPE again opta for efficiency, bu: could easily be changed to explore the additional eearch apace of replanning after the removal of wedges.

SIPE also reduces the warch space by generating only one candidate wedse. It gives up taking advantage of the serendipitous effect if this wedze does not work. The candidate wedge is generated by following anceator links from the node given to POP-REMOVE (which aupposedly has a purpose chat hae become true serendipitously), as long as some main effec: of the candidate node is made true by one of the predicates in the list of given predicases (that have unexpectedly become true). The candidate node found in this manner determines the candidate wedge. The wedge is rejected immediately unless all its main effects are true in the given list of predicates.

Figure 4 uses the example of geting John to the airport to help illustrate this selection process. This example depicts a frequently occurring case in which the last action at one level of a wedge achueves the main effect of every level above that. Por example, at Level 1 the goal is only to gei John to the airport. At Level 2, after the choice hae been made to take the taxi, the last node will achieve getting both John and the taxi to the airport. If Leval $\boldsymbol{i}$ pians the mechanics of leaving the taxi, the last node there might contain all these higher-level effects an well as the thianer state of John's walles.

The above neloction procene requirea that all goals generated at a higher level and achieved in the candidate wedge be achieved before the wedge becomes a candidate, while goale generated at a lower level than the top of the candidate wedge noed not have boen achioved serendipitously. Thua, for Wedge 2 to be selected in Figure 4, the serendipitous effecte must include (AT JOHN AIRPORT) from the highar level bat need aay nothing aboat how much caeh Joha hat aince that is at a lower leval. (It is aseumed that, at long a the highest-level goal is achieved, we do not care ai - it the lower-leval goale that were necosasy to bring this about.) The main effects of higher-level nodes that are achieved within a candidate wedge are astily checked bocause they are copied down an effecta of the node that achievee them. Thun, checking to verify that all main effects of the candidate wecige are trce anaures that all important higher-level effects will be true. La the example as ahown, Wedge 2 can never t. . iected by this eelection process bocause Wedge 1 will work whenever Wedge 2 doea. However, in another example the effecte of Wedge 1 might be achiep... : Level 2 before Wedge 2 so that Wedge 2 might then be selocted.


Figure 4: Hierarchical Wedgee with a Common Lat Action

## 6 Guiding the Replanning

The replanaing actions of the proceding section form the basin for SIPE'S general replanning capability and for a language capable of apocifying dornainapecific arror recovery instructions that has heen designed but not implemented. The latter could be thought of an instructiona for guia.ug the search of the general raplanner. This nection describes the automatic replanner and briefy outlinas the error recovary operatorn.

### 6.1 The General Replanner

The general replanner takea a list of problems as well as posible cerendipitous affects from the problem recogniser, and calle one or more of the replanaing actione in an attempt to solve the problem. It firt checks that a listod problem is atill a problem, aince the REINSTANTIATE action may solve many problems at once.
If the problem is a purpoee that is not baing achieved, the syatem tries a REDO, which inserts the unachieved purpose as a GOAL node after the MOTHERNATURE node. If the probiem is a provious phantom not beiag maintained, SIPE Grst tries REINSTANTIATE and, if that faile, it calle RETRY. The idea is that, if there is another object around with all the decirod propertios, it wovid be casier to nee that object than to reachieve the desired mate with the original objost. If a PROCESS node han an unknown variable a an argument, INSERT-CONDITIONAL in called. If a futare phantom is ao longer true, RETRY is called. As with maintaining phantoms, REINSTANTIATE may be more appropriate, but, in both caves, this depende entirely on the domain; thus the colection here is arbitrary. For preconditions that are not true, the general replanner firat calle REINSTANTIATE and, if that faile, calle POP-REDO. U parallol posteonditions are not true, the general roplanser calls INSERT-PARALLEL with the appropriate parallel goalh.
While a general replanning capability is a significant achievement, one cannot expect very impreenive performance from a replanner that doea not have domain-spocific information for dealing with errors. For example, whether or not REINSTANTIATE is likely to succoed will be depeadent on the domain. The automatic repianner makes reaconable guesese at what might be a good choice in the domaine on which SIPE has boen tested. Siace it meraly choowet a replanning action for each type of problem that is found, it is very simple and could easily be rewritien for different domains.

### 6.2 Error Recovery Language

We aleo have deaigned an extension of the operator description language that enables instructions for handliag foresecable errort to be included in operatore. This will allow encoding of domain-apecific knowledge for guiding the search of the general replanner (or even avoidiak the gearch altogether). The error recovery operators will have the same syntax an all other SIPE operators, with nome new additions made to chis language as described below. Ite plots of these operatora will include references to the replanning actions in Section S. SIPE's ability to apecify conditional plane in operators can be used to try a second replanning action if the firat one faila.
The error recovery operators will match their argument list to the argumente of the node being executed wo that original problem variables can be bound to the variables in the operators. There will be two ways to invoke these operatora: one for general operators that rolve problems that have been recognised, and one for more specific operatore that act directly on unexpected predicates. Both are described below.
The general operatore will be applied after a MN node in added and problema have been found by the problem recogniser, but before the general replaaner is called (see Figure 1). They will be applied to each of the problems in tura. Like deductive operators, error recovery operatora will have a TRIGGER elot $[10]$ to determine when they should be applied. The trigger will be a combination of keywords and predicates, with the keyworde referriag to the six types of problems. These triggers will match when their keyword matches the problem being tried and any predisite in the trigger matches the appropriate predicates given in the problem. These operators may aleo have normal preconditions, which will be matched (in the normal manner) in the aituation specified by the MN node. If any general error-recovery operator matches a given problem (i.e., both the trigeer and precondition mateh), then the general replanner aimply uses the instructions in the plot of the operator to chooee the replanning setions to perform (rather than applying ite own actions). Thus domain-specific guidance is supplied to the general replanner. It would be eany for such an operator, for example, to always force or prevent a REINSTANTIATE for a certain type of error uader certain conditions.
Specific error-recovery operatort are afylied directly to predicates at they are inputted to the execution monitor before the problem recogniser is called. This thould not be expensive because only oporators mentioned in the ERROR alot of the action beiag executed are tried (eee below). This ability seme attractive, since is can save a lot of effort when there is good domain-depeadent error-recovery information available. When a specific operator matchea an unexpected predicate, it may be poesible in certain domains to aimply apply the operator and aceume that it will colve any problems caused by the unexpected predicate, thus circumventing the normal problem detection mechanism. If this option is chosen, SIPE simply amumes these error recovery operators are correct. Normal operation would involve checking for problems as usual after the application of apecific operatorn.
Nodes in plote of regular operators will be able to apecify an ERROR alot that gives names of apecific error-recovery operators. When a node with an ERROR slot is being executed, the execution monitor will apply the operators linted in the error alot immediately to any anexpected predicate that is inputted during execution. Matching will be the same as for general operatorn, except that there are no keyworda in the trigger. If one of these operatort matches, the replanaing actions in its plot will be carried out immediately. There may or may not be an option to preclude further problem detection on a predicate that hat been so matched.

Ae an example of the intended use of a apecifte arror recovery operator, coander the problom of dropping a cerow that REINSTANTIATE solved in the gemorni replanaer. Suppone it in always dosirable to return to the bopper after droppiag a ecrow duriag the procoen of trameporting it. The aode for moviag OBJECT1 so LOCATION1 would have an error alot that liated the DROP-SCREW operator. This apecific arton-recovary oparator would matel whozaver the haad auddealy becomee ampty and OBJECTI is a cerw. The plot of this operator could then apecify a REINSTANTIATE of the variable OBJECTI (which will still be conatrained to be a screw) followed by an INSERT of a GOAL node to be schioved (AT OBJECTI LOCATIONi). Une of this operator could solve all problems of dropping a scruw so that mee of both the problem recogniser and geaeral replenner could be avoided.

## 7 Examples

This ecetion presente two simple examples of SIPE monitoring the execution of a simple plan, then replanning when thinge do not go an expected. SIPE hae bean teated on larger and more complex problems than thoes presented here. Thay involve a atanderd blocke world with ON and CLEAR prodicaten and a PUTON operator. ae described in more detail eloowhere [10]. The ueer inputs only what is explicitly mentioned in boidface below; averything elee in senerated automatically by the ayotem. Thin firat problem was conatructed to thow the ancceafol une of the REINSTANTIATE rapianaing action, and the escond shows how the system inserts a newly created subplan duriag the replanning process.

Pigure 5 shows the initial world atate and the original problem. The problem is to got A on C in parallel with getting any blue block on any red block. In the initial world B1 and B2 are the only blue blocke (they are both on the table) and R1 and R2 are the only red blocks (R! is on B1 and R2 is on the table and cleas). Since $A$ and $C$ are both clear initially, SIPE quickly finds a twometion plan of putting $A$ on $C$ in parallel with putting B2 on R2, ae shown in Figure 6.
This plan is then given to the execution monitor module of SIPE, which aske if P197 or P168 is to be executed firat. The user types P197 and the ayetom atks for unexpected effects. In thin cace the user types (ON D R2) followed by NIL to show one unexpected effect, namely D has suddealy appeared on top of R2. This croates a MN node after P197 which aleo hae the following effecta deduced by the syatem: -(ON D TABLE) 1 -(CLEAR R2). The problem recogniser is called and it finds only one problem, namely the PHANTOM node Pies in the parallel branch wat being maiatained but is no longer true. This is given to the general replanner which frat trian REINSTANTIATE. This succoede as the OBJECT1 variable in the PHANTOM node an be rebound to R1 without causing any new probleme in the plan. The plan in Figure 7 is paeed from the planning module beck to the execution monitor (without ohowing phantom nodes and mainstep alots). Pl 168 is then executed without any unoxpected effecte and the goal ia achieved. Note that the original plan was retained in its entirety and that B2 wat placed on R1 inatead of R2, thus achiaving the original goal of getting $A$ on $C$ and any blue block on any red block.
The second problem is the same as the frot, excopt that the variable REDBLOCK1 in conatrained not to be R1 (by apecifying IS NOT R1 in the original problem). This will cause REINSTANTIATE to be attempted but fail, since R2 is the only other rei block. The original plan produced by SIPE is the eame and the unexpected situation input by the ueer is the same. The problem rocognizer again paeses the same prublem to the general replanner. This time SIPE tries REINSTANTIATE and faile, 00 it calls RETRY, which csuses the (CLEAR R2) phantom in Figure 6 to be made into a goal. The planaer colves this by producing a plan that pute D back on the table before B1 is placed on R2. The subplan shown in Figure 8 raplaces the (CLEAR R2) phantom node, P165, on the parallel branch before the PU'TON B2 R2 node in Figure 6. Without unexpected eventa, the plan so conatructed then executee correctly to achieve the original goal. Alternatively, more unexpected occurrences could be given during execution of the nawly conatructed plan and SIPE would again go through a similar loop of finding and fixing problems until the original goal is achieved.

## 8 Comparison with Other Systems

There is very little previous work in this area, aince most domain-independent planning aystems du not addrese the problem of replanning. Two that do $|3,6|$ are discussed below. Tute's NONLIN $|8|$, and Vere'r DEVISER $\{9 \mid$ do not concern themselves regarding execution. PLANX10 $|1|$ lista "plan revision strategies" as an arez for future work, but doee not appear to do replanning currently. McCalla and Schneider'a ELMER (5) has a module called the "executor" and claime to take an integrated view of planning and execution. The executor adde more detail to the plan by aimulating execution. For example, secondary plans are added in parallel with the original plan $|4|$ to provide a demon-like capsbility for handling certain aituationa that may arise. Thu is not replanning, but rather a more detailed level of planning, albeit with complex planning operations. The executor effectively produces complex, conditional plans (with poasibly complex parallel interactions) for situations it formees. It does not accept arbitrary input during execution and then replan by changing the original pian as SIPE does. In fact, the authore mention that "to allow replanning after a plan goes awry" $[\mathbf{B} \mid$ is a future atep in their rese arch.
Sunoman's HACKER $|7|$ does modify plans (as do most planners that handle parallel actione), but does not deal with unexpected occurrences. HACKER produces plans that are not correct, then simulates them to detect errors. HACKER then solves some of these errors by using afew simple actions, such as reordering parallel actione or reachieving aubgoale that have not been maintained. Thug, the program is actually dealing with expected, not noexpected, occurrences. SIPE generates correct plans to begin with, then modifies them on the basis of arbitrary unexpected occurrences. What HACKER does with regard to plan modification is analogous to what the critica do in the atandard planning module of SIPE. While some of the problems found by auch critics are similar to those found by the problem detector in SIPE (e.g., previous phantome not being protected), they are only a aubset. SIPE aleo providea a richer set of replanning actions for modifying plans.
The PLANEX system at SR! International $\{2\}$ wat used to monitor the execution of STRIPS plans that were represented in triangle tables. PLANEX doet not do replanning Lecause it never changes the sequence of actiona in the plan. However, it does allow for a weak version of the REINSTANTIATE action in SIPE where a variable can be reinatantiated and the same plan resiarted. Without SIPE's ability to post constraints on variables, this is leas useful. PLANEX usea the triangle table representation to determine the lateat point in the plan where execution could begin in the current situation (unexpected or expected), including both the executed and unexecuted portione of the plan in this calculation. If unexpected occurrences create a situation in which reatarting the plan from some point other than the current execution point would colve the problem, PLANEX would do this. (Note this may involve redoing previous actions or skipping actiona that had been planned.)
Although PLANEX can reatart the original plan at a different point, this chould not be construed as replanning. Moreover, it is not likely to be useful in a realistic domain. The world is not so benign an to frequently have onexpected cecurrences produce situations in which ones's original plan is still applicable exactly as is from some point. With very high-level examples (as in $|2|$ ), this may occacionally happen, but it will happen only rarely with detailed plans. For example, an action such as "pick up block $B$ (wherever it may bo)" can simply be repeated when $B$ is accidentally dropped and its new location is unknown. However, if the robot must plan to go to the location of $B$ before picking it up, the original plan will be applicable only in the unlikely event that $B$ is accidentally dropped onto its original location.
Hayes's aystem [3] and Sacerdoti'n NOAH | 6 | have addreased the replanning problem. However, the approaches used in both these aytems are considerably simpler and less powerful than that of SIPE. For example, NOAH does not allow the input of arbitrary predicates, so the general replanning problem never


Figure S: Initial Blocke World and Problem to be Solved


Figure 0: Initial Plan Produced by SIPE


Figure 7: New Plan Produced for Continuing Execution


Figure 8: Subplan for Replacing PHANTOM P185
arieea. It does permit the user to aperify that whole wedgen had been executed at once, and allows a node that han juat been executed to be planned for again if it faile. This ensentially provides one limited replanaing ection that in ueful only in very apecific situstione.

Hayes's aystem does allow the input of some information about unexpected situationa. It is not clear what types of information can be provided, but they appear lese general than the arbitrary predicates sceepted by SIPE. The system's only replanning action is to delate part of the plan. Thie permita the planner to renchieve higher-level goale, but they muat be the aame higher-level goale that wore already present in the plan. The ayotem deletes everything that dapended on any effect of a decision that in no loager valid. This will in general be wasteful, since mach of the plan may be removed unneceagrily. If only one of the many affects of an action has failed, subpians dependiag on the unfailing affects do not need to be deleted. SIPE would keap such subplane in the plan (and find any probleme that may have beon generated within them).
SIPE provides a much more poweful replanaing capability than aither of these oyotems. It allows the input of arbitrary predicatea, computen the exteat to which thee affect the reat of the plan, only finde complections that are rually problomatical, and uaes a large aumber of replanniag actions (including REINSTANTIATE) to remedy problems in waye that enable much of the original plan to be maintained.

## 9 Conclusion

### 9.1 Summary

Given correct information about unexpected evente, SIPE is able to determine how this affecte the plan boing executed. In manv cases, it in able to rotain most of the original plan by making changes in it to avoid problome caused by these unexpected evente. It in aleo capable of ahortaniag the orisinal plan when cerendipitous events oceur. It cannot solve di. icult problems involving drastic changea to the expected atate of the world, bat it does handle many types of amall errora that may crop up frequently in a mobile pobot domain. The execution-monitoring package doee this without the necescity of planniag in advance to check for such errore.

The major contribution of thie work is the development of a general set of replanaing actione that are used an the basis of an automatic replanner, as woll as the basis of a language for apecifying domain-dependent error recovery information. These actions provide sufficient power to alter plane in a way that often retains much of the original plan, (e.s., the REINSTANTIATE action). The general replanner sttempts to solve all problema that are found. It is unlikely to be very succeaful unless it is adapted to particular domains. The desipn of the language for error recovery operatore allows both for operatore that will hande very apecific situations and for thue that will give more general advice to the replanner.

The success of these mechaniams can largely be attributed to takir. advantage of the rich atructure of SIPE'a planner and its plans. Often, the raplanner calle the atandard planning system as a subroutine. In this way it can take advantage of the efficient frame-reaconing mechaniams in SIPE to discover problems and potential fixes quickly, applying its deductive capabilities to provide a reasonable eolution to the truch maintenance problem. The fxee suggested by the repianner need involve only the insertion of acw coals inte the plan, since calliag the planner an a subroutine will solve these goale in a manner that amures there will be no conflicts with the reat of the plan. SIPE' execution-monitoring capabilitiee make extemive uce of the explicis representation of plan rationale. The problem detector makes uses of the information encoded in MAINSTEP elote, phantome, and preconditions to quickly find all the problems with a plan. Furthermore, it does not remove pares of the original plan unlese the parts are actually problematicel. The raplanning actions make uee of conatraints, alternative contexte, and wedges in SIPE whenovar they consider removing part of the plan.

### 9.2 Issucs and Limltations

From the beginning, the rationale behind SIPE her been to place enough limitations on the representation so that planaing can be done efficiently, while retaining enough power to atill be useful. This motivation underlies most of the design decisione that have been made ia implementing the aystem, including the detign of the replanning module. For example, REINSTANTIATE and POP-REMOVE are limited to prevent the exploration of large search apacee. The use of SIPE's deductive capability to solve the truth maintenance problem also reflects our commitment to this deaign philosophy. The replanning capabilities have proved useful in two test domains.
The major limitations of this research stem from the assumption of correct information about unexpceted events. This avoide many dificult problems, the most important of which in generating the high-level predicatea used by SIPE from information provided by the ceasore. This appears to be the most critical isaue in getting a high-level planner such as SIPE to control a mobile robot. Part of the problem in heuristic adequacy - the robot cannot wait ten minutes for a vision module to turn pixels into predicatea while the world ia changing. Other questions that have not been diacusced here are deciding how much effort to expend checking facts that may be suspect, and modeling uncertain or unreliable sensors. Finding solutivas to these problems is , $\{$ crucial importance to the task of endowing a mobite robot with execution-monitoring rapabilitiea.

## Acknowledgments

Many people influenced the ideas expressed in this paper. Special thanks go to Michael Georgeff for many enlightening discuasions.

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# Incremental Planning to Control a Blackboard-Based Problem Solver 

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#### Abstract

To control problem solving activity, a planner must resolvguncertainty about which specific long-term goals (solutions) to pursue and about which sequences of actions will best achieve those goals. In -this piper, we thesritice a planner/that abstracts the problem solving state to recognize possible competing and compatible solutions and to roughly predict the importance and expense of developing these solutions. With this information, the planner plans sequences of problem solving activities that most efficiently resolve its uncertainty about which of the possible sosibtions to work toward. The planner only details actions for the near future because the results of these actions will influence how (and"whether) a plan should be pursued. As problem solving proceeds, the planner adds new details to the plan incrementally, and monitors and repairs the plan to insure it achieves its goals whenever possible. Through experiments, illustrate how these new mechanisms significantly improve problem solving decisions and reduce overall computation. What briefly discuss owe currentrescarch directions, including how these mechanisms can improve a problem solver's real-time response and can enhance cooperation in a distributed problem solving network.


## 1. Introduction



A problem solver's planning component must resolve control uncertainty stemming from two principal sources. As in typical planners, it must resolve uncertainty about which sequence of actions will satisfy its long-term goals. Moreover, whereas most planners are given a (possibly prioritized) set of well-defined long-term goals, a problem solver's planner must cften resolve uncertainty about the goals to achieve. For example, an interpretation problem solver that integrates large amounts of data into "good" overall interpretations must use its data to determine what specific long-term goals (overall interpretations) it should pursue. Because the set of possible interpretations may be intractably large, the problem solver uses the data to form promising partial interpretations and then extends these to converge on likely complete interpretations. The blackboard-based problem solving architecture developed in Hearsay-II permits such data-directed problem solving [1].

In a purely data-directed problem solver, control decisions can be based only on the desirability of the expected immediate results of each action. The Hearsay-II system developed an algorithm for measuring desirability of actions to better focus problem solving 2|. Extensions to the blackboard architecture unify data-directed and goal-directed control by representing possible extensions and refinements to partial solutions as explicit goals, 3|. Through goal processing and subgoals, sequences of related actions ran be triggered to achieve important goals. Further modifications separate control knowledge and decisions from problem solving activities, permitting the choice of problem solving actions to be influenced by strategic considerations t. However. note of these approaches develop and use a high-level view of the current problem solving situation so that the problem solver can recognize and work toward more specific long-term goals.

In this paper, we introduce new mechanisms that allow a blackboard-based problem solver to form such a high-level view. By abstracting its state. the problem solver can recognize possible competing and compatible interpretations, and can use the abstract view of the data to roughly predict the importance and expense of developing potential partial solutions. These mechanisms are much more flexible and complex than those we previously developed : 5 ; and allow the recognition of relationships between distant as well as nearby areas in the solution space. We also present new mechanisms that use the high-level view to form plans to achieve long-term goals. A plan represents specific actions for the near future and more general actions for the distant future. By forming detailed plans only for the near future, the problem solver does not waste time planning for situations that may never

[^7]arise; by sketching out the entire plan, details for the near-term can be based on a long-term view. As problem solving proceeds, the plan must be monitored (and repaired when necesary), and new actions for the near future are added incrementally. Thus, plan formation, monitoring, modification, and execution are interleaved $\operatorname{~} 6,7,8,9,10 \mid$.

We have implemented and evaluated our new mechanisms in a vehicle monitoring problem solver, where they augment previously developed control mechanisms. In the next section, we briefly describe the vehiele monitoring problem solver. Section 3 provides details about how a high-level view is formed as an abstraction hierarchy. The representation of a plan and the techniquea to form and dynamically modify plans are presented in Section 4 . In Section 5 , experimental results are discussed to illustrate the benefits and the costs of the new mechanisms. Finally, Section 6 recapitulates our approach and describes how the new mechanisms can improve real-time responsiveness and can lead to improved conperation in a distributed problem solving network.

## 2. A Vehicle Monitoring Problem Solver

A vehicle monitoring problem solving node, as implemented in the Distributed Vehicle Monitoring Testbed (DVMT), applies simplified signal processing knowledge to acoustically sensed data in an attempt to identify, locate, and track patterns of vehicles moving through a two-dimensional space 11. Each node has a blarkboard-based problem solving architecture, with knowledge sources and levels of abstraction appropriate for vehicle monitoring. A knowledge source (KS) performs the basic problem solving tasks of extending and refining hypotheses (partial solutions). The architecture includes a goal blackboard and goal processing mudule, and through goal processing a node forms knowledge source instantiations (KSls) that represent potential KS applications on specific hypotheses to satisfy certain goals. KSls are prioritized based both on the estimated beliefs of the hypotheses each may produce and on the ratings of the goals each is experted to satisfy. The goal processing component also recognizes interactions between goals and adjusts their ratings appropriately; for example. subgoals of an important goal might have their ratings boosted. Goal processing can therefore alter Kisl rankings to help focus the node's problem solving actions on achieving the subgoals of important goals 3 .

A hypothesis is characterized by one or more time-lorations (where the vehicle was at discrete sensed times), by an event-elins (classifying the frequency or vehicle type). by a belief (the confidence in the accuracy of the hypothesis), and by a blackboard-level (depending on the amount of processing that has been done on the data). Synthesis KSs take one or more hypotheses at one blackboard-level and use event-class constraints to generate hypotheses at the next higher blackboard-level. Extension KSs take several hypotheses at a given blackboard-level and use constraints about allowable vehicle movements (maximum velocities and accelerations) to form hypotheses at the same blackboard-level that incorporate more time-locations.

For example. in Figure 1 nach blackboard-lezel is represented as a surface with spatial dimensions $x$ and $y$. At blackboardlevels (signal level) there are 10 hypotheses, each incorporating a single time-location (the time is indicated for each). Two of these hyperbeses have been synthesized to blarkboard-level $g$ (group level). In turn, these hypotheses have been synthesized :o blackboard-level $v$ (vehicle level) where an extension $K S$ has connected them into a single track hypothesis, indicated graphically by connecting the two tocations. Problem solving proceeds from this point by having the goal processing component form goals (and subgoals) to extend this track to time 3 and instantiating KSls to arhieve these goals. The highest rated pending KSI is then invokod and triggers the appropriate KS to execute. New hypotheses are posted on the hlackboard, causing further goal processing and the rycle repeats until an acceptable track incorporating data at each time is created. One of the potential solutions is indicated at blackboard-level $v$ in Figure 1.

 tumes llypotheses at ligher blackboard-levels are synthesized from lower level data. and 1 petentiai whition is allistrated with a hoted track at blackboard-level $v$

Figure 1: An Example Problem Solving State.

## 3. A High-level View for Planning and Control

Planning about how to solve a problem often requires viewing the problem from a different perspective. For example, a chemist generally developes a plan for deriving a new compound not by entering a laboratory and envisioning possible sequences of actions but by representing the problem with symbols and using these symbols to hypothesize poseible derivation paths. By transforming the problem into this representation, the chemist can more easily sketch out possible solutions and spot reactions that lead nowhere, thereby improving the decisions about the actions to take in the laboratory.

A blackboard-based. vehicle monitoring problem solver requires the same capabilities. Transforming the node's problem solving state into a suitable repreaentation for rlanning requires domain knowledge to recognize relationshipa-in particular, long-term relationships - in the data. This transformation is accomplished by incrementally clustering data into increasingly abstract groupa based on the attributes of the data: the hypotheses can be clustered based on one attribute, the resulting clusters can be further clustered based on another attribute, and so on. The transformed representation is thus a hierarchy of clusters where higher-level clusters abstract the information of lower-level clusters. More or less detailed views of the problemsolving situation are found by accessing the appropriate level of this abstraction hierarchy, and clusters at the same level are linked by their relationships (such as having adjacent time frames or blackboard-levels, or corresponding to nearby spatial regions).

We have implemented a set of knowledge-based clustering mechanisms for vehicle monitoring, each of which takes clusters at one level as input and forms output clusters at a new level. Each mechanism uses different domain-dependent relationships, including:

- temporal relationships: the output cluster combines any input clusters that represent data in adjacent time frames and that are spatially near enough to satisfy simple constraints about how far a vehicle can travel in one time unit.
- spatial relationships: the output cluster combines any input clusters that represent data for the same time frames and that are spatially near enough to represent sensor noise around a single vehicle.
- blackboard-level relationships: the output cluster combines any input clusters that represent the same data at different blackboard-levels.
- event-class relationships: the output cluster combines any input clusters that represent data corresponding to the same event-class (type of vehicle).
- belief relationships: the output cluster combines any input clusters that represent data with similar beliefs.

The abstraction hierarchy is formed by sequentially applying the clustering mechanisms. The order of application depends on the bias of the problem solver: since the order of clustering affects which relationships are most emphasized at the highest levels of the abstraction hierarchy, the problem solver should cluster to emphasize the relationships it expects to most significantly influence its control decisions. Issues in representing bias and modifying inappropriate bias are discussed elsewhere :12!.

To illustrate clustering, consider the clustering sequence in Figure 2, which has been simplified by ignoring many cluster attributes such as event-classes, beliefs, volume of data, and amount of pending work; only a cluster's blackboard-levels (a cluster can incorporate more than one) and its time-regions (indicating a region rather than a specific location for a certain time) are discussed. Initially, the problem solving state is nearly identical to that in Figure 1, except that for each hypothesis in Figure 1 there are now two hypotheses at the same sensed time and slightly different locations. In Figure 2a, each cluster $c_{n}^{l}$ (where $l$ is the level in the abstraction hierarchy) corresponds to a single hypothesis, and the graphical representation of the clusters mirrors a representation of the hypotheses. By clustering based on blackboard-level, a second level of the abstraction hierarchy is formed with 19 clusters (Figure 2b). As is shown graphically, this clustering "coliapses" the blackboard by combining clusters at the previous abstraction level that correspond to the same data at different blackboard-levels. In Figure 2c, clustering by spatial relationships forms 9 clusters. Clusters at the second abstraction level whose regions were close spatially for a given sensed time are combined into a single cluster. Finally, clustering by temporal relationships in Figure 2d combines any clusters at the third abstraction level that correspond to adjacent sensed times and whose regions satisfy weak vehicle velocity constraints.

The highest level clusters, as illustrated in Figure 2d, indicate four rough estimates about potential solutions: a vehicle moving through regions $R_{1} R_{2} R_{3} R_{4} R_{5} R_{5 ;}$. through $R_{1} R_{2} R_{3} R_{1} R_{5}^{\prime} R_{5}^{\prime}$, through $R_{1}^{\prime} R_{2}^{\prime} R_{3} R_{4} R_{5} R_{6}$, or through $R_{1}^{\prime} R_{3}^{\prime} R_{3} R_{4} R_{5}^{\prime} R_{6}^{\prime}$. The problem solver could use this view to improve its control decisions about what short-term actions to pursue. For example. this view allows the problem solver to recognize that all potential solutions pass through $R_{3}$ at sensed time 3 and $R_{4}$ at sensed time 4. By boosting the ratings of KSIs in these regions. the problem solver can focus on building high-level results that are most likely to be part of any eventual solution.

In some respects. the formation of the abstraction hierarchy is akin to a rough pass at solving the problem, as indeed it must be if it is to indicate where the possible solutions may lie. However, abstraction differs from problem solving because it ignores many important constraints needed to soive the problem. Forming the abstraction hierarchy is thus much less computationally expensive than problem solving, and results in a representation that is too inexact as a problem solution but is suitable for control. For

(a)

(b)

| Cluster | Time-regions | Bleckbourlt- Sube lunters levels | [ ${ }_{1}$ |
| :---: | :---: | :---: | :---: |
| $c_{i}^{3}$ | $\left(1 R_{1}\right)\left(2 R_{2}\right)$ |  | [ $A_{3}$ 里 |
| ${ }^{\frac{1}{2}}$ | $\left(1 R_{1}^{\prime}\right)$ $\left(, R_{3}^{\prime}\right)$ | $c_{3}^{2}, c_{4}^{2}$ | $R_{2} R_{0}$ |
| $c_{1}$ | i $2 R_{j}^{\prime}$ ) | $3 \quad$ cinc | $\left[\overrightarrow{R_{2}^{\prime}}\right]$ <br> $R$ R |
| . 3 | $\left(6 R_{6}^{\prime}\right)$ | cis.ris | R $R_{0}^{0}$ |

(c)

(d)

A sequence of clustering steps are illustrated both with tablea (left) and graphically (right). ${ }_{6}$ represents cluster 1 at level $/$ of the abstraction hierarchy. In (a), each cluster is a hypothesis. These are clustered by blactboard-level to get (b); note that graphically the levels have been collapsed into one. These clunters are then grouped by apatial relationahips to form (c). which in turn is clustered by temporal relationships to form (d).

Figure 2: An Example of Incremental Clustering.
example, although the high-level clusters in Figure $2 d$ indicate that there are four potential solutions, three of these are actuaily impossible based on the more stringent constraints applied by the Kis. The high-level view afforded by the abstraction hierarchy therefore does not provide answers but only rough indications about the long-term promise of various areas of the solution space. and this additional knowledge can be employed by the problem solver to make better control decisions as it chooses its next task.

## 4. . Incremental Planning

The planner further improves control decisions by intelligently ordering the problem solving actions. Even with the highlevel view, uncertainty remains about whether each long-term goal can actually be achieved, about whether an action that might contribute to achieving a long-term goal will actually do so (since long-term goals are inexact), and about how to most economically form a desired result (since the same result can often be derived in different ways). The planner reduces control uncertainty in two ways. First, it orders the intermediate goals for achieving long-term goals so that the results of working on earlier intermediate goals can diminish the uncertainty about how (and whether) to work on later intermediate goals. Second, the planner forms a detailed sequence of steps to achieve the next intermediate goal: it determines the least costly way to form a result to satisfy the goal. The planner thus sketches out long-term intentions as sequences of intermediate goals, and forms detailed plans about :he best way to achieve the next intermediate goal.

A long-term vehicle monitoring goal to generate a track consisting of several time-locations can be reduced into a series of internediate goala, where each intermediate goal represente a desire to extend the track satisfying the previous intermediate goal inth a new time-location.' To determine an order for pursuing the poseible intermediate goala, the planner currently usea three domain-independent heuristica:

Heuristic-1 Prefer common intermediate goala. Some intermediate goals may be common to several long-term goala. If uncertain about which of these long-term soala to pursue, the planner can poatpone its decision by working on common intermediate goals and then can use these results to better distinguish between the long-term goals. This heuriatic is a variation of leat-commitment [13].

Heuristic-2 Prefer lesa costly intermediate goale. Some intermediate gosh may be more coatly to achieve than others. The planner can quickly estimate the relative coats of developing resulta in different areas by comparing their corresponding cluaters at a high level of the abstraction hierarchy: the number of event-claness and the spatial range of the data in a cluater roughly indicates how many potentially competing hypotheses might have to be produced. This heuristic causes the planner to develop results more quickly. If these results are creditable they provide predictive information. otherwise the planner can abandon the plan after a minimum of effort.

Heuristic-3 Prefer diseriminative intermediate goals. When the planner must discriminate between possible long-term goals, it should prefer to work on intermediate goals that most effectively indicate the relative promise of each long-term goal. When no common intermediate goals remain. therefore, this heuristic triggers work in the areas where the long-term goals differ most.

These heuristics are interdependent. For example, common intermediate goals may also be more costly, as in one of the experiments described in the next section. The relative influence of each heuristic can be modified parametrically.

Having identified a sequence of intermediate goals to achieve one or more long-term goals, the planner can reduce its uncertainty about how to satisfy these intermediate goals by planning in more detail. If the planner possesses models of the KSs that roughly indicate both the costs of a particular action and the general characteristics of the output of that action (based on the characteristics of the input), then the planner can search for the best of the alternative ways to satisfy an intermediate goal. We have provided the planner for our vehicle monitoring problem solver with coarse KS models that allow it to make reasonable predictions about short sequences of actions to find the sequences that best achieve intermediate goals. ${ }^{2}$ To reduce the effort spent on planning, the planner only forms detailed plans for the next intermediate goal: since the results of earlier intermediate goals influence decisions about how and whether to pursue subsequent intermediate goals, the planner avoids expending effort forming detailed plans that may never be used.

Given the abstraction hierarchy in Figure 2, the planner recognizes that achieving each of the four long-term goals (Figure 2d) entails intermediate goals of tracking the vehicle through these regions. Influenced predominantly by Heuristic-1, the planner decides to initially work toward all four long-term goals at the same time by achieving their common intermediate goals. A detailed sequence of actions to drive the data in $R_{3}$ at level $s$ to level $v$ is then formulated. The planner creates a plan whose attributes (and their values in this example) are:

- the long-term goals the plan contributes to achieving (in the example. there are four);
- the predicted, underspecified time-regions of the eventual solution
(in the example, the time regions are ( $1 \quad R_{1}$ or $\left.R_{t}^{\prime}\right)\left(2 R_{2}\right.$ or $\left.R_{2}^{\prime}\right)\left(3 R_{3}\right) \ldots$ );
- the predicted vehicle type(s) of the eventual solution (in the example, there is only one type of vehicle considered):
- the order ot : rmediate goals (in the example, begin with sensed time 3, then tume 4 , and then work both backward to earlier times and forward to later times):
- the blackboard-level for tracking, depending on the available knowledge sources (in the example, this is level $v$ );
- a record of past actions, updated as actions are taken (initially empty);
- a sequence of the specific actions to take in the short-term (in the example. the detailed plan is to drive data in region $R_{3}$ at level sto level $u$ ):
- a rating based on the number of long-term goals being worked on, the effort already invested in the plan, the average ratings of the KSIs corresponding to the detailed short-term actions. the average belief of the partial solutions previously formed by the plan, and the predicted beliefs of the partial solutions to be formed by the detailed activities.

[^8]As each predicted action is conseculively pursued, the record of past actions is updated and the actual rasulta of the action are compared with the general characteristica predicted by the planner. When these agree, the next action in the detailed short-erm sequence is jerformed if there is one, otherwise the planaer develope another detailed sequence for the next intermediate goal. In our example, after forming reaulta in $R_{3}$ at a high blackboard-level, the planner forma a sequence of actions to do the same in $R_{8}$. When the actual and predicted resulte dinagree (oince the planner's modete of the KSe may be inaceurate), the planmer meat modify the plan by introducing additional actiona that can ges the plan beck on track. Wno such actions exist, the plan ia abortad and the next highest rated plan is pursued. U the planner exhausta its plans before forming a complete solution, it reforme the abatraction hierarchy (incorporating new information and/or cluatering to streme different problem attribetas) and attempta $\omega$ find new plans. Throughout this paper, we asume for simplicity that no important new information arrives after the abstraction hierarchy is formed: when part of a more dynamix environment, the node will updace ite abatraction hierarchy and plans whenever such information becomes available.

The planner thus generates, monitors, and revises plans, and interleaves these activities with plan execution. in our example. the common intermediate goals are eventually satiffied and a separate plan must be formed for each of the alternative ways to proceed. After finding a partial trark combining data from sensed times 3 and 4 . the planner decidea to extend this track backward to sensed time 2. The long-term goals indirate that work sho ild be done in either $R_{1}$ or $R_{2}$. A plan is generated for each of the two possibilities, and the more highly rated of these plans is followed. Note, however, that the partial track already developed can provide predictive information that, thrnugh goal processing. can increase the rating of work in one of these regiona and not the other. In this casp, constraints that limit a vehicte's turning rate are used when goal processing (subgoaling) to increase the ratings of KSI's in $\boldsymbol{R}_{2}^{\prime}$, thus making the :plan to work there next more highly rated. ${ }^{3}$

The planner and goal processing thus work in tandem to improve problem solving ;erformance. The goal procensing uses a detailed virw of loral interactions hetween hypotheses, goals. and KSls to differentiate between alternative actions. Goal processiag can be computationally wasteful, however. when it is invoked based on strietly local eriteria. Without the knowledge of long-term reasons for building a hypothosis, the problem solver simply formagonis to extend and refine the hypothesis in all poasible ways. These goals are further processed (subgnaled) if they are at certain blackboard-levels, again regardless of any long-term justification for doing so. With its long-term view. the planner can drastically reduce the amount of goal processing. As it pursues, monitors. and repairs plans. life planner identifies areas where goals and subgoals rould improve its decisions and selectively invokes goal procrssing to form only those goals that it needs. As the experimental results in the next section indicate. providing the planner with the ability to control goal processing can dramatically reduce control overhead.

In summary. we have developed mechanisms that permit incremental planning of problem oolving artivities in a blackboardbased problem solver. These mechanisms interleave planning and execution, monitoring plans and replanning when necesaary. We base these merhanisms on having a high-ievel, long-term view of problem solving and on having acceptable modela of problem solving artions. Furthermore. note that incremental planning may be inappropriate in domains where details about actions in the distant future can highly constrain the options in the near future. In these domains, constraints must be used to detail an entire plan before acting 13 . However. in unpredictable domains, incremental planning, plan monitoring, and plan repair are crucial to effective control since plans about the near future cannot depend on future states that may never arrive.

## 5. Experiments in Incremental Planning

We illustrate the advantages and the costs of our planner in several problem solving situations, shown in Figure 3. Situation $A$ is the same as in Figure 2 except that each region only has one hypothesis. Also note that the data in the common regions is most weakly sensed. In situation B. no areas are common to all possible solutions, and issues in plan monitoring and repair are therefore stressed. Finally, situation C has many potential solutions, where each appears equally likely from a high-level view.

When evaluating the new mechanisms. we consider two important factors: how well do they improve control decisions (reduce the number of incorrec: decisions), and how much additional overhead do they introduce to achieve this improvement. Since each control decision causes the invocation of a KSI. the first factor is measured by counting KSls invoked-the fewer the KSIs, the better the control decisions. The second factor is measured as the actual computation time (runtime) required by a node to solve a problern, representing the combined costs of problem volving and control computation.

The experimental results are summarized in Table 1. To determine the effects of the new mechanisms, earh problem situation was solved both with and without them, and for each case the number of KSls and the computation time were measured. We abo measured the number of goals generated during problem solving to illustrate how control overhead can be reduced by having the planner control the goal processing.

[^9]

A


B


C


Three problem sulviag sut ratione are ilmplayed The pamble tracke (fownd in the aboeraction
 for earh attulimg afe elven

Figure 3: The Experimental Problean Situations.

Fixperimenta E. 1 and E2 illustrate how the new mechanisms can dramatically redure both the number of KSle involed and the computation time needed to solve the problem in situation $A$. Without these mechanisms (El), the problem solver begins with the most highly sensed data $\left(d_{1}, d_{2}, d_{5}\right.$, and $\left.d_{n}\right)$. This incorrect data actually corresponds to notse and may have been formed due to sensor errors or echoes in the sensed area. The problem solver attempts to combine this data through $d_{3}$ and $d_{4}$ but fails berause of turning constraints, and then it uses the results from $d$ and $d_{4}$ to eventually work its way back out to the moderately sensed correct data. With the new mechaniams (E2), problem soiving begins at $d_{3}$ and $d_{4}$ and, because the tract formed ( $d_{3} d_{4}$ ) iriggers goal procesaing to stimulate work on the moderate data, the solution is found much more quickly (in fact, in optamal time 14). The planner controls goal processing to zenerate and process only those goals that further the plan; if goal proceasing is done independently of the planner (E3), the ovephead of the planner coupled with the only slightly diminished goal proceasing overhead (the number of goals is only modestly reduced, comparing E3 with E1) nullifies the computation time saved on actual problem solving. Moreover, because earlier, leas constrained goals are subgoaled, control decisions deteriorate and more KSLa must be invoked.

The improvements in experiment $E 2$ were due to the initial work done in the common areas $d_{3}$ and $d_{4}$ triggered by Heuristic-l. Situation $A^{\prime}$ is identical to situation $A$ except that areas $d_{3}$ and $d_{4}$ contain numerous competing hypotheses. If the planner initially works in those areas (E5), then many KSls are required to develop all of these hypotheses - fewer KSls are invoked without planning at all (EA). However, by estimating the relative costs of the alternative intermediate goals, the planner can determine that $d_{3}$ and $d_{1}$. although twice as common as the other areas, are likely to be more than twice as costly to work on. Heuristic- 2 overrides Hruristic-1. and a plan is formed to tevelop the other areas first and then use these results to more tightly control processing in $d_{1}$ and $d_{4}$. The number of KSis and the computation time are thus redured (E6).

| Expe | Situ | Plan? | KSLs | Rtime | Goala | Commert: |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| El | 1 | no | 58 | 1\%2 | 262 | - |
| $E 2$ | 1 | yes | 24 | 31 | 49 | - - |
| E. 3 | 1 | yes | 32 | 194 | 203 | independent goal procestina and planame |
| F. 4 | $\mathrm{A}^{\prime}$ | no | 58 | 191 | 284 | nowe in $d_{1}, d_{4}$ |
| F.S | $A^{\prime}$ | yea | 54 | 173 | 112 |  |
| Es | $\mathrm{A}^{+}$ | yeu | 18 | 16.5 | 71 |  |
| $\because$ | B | no | $\rightarrow 1$ | :14 | 371 | - |
| Es | B | yea | 45 | 11.8 | 40 | - |
| E9 | 8 | yes | 45 | 306 | 257 | independent goal proceating and planman |
| E10 | $C$ | no | 85 | 29.8 | 465 | - |
| EII | C | yes | 44 | 19.3 | 75 | - |
| Legend |  |  |  |  |  |  |
| Situ: |  | The problem situation. |  |  |  |  |
| Plan?: |  | Are the new planaine mechanisms used? |  |  |  |  |
| KSla: |  | Number of KSIs tavoked to find solution. |  |  |  |  |
| Reime: |  | The cotal runtime (computation time) to had solution (in minuted). |  |  |  |  |
| Goala: |  | The number of goats formed and processed. |  |  |  |  |
| Commerta: |  | Additional apecte of the experiment. |  |  |  |  |

Table 1: A Summary of the Experimental Results.

In situation B, two solutions must be found, corresponding to two vehicles moving in parallel. Without the planner (E7), problem solving begins with the most atrongly sensed data (the noise in the center of the area) and works outward from there. Only after many incorrect decisions to form short tracks that cannot be incorporated into longer solutions does the problem solver generate the two solutions. The hish-level view of this situation, as provided by the abstraction hierarchy, allows the planner in experiment E\& to recognize six possible alternative solutions, four of which pasa through $d_{3}^{\prime}$ (the most common area). The planner initially forms $\boldsymbol{\rho}_{\text {an }} n_{1}$. plan $n_{1}$, and $\boldsymbol{p l a n} n_{3}$, beginning in $d_{3}, d_{3}$, and $d_{1}$ reapectively (Heuristic-l trigers the preference for $d_{1}$, and subeequently Heuristic- $\mathbf{3}$ indicates a preference for $d_{s}$ and $d_{j}$ ). Since it covers the most long-term goals, plan $\boldsymbol{p}_{1}$ is pursued first-a reasonable strateay because effort is expended on the solution path if the plan succeeds, and if the plan fails then the larteat pomitle number of candidate solutiona are eliminated. After developing $d_{3}$, plan, is divided into iwo plans to combine thia data with either $d_{2}$ or $d_{2}$. One of these equally rated plans is chowen arbitrarily and forms the track $d_{2} d_{1}$, which then muat be cornbined with $d_{1}$. However. because of vehicie turning conatrainta, only $d_{1} d_{1}$ father than $d_{1} d_{2} d_{3}$ is formed. The plan monitor hage an error, an attempt to repair the plan faila, and the plan aborta. Similarly, the plan to form $\mathcal{C}_{1} \mathcal{C}_{2} \mathcal{C}_{4}$ eventually aborts. Plang ia thea invoked, and after developing $d_{1}$ it finda itat $d_{2}$ han already been developed (by the first aborted plan). However, the plan monitor detects that the predicted reault, $d_{1} d_{2}$ wat not formed, and the plan is repaired by inserting a new action that takea advantage of the previous formation of $d_{1} d_{1}$ to generate $d_{1} d_{2} d_{s}$. The predirtions are then more than satisifed, and the plan continuea until a solution in formed. The plan to form the other solution is s . . fly succemafully completed. Finally, note once again chat, if the planner does not control goal proresaing (E9), unnecesaary overhead costa are incurred, alchough this time the control decisions (KSis) are not degraded.

Situation C also represents two vehicles moving in parallel, but this time they are clower and the data pointa are all equally well sensed. Without the new mechaniame (E10), control decisions in this situation have little to so on: from a local perapeative. one area looks as good an another. The problem solver thus develops the data points in parallel, then forma all tracks between pairs of points, then combinen these into larger tracks, until finally it forms the two solution tracks. The planner usea the possible solutiona from the abatraction hierarchy to focus on generating longer tractar woner, and by monitoring its actions to extend its tracks, the planner more quickly recognizes failed extensiona and redirecta proceasing toward more promising extenaions. The new mechanisms thus improve control decisions (reduce the KSta) without adding exceasive computational overhead (Ell). However, the planner must consider 32 possible solutions in this case and does incur significant overhead. For complex situations, additioaal control mechanisms may be needed by the planner to more flexibly manage the large numbers of powibilities.

## 6. The Implications of Abstraction and Planning

We have deacribed and evaluated merhanisms for improving control decisions in a blackboard-hased vehicle monitoring problem solver. Our approach is to develop an abstract view of the current problent solving situation and to use chia view to better predict both the long-term significance and cost of alternative actions. By recognizing and planning to achieve long-term goals, problem solving is more forused. By using the abstraction hierarchy when making planning decisions, problem solving can be more cost effective. Finally, by interleaving plan generation, monitoring, and repair with plan execution, the mechanisms lead to more versatile planning, where actions to achieve the system's (problem solving) goals and actions to satisfy the planner's needs (resolve ts own uncertainty) are integrated into a single plan.

This approach can be generally applied to blackhoard-based problem solvers. Abstraction requires exploiting relationships in the data - relationships that are used by the knowledge sources as well-such as allowable combinations of speech sounds 1 or how various errands are related spatially or temporally $t 4^{4}$. Planning requires simple models of KSs , recognition of intermediate goals (to extend a phrase in speech. to add another errand to a plan), and heuristics to order the intermediate goals. We believe that :nany if not all blackboard-haved problem solvers/and more generally, problem solvers whose long-term goals depend on their current vituationl could incorporate similar abstrartion and planning mechanisms to improve their control decisions.

The benefits of this approach extend beyond the examples demonstrated in this paper. For exampie, goal satisfaction and problem wolving termination are important issues in blackboard-based problem solvers. Given its underspecified goals of forming "good" wolutions with its input. how does the probiem solver recognize when it has found such solutions or when it can improve d solution? The more global view of the problem provided by the abstraction hierarchy helps the problem solver discover areas where improvements are possibie and potentially worthwhile. The enumeration of possible solutions, and the success or failure to achueve them. similarly improves the problem solver's ability to determine when a solution is the best of the possible alternatives.

These mechanisms also nelp a problem solver to make informed decisions about how best to solve a problem inder real-time constraints. The KS models provide estimates of the cost (in time) of possibie activities so that the amount of time to achieve the next intermediate goal can be predicted. By exploiting the similarities between intermediate goals, moreover, these predictions can be generalized over all intermediate goals (making allowances for more or less costly areas as indicated by the abstraction hierarchy) and the tirne needs for the entire plan can be predicted. With this prediction, the planner can modify the plan (eliminate actions that unnecessarily increase the belief in a hypothesis, replace expensive actions with actions that inexpensively achieve leas exact results) until the predicted time costs satisfy the time constraints.

[^10]Finelly, plaanias and prediction are vital to cooperation amons problem solvers. A networt of such problem solvers that are cooperatively solving a single problem could communicate about their piana, indicating what partial solutinas they expect to generate and whea. With this information, aech problem solver can coordinate its activitiea with the others to generate and exchange moful reutts more enficiently, thereby improving setwork problem solvias performance $(12,14,3)$. In emence, the problem solvers together form a diatributed plan. The uee of incremental plaaning, plan monitoriag, and plan repair in particularly appropriate in such domaize due to the inherent unpredictability of future actions and interactions.

The mochaniense we have outlined in thie paper provide the besis for theee posibilities. We are currently augmenting the mechanisue with capabilisime to perform in more complex, dyanmic enviroaments: to modify the abetraction hierarchy when important asexpected situationa arive and to model and plan for poleatial future situationa (the arrival of more data to be procemed, the actions of other problem solvers). Our new mechaniama, though they addrem imuen previously neglectod, should be intagrated with other control techaiques to be fully Rexible, an seen in experiment Eli. The combination of our mechanisma and gool procesing has proved fruitful, and we believe that our mochanieme could similarty benefit by being integrated with oher control approsches such ar a blectboard architecture for control [4|. Beood on the resules we have outlined in this paper, we anticipate that the further development of mecheniams for developing abstract view and for incremental planniag to control bleckboard-beeed problem sotvers will greatly enhance the performance of these problem solving systems. will lead to improved reat-time response and to better coordination in distributed problem solving networks, and will increase our understanding of planning and axtion in highly uncertain domaina.

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# Knowledge Representation System for Assembly Using Robots 

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## 1．0 Inerobwetion

nescmbly robots combs me the benefits of speed and accuracy with the capability of adaptation to changes in the work environment．However，an impediment to the use of robots is the complexity of the man－anchine interface．This interface can be improved by providing a means of using apriorism knowledge and reasoning capabilities for controlling and monitoring the take performed by robots．

Robots ought to be able to perform complex assembly tasks with the help of only supervisory guidance from human operators．For such supervisory guidance，it is important to express the commands in terns of the effects desired，rather then in terms of the motion the robot must undertake in order to achieve these effects．

A suitable knowledge representation can facilitate the conversion of task level descriptions into explicit instructions to the robot．arch a system mould use symbolic relationships describing the apricci information about the robot，fth environment，and the take specified by the operator to generate the commands for the robot．
however，the knowledge representation system should provide a scheme for building assembly models in an environment that maintains the knowledge of the spatial and functional／relationships among the engineering parts comprising an assembly．The system then prevents the user from violating these relationships unless specifically med to do so．Thus，the knowledge representation my ten has amor for specified constraints and prevents actions whose aide effects cause the violation of these constraints．

The modeling method provides an organizational structure that maps the relationship e between the components of an ascobly into a aet of constraints．These constraints ark simplified using a number of rules to determine the available degrees of freedom for a component．When a no relationship is to be established，the required motion is calculated and the feasibility of this motion is giterained by checking again et the available degrees of freedom．

## 2.0 tam Level Programing

Task level programing is an attempt to make use of structured programing together with an information base that enables the use of apriori knowledge to interpret the commands issued by the programer．Examples of task－ oriented programing languages include AURGASS，RAP第 and lam．

The process are expressed in terms of the drained effects on the objects．Thus，a task is completed not when a mipulator has completed a series of motions，but when the objects have reached the desired configuration．The knowledge representation aster converts the task level specifications into manipulator level commands．

AUTCPASS（ll accepts the steps of asseably／as input from the user and with the help of a model of the world， converts these into a propr am that a manipulator can process．However，the yean does not recognize changes in relationships occurring due to manipulator actions．The user is responsible for specifying physically realizable operations and also for informing the system of changes in the attachment relationships．

RAPT（3）developed at edinburgh，hap concentrated on specifying the spatial relationships and reasoning about them．The relationships between objects such as＂Bl against BR ＂and＂C1 against C2＂，etc．，are converted into algetrale equations．These equations can be expressed in terms of operational parameters，such as joint angles of the manipulator or in terms of the degree of freedom for the objects．the solution of these simultaneous non－linear equations is quite complicated and time consuming．However，some efforts at recognizing stereotypes and applying standard solutions have been successful．The emphasis here has been on representation of relationships；however，the geome tical representation is at present incomplete．Hence，path planing， collision detection，etc．，cannot be ifplemented using this system alone．

LAMA（3）uses general program plans that are expanded by details of individual assemblies．Polyhedral representation for object permits trajectory planning and collision detection．The constraints are expressed in terms of parameters that are unspecified element of partially filled transformation matrices．The constraints are represented as constraint planes，which indicate the boundaries of permissible motion．Ranges of par meters are calculated using volume interactions and constraint plane interactions．The assumption is made that
different constraints on an object are non-interfering. This assumpion is not required in the syeter we propose, as the bodies alwaya move within the specified constraints and inconalistent constrainta cannot be establ lehed in a constralite mforcing enviroment.

LM (4) end Feature Descriptor (5) repcesent other exemples of task level programing.
Fhimen ( 9 propeed a task planner for robot construction take in a blocke' vorld domain. The intormation contained in the object's motion was not used. The feasibility of the final atate is ceternined fromatability viewpoint, but the motion trom the initial co the tinal state is not considered. A task planner builtina knowledge base that coniders the feasibility of object motion can be integrated with erajectory planning symen.

In the proposed knowledge representation gysten, a conetralnt entorcing environment is provided at the level of the database. This approach permits supporting a task planner an well as interactive sessions with the progr mer. This appronch varies from the traditional appromeh by transferiing acme aupervisory capability to the computer. The feasibility of motion in a constralint enforcing enviroment is deteralned by mapping relationshipe into a constraint set, simplifying the conetraint, and interpeting the resulting conetraints.

## 3.- Cecmetricel Model

Bonogencous transocmations are used to express the position and oriantation information for objects. The position and orientation with respect to the world reference frame is atored with each object. Hence, this information in updated only when the object moves. The priaitives, es., blocks and cylinders, are defined in teras of the centroid position and faces. Loci definitions are used as they aresufficient for the reasoning involved in constraint aimplification as discussed later.

An assembly is defined recursively as an object consisting of other objects. By imposing the restriction that these primitives cannot move relative to one another, we can create rigid asserablies. We can represent mechanisms by peraitting specifiedrelative degrees of freedom for the primitives that combine to form the assembly.

### 4.0 Relctionshipa

The objects can be related to one another in a variety of ways with regard to ralative motion. These relationshipe are shown in piqure 1 . The contact relationships involve both concave and convex surfaces. $A$ contact relationship between two convex surfaces is called "against". The against relationship can involve a surface, edge, or a point contact between two bodies. A contact relationship between a concave and a convex surface is called a "fit".


Figure 1: Motion Relationstips

### 4.1 The Ogainst Relationship

We will first define against" in aestrictive sense and then generalize the definition. mainst is a relationship established between two plane surfaces, such that translation is permitted in the common plane and rotation is permitted about an axis normal to this comon plane. An exmple of against relationsip is shown in Figure 2. This "against relationshipexists between face fl of block Bl and face f2 of Block B2.

This definition of "against" can then be extended to cur ved convex surfaces, where the contact area ceducea from the plane to a ilne. The object with the curved surface retains any rotation capabilities that it had about


Figure $2: 81$ mainat 82 (n, 2)
the body axis normal to the curved crosesection, prior to ectablishing the "againat" gelationahip. siailar definttion applies for apheriond surfacel whare the contact reduces to a point.

### 4.2 Th Fite mationalp

 shaft. Imeert" is the driver function to comand the establishent of a "ft" relationship. The hole and shaft can be of various crose-sections. The pre-condition for establiahaent of a "fit" relationshipia that the profiles of the convex and the concave surfaces must match.

In a "fit" relationship, the body containing the hole and the body containing the shaft are restrained to sotate about the holeshaft axis if the profilea are circular. A tranalation along this axis is also peraited.

Many apacial casen of "fite can then be defined. A restricted "fit" relation might permit only rotational freedon A rectengular key, on the other hand, is permitted tranalation but no rotation.

Representation of other relationshipe such as threadedjoints, etc., can becompliahed by using these basic relationships. Peraisable translation and rotation would no longer be independent of each other, but would be related by parameters auch as the thread pitch.

The contact relationahipe of "againet" and "fit" and other functional relationshipe in the motion domain map into a set of motion conatralnta that represent the degrees of freedom for the objects. These conitraints are represented in our syecem in terme of new primitives which are necessary for reasoning about motion. thase new constraint relationahipe represent a general set into which all the relationahipe that pertain to motion cen be mapped. An example of this mapping is shown in Figure 3 .

degn specificd melatiowsilp
comstraint mepreszmintion

## Figure 3: Mapping of Relationships to Constraints

### 5.0 Drivers

The driver functions that command the motion execution are of two types. The first type is the "move" function. The "move" function is used when same explicit motion is to be implemented by specifying the anount of motion or by apecifying the final destination. The second type of drive functiona are those which coinand the estabilshant of relationships. These functions determine the motion required to establish the relationship, find out if the motion is possible, and, if so, then send out the necessary inatructions to the robot to implement the motion. For the two contact relationships. "against" and "fit", the driver functions are respecti vely called "etablish-against" and "insert".

Determination of motion feasibility involves a number of stepa. This is showninfigure 4 the relationships of the body with other bodies, represented as constraints, are simplified to deteraine the translational and rotational degrees of freedom. If the entire desired motion ia not possible with the available degrees of freedan, then an attempt is made to find one or more neighboring bodies such that this set of bodies can together accomplish the remaining motion. This strategy is explained in Section \&

### 6.0 Representation of Motion Constrainta

We need a representation that is sufficient for the reasoning involved in object manipulation, and is also simple and efficient to manipulate. We have departed from the conventional manner (popplestone, Taylor, Mazer, etcd of converting all the constraints into parametric equations and solving then over all the objects under constider ation.

```
Let ua define a few symbols first:
```

T: representa a Translation degree of freedom
R: represents Rotation degree of freedom
L: stands for a line. This is used as a memonic for the axis of rotation or line of tranalaticn.
P: stands for Plane.


Pigure 4: Feasibility of Motion

These mincondca are combined in the following ways to give the constraints that are established by establishagainst" and "insert" operations:


The direction vector that is required with all of these oodes is called the constraint vector (CV). Since the magnitude represented by this vector is insignificant, it is stored as a normalized vector. This direction is interpreted differently depending on the presence of $P$ or $L$ in the code.

Besides the above constraint primitives, wemust define afew additional constraint classes in order to implement working set. These include three other constraints--"Stationary," attached," and "rixed"-and the unconstratned condition, Tree". These are clarified as follows:

The Gtationary constraint indicates that the object has a particular position and orientation ane these cannot change.

Object $A$ is said to be "Attached" to object $B$ when $A$ and $B$ alway move together. In other words, $A$ is rigidly connected to $B$ Every object must store aist of all other objects that are "Attached to it. When the move comend is to be executed for an object, all the "Attached" objects should move together.

As opposed to the "Stationary and Attached" conatraints which are specified by the user, "pixed" is a constraint derived from a set of user specified constraints. "ixade represents the condition when a combination of constrainte applied to an object results in no freedam of moviment for the object when considered as a single entity. However, the implication is that the object could move if it were to move together with one of the constrained objects. This is clarified by the example shown in figure 5 .

Consider the case when the following constraints are established:
B1 ACAINST B2 (E2 E1)
B2 AGADISTB3 (F3 F4)
The features involved are indicated within parentheses.
Under these two constraints, $\frac{B 2}{}$ can translate along a ine in and out of the paper. Nose we



$$
\begin{aligned}
& \text { al manst pirgin }
\end{aligned}
$$

Figure 5: 82 "Fixed" By Constraints
and cannot move by itself. However, any of the conbinations ( $B 2, B 4$ ), ( $B 2,83$ ), and ( $B 2$, B1) can move along different directions.

The constraints only describe the restrictions that the enviroment has on the object's motion. Tre driver functions, the "move" comand, and the comands to establish relations use these constraints to deternine vacther the body can be moved or same other body needs to move along with the body prior to trar:smitting the comand to the robot. By using these constraints, we can build assemblies in a constraint enforcing enviroment where the user or the task planner is prevented from violating any previously specified constraints, unless the syter is specifically asked to do so.

In order to determine the available degrees of freedom, we willfirst show how to simplify the notion constraints in the next section. Following that, we will develop a method of determining the available dageea of freedom from the specifled constraints.

### 7.0 Simplification of Conatraints

Same rules for simplifying constraints are presented here. These apply to the case of rectangular blocks and cylinders. The extension to other bodies involving curved surfaces and to more general profiles is posfale.

The rules may not represent a complete set, but are sufficient to demonstrate the feasibility of the approach. Thie rules which follow arefor a prototypical object Bl wich is being conmanded to move by ame driver function ( $-\boldsymbol{t}$ signifiea is mapped into):

1. IF (TPR CVI) AND (TPR CVZ) AND CVI $X C V 2 \neq 0$

THEN (TL CN AND CV $=\mathrm{CVI} \times \mathrm{CVI}$
Consider the motion for Bl in the following exaples (figure 6 ).
$B 2$ AGAINST B1 $(F 2, F 1) \rightarrow(P R C V 1)$
$B 3$ AGAINST $34(F 4, P 3) \rightarrow(P R C V 2)$

Since CV1 and CV2 are orthogonal to each other and lie in the plane, gl can translate along a ine in and out of the plane.
2. IF (IPR CVI) AND (TPR CVI) AND CVI $X C V I=0$

THEN (TPR CVI) $C R$ (TPR CV2)
Consider the motion for Bl in the following example (Pigure 7a).
82 AGAIKST BI $(F 2, F 1) \rightarrow$ (TPR CVI)
B3 AGAINST BI (P4, F3) $\rightarrow$ (PPR CV2)




Figure 7a: B2 Againet Bl (R, R) B3 Agalnet BL (P4, F3)

BL can acill translate in the plane perpendicular to CVI and rotate about CV1. Using CVI or CV2 does not aater A shown in Figure 7 , CV , and CV 2 can also be pointing in the same direction.

 Bl Against $\operatorname{Bl}(\mathrm{FA}, \mathrm{F} 3)$
3. IF (TLR CVI) AND (TLR CV2) AND CVI $X C V 2 \neq 0$

THEN FIX
Conalider the motion for bl in the following examle for a fit" relationship between a shat and a hole (Figur, 8).
$B 2 \operatorname{ITHS} \operatorname{BL}(52, H L) \rightarrow(T L R$ CVI)
81 FITS B3 (S1, H3) $\rightarrow$ (TLR CV2)
Bl cannot move all by itself. However, Bl and BO can move together as can Bland Bo.
4. IF (TLR CVI) AND (TLR CV2) AND CVI $X C V 2=0$

THEN (TLR CV1) OR (TLR CV2)
Consider the motion for Bl in the following example (Figure 9.
B1 PITS $\mathrm{B} 2(S 1,[2) \rightarrow$ (TLR CV1)
BI PITS B3 (S1, H3) $\rightarrow$ (TLR CV2)
Until now, we tave been considering infinite lines and planes and, hence, no mention has teen made regarding th depth of the hole and the length of the shaft. These par aneters will be introduced later as factors necessar for deternining the possibility of establishing relations required for object manipulation.


Figure 8: 82 Fite $\operatorname{Bl}(82, \mathrm{HL})$ M1 Fits B3 (S1, H3)


Figure 9: Bl Fits B2 (Sl, HR) Bl Fita 83 (S1, H3)
5. IP (TPR CV1) AND (TLR CV2) AND CV1 $X$ CV2 $=0$

THEN (RL CV1) OR (RL CV2)
Consider the motion for Blin the following exame (Figure 10 ).
$B 2$ AGA INSTB1 $(F 2, F 1) \rightarrow(T P R C D)$
BL FITS B3 (S1,HD $\rightarrow$ (TLR CV2)


Figure 10: 82 Againte $\operatorname{Bl}(12, F 1)$
只 Fica 83 (S2, H3)
Bl can rotate about CV1. B2 AGANST Bl permits translation in plane while Bl FITS B3 permits translation along the normal to the same plane. The net result is that Bl cannot translate at all.
6. $\quad$ (IPR CVI) AND (ILR CV2) AND CV1 $\times$ CV2 $\neq 0$

THEN IF CV1 - CV2 = 0, THEN (TL CV2), ELSE PIX
Consider the mocion for Bl in the following example (Figures lla and llb).


Fiqure lla: bl moimet n2. (t?, t2)
Al pitn (s) (: 1, III)


II fit: lis (: in, III)
H ACAINET B2 (HI, F2) $\rightarrow$ (TPR CVI)

 CVI CV2 $\neq 0$, 111 cannot move at all.
7. IP staticna ry and any onf: cinstim int
tigen stationary
If a body is stationary, it cannot mowe at all.
8. if fix and any one constilaint exchbt stationary
tiven fix
If a hody is onnstrained so that it cannot move by itself, then constraining it furthor will mithancp the fix constraint.
9. if frge and any onfe constra int c
then C
Fiep refers to an unconstrained state and, hencr. $C$ is the only one onnstraint.
 discussed atove. When motion feasibility is to be determined, the different relationifice are firet samplifiof
 simplified in order to determine the feasitility of motion of tho group as whal $\cdot$.


 the ohject changes, since the number of updates reruired is rodiucod.


## 2. 0 Strateqy for Implamenting motion and for Sistablishment of Eelationahips




position and orientation is desired, the motion matrix is given by
MOTION = (CLD-POSITION) ${ }^{-1}$ (NEW-POSTTION)

- using the homogeneous transformation notation. This is represented by the diagram in Figure 12.


Figure 12: Calculation of Motion Matrix

## 8. 1 Establisbing Relations

Driver functions such as "establish-against" and "insert" are used to establish relations between objects. These commands are used to calculate the necessary motion matiax which is used tombe the budy in or det to establish a relationship.

Consider the diagram shown in Piqure 13. Let us define the terms used in this diagram:
POSBl: position of object Bl in the world
alo-pOSB2: position of object B2 before moving
NEW-PCSB2: position of object B2 after the relation has been est blished. This is undetermined at this stage.
MOTION: The motion reguired of $\quad$ bject $B 2$ to establish the relation. This is yet to be determined.
FEAT1, FEAT2: position of features on todies $\overline{B l}$ and $B 2$ that are involved in this relation.
REL-ESTABLISH: a matrix that orients the constraint vectors to establish the relevant relation.


Figure 13: Establishing Relations
In the case of the "against" celationship, the rel-ESTABLISH matrix would define the rotation needed to bring the normal vectors associated with the twofaces in line with each other. In the case of the fit" relationship, it is the matrix that brings the twi axial vectors in ine with each other.

To determine REL-ESTABLISH, consider the example in Figure 14. Consider two unit normal vectors Nl and N2, associated with the faces F1 and F2 of two objects R1 and $P$.


Figure 14: Definition of REL -ESTABLISH
N1 and N2 are two vectors to be aligned. A rotation of Nl by an angle $=$ cos -1 (NL N about the direction N1 $x$ N2 will bring them into line with each other. This rotation is the one defined by KEL-ESTABI, ISH (Equation 1). An additional translation is required to bring the two faces together. NEW-posB2 inoorporates the oombined rotation and translation of this new position for $B 2$ (Equation 2 . Morian is calculated using this new rosition NEW-POSB2 (Bquation 3).


The foliowing "loop equetion" can be used at any time to determine if the selation has been established. it the equality is satisfied, then the relation has been eatablished.
$[I D E N T I T Y]=(P O S B 1)($ PEAT1 $)(P E A T 2)^{-1}(P C B B 2)^{-1}$

The next task is then tofind out if this motion is feasible in the presence of the algeady established constradints.

However, there is one factor that stili needs to be considered. The abovementoned stategy for establishing relations will work only if object bi does not move while object bi is being moved. if okject bl moves in the process of achievirg the required motion for object a2, then therelation is rot jet established. This is explained by the following example (see figure 15).


Fiqure 15: Example of Unachitevatle Relationship

The motion needed by 82 to establish this relation is calculated. and in this case, it is simply a translation in the $x$ ditection. When this motion is attempted, it isfound to be feasitle and, hence, is implamented. But since Rland Bo are "attached", glwillalso move and, hence, the desired relationship will not be established. In fact. for this example, the relation can never be established since the two todies are attached and. hence, move by the 5 ame amounts.

This problem is resolved by using an iterative proondure. Consider the diagram shown for the general case infigure 16.

In the first attempt, motion $M$ is the required movement of object b2 to establish the relarion. However, in the process of planning motion M. motion Mlof object blalso occurs. At the end of motion m. the loop equation" (Bquation 4) is used to check if the relation has been established. If the relation is estati :shes, the new motion matgix will indicate zero translation and rotation. If the relation is not estatlashed, tren an iteration is performed in which NP2 is redefined as OP2, Pl is redefined as pl, the new tel-estatlish matrix is given by fe', and the motion matrix is cecomputed. This is shown as M 2 in figure 16 . By comparing the motion M with motion m2 (as shown later), one can find if any progress has ben made towards establishing the relatan. If no progress is made as in the example in figure 15 , then we conclude that the new relation cantict te established in tre presence of the already established relation.

In the above iterative approach, it is necessary to determine if progress has been achieved in tiangag tat two objects closer to each other. We assume that the iteration is successful if the new motion prediction is "smaller" than che initial" motion prediction. A mechanism to compare an initial motion prediction with a ner motion peediction is described in moce detail in reference (1).


Pigure 16:
P1 - old position of body 1
P1. - new position of body 1
M1 - motion of body 1
F1 - feature of body 1
F2 - feature ol body 2
OP2 - old position of body 2
NP2 - new position of body 2
NP2' - new position desired for body 2
M - motion of body 2
M2 - mokion needed for body 2
RE - Old REL-ESTABLISH
RE' - new ReL-ESTABLISH

### 9.2 Feasibility of Motion

In order to decide whether aparticular body motion is feasible or not, we need to consider the conetraines to which the body is subjected. We heve shown how to simplify these different constraints in order to determine the posaible degrees of freedom. In order co facilitate the process of deterilining motionfeasibility, the translation and rotation are each handled separately.

If, after simplification has ben performed, the constraint code includes The then a part of the transiotion needed can be accomplished by moving the body along the direction given by the constralnt vector. li the onnstraint code includes TP, then a part of the motion is achleved by translating in the specified plane. If the -ntire desired translation is not possible, then one cannot achieve the entire motion by moving this object alone. One must then consider moving a neighbotingobject alongwith the body of interest. This strategy is described later.

The rotation part of given motion matilx is converted into an equivalant angle about an axis located in the world reference frame. The constraint vector associated with the rotation degree of freedom is then crapared with this equivalent axis. If the two coincide, then the rotation is posiblef detmise, the rotatior. is not prssible by this body alone.

If the body is found to be constrained frow moving when considered alone, then the follawing atrategy for considering cooperative motion with neighboring objects is pursum. A search similar to a breadth first search is used. This is best explained by an example isee figure 17 .


Figure 17: Cooperative Motion

Consider an atempt to move the object Bl. suppose the following zelations already xist:

It If in contact ulth and 0.
22 Is in comenct with 34 mid 85.
 then in attapt is mate to conalder the motion of al and $\mathbf{B 2}$ together. In the constraint ainplification that is
 motion.
 If the entige motion if still not ieplemented, then B1, m2, and mare conaidered together. The search order
 ( $\mathrm{BL}, \mathrm{B2}, \mathrm{B3}, \mathrm{BA}, \mathrm{BS}$ ).
 the object of interest. The governing rule in this searchis to always attempto move ainimum nuber ot bodia price to attemping a 1 aspr set.

At each stage in the search, motion is carifed out as far ae posible. If there is atili sose motion :equired, then the search continues. As result, the total desired motion for bl migh be achieved partiy by al


If a constraint egtationary is ancountered in the search, the search along the path is imandately cerminated. Note that all the attached bodien are always considered together in this motion feasibility paradig.

The servcture is deacribed as a tree didgran. However, the generalimed wersion can be graph fee Figure 10). This graph can be converted into tree by repeating appropriate nodee (see Figure 1 B).


Figure 18: Conversion of Graph to a Tree

### 8.3 Motion of Acemblies

To determine the motion posaibilities for an asaembly, we have totreat them atim at tached" group of primitives. Westore the position and orientationinformation for each primitive. Whenthe asseably has to nove, all the primitives must sove by the sme mount.

## 0. 4 lunge of motion

In our discussion mo far, we have considered the conatrainta at besicaliy degrees of freadom. foc intence, TPR indicates that translation is allawed in a plane and rotation is peraitted about the normal to the plane. However, thee motions are not infinite. These motions are limited by the finite dimensions of the bodies invol ved in the relationship.

### 9.0 Implematation

The knowledge representation ystem described here has been implement using the fiavar system on the imi Lisp Machine. A typical data network is shown in figure 19. In the constralnes discussed above, we did not include the range of motion. Geometricalgorithas to detect colilision of objects and overlapping of taces, overlapping of two holes, etc., are implamented as teata to be conducted before sending any motion command to the robot. This schene provides for expandebility of the syetem to inoorporate checks such as tolerance motching, striace finish matching, checking types of fit, calculating inertia properties, etc. Same exaplea cmibe fourd ingeference (3).

The systen described thug far is a conatraint enforcing syetem. The breaking or making of relationshipe at a side effect of motion is not considered. This consideration of dynamic constraint propegation requires a conetraint network on alobal scale which was indeed designed and implemented (1). but is outaide the scopp of this paper.


et Pry


Figure 19: Asacmbly Model


The rectangle indicate data objects. The ovals indicate information slots. oval consecting two data objecte Indicate infornation siots in both data objecte.

Figure 19bs Data Network for model shown in Figure is
Noter This is not a complete network as some repetitive structures are not shown.
10.0 Conclusion

$$
+1 E
$$

$$
\theta p^{2}+\cdots, i+6 i+i
$$

 higher level constructs hides the robot specific programing details from the user by autierging then into the domain epecific knowledge base. The constraintenforcing environamt provider for an on-line debuging facility which oper ates at the conceptual tak level rather than te the programe yntax level. Before sending a comand to robot, the command is mimulatedinthe worldmodel, and this providet for real time er rot preventive capability. With the addition of dyname contraint propagation, the knowledge representation aysten beaes


## 11.0 sehnoeledginut

Inis work wes supported in pert by National Science Foundation PYi Grant pmis351827.

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# Real Artificial Intelligence for Real Problems 

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This is a sumary of a pansl discusesion that wes held at the conclusion of the sevions on artificial intelligence (AI). Panel-members more: R. Cheereman, MSA Anes: S. Harmon, frobot Intelligence International: V. Hxpmard, MoGill Uniswersity; $K_{r}$ Kentif, FMC; 1. Gopenheim, GM; A. Sanderson, aMJ: and D. Whleins..SRI.

The following questions were discussed loy-the osnel: 1) What other research areas need to he addreseed to extend your work? 2) In your research experience, have you found it ftuitful to choose application-domains from roal-torld scenarios? 3) The felerobot progrmm needs to merge task planning and spatial reasoning. What do you think needs to be done to achieve this merger? 4) Is the blockg world really simple/trivial? Responses and discusision on these issyys are sumarized, bolas.

The blocles world dgain is definttely not simple. Meny AI planning systems have solved many mopects of the toy-blocks world. Hewever, the real-biocks world is largoly unsolved. The real-blocies world problem includes the following subproblems: 1) Ropresenting germetries of the workrsell, the mbot(s), the objocts, sssemblies of objocts; 2) Practical technirues for efficiently reesoning with there rapresentations; 3) Developing mathematical and symbolic models of sensor handware and interpretation of seneor behavior; 4) Developing mathematical and symbolic models of external agents like friction, dynamics, kinamatics and wear and tear. Although these areas are undergoing active rosparch, rosults are far frm complete.

Nowadays, it is imperative to choose drmains from real-world scenarios. This helps make for good demonstrations. However, to actually make it realistic, the application must also include one or more (to a certain manageable degree) subproblems of the real-blocks world (discussed above). An excaniole of such a ralistic problem is to investigate an AI task planner which also reasons about the (mumeric) grometiric representations and plans actions kased on real, actual, existing mhot hamware. Even if the AI task planner is simple, solution of the crmbined problem adds a lot to the state-nt-the-art.

One of the immortant unsolved problems is that of integration of diverse technologies in the fields of mathematics, omputer science, monitive science, artificial intelligence, mechanical engineering, mborins, elertrical engimering, etc.

Integration of AI with non-AI systems is something which is talked about a lot and mostponed. Mary theoretical suggestions rin exist. The general feeling seems to be that the integration will not be a batch-mode integration ("do-this--then-do-that"). It will be a multi-contact, heavily coupled interaction. The important thing is to start integrating and failing. Fall enougt, and often enough, so enough an be learmed.

NFITMKN

# TRAJECTORY PLANNING FOR MANIPULATORS 

# The Use of 3-D Sensing Techniques for On-Line 

 Collision-Free Path PlanningV. Hayward, S. Aubry, and Z. Jasiukajc McGill University<br>Montreal, Quebec, Canada H3A2A7

## 3. Proposed Ideas

We would like to suggest a few new ideas to the problem of collision prevention, in the view of their use in an online control system. We mean by that, trajectory generation techniques that allow the computation of collision-free trajectories in the same amount of time as require by the motion of general purpose robots, using limited computational resources. At this point, we would like to draw an analogy with the problem of theorem proving in computational logic. This problem is undecidable, that is to say, if the proposition submitted to the system is in fact false, the result cannot be obtained in a finite number of steps.

The search space to explore in order to prove a proposition can become arbitrarily large. If the proposition under study in indeed provabie, efficient methods in theorem proving attempt to use powerful heuristics to reduce the search space. These heuristic methods are called "weak methods" because they do not guarantee success, but are likely to converge in most interesting cases. If the proposition is false, this conclusion may not be reachable in a finite number of steps, and one has to resort to cut the search at some arbitrary point and to assume that the proposition probably is false.

In path planning, there are many heuristics available, and we suppose a limited amount of available computations, hence the architecture described below, based on a collection of weak methods.

### 3.1. Computational Architecture

We require the system to be extensible in the sense defined by Brooks (Brooks 1986). As researchers are devising new methods to calculate collision free trajectories, we would like to be able to integrate these advances while causing a minimum of disturbances to existing and working parts of the system.

The following diagram illustrates the design concept of an extensible architecture. The question of whether each of the methods will reside on one or several processors is of litthe importance. What is is important is to design them as peers such as they can accept the same input and produce the same output formats. The crucial point is not to attempt to parallelize the computations of one particular method, because we know that many of them require exponential times to execute, but to parailelize the methods between them so that we obtain a natural selection of the most appropriate for the situation at hand.

The computations should be done at a least two levels. The top-level searches for collition-free trajectories, using any of the methods available. Input to the high-level is data obtained from global sensor measurements as discussed in the section "Sensors," or from pre-determined models obtained from data-bases. The lowest level is a local collision detector that also uses either sensor information or pre-determined model information. We will require an efficient collision detector to certify the proposed trajectories, or use on-line proximity sensor mounted on the arm to locally modify the trajectories as they are executed. In the later case, we take the chance that the motion may never terminate.

### 3.2 A Variety of Heuristics: Archetypical Motions

The study of robot motions shows that given an approximate description of a robot's environment, and given the initial and final configurations, robot motions can be classified into classes of archetypical motions. A preliminary analysis shows that collision-free motions bear a strong relationship with the structure of the workspace. This relationships can be exploited to built a system that infers plausible motions. In this framework, the problems under study are:

- Classification of obstacles according to the influence that their shape might have on the motions: small, large (with respect to the robot), compact, elongated, flat, etc... Interesting simple cases are: spheres, infinite cylinders, hall spaces, and holes.
- Classification of the relations between these obstacles with respect to the robot elements: proximity, position with respect to the elbow, under, above, on the left, on the right, etc... Inference will occur on criteria of this kind.
- Classification of typical motions: retraction, extension, sweep, wrist re-orientation. The consequences of these motions are explicit: if joint No 1 turns in the positive direction and the arm is stretched, then end-effector sweeps on the left; if the arm retracts and is in the elbow up configuration, then the end-effector moves inward, and the elbow moves upward, etc...
Once the scene and the robot attitude are encoded in terms of facts and rules, motions can be generated by automatic inference.

> GLOBAL SEVSORS or CAD DATABASE MULTI-RESOLUTION SCENE DESCRIPTION METHOD.I METHOD-2 METHOD-3 ... METHOD-n CERTIFICATION and SELECTION, or LOCAL ON-LINE MODIFICATION

### 3.3 Jolnt Decoupling

Joint decoupling is another way to attack the problem. The observation of certain collision avoiding motions reveals that motion planning can be performed by planning the motions of the joints independently. During such a motion, in the reference frame attached to each link of a robot, all the obstacles appear as moving obstacles. The task consists for each link to plan a one-dimensional trajectory in its own coordinate frame with a time-varying environment. We know from (Kant 1984 and 1986) that such a planning is possible, by planning the velocity along a predetermined path. This algorithm finds solutions in a large number of cases, when the priority among the set of joints is adequately determined. The problem is formaly equivalent to moving multiple objects as in (Erdman 1986).

### 3.4 Piece-Wise Trajectory Decomposition

Another heuristic method can be described as follows. If the arm is to move from point $A$ to point $B$, a trajectory is generated at the first iteration using a very simple scheme: a linear joint interpolated motion between $A$ and $B$, for example. The trajectory is then verified. In case of collision, an intermediate knot point is generated by the closest non interfering position. The initial segment is then split and the process recursiveiy iterated on the sub-segments.

### 3.5 The generatetest-refine architecture

We have just listed three powerful heuristics to reduce the search space of the problem. There exist others. We can augment the power of these heuristics by feeding back to a motion planner information provided by the collision detector in case of the failure of a plan, or information provided by a merit estimator, in case of success. The system is left interating during the allocated time period, the last best solution being retained.

### 3.5 Good Collision Detectors

Of what precedes, we require a good quality collision detector, that is to say, one that does not require exponential nor polynomial times to perform and one that uses multiresolution algorithms. This problem has been examined in (Hayward 1986). One approach is to perform the modeling the robot in terms of contro' points scattered on its boundary. Collision detection can be then performed by showing that all the control points are in free-space. (note that there is no need to worry about rotations). A multi-resolution system can then be easily obtained.

The quality of the result augments with the allocated running time and the CPC power. Methods for generating multi-resolution rontrol points are indicated in (Bhan 1986). Octree encoding methods provide very naturally for multiresolution algorithms, however, we have other schemes under consideration because octree make no use of the coherence that might be present in a scene and therefore can lead to great inefficiencies.

## 4. Sensors

"Model Building Sensing" is used to gather global threedimensional information from the environment. In a robotic context, the sensors perform a "surveying" function, providing information to be used by the path planning module. This is quite different from the on-line uses of sensors in which the local environment is continuously sampled so as to avoid crashes. In particular, model building sensors must operate over a wider range than their servoing counterparts.

### 4.1 Global Sensors versus Proximity Sensors

The chosen sensor must be either a proximity sensor attached to a "roving" arm or it must be capable of acquiring three-dimensional information at a distance. In the first case, the accuracy is limited only by that of the manipulator. However, control problems are likely to crop up for complex environments where concavities abound. Such problems arise because the environment is not known a priori: in fact, the environment is difficult to explore precisely because it is not known! Consequently, such a process is likely to be a slow one.

We contend that such proximity methods are only advisable when the task environment is so intricate that spatial considerations prevent larger apparatus such as those we describe below to conveniently operate. Suppose for example that we wish io model the bottom of a narrow, oblong cavity inside a giver object. We can safely assume that our robot arm can indeed penetrate the cavity and orient itself within it, since otherwise there would be little point in modeling it. Using the very manipulator which is to perform the robotic task is then the most direct way to model the task environment.

### 4.2 Acquiring 3-D Information

Techniques developed for acquiring three-dimensional information at a distance are still the preferred answer to auto mated model-building in most cases. These techniques can be either photometric or telemetric.

### 4.2.1 Photometric Techniques

Photometric techniques attempt to infer distance from photographic images. But such images map intensity, an extrinsic characteristic of the three-dimensional world, onto the the two-dimensional plane along the lines of a perspective projection. The task of recovering the correct interpretation for a given image is then a formidable one since it requires that the perspective ambiguity (lines map into points) be resolved from the intensity cue alone; formally, this task consists of inverting an illumination-reflectance operator / which maps the three-dimensional scene to the image plane. The socalled "shape-from" techniques attempt to perform that difficult inversion using a combination of analytical worix (Horn 1968; Horn 1975; Ferrie 1986; Levine, O'Handley and Yagi 1973), and of higher-level cognitive processes (Rosenburg, Levine and Zurker 1978; Bajcsy and Lieberman 19it: Shirai 1973: Marr 1976). These methods have been much investigated in part for their similarity to human visual processing, but also because they do not require sophisticated optical hardware.

### 4.2.2 Telemetric Techniques

In contrast with photometric techniques, telemetric techniques usually require specialized hardware but are much easier to analyze in return and therefore constitute a much preferable means for automatic three-dimensional scene acquisition. The goal here is to build "range images": a range image maps the distance of the closest point in the scene to every node of an orthographic grid the size of that scene. These images are usually constructed by monitoring patterns of points (Hasegawa 1982; Ishii and Nagata 1976), lines (Oshima and Shirai 1979: Sato and Inokuchi 1985), or grids (Potmesil 1979) of light which are successively projected onto the scene and reflected to a sensor located at or near the light emitting device (often a laser). Either positional analysis of the returning rays or sime-of-flight discrimination can now be used to infer the range of the closest obstacle. In the first case, simple geometrical relationships relating emitted and returned rays yield the sought distance in a process called triangulation. In the second case, the time taken by light rays to travel from and back to the emitting laser source allows us to calculate that same distance. Needless to say, the practicability of the latter method is limited by the very sophisticated electronies that the enormous speed at which light travels requires (Lewis and Johnston 1977; Nitzan. Brain and Duda 197\%.)

An alternative time-of-flight method uses sound waves instead of light rays berause of their more manageable speed. Although simple in principle, the method suffers from various engineering problems such as the need for frequent recalibration. the diffirulty experienced in focusing sound waves, as well as their hard-to-model reflective properties.

In su:nmary, the "safest" and most accurate methods of acquiring distance information seems at present to be triangulation. However. one should not discount ultrasonic time-of-Hight methods which are already commercially available. Further. many researchers believe that laser time-offlight methods will soon present itself as the most viable method since it offers in theory the greatest absolute accuracy. The interestid reader should refor to the excellent resiew by Jarvis (Jarvis 1983) for further reading on range acquistion techmques.

## 5. Conclusion

In this paper, we have presented an overview of methods related to the collision prevention for manipulators with revolute joints. It has been shown that it is a difficult problem in its generality and we have proposed a computational architecture based on an analogy with an another domain of Artificial Intelligence.

## 6. Acknowledgements

The ideas in this paper were initially formulated when the first author was developing a Cartesian-based collision detector while at C.VRS (France). and by the second author while studying 3D sensing techniques at McGill University. Further contributions are due to conversations with Kamal Kant at McGill University, and to an inspiring lecture delivered by R. A. Brooks.

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# Collision-Free Trajectory Planning Algorithm for Manipulators 

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## 1. Abstract

Collision-fres trajectory planning for robotic manipulators is investigated, in-thie-pepen. The task of the manipulator is to move its end-effector from one point to another point in an environment with polyhedral obstacles. An online algorithm is developed based on finding the required joint angles of the manipulator, according to goals with different priorities. The highest priority is to avoid collisions, the second priority is to plan the shortest path for the end effector, and the lowest priority is to minimize the joint velocity for smooth motion. The pseudo-inverse of the Jacobian matrix is applied for inverse kinematics. When a possible collision is detected, a constrained inverse kinematic problem is solved such that the collision is avoided. This algorithm can also be applied to a time-variant environment.

## 2. Introduction

Ordinary tasks for a robotic manipulator are to move its end effector from an admissible point to another admissible point in an environment with obstacles. For that, the initial and final configuration of the manipulator are often given for the trajectory planning. Usually. there are infinite paths for the end effector. Even for a specific path of the end effector, there are still infinite trajectories possible for the manipulators. However, some of the trajectories are not feasible becasue of arm geometry, obstacles, and some kinematic or dynamic constraints. Even with the kinematically feasible trajectories, some computational or logic problems in the algorithms may make therm impractical.

## 3. Trajectory Planning

In order to move from one point to another, in the task space, one needs to solve for the angular information from the spatial information, using the inverse kinematic relationship. Consider a robotic manipulator with $n$ degrees of freedom. Let the kinematic relationship between joint angles and the end-effector position and orientation be given by

$$
\begin{equation*}
x=r(q) \tag{1}
\end{equation*}
$$

where $X$ is the $m$-dimentional vector of the end-effector position and orientation, and $q$ is the n-dimensional vector of joint angles. For a kinematically redundant manipulator, the dimension of $q$ is greater than the dimension of $X,(n>m)$. Differentiating the above relation, we get

$$
\begin{equation*}
\dot{x}=J(q) \dot{q} \tag{2}
\end{equation*}
$$

where $J(q)$ - df/dq is the man Jacobian matrix. [7]. For a redundant manipulator, the Jacobian matrix will have more columns than rows. Moreover, the inverse of such non-square matrix is not defined in a regular sense. However, useful solutions of equation (2) can be round, by using the generalized-inverses of the jacobian matrix $J$, and is given by

$$
\begin{equation*}
\dot{q}=J^{\dagger} \dot{x} \cdot\left(I-J^{\dagger} J\right) z \tag{3}
\end{equation*}
$$

where $J^{\dagger}$ - $J^{T}\left(J J^{T}\right)^{-1}$ is the pseudo-inverse or $J, I$ is the non identity matrix, and 2 is an arbitrary $n$-dimensional vector. When the vector 2 is selected to be zero, equation (3) reduces to

$$
\begin{equation*}
\dot{q}=J^{+\dot{x}} \tag{4}
\end{equation*}
$$

which gives the best approximate solution to equation (2). This is in the sense that if $q$, is the solution of (2), given by (4), then $\left\|\dot{q}_{0}\right\|<\|\dot{q}\|$. where $\dot{q}$ is any other solution of (2) that is given by (3). [5-6]. It should be noted that, such minimum norm, or best approximate solution, is defined when there is no restriction in the task space. This means that, in a restricted environment, the above mentioned best approximate, solution may not be feasible for application and may result in collision.

Let us define collision point to be the point on the manipulator body whion has the potential to collide with the obstacle. The collision-free trajectory planning problem here is to develop an algorithm, for the on-line deteruination of the required joint angle rates, $q_{\text {, for }}$ aafe manipulator motion. The approach is to continuously monitor the task apace, for deteoting possible collisions. If no potential collision is detected, the required jolnt angle rates are generated, using the best approximate solution, to move the end-affector on a shortast distance. But when a potential collision point is detected, the trajectory is modified in order to avold collision.

## 4. Obstacle Aroldance

In order to avold obstacles, one needs to use the kinematic relationship for the collision polnts, simblar to that of equations (1) and (2). Let the potential colifision point, in the task space, be denoted by $X_{c}$. Then, similar to equation (2), we can write

$$
\begin{equation*}
\dot{x}_{c}=J_{c}(q) \dot{q} \tag{5}
\end{equation*}
$$

where $J$ is the mxn Jacoblan matrix for the collision politt. The Inverse kinematic solution to the above is similar to (3) and is given by

$$
\begin{equation*}
\dot{q}=J_{c}^{\dagger} \dot{x}_{c}+\left(I-J_{c}^{\dagger} J_{c}\right) z^{\prime} \tag{6}
\end{equation*}
$$

where $J_{c}^{\dagger}$ is the pseudo-inverse of $J_{c}$ and $z$ is an arbitrary $n$-dimensional vector.
Now, the problem or oostacle avoldarce is that, when a potentlal collision is detected, the nighest priority is to avold the obstacie, and, if needed modify the position of end-effector. In order for the trajectory planning to have minimum norm, we choose $2=0$ as in (4). On the other hand we choose 2 '=0, like in (6). to account for both collislon avoldance and trajectory planning. From (3) and (6), a minimum norm soiution ror $2^{\prime}$ is

$$
z^{\prime}=\left[J\left(I-J_{c}^{\dagger} J_{c}\right)\right]^{\dagger}\left[\dot{x}-J J_{c}^{\dagger} \dot{x}_{c}\right]
$$

Plugging this back into (6), we get
$\dot{q}-J_{c}^{\dagger} \dot{x}_{c}+\left(I-J_{c}^{\dagger} J_{c}\right)\left[J\left(I-J_{c}^{\dagger} J_{c}\right)\right]^{\dagger}\left[\dot{x}-J J_{c}^{\dagger} \dot{x}_{c}\right]$.
Then, using the following identity, [6]

$$
\left(I-J_{c}^{\dagger} J_{c}\right)\left[I\left(I-J_{c}^{\dagger}\right)\right]^{\dagger}=\left[J\left(I-J_{c}^{\dagger} J_{c}\right)\right]^{\dagger}
$$

we get

$$
\begin{equation*}
\dot{q}-J_{c}^{\dagger} \dot{x}_{c}+\left[J\left(I-J_{c}^{\dagger} J_{c}\right)\right]^{\dagger}\left[\dot{x}-J J_{c}^{\dagger} \dot{x}_{c}\right] . \tag{7}
\end{equation*}
$$

The above relation, generates the joint angle rates $q$ such that the obstacle is avolded and the end effector velocity is modified, for the on-line trajectory planning.

Now the question is how and in what direction the end effector spatial velocity should be changed. For the algorithm to be fast and Implementable, a flnito search for the min!mua norm solution is considered. The value of $\dot{X}_{c}$ is preselected oy the user, and the value or $\dot{x}$ modification is also preselected by a value of $c$. The value of $q$ may now be found by examining seven different directions for rate modiclcation, e.g., e-variation in plus or minus X. Y.Z coordinates and also no modification. The smallest norm $\|\dot{q}\|$ is then chosen for trajectory modification, from seven different possibllities.

The overall algorithm is such that when no potential collision ls detected, a minimum norm solution $q$ is planned according to (4). But, when a potential collision is detected, the obstacle is avolded and the path in modifled according to (7).
in order to develop the algorithm, it is assumed that:
(1) the solution extsts
(2) the obstacles are represented by polyhedrals.
(3) the geometrical information about the task area is known, e.g., using sensory systems, the positions of obstacles and manipulator links are known.
(4) the potential collision point on the manipulator links can be detected.
(5) only a single collision may occur, and will be detected, at any given time.

## 5. The Al corithe

The following sumarizes the steps, Involved in the proposed algoritha, for the on-1Ine colilsion-free trajectory planning of the robotio manipulators. The steps of the algorithm are:
(1) Deteralne minimum-lensth path for the ond-effector, from the curront position to target position.
(2) Check if there is a potential collision point. If there 19, go to atep (5), otherwise cont inue.
(3) No potential collision is detected. Make an incremental move acoording to the joint angle rates vector 9 . given by equation (4).
(4) If the end-effector has not reached the target, go to atep (2). If it has reached the target, go to step (7).
(5) Potential collision is detected. Make an incremental move according to the joint angle rates vector $\dot{q}$, given by equation (7), such that $\|\dot{q}\|$ is minimized in a rinite search.
(6) Go to step (1).
(7) Stop.

## 6. Conclusion

On-line, collision-free trajectory planning is discussed. An algorithm, which utilizes sensed information about the configuration of the aanipulator and obstacles, is developed based on the task priorities. The order of the task priorities ares to avold collision, to plan the shortest path for the end effector, and to choose the minimum norm solution. The algoritha is fast and could be implemented on robotic manipulators for on-line trajectory planning.

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# Task Planning and Control Synthesis for Robotic Manipulation in Space Applications 

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## 1. Abstract

Space-based robotic systems for diagnosis, repair and assembly of systems will require new techniques of planning and manipulation to accomplish these complex tasks. This-papor-summarizes results of work in assembly task representation, discrete task planning, and control synthesis which provide a design environment for flexible assembly systems in manufacturing applications, and which extend to planning of manipulation operations in unstructured environments Assembly planning is carried out using the AND/OR graph representation which encompasses all possible partial orders of operations and may be used to plan assembly sequences. Discrete task planning uses the configuration map which facilitates search over a space of discrete operations parameters in sequential operations in order to achieve required goals in the space of bounded configuration sets.

## 2. Introduction

Space-based robotic systems will be required to perform tasks involving dexterity, perception, and planning. Telerobotic systems integrate human perception and human planning capabilities in order to accomplish tasks. Autonomous systems will increasingly require imbedded task planning systems with accompanying sensory integration, control synthesis, and system architecture to support goal-directed activities in an uncertain environment. Diagnosis, repair, and assembly are tasks which will be essential to the maintenance of space-based systems and require both complex manipulation as tell as reasoning about system configuration and system functionality. This paper reviews our recent work on task planning for assembly systems and discusses its implications for the development of robotic systems for assembly, maintenance, repair, and transport tasks.

Both manned and unmanned spacecraft require a variety of maintenance and repair tasks including materials handling, diagnosis of faults. reasoning about the origin of faults, hypothesis formation and testing, planning and executing repair procedures, disassembly, assembly, And replacement of parts. Currently these tasks may be accomplished in a limited way by on-site manual and teleoperated systems. As the number, complexity, cost, and importance of these spacecraft increases. autonomous systems which can provide service and maintenance on a routine basis will become essential.

Diagnosis and repair are problems in reasoning as well as manipulation, Any successful approach to these issues requires a representation of the task and an automated reasoning system which enables a decomposition of the problem into feasible sensing and manipulation procedures. Our work on assembly planning is based on several generations of assembly workeells which wd built and demonstrated for manufacturing applications [1]. These flexible worked 1 s incorporated multiple robot arms, vision, tactile and force sensing to accomplish tasks in electronic assembly, wire harness assembly, and assembly of instrument products such as copiers and printers.

Our experience with implementing tasks on these prototype workcells is the basis for current research on the development of tools for efficient design, programming, and implementation of complex systems. Task representation, decomposition, and sequencing [2,3,4], discrete task planning, [8] and adaptive control and learning techniques [9] are principal issues which are currently being addressed. Embedding such adaptation and learning procedures in the control and planning hierarchy is fundamental to successful implementation in uncertain envifonments. In this paper, we summarize an approach to assembly task representation and sequencing, and describe in more detail the use of the configuration map as a tool in discrete task planning.

The control functions of the system are allocated hierarchically into Strategic, Tactical, Operational, and Device levels. The control synthesis problem is to map the control hierarchy onto the set of feasible assembly plans in order to achieve desired performance. In this procedure, we seek to iteratively adjust the assignment of system resources subject to task precedence and configuration tolerance contraints. This procedure requires the definition of motion strategies and motion primitives which can be employed. We have developed a detailed understanding of sensorless manipulation strategies $[5,6,7,8]$ which facilitate planning of sliding, pushing, and grasping operations. We are studying control structures for vision, tactile, and force feedback [9], and have demonstrated feasibility of adaptive control strategies for visual servoing. This work on sensor-based control is currently being extended to employ learning algorithms at the level of the motion primitive in order to improve performance by local adaptation in the face of uncertainty in the task environment. We have formulated an approach to quantitative description of task uncertainties using entropy methods [10], and have investigated the use of this parts entropy approach for planning strategies. We have also developed and demonstrated a new approach to arm signature analysis which improves the identification of kinematic models of manipulator structures and increases the resulting positioning accuracy [11].

Implementation of robotic systems in either a telerobotic or autonomous mode will require many of these planning, control, and manipulation capabilities. Task decomposition and control hierarchy have not been studied sufficiently for the telerobotic case. Development of motion primitives and planning of fine-motion strategies are important topics for research. The addition of adaptive and learning strategies to teleoperator systems is also important. The evolution of autonomous systems from telerobotic systems will require more effective models of human task planning strategies and task representation. The design of the components and tools of the space-based environment will depend on a consistent task representation which evolves to accept autonomous manipulation.

## 3. Assembly Task Representation

In our approach to assembly system design, $[2,3,4]$, the planning of assembly of one product made up of several parts is viewed as a path search in the state space of all possible configurations of that set of parts. A syntax for the representation of assemblies has been developed based on contact and attachmem relations. A decomposable production system implements the backward search for feasible assembly sequences based on a hierarchy of preconditions: (1) Release of altachments, (2) Stability of subassemblies, (3) Separability of subassemblies, including (a) Local analysis of incremental motion, and (b) Global analysis of feasible trajectories. Because there are many configurations that can be made from the same set of parts, the branching factor from the initial state to the goal state is greater than the branching factor from the goal state to the initial state. The backward search is
therefore more efficient and corresponds in this case to the problem of disassembling the product using reversible operations. The resulting set of feasible assembly sequences is represented as an AND/OR graph and used as the basis for eaumeration of solution trees satisfying system and performance requiremeats.

Figure 1 shows an example of an AND/OR graph representation of assembly sequences for a simple product with four parts. Each node in the graph corresponds to a subassembly and is described in the representation by a relational seructure using the syntax of contacts and attachments. The hyperares correspond to the disassembly operations, and the successor nodes to which each hyperare points correspond to the resulting subassemblies produced by the disassembly operation. For most products, the assembly operations usually mate two subassemblies, and the resultiag hyperares are typically 2 -connectors as in this example.


Figure I. AND/OR graph representation of assembly plans for a simple product.

A solution tree from a node $N$ in an AND/OR graph is a subgraph that may be defined recursively as a subset of branching hyperarcs from the original graph. The AND/OR graph representation therefore encompasses all possible partial orderings of assembly operations. Moveover, each partial order corresponds to a solution tree from the node corresponding to the final (assembled) product. The AND/OR graph representation therefore permits one to explore the space of all possible plans for assembly or disassembly of the product. The problem of selecting the best assembly plan may therefore be viewed as a search problem in the AND/OR graph space, and for some given evaluation function on the graph, generic search algorithms such as AO* [12] may be used. In practice, the development of such an evaluation function is very difficult since it would often depend explicitly on implementation issues such as choice of devices and underlying control strategies. We have explored the assignment of weights to hyperarcs using criteria of (a) operation complexity, and (b) subassembly degrees of freedóm, or parts entropy [10]. Such an approach is viewed as a preliminary search procedure which may narrow the search space for later detailed examination using implementation details. In the simple examples studied, the resulting ranking of candidate assembly sequences was consistent with intuitive assessment of complexity.

The representation of assembly plans is particularly important for systems which do online planning or scheduling. Previous studies of online planning problems [13] have used discrete sequence representation or precedence diagrams of operations. In the precedence diagram formalism, typically no single partial order can encompass every possible assembly sequence. The AND/OR graph represents all possible partial orderings of operations, and each partial order corresponds to a solution tree from the node corresponding to the final product. We have illustrated the use of the AND/OR graph for online scheduling of a simple robotic workstation with random presentation of parts [2]. The resulting analysis showed a relative improvemens in efficiency (number of operations required) from fixed sequence operation of $6 \%$ for precedence diagrams and $18 \%$ for the AND/OR graph. The principal advantage in this example was the reduced need for buffering and corresponding retrieval of parts.

The AND/OR graph representation provides a framework for the planning and scheduling of operations sequences. The protlems of testing, disassembly, repair, and assembly all benefit from a unified representation which encompasses partial ordering of procedures. Preliminary search of the task space may reduce the candidate subtrees substantially, but the development of final plans typically involves directly the implementation and specification of the underlying devices and motions. In the next section we describe a tool for discrete task planning which facilitates exploration of alternative sequences of operations at the level of parts configurations.

## 4. Discrete Task Planning

A sequence of assembly or disassembly subtasks is implemented by performing operations on the parts using system resources such as robot hands, fixtures or sensors. The allocation of these resources and the synthesis of control programs to coordinate them must be developed in a second level of planning. In general, such operations require detailed motion planning of individual devices and is extremely difficult. In this section, we describe a definition of discrete operations which lend themselves to planning through manipulation of the configuration map relating input and output configuration states.

Any subtree of the AND/OR graph may be thought of as a subtask precedence graph.
and each branch of the subtask precedence graph defines a process in the configuration space of the parts. An assembly operation can then be defined by:

## Assembly Operation:

Given $e^{0}=\left(C_{i}^{\circ}, C_{j}{ }^{\circ}\right) \in C$, control manipulation, sensing, and computation to achieve $c^{f}=\left(C_{i}^{f}, C_{j}^{f}\right) \in T$, then
execute operation,
where $\mathbf{T}=$ tolerance set,
$T \subseteq C=C_{i} \times C_{j}$ for entitiea $i, j$,
is the set of configurations (region of configuration space (14)) for which an operation on $\mathrm{i}, \mathrm{j}$ can be successfully performed.

This definition emphasizes the basic problem in assembly as the control over configuration uncertainty in order to meet tolerance requirements of successive operations. While it is possible to define probability distributions over configurations of parts, in practice, it is very difficult to accurately estimate such distributions, and it is cumbersome to propagate the effect of such distributions through successive operations in a sequence. The configuration map used here provides a tool to compute the effect of operations on bounding sets of configuration points.

A bounding set $B(v)$ is defined as

$$
B(v)=\{\text { possible outcomes of } v\rangle
$$

where $v$ is a bounded variable. We can define in turn:
Joint bounding set: $B\left(v_{1}, v_{2}, \ldots, v_{d}\right)$
Conditional bounding set: $B\left(v_{1} \mid v_{2}=\eta\right)=\left\{v_{1} \mid\left(v_{1}, \eta\right) \in B\left(v_{1}, v_{2}\right)\right\}$
Sum of bounding sets: $A+B=\{v \mid v=a+b$ for $a \in A, b \in B\}$
Scalar multiplication: $c \mathbf{A}=\{\mathbf{v} \mid \mathbf{v}=\mathrm{ca}$ for a $\in \mathbf{A}\}$.

An operation which alters the configuration of a part may be described by a mapping between the initial configuration, $\theta_{i}$, and the final configuration $\theta_{f}$. An operation with a unique mapping occupies a single point in ( $C$-space $x C$-space) and completely defines the change in configuration state of the system. In this case, planning of operations reduces to planning of unique trajectories in configuration space. As discussed above, such unique mappings are often of limited use due to the uncertaint $y$ in configurations and the finite tolerance of operations. Then, states of the objects may be described by bounding sets of points in the configuration space.

The configuration map $M\left(A_{i}, B_{i}\right)$ describes a single operation which maps a bounded set of input points to a bounded set of output points:

$$
M\left(S_{1}, S_{2}\right):\left\{S_{1}\right\} \rightarrow\left\{S_{2}\right\}
$$

The configuration map takes on logical values in (C-space $\times$ C-space) where each logical ' 1 ' defines a feasible mapping. The configuration map for a rigid part is a function of twelve dimensions, although in many cases these degrees of freedom are not of equal interest.

The usefulness of the configuration map representation of operations lies in the ease of combining sequential operations. An operation $M_{1}\left(\Theta_{2}, \alpha\right)$ followed by an operation $M_{2}\left(\alpha, \theta_{f}\right)$ is defined as:

$$
M_{12}\left(\theta_{1}, \theta_{f}\right)=M_{1} M_{2}=U_{a}\left\{M_{2}\left(\alpha, \theta_{f}\right) \cap M_{1}\left(\theta_{i}, \alpha\right)\right\} .
$$

Sequences of alternative operations may therefore be compared using simple relations.

The configuration map is particularly useful in cases where inputs and outputs may be partitioned into bounded sets. If we identify N subintervals B of the output space and $\mathbf{N}$ subintervals of $\mathbf{A}$ of the input space, then a symbolic mapping:

$$
M^{\prime}=U_{j}\left\{A_{j} \times B_{j} \mid M\left(\theta_{i}, \alpha\right)>0\right\} .
$$

defines bounded regions of the configuration map associated with transformations of bounded sets due to a given operation. A useful instance of the bounded set map occurs when we let:

$$
A_{j}=U_{a \in B_{j}}\left\{\theta_{i} \mid M\left(\theta_{i}, \alpha\right)>0\right\} .
$$

Then the configuration mad

$$
M^{\prime}=U_{j} A_{j} \times B_{j}
$$

is rectangular and the operation is completely defined by the symbolic map and the definition of the underlying sets.

The product of rectangular configuration maps is completely defined by bounding set operations:

$$
M_{1} M_{2}=U_{j}{ }^{2} B_{j} \times\left\{U_{k \in}{ }^{21} c_{j}{ }^{1} A_{k}\right\}
$$

where

$$
{ }^{21} C_{j}=\left\{\left.k\right|^{2} A_{j} \cap{ }^{1} B_{k} \neq \varnothing\right\}
$$

is the resulting configuration map product.
Figure 2 shows an example of a peg insertion operation in two dimensions, This type of problem has been studied from the point of view of trajectory planning in configuration space [15]. The configuration map shown in figure 2 is derived from such a trajectory analysis and summarizes the input-output relations in a manner which permits the resulting discrete operation to be integrated into task plan. A
different configuration map is developed for each set of discrete operations parameters, and the ability to form configuration map products permits search over the space of operations sequences. In figure 2 , the $x$ position of the pes is regarded as the independent variable of the map, and the initial z-position of the peg is fixed for a given configuration map. The operation moves the peg in a -z direction using a compliant move and directional uncertainty represented by the velocity cone (16).

The resulting configuration map in figure 2 has three output bands corresponding to successful insertion, miss-to-the-left, and miss-to-the- right. These three bands occur consistently for different parameter values. Five input bands may then be reconstructed and labelled defining a partitioning of the input configuration space. The resulting map may be 'rectangularized' as shown by the dotted areas, and in that form the symbolic mapping provide a complete description of the operation and a basis for search procedures.

CONFIGURATION SPACE


CONFIGURATION MAP


Figure 2. Configuration map for peg in hole example.

An example of a product of configuration maps is shown for a different set of operations in figure 3. Each of these maps is derived from our analysis of sliding objects $[5,6,7]$ and corresponds to the orientations of a polygonal object being pushed by a two-dimensional fence of finite length. Equivalently, the object may be moving on a conveyor belt past a fixed fence. The independent variable in each map is the object orientation while the operation parameter is the fence angle. The uncertainty represented by the finite width bands in the maps is a result of the unknown support distributions of the objects. In [5,6,7] we derived bounds on the rates of rotation of such objects and have used these to compute the configuration maps for this example. The product of configuration maps therefore defines the bounds on the sets of orientations resulting from successive fence pushing operations, and can be used as a planning tool for designing sequences of fence push operations to achieve required goals.

For discrete tasks, the space of all operations sequences may be represented by a tree. Ares correspond to operations, and each node represents a set of possible configuration states after execution of all the operations on the path from the root to that node. Figure 4 illustrates one such tree which corresponds to sequences of fence pushing operations for fences of different angles operating on the object shown in figure 3. The possible configurations of a part at a given node are obtained by multiplying the configuration maps for the operations on the path from the root to the node. Traversing the tree in order to search it is facilitated by the ease with which products of multiple configuration maps can be compuied using the code sets.


Figure 3. Product of two configuration maps.


Figure 4. Tree search ror operations sequence.


Figure 5. Resulting sequence of fence push operations.

Each node is labelled with the subset of the indices j of B for the bands B for the fence angle of the preceding arc. The goal of this task was to reduce the set of possible configurations to a narrow range of orientation, and a search strategy was implemented to reduce the number of output bands to one using the minimum number of operations.

Searching this tree of discrete operations exhaustively is computationally difficult due to the high branching factor which results from the available set of fence angles at each step. Two techniques have been developed to make this search feasible. First, there are systematic relations among bands for different operations parameters. Since there are only a few distinct code sets for the output arcs, it is of ten possible to systematically choose the subset of arcs which need to be followed among these outputs. Second, branches of the tree which develop code sets which have occurred previously in a shorter route may be pruned during search.

Implementation of these search techniques permits solution of the fence sequence design problem with the resulting design shown in figure 5 . This parts feeder design will align parts with the geometry shown in figure 3 independent of the input orientation. Bounds on the orientation of the resulting single band are also derived from the procedure. The output part is then aligned for acquisition or handling by a robot. Computation for this search problem requires a few seconds of computation.

## 5. Conclusions

In this paper, we have reviewed several results in assembly representation, discrete task planning, and their relation to underlying control strategies. These methods of planning and manipulation are important for applications which will require autonomous systems to carry out complex tasks in diagnosis, repair, and assembly in space. The development of such analytical tools and their demonstration in prototype systems will be an important part of the cvolution of telerobotic and autonomous systems for space applications.

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# Using Automatic Robot Programming for Space Telerobotics 



This paper describes the interpreter of a task level robot programming system called Handey Handel is a system that can recognize, manipulate and assemble polyhedral parts from given only a specification of the goal. To perform an assembly, Handey makes use of a recognition module, a gross motion planner, a grasp planner, a local approach planner and is capable of planning part re-orientation. The possibility of including these modules ina telerobotics work-station is discussed.

## 2 Introduction

The projected increase in the use of robots in space will make their increased autonomy essential. Direct teleoperation of robots in complicated, repetitive tasks, such as those found in space, can be very tedious. Robot autonomy would relieve the operators from unnecessary fatigue as well as improving reliability and cost [1].

One step towards improving the autonomy of robots consists of having a system capable of planning simple grasp and assembly operations. This goal seems to be a fairly simple and short term objective and yet, it has only been achieved for very well structured environments. The early research on automatic planning of robot operations $[2,3,4]$ focused exclusively on simple situations involving completely modeled environmints.

More recent work has now made it possible to design systems working in much more general environments, including environments with significant uncertainty. These environments resemble the type of environment one can expect to find in the vicinity of a space station. This type of environment can include complex parts and six degree-offreedom revolute arms, all of it modeled with a CAD system. In this paper we describe Handey, a new system that embodies many of the fruits of this more recent research. While Handey still makes strong assumptions about its environments and tasks, we believe that these assumptions are realistic enough so that Handey can contribute towards improving the tele-operation of manipulators.

## 3 Handey overview

Handey is a task-level [5] robot programming interpreter, that is, the commands given to the system are not robot motions or gripper operations as in VAL or LM $[6,7]$ but simply by describing a certain desired state of an assembly. For example, a full sequence of robot motions, gripper operations and sensor calls can be replaced in a task-level robot programming system by a single statement: PLACE PART A on PART B. The interpreter is responsible for planning and carrying out the detailed motions and other actions which lead to this assembly. In its current stage of development, however, Handel makes use of the "perfect world" hypothesis and so, does not take in account problems related to uncertainty or unexpected events. For example Handey does not


Figure 1: Experimental setup
include a compliant motion planner which would plan assembly strategies in the presence of uncertainty ( $8,9,10$ ), and does not provide program venification techniques based on uncertainty propagation to patch a predefined plan [11,12]. This remains as future work.

Figure 1 represents a scene used during the development of the system, the doted line on the table shows the limits of the field of view of the laser range finder (this area is called the $V$-area in the rest of the paper).

Part $A$ is assumed to be located in the $V$-area. Nothing is assumed concerning this area: part $A$ can be in any location and it can be partially obscured from the range finder by other objects. Except for part A it is not necessary for parts entirely located in the $V$-area to be modeled in the CAD system. The location of part $B$ is assumed to be known, as are the locations of obstacles in the workspace outside of the V -area, such as the laser-camera device, which we plan to use in the future to find the exact relative position between the gripper and the part.

The user describes the final assembly with a set of relationships between geometric features of parts $A$ and $B$, then he activates the interpreter. The following is a typical sequence performed by the interpreter to plan the detailed motions and operations necessary to achieve the assembly.

Determining the Final Location. Based on the specified symbolic geometric relationships between parts A and B , a geometric transform representing the relative location between A and B after the assembly is computed.

Recognizing and Localizing Part A. A model-based vision algorithm is executed to determine the location of part $A$.

Planning a Grasping operation. The grasp planner first tries to find a grasp which permits placing part $A$ directly on part $B$. If this is not possible, a regrasping operation will be planned later.

Planning the first gross motion. A collision-free path is planned from the initial position to a point on the boundary of the V-atrea.

Planning a collision free approach. Since the scene in the V -area is not modeled in the CAD system, it is not possible to find a collision free path by the method
used for gross-motion planning. Another plamer, using the data provided by the range finder, plans a path for the gripper among the obstacles which are located in the V -area.

Planning re-orientation. In many occasions it is not possible to grasp and to assemble the part keeping the same relative position between the part and the gripper. In this case a regrasp operation is planed in a obstacle free portion of the work space.
Planning the remaining gross motions. The regrasp planner produces a number of intermediary locations to be reached by the robot during the re-orientation phase, this phase computes all the paths necessary the perform the regrasping and the path to the final destination.

## 4 Functional description of Handey

Handey is composed of several modules, most of these modules correspond to substantial pieces of code.

### 4.1 Experimental Environment

The hardware-dependent software primitives provide a way for the planner to ignore the details needed to operate real-world equipment. It is crucial that these modules be quite good, since they determine the overal system's precision in localizing and moving parts and, as a consequence, will determine the success of the experimentats. Handey makes use of a very limited number of such hardware dependent primitives.

- Range finder calibration: this primitive eliminates non-linearity due to the technology of the laser sensor and scanner hardware. It also determines the scale factors between the sensor and the model.
- Robot Vision calibration: this primitive determines accurately the relative location between the reference frames associated with the robot and the range finder.
- Depth map acquisition: this primitive activates the range finder and returns a depth map in standard units.
- Joint motion: in its current version Handey makes use of one robot motion primitive: "MOVE-JOINT TO (q1,q2,....q6)" to command a coordinated motion of the robot. The gripper is operated in a binary mode.


### 4.2 The world modeling system

The world modeling system is used to construct polyhedral models of the parts involved in the assembly including the obstacles (table, ceiling, etc.) and the robot. It is also used to maintain a model of the world during the planning. Once geometric models have been created it is possible to use the following primitives to create, modify or interpret a scene:

```
- assign a location to a part,
- express the location of a part in a different reference frame,
- affix and unfix parts from the gripper of the robot
```



- Face A of Part A is Parallel to Face A of Part B
- Face $C$ of Part $A$ is Against Face $C$ of Part $B$
- Face B of part A is Against Face B of Part B

Figure 2: Describing the final assembly


Figure 3: Depth-map produced by the laser range finder

### 4.3 Computing the final location of part $A$

The user can describe symbolically the next assembly step by specifying a set of geometric relationships which should hold between geometric features of part A and geometric feat ures of the sub-assembly. Figure 2 represents the set of relationships used to describe the final location of part $A$.

The set of geometric relationships is then translated into a set of algebraic equations [13]. The system then solvesthe the set of equations to compute the relative position between the part and the sub-assembly. At the present time this feature is not integrated into the on-line version of Handey but works separately.

### 4.4 Locating part A

The range finder is activated and produces a depth map (figure 3). The map is then processed as if it was an image except that the brightness corresponds directly to the elevation above the work table.

A standard edge operator [15] is run over the image and extended linear segments are identified in the resulting array. Note that this process identifies 3D edge segments.


Figure 4. Matching model edges with scene edges
not just their projection in an image. The method used for object localization is a simple hypothesize-verify algorithm based on matching linear segments in the depth map to edges in the polyhedral model of the part. This method is a variation of the method described in Lozano-Pérez and Grimson [16] using edge data instead of face data. Figure 4 represents one matching between edges of the scene and edges of the model.

### 4.5 Planning collision-free motions

At a number of points in the operation of the system, a collision-free path is required from one specified location to another. Handey uses a simplified version of the path planner described in Lozano-Pérez [17]. This path planner uses the robot's joint space as the configuration space. The version of the path planner used by Handey never computes configuration spaces of dimension greater than three, but it allows motions requiring six degrees of freedom. Essentially, we assume that a path from the start to the goal exists such that the last three joints of the arm retain their starting values until some intermediate point where they change their values at the goal and never change after thai. It is easy to construct cases where this assumption will fail. but it works in a large percentage of actual cases.

The actual planning proceeds as follows: An approximate arm model is built in which the last three joints are replaced by a box. This box must be large enough to enclose the last three links. the hand and any object in the hand, not only at their start and goal position but also at the intermediate positions between the two. The three-dimensional configuration space for this model can then be built. We find the closest free point in this configuration space to both start and goal positions, a path is then found between these two free points. Note that the complete robot is guaranteed to be safe along this path. for the whole range of values of the last three joints between the start and the goal. Therefore we can simply interpolate the values of the last three joints between the start and the goal values. Then, we plan a path using the original model of the robot between the free point and the start point itself. We also plan a path from the free point closest to the goal itself. In these two paths the value of the last three joints are fixed. The concatenation of these three paths form the desired path. Figure 5 represents the path found the final motion.


Figure 5 Example of a path generated by the path plannor


Figure 6 Back-projection of parts

### 4.6 Grasping

Once the part has been located Handey chooses a grasp. This operation has to take in account several constraints:

- the grasp should be stable,
- a path should exist inside the $V$-area to reach the grasp,
- the grasp should permit assembling the part once it is in the gripper.

The last constraint can be satisfied by "back projecting" all the obstacles in the $V$-area. After this operation virtual obstacles exist in the $V$-area, these obstacles thave the same relative location with part $A$ that the real obstacles will have in the final sub-assembly. If one can find a grasp in this environment then it is guaranteed than the grasp will permit assembling the part (figure 6 ).

### 4.6.1 Finding a stable grasp

In its current version, Handey uses a grasp planner designed for a gripper equipped with parallel jaws. A future version of Handey will include a more sophisticated planner designed for the three fingers JPL-Stanford-MIT hand 14]. Currently, the planner


Figure i cirasp-pointa associated with one face


Figure is Angular range associated with a grasp
associates two grasps for each locally convex edge of the model. A grasp is defined by one of the face adjacent to the edge and a grasping point. The grasping point is located on the face at a prespecified distance from the edge. Figure $i$ represents all the grasping points of one face of part $A$.

To be valid a grasp should satisfied three conditions:

1. a parallel face should exist and should permit a grasp (mutual visibility : $18 j$ ) with an allowed distance between the two faces.
2. The gripper should be capable of some rotation around the grasping point (grasping range),
3. an inverse kinematic solution should exist at the grasping point.

The grasping range can be computed using a submodule of the path planner. The grasps are sorted with such grasp permitting the most vertical approach. Figure 8 represents the gripper into two end-points of an angular range.

### 4.6.2 Planning motions in the $V$-area

Since no information on obstacles exists in the world-modeling system for the $V$-area. we must : ake in account the presence of objects reflected in the depth map. For this purpose we use a planner specialized for planning the motion of the hand in the grasp plane. The grasp plane is a plane parallel and at equal distance from the two faces defining the grasp. When approaching a grasp the fingers remain parallel to the grasp


Figure 9: Planning a path in the grasp plane
plane and centered about it but, otherwise, are free to rotate and translate in the plane. Obstacles are projected into this plane to reflect the possibility that they collide either with one of the two fingers, the cross-piece of the hand, or the force sensor.

The planner uses a method loosely modeled on the potential field method for obstacles avoidance [19]. The goal of the grasp planner is to bring a gripping point located between the fingers as close as possible from the ह. asping point without colliding. The grasping point attracts the gripping point of the grippir while projected obstacles on the grasping plan repel the boundaries of the projected gripper parts (fingers, cross-piece and force sensor). These pseudo-forces are combined in such a way that the gripper is guided toward the goal in X,Y, $\Theta$ on the grasp plane. The initial position and orientation of the gripper is given by the grasping planner. Figure 9 represents the evolution from the initial position toward the goal.

### 4.7 Regrasping

Back-projected objects are artificially added to the depth map so that the final grasp will also permit the assembly. However this may constrain the problem so much that a feasible solution cannot be found. Handey will backtrack among sorted grasps a limited number of times before giving up and trying to find a solution with regrasping. The $V$-area path planner is then called without back-projected parts and a regrasping operation is planned.

For each part the grasp planner uses two data structures: placentents and grasps.

1. A placement is a way of placing the part at a particular location on the work table. This location is chosen in an area known to be free of any oostacle, the regrasping will take place in this area. A parameter $p$ is associated with each placement $P$. Changing this parameter corresponds to rotating the part on the table around a vertical axis. All the placements $P_{1}$ are computed automatically by computing the stable faces of the convex hull of the part.
2. A grasp is defined by a parameter $\theta$ associated to each grasp 6. Changing this parameter corresponds to rotating the gripper along an axis perpendicular to the


Figure 10: Finding successive plarements and grasps
grasping face and containing the grasping point. The set of grasps $G$, is also computed automacically.

In order to plan a regrasping operation it is necessary to compute all the vertices of the "regrasping graph". A vertex consists of a data structure defined by a pair $P_{1} G$, having a non-empty $\theta \varphi$ map. A map is built by sampling $\varphi$ and $\theta$ over the interval $(0.0,2 \pi)$. Each $\varphi, \theta$ specifies a single position of the gripper. To be valid, a pair should correspond to a position of the gripper where a solution of the inverse kinematic exists. The map is the set of all the valid $\theta \varphi$ pairs.

There are two operations necessary to perform a re-orientation.

1. Moving from one placement $P_{1}$ to another $P_{k}$. This is possible when a grasp $G_{\text {, }}$ exists. such that the maps associated with $P_{1} G$, and $P_{k} G$, have at least one valid $\theta 9$ pair.
2. Changing from one grasp $G$, to another grasp $G_{k}$. This is possible when a placement $P$. exisis such that the map associated with $P_{1} G_{j}$ and $P_{1} G_{k}$ has at least one valid $\theta \varphi$ pair.

The regrasp planner is given an initial position of the part inside the gripper and a final $G!, p$ : grasp which permits the assembly operation. The goal of the regraspplanner is to find a way through various placements and grasps between the initial and the final grasps. This is represented in figure 10. Horizontal arcs in the figure represent motions of the part from one placement to another and vertical arcs represent motions of the gripper to change the grasp.

## 5 Applications to Telerobotics

Uising telemanipulators in earth orbit has long been recognized as a difficult task. The trend has been to increase the level of commands available to the operator (20]. Prototype telerobotics workstations have been built integrating such high level teleoperation cummands. For example, hybrid-control permits the compuier to maintain a drill on a given axis while the operator can concentrate controlling the force necessary to perform the drilling operation.

Based on our experience we believe that it is not unrealistic to add some of the capabilities of Handey to such a work-station. As explained in 4.1 the number of primitives used by Handey is fairly limited and should be available in such a workstation anyway.

The most limitating factor being the possibility of including a range finder on the mobile remote servicer (RMS). Once integrated, one can imagine to use a system such Handey in three modes.

- autonomous mode: The operator describes the next assembly step. The system computes the sequence of operations and sensor calls to perform the assembly a graphic simulation is presented to the operator before actual execution.
- partially automatic mode $\ln$ this mode the operator asks the system to plan certain portions of the assembly, for example, the system can plan the trajectory to align two axes so that a drilling operation can take place under the control of the operator.
- monitoring In this mode the operator first describes what result he expects to achieve.' The system monitors the task and sends an alarm when it detects that the present configuration of the system makes it difficult or impossible to reach the goal. For example grasping a part in a way that the final assembly or an intermediate path is difficult or impossible.


## 6 Conclusion

Watching Handey performing an assembly is always astonishing and fun, no operator would ever program a task the way our system does. Potentially, we believe that future versions of Handey could be more efficient in performing assembly tasks than typical operators. It could plan paths more effectively in term of time, energy, and safety. It would be less likely to make a mistake such as grasping a part and not being able to move it at a later stage of the assembly because of mechanical stops or collisions. Handey is based on well-establish geometric principles which can make it a robust system. For this reason. it is possible to think of the Handey interpreter as a target system for higherlevel planners. The current Handey implementation on a Lisp Machine is still quite slow; it takes approximately 10 min to plan a single pick and place operation. But, we believe that is possible to reduce this time significantly simply by reimplementing it in a machine with fast floating point hardware.

Telerobotics is often presented as feasible alternative to an infeasible autonomous robot. This is certainly true at the present time, but the contrary may be true in the future, that is, the technology necessary to achieve good tele-presence may be more sophisticated than the technology necessary to provide on-board intelligence and dexterity.

## 7 Acknowledgments

This work was founded primarily ty the Office of Naval Research under contract N00014-85-K-0214. Additional support was provided by an NSF Presidential Young Investigator Award (Lozano-Pérez) and the French CNRS (Mazer).

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MANIPULATOR CONTROL

# Manipulator Control: An Overview 

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The next generation of robot manipulators will exhibit rudimentary intelligence by acquisition and perception of sensory data and operation in partially unknown environments with some degree of autonomy. One of the crucial components for realizing these capabilities is a sophisticated manipulator control system. Consequently, research in advancing the control of manipulators is an essential step towards the development of future robots.

The topic of "Manipulator Control" was chosen as one of the central themes of the NASA Workshop on Space Telerobotics. Six sessions containing 38 papers were devoted to manipulator control. In the course of presentations and subsequent panel discussions, it became evident that the topics under manipulator control may broadly be classified into two categories.

The first category contains topics which are largely resolved by now, such as advanced control methodologies for single-arm robots in free motion. Theoretical research in these areas has been pursued actively during the past decade and has reached a modest level of maturity. Nevertheless, there are very few practical implementations of the advanced control techniques, even at the academic level. It was generally felt that more emphasis should be placed on implementing advanced control methodologies on robot manipulators.

The second category covers those topics which are partially known at the present time. These include force control, coordinated multiple-arm control and control of redundant and flexible arms. Although some partial results have been reported on these topics, considerable research is still much needed. For instance, in the area of force control, when the robot makes contact with the environment, dynamic models which adequately represent this interaction are not yet completely developed. In a similar manner, optimum coordination and task allocation among multiple cooperative arms are not yet resolved. It was generally believed that support of fundamental theoretical research on these topics is currently needed.

In summary, the manipulator control sessions at the Workshop were successful in providing a forum for technical interaction among researchers in robot control. Furthermore. the findings of the Workshop were beneficial to the robotics community and in particular to NASA in enhancing its perception of manipulator control technology and in identifying the directions of future research and development in robot control.

# Adaptive Force-Position Control for Teleoperated Manipulators 

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An adaptive controller with self-tuning can be designed for teleoperated robotic manipulators by determining a time-series model for the function of the teleoperator. Specifically, the position and force exerted by the operator are modelled for determining the derived values for the trajectory of the endeffector of the manipulator. Thus, the adaptive controller can be designed by following the steps which have previously been presented for the controller design of the gross motion.

## 1. Introduction

A teleoperated robotic manipulator refers usually to a system in which an operator equipped with sufficient senmors, effectors and computer intelligence can make the manipulator perform complex tasks either under human supervision or autonomously. The human operator supervises the robotic system which is performing low-level tasks by intermittently monitoring and/or reprogramming the computer. Thus, the teleoperator can increase the level of intelligence of the overall system.

The teleoperator functions in the system as the "master" and the manipulator as the "slave". The teleoperator is mainly interested in the motion of the end-effector, and not so much in the motion of the intervening segments. Although the control of manipulator motion is comenonly performed in the joint space, it can also be accomplished directly in the Cartesian coordinate system [5]. If the motion of the master (the teleoperator) is described in the Cartesian world coordinate system, the slave can be made to follow the motion of the master by controlling the motion of the slave (manipulator) in the Cartesian base coordinate system. In the preliminary work described here, we will use this approach to control the force exerted by the end-effector of the manipulator, and its gross motion. We will construct an adaptive self-tuning controller for the control of the force and position of the end-eflector.

The overall system is first described briefly, and the problem formulation is given. A time-scries model for the motion of the teleoperator is then developed. This model is used to predict the desired motion of the manipulator. An adaptive controller is then designed to make the manipulator follow the desired motion without the supervision of the operator.

## 2. Teleoperated Robotic Manipulator System and Problem Statement

The overall system consists of a robotic manipulator with an end-effector, computer, and a teleoperator, whose arm and hand are constrained to have the same configuration as the manipulator. The hand is assumed to possess two (jaw-like) fingers. It will be assumed in this preliminary study that the position of the hand as well as the force and moments exerted by the hand can be measured.

The measurements of the position of the hand and the forces (moments) exerted by the hand on the object will be described as a multivariate discrete time-series model. The parameters of this model are estimated recursively by the least squares error method on-line. The resulting model will be used to predict the desired values of the variables for controlling the manipulator, and its end-effector.

The dynamics of the manipulator will also be modelled by means of a multivariate stochastic discrete time-series equation with unknown parameters. This vector difference equation is used as the basis in designing an adaptive selftuning controller for the manipulator motion.

The problems to be considered in the following consist of constructing (i) a discrete time-series model for the desired values of the position and forces for the end-effector; (ii) a multivariate auto-regressive ( $A R X$ ) model with exteranal inputs for designing an adaptive self-tuning controller for the dynamics of the manipulator. The difference equation model for the desired values of the position and force are based on the measurements which become available when the teleoperator performs a task (i.e., teach-by-doing). The ARX-model is determined on the basis of the measurements available from the position sensors and the force sensors of the manipulator. We will assume that the forces exerted by the end-effector on the object are "soft", i.e., that they can be modelled by linear springs.

## 2. Mathematical Model for Teleoperated Manipulator System

In order to make the end-effector follow the path determined by the teleoperator, and exert the force (torque) specified by the same operator, the values of these variables will be measured as a function of time. The future values of these variables can be predicted by using a time-series model constructed on the basis of the measurements. Suppose
that the teleoperator exerts a force $\Gamma^{d}(k)$ at time $k T$ ( $T=s a m p l i n g$ period) on an object, while following a trajectory passing through the points $p^{d}(k)$ expressed relative to a chosen Cartesian world coordinate system. These values can be modelled by time-series models:

$$
\begin{align*}
& f^{\prime}(k)=A_{o}^{d}+\sum_{i=1}^{n} A_{i}^{d} f^{d}(k-i)+\xi^{d}(k)  \tag{1}\\
& p^{d}(k)=B_{o}^{d}+\sum_{i=1}^{n} B_{i}^{d} p^{d}(k-i)+\eta^{d}(k) \tag{2}
\end{align*}
$$

where the equation error is signified by $\xi^{d}(k)$, and $\eta^{d}(k)$; the unknown parameters in the matrices $A_{j}^{d}$, and $B_{j}^{d}, j=$ $0,1, \ldots, n$ are estimated by the least squares error method on the basis of the measurements.

The one-step ahead predicted values for the force $f^{d}$ and $p^{d}$ are computed by the following equationa:

$$
\begin{align*}
& \mathrm{S}^{\mathrm{J}}(k+1 \mid k)=\hat{A}_{0}^{d}+\sum_{i=1}^{n} \hat{A}_{i}^{d} \mathrm{~S}^{d}(k-i+1)  \tag{3}\\
& \mathrm{p}^{d}(k+1 \mid k)=\mathrm{B}_{0}^{d}+\sum_{i=1}^{n} \hat{B}_{i}^{d} p^{d}(k-i+1) \tag{4}
\end{align*}
$$

where the terms on the right are known from the measurements and the calculations of the parameter estimates.
To construct a controller for the manipulator, an ARX-model is used as the basis of the design. If the position of the end-effector relative to the Cartesian base coordinate system $p(k)$ at time $k$ ' $T$ is measured (e.g., encoder readings), while it exerts a force $f(k)$ on an object, then an ARX-inodel for the measuremente can be written as:

$$
\begin{align*}
& p(k)=C_{0}+\sum_{i=1}^{m} C_{i} p(k-i)+G_{1} u(k-1)+e_{1}(k)  \tag{5}\\
& r(k)=D_{0}+\sum_{i=1}^{m} D_{i} f(k-i)+E_{1} u(k-1)+e_{2}(k) \tag{0}
\end{align*}
$$

where the modelling errors are denoted by $e_{1}(k)$ and $e_{2}(k)$; the unknown parameters in the matrices $C_{j}$, and $j, j=0, \ldots, m$ $2 r e$ estimated recursively on line by the least squares error method on the basis of the available measurements.

The desired values for the position and force are related to the values $\mathrm{p}^{\mathrm{d}}(\mathrm{k})$ and $\mathrm{f}^{\prime}(k)$ determined by equations ( 1 ) through (4) for the teleoperator. By applying appropriate coordinate transformation, which relates the Cartesian world cogrdinate system used in describing ( $\rho^{d}(k) \mid$ and $\left\{\left\{^{\prime}(k)\right\}\right.$, to the Cartesian base coordinate system, the desired values \{ $\overline{\mathbf{p}}(\mathbf{k})\}$ and $\{\mathbf{r}(\mathbf{k})\}$ expressed in the base coordinate system can be determined.

Having obtained the desired values for the manipulator motion, an adaptive self-tuning controller can next be designed. It is determined by minimizing the following performance criterion:

$$
\begin{equation*}
I_{k}|u|=E\left\{\left\|p(k+1)-\bar{p}^{d}(k+1 \mid k)\right\|_{(1-s)}^{2}+\left\|r(k+1)-r^{-1}(k+1 \mid k)\right\|_{s}^{2}+\|u(k)\|_{R}^{2}\right\} \tag{7}
\end{equation*}
$$

where the matrix $S$ represents the selection matrix for determining when the force-servoing or position-servoing will be used. The norm refers to the usual generalised Euclidean norm. The expectation operation is conditioned on the available measurements.

The problem is solved by minimizing $I_{k}(u)$ in equation (7) subject to the plant equation constraints given by equations (5) and (B).

The minimising controller $u^{\circ}(k)$ is specifed by the following equation:

$$
\begin{align*}
& \hat{G}_{1}^{\prime}(I-S)\left|\dot{C}_{0}+\sum_{i=1}^{m} \dot{C}_{i} p(k+1-i)+\hat{G}_{1} u^{\cdot}(k)-\bar{p}^{d}(k+1 \mid k)\right|+ \\
& \quad+\hat{E}_{1}^{\prime} S\left[\hat{D}_{0}+\sum_{i=1}^{m} \hat{D}_{i} f(k-i)+\hat{E}_{1} u^{\cdot}(k)-f^{-d}(k+1 \mid k) \mid+R u^{\cdot}(k)=0\right. \tag{8}
\end{align*}
$$

Equation (8) can be solved for the control input $u^{\circ}(k)$ in the feedback form. It should be observed that the desired trajectory must also be updated according to equations (1) through (4), with the model parameters.

Simulations studies are currently being conducted to demonstrate for the feasibility of the approach.

## 4. Conelusions

An adaptive self-tuning controller design has been presented for the operation of teleoperated manipulators. The approach is first to model the function of the teleoperator. Then, the controller design is performed on the basis of the desired trajectory determined. Simulation studies are currently being conducted using the approach.

Acknowledgement
This work has been supported in part by National Science Foundation Grant COR 8500022.

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# Adaptive Control of Dual-Arm Robots 

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 adepitve controllers unease that the end-affector position e of beth arisen track

 strategy, the adaptive controller of one art controls andeffecter motion in


 adaptive controllers enema the both end-sfiectore trios reference position trajectories willie timitameonsty aphyiat desired forces on the load. In all


 controllers do a ot require the complex mathention model of the art dynamics

 computationally fast for on-lige implementation with high stapling rates.

## 1. Inisoducisian

Daring the pest decade. robot manipoletori have bees utilized in industry for performing simple tasks, and
 various complex task both in ladustry and la hazardous arifoameats. Nevertheless, preseat-day robots can be considered at best as "handicapped" operator a due to their giagle-ari etreotare. It is evident that
 handing larger loads. Deal -arm robots will therefore have capabilities which may match those of ambidextrous haman operators in dexterity ad efficiency.



 describe a method for coordinated control of two erna. Thong and hah [5,6] obtain constrained relations and control laws for two coordinated arms. Tara et al. [7] employ the exact linearization technique for dual -arm control. Hayati [8] proposes a method for controlling deal-ara robots based on partitioning the load better
 Lin and hymn [10] describe positional control scheme for two cooperating robot arms.

The control architecture considered in this paper ls based on the tri-level hierarchical control of dual-
 performed and decomposes the task into appropriate subtask for the right and left ara. In the latermediate level, each abtask is transformed into a sequence of aynchronome desired trajectories of ead-affectors motions and applied forces. The low level la concerned lith the erection of the desired trajectories and employs
 desired motion and operates in "millisecond" timescale, the faternediate level "determines the motion desired $1 \because 1 / 159$



The present peper losesithe thren centrol etrateglen fer lov-iovel adaptive ceatrol of copperative deal-are










## 2. Rothton-Ponifion Contral Prapery





 gegted throgit the losd.

## 

 a

$$
\begin{equation*}
M(X) \dot{X}+N(X, \dot{X})+O(X)+E(\dot{X}) \pm-Y \tag{1}
\end{equation*}
$$

Where the ebove terns art defiaed se:

| $\mathbf{X}, \dot{\mathbf{x}}, \overline{\mathbf{z}}$ | - axl vectors of end-affector position, volootig and acoelerntion fa fired task-related Cartecian frame of reference |
| :---: | :---: |
| F | - asl vector of "virtial" Cartesian foroes spplied to the ondeffector as the control lapat |
| $\boldsymbol{M ( I )}$ | - max eymetrio poitive-deftatte Cartestan mes metrix |
| $N(X, \dot{x})$ | - mil Cartosian Coriolis and contrifegal foroe veotor |
| $\theta(2)$ | - nsl Cartesian travity losding veator |
| $\boldsymbol{B}(\dot{X})$ | - asl Cartesias friction forde vector |
| 1 | - axl vector of forcen and torques enerted by the end-effector on the load |



 end-affector position vector $X$ iracks the anl vector of desired trajectory $X_{d}$ despite the disterbace force $f$. For sech matipelator arm. let asply the liaear adaptive poition control lev (12)

$$
\begin{equation*}
P(t)=d(t)+\left[X_{p}(t) E(t)+Z_{V}(t) \dot{B}(t)\right]+\left[C(t) X_{d}(t)+B(t) \dot{X}_{d}(t)+A(t) \dot{X}_{d}(t)\right] \tag{2}
\end{equation*}
$$





accorlizg to the folioviag adeptation lave:

$$
\begin{align*}
& d(t)=d(0)+b_{1} \int_{0}^{t} t(t) d t+b_{2} I(t)  \tag{3}\\
& E_{p}(t)-E_{1}(0)+c_{1} \int_{0}^{t} f(t) E(t) d t+e_{2} z(t) E(t) \tag{4}
\end{align*}
$$

$$
\begin{align*}
& C(t)=C(0) \cdot \sigma_{1} \int_{0}^{t}(t) X_{j}(t) d t \cdot v_{2} s(t) I_{j}(t)  \tag{6}\\
& B(t)=E(0)+T_{1} \int_{0}^{t} f(t) \dot{X}_{f}(t) d t+T_{2} f(t) \dot{X}_{f}(t)  \tag{7}\\
& A(t)-A(0)+\lambda_{1} \int_{0}^{t} r(t) \ddot{X}_{j}(t) d t+\lambda_{2} z(t) \ddot{X}_{j}(t) \tag{B}
\end{align*}
$$

vhere

$$
\begin{equation*}
r(t)=v_{p} E(t)+\eta_{v}(t) \tag{9}
\end{equation*}
$$





 aseipalator ife. the coatrol lat in joint apace is fiven by

$$
\begin{equation*}
T(t)=J \cdot(\theta) P(t)=J \prime(\theta)\left(d(t)+\mathcal{K}_{p}(t) B(t)+X_{V}(t) \dot{z}(t)+C(t) \mathbf{X}_{d}(t)+B(t) \dot{x}_{d}(t)+A(t) \ddot{X}_{d}(t)\right) \tag{10}
\end{equation*}
$$





 relative to the adaptation lavi. It is sean that the faclision of the diatmbance force fin the robot model
 teall.
 sdeptive position costrollerg, we expect the esd-effectorito track the destred position trajectories despite the iateraction forces and torquet exerted throgh the load. it mate be aoted that atace the force on the load
 trejectories ere aot plamed in coordiantion or ere not tracked olosely.

## 3. Posibion-liybrid Contrel Sirategy

In this sestion, the position-hobid control etretesy for deel-arm agipalators vill be studied is vich
 Figere 4. In other vords. for the left arm. the ead-effector poition ig required to trick a desired trijectory
 -ffector end the loed aist be coitrolled in the directions coistrifed by the losd. while the ead-affector position is to be controlled sifeteacougly in the free directions. This apatrol strategy can be epplied when one robot arim is confiaed to operate only in poition control mode vheres the other arm can be controlled fil aybid comtrol mode.

### 8.1 Enshin cmitedier for Lift are



 loft ere shert ia Figure is fiven by [12]

 trackiag-orger veetor and the terme in equetion (11) ore adapted at follows:

$$
\begin{align*}
& \bar{T}(t)=\boldsymbol{T}(0)+t_{2} \int_{0}^{t}(t) d t+s_{2} r(t)  \tag{12}\\
& T_{p}(t)=E_{7}(0) \bullet c_{1} f_{0}^{t}(t) E(t) d t * c_{2} r(t) E^{\prime}(t) \tag{13}
\end{align*}
$$

$$
\begin{align*}
& \bar{C}(t)=\boldsymbol{Z}(0) \cdot v_{1} f_{0}^{t} r(t) \dot{L}_{i d}^{\prime}(t) d t+y_{2} z(t) I_{i d}^{i}(t)  \tag{15}\\
& \boldsymbol{T}(t)=\bar{T}(0)+T_{1} f_{0}^{t} r(t) \dot{x}_{i d}(t) d t+T_{2} r(t) \dot{i}_{i}(t)  \tag{16}\\
& \bar{X}(t)=\bar{A}(0)+\lambda_{1} f_{0}^{t} r(t) \bar{X}_{l_{d}}(t) \Delta t+\lambda_{2} r(t) \ddot{X}_{l_{d}}(t)  \tag{117}\\
& \text { There } \\
& z(t)-\nabla_{p}(t)+\nabla_{t}(t) \tag{18}
\end{align*}
$$

sad the symbola are defined in seotion 2.

### 3.2 Frbrid Controlier fer Bisht An









 treckiat of desired posistion trajectories in the free directions.

The dyanic model of the right arm in the constraist dizections can be vitita as [13]
$A(X, \dot{x}) \ddot{P}(t)+B(X, \dot{X}) \dot{P}(t)+Z(t)+C_{p}(\bar{Y}) \pm f_{z}=F_{z}(t)$




 to the mybid controller.
 architectare. For the right ara, the linear adeptive force control lat in the coastatat directions fis give by [13]

$$
\begin{equation*}
\nabla_{E}(t)=P_{F}(t)+A(t) \cdot \mathcal{E}_{7}(t) \varepsilon(t) \cdot E_{I}(t) f_{0}^{t}(t) \Delta t-E_{F}(t) \dot{z}(t) \tag{26}
\end{equation*}
$$



 staptel as fellous:

$$
\begin{align*}
& A(t)=A(0)+s_{1} f_{0}^{t} f(t) d t+s_{2} f(t)  \tag{21}\\
& E_{1}(t)=E_{1}(0) \cdot L_{1} \int_{0}^{i}(t) \text { Eev(t) ct } c_{2}(t) E v(t)  \tag{22}\\
& E_{0}(t)-E_{1}(0)+H_{1} f_{0}^{t}(t) E(t) d s+H_{2}(t) E(t)  \tag{23}\\
& L_{v}(t)=E_{-}(0)-7_{1} \int_{0}^{t} f(t) \dot{z}(t) d t-r_{2}(t) \dot{z}(t) \tag{24}
\end{align*}
$$

-bere

$$
\begin{equation*}
T(t)=\Psi_{I} E(t)+\nabla_{p} E(t)-v_{t}(t) \tag{25}
\end{equation*}
$$


 defigmer to refleat the rilative oignifleanee of E . E and t .

The dyminto modal of the githt arm tate free directions oan be vitten as [13]

$$
\begin{equation*}
A_{0}(\underline{X}, \dot{X}) \ddot{Y}(t)+B_{0}(X, \dot{X}) \dot{I}(\varepsilon)+C_{0}(X, \dot{X}) Y(t)+C_{f}(P) \pm X_{y}=\nabla_{Y}(t) \tag{26}
\end{equation*}
$$



 firections fe fives by

$$
\begin{equation*}
F_{y}(t)=\tilde{d}(t)+X_{p}(t) E_{p}(t)+\tilde{E}_{V}(t) \dot{B}_{p}(t)+\tilde{C}(t) \mathbf{L}(t)+\mathbb{E}(t) \dot{\mathbf{E}}(t)+X(t) \underline{B}(t) \tag{27}
\end{equation*}
$$



 given by

$$
\begin{equation*}
T_{r}(t)=I_{f}^{\prime}\left(\theta_{g}\right)\binom{F_{z}(t)}{F_{Y}(t)} \tag{28}
\end{equation*}
$$






 the adaptation scheme. Thes the isolsation of the disturbsace forces fand fin the robot modela (19) and (26) does not affect the controller performace.

We conclade thet meing the position-hybid control strateg, the loft ead-offector vill track the desired





## 









## 

 be Wittes es

$$
\begin{equation*}
T(t)=J \cdot(\theta)\binom{F_{8}(t)}{F_{5}(t)} \tag{29}
\end{equation*}
$$

 "virtual Cartestan forces spplied to the ed-effector in the zoestraint direotiosa (Z) and fres diroctione (T), respectively. ha shown in Figure j, the foree control lav is given by
$I_{g}(t)=P_{g}(t)+d(t)+I_{I}(t) f_{0}^{t} Z_{g}(t) d t+I_{8}(t) E_{g}(t)-E_{T}(t) Z(t)$
 1ate ase:

$$
\begin{aligned}
& d(t)=d(0)+\delta_{1} \int_{0}^{t} q(t) d t+\delta_{2} d(t) \\
& I_{I}(t)=I_{I}(0)+a_{1} \int_{0}^{t}(t) E_{2}^{*}(t) d t+a_{2} q(t) E_{z}^{*}(t) \\
& E_{p}(t)=E_{p}(0)+B_{1} \int_{0}^{t} q(t) E_{i}(t) d t+H_{2 q}(t) E_{i}(t) \\
& Y_{V}(t)=L_{v}(0)-r_{1} \int_{0}^{t} q(t) \dot{Z}(t) d t-r_{2} q(t) \dot{z}(t)
\end{aligned}
$$

-here

$$
q(t)=\nabla_{I} E_{z}^{\bullet}(t)+\nabla_{p} E_{2}(t)-\nabla_{\nabla}(t)
$$



The position control lev is expressed as
 adaptation laws are:

$$
\bar{d}(t)=\bar{d}(0)+\bar{b}_{1} \int_{0}^{t} r(t) d t+\bar{\delta}_{2} r(t)
$$

$$
\begin{aligned}
& F_{7}(t)=\bar{E}_{7}(0)+\bar{W}_{2} f_{0}^{t}(t) \dot{E}_{f}(t) d t+\bar{T}_{2} z(t) t_{f}(t) \\
& \boldsymbol{C}(t)-\boldsymbol{Z}(0)+\bar{T}_{2} f_{0}^{t}(t) E(t) \Delta t+\overline{\mu_{2}} z(t) \Sigma^{\prime}(t) \\
& T(t)=T(0)+\bar{\gamma}_{2} f_{0}^{t} z(t) t(t) d t+\overline{\mathcal{F}_{2}} z(t) t^{\prime}(t) \\
& \bar{X}(t)=\bar{X}(0)+\bar{X}_{1} f_{0}^{t} z(t) \bar{\Sigma}^{\prime}(t) d t \cdot \bar{\lambda}_{2} z(t)^{\infty}(t)
\end{aligned}
$$

vhere

$$
=(t)=\nabla_{H}(t)+\dot{T}_{j}(t)
$$





 position and foree is required.

## 

Trree adeptive control strategios for cooperative dual-ars robote are coseribed la this paper. In tase
 an adeptive oostrolier in the lov level of the control hierarohy. Eeol oostroller easures that the controiled


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## 6. Achagletrmat

The research deseribed in this peper fas performed at the Jet Propalisoa Laboratory, Califorala Institate


## 7. Referemen

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Figure 1. Tri-level Hierarchical Control of Dual-Arm Robot


Figure 2. Position-Position Control Strategy


Figure 3. Adaptive Position Control System


Figure 4. Position-Hybrid Control Strategy


Figure 5. Adaptive Force Control Syst:m


Figure 6. Hybrid Position/Force Control System


Figure 7. Hybrid-Hybrid Control Strategy

# Design of a Reconfigurable Modular Manipulator System 

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#### Abstract

Using manipulators with a fixed configuration for specific tasks is appropriate when the task requirements are known beforehand. However, in less predictable situations, such as an outdoor construction site or aboard a space station, a manipulator system requires a wide range of capabilities, probably beyond the limitations of a single, fixed-configuration manipulator. To fulfil this need, weave been working on a Reconfigurable Modular Manipulator System (RNMS). + Ce......


Unlike conventional manipulators with fixed configurations, The RMMS win utilize a stock of interchangeable link and joint modules. Given requirements such as the workspace, dynamic atcurapl, and the payload required to accomplish a task, the RMMS will design the most appropriate manipulator configuration elect suitable modules form the inventory, generate an assembly procedure, configure the controller, and finally apply the fosultint manipulator to the task. In this way, the RMMS will fill a far wider requirement space than any single manipulator. It also inherently easy to maintain and transport, since it can be readily assembled and disassembled.

PeRe rc. $+\cdots$
we have designed and are constructing a prototype RMMS. The prototype currently consists of two joint modules and tour link modules. The joints utilize a conventional harmonic drive and torque motor actuator, with a anal servo amplifier included in the assembly. A brushes resolver is used to sense the joint position and velocity. For coupling the modules together, we use a standard electrical connector and V-band clamps for mechanical connection, although more sophisticated designs are under way for future versions. The joint design yields an output torque to 50 ft -lb at joint speeds up to 1 radian/second. The resolver and associated electronics have resolutions of 0.0001 radians, and absolute accuracies of $\pm 0.001$ radians. Manipulators configured from these prototype modules will have maximum reaches in the 0.5 to 2 meter range.

The real-time RMMS controller consists of a Motorola 68020 single-board computer which will perform real time servo control and path planning of the manipulator. This single board computer communicates via shared memory with a SUN3 workstation, which serves as a software development system and robot programming environment. Ya have designed a bus communication network to provide multiplexed communication between the joint modules and the/computer controller. The bus supports identification of modules, sensing of joint states, and commands to the joint actustor. This network has sufficient bandwidth to allow servo sampling rater in excess of 500 Hz .


## 1. Introduction

Paly applications of robotics in space will be different from the typical applications found in industry. Unlike conventional industrial mimpulators which work in a precisely known and controlled environment, a space robot is envisioned as a backup astronaut, performing construction, maintenance, and experimentation. Such a robot must be capable of a wide range of tasks, from small scale, high precision operations such as replacing electronic components in faulty equipment, to immense scale, such as assembling room sized structures into a space station. Many of these tasks will be poorly defined or completely unexpected, particularly maintenance operations.

It is inconceivable to develop a single manipulator whist meets even those requirements we can predict as necessary for a space manipulator. The level of dexterity and versatility; nonviried by current manipulator technology is sufficient tor only very constrained or well known operations. Nor is it practical:, given the expense of transporting material to space, to maintain a large number of different manipulators at potential task sites. Our solution is the development of a manipulator system, which can be readily adapted to meet the individual constraints of each particular application as it occurs.

Dusigning a manipulator for a single, specilic task is conceptually the same proceas as that employed now for conventional robot applicationa. However, at present the process of deaigning and manulacturing robol systom for a particular application bas generally too long to be practical for the highly variable and urgent tagks that arise during space missions. To achleve the deelred range of robot capabilities, streamlining-and automation of the system configuration process are required.

In order to explore tnis approach, we are currently deaigning a Reconfigurable Modular Manipulator System, or RMMS. The RMMS ts a collection of manipulator components or maduies (links, Joints, actuators, and end effectors), with a wide range of performance (length, strength, torque, speed, resolution, erc) utilizing common electrical and mechanical interfaces. This deaign allows a large number of different manipulators to be assembled, at the task site, from a smail invertory of componenta. In paralled with the development of reconflgurable hardware, a similar software effort is underway to automate the generation of servocontrollor and path planning algorithms, provide a simple means of programming the manipulator task, and actually syntheaize a workable manipulator conliguration for a given task. Such a manipulator can thus be custom tailored to perform a apecific tack, and then broken down and re-used in a different conliguration when a new lask arteee.

In a sense, the RMMS concept is an extension of conventional interchangeable manipulator end tools. To date, however, the major efforts in this area have beun in the development modular hardware, so that a robot manufacturer can produce various manipulator configurations Irom standard sub-aseemblies [1], or to allow remote maintenance of manipulators in hazardous (ce. radioactive) environments [2]. In contrast, the aim of our RMMS effort is to develop a manipulator system which is modular and reconligurable by the user al the task site.

## 2. Design Philosophy and Implementation

An RMMS consists of the same major subsystems as those found in conventional manipulators:

- A pnysical structure made up of joints and links.
- Servo systems lor each joint, consisting of actuators, transmissions, and sensors.


## - A computer controller and programming anvironment.

The major differonce belween an RMMS and a conventional manipulator are the standardized component interlaces. This includes the mechanical mating of maniputator modules, the format of data communication, the communication protocols between hardware and software, and between various levels of software. Although adopting such standards impose inherent restrictions on the design of the actual components, this disadvantage is far ollset by the interchangeability of manipulator components and the capability for rapid reconfiguration. In the following subsections, we discuss the conceptual design of each major component and inierface in the RMMS we are devoloping, and the actual implementation in the prototype system.

### 2.1. Link and Joint Modutes

The mechanical modules making up an RMMS are divided into two groups, joints and links. Links are simply structural elements, and joints are servo mechanisms, made up of sensors and actuators. When we represent the kinematics of a manipulator by a series of transformation matrices representing its links and joints, links have fixed transformation matrices, while joints have variable ones (a function of the joint variable). Electrical power is bussed and communication is multiplexed over a small number of conductors permanently instalted in each module, allowing for simple assembly without custom cabling.

One implication of this modular joint design is that the entire joint actuator must be packaged within the joint module. Each joint module must include a motor (or some type of actuator), a transmission mechanism, a position sensor, and the necessary power electronics to control the motor. Although these design constraints limit the power which can be generated by the joint due to the limited size of the motor, transmission, and power amplifier, this is not viewed as a major short comming of the design, particularly for space applications. By properly selecting the transmission reduction ratio, high torques at low speeds can be obtained, which is appropriate for manipulating massive objects in near zero gravit, (and also on earth) as long as speed of operation is not critical.


Figure 2.1: Modular Joint Assemblies

For simplicity and convenience, we are considering only the two common types of revolute jotnt in our RMMS. Theee two typee ere rotate, and pivot, and are distinguished by the orientalion of the joints link axee with the joint axts. Both tyoes of foint are shown schematically in Figure 2.1. A rotate type joint has link exes which are co. linear with each other and with the foint axis. A pivot has Hnk axes which are both perpendicular to the joint axie.

Our current deaign for a pivot joint ts shown in the photograph in Figure 2.2, and in the aection drawing in Figure 2. The joint actuator ts a conventional servo motor and linear amplilier driving a harmonic drive with $100: 1$ reduction ratio. This deaign yialdse maximum output lorque of $60 \mathrm{~h} \cdot \mathrm{lbf}$, and maximum axis speed of 1 radian/second. Also integral with the joint aseembly is e brushiess resolver mounted coaxially with the output shaft, providing position leedback with an accuracy of $\pm 0.001$ redian. If lower position resolution is acceptable, lower resolution (and lass expensive) senaing dectronics can be installed in the joint


Figure 2-2: CMU RMMS Prototype Pivot Joint


Figure 2-3: Section View of an RMMS Joint
module. A wire windup allows the resolver (and output shaft) to turn up to $480^{\circ}$ before damaging the resolver electrical connections. All of the actuator components are packaged in a sub assembly of the joint inodule. allowing a number of different types of module to be based on common patts. The total weight of the joint is 25 lbs . A more compact and lighter version of this joint, as well as several kinematic variations on this tasic design are currently under deveiopment.

### 2.2. Joint - Link intertace

in order to aseombte the joint and link modules into a manipulator, a method of mechanically coupling the modulas is reouired. This coupling must both align the modules, and lock them together with sulficient strength to tranemit the internal forcee generated by the movement of the manipulator. In addition to structurally coupling the modules together, this interface must also electricatly couple the modules, and be able to eense the coupling ornentation of succeseive modules.

The current interface design ts thown in the photograph in Figure 2.4. An arrangement of pins and hoves firntt the coupling orientation to four, equally spaced positions. An LED in one kange and fouf photolranmiators in the other allow the centrotter to sense which of the four possibie orientations is in use. A commerrial V.band clemp couples the two fienges logether. As shown in the figure, the same coupling flange in an integral part of the link modules. Although rudimentary, this deagn provides the necessary functionality for the module interface. However it is not easily operated. Future versions will make use of either quick pelease V.band clamps. or a more sophraticated deevgn with an automated locking mechenizn to allow automatic "peg.in.hole" type coupling.


Figure 2.4: Prototyde Module Interface

### 2.3. Communication Interface

As mentioned. each joint will contain the power and sensor electronics for the actuator. In order to control the jomt actuators and obtain sensor leedback. a conimunication tink between the poi.tt modules and a computer controller is required. in order to allow standard connectoring between joint modules. this communication link must be implemented using a fixed number of conductors. yet be capable of supporting an arbitrary number of modules. This inplies a multiplexed communication link, similar to a computer bus or LAN.

Cue to the high overhead associated with existing LANs, our prototype ulilizes a bus type implementation. The design is shown schematically in Figure 2.5. The bus design is based on a conventional 8 bit bi directional data bus, an additional 5 control lines. and a rather unconventional 4 bit daisy chained address bus. The daisy chained address bus provides automatic node addrese configuration. that is. the first module in the manipulator is node address 1 , the second module is node address 2, and so on. Thia is accomplished by including a "subtract one" circuit in each module which is in the gath of the node address lines. Each joint can thus detect "address erquals zero" as the node address. Due to the low data rate of the bus (current bus clock is 500 KHz ), the propagation delay added by the subtract circuit is negligible.

### 2.4. Soflware Controller

In order to augment the capabilities of the RMMS hardware. a reconfigurable control and path planning algorithm are required. These algorithms would allow the control computer to automatically synthesize a control program for a given manipulator configuration and task requirement. This entails automatucally deriving the manipulator forward and inverse kinematics, and inverse dynamics given the kinematic and dynamic parameters of the modules. This is one of the major research area of our AMMS project.


Figure 2.5: Alanipulator Communication Bua Logic


Figure 2-6: Schematic of RMMS Computing Architecture

### 2.5. AMMS Computing Architecture

The reartime control soltwee in our RWMAS runs on a dedicated controfler CPU, with a hardware interlace to the inter-modute
 commands from a second, master CPU. The intent of this erchitecture is to have the manipulator controler appear as a peripherel In a standard microprocescor syitem. Communication between the master and controller CPU will be wa a number of memory mapped control and statuse registers mantained in shared memory on the controllor CPU. This allows the controliter CPU to be readity integretad into any mater CPU syatiom sharing the same syetom bue.
The present implementation of this architecture is shown in Figure 2.6. The controtier CPU is an lronice angle-boerd computer, based on a Motorola 68020 proceseor and VME bus, with I MByte of dual ported RAM. The master CPU is a SUN3 workstation. alac based on the Motorola $6 e 020$ and VME bus. The similurity between the two machines athows un to use the aame erifor and compiler for both processors, simplitying software development and inter-proceasor communication. The interface to the mentpulator communlcation network ie via the VMX bus interface included on the lronica. The VMX bus is a recognized extenaion to the VME bus, intended as a local 10 bus in multiproceseor systoms euch an this.

## 3. Span of Possible RMMS Configurations

In order to illustrate the range of capabilities of an RMMAS. we have eatimated several performance specilications for alt of the manipuintors possible given a reasonable size inventory of componente. This set of specifications is used to clessty the possitio manupulators into groups wht similar characteristics. By noting the number of different configurations within each class, we can gair an appreciation for the versatlity provided by an RMMAS.

### 3.1. Module Inventory

Based on our current design effort, let us consider an example RMMS with a module inventory consisting of:

- 25 joints, conaiating of 5 sets with maximum foint torques of: $10 \mathrm{Nm}, 30 \mathrm{Nm}, 60 \mathrm{Nm} .120 \mathrm{Nm}$ and 200 Nm . Each set consests of 3 puvot and 2 rotate joints.
- 12 inint position sensors. consisting of 3 each of the followng resolutions. 10 but, 12 bit. 14 bit and 16 bits.
$\bullet 20$ links, consuating of 4 each of the following lengths: $0.1 \mathrm{~m}, 0.2 \mathrm{~m}, 0.5 \mathrm{~m}, 1.0 \mathrm{~m}$ and 2.0 m .
3.2. Obtainatle Configurations and Range of Specifications

To limet the scope of this analysis. let us make the followng asamptions:

- The mampulators will be 3 degree-of freedom robots, made up of revolute foints, and will end in a conventional tirce axis (rotate.pavot-rotate) wrist. Thus the final jount of the tiree axie manipulator should not be a rotate foim, ws this would be redundant.
- Two types of revolute joint assemblies will be considered: puot and rota: ? with are shown in figure 2.1.
- Consacutive pivot joints must have either parallel or perpendicular axis orientatione fin serms of the Denavit. Hartenburg convention a $=0^{\circ}$ or $90^{\circ}$ ).
- The onentation of the manipulator with respect to the world will be ignored.

Within these assumptions, there we seven basic meeningful manipulator configurations, which are shown in Figure 3-1. Within each of the seven configurations, numerous combinations are possubie by using links of assorted lengins and assorted joint module deangns, resulting in a wide range of manipulator performance.

The manipulator specifications we have considered in comparing the cotainable manipulator configurations are reech. tip force. and tup position accuracy. The specilications were delined as loilows:

- Reach: length of the manipulator when extened. We use the sum of the link lengths as an estimate.
- Tip lorce: the minumum of the three joint torque capacities divided by each fonts longest moment arm. Measured in Newtons.
- Accuracy. the root mean square value of the uncertainty in tip position due to the error in measuring the joint positions. Mensured in meturs.


Figure 3-1: Meaningfut Manipulator Configurationa

We have calculated the mumber of obtaineble erm conliguratione with various combinatione of tip force, accuracy, end reech. The results ere given as hictograms in Table 3-1, which show how the obeainable conflgurations are diatributiod over the range of sepecilicationa.

We can incerpret the histograme in Table 3-1 by considering the claesification of typical robot tadk and performence requrements shown in Table 3-2. Ignoring speed requirements (which are generally impoeed by economic thru-out conmraints rather than the abilly to perform a task) we can twe that the obtanable renge of menipuletors apans cach of the four major ctamee of tank.

position error

| 0.05 m | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 16 | 0 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 0 | 0 | 0 | 0 | 0 | 8 | 20 | 12 | 0 |  |
|  | 0 | 0 | 0 | 0 | 0 | 0 | ? | 60 | 44 | 0 |  |
|  | 0 | 0 | 0 | 0 | 16 | 38 | 355 | 137 | 15 | 0 |  |
|  | 0 | 0 | 0 | 16 | 303 | 114 | 723 | 120 | 97 | 0 |  |
|  | 0 | 0 | 0 | 388 | 1751 | 521 | 360 | 217 | 1 | 0 |  |
|  | 0 | 0 | 138 | 2115 | 2604 | 294 | 934 | 88 | 46 | 0 |  |
|  | 0 | 4 | 2537 | 789 | 995 | 345 | 478 | 214 | 71 | 0 |  |
|  | 0 | 2616 | 4817 | 2844 | 4715 | 832 | 1528 | 294 | 98 | 0 |  |
| $0.005 \pi$ | 448 | 5892 | 6596 | 3256 | 3504 | 544 | 918 | 194 | 48 | 0 | reach |
| 0.5 - 5.0 |  |  |  |  |  |  |  |  |  |  |  |

Table 3-1: Mistogram of Obtainable Manipulator Contiguration Distritutions

## 4. Summary





 meh a eypem.
W. ase now buiding a mals pinks prototype. As of danuary 1837, two joint module and four inik modulas have been machined, and ere now being holegrated with a computer control sytiem. At the same time, theoratical work is under way to devilop agortinme for generating menipulator kinemetice, dynamics, and control. A malior effort ba being mede to reduce the stee and woight of the current folit deaign, which to a miviting factor in both manipulator apeed and payloed. Firat aperation of the

| $\begin{gathered} \text { IAse } \\ \text { spreificarion } \end{gathered}$ |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | alach (aters) | Parloan ( 4 y ) | accumacr (ma) | 30110 (0/5ec) |
| C1Ass 1 | - 3 - 18 | 0110 | 0080.3 | 0.10 |
| Class II | . $78 \cdot 1.5$ | 10.90 | 0120 | 0.2.0 |
| class ItI | 0.78-1. | 15.230 | 20.150 | $02 \cdot 1.0$ |
| class it | 1.10 | 10.90 | 20180 | 0.6.2.0 |

> Examples of typtcal class applications:
> Class 1 - precision assembly, princed circuit component insortion.
> Class il - small part handing. pick and place assembly. loading and unloading of machine cools
> Class III - large part handing. high contact force operations, surface grinding and deburring
> Class IV - sean tracking for welding or sealing. spray painting
> Table 3-2: Manipulator Task Classification
system is expected in the second quarter of 1987.

## Acknowledgments

The authors would particulerty like to thank Mark DoLous for his contributions to the RMMS design, and for machining and aseembling the prototype RMMS hardware in time to appeer in this presentation. We would also like to thank Regis Holfman, Laura Kelmar, Pradeep Khoela, and Jim Moody for contributing their experience and ideas to the RMMS design, and a great deal of eflort to its continual development and implementation.

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# Nonlinear Feedback Control of Multiple Robot Arms 

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## 1. Abstract

In-thie-pepes-we-moded multiple coordinated robot aras/by considering the arms (1) as closed kinematic chains and (2) as a force constrained mechanical system working on the same object simultaneously. In both formulations a new dynamic control method is discussed. It is based on a feedback linearization and simultaneous output decoupling technique. Applying a noil ear feedback and a nonlinear coordinate transformation, the complicated model of the multiple robot arms in either formulation is converted into a linear and output decoupled system. The linear system control theory and optimal control theory are used to design robust controllers in the task space. The first formulation has the advantage of automatically handling the coordination and load distribution among the robot arms. In the second formulation, by choosing a general output equation we could superimpose the position and velocity error feedback with the force-torque error feedback in the task space simultaneously.

## 2. Introduction

The notion of "multiple robot arms" originates from two everyday scenarios. The first scenario is an authropomorphic one by noting that humans have two arms and hands and everyday manual work is normally performed by twohanded humans. In fact, manual activities and tasks are normally perceived and designed such that they assume two-handed humans; a one-handed person is a handicapped person from that point of view. Thus, in order to replace humans with robots to perform normal manual activities it seems natural to visualize and design robots with two arms and hands. The second scenario is an industrial one by noting that production lines in industry assume an organized distribution of manipulative activities along the production line that can be carried out by a distributed set of robot arms in a proper arrangement.

Scenarios of multiple robot arms are also assumed and predicted for space applications in a natural way. Space station assembly, maintenance and servicing will require the in-site manual work of EVA astronauts in the initial operational configuration. This manual work also includes the simultaneous activities of two or more EVA astronauts in the handling or assembly of large structural, elements in space. Most satellite servicing and maintenance operations also assume two-handed manual work of EVA astronauts. Thus, the objective of decreasing eva activities in Earth orbit by introducing and increasing robot activities there requires the consideration and the design of the control of multiple robot arms.

The technically interesting and challenging problems in the control of multiple robot arms arise when (i) the work envelopes of two or more robot aras overlap and (ii) two or more robot arms simultaneously work on the same object in a presumably cooperative manner to perform a given task which cannot be performed by one am e only.

The control problem of two or multiple robot arms has been studied by many investigators [1-12]. Although the control problem of two or multiple arms is complex, some examples of applications, such as a two-arm lathe loader, a two -arm robot press loader/ unloader, and two single-arm robots working together to handle stamping prese loading and unloading, are given by Chimes [1]. In these applications, the problem is solved specifically. The system design is based on a solid understanding of the problem.

Hemami and Wyman [2] investigated the problem of force control in closed chain dynamic systems. In their work, the dynamic system is linearized about an operating point and linear feedback is used to maintain the forces of constraints. The validity of the method is restricted to a rather small neighborhood of the operating point in which the dynamic system can be linearized. orin and oh (3) considered the control of force distribution in robotic mechanisms containing closed kinematic chains. The problem of solving for the input joint torques ir om a given trajectory is underspecified. The linear programing has been used to obtain a solution which optimizes a weighted combination of energy consumption and load balancing. The dynamic equations of the mechanisms are excluded from the control method. The stability of the control algorithm is in no way ensured. Ishida [4] developed a force control technique which uses a wrist force sensor to measure the interactive force between two arms. The parallel transfer task and the rotational transfer task are.considered only. The control
algorithm is derived for both master/slave mode and indistingulehed mode (the same atatus mode). Fujil and kurono (5) proposed the method of virtual reference. This method consists of the identification of the joint control mode required to perform a desired Carteaian motion. The control loop at each joint uses only position feedback and no compensation for the coupling between joints.

Alford and Belyou [6] have deaigned a hierarchical computer control atructure for two puln robot arns operating in a master/slave mode. The proposed coordinated control system has joint position predictors, a coordinate transformation, and a slave comand modifier. An explicit control algorithm is derived and tested/implemented for an experimantal path: a etraight ine in the vertical direction. However, the question on how to define the prediction function, the transformation, and the modification function is left open in the paper, and the dynamics of the arme is excluded from the algorithm.

When two robot aras work on an object certain constrainta must be aatiafied in order to carry out a mooth, coordinated oparation. zheng and Luh [7] have derived a set of holonomic constraints on positions and orientations of the and effectors for two robots in three apecific working conditions, namely, handling a rigid-body object, handilng a pair of pliers, and handing an object having a spherical joint. The result is extended to the constraints between foint velocities and accelerations of the two robots for the three above mentioned cases [8].

Considering tasks of transferring an object by holding it with two robot arma, Lim and Chyung [9] introduced a poaition control method using kinamatic relations between the object and the two robot arme. By first specifying the trajectory of the object, the differential changes of each robot hand are computed from the differantial changes of the planned path. The commands or difforential changes of each joint of the two robot arak are generated by applying the inverse Jacobian matrix. The mathod is simple but applicabie only when the involved motion is vary slow. graund and Hoyer [10-12] proposed a hierarchical control method for collision avoidance in multi-robot eysteme. The method adopts a hierarchical coordinator and is systematic. However, an algorithm is needed to design the couplings among robots. Vukobratovic and Potkonjak [13] described a method which can be used to obtain the closed chain dynamics of two coordinated robot aras. Howover, the reaction force and reaction moment between the two arme are retained in the final equations. Hayati [20] extended the idea of hybrid position/force control to the multi-arz case. Based on equations of motion for a multi-arm syster, which are derived in a constrained coordinate irame located at the grasped object, a controller is designed to cooperate $n$ robot arms such that the load is shared among the arme in a non-conflicting way. A minimization of the magnitude of forces and torques is performed to decide how much each robot arm should contribute. It appears that the existing coordinated control methods fall in lack of either systematic synthesis of the control system or full censideration of robot arm dynamics.

In this paper we concentrate on the application of nonlinear feedback to the control of multiple robot arms. Previously we derived a general algorithm for the control of a single rigid robot arm through nonlinear feedback and state transformation reaulting exact aystem iinearization and simultaneous output decoupling [15,16]. our control deaign technique elevates the robot arm eervo problem from the joint space to the task space with three important consequences. (i) on the joint level our scheme computes and commands drive forces or torques on their actuator-equivalent quantities (current, voltage, pressure). (ii) The robot arm system in the task space is considered as a linear system, and the powerful tools of ifnear control theory, including optimal control, are applicable to robot arm controller design in the task apace. (iii) Our controller can directly respond to task space commands provided that these commands are formulated in form of elosed time functions. The question discussed in this paper is: how can our control method be applied to the control of multiple robot arms.

We are discussing $=w o$ modeling approaches. In tho first approach, we model the multiple arm system as a single system, that is, as a closed loop kinematic chain. In the second approach we retain the single arm models, but we introduce task constraints and force-moment measurements in the control scheme. The paper concludes with a brief discussion of computational architectures that are needed to implement our control technique for the control of multiple robot arms.

## 3. Closed Chain Formulation

As the first approach to coordinated control of multiple robot arms, we consider the multiple robot arms as a single mechanical system consisting of kinematic clesed chains. For tasks of lifting a heavy workpiece using robot arms, two or more robots are required if the workpiece is out of loading limit of any available robot arm. Suppose that m robot arms are used in such a task and that they all grasp on the same object (workpiece) in order to lift it, turn it, etc. Our primary concern is to obtain a dynamic model of these robots for the control purpose. Since they grasp on the same object, the dynamic behavior of one robot is not independent of the dynamic behavior of the other robots any more. A unity of mechnical system is rather formed by the robot arms involved and by the grasped object.

We will derive the Lagrange's equations of motion for this mechnical system. Those equations will eerve as a model of the systen to design control algorithme. For the mobots
of considaration, we name then robot 1 , robot $2, \ldots$, and robot $m$, raspectively, Wo ascure that robot 1 has $n_{i}$ ilnks. We also assume that each robot innuly graspe the object so that there is no movement between ite end effeotor and the object. closed ahains are formed in wuch a consiguration by the mobot arna, the object, and the ground. Motice that the object and the last ilnks of the robot arnis become a single iink. From the Kutzbach-Grubler criterion (17), the degrees of errecion of apatial linkage etructure connected by jointa vith each joint poscesaing one degree of freedom are given as follows

$$
\begin{equation*}
p=6(1-1)-51 \tag{1}
\end{equation*}
$$

whare if the number of linke and $f$ is the number of joints. This formula rezlects the fact that ach moving link has six degreas of freedom and the fixed link (the ground) has none, and that cach joint of one degrae of freedom causes a lome of five degrees of freedon for a link. for our case of mobots, the degrees of freedom of this entire mechanical syeten is then

$$
\begin{equation*}
p=6\left\{\sum_{k=1}^{m}\left(n_{k}-1\right)+1\right\}-5 \sum_{k=1}^{m} n_{k}=\sum_{k=1}^{m} n_{k}-6 n+6 \tag{2}
\end{equation*}
$$

where $n_{K}$ is the number of ilnks of robot $k$. If three robot arme are involved to perform a task, Table 1 shows 10 different combinations of three robot aria with five, aix or eeven degrees of freedom.

Before proceeding, let us define some notations that will be uned in the rest of this section.

$$
\begin{array}{ll}
\theta^{1}=\left[\theta_{1}^{1} \theta_{2}^{1} \ldots \theta_{n_{1}}^{1}\right]^{\prime} & : \text { joint variables of robot } 1 \\
\theta=\left[\left(\theta^{1}\right)^{\prime}\left(\theta^{2}\right)^{\prime} \ldots\left(\theta^{m}\right)^{\prime}\right]^{\prime} & \text { : foint variables of the mechanical system } \\
q=\left[q_{1} q_{2} \ldots q_{p}\right]^{\prime} & \text { : generalized coordinates } \\
\tau=\left[\tau_{1} \tau_{2} \cdots \tau_{p}\right]^{\prime} & \text { : generalized forces corresponding to } q \\
F^{1}=\left[F_{1}^{1} F_{2}^{1} \ldots F_{n_{1}}^{1}\right]^{\prime} & \text { : joint force/torque of robot } i \\
F=\left[\left(F^{1}\right)^{\prime}\left(F^{2}\right)^{\prime} \ldots\left(F^{m}\right)^{\prime}\right]^{\prime} & \text { : joint force/torque of the nechanical aystem } \\
n=n_{1}+n_{2}+\ldots+n_{m} . &
\end{array}
$$

The generalized coordinates $q$ can be chosen arbitrarily as long as they are linearly independent of each other. They are functionally related to the joint variables $\theta$. We denote the relation by

$$
\begin{equation*}
q=Q(t) \cdot \tag{3}
\end{equation*}
$$

Knowing the generalized coordinates $q$, the configuration of the mechanical syatem, thus the joint variable 9 , is uniquely determined. We denote such inverse relation by

$$
\begin{equation*}
\theta=\theta(q) . \tag{4}
\end{equation*}
$$

With the above notations, the Lagrange's equations of motion for the mechanical system are described by
where $L$ is the Lagrangian of the whole mechanical system. Equation (5) is a genaralization of the equations of motion of two robot arns presented in [14].

Wo masign a coordinate tram to each link of every robot arn. We locate a vorld coordinate trame in the comen vork apace of the a robots. In the procese of expresaing the kinotic and potential onergies of the mectanical oyetem, wivide the mase of the object into - parte. Each rebot is remponaible for one part of the objeot mase by adding it to the naise of the last ilnk. After carrying out the derivations of the Lagrangian function, ve obtain, the dynamic equations of the mechanical bywten

$$
\begin{equation*}
D(q) \ddot{q}+E(q, q)+\sigma(q)=J_{\dot{\theta}} T \tag{6}
\end{equation*}
$$

whare

$$
\begin{aligned}
& D(q)=J_{\theta}^{0} \dot{D}(\theta(q)) J_{\theta} \\
& J_{0}=\frac{\partial \theta}{\partial q} \\
& \dot{D}(\theta)=\left[\begin{array}{lllll}
D^{2} & & & \\
& D^{2} & & 0 & \\
& & & & \\
& & & & \\
& & & & D^{m}
\end{array}\right]
\end{aligned}
$$

$D^{r}=\left(D_{i j}^{r}\right)$ is the inortia matrix of robot $r$


$\mathbf{E}_{i}=\left[\begin{array}{lllll}\mathbf{E}_{1}^{1} & & & & \\ & \mathbf{E}_{1}{ }^{2} & & & \\ & & \cdot & \\ & & & & \\ & & & & \\ & & & E_{1}\end{array}\right] \quad, \quad,=1, \ldots, n$
$E_{i}^{r}=\left(E_{i j k}^{r}\right)$ is the coofficient of centripatal ( $y=k$ ) or Coriolis ( $\left.j \neq k\right)$ force of robot $r$


$$
\sigma(q)=-J^{1}\left[\begin{array}{c}
a^{2} \\
c^{2} \\
\cdot \\
\cdot \\
\cdot \\
c^{m}
\end{array}\right] \quad c^{x}=\left[\begin{array}{c}
c_{1}^{x} \\
\cdot \\
\cdot \\
\cdot \\
c_{n_{r}}^{x}
\end{array}\right] \text { La the gravity force of rolot } x
$$

$$
G_{i}^{T}=\sum_{k=1}^{n_{r}} \quad m_{k}^{T} \quad \frac{\partial T_{k}^{T}}{\partial \partial_{i}^{T}} \bar{r}_{k}^{\Gamma}
$$

In the above definitions, $T_{1}^{T}=\lambda_{01}^{T} \lambda_{12}^{T} \ldots \lambda_{(i-1) 1}^{T}$ where $\lambda_{i f}^{T}$ is the Denavit-Hartenberg homogeneous trar.eformation matrix trom coordinate frame 1 to coordinate frame of robot ri mis
1s the mase of $11 n k 1$ of robot $r \bar{F}_{1}^{r}$ is the mase canter of 1 ink 1 of robot r: $I_{1}$ is the peadoinartia matrix of link 1 of robot rig is the acceleration of gravity, defined to be a $4 \times 1$ coluan vector with the last component being equal to zero.

Equation (6) characterizen the dynamic behavior of the whole machanical gystem. However, this equation is nonlinear, coupled, and complicated. It poses great difficulty in controller designs. We propose to iinearize and output decouple the seter (6) using a nonlinear feedback and nonlinear coordinate transformation. Let us introduce a state space varlable $x$ by setting

$$
\begin{aligned}
& x_{1}=q_{1}, \quad x_{1+p}=q_{1} \quad, \quad 1=2,2, \ldots, p \\
& x^{2}=\left[x_{1} x_{2} \ldots x_{p}\right]^{\prime} \quad, \quad x^{2}=\left[x_{p+1} \cdots x_{2 p}\right]^{\prime} \\
& x=\left[\begin{array}{c}
x^{1} \\
x^{2}
\end{array}\right]
\end{aligned}
$$

The dynamic equation (6) can be written as

$$
\begin{align*}
x & =\left[\begin{array}{c}
x^{2} \\
-D^{-1}\left(x^{1}\right)\left[E\left(x^{1}, x^{2}\right)+G\left(x^{1}\right)\right]
\end{array}\right]+\left[\begin{array}{cc}
0 \\
D^{-1}\left(x^{2}\right) & J_{0}^{\prime}
\end{array}\right] F \\
& =I(x) \rightarrow g(x) F \tag{7}
\end{align*}
$$

We take the position (orientation) of the object handied as the syster output

$$
\begin{equation*}
y=h\left(x^{1}\right)=\left[h_{1}\left(x^{2}\right) h_{2}\left(x^{1}\right) \ldots h_{p}\left(x^{2}\right)\right]^{\prime} \tag{8}
\end{equation*}
$$

For the nonlinear feedback, the so-called decoupling matrix is [15,16]

$$
A(x)=J_{n}\left(x^{2}\right) D^{-1}\left(x^{1}\right) J
$$

where $J_{h}$ is the Jacobian matrix of $h$. The nonlinear feedback has the form

$$
F=x(x)+\beta(x) u
$$

where $x(x)$ and $f(x)$ are determined fron the following two algebraic equations [15, 16]

$$
\begin{align*}
& A(x) \quad x(x)=-L_{f}^{2} h  \tag{9}\\
& A(x) \quad B(x)=Y \tag{10}
\end{align*}
$$

In the above equations, $L_{f}^{2} h$ is the eccond order Lie derivative of $h$ along $R$,

$$
r=\left[\begin{array}{ccccc}
r_{1} & & & \\
& & r_{2} & & \cdot \\
& & \cdot & & \\
& & & & \\
& & & & r_{p}
\end{array}\right]
$$

$y_{1}=(11 \ldots 2)$ is a $2 x x_{1}$ now vector with all ontrien equal to 1 and $m_{1}, 1=1, \ldots p$ are chosen such that $m_{1}>0$ and $m_{1}+m_{2}+\ldots+m_{p}=n$. The Index $m_{1}$ is ascociated with the fact: that a total number of $n$ Independent actuators (inputs) are to be divided into $p$ groupe to control $p$ outpute. The required nonilnear coordiante tranafornation is given by (15,16)

$$
\phi(x)=\left[h_{1} L_{\varepsilon} h_{1} \cdots h_{p} L_{\varepsilon} h_{p}\right]^{\prime}
$$

since both equations (9) and (10) are underdeternined, there are infinite many colutione for tham. Any solution sarves the purpose of innearisation and decoupiing provided that $B(x)$ is levertible. A solution to equation (9) is given by (18)

$$
\begin{equation*}
\alpha(x)=-\lambda^{+}(x) \varepsilon_{f}^{2} h(x) \tag{11}
\end{equation*}
$$

Where $\lambda^{+}-\lambda^{\prime}\left(\lambda \lambda^{\prime}\right)^{-1}$ in the generalized inverse of $A(x)$. The general solution to equation (10) is [18]

$$
\begin{equation*}
B(x)=\lambda^{+}(x) \gamma+\left(I-\lambda^{+} \lambda\right) H \tag{12}
\end{equation*}
$$

Where $H$ is an arbitrary matrix which is to be chosen to make $\beta(x)$ invertible.
after applying the nonlinear feedback and the nonlinear coordinate transformation, the original eystem (7) with output (8) is converted into the following linear and decoupled mystem

$$
\begin{align*}
& z=A z+B u  \tag{138}\\
& y=C z \tag{13b}
\end{align*}
$$

where

$$
\begin{aligned}
& \boldsymbol{A}_{1}=\left[\begin{array}{ll}
0 & 1 \\
0 & 0
\end{array}\right], \quad B_{1}=\left[\begin{array}{l}
0 \\
r_{1}
\end{array}\right], \quad c_{1}=\left[\begin{array}{ll}
1 & 0
\end{array}\right], 1=1, \ldots, p .
\end{aligned}
$$

Note that the obtained linear system (13) conaists of $p$ independent subaystems. The control problen of the whole mochanical system is then eimplified to a deaign problen of individual subsystems. The ith subsystea is defined by

$$
\left.\left.\begin{array}{l}
{\left[\begin{array}{l}
z_{21-1} \\
z_{21}
\end{array}\right]=\left[\begin{array}{ll}
0 & 1 \\
0 & 0
\end{array}\right]\left[\begin{array}{l}
z_{21-1} \\
z_{2 i}
\end{array}\right]+\left[\begin{array}{c}
0 \\
1
\end{array}\right] u^{1}} \\
y_{1}=[1
\end{array} 0\right] \quad\left[\begin{array}{l}
z_{21-1}  \tag{14b}\\
z_{21}
\end{array}\right], 1=1, \ldots, p\right) .
$$

where $u^{1}$ is the 1th group input with $m_{1}$ componante. To stablile the abbysten (14), wo introduce a constant reeaback $u^{1}=-x^{1} g^{1}+v^{1}$ with

$$
x^{1}=\left[\begin{array}{cc}
0 & 0 \\
k_{11} & k_{12}
\end{array}\right]
$$

whare $\varepsilon^{1}=\left[s_{21-1} z_{21}\right]^{\prime}$, and $v^{i}$ is the new raference input. With such a constant feadback, subeystem (14) becomes

$$
\begin{align*}
& {\left[\begin{array}{l}
z_{21-1} \\
z_{21}
\end{array}\right]=\left[\begin{array}{cc}
0 & 1 \\
-k_{11} & -k_{12}
\end{array}\right]\left[\begin{array}{l}
z_{21-1} \\
z_{21}
\end{array}\right]+\left[\begin{array}{l}
0 \\
y_{1}
\end{array}\right] v^{1}}  \tag{15a}\\
& y_{1}=[1 \quad 0]\left[\begin{array}{l}
z_{21-1} \\
z_{21}
\end{array}\right], 1-1, \ldots, p_{1} \tag{25b}
\end{align*}
$$

or in compact tora

$$
\begin{aligned}
& z^{1}=\bar{\lambda}_{i} z^{i}+B_{1} v^{1} \\
& y_{1}=c_{1} z^{1}
\end{aligned}
$$

where $\bar{\lambda}_{1}$ can be casily identified from equation (15a). For the above aystem (15), the damping ratio $\xi$ and the natural frequency $\omega_{n}$ are related with the feadback gaine by

$$
\omega_{n}^{2}=k_{11} \quad 2 E \omega_{n}=k_{12}
$$

We now consider equation (15) as the new mathematical model of the real syatem which is exactly innearized, output decoupled and etabilized. The desired (nominal) input to each subsystem can be derived from the following system

$$
\begin{align*}
& {\left[\begin{array}{l}
z_{21-1}^{d} \\
z_{21}^{d}
\end{array}\right]=\left[\begin{array}{cc}
0 & 1 \\
-k_{11} & -k_{12}
\end{array}\right]\left[\begin{array}{l}
z_{21-1}^{d} \\
z_{21}^{d}
\end{array}\right]\left[\begin{array}{l}
0 \\
\gamma_{1}
\end{array}\right]\left(v^{1}\right)^{d}}  \tag{16a}\\
& y_{1}^{d}=\left[\begin{array}{ll}
1 & 0
\end{array}\right]\left[\begin{array}{l}
z_{21-1}^{d} \\
z_{21}^{d}
\end{array}\right], 1=1, \ldots, p \tag{26b}
\end{align*}
$$

where the superscript "d" indicates "deaired" quantities. From equation (16), the desired input can be obtained in terms of the desired task apace trajectory.

$$
\begin{equation*}
r_{i}\left(v^{i}\right)^{d}=y_{1}^{d}+k_{12} \dot{y}_{1}^{d}+k_{11} y_{1}^{d}, i=1, \ldots, p . \tag{17}
\end{equation*}
$$

It is observed that the left hand side of equation (17) is the sum of $m_{i}$ inputs in task space computed rrom the planned trajectory. For a given planned trajectory, at any instant tine the right hand aide of equation (17) ie a given value. Applying the generalized inverne, wa obtain [18]

$$
\begin{equation*}
\left(v^{1}\right)^{d}=r_{i}\left(\quad y r_{i}\right)^{-1}\left(y_{1}^{d}+k_{i 2} \dot{y}_{1}^{d}+k_{i 1} y_{i}^{d}\right) . \tag{18}
\end{equation*}
$$

Note that in our control deaign methodology the actual control vector is the task space command as formulated by equation (17). On the joint level, our methodology computes drive forces or torques for the individual actuators, and the servo design is on the task level.

Let the output error be defined as follows:

$$
\bullet_{1}-\left[\begin{array}{l}
e_{11} \\
e_{12}
\end{array}\right]-\left[\begin{array}{l}
y_{1}-y_{1}^{d} \\
\mathbf{y}_{1}-y_{1}^{d}
\end{array}\right]
$$

where $y_{1}$ and $\varphi_{1}$ are the raal (meatured) values, and $y_{1}^{d}$ and $\xi_{1}^{d}$ are the dealred values. 20 ellminate the output error $e_{i}$, we utilize an optimal error correcting control loop by minimising the following cost functional

$$
J=\int_{0}^{T}\left[\left(\Delta v^{1}\right) \cdot R \Delta w^{1}+e_{1}(t) \cdot Q e_{1}(t)\right] d t+e_{1}(T) \cdot s e_{1}(T)
$$

The optimal corraction is givan by

$$
\begin{equation*}
\Delta v^{1}=-R^{-1} \Delta_{f} P(t) e_{i}(t) \tag{19}
\end{equation*}
$$

where $P(t)$ is a poiltive definite solution of the Riccati equation
$\dot{P}(t)=-P(i) X_{1}-X_{1} P(t)+P(t) B_{1} R^{-1} B_{1} P(t)-Q$
$r(t)=8$,
with

$$
\bar{\lambda}_{1}-\left[\begin{array}{cc}
0 & 1 \\
-k_{11} & -k_{12}
\end{array}\right] .
$$

The overall structure of the controlier dealgn is depicted in Figure 1.

## 4. Corce Control Appraach

In this approach, we conaider the dynamice of each robot aeparately, but we pose constrainte on the dynamic equations by introducing the interactive force and interactive moment among the robot arms.

We have prople ad torce control approach to the coordination of two robot arna performing a single task (19). The coordination betwean two robot arms is achieved by monitoring the interactive force and moment at the end effectorm. Now we extend this mathod to multi-arm case.
suppose that mobot arma ( $m \geq 2$ ) are working on an object, e.g., lifting or turning a heavy workpiace. The problea we are dealing with is to sind a control algorithr for mobots such that the task is performed in a coordinated faehion. We aseume that each rabot han a force (torque) sensor installed at its end affector. Using force control approach, the coordination among m robot arms is realized by regulating the force and moment applied to $t$ object by each robot. With the ald of proper tamk planning, mobot arme are able to move. a non-conflicting way.

The dynamic equations of a syoter of mobot anse are given as follows

$$
D_{i}\left(q^{1}\right) \ddot{q}^{1}+E_{i}\left(q^{1}, q^{i}\right)+J_{i}\left(q^{1}\right) r^{1}=\tau^{i}, 1=1,2, \ldots,=
$$

where $q^{1}$ ia an $n_{1}$-dinensional joint variable vector of robot $1, n_{1}$ in the degreas of ereedon of robot $1, f^{i}$ is an $n_{1}$-dimensional vector of the force and moment measurements of robot 1 , $T^{1}$ is an $n_{1}$-dimensional joint torque (force) vector of robot 1 , and $J_{1}$ is the Jacobian matrix of robot 1 .

Now we introduce a state variable $x$ by letting

$$
x^{1}=q^{1}, \quad x^{m+1}=q^{1}, \quad 1=1, \ldots, m,
$$

1.e.,

$$
x^{1}=\left[\begin{array}{llll}
x_{1} & x_{2} & \ldots & x_{n_{1}}
\end{array}\right]^{\prime}=\left[\begin{array}{llll}
q_{1}^{1} & q_{2}^{1} & \ldots & q_{n_{1}}^{1}
\end{array}\right]^{\prime}=q^{1}
$$

$$
x^{2 n}=\left\{x_{n+n_{2}}+\ldots+n_{m-1}+1 \cdots x_{2 n}\right\}^{\prime}=\left\{q_{2}^{m} \ldots q_{n_{n}^{m}}^{m}\right\}^{\prime}=q^{m}
$$

where $n=n_{1}+n_{2}+\ldots+n_{n}$ then $x$ is a $2 n-d i m e n s i o n a l$ vector partitioned into $2 m$ blocke

$$
x=\left[\begin{array}{llllll}
x_{1} & x_{2} & \cdots & x_{n} & x_{n+1} & \cdots \\
x_{2 n}
\end{array}\right]^{\prime}=\left[\begin{array}{c}
x^{2} \\
\vdots \\
\vdots \\
x^{m} \\
x^{m+1} \\
\vdots \\
\vdots \\
x^{2 n}
\end{array}\right],
$$

$$
\dot{x}=\left[x_{1} \ldots x_{n}\right]
$$

With the first m blocks (corresponding to the first $n$ componenta $\bar{x}$ ) representing the foint positions of robote and with the last blocke representing the joint valocities of a robots.

The dynamic equations of mabne can now be written in terms of atate variable $x$ as followe:

or $\quad \&=f(x)+g(x) \tau$
where $f$ ang $g$ can be easily identified from the above equation. We take the output equations of the form

$$
\begin{aligned}
& x^{2}=\left\{x_{n_{2}+2} \cdots x_{n_{1}+n_{2}}\right\}^{\prime} \cdots\left\{q_{2}^{2} \ldots q_{n_{2}}^{2}\right\}^{\prime}=q^{2}, \\
& x^{\#}=\left\{x_{n_{2}+\ldots+n_{n-1}+1} \cdots x_{n}\right\}^{\prime}=\left\{q_{1}^{M} \cdots q_{n_{n}}^{m}\right\}^{M}=q_{1} \\
& x^{m+1}=\left[x_{n+1} \cdots x_{n+n_{1}}\right\}^{\prime}=\left[\phi_{2}^{1} \ldots q_{n_{1}}^{1}\right)^{\prime}=q^{1} \\
& x^{m+2}=\left\{x_{n+n_{2}+1} \cdots x_{n+n_{2}+n_{2}}\right\}^{\prime}=\left\{\phi_{2}^{2} \cdots \dot{4}_{n_{2}}^{2}\right\}^{\prime}-4^{2}
\end{aligned}
$$

$$
y=h(x)=\left[\begin{array}{c}
h^{1} \\
h^{2} \\
\vdots \\
\vdots \\
h^{m}
\end{array}\right]=\left[\begin{array}{c}
w_{p}^{1} p^{1}+w_{F}^{2} F^{1} \\
w_{p}^{2} p^{2}+w_{p}^{2} p^{2} \\
\vdots \\
w_{p}^{m} p^{m}+w_{p}^{m} p^{m}
\end{array}\right]
$$

mere $w_{p}^{1} m_{p}^{1}, 1-1, \ldots, m$, are the velghting matrices, and $p^{i}$ is the position and orientation vector of robot 1 in the vorld coordinate frame. The dimenaion of output vector $y$ is $n$.

Equation (20) ropresente nonlinear and coupled aystem with output (21). Using a nonlinear feedback $\tau=\alpha(x)+B(x) u$ and a nonlinear coordinate transformation $T(x)$, we are able to ilnearize and output decouple the eysten (20). The $a(x)$ and $g(x)$ in the nonlinear feedback are given by

$$
\begin{align*}
& a(x)=-\lambda^{-1}(x) I_{f}^{2} h  \tag{22}\\
& B(x)=A^{-1}(x) \tag{23}
\end{align*}
$$

where

$$
A(x)=\frac{\partial h}{\partial \dot{x}}\left[\begin{array}{ccc}
D_{1}^{-1}\left(x^{1}\right) & & \\
\cdot & & 0 \\
& \cdot & \\
& & \\
& & D_{m}^{-1}\left(x^{m}\right)
\end{array}\right]
$$

The nonlinear transformation is given by

$$
T(x)=\left[\begin{array}{c}
h_{1}  \tag{24}\\
L_{f} h_{1} \\
\vdots \\
\vdots \\
h_{n} \\
L_{f} h_{n}
\end{array}\right]
$$

Application of the nonlinear feedback and the nonlinear coordinate transforeation converts the system (20) with the output (21) into the following linear and decoupled syatem

$$
\begin{equation*}
z=\lambda z+B u \tag{25a}
\end{equation*}
$$

$$
\begin{equation*}
y=C z \tag{25b}
\end{equation*}
$$

where

$$
\begin{aligned}
& z=\left[z_{1} \ldots z_{2 n}\right]^{\prime}, u=\left[u_{1} \ldots u_{n}\right]^{\prime}, y=\left[y_{1} \ldots y_{n}\right]^{\prime},
\end{aligned}
$$

$$
\begin{aligned}
& \lambda_{1}=\left[\begin{array}{ll}
0 & 1 \\
0 & 0
\end{array}\right], \quad B_{1}=\left[\begin{array}{l}
0 \\
1
\end{array}\right], \quad c_{i}=\left[\begin{array}{ll}
1 & 0
\end{array}\right], \quad i=1,2, \ldots, n .
\end{aligned}
$$

Note that system (25) consists of $n$ indepandent subsystoms. Likevise as in the closed chain formulation, for each eubsystem we can design a constant feadback to stablilie it and deisign an optimal orror-correcting loop to ellminate the output error. The overall controller structure is shown in rigure 2.

## 5. Conclunion

Our approaches to the control problem of multiple robot arns are motivated by the desire of making rigorous use of the dynamics of robot arma involved in the task. The closed chain approach is initlated from the fact that the dynamic behaviore of the robot arma are not independent of each other any more if they grasp on a common object. In this approach, the multiple robot armare modeled as a single mechanical system by choosing a set of ganeralized coordinates whose number equals the number of degrees of freedon of the whole systen. Figure 1 shows the schematic etructure of the controller for the closed chain approach as implemented on computers. Fron the initial physical task, the task planning of the upper left block in Figure 1 produces a trajectory in the task space expressed an a mooth function of time. The comeand generator block realizes equation (18) and ylelds the desired refarence input. The lower left block is the implemantation of the optimal orror correction described by equation (19). It takes the task epace error as its input, and produces the optimal correction as its output. The $Q(9)$ block to the right of the multiple robot arns establishes the generalized coordinates as well as their time derivatives srom the measured joint positions and velocities of the robot arms. The bulk of the controller is the nonlinear feedback block which computes the joint driving torques or forces. Because the dynamic profection functions $D^{i}, E_{1}$, and $G^{1}$ are derived in terme of the folnt variables, it may be convenient to use the foint variablea in addition to the generalized coordinates for computing the nonlinear feedback.

Dlffarent from the closed chain approach, the force control approach asauman that aach robot arm has a force and moment sensor located at the ond effector. The iorce and moment measurements are introduced into the dynamic equations and output (task) equations. This is schomatically depicted in Figure 2. The measurements $F^{1}, F^{2}, \ldots, F^{m}$ are transmitted to the nonlinear feedback block, the output $h$ block, and the coordinate transformation $T$ block. The three blocks to the left of the nonlinear feedback block in Figure 2 are structuraliy similar to those in Figure 1.

Using the results from differential geometric system theory, we are able to inearize and to decouple the complicated dynamic equations of multiple robot arms including the object held by the arms. Independent of the approach being taken, we eventually deal with a linear and decoupled eystem. Thus we can have a unified design technique for coordinated control of multiple robot arms.

It should be noted that both methods used in this paper are aystematic and are robot arm independent. The most important feature is that the control algorithms are task independent, that is, there is no need to change the structure of the controller or even the parameters of the controller from task to task. As natural as would be, the change of tasks only causes the adjustment of the input command which is conveniently given in the task apace rather than is. the joint space. The two control methods can be used in slightly different eituations. For example, if the robot arms are loosely connected through the object, the force control approach is preferable; if the robot arms are mechanically locked while transferring the object, the closed-chain approach is more likely a solution.

Each control scheme naturally leads itself for computational implementation using distributed computing system, possibly in multi-bus architecture. Figures 1 and 2 provide a high level structure of computational implementation requirements. The details of the implementation require a deeper analysis.

## 6. ACTNQWLEDGEMENT

The research described in this paper was jointly performed by Washington University, St. Louia, Misaouri, and the Jet Propulsion Laboratory, California Institute of Technology, Pasadena, California, end was jointiy eponsored by the National Science Foundation and the National heronautics and Space Administration.

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Table 1．Degreen of freedo of the cloed chaine formed by three robot arme

| cases | $n_{1}$ | $n_{2}$ | $n_{3}$ | 1 | 1 | $n_{2}+n_{2}+n_{3}$ | $p$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 7 | 7 | 7 | 20 | 21 | 21 | 9 |
| 2 | 6 | 7 | 7 | 19 | 20 | 20 | 6 |
| 3 | 6 | 6 | 7 | 10 | 19 | 19 | 7 |
| 4 | 5 | 7 | 7 | 10 | 19 | 19 | 7 |
| 5 | 5 | 6 | 7 | 17 | 10 | 16 | 6 |
| 6 | 6 | 6 | 6 | 17 | 10 | 10 | 6 |
| 7 | 5 | 6 | 6 | 16 | 17 | 17 | 5 |
| 6 | 5 | 5 | 7 | 16 | 17 | 17 | 5 |
| 9 | 5 | 5 | 6 | 15 | 16 | 16 | 4 |
| 10 | 5 | 5 | 3 | 14 | 15 | 15 | 3 |



Fig. 1 Schematic Control Structure of the Closed Chain Approach


Fig. 2 Schematic Control Structure of the Force Control Approach

# Dynamics and Control of Coordinated Multiple Manipulators 

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## Memact

 vhiol are heldiag a ingle odrot and therefort form olosed kineaste ohela.
 csemed to be held rigitiy by a robot end-offectors. The cerivation le besel on

 consileret then one or more of the dogregs of freedor of the object is redseed
 oqutione, decompliag controller is foaigned to control both the poition and the iateraction forees of the object fith the eaviroanest. Finaliy, -qumererter


## 1. INTRODOCTION

The topie of maltipie robot control ts roletively mev la robotig researel. The exteasion of robot cortrol






 meterity. In this paper ve vill oaly addrese the cogrol problem associated with aitiple anatpaletori handiting




 force servoed arn is coatrolled in conflient mode to foliov the mator (position gervoed) arm. Additional feed






 torqeet sxt compred to dyfive the arme.

 may also be cometray fed from motion in one or more dimesion oy an extermi eavirometi. Equitluas of motion





 manipaletorstis presented to validate the emiyait.

## 2. bmunice of mititli coortaytio monot muifolatoes



 grasped ofject may the lit comtict vith efgil eaviromont.





Figuge 2.1.



$M(g) \underline{g}+\boldsymbol{g} \cdot \dot{g})+\boldsymbol{g}(\underline{g})=I$



 vectors. The above equetion applies oniy to idealized frifotionlese gigid agme.


 flimetrates one of these parte together fith the last llak of arin 1 .


Pigure 2.2. Sohomatic draviag of the partitioned load and its integration with the last liak of an 1.
 tion in the E frame is obtalmed from:

Since the aras are gomaderad to be gigidy attached to the load, there la comatat trasaformation relatiag the 1 and $E$ frames, Fr. It then follows thet







 cosivel abmply in gillisiag the mandpelator Jaeoblan as (S,10):

$$
\begin{equation*}
i=J(n) i \tag{2.5}
\end{equation*}
$$





$$
\begin{equation*}
x=\dot{J}(g) i+J(g) \tag{2.6}
\end{equation*}
$$

Smetituting (rom (2.6) into (2.1). we obtata

$$
M(g) J^{-1}(\underline{g}) \llbracket \underline{j}-\dot{j} \dot{j}+I(\underline{g}, \dot{g})+g(g)=I
$$

05

$$
\begin{equation*}
M_{2}(\Omega) \sum+Y_{n}(\Omega, \dot{g})+g_{n}(\Omega)=E \tag{2.7}
\end{equation*}
$$

Vhere

$$
\begin{align*}
& M_{g}(g)=J^{-T}(g) M(g) J^{-1}(g)  \tag{2.8}\\
& Y_{2}(a, q)=J^{-1}\left[Y(g, \dot{\underline{g}})-M(\underline{q}) J^{-1}(\underline{q}) \dot{j}(\underline{q})\right. \text { il }  \tag{2.9}\\
& g_{g}(g)=J^{-r}(g) G(g) \tag{2.10}
\end{align*}
$$

In equation (2.7). E rapresegts an equivalont gemeralized force vootor applied at the end-potat (D-frame). Let
 of notion are therefore


$$
\sum_{i=1}^{i} \ell_{1}=0
$$

To equetion of motion, therpfore, mey be obtaleod by simply addig the oquations (2.11)

In order to binpiliy the motation, let us dofine

$$
\begin{equation*}
n_{n}(g)=\sum_{i=2}^{m} n_{2 i}\left(g_{i}\right) \tag{2.13}
\end{equation*}
$$

$$
\begin{equation*}
Y_{2}(g, g)=\sum_{i=1}^{n} \underline{Y}_{21}\left(g_{1} \cdot \dot{q}_{1}\right) \tag{2.14}
\end{equation*}
$$

$$
\begin{equation*}
g_{2}(g)-\sum_{i=1}^{m} g_{21}\left(g_{1}\right) \tag{2.15}
\end{equation*}
$$

Where

$$
\mathbf{P}=\sum_{i=1}^{n} \mathbf{R}_{i}
$$

$$
g=\left(q_{1}^{T}, \varepsilon_{2}^{T}, \ldots . . g^{T}\right)
$$

Now, equation (2.12) can be writtor is the inpler forn

$$
\begin{equation*}
n_{2}(\underline{g}) \dot{Z}+Y_{I}(9, \dot{g})+g_{I}(g)-E \tag{2.16}
\end{equation*}
$$





 conetratar equtions

$$
z=E_{1}\left(g_{2}\right)=E_{2}\left(\varepsilon_{2}\right) \ldots=I_{1}(\Omega)
$$

Where $\mathrm{E}_{\mathrm{i}}$ is the forvard kimometce folation for ism 1.

## 






 © 10 gives from

$$
\underset{\mathrm{C}}{\mathrm{E}}=\left[\begin{array}{c|l}
\mathrm{E}_{\mathrm{B}} & \mathrm{E}_{\mathrm{g}_{\mathrm{aOn}}} \times \mathrm{E}_{\mathrm{C}}  \tag{3.1}\\
\hdashline 0 & \mathrm{E}_{\mathrm{B}}
\end{array}\right]
$$

Wher
 $X_{0}$ denote the position/oriestation of the C-frame. Then

$$
\begin{align*}
& \dot{i}_{c}-\mathrm{C}_{\mathrm{B}} \dot{\mathrm{i}} \tag{3.3}
\end{align*}
$$

and hesce equation (2.16) is

$$
\begin{equation*}
X_{\mathbf{x c}}(g) Z_{a}+Y_{\mathrm{Ic}}(g . \dot{g})+Q_{\mathrm{xc}}(g)=E_{c} \tag{3.5}
\end{equation*}
$$



$$
E_{c}=\left(\frac{\mathrm{E}}{\mathrm{C}}\right)^{T} \mathrm{E}
$$



$$
\begin{equation*}
t\left(x_{a}\right)=0 \vee t 20 \tag{3.6}
\end{equation*}
$$

If تe denote by fothe geacralized constralat force vector, then the equation of motion it

The constraint force $\mathcal{L}_{\mathrm{c}}$ can be obtalaed by soting thet the virtul work performed by fols equal to zero and by mains the Lagrage maltiplier method as discossed in [6] and [7].

The stetement of virtual work is

$$
\begin{equation*}
f_{q}^{T}{ }^{8} x_{c}=0 \tag{3.8}
\end{equation*}
$$

From the constraint ofation (3.6) wo uan witie

$$
\begin{equation*}
\frac{\phi\left(x_{0}\right)}{0 x_{0}} \Sigma_{0}=0 \tag{3.8}
\end{equation*}
$$



$$
\left[\Sigma_{\theta}^{T}-\lambda^{T} \frac{\partial\left(x_{\theta}\right)}{-\frac{\Sigma_{0}}{}}\right) \delta \Sigma_{\theta}=0
$$



$$
\begin{equation*}
L_{0}=D^{T}\left(z_{0}\right) \lambda \tag{1.10}
\end{equation*}
$$

Thore

$$
\begin{equation*}
D\left(x_{e}\right)=\theta\left(x_{0}\right) / \phi x_{0} \tag{3.11}
\end{equation*}
$$

To find $\lambda$. we difforentlete (3.6) चith respegt to time twice

$$
\begin{align*}
& \dot{i}\left(x_{d}\right)=D\left(x_{Q}\right) \dot{x}_{d}=0  \tag{3.12}\\
& \dot{D}\left(x_{d}\right)=\dot{\eta}\left(x_{d}\right) \dot{x}_{d}+D\left(x_{d}\right) \dot{x}_{d}=0 \tag{3.13}
\end{align*}
$$

and sebetitete for $\mathbf{X}_{0}$ 8con (3.7)
whel gives $\lambda$ se
 cooperating cobot with orteral notion cometraints on the object.

## 4. CONTEL OP MULTIPLE COOPREATINO 2OBOTS

 to coatrol the goeition of the object and iatersction foroes of the objeot fith the exterasi enviroameat. The

 by tatroduclag a decoupliag gemeralized force such thet

$$
\begin{align*}
& +Y_{20}(\underline{g}, \dot{g})+g_{50}(g) \tag{4.1}
\end{align*}
$$

Where fod is the desired fatersetion force vector between the object and the externel eaviroment.



 subsace and the last elemeats bolong to the force costrolled subspace. This meas that, for example,

$$
\begin{align*}
& X_{04}=\left[\frac{x_{0 e}}{0}\right]!{ }^{6-m} \tag{4.2}
\end{align*}
$$

and

$$
\mathrm{s}_{\mathrm{ad}}=\left[\begin{array}{c}
0  \tag{4.4}\\
\frac{\varepsilon_{0 d}^{0}}{}
\end{array}\right]^{!-n}
$$

Lot us partition the meat metris at

$$
u_{20}=\left[\begin{array}{ll}
n_{11} & m_{12}  \tag{4.3}\\
n_{21} & m_{22}
\end{array}\right]
$$



$$
x_{26}^{1}-\left[\begin{array}{ll}
u_{11}^{\prime} & u_{12}^{\prime}  \tag{4.6}\\
\mu_{21}^{\prime} & m_{22}^{\prime}
\end{array}\right]
$$

 3.11) ons be writton as

$$
D\left(x_{0}\right)=\left[\begin{array}{llll}
0 & 1 & x_{\text {nin }} \tag{4.1}
\end{array}\right]
$$

and

$$
\begin{equation*}
\dot{D}\left(\mathrm{I}_{0}\right)=0 \tag{4.8}
\end{equation*}
$$


 cooderation, and veloolty vectors as


$$
r_{f d}=\left[\begin{array}{ll}
0 & 0  \tag{4.10}\\
0 & k_{f p}
\end{array}\right], x_{f p}=\left[\begin{array}{ll}
0 & 0 \\
0 & k_{f d}
\end{array}\right] . x_{p p}=\left[\begin{array}{ll}
k_{p p} & 0 \\
0 & 0
\end{array}\right], r_{p d}=\left[\begin{array}{ll}
k_{p d} & 0 \\
0 & 0
\end{array}\right]
$$




The reaction foreo $f_{0}$ can be conpeted by anbstitetiat from equation (4.11) for Eo it equetion (3.10) and (3.14). After som manipalaifum vi int equations, one obtains

Sebetititias from (4.9) for $\mathcal{L}_{0}$, Te heve

$$
z_{02}=z_{f f}^{\prime}+k_{f t} \dot{i}_{f}+k_{f_{p}} \mathbf{e}_{f}
$$

$\mathbf{k}_{\mathrm{fc}} \mathbf{i}_{\mathrm{f}}+\left(\mathbf{k}_{\mathrm{f}}+\mathrm{I}\right)_{\mathbf{l}_{\mathrm{f}}}=0$
 canstan can be desiguod by seloctiag the proper kid and kf diagomal gela metrioes.


$$
n_{x c}(p)\left[\begin{array}{c}
z_{p}+k_{p d} i_{p}+k_{p p} i_{p}  \tag{4.13}\\
0
\end{array}\right]=0
$$

Tis means thet in the poition subspace, the maliple maipulator system oan be controlled by the proper chotce of the $k_{p d}$ and $k_{p p}$ atitices.

To sumerize, a searalized force vector as given by equation (4.1) Fas found such that both the position of the object ad its interaction forces fith an eaviroanent can be controlled. Note thet thit equation does sot apealify how the gearilized force vector vill be realized by aystem of redundant actuatorit of mitiple
 meessery acteation for its own liaks ples porifon of the object's. Similarly, one can deternine a priori how
 enviromeat (for more details, please see lif. (i). The lateral forces at the origin of the E-framen also be controlled in the position subpace by addiag force vectors to $\mathrm{F}_{\mathrm{i}}$ (see equation 2.17) anch that oae has

$$
F=\underbrace{\prime}_{1} \boldsymbol{F}_{\mathfrak{t}}+\mathbf{E}_{\mathbf{I}})
$$

or

$$
\sum_{i=1}^{m} \mathbf{E}_{\mathbf{i}}=0
$$

 ville realifing the desired interal forces.


Fig. 5.1 Schematic Dreviag of asir of Tvo-Link Cooperating Manipolators

## 5. EXNXPLE



 degrest of freedom. the object vis isanad to be tin contact fith the upper links only through fateraction forc rather than throagh forces and i.orquat.




 interaction force betwoen the object end the unironvent vas set equito zero in the second cese.
 on the dyanaics equations (such as those developed in this peper) or by asiming the enistance of stiff eprin (or spring-dashpot) at the contact points. Since in actril erperiments, one uses forceltorque sensors to obta

 enviconaent. Figure 3.3 shows the overall imulation block diagram.

(a)

(b)

Figure 5.2 Detailed modeliag of the conaections between (e) the aras and the object, and $(b)$ the object and the exterail constraint


Figure 5.3 Siaplified block diegran of che simalation study

Although this paper does not atempt to addres the digital control asectsof the probleng the imistion tin Fas mede more realistic by separating the coatinnons and discrete parts as shown in Fignre 5.3 .

Figures 5.4 through 5.6 show the resposes of the syoten for the followiag set of parameters ad control gains.

| link leagths | -. 04 [m] . equal for all 10 |
| :---: | :---: |
| link mases | $-4.0\left[\mathrm{~K}_{8}\right] \quad$ - for lower liaks $-2.0[\mathrm{~K}] \quad$. for upper links |
| object mess | -2.0 [ Ef ] |
| Iattial joiat angleas | $\begin{array}{lll}\theta_{11}=30^{\circ}, & \theta_{12}=120^{\circ} \\ \theta_{21}=150^{\circ}, & \theta_{22}=-120^{\circ}\end{array}$ |
| Position loop geins | $k_{P P}=4900.0$, $k_{p d}=98.0$ |
| Force loop gains | $\mathbf{t}_{\mathrm{fp}}=1.0 \quad$, $\mathbf{k}_{\mathrm{fd}}=0$ |
| Spring constants | 500,000 [ $\mathrm{N} / \mathrm{m}$ ] for all of the springs |
| Damping constants | $140.00[\mathrm{~N} /(\mathrm{m} / \mathrm{sec}) \mathrm{l}$ |
| Saepling period | 1-180 |

Figure 5.4 shows the tracking capability of the object aved by the aris. Since tho difference betwen the
 position of the upper links. Figure 5.5 shows the tracking of the object in the second case (ainor constrained

 be understood thit several inportant practical probleas such as friction, flemibility of the links and joints,
 necestary to include such effecti.

PLOT of Load Position vs. TIME


Pigure 5.4 Pure Poistion Costrol vith Cooperatize Ares

PLOT of Load Position vs. TIME


Figure 3.5 Position Trackise in Poaition/force Control with Cooperatiag Aras

LOAD-ENVIRONMENT INTERACTION FOROES vS. TIME


Figure 5. 6 Force Tracking in Position/Force Control vith Cooperating Aras
6. CONCLUSIONS

This paper presented theory for the position and force control of enjtiple matpalators holding an objet

 mybid position/force control concept to the case of maltiple aras. Simple but realistic aimalation stadies confiraed that the developed control concept resift in excelleat position tracking and force control.

## 7. ACguTHE!. TI




1. nerasucts
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A Survey of Adaptive Control Technology in Robotics

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#### Abstract

This paper review the previous work on the adaptive control of robotic system Although the field is relatively new and does not yet represent a mare discipline, considerable attention for the design of sophisticated robot controllers has occurred. In chis presentation, adaptive control methods are divided into model reference adaptive systems and self-tuning regulators with further definition of various approaches given in each class. The similarity and distinct features of the designed controllers are delineated and tabulated to enhance comparative review.


## 1. INTRODUCFION

Control of robotic manipulators is challenging problem mainly due to ene nonlinear and coupled nature of the system dynamics. A considerable amount of valuable work has bern produced in the dynamic formulation and the control of these systems within the last two decades. Since the pioneering works of licker (if. Hooker and Mazgulies $\{2]$, and kahn and Roth $[3]$ on the formulation of dynamics, zesearchass have concentrated on the eficicient computer implementation and numerical construction of the dynamic equations. while the wort on the efficient dynamic equation algorithm e is still going on, control of manipulators has also received significant attention. Over che years, literature on the manipulator control methods using optimization, linearization, nonlinearity compensation, and recently, adaptive techniques has become quite rich.

This paper reviews the accumulated work in the area of adaptive control as applied to robotics. The reader should note that adaptive control in itself is not yet a mature discipline in systems heory. Also, since some of the existing cools in adaptive control are strictly for linear and/or time-invariant systems. their application to robotics deserves special attention. The immaturity of adaptive control is best demonstrated by the back of a definit. in of adaptive control agreed to by the leading researchers ill.

According t. Webster's dictionary, to adapt means "o adjust (onesel fl to new circumstances". Adaptive control, then, in essence, is used co mean a sophisticated, Flexible control system relative to the conventional fixed feedback system. An adaptive system will assure quality system performance when large and unpredictable variations in the plant dynamics or loading occur. Although our aim is by no means to establish fine missing definition, since the robotics community seems to have reached a consensus on what is meant by adaptive control, we will give our definition to illustrate our interpretation of adaptive control.

Definition: A feedback control system is adaptive if the gains are selected with the on-line information of plant outputs and/or plant state variables.

This definition is depicted in block diagram format in Figure 1 . The above definition encompasses all the previous work on the adaptive control of manipulators currently available to us.

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``` ertenaive application co robotice we qiven by buboutty and penforgen in is7y [3]. Eince
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``` conerol mehode applied to robotice my be categorised under the delifn of
1. Model Referene Adeptive systene (Mins)
11. Solf-tuning Jegulatore (ETh)
The Lollowing methods are used in ene design of Masi
i. Local paranetic optimisation
11. Lyapunov's second method
111. Hyperstability theory
1v. Silding control theory
The STR design procedure may be divided into ehree erepe:
1. Selection of paramatric atructura to represent the robotic gyeten via discete-time modeling
11. On-iine estimation of systen paraneters uning the least squares, extended leat squates or maximum 1ikelihood mehode
11i. On-line controller design based on the estinated systen parameters vin extended minimu variance or pole-zero placomant technigues
BLock diagrame of mRAS and stitare lilustrated in Figures 2 and 3 . Note that the dotted boxes in thege ilgures may be reduced eo the regulator block in Figure l. After a brief review of syitem dynamice, the related background work is presented below.
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Flquze 2. Slock oieqzan of model Reforence Mdaptive syetem


Fiqure 3. Dock jieqran of Self-tunine Requiatar

## 2. SYSTRM DYMAIICS

Dynanle equatione of an $n=11 n k$, n-degree-of-ireedon, spatial, serial robot arm with rigid liaks are given by

$$
\begin{equation*}
A(0) i=F(0, \delta) i-G(0) \theta-u \tag{1}
\end{equation*}
$$

 qeneralised inertia metrix, $-f=-F(0, i) i \mathrm{f}^{\boldsymbol{n}}$ represents the inertia torques due co centrifugal and corioliis acceleratione, $-g=-G\left(o f \operatorname{cin}^{n}\right.$ it the gravity loads as seen se the joints where $G(f) \sin ^{n \pi n}$ is non-unique, and $U_{i} R^{n}$ is the control. In state-space representation, Eq. (i) can be given by

$$
i \cdot\left[\begin{array}{cc}
0 & 1  \tag{21}\\
A^{-1} G & A^{-1} r
\end{array}\right]:+\left[\begin{array}{c}
0 \\
A^{-1}
\end{array}\right] u
$$

Mote that functional dependencien are dropped for elarity. If each actuator (D.C. motor) is modeled as a second-order, ilnear, timelnvariant subsyetem ineqlecting the armature inductancel, and is coupled with the anipulator dynamics. the previously defined state vector. $x$. Will be preserved and the control will be the actuator input voltage. in this case. Eq. $(2)$ takes the following form

$$
\dot{x}=\left[\begin{array}{cc}
0 & I  \tag{3}\\
A^{-1}\left(G-E_{1}\right) & A^{-1}\left(P-E_{2}\right)
\end{array}\right] x+\left[\begin{array}{c}
0 \\
A^{-1} E_{3}
\end{array}\right]
$$

where $A=A+J$ is the combined inertia matrix with $J=d i a g[J]_{k} J_{x}$ ic the rotor inertia of the $k^{t h}$ actuator referred to output ahaft, $E_{1}, E_{2}$ and $E_{3}$ are diagonal, positive definite constant matrices and functions of various actuator/gear train parameters.

Although most of the works do not include the actuator dynamics, the above, simplified form may be subatituted, since the form of the equations remains the same. Depending on the adaptation algorith, these constant actuator parameters may ither be included in the on-line identification schema or assumed known. In our presentation, the generic $u$ will represent the suitable contzol leither the effective input torques or the voltages). The only exception is [6] where third-order actuator dynamics is studied in addition to the above simplified form. The dynamic oquations, Eq. (2) or (3), way be given in terms of the robot-hand coordinates expresed in a sixed reference frame ltask-oriented coordinates) and adaptive controllers may be designed tor this system [6,7,8].

## 3. MRAS-BASED CONTROLLERS

In MRAS design, usually a second-oider, linear, time-invariant, continuous-time reference model if selected for each link of the serial robot. Then, a control law is derived to force the robot to behave like the selected model. As mentioned earlier, local parametric optimization $\{5,9]$, Lyapunov's second method [101, hyperstability $111,12,13]$, or the sliding control theory $[14,15,16]$ is usually employed to achieve the goal.

In 1979, Dubowsky and Desforges (5) implemented the local parametric optimization technique on robot arm. In their formulation, each servomechanism is modeled as a second-order, single-input, single-output system, neglecting the coupling between system degrees of freedom. Then for each degree-of-freedom, position and velocity feedback gains are calculated by an algorithm which minimizes a positive semi-definite error function utilizing the steepest descent method. stability ia investigated for the uncoupled, linearized syetem model. This work represents the first implementation of adaptive control to robotics.

The recent works have concentrated on the designs based on the Lyapunov's second method and the hyperstability theory. In the most general case, these control mothods yield the following control seructure $u_{p}$ :

$$
\begin{equation*}
u_{p}=s_{1} \xi_{1}\left(A_{P}, F_{P}, G_{P}, x_{P}\right)+s_{2} f_{2}\left(x_{i}, x_{P}, x_{F}, u_{F}\right)+j_{j} f_{j}\left(m_{j}, x_{P}, x_{F}, \dot{x}_{F}\right) \tag{4}
\end{equation*}
$$

where subectipti $p$ and $r$ represent the plant and reference model. respectively, $i_{i}$ is either
 unknown system perameter like the payload, link aas contant, center of graviey location. etc., where combination of these constant parameters or a nonlinear tern is to be estimated in the adaptation process, and $j=1,2, \ldots, k$, where $k$ depends on the specific controlier design. Although some controllers call for plant joint aceelerations, they are not shown in Eq. (4).

The first term $\mathrm{f}_{\text {: }}$ in Eq. (4) describes the nonlinearity compensation. it may represent the complete manipulacor dynamics as in 117.101 , or only the gravity terms and the jacobian as in $\{7]$. A controller with $j_{1}-1$ and $j_{2}-s_{3}=0$ indicates only a nonlinearity sompensation. The second term in up represents the seedback portion of the controller. Tha
 the control structures of $\{16,19,20,211$ among others. The third erm in up includes the portion of the conerol where systen parameters are estimated [17,18,221. Slotine (18), for example, includes all the componenes into his controlier, theretore, $s_{1}=s_{2}=s_{3}=1$. Takegaki and Arimoto's control strategy $17 \mid$ may be sumerized by $\delta_{1}=\delta_{2}=1$ and $\delta_{3}=0$. Horowitz and Tomizuka's $|22| s_{1}=0, s_{2} \cdot \delta_{3}=1$, etc.

Various mRAS coneroi seructures are sumarized in Table l. This table differentiates the mechods which require explicit calculation of dynamic equations (Monlinearity Compensationl from the methods which adeptively estimate the plant parameters on-line Ifncorporation of Plant parameter Estimationl. However, further distinction is needed in the later group, since while on approach explicitiy idantifies the nonlinear terms las ia $A$, 6 , and pwith reference to Eq. (21), and estimates them on-iine, the recent methods treat the constant robot parameters as unavailable, estimate and compensace them in their algorithms. Some methods choose nonlinear feedback matrices in their controllers (H, $L$, $S$ in Table il without incorporating the explicit system parameter estimations.

The arly works presented in rable 1 have generaily avoided the nonlinearity compensation and opeed for the assumption that the nonlinear system parameters vary slowly in sime. On this basis, a stability analysis is qiven for the systam. This assumption almose certainly 13 too restrictive. since the nonlinear manipulator system parameters are cunctions of the jcint position and velocities. The faster the robot movement is, the more rapidly the system parameters will vary. The objective on the other hand, for the more sophisticated control methods is to enable fast robot movements with high precision. As a result of revolutionary advances in the microprocessor industry with prices steadily coming down, the possibility of real-time implementation of computation intensive algorithms is steadily improving. Recently, Wander and Tasar i23,241 have implemented the complete dynamic equations $\{25$ ) of $6-1 i n k$, general architecture robot arm in 6.5 sailii sec. (150 $H z)$ in explicit form without using recursion. Their algorithm is able to treat an n-degree-ot-freedom (DOF) serial system of completely general parameters (43 milli sec. for (2-DOF). They have lmplemented the algorithm on an Analogic ap-500 array processor. Further work on the comparative analysis of various computer architectures is underway at the University of rexas at Austin.

Some of the most recent works include nonlinearity compensation along with a feedback portion and parameter identification ieatures $\left[6,17,181\right.$. Once the control has the form $u_{p}=$ $\bar{A}_{p}\left({ }^{\prime}{ }_{p}\right) u_{p}$, where subscript $p$ denotes the plant, $\bar{A}_{p}$ is the on-line calculated generalized inertia matrix and $u_{p}^{\prime}$ is yet to be selected, generally, global stability of the closed-loop system can te shown provided that $\lambda_{p}^{-1} \bar{\lambda}_{p}=I$, where $I$ is the identity matrix of order $n$, is maintained $\{6$ ]. Otherwise, in reference to rable 1 , all methods without nonlinearity compensation need to assume system parameters stay constant during the adaptation.

Table 1. Summary of MRAS Controllers in the Literature

|  | CONTROLLER CHARACTERISTICS |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | MONLINEARITY COMPENSATION | GINEAM I ML FEDDACK 1 METHOD | INCORPORATM of Plant panameten ${ }^{2}$ ESTIMATIOM | ASSUMED SYS. PARAMETERS star const. DURING ADAPTATIOM 3 |
| Duboweky and DeaForges 1970 [5] | - | 0 | -* | $\checkmark$ |
| Horowitz and Tomizuka 1980 [22] | -* | C | $N$ | $\checkmark$ |
| Takegakl and Arimoto 1981 [7] | 0 | L | -- | $\sqrt{ }$ |
| Batestrino.ot al. 1983 [19] | - | H | -- | $\checkmark$ |
| Stoten 1983 [21] | -* | H | -* | $\checkmark$ |
| Balegtrino.et al. 1s84 [16]. | -* | H,S | -* | $\checkmark$ |
| Nicosia and Tomel 1894 [27] | -- | H | -- | $\checkmark$ |
| Vukobratovic.at at.1985 [30] | ** | H | $\cdots$ | $\checkmark$ |
| Landeu 1985 (36) | $\cdots$ | C | N | $\checkmark$ |
| Whyte 1985 (20] | -* | L, C | -* | $\checkmark$ |
| Eelemt 1905 [20] | - | L | -a | $\checkmark$ |
| Lim and Esiaml 190s [20] | $\checkmark$ | $L$ | ** | $\cdots$ |
| - | ** | L | $\cdots$ | $\checkmark$ |
| Hsia 1988 [29] | $\cdots$ | H,C | $\cdots$ | $V$ |
|  | $\cdots$ | C | $N$ | $\checkmark$ |
| Craig, ot at. 1986 (17) | $\checkmark$ | -- | $V$ | $\cdots$ |
| Sloline 1986 [18] | $\checkmark$ | L.C.S | $v$ | $\cdots$ |
| Tosunoglu and Tesar | A | L, H,C | $\checkmark$ | $\bullet$ |
| 1986 [8] | $\checkmark$ | L,H,C | $\checkmark$ | -- |

Kry to Bemertal
: Calculates complete or partial nonlinear dynemica on-line.
: Robot link lengthe, mass contents, setuotor parameters, etc., it not otherwise specifled.
3: If "yes", stability anclyate besed on this aseumption.
G : Gravity load compensated; also requires on-line Jacobien calculation.
A : Requires only the on-line calculation of the inertio matrix.
: Nonlinear-gain feedbeck using local parametric optimization.
: Constant-gein teedback.
Nonlinear-gain feedback using Lyapunov'e aecend method.
Norlinear-gain feedback using hymaratability theory.
Nonilinear-gain feedback using sliding theory.
: Structure of nonlinear system paramefers (funetiona of state variables) are explicitly assumed known and are adaptively estimated; stability analysis based on hyperstability theory.
v : Yes.

- : No.

Balestrino, et al. [19] have developed an adaptive controller which produces discontinuous control signals leading to chattering. stability analysis is presented using hyperstability theory. In (161, Balestrino, et al. present three methods; the firat is based on the theory of variable-structure systems, the aecond on the hyperstability and the third is a combination of the first two methods. Designed controllers produce high-frequency chatter which is highly undesirable since higher order dynamic modea may be excited. Numerical simulations show an extremely high frequency of sign switches in the plant input. prohibiting its phyaical realization. Stoten 121 ] formulatea the MRAS problem closely following the procedures in $[9]$ and simulate the algorithm for a l-ink manipulator.

Horowitz and romisuka [22] study the adaptive control of a 3-1ink arm. Gravity effects and the mass and inertia of the first link are neglected. Each nonlinear terim in the dynamic equations is identified a priori, treated as anknown, and estimated by an adaptation algorithm. For the modeled system and the designed controller, stability analysis is given by Popov's hyperstability theory. Later, Anex and Hubbard [26] have experimentaliy implemented this algorithm with some modifications. System response to high speed movements is not tested, but practical problema encountered during the implementation are addressed in detail.

Takegaki and Arimoto [7] propose an adaptive method to track desired trajectories which are described in the task-oriented coordinates. The suggested controller compensates gravity terms, calculates the Jacobian and the variable gains, but does not compensate the manipulator dynamics completely. System stability is assured if the manipulator hand velocity is sufficiently slow, i.e., nonlinear system parameters change slowly.

Nicosia and Tomei $\{27]$ derive control laws using the hyperstability theory to follow a linear, time-invariant reference model. The plant (manipulator) parameters and the payload are assumed known and are not identified. Their controller does not produce chatering and is relatively easy to implement. Lim and Eslami $\{201$ propose controller designs based on Lyapunov's secord method. The author's objective is to control the linearized dynamic equations with the developed controliers; hence, assuring the stability of the linearized system. Later, nonlinearity compensation is suggested to enhance the system response. Whyte (28! designs an adaptive controller via Lyapunov's second method. The algorithm does not require any knowledge of the manipulator dynamics and selects nonlinear gains in the feedback loop to follow the reference model. System stability is shown, provided that the parameter changes are slow. Hsia [29] reviews the current methods used in adaptive control and gives brief formulations for each method. Vukobratovic, et al. (30), review local parametric optimization and hyperstability-based methods and choose not to include the approaches based on Lyapunov's second method in their book on the non-adaptive ard adaptive control of manipulators.

Tosunoglu and Tesar [6] select a generalized nonlinear reference model which represents ideal robor dynamics. The plant, the actual robot whose system parameters may not be exactly known, is then forced to behave like the reference model to follow the desired trajectory. The advantage of the nonlinear reference model is that the adaptation process ceases once the nominal trajectory is recovered. (Such is not the case when linear models are selected.l Error-driven dynamics is derived and the system is augmented to include the integral feedback feature to eliminate the parameter discrepancies between the plant and the reference model, and the disturbances acting on the system. It is shown that the controllers designed in this work via Lyapunov's second method also produce hyperstable systems. Simulations demonstrate successful trajactory tracking on 3- and 6-link, spatial manipulators under unknown payloads and estimated system parameters link lengths, masses, inertia components, payload, etc.l. The authors also provide comparative analyses of the effect of integral feedback and various controller update rates, 60 to 200 Hz .

Craig, Hsu and Sastyy [17] take an interesting approach in designing an adaptive controller using the Lyapunov's second method. In this work, the structure of the terms in the dynamic equations is assumed known, but their numerical values remain unknown. They partition the dynamics into xnown and unknown portions and estimate the unknown parameters along with compensation for the nonlinearities. Global stability is proved ty assuming tha: a matrix function of the plant joint position, velocity and accelerations is bounded. Althoigh all the terms which are functions of positions are bounded, velocity and accelerations may increase without bounds; thus, maixing the matrix unbounded. However, modifications in the controller structure may alleviate this problem. Their numerical simulations identify iink masses and coulomb friction coefficients for a two-link manipulator with encouraging results.
slotine and $L i$ [18] derive a control law with full feedforward dynamica compengation, PD qeedback and on-ilne payload and manipulator parameter estimation using Lyapunov's second method. since this control scheme does not eliminate the steady-state errors, the authors restrict the steady-state position arrors to lie on a sliding surface. This modification, In turn, causes the lose of numerical efficiency where, interestingly, the authors make use of the recursive computation feature of the manipulator dynamics. Later, an approximate implementation is suggested to improve the numerical efficiency. payload parameter identification is simulated on a two-link manipulator.

Once these currant methods are refined, application to manipulators with higher degrees of freedom will naturally follow. Determination of the structure of the constant terms ffor identification) for manipulators with higher number links may be achieved with symbolic generation of dynamic equations, but the effect of increased number of terms wili require further investigation.

## 4. SELF-TUNING REGULATOR (STR) BASED CONTROLLERS

In this method, typically, nonlinear manipulator dynamics is linearized about a nominal trajectory and then discretized. The discretized model gives the structure of the parametric model whose parameters need to be estimated on-linc. The parametric model is given by the following multivariable difference equation

$$
\begin{equation*}
y(k)=\theta^{T}(k-1)+\theta(k) \tag{5}
\end{equation*}
$$

where $Y(k) c R^{n}$ is the system output, $k$ is the sampling time counter, $\theta c R^{n \times(2 n m+1)}$ contains the parameters to be identified, $C^{(2 n m+1)}$ represents the combined system input and output vector, ecin is a random, zero-mean Gaussian white noise, and $m$ is the order of the estimation model.

Parameter estimation is based on the system identification techniques using the sampled input-output data. Although such techniques include the least squares, extended least squares and maximum likelihood methods; the recursive least squares method is extensively used because of its lower computational requirements [8,25,29-36]. The recursive least squares estimation is given by

$$
\begin{equation*}
\hat{\theta}_{i}(k)=\hat{\theta}_{i}(k-1)+P(k) \neq(k-1)\left[y_{i}(k)-\hat{\theta}_{i}^{T}(k-1)+(k-1)\right] \tag{6}
\end{equation*}
$$

where

$$
P(k)=\frac{1}{\lambda}\left[P(k-1)-\frac{P(k-1) \phi(k-1) \phi^{T}(k-1) P(k-1)}{\lambda+h^{T}(k-1) P(k-1) \lambda(k-1)}\right)
$$

$\hat{\theta}_{i}(k)$ rapresents the estimate of the $i^{\text {th }}$ row of $\theta$ definad in Eq. (5), $P(k)$ is a square symetric matrix of order (2nm+1), and $0<\lambda<1$ is an exponential forgetting factor.

Once the parameters are identified at each sampling time, regulator parameters are estimated using the extended minimum variance $i 8,30,321$, or pole-rero placement techniques [29,33,35]. The method described above is known as explicit or indirect STR. If the regulator parameters are estimated directly by a reparameterization of the process model. the model is called implicit or direct STR. Usually implicit STR algorithms cancel all process zeros making them suitable only for minimum phase systems.

Koivo and Guo [32] assume an autoregressive model and identify system parameters using the recursive least squares technique. They design an extended minimum variance controller for the estimated model. The method chooses a quadratic performance index in terms of the state error vector and the system control vector and minimizes it relative to admissible controls while satisfying Eq. (5). Their simulations include decoupled and partially coupled parametric model structures. They report that the parameter convergence is faster In the decoupled case, and that no significant improvement in the system response is obserred for the coupled model. This is rather interesting, because the amount of on-line calculations is considerably reduced for the decoupled case. Also reported is the fact that the model and the controller parameter identifications may not converge fast enough while the robot motion takes place.

Lee (8) derives the perturbation equations, discretizes them and estimates the sysem parameters using the recursive least aquares method. Then in adaptive controller is designed using the extended minimum variance technique. The parameter identification requires the estimation of $6 n^{2}$ parameters on-iine $(216$ for a $6-1 i n k$ manipulator). Lee provides a detailed breakdown of the computational requirementa and concludes that for a 6-1ink manipulator the control scheme can be updated at about 56 Hz on a PDP-11/45.

Heia [29] reviews the STR formulation on a decoupled model. Karnik and Sinha [35] develop an STR based on a non-minimum phase model which assigns the system poles while retaining all the zeros. Thia algorithm is developed for a unIMATE-2000 robot. Landau (9,36) and Vukobratovic, et al. [30] review various STR designs in detail.

In general, discrete-time formulation is ospecially suitable for computer control. However, on-line estimation of all system parameters and the control design make STR computationaliy intensive. Astrom [4] reports that numerical estimation techniques tend to be numericaliy unstable as the number of parametezs increases in the system model. In this case, the complete system is parameterized. However, the papers reviewed in this work do not raise the question with regard to numerical instability although they do indicate the importance of initial parameter selections. In STR methods, convergence of estimated parameters in the adaptation process is guaranteed if the system parameters are constant. Since the actual robot model parametera are nonlinear functions of the state vector, the question of system parameters varying slowly in time again arises in the STR methods.

## 5. CONCLUSIONS

Adaptive control of robotic systems has received significant attention within the past few years. A class of control laws based on the MRAS design realize the adaptation through signal synthesis with a complately known parameter structura, while some methods select a subclass of the parameters for identification for reduced computational burden. All adaptive controllers via STR design and some MRAS-based methods estimate the complete (nonlinear) system parameters on-ine.

Stability analysis usually relies on the condition that the nonlinear system parameters vary slowly. This condition is removed if a nonlinearity compensation component is also incorporated i the controller. The most recent works, which exploit the special structure of manipulator dynamics, seem to favor this feature. The use of state-of-the-art microprocessor technology along with the sophisticated dynamic formulation algorithms strongly indicate that real-time implementation of dynamic compensation is rapidly becoming Seasible.

Further research to perfect the existing algorithms and to provide rigorous stability proofs, which will improve the maturity of the adaptive control, is still needed. Although today's industrial robots employ linear controllers to accomplish a number of useful tasks, fast and precise robot movements remain to be implemented. Development of laboratory test beds and implementation of the developed adapeive controliers on robotic manipulators are also crucial, since only after successful demonstrations will technology transfer be possible.

## 6. ACKNOWLEDGEMENTS

This work is partially funded by the U.S. Department of Energy (Grant No. DE-FGO2-86NE37966) and the NASA Johnson Space Center (Grant No. NAG 9-188). The authors gratefuliy acknowledge the funding agencies.

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# Simple Robust Control Laws for Robot Manipulators, Part I: Non-adaptive Case 

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## Abatract

A new class of exponentially stabilizing control laws for joint level control of robot arms is introduced. it has been recently recognized that the nonlinear dynawics associated with robotic manipulators have certain inherent passivity properties. More specifically, the derivation of the robotic dynamic equations from the Hamilton's principle gives rise to natural Lyapunov functions for control design based on cotal energy considerations. Thrnugh a slipht modification of the energy Lyapunov function and the use of a convenient lema to handle third order terms in the lyapunov function derivatives, closed loop exponential stability for both the set point and tracking control problem is demonstrated. The exponential convergence property also leads to robustness with respect to frictions, bounded modeling errors and instrument noise. In one new design, the nunlinear terms are decoupled from real-time meisurements which completely removes the requirement for on-line computation of nonltnear terms in the concroller implementation. In general, the new class of control laws offers alternatives to the more conventional computed torque method, providing trade offs between robustness, computation and convergence properties. Furthemore, these control laws have the unique feature that they can be adapted in a very simple fashion to achieve asymptotically stable adaptive control.

## 1. Introduct ton

The problem of joint level control for the multi-link rigid articulated robot arm is addressed in this paper. Accurate measurements of the joint variables, efther angular or displacement, and the joint velocities are assumed avallable. Traditionally, this problem has been treated by the PID algorithm. Since the justification of using PID control is based on elther linearizatica or some local stability argument [l], its applicafion is limited to small angle maneuvers. Large excursions usually require partitioning a desired trajectory into intermediate points and PID control is used to drive the arm between adjacent points. This approach is less than satisfactory since global stability and adequate performance are not guaranteed. This then motivates the computed torque method [2] which compensates for nonlinear terms in the robot dynamics. Assuming that the robot dynamics are known exactly, the compensated system apoears like a decoupled system of double integrators and the closed loop dynamics can be shaped into desirable forms.

A different approach has been advanced in the past few years. It is based on the recognition chat robot arms belong to the class of natural systems, which means time invariant, unconstrained and lying in a conservative force field [3]. It is natural to investigate if the structure specific to this class of systems can be exploited in controller design. It has long been known ( 3,4 and earlier) that nega:ive proportional (generalized position) and derivative (generalized velocity), or equivalently PD, feedback globdlly asymptotically stabilized natural systems. The stability analysis is based on a lypaunov function motivated by total energy considerations. Application of this result co robot arms has been relatively recent. In particular, global asymptotic stability under goint level PD feedback with gravity compensation has been shown [5-8]. Application to the tracking problem is more difficult due to the time varying nature of the problem, Mo:e specifically, the stability analysis requires a generalization of the invariance principle to time-varying systems; this issue has been partially addressed in [9-10].

In this paper, we will introduce a new class of exponentially stabilizing control laws for both the set poine and tracking control problems. The stability proof is achieved by making use of a particular class of energy-like Lyapunov functions in conjunction with a useful lema for addressiar, the higher order terms in the ivapunov function derivatives. In the set point control case, lyapunov functions based on various artificial potential fields are used to derive control laws possessing desired properties. These include set point controllers having simple PD or PD+bias structures and the ability to handle joint stop constraints. In the tracking control case, this new class of Lyapunov functions avoids the need for a generalized invariance principle, which, as mentioned above, has been the major source of difficulty in existing approaches. This leads to a new class of exponentially stabilizing tracking control laws. In one design among thisinew class. the nonlinear terms are decoupled from real-time measurements whith completely removes the requirement for on-line computacion on nonlinear terms in the controller implementation. This result is believed to have no counterpart in the present day literature. in general, the new class of control laws offers alternatives to the more conventional computed torque method, providing tradecffs between robustness, computation and convergence properties. Furthermore, these control laws have the unique property that they can be adapted in a very simple fashion to achteve asymptotically stable adaptive control. This last property will be elaborated on in the companion paper [13]. The closed loop exponential stability also allows the robustness property to be established with respect to viscous and Coulomo friction, bounded modeling error and instrument noise.

This paper is organized as follows: Some background derivations, identities, notations, lemmas and relevs results in the literature are covered in Sec. 2. Several useful set point controllers based on different artificial potential energies are presented in Sec. 3. A new Lyapunov function is also introduced to demonatrate exponential convergence. In Sec. 4 , new family of exponentially stabilizing tracking control lawe are derive We will discuss the trade off between the ase of implementation and the atrength of asumptions for these controllers. Their robustness properties are also analyzed in this eection. Finally, concluaiona are drawa in Sec. 5 togather with table sumarizing all of the controllers presented in this paper and the conditions for stability. Due to the size limitation of this paper, the detail derivations are given in $k j$.

## 2. Background

### 2.1 Robot Dynamic Equation

In this section, the dynamic equation of robot manipuiator is derived. at the first glance, it appears as a complex, tightly coupled set of nonlinear equations. However, based on derivation from Hamiltonian principle, the nonlínearity actually contains a great deal of structure. As a result, some important identities are developed in the nexr section on which the rest of the paper is bssed.

An n-link rigid robot arm belongs to the class of so-called natural systems with the kinetic and pocential energies given by
$T=\frac{1}{2} q_{2}^{T} M\left(q_{1}\right) q_{2}$
$U=-q_{2}{ }^{T} u+g\left(q_{1}\right)$
where
$T=k i n e t i c$ energy, $U=$ potential energy, $q_{1}$ - joinc angle or position vector $\varepsilon R^{n}$, $q_{2}=$ Joint velecity vector $\varepsilon R^{n}, M\left(q_{1}\right)=$ mass inertia matrix $\varepsilon R^{n \times n}, g\left(q_{1}\right)=g r a v i t a t i o n a l$ potential energy, $u=$ joint torque force vector $\varepsilon R^{n}$

Note that since all the analysis is done at foint level, the arm can be redundant (wore than 6 foints). To derive the differential form of the robot dynamics, first set up the Lagrangian

$$
L=T-U
$$

Then apply the Lagrange's Equaticn

$$
\frac{d}{d t}\left(\frac{\partial L}{\partial q_{2}}\right)-\frac{\partial L}{\partial q_{1}}=0
$$

This gives the dynamic equation of robot motion:

$$
\begin{equation*}
\dot{q}_{1}=q_{2} \tag{2.2}
\end{equation*}
$$

$$
M\left(q_{1}\right) \dot{q}_{2}=-C\left(q_{1}, q_{2}\right) q_{2}-k\left(q_{1}\right)+u
$$

where

$$
C\left(r_{1}, q_{2}\right)=\sum_{i=1}^{n}\left[\left(e_{1} q_{2}{ }^{T} M_{1}\left(q_{1}\right)\right)^{T}-\frac{1}{2}\left(e_{1} q_{2}{ }^{T} M_{1}\left(q_{1}\right)\right)\right]
$$

$$
e_{1}=\text { fth unit vector }
$$

$$
M_{1}\left(q_{1}\right)=\frac{\partial M\left(q_{1}\right)}{\partial q_{11}}
$$

$$
k\left(q_{1}\right)=\frac{\partial g\left(q_{1}\right)}{\partial q_{11}}
$$

$$
q_{11}=\text { ith cumponent } u f q_{1}
$$

The term $C\left(q_{1}, q_{2}\right) q_{\text {, }}$ represents the coriolis and centrifugal forces and $k\left(q_{1}\right)$ reprasents the gravity load. Note that $C\left(q_{1}, q_{2}\right)$ is determined entirely from the mass-inertia matrix. Many desirable properties, for example, inherent passivity, well-posedness of solution (no finite escape under any bounded control), existence of solution to optimal control problem (14) etc. are the consequence of this additional structure. Other important properties of (2.2) include that $M\left(q_{1}\right)$ and $M_{1}\left(q_{1}\right)$ are symetric and $M\left(q_{1}\right)$ is positive definite, for all $q_{1} \in R^{n}$. For later use, the matrices $C\left(q_{1}, q_{2}\right)$ and $M_{1}\left(q_{1}\right)$ are interpreted as $R^{n x n}$ valued function of two n-vectors ( $q_{1}$ and $q_{2}$ ) and one n-vector ( $q_{1}$ ), resfectively.

### 2.2 Some Useful Identities

Some key identities that will be used throughout this paper and the companion paper [13] are derived in this section. First define some notations:

$$
\begin{align*}
& M_{D}\left(q_{1}, z\right)=\sum_{i=1}^{n} M_{1}\left(q_{1}\right) z e_{1}^{T}  \tag{2.6}\\
& \dot{M}\left(q_{1}, q_{2}\right)=\frac{d}{d t} M\left(q_{1}\right)  \tag{2.5}\\
& J\left(q_{1}, z\right)=\sum_{1=1}^{n}\left[\left(e_{1} z^{T} M_{1}\left(q_{1}\right)\right)-\left(e_{1} z^{T} M_{1}\left(q_{1}\right)\right)^{T}\right] \\
& r\left(q_{1}, q_{2}, q_{2 d}\right)=\left(q_{2}-q_{2 d}\right)^{T}\left[\frac{1}{2} \dot{M}\left(q_{1}, q_{2}\right)\left(q_{2}-q_{2 d}\right)-C\left(q_{1}, q_{2}\right) q_{2}\right] \\
& q_{1 d}, q_{2 d}=\text { deaired joint position and joint velocitiea } \\
& \Delta q_{1}=q_{1}-q_{1 d}  \tag{2.8}\\
& \Delta q_{2}=q_{2}-q_{2 d}
\end{align*}
$$

Again, $M_{D}$ and $J$ are regarded as $\mathbb{R}^{\mathrm{nxn}}$ valued function of two $n$-vector arguments. Note that $J$ is a skew eymmetric matrix, i.e.. $J+J^{T}=0$.

Idenciey 1

$$
\begin{equation*}
\dot{M}\left(q_{1}, q_{2}\right) z=M_{D}\left(q_{1}, z\right) q_{2} \tag{2.9}
\end{equation*}
$$

Identicy 2

$$
\begin{equation*}
c\left(q_{1}, z\right) z=\frac{1}{2}\left(M_{0}\left(q_{1}, z\right)-J\left(q_{1}, z\right)\right) z \tag{2.10}
\end{equation*}
$$

Identity 3

$$
\begin{equation*}
J\left(q_{1}, z\right)=M_{D}{ }^{T}\left(q_{1}, z\right)-M_{D}\left(q_{1}, z\right) \tag{2.11}
\end{equation*}
$$

Identity 4

$$
\begin{equation*}
M_{D}^{T}\left(q_{1}, x\right) y-M_{D}^{T}\left(q_{1}, y\right) x \tag{2.12}
\end{equation*}
$$

Identity $S$

$$
\begin{align*}
r\left(q_{1}, q_{2}, q_{2 d}\right) & =\frac{1}{2} \Delta q_{2}^{T}\left(J\left(q_{1}, q_{2}\right) q_{2 d}-M_{D}\left(q_{1}, q_{2 d}\right) q_{2}\right)  \tag{2,13}\\
& =\frac{1}{2} \Delta q_{2}^{T}\left(J\left(q_{1}, q_{2 d}\right) q_{2}-M_{D}\left(q_{1}, q_{2}\right) q_{2 d}\right)  \tag{2.14}\\
& =\frac{1}{2} \Delta q_{2}^{T}\left(J\left(q_{1}, q_{2 d}\right) q_{2 d}-M_{D}\left(q_{1}, q_{2}\right) q_{2 d}\right)  \tag{2.15}\\
& =\frac{1}{2} \Delta q_{2}^{T}\left(J\left(q_{1}, q_{2}\right) q_{2}-M_{D}\left(q_{1}, q_{2 d}\right) q_{2}\right) \tag{2.16}
\end{align*}
$$

Identity 6

$$
\begin{align*}
& M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}-C\left(q_{1}, q_{2}\right) q_{2}-\frac{1}{2}\left(J\left(q_{1}, q_{2}\right) q_{2 d}-M_{D}\left(q_{1}, q_{2 d}\right) q_{2}\right) \\
& =M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}-C\left(q_{1}, q_{2}\right) q_{2}-\frac{1}{2}\left(J\left(q_{1}, q_{2 d}\right) q_{2}-M_{D}\left(q_{1}, q_{2}\right) q_{2 d}\right) \\
& \left.=\frac{1}{2}\left(M_{D}^{T}\left(q_{1}, \Delta q_{2}\right) \Delta q_{2}+M_{D}^{T}\left(q_{1}, q_{2 d}\right) \Delta q_{2}-M_{D}\left(q_{1}, q_{2 d}\right) \Delta q_{2}+M_{D}, q_{1}, \Delta q_{2}\right) q_{2 d}\right)  \tag{2.17}\\
& M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}-C\left(q_{1}, q_{2}\right) q_{2}-\frac{1}{2}\left(J\left(q_{1}, q_{2 d}\right) q_{2 d}-M_{D}\left(q_{1}, q_{2}\right) q_{2 d}\right) \\
& =\left(M_{D}^{T}\left(q_{1}, q_{2 d}\right)-M_{D}\left(q_{1}, q_{2 d}\right)\right) \Delta q_{2}+\frac{1}{2} M_{D}^{T}\left(q_{1}, \Delta q_{2}\right) \Delta q_{2}+\frac{1}{2} M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2 d}  \tag{2.18}\\
& \\
& M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}-C\left(q_{1}, q_{2}\right) q_{2}-\frac{1}{2}\left(J\left(q_{1}, q_{2}\right) q_{2}-M_{D}\left(q_{1}, q_{2 d}\right) q_{2}\right)  \tag{2.19}\\
& =\frac{1}{2}\left(M_{D}\left(q_{1}, \Delta q_{2}\right) \Delta q_{2}+M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2 d}\right)
\end{align*}
$$

2.3 Important Lemmas

In this section, two important stability lemmas are presented that will play pivotal roles in later sections. The first lema is essentially a local stability theorem that establishes a region of convergence. The second lema generalizes the first result to the Lagrange stability case. The proofs of these lemas are given in [23].

Leman 2-1

## Given a dymaical syotea

$\dot{x}_{1}-\varepsilon_{1}\left(x_{1}, \ldots, x_{N}, t\right), x_{1} c n^{n_{1}} ; t \geq 0$
Lat $f_{i}^{\prime}$ s be locally Lipschitz with rasect to $x_{1}, \ldots, x_{N}$ uniformiy in $t$ on bounded intervele and concinuous in $t$ for $t \geq 0$.

$$
\text { Suppose a function } v: R^{n_{1} x \ldots n_{n}} \times x_{+} \rightarrow x_{+} \text {is given such that }
$$

$$
v\left(x_{1}, \ldots, x_{N}, t\right) \cdot \sum_{i, j-1}^{N} x_{i}^{T} P_{1 j}\left(x_{1}, \ldots, x_{j}, t\right) x_{j},
$$

$v$ is positive definite in $x_{1}, \ldots, x_{N}$
$\dot{v}\left(x_{1}, \ldots, x_{N}, t\right) \leq \sum_{1 \in I_{1}}\left(a_{1}-\sum_{j \in I_{I_{1}}} r_{1 j}\left\|x_{1}(t)\right\|^{k_{1 j}}\right)\left\|x_{1}(t)\right\|^{2}$
where $a_{1}, \gamma_{1 j}, k_{1 j}>0, I_{2_{1}} \in I_{1} \in\{1, \ldots, N\}$
Let $\xi_{i}>0$ be such
$\xi_{1}\left\|x_{i}\right\|^{2} \leq v\left(x_{1}, \cdots, x_{N}, t\right)$
Let $v_{0} \triangleq v\left(x_{1}(0), \ldots, x_{N}(0), 0\right)$
If $\vee 1 \in I_{1}$,
$1 \in I_{1}$,
$a_{1}=\sum_{j \in I_{2_{1}}} \gamma_{1 j}\left(\frac{v_{0}}{\varepsilon_{j}}\right) \frac{k_{11}}{2}$
then $v \lambda_{1} \in\left(0, a_{1}-\sum_{j \in I_{2_{1}}} \gamma_{1 j}\left(\frac{v_{0}}{\varepsilon_{j}}\right) \frac{k_{11}}{2}\right)$,
$\dot{v}\left(x_{1}, \ldots, x_{N}, t\right) \leq-\sum_{1 \in I_{1}} \lambda_{1}\left\|x_{1}\right\|^{2} \quad v \in \geq 0$
In the above lema, we choose to bound over $\sum_{j \in I_{2_{1}}} Y_{1 j}\left\|x_{j}(t)\right\| \|_{1 j}$
(Condition (2.20) reflects that choice). This choice is arbitrary; in fact, we can extract any quadratic cross term from

$$
\sum_{j \in I_{2}}^{\gamma_{1 j}}| | x_{j}(t)| |^{k_{1 j}}\left\|x_{1}(t)\right\|^{2}
$$

and overbound the rest. After completing square stability condition aimilar to (2.22) can be acated. We do not pursue this generalization here.

## Letrua 2-2

Suppose in Leuma 2-1,
$\dot{v}\left(x_{1}, \ldots, x_{N}, t\right) \leq-\sum_{i \in \frac{1}{I_{1}}}\left(a_{i}-\sum_{j \varepsilon I_{2_{1}}} r_{i j}\left\|x_{j}\right\|^{k_{i j}}\right)\left\|x_{i}\right\|^{2}+0$
$I_{1}=\{1, \ldots, N\}$
Let $p=\sup _{x_{1} \in R^{n_{1}, t \in R+}}\left\|P\left(x_{1}, \ldots, x_{N}, t\right)\right\|<\infty,[P]_{1 j}=P_{1 j}$
If $V \notin \varepsilon I_{1}$
$1 \varepsilon I_{1}$
$a_{1}>\frac{\rho p}{V_{0}}+\sum_{j \varepsilon I_{2_{1}}} Y_{1 j}\left(\frac{V}{\xi_{j}}\right)^{\frac{k_{1 j}}{2}}$
then
Limsup $V\left(x_{1}(t), \ldots, x_{N}(t), t\right) \leq \frac{0}{\lambda}$
where $\lambda=\frac{1}{0} \min _{1 \in I_{1}}\left(\alpha_{1}-\sum_{j \in I_{2}} Y_{i j}\left(\frac{v}{\xi_{j}}\right)^{\frac{k_{1 i}}{2}}\right)$
Furthermore, the convergence ${ }^{1} 0$ the set $\left\{\left(x_{1}, \ldots, x_{N}\right):\left\|x_{k}\right\|^{2} \leq \frac{D}{F_{k}}, k E I_{1}\right\}$ is exponential with rate $\lambda$.

The utility of this reault is mainly in robustness analyals. Basicaliy, given a bounded eet of posaible initial condicione, excluding a neighborbood of the origia, of mate be large enough ia the sance of (2.25) for the trajectorfes to be uniforaly bounded. What if $v_{0}=0$, We caa shift $t-0$ to some finite time later when $V_{t} \neq 0$. In practice, a neighborhood around origin can uaully be excluded alnce some robuat locally atable control algorithe euch as Pid eake over.

### 2.6 Recent Reaults

Some of the recent reaulta rolated to lyapuno analysis of robot aystem are reviewed in this section. For the set point control problen, [5-10] has applied the reault that linear negative feedback of generalised position and velocities globally asyaptotically stabilizes natural gyatem to robot eanipulatora. We wili rencate this result, mention work for the tracking problem in $[9,10,18)$ and atate some open iseues that will be addressec in the remainder of this paper and in \{13\}. Theorem 2-1

Consider (2.2) with the control law
$u=-K_{p} \Delta q_{1}-K_{v} q_{2}+k\left(q_{1}\right), K_{p}>0, K_{v}>0$
The null atate of $\left(\Delta q_{1}, q_{2}\right)$ - ayetem is a globally asymptotically atable equilibriua.
The main idea of this approach is to shape the potential field in such a way that it is globally convex and attains a global minimum at $\Delta q_{1}-0$. Complete damping (in the terminology of (31) is added through the derivative feedback to drive the system to the minimum potential energy atate which by design is the desired state. To be specific, suppose the desired potential field is $U *\left(\Delta q_{1}\right)$. The tocal energy under this potential is

$$
\begin{equation*}
V=I+U * \tag{2,29}
\end{equation*}
$$

## Rewrite $V$ as

$$
V=I+U^{0}+U^{\star}-U^{0}-V^{0}+U^{\star}-U^{0}
$$

where $v^{\circ}$ is the original potential energy without external force fields, and $v^{0}$ is che correaponding cotal energy. Let $p=M\left(q_{1}\right) q_{2}$ be the generalized momentum. From Hamilion' equation,

$$
\begin{align*}
\dot{V} & =\left(\frac{\partial V^{0}}{\partial p}\right)^{T} u+q_{2}^{T}\left(\frac{\partial U^{\star}}{\partial \Delta q_{1}}-\frac{\partial U^{0}}{\partial \Delta q_{1}}\right) \\
& =q_{2}^{T}\left(u+\frac{\partial U^{\star}}{\partial \Delta q_{1}}-\frac{\partial U^{0}}{\partial \Delta q_{1}}\right) \tag{230}
\end{align*}
$$

Hence, to drive the desired total energy to its minimum state, we can select ([5])

$$
\begin{equation*}
u=-k_{v} q_{2}-\frac{\partial u^{*}}{\partial \Delta q_{1}}+\frac{\partial U^{0}}{\partial \Delta q_{1}} \tag{2.31}
\end{equation*}
$$

Then $\dot{V}=-q_{2}{ }^{T} K_{v} q_{2}$. From the fact that $-2 \dot{q}_{2}{ }^{T} K_{v} q_{2}$ is uniformly bounded $(\dot{V} \leq 0), \mathcal{Z}_{2}(t) \rightarrow 0$ as $t \rightarrow$ [19].

$$
\dot{p}=-\frac{\partial v^{\circ}}{\partial q_{1}}+u
$$

$$
=-\frac{\partial T}{\partial q_{1}}-\frac{\partial U^{\circ}}{\partial \Delta q_{1}}+u
$$

$$
\begin{equation*}
=-\frac{\partial T}{\partial q_{1}}-K_{v} q_{2}-\frac{\partial U^{\star}}{\partial \Delta q_{1}} \tag{2.32}
\end{equation*}
$$

Since $\frac{\partial T}{\partial q_{1}}-0, \frac{\partial U *}{\partial \Delta q_{1}} \rightarrow 0$, also. Hence, if $U^{*}\left(\Delta q_{1}\right)$ is globaliy convex with minimum at $\Delta q_{1}=0$, $\Delta q_{1}(t) \rightarrow 0$ as $t \rightarrow-$. If $U *\left(\Delta q_{1}\right)=\frac{1}{2} \Delta q_{1}{ }_{T_{p}} \Delta q_{1}$. Theorem $2-1$ is immediacely obtained.
Obviously, any other potential field convex in $\Delta q_{1}$ that has global minimum ac $\Delta q i=0$ (and no local ainima) can be used. We will use this idea in the next section to address the joint stop isaue.

This control law is very appealing in its simplicity and obvious robuseness with respect to modeling error in ass matrix, and centrifugal and coriolis terme. Generalization to the tracking problem is partially addressed in [9, 10]. A control algorithm iw given in [9], but it lacks stability analysis. In [10], Metrosov's Theorem [1i] is used. A question remaina on tha necessity and applicability of the Metrosov's Theorem to the tracking problem. One version of the tracking control lav 15. Section 4 is the same as in [10], but the stability issue is resolved more completely. Nonadaptive vergion of the tracking concrol laws in $[10,18]$ do yield glabal asymptotic stability. However, the simple structure of (2.28) is lost; even for set point control, full model information is reeded.

Based on the above very brief revieu of the currently available pertinent results, it is evident that the folloulng latuee remaln open:

1. Can we get avay with no grovity information, thus achieving a "universal" (arm independent) set point concrol law?
2. Computed torque achieves exponential stability. Are acheaes besed on energy lyapunov functions inherencly ioferior (e.g., only anyptotic atability is poasible) or have we not been clever anough in choosing the lyapunov functions?
3. The tracking problem producee a timevarying ayatem. Can the Invariance Frinciple atill be applied?
4. How far can we reduce the on-line computation requirement (thus allov incraasing performane) for boch set point and cracking problease? What ta the price to be paid?
5. Can ve ensure any robustneas properties with reapect to friction, inatrumant noise, modeling errors?
6. How does one incorporate joint stop conetrainte?
7. Would these scheses (set point and tracking controls) etill work 11 unknown parametera are adapted?

The rest of this paper will be devoted to ansvering lasues $1-6$. The last item is addressed in $\{13\}$ and $\{24\}$.

### 2.5 Computed Torque from Lyapunov Perspective

In Section 2.4, we introduced the total energy Lyapunov function (2.29) to derive a alaple set point control law. The coaputed corque method can also be motivated in the ame manner vith different Lyapunov function. For generality, we will conaider the tracking case. Let

$$
\begin{equation*}
v\left(\Delta q_{1}, \Delta q_{2}\right)=\frac{1}{2}| | \Delta q_{2} \|^{2}+\frac{1}{2} \Delta q_{1}^{T} K_{p} \Delta q_{1} \tag{2.33}
\end{equation*}
$$

Calculate derivative along solution,

$$
\dot{v}=\Delta q_{2}^{T}\left(M^{-1}\left(q_{1}\right)\left(-C\left(q_{1}, q_{2}\right) q_{2}-k\left(q_{1}\right)+u\right)-\dot{q}_{2 d}+k_{p} \Delta q_{1}\right)
$$

If the computed corque control is used

$$
\begin{equation*}
u=k\left(q_{1}\right)+C\left(q_{1}, q_{2}\right) q_{2}+M\left(q_{1}\right)\left(q_{2 d}-k_{p} \Delta q_{1}-k_{v} \Delta q_{2}\right) \tag{2.34}
\end{equation*}
$$

then

$$
\dot{v}=-s q_{2}^{T} K s q_{2}
$$

From the same line of reasoning as before, the closed loop system is globally asymptotically stable. However, ve know that the closed loop system is linear, therefore it ia exponentially stable. This meane that we shoula look for betser Lyapunov function. An obvious choice ia to add in a croas cerm in (2.33). Then

$$
\begin{equation*}
v\left(\Delta q_{1}, \Delta q_{2}\right)=\frac{1}{2}\left\|\Delta q_{2}\right\|^{2}+\frac{1}{2} \Delta q_{1}^{T}\left(K_{p}+c K_{v}\right) \Delta q_{1}+c \Delta q_{1}{ }^{T} \Delta q_{2} \tag{2.35}
\end{equation*}
$$

where $c$ is small constant so that $V$ is positive definite. Take derivative and apply (2.34),

$$
\begin{aligned}
\dot{V} & =-\Delta q_{2}{ }^{T} K_{v} \Delta q_{2}+c \Delta q_{2}^{T} K_{v} \Delta q_{1}+c\left\|\Delta q_{2}\right\|^{2}-c \Delta q_{1}^{T} K_{p} \Delta q_{1}-c \Delta q_{1}^{T} K_{v} \Delta q_{2} \\
& =-\left(\sigma_{\min }\left(K_{v}\right)-c\right\rangle\left\|\Delta q_{2}\right\|^{2}-c \Delta q_{1}^{T} K_{p} \Delta q_{1}
\end{aligned}
$$

which shows elosed lood expronential tability.
Note that in (2.34), in contrast to (2.28), even for set point case, full model nonlinearity cancellation is needed. The approach in this paper is to use the energy lyapunov function instead of (2.33) to generate control laws. We will see in later sections that this affords much larger class of controle which concains much simpler structure in certain cases (especially for set point control).
3. New Results on PD Set Point Control
3.1 Simple PD Concrols

In this section, whill explore using different $u$ e in the controller design. The folluwing has been suggested in [5]:

$$
\begin{equation*}
U *\left(\Delta_{q_{1}}\right)=\frac{1}{2} i q_{1}{ }^{T} K_{p} \Delta q_{l}+g\left(\Delta q_{1}+q_{l d}\right)-g\left(q_{l d}\right)-\underset{220}{T} k\left(q_{l d}\right) \tag{3.1}
\end{equation*}
$$



$$
\begin{equation*}
v=-K_{p} \Delta_{q_{1}}-X_{v} q_{2}+k\left(q_{1 d}\right) \tag{3.2}
\end{equation*}
$$

Suppose each joint is conazrained between joint etops:

$$
\begin{equation*}
q_{11}^{(l)} \leq q_{11} \leq q_{11}^{(h)} \tag{3.3}
\end{equation*}
$$

and the set point is in the interior of the joint inputs:

$$
\begin{equation*}
q_{11}^{(l)}<q_{1 i} \leq q_{11 d} \leq q_{11}<q_{11}^{(h)} \tag{3.4}
\end{equation*}
$$

Let the desired potential function be

$$
\begin{equation*}
U^{*}\left(\Delta_{q_{1}}\right)=\frac{1}{2} \Delta_{q_{1}}^{T} K_{p} \Delta_{q_{1}}+\sum_{i=1}^{n}\left(H_{1}\left(\Delta_{q_{1 i}}+q_{11 d}\right)+L_{i}\left(\Delta q_{11}+q_{1 S d}\right)\right) \tag{35}
\end{equation*}
$$

ware $H_{1}$ and $L_{1}$ are appropriate upper and lowar barriar potontial functione for joint 1 [ 23 .
Then, $\dot{* q_{1}}=0$ 1s a globel ainimue of $\mathrm{U*}\left(E q_{1}\right)[23]$. From (2.31)

$$
\begin{equation*}
u=-K_{v} q_{2}-K_{p} \Delta q_{1}-\dot{H}\left(\Delta q_{1}\right)-i\left(\Delta q_{1}\right)+k\left(q_{1}\right) \tag{3.6}
\end{equation*}
$$

Similarly, if $K_{p}$ is auficiencly large $\left(K_{p}+\frac{\partial k}{\partial \Delta q_{1}}\left(\Delta q_{1}+q_{I d}\right) \geqslant 0\right)$, the following control law also achieves global asymptotic stabllity:

$$
\begin{equation*}
u=-K_{v} q_{2}-K_{p} \Delta q_{1}-\dot{H}\left(\Delta q_{1}\right)-i\left(\Delta q_{1}\right)+k\left(q_{l d}\right) \tag{3.7}
\end{equation*}
$$

Control laws (3.2), (3.6) and (3.7) still require inforsation an the gravity load. It in intereating co ask if this last piece of model information can be removed. This case corresponds to the desired potential energy

$$
\begin{equation*}
U *\left(\Delta q_{1}\right)=\frac{1}{2} \Delta q_{1} T K_{p} \Delta q_{1}+g\left(\Delta q_{l}+q_{l d}\right) \tag{3.8}
\end{equation*}
$$

The corresponding concrol law is

$$
\begin{equation*}
u=-K_{p} \Delta q_{1}-K_{v} \Delta q_{2} \tag{3.9}
\end{equation*}
$$

From section 2.4,

$$
\frac{\partial U^{\star}}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)=K_{p} \Delta q_{1}+k\left(\Delta q_{1}+q_{1 d}\right) \rightarrow 0
$$

This implies

$$
\begin{align*}
& \text { If } K_{p}+\frac{3 k\left(q_{1}\right)}{3 q_{1}}>0 \forall q_{1} c R^{n},-K_{p} k\left(\Delta q_{1}+q_{1 d}\right) \text { is a concraction map in } \Delta q_{1} \text {. Then } 1!q_{1} \text { * such thac } \\
& K_{p}\left(q_{1}{ }^{*}-q_{1 d}\right) \div k\left(q_{1} *\right)-0  \tag{3.10}\\
& \lim _{t \rightarrow 0} q_{1}(t)=q_{1}
\end{align*}
$$

This result suggesta a very simple, robust and practical control scheme. The feedback gain $K_{p}$ can be chosen large enough to justify the use of PID control [1, ch. 6 of 19] which is locally gtable. $p$ Typically, $k(q$, and $\frac{\partial k\left(q_{1}\right)}{J q_{1}}$ are composed of erigonometric functions, therefore, they are uniforaly bounded.

### 3.2 PD Control with Exponential Convergence Race

Uee of the Invariance Principle in Section 2.4 only shows aumptotic stability. Some guaranteed rate of convergence is highly desirable not just for performance reasons but for robustness analysis and adaptive control also. In Section 2.5. a Lyapunov function with cross cera has been used to show exponential stability. This augeats a similar modifiation here. The result is summarized below.

## Theorea 3-1

Given the control lav (2.31)

$$
\begin{equation*}
u=-K_{v} q_{2}-\frac{\partial u^{*}}{\partial q_{1}}+\frac{\partial u^{0}}{\partial q_{1}} \tag{2.31}
\end{equation*}
$$

Suppose $7 v \geqslant 0$ such that

$$
\begin{equation*}
\Delta q_{1}^{T} \frac{\partial U *}{\partial \Delta q_{1}}\left(\Delta q_{1}\right) \geqslant\left\|\Delta q_{1}\right\|^{2} \tag{3.11}
\end{equation*}
$$

and $U *\left(\Delta q_{1}\right)$ has a slobal minimu at $\Delta q_{1}$ - 0 . then the closed loop ( $\Delta q_{1}: q_{2}$ ) syatem is exponentially stable.

## Proof:

Modify the total energy Lyapunov function (2.29) to

$$
v=T+U W^{\prime}+c \Delta q_{l}^{T} p+\frac{1}{2} c \Delta q_{l}^{T} K_{v} \Delta q_{l}
$$

where $c$ is amall constant so chat $V$ is positive definite in $p$ and $q_{1}$. Without loss of generality, U* can be considered poaitive definite in $q_{1}$ (by adding an appropriate constant). Then frow (2.32),

$$
\begin{aligned}
\dot{v} & =q_{2}^{T}\left(11+\frac{\partial U}{\partial q_{1}}-\frac{\partial U^{0}}{\partial q_{1}}\right)+c q_{2}^{T} p+c \Delta q_{1}^{T}\left(-\frac{\partial T}{\partial q_{1}}-\frac{\partial U^{0}}{\partial q_{1}}+u\right)+c q_{2}^{T} K_{v} \Delta q_{1} \\
& =-q_{2}^{T} K_{v} q_{2}+c q_{2}^{T} M\left(q_{1}\right) q_{2}-c \Delta q_{1}^{T} \frac{\partial T}{\partial q_{1}}-c \Delta q_{1}^{T} \frac{\partial U}{\partial q_{1}}
\end{aligned}
$$

Let

$$
\begin{equation*}
\cup: \sup _{q_{1} \in R^{n}}\left\|M\left(q_{1}\right)\right\| \tag{3.12}
\end{equation*}
$$


From Lemma 2-1, $\lambda_{2} \in\left(0, a_{2}-{ }_{2} \lambda_{2}\left(\frac{V_{0}}{F_{1}}\right) \frac{1}{2}\right)$,

$$
\dot{v}=-x_{1}| | s_{1}\left\|^{2}-x_{2}\right\| \Delta q_{p} \|^{2}
$$

Note

$$
\begin{equation*}
\frac{\partial T}{q_{1}}=\frac{1}{2} \sum_{1=1}^{n} e_{1} q_{2}^{T} M_{1}\left(q_{1}\right) q_{2}=\frac{1}{2} M_{D}^{T}\left(q_{1}, q_{2}\right) q_{2} \tag{3.14}
\end{equation*}
$$

Then

$$
\dot{v} \leq-\left(\sigma_{m i n}\left(K_{v}\right)-c u\right)| | a_{2}\left\|^{2}-c v\right\| \Delta a_{1} \|^{2}-\frac{c}{2} \Delta q_{1}{ }^{T} M_{D}^{T}\left(q_{1}, a_{2}\right) a_{2}
$$

Define

$$
\begin{equation*}
n_{1}=\sup _{q_{1}} \sum_{i=1}^{n}\left\|M_{i}\left(q_{1}\right)\right\| \tag{3.15}
\end{equation*}
$$

Then

$$
\dot{v}=-x_{2}\left\|\Delta q_{q_{1}}\right\|^{2}-x_{2}\left\|\Delta q_{2}\right\|\left\|^{2}+y_{21}\right\| \Delta_{q_{1}}\| \| \Delta q_{2} \mid \|^{2}
$$

where

$$
\begin{aligned}
& 1_{1}=c \\
& 1_{2}=\min _{v}\left(K_{v}\right)-c_{4} \\
& v=\frac{1}{2} c \eta_{1}
\end{aligned}
$$

Choose

$$
\begin{equation*}
c \cdot J_{\min }\left(K_{v}\right)\left(u+\frac{1}{2} \eta_{1}\left(\frac{v}{k_{1}}\right)^{\frac{1}{2}}\right)^{-1} \tag{3.16}
\end{equation*}
$$

where $v_{0}=\left.v\right|_{t=0}$ and
for some $\lambda>0$. Hence, the closed loop aystem ia exponentially atable.
Given any $U *$ according to (3.11), $K$, 0 and initial condition, there alvays exiets chat satiafies (3.16). Even though $c$ is not needet in the implementation, its maximum allowable aize affects the convergence rate. The artificial pocantiala $U * \in 1 / 2 \Delta q_{1} T K_{p} \Delta q_{1}$, (3.1) and (3.5) all aatiafy the assumptions of Theore 3-1. Therefors, the corresponding closed loop aysteme are exponentially stable. For the potential given by (3. 8) and $K_{p}$ large enough $\left(X_{p}+\frac{\partial k}{\partial q_{1}}>0\right)$, we can add a constant to $U *$ so
thet $U *$ is positive definite $\ln q_{1}-q_{1}^{*}$ and (3.11) is satisfied for $\Delta q_{1}-q_{1}-q_{1}^{*}$, where $q_{1}$ solves (3.10). Then Theorem $3-1$ implies exponential convergence of $q_{1}$ to $q_{1}^{*}$ which is within $\sigma_{m i n}\left(k_{p}\right)$ sup $q_{1} k\left(q_{1}\right) \|$ from the true $q_{1 d}$.
4. Nev Results in Tracking Control

### 4.1 Exponentially Stable Algorithme

Frequently a robot is required to follow a prespecified pach for continuous action at the end effector (e.g., arc welding), tracking of a moving target (e.g. pick and place operation from conveyer belt) or other high level objectives (e.g., time optimality, colilian avoidance, arm singularity avoidance, etc.). This can be posed as the problem of cracking the desired positions and velocities ( $q_{1 d}, q_{2 d}$ ) by ( $q_{1}, q_{2}$ ). In this section, we will extend the basic ideas put forth in Section 3 to the tracking problem. The error equation is now in the form

$$
\begin{align*}
& \Delta \dot{q}_{1}=\Delta q_{2} \\
& M\left(q_{1}\right) \dot{\Delta q_{2}}-C\left(q_{1}, q_{2}\right) q_{2}-k\left(q_{1}\right)+u-M\left(q_{1}\right) \dot{q}_{2 d} \tag{4.1}
\end{align*}
$$

We wil: first stace several direct generalizations of Theorem 3-1. An energy type Lyapunov function similar to (2.33) used in Section 2.5 to motivate computed torque is used here.

## Theorem 4-1

Conaider (4.1) with the control law

$$
\begin{equation*}
u=-K \nu q_{2}+k\left(q_{1}\right)-\frac{\partial U \star}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}-D\left(q_{1}, q_{2}, q_{2 d}\right) \tag{4.2}
\end{equation*}
$$

where $D$ is given by any one of che following expressions

$$
\begin{align*}
& D\left(q_{1}, q_{2}, q_{2 d}\right)=\frac{1}{2}\left(J\left(q_{1}, q_{2}\right) q_{2 d}-m_{D}\left(q_{1}, q_{2 d}\right) q_{2}\right)  \tag{4.2a}\\
& D\left(q_{1}, q_{2}, q_{2 d}\right)=\frac{1}{2}\left(J\left(q_{1}, q_{2 d}\right) q_{2}-M_{D}\left(q_{1} \cdot q_{2}\right) q_{2 d}\right)  \tag{4.2b}\\
& D\left(q_{1}, q_{2}, q_{2 d}\right)=\frac{1}{2}\left(J\left(q_{1}, q_{2 d}\right) q_{2 d}-m_{D}\left(q_{1}, q_{2}\right) q_{2 d}\right)  \tag{4.2c}\\
& D\left(q_{1} \cdot q_{2} \cdot q_{2 d}\right)=\frac{1}{2}\left(J\left(q_{1}, q_{2}\right) q_{2}-M_{D}\left(q_{1}, q_{2 d}\right) q_{2}\right) \tag{4.2d}
\end{align*}
$$

Assume $\mathrm{I}_{v}$ > 0 such chat

$$
\begin{equation*}
\Delta q_{1}^{T} \frac{\partial U^{*}}{3 \Delta q_{1}}\left(\Delta q_{1}\right)>v\left\|q_{1}\right\|^{2} \tag{4.3}
\end{equation*}
$$

and $U *\left(S q_{1}\right)$ is positive definite in $: q_{1}$. Then the nuli state of the $\left(1 q_{1}, \Delta q_{2}\right)$ system is a globally exponencially stable equilibrium.

Proof: l'se the following Lyapunov function
where $c$ is a small constant, such that $V$ is positive definite in $\Delta q_{1}$ and $\Delta q_{2}$. Take derivative along solution:

$$
\begin{aligned}
\dot{v}\left(\Delta q_{1}, \Delta q_{2}\right) & =\Delta q_{2}^{T}\left(M\left(q_{1}\right) \Delta \dot{q}_{2}+\frac{1}{2} M\left(q_{1}, \Delta q_{2}\right) q_{2}+\frac{\partial U}{\frac{\partial}{\Delta q_{1}}}\left(\Delta q_{1}\right)+c M\left(q_{1}\right) \Delta q_{2}\right. \\
& \left.+c K_{v} \Delta q_{1}\right)+c \Delta q_{1}^{T}\left(M\left(q_{1}\right) \Delta \dot{q}_{2}+M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}\right)
\end{aligned}
$$

Substitute (4.1) and (4.2) and use (2.7)

$$
\begin{aligned}
\dot{v}\left(\Delta q_{1}, \Delta q_{2}\right) & =\Delta q_{2}{ }^{T}\left(-\alpha q_{1}, q_{2}\right) q_{2}-k\left(q_{1}\right)+u-M\left(q_{1}\right) \dot{q}_{2 d}+\frac{1}{2} M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2} \\
& \left.+\frac{\partial U}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)+c M\left(q_{1}\right) \Delta q_{2}+c K \Delta q_{1}\right) \\
& +c \Delta q_{1}{ }^{T}\left(-c\left(q_{1}, q_{2}\right) q_{2}-k\left(q_{1}\right)+u-M\left(q_{1}\right) \dot{q}_{2 d}+M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}\right) \\
& =r\left(q_{1}, q_{2}, q_{2 d}\right)-\Delta q_{2}{ }^{T}\left(X_{v}-c M\left(q_{1}\right)\right) \Delta q_{2}-\Delta q_{2}{ }^{T} D\left(q_{1}, q_{2}, q_{2 d}\right) \\
& +c \Delta q_{1}{ }^{T}\left(M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}-c\left(q_{1}, q_{2}\right) q_{2}\right) \\
& -c \Delta q_{1}{ }^{T} \frac{\partial U *}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)-c \Delta q_{i}{ }^{T} D\left(q_{1}, q_{2}, q_{2 d}\right)
\end{aligned}
$$

Apply Identity $5, r-\Delta q_{2}{ }^{T}=0$. Define $n_{1}, n_{2}$ as followa

$$
\begin{aligned}
& n_{1}=\sup _{q_{1} \in \mathbb{R}^{n}} \sum_{1-1}^{n}\left\|m_{1}\left(a_{1}\right)\right\| \\
& n_{2}=\sup _{q_{2 d}}\left\|a_{2 d}\right\| n_{1}
\end{aligned}
$$

From Identity 6 ,

$$
\begin{aligned}
& \mid c \Delta q_{1}{ }^{T}\left(M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}-c\left(q_{1}, q_{2}\right) q_{2}-D\left(q_{1}, q_{2}, q_{2 d}\right) \mid\right. \\
& \leq c\left\|\Delta q_{1}\right\|\left(a q_{2}\left\|\Delta q_{2}\right\|+\frac{n_{1}}{2}\left\|\Delta q_{2}\right\|^{2}\right)
\end{aligned}
$$

where $=\frac{-3}{2}$ for (4.2a), $\mathrm{a}-\frac{3}{2}$ for (4.2b), a $\frac{5}{2}$ for (4.2c), a- $\frac{1}{2}$ for (4.2d)
Hence.

$$
\begin{aligned}
\dot{v}\left(\Delta q_{1}, \Delta q_{2}\right) \leq & -\left(\sigma_{m i n}\left(K_{v}\right)-c p\right)\left\|\Delta q_{2}\right\|^{2}-c v\left\|\Delta q_{1}\right\|^{2} \\
& +c\left\|\Delta q_{1}\right\|\left(\Delta n_{2}\left\|\Delta q_{2}\right\|+\frac{n_{1}}{2}\left\|\Delta q_{2}\right\|^{2}\right)
\end{aligned}
$$

Completing square for the cross term,

$$
\begin{equation*}
\dot{v}\left(\Delta a_{1}, \Delta q_{2}\right) \leq-a_{1}\left\|\Delta q_{1}\right\|^{2}-a_{2}\left\|\Delta q_{2}\right\|^{2}+r_{21}\left\|\Delta q_{1}\right\|\left\|\Delta q_{2}\right\|^{2} \tag{4.5}
\end{equation*}
$$

where

$$
\begin{aligned}
& a_{1}=c\left(v-\frac{1}{2} a{n_{2}}^{2}\right) \\
& a_{2}=a_{\min }\left(K_{v}\right)-c\left(\mu+\frac{1}{2} \frac{a n_{2}}{\rho^{2}}\right) \\
& r_{21}=\frac{1}{2} c n_{1}
\end{aligned}
$$

Given $\nu$, choose $o^{2}<\frac{2 a_{1}}{a n_{2}}$. By Lemma 2-1, for

$$
c<o_{m i n}\left(x_{v}\right)\left(u+\frac{1}{2} \frac{a n_{2}}{\rho^{2}}+\frac{1}{2} n_{1}\left(\frac{v_{o}}{r_{1}}\right)^{\frac{1}{2}}\right)^{-1}
$$

$\left(v_{0}, \xi_{1}\right.$ are as defined from the proof of Theorem 3-1) and $v_{2} \varepsilon\left(0, \alpha_{1}-\gamma_{21}\left(\frac{v_{0}}{\varepsilon_{1}}\right)^{\frac{1}{2}}\right)$
$\dot{v} \leq-a_{1}| | \Delta q_{1}\left\|^{2}-\lambda_{2}| | \Delta q_{2}\right\|^{2}$
$\leq-\lambda v$
for some $\lambda>0$. Hence, the closed loop system is exponentially stable.
A common Lyapunov function used for cracking problem has been $[9,10]$
$v\left(\Delta q_{1}, \Delta q_{2}\right)=\frac{1}{2} \Delta q_{2}^{I} M\left(q_{1}\right) \Delta q_{2}+U *\left(\Delta q_{1}\right)$

In this case, a generalization of Invariance Principle to time-varying case is required. There are two possibilities. The result in (Theorem A.7.6, 21) appears proaising, but we must verify that (4.1) is positive precompact (in the sence defined in [21]). A more direct route is to use [Leman 1, 22] which staces that if $\Delta \dot{q}_{2}$ and $\Delta \dot{q}_{2}$ are both bounded uniforaly in $t$ (it follows frow $\dot{V} \leq 0$ ), then $\Delta q_{2}(t) \rightarrow 0$ implles $\Delta \dot{q}_{2}(t) \rightarrow 0$.

Note that $U^{*}\left(\Delta Q_{1}\right)$ does not depend on cine explicitly. This restriction eliminates some of the candidates used in set point case. How to generalize to the case of $U^{*}\left(\Delta q_{1}, t\right)$ and $\frac{\partial U^{*}}{\partial t}\left(\Delta q_{1}, t\right)$ not necesearily negative semidefinite is cursently under investigation

Concrol laws (4.2a-d) all have same stability property nominally. When 92 is a very noise measurement, as is typically the case, ( $4.2 c$ ) which only uses $q_{2}$ once may have better robuatnesa.

Note that all the controllers have etructures very similar to computer torque; in fact, if all occurrences of $\mathrm{q}_{2 \mathrm{~d}}$ are replaced by $\mathrm{q}_{2}$, then the nonlinear compensation is exactly the same as the case of computer torque. However, in their present forms, ( $4.2 a-d$ ) cannot take advantage of well known recursive algorithme for inverse dynamics computation (11, 12]. Therefore, we next present silghtly modified versions that can be impleaented with these algorithma.

## Corollary 4-1

Consider (4.1) with the control law
$u=-K_{v} \Delta q_{2}+k\left(q_{1}\right)=\frac{\partial U^{*}}{\partial \Delta q_{1}}\left(\Delta q_{2}\right)+M\left(q_{1}\right) \dot{q}_{2 d}+C\left(q_{1}, q_{2 d}\right) q_{2 d}$
where $U *\left(\Delta q_{1}\right)$ satisfies the same assumptions as in Theorem 4-1.
If
$\sigma_{m i n}\left(K_{v}\right)>\frac{n_{2}}{2}$
then the null state of the $\left(\Delta q_{1}, \Delta q_{2}\right)$ system is a globally exponentially stable equilibrium.

## Corollary 4-2

Consider (4.1) with the control law
$u=-K_{v} \Delta q_{2}+k\left(q_{1}\right)-\frac{\partial U^{*}}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}+C\left(q_{1}, q_{2}\right) q_{2}$
Where $U\left(\Delta q_{1}\right)$ satisfies the assumptions as in Theorem $4-1$. Given a set of possible initial conditions, if $K_{v}$ is sufficiently large, then the closed loop system is exponentially stable with respect to that set.

If $U *\left(\Delta q_{1}\right)=\frac{1}{2} \Delta q_{1}^{T} K_{p} \Delta q_{1},(4.7)$ is actually a modification of the computed corque mechod with $K_{p}$. $K_{v}$ replacing $M\left(q_{1}\right) K_{p}, M\left(q_{1}\right) K_{v}$.

So far we have generated many control laws that are similar to computed torque. However, the ones fust requiring $K$ > $0, v>0$ are not easily implementable, and the easily implementable ones need stronger conditions ( $K_{v}$ sufficiencly large). The next concrol law that we shall present is of very appealing structure: the real time undate computations are linear and the off-ifine computation can take advantage of efficient algorithms (e.g., Newton-Euler type). The trade-off is that $X_{v}$ and $v$ must both be large enough for a given set of initial conditions.

## Theorem 4-2

Consider (4.1) with the control law
$u=-K_{v} \Delta q_{1}+k\left(q_{1 d}\right)-\frac{\partial U^{*}}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)+M\left(q_{1 d}\right) \dot{q}_{2 d}+C\left(q_{1 d}, q_{2 d}\right) q_{2 d}$
where $U *\left(\Delta q_{1}\right)$ satisfles the assumptions as in Theorem 4.1. Given a set of possible initial conditions, if $K$ and $u$, are sufficiencly large, then the closed loop system is exponentially stable with respect to that set.

Tvpically, $q_{2}(0)=q_{2 d}(0)=0$ and $\Delta q_{1}(0)$ is always within $2 \pi$. Hence $v_{0}$ is bounded above, and the result is essentially a global one. This scheme requires both $v$ and $K_{v}$ large enough. This requirement is made easier by shifting the computational burden to offline thus allowing very high sampling rates which in tirn means high gains can be tolerated.

## 4. 2 Robustness

Lyapunov analysis provides a useful approach to study che robustness issue. Robustness is a much abused word in the literarure. Here, we use insensitive design to mean preservation of stability (in the sense of Lagrange) under sufficiently small perturbations. Furthermore, the size of the ultimate bound should vary continuously with the size of perturbations. By robust design we mean a controller design that preserves stability under prescribed size of perturbations. In this section, we will examine frictions, both viscous and Coulomb type, bounded modeling error and bounded actuator and sensor noises.

Friction forces can be approximately modeled by Coulomb friction, or dry friction, and viscous friction due to oil lubrication. Por joinc 1 , the frictional force is given by

$$
\begin{equation*}
f_{f r 1 c}^{(1)}=F_{11} \operatorname{sen}\left(q_{21}\right)-F_{21} q_{21} . \quad F_{11}, F_{21}>0 \tag{4.9}
\end{equation*}
$$

Equation (4.1) is then

$$
M\left(q_{1}\right) \Delta \dot{q}_{2}=-c\left(q_{1}, q_{2}\right) q_{2}-k\left(q_{1}\right)+u-M\left(q_{1}\right) \dot{q}_{2 d}-F_{1} \operatorname{sgn}\left(q_{2}\right)-F_{2} q_{2}
$$

where $F_{1}$ and $F_{2}$ are diagonal matrices wieh elemente $F_{1 i}, F_{21}$, raspectively, sgn( $q_{2}$ ) represents a vector with elements $\operatorname{sig}\left(q_{21}\right)$.
From [23], the set point controller is both insensitive and robust. The tracking controllers are also insensitive with respect to frictions. For robutt design for a given level of friction, $\sigma_{m i n}\left(K_{v}\right)$ and $v$ should be chosen large enough.

Next we consider modeling error in controller implementation. Model parameters $k, M, M_{D}$ can all be written innearly in constant parameters. Assume bounded errors are incurred in these parameters. Control laws (4.2) are in the form

$$
u=-k_{v} \Delta q_{2}-\frac{\partial U}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)+k\left(q_{1}\right)+N\left(q_{1}\right) \dot{q}_{2 d}-D\left(q_{1}, q_{2}, q_{2 d}\right)+\Delta_{1}+\Delta_{2}
$$

The additional terms in $\dot{v}$ are
$-\left(C \Delta q_{1}+\Delta q_{2}\right)^{T}\left(\Delta_{1}+\Delta_{2}\right)$
which, after overbounding [23], becomes
$\left(c\left|\mid \Delta \Delta_{1}\|+\| \Delta q_{2} \|\right)\left(\delta_{1}+\delta_{2}+\delta_{3}\left\|\Delta_{q_{2}}\right\|+\delta_{4}\left\|\Delta_{q_{2}}\right\|^{2}\right)\right.$
After completing squares, the overbound over the extra terms in $\dot{v}$ become

$$
\begin{aligned}
& \frac{c}{2}\left(\delta_{3} 3_{3}^{2}+\frac{\left(\delta_{1}+\delta_{2}\right)}{s_{1}^{2}}\right)\left\|\Delta q_{1}\right\|^{2}+\left(\delta_{3}+\frac{c s_{3}}{2 s_{3}^{2}}+\frac{\delta_{1}+\delta_{2}}{2 s_{2}^{2}}\right)\left\|\Delta q_{2}\right\|^{2} \\
& +\delta_{4}\left\|\Delta q_{1}\right\|\left\|\Delta q_{2}\right\|^{2}+\delta_{4}\left\|\Delta q_{2}\right\|^{3}+\frac{1}{2}\left(\delta_{1}+\delta_{2}\right)\left(c s_{1}{ }^{2}+s_{2}{ }^{2}\right)
\end{aligned}
$$

For $5_{1}$. $\xi_{5}$ sufficiently small, $3 a_{1}, a_{2}, n>0$ such that

$$
\dot{v} \leq-a_{1}\left\|\Delta_{q_{1}}\right\|^{2}-a_{2}\left\|\Delta_{q_{2}}\right\|^{2}+c\left(\frac{n_{1}}{2}+s_{4}\right)\left\|\Delta_{q_{1}}\right\|\left\|\Delta_{q_{2}}\right\|^{2}+\delta_{4}\left\|\Delta_{a_{2}}\right\|^{3}+0
$$

By Lemma 2-2, if $v_{0}>0$ and $\delta_{1}, \delta_{5}$ are small, the system remains Lagrange stable and the ultimate bound vanishes as $\delta_{1}, \delta_{5}{ }^{0}+0$. Hence, the design is insensitive with respect to modeling errors. For robust design for a given level of modeling uncertaincies, $G_{m i n}\left(K_{v}\right)$ and $v$ should be large enough.

Finally, suppose bounded errors $\varepsilon_{1}, \varepsilon_{2}$ and $\varepsilon_{3}$ are incurred in $q_{1}, q_{2}$ and $u$, :espectively. Control laws (4.2) are now in the form

$$
u=-k, \Delta q_{2}-\frac{\partial U^{*}}{\partial \Delta q_{1}}\left(\Delta_{q_{1}}\right)+k\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}-D\left(q_{1} \cdot q_{2} \cdot q_{2 d}\right)+\Delta_{1}+\Delta_{2}
$$

Follow the same steps as before, we overbound the extra terms in $\dot{v}$ by sums of $\left\|\Delta q_{1}\right\|^{2},\left\|\Delta q_{2}\right\|^{2},\left\|\Delta q_{1}\right\|\left\|\Delta q_{2}\right\| \|^{2}$ and constant terms. Again use Lemma 2-2 to conclude that the controller is insensitive to bounded instrument noises and robust for a given level of noise if $\sigma_{m i n}\left(K_{v}\right)$ and $v$ are sufficiently large.

As an aside. it should be noted that similar results as derived here hold for computed torque techniques also. For the case of set point control under friction, the insensitivity and robustiness properties of the design here do not follow directly from the analysis.

There are obviously many more practical implementation issues not addressed here: sampling effect, actuator saturacion, joint and arm flexiblitites and instrument dynamics. They are currently under tivestigacton in the same framework.

## 5. Summary

We have introduced a new class of exponentially stabilizing control laws for the joint level control of robot manipulators (sumarized in Table $\bar{f}$ ). The stability result is achieved by making use of a particular class of energy-like Lyapunov functions (of the form (4.4)) In conjunction with a useful lemma (iemma 2-1) for addressinp, higher order rerms in Lyapunov function derivatives. This approach avoids the need for a generalization of the invariance principle ro time-varying systems, which has been the major source of difficulty in the past $\{9,10]$.

In the set point control case, by incorporating artificial potentisl flelds in the lyapunov function, we have derived a class of exponentially scabilfzing, PD + potential shaping type of control laws. Several useful potential flelds have been examined resulting In simple structures: $P D$ and $P D+b i a s$, and the ability to handle joint stop constraints with $P D+j o t n t-s t o p-b a r i f e r ~ c o n t r o l l e r . ~$

In the tracking control case, the modified Lyapunov function leads to new clase of exponentially stable control laws. This class of control laws offers an alternative to the convertional conputed torque method and provides trade-offs between on-line computation (which directiy relates to performance chrough maximua sampling rate) and condition for stabllity. In one new design, (4.8), the nonlinear structure is decoupled from the real-time measureaents which completely removes the requirement for on-line nonlinear computation. The chart below illustrates the trade-offs in the various tracking control laws.


The framework of Lyapunov stability analysis also allows robustness issues to be directly addressed. Specifically, insensitivity property (preservation of stability under small perturbation) and conditica for robuse design (preservacion of stability under a specified amount of perturbation) fcr chis new class of controllers have been derived with respect to viscous and Coulomb friction, modeling error and bounded instrumenc noise.

The new stability anslysis and controller design teqhniques presented in this paper open up many =romising avenues for future research. In particular, our current research directions include: ways to incorposte timevarying artificial potencial ftelds in the tracking problem and the generallzation of the exponentialis stabilizing joint-level control laws te the task space.

## Acknowledgemenc

The au:hors would like to thank Dr. K. Kreutz for many helpful discussions. This research has been performed ar the Jet rropulston Laboratory, Calfornia Institute of Technology under contract with the National ieronautics and Space Administration.
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| Equation | Nuaber Control laus |
| :---: | :---: |
| (2.31) |  |
| (2.28) | $\cdots \cdots k_{v q_{2}}-k_{p} a_{a_{1}}+k\left(a_{1}\right)$ |
| (3.2) | $u=-k_{v} q_{2}-k_{p} \Delta_{q_{1}}+k\left(q_{1 d}\right)$ |
| (3.6) | $\cdots-k A_{2}-k_{p} \Delta_{a_{1}}-\dot{H}\left(s_{1}\right)+i\left(a_{1}\right)+k\left(a_{1}\right)$ |
| (3.7) | $w=-K_{v} q_{2}-K_{p} \Delta_{q_{2}}-\dot{H}\left(s_{q_{1}}\right)-\dot{L}\left(c_{s_{1}}\right)+k\left(q_{1 d}\right)$ |
| (3.9) | $u=-K_{v} a_{2}-K_{p} b_{4}$ |
| (2.34) | $u=M\left(a_{1}\right)\left(-K_{p}+a_{1}-K_{v} \dot{-a}_{2}-\dot{q}_{2 a}\right)+M\left(a_{1}\right)+C\left(a_{1} \cdot q_{2}\right) a_{2}$ |
| (4.24) |  |
| (4.2b) | $u=-x_{v} \leq a_{2}-\frac{\partial v}{\partial a_{1}}\left(a_{1}\right)+M\left(a_{1}\right) \dot{q}_{2 d}-\frac{1}{2}\left(נ\left(q_{1} \cdot a_{2 d}\right) q_{2}-M_{D}\left(a_{1} \cdot q_{2}\right) q_{2 d}\right)$ |
| (4.2-) | $\left.u=-k_{v i a_{2}}-\frac{\partial u}{3 x_{1}}\left(i_{1}\right)-M\left(a_{1}\right) \dot{a}_{2 d}-\frac{1}{2}\left(1 a_{1}, a_{2 d}\right) a_{2 d}-M_{D}\left(a_{1} \cdot a_{2}\right) a_{2 d}\right)$ |
| (4.2d) | $u=-K_{v} q_{2}-\frac{\partial u *}{\partial a_{a_{1}}}\left(a_{q_{1}}\right)+M\left(a_{1}\right) \dot{q}_{2 d}-\frac{1}{2}\left(J\left(a_{1} \cdot q_{2}\right) a_{2}-M_{0}\left(a_{1}, a_{2 d}\right) a_{2}\right)$ |
| (6.6) | $u=-x_{v} \Delta q_{2}-\frac{i U^{*}}{\partial \Delta q_{1}}\left(\Delta q_{1}\right)+k\left(q_{1}\right)+M\left(q_{i}\right) q_{2 d}+C\left(q_{1} \cdot q_{2 d}\right) q_{2 d}$ |
| (4.7) | $u=x_{v s a_{2}}-\frac{\mu *}{\partial \Delta a_{1}}\left(\Delta a_{1}\right)+M\left(q_{1}\right)+M\left(q_{1}\right) q_{2 d}+C\left(q_{1} \cdot q_{2}\right) q_{2}$ |
| (6.8) |  |

# Simple Robust Control Laws for Robot Manipulators, Part II: Adaptive Case 

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## 1. Abstract

A new class of asymptotically stable adaptive control laws is introduced for application to the robotic manipulator. Unlike most applications of adaptive control theory to robotic manipulators, this analysis addresses the nonlinear dynamics directly without approximation, linearization, or ad-hoc assumptions, and utilizes a parametrization based on physical (timeinvariant) quantities. This approach is made possible by using energy-like lyapunov functions which retain the nonlinear character and structure of the dynamics, rather than simple quadratic forms which are ubiquitous to the adaptive control literature, and which have bound the theory tightly to linear systems with unknown parameters. It is a unique feature of these results that the adaptive forms arise by straightforward certainty equivalence adaptation of their nonadaptive counterparts found in the companion to this paper (ie.. by replacing unknown quantities by their estimates) and that this simple approach leads to asymptotically stable closed-loop adaptive systems. Furthermore, it is emphasized that this approach does not require convergence of the parameter estimates (i.e., via persistent excitation), invertibility of the mass matrix estimate, or measurement of the joint accelerations.

## 1. Introduction

In past years, many papers have appeared on the application of adaptive control theory to robotic manipulators (cf., \{2]-\{7], and Hestia [8] for overview). It is a general property of adaptive designs based on Lyapunov's Direct Method, that the lyapunov/function is chosen as a simple quadratic type, wellknown and well studied in the standard adaptive control literature [12](13). However, this particular Lyapunov function was originally motivated for applications to the standard adaptive control problems (1.e., linear systems with unknown parameters), gid not for nonlinear dynamical systems. Hence, applications of standard adaptive control techniques to robotic manipulators invariably require rae dynamics to be considered as linear. This in turn, requires the use of ad-hoc assumptions and/or analysis techniques including 1) treatment of position dependent quantities as unknown constants, for which they gust be assumed to vary slowly with time; 2) inearization of the system about some local operating point-valid only for small excursions from nominal; 3) the use of linear decoupled models for the links, which neglects nonlinearities and crosscoupling effects; and 4) neglecting the nonlinear and ime-varying dynamics completely by assuming the plant is linear. Hence, stability results based on these assumptions are questionable, and a rigorous proof of stability for adaptive control of robotic manipulators remains unresolved.

A recent exception to the above criticism is due to the work of Craig. Han and Sastry [9]. Here, a useful "linear in the parameters" formulation is exploited to simplify the analysis, and to demonstrate global convergence of an adaptive version of the computed-torque control law - without approximation to the nonlinear dynamics. However, the resulting adaptive controller requires the invertibility of che mass matrix estimate (which is not guaranteed a-priori), and measurement of the joint accelerations (which is generally unavailable). It is suggested in [9], that the former can be handled by projecting parameter estimates into known regions of parameter space for which the mass matrix inverse exists, and in which the true parameters are required to lie. However, knowledge and calculation of such regions is not straightforward and appears to be a weakness of the method.

In this paper, the "I near in parameters" formulation of [9 ]is used in conjunction with a different Lvapunov function. Here, the choice of Lyapunov function is more closely related to the energy of the system, and better retains the nonlinear structure d nd character of the dynamics. In addition, many problems associates with adapting the computed-torque control law directly are avoided by making use of the new class of exponential iv stabilizing controllers introduced in [1]. Although these controllers are very similar in form to the computed torque method, they have many advantages in the nonadaptive case (cf., [1]), and have the unique property that they can be made adaptive by using a straightforward certainty equivalence approach (ie., by replacing unknown quantities by their on-line estimates). Furthermore, the class of adaptive systems defined in this manner can be shown to be asymptotically stable without approximation to the nonlinear: manipulator dynamics. This approach does not require convergence of parameter estimates (i.e., via persistent excitation), invertibility of the mass matrix estimate, or measurement of joint accelerations.

In the most recent literature (1.e., preprints, conference papers, etc.) there appears to be other work currently taking place which combines the linear in parameters formulation with a new Lyapunov function [10], [11]. Although this work is very new and is evolving very rapidly, we will try to contrast our results where possible, and provide an overall perspective.

The format of the peper is at follow. In sec. 2 the resulta of [I] are reviewed and sumentized an required for treatment of the adaptive control caes. In Sec. 3, agyptotic etability ia proved for che clise of sygten arising fros certainty equivalence adaptation of the control law in [i]. gilehtiy tangential to the min thruat of the paper is the analysis in Sec. 4 of the edaptive computed torque mathod. Since the computed-torque control lav is videly established in the literatura, and widely applied in practien, it is useful to apply the techniques developad herein to aes to what extent it can be ande adaptive and to whe extent stability can be guaranteed. In Sec. 5 . several remarks are made pertinent to the new adeptive desige, and conclusiona are given in sec. 6.

## 2. Background and Rotation

### 2.1 Manipulator Dynamice

The well-known Lagrange-Euler equations of motion for the n-joint annipulazor ia given as follow,

$$
\begin{align*}
& \dot{q}_{1}=q_{2}  \tag{2.1}\\
& M\left(q_{1}\right) \dot{q}_{2}=-C\left(q_{1}, q_{2}\right)-k\left(q_{1}\right)+u \tag{2.2}
\end{align*}
$$

where
$C\left(q_{1}, q_{2}\right)=\sum_{i=1}^{n}\left[\left(e_{i} q_{2}{ }^{T} M_{i}\left(q_{1}\right)\right)^{T}-\frac{1}{2}\left(e_{1} q_{2}{ }^{T} M_{1}\left(q_{i}\right)\right)\right]$
$e_{i} \Delta i^{\text {th }}$ unit vector
$M_{1}\left(q_{1}\right)=\frac{\partial M\left(q_{1}\right)}{\partial q_{11}} ; q_{11} \Delta_{1}^{\text {th }}$ component of $q_{1}$
$k\left(q_{1}\right) \Delta$ gravity load
Here, uef ${ }^{n}$ is a generalized torque vector, $q_{1}, q_{2}, \dot{q}_{2} \subset R^{n}$ are genersilized joint poaition, velocity ard acseleration vector, (e.g., $q_{1}$ is an angle or distance for a revolute or priamatic joint, reapectively, $M\left(q_{1}\right) \subset R^{n \times n}$ is the symetric positive definite mase inertia matrix; $C\left(q_{1}, q_{2}\right) \in R^{n}$ is the Coriolis and centrifugal force vector; and $k\left(q_{1}\right) \in R^{n}$ is the gravitational load vector.

### 2.2 Some Useful Identities

Let the following notations be detined,
$M_{D}\left(q_{1}, z\right)=\sum_{i=1}^{n} M_{i}\left(q_{1}\right)=e_{n}^{T}$
$\dot{M}\left(q_{1}, q_{2}\right)=\frac{d}{d t} M\left(q_{1}\right)=\sum_{i=1}^{n} M_{1}\left(q_{1}\right) \quad{ }_{1}{ }^{T} q_{2}$
$J\left(q_{1}, z\right)=\sum_{i=1}^{n}\left[\left(e_{1} 2^{T} M_{1}\left(q_{1}\right)-\left(e_{1} 2^{T} M_{1}\left(q_{1}\right)\right)^{T}\right]\right.$
$\Delta q_{1} \Delta q_{1}-q_{1 d}, \Delta q_{2} \Delta q_{2}-q_{2 d}$
$q_{1 d}, q_{2 d} \triangleq$ desired foint position and velocities reapectively $\left(q_{2 d}-\dot{q}_{1 d}\right)$

$$
\tau\left(q_{1}, q_{2}, q_{2 d}\right)=\Delta q_{2}^{\tau}\left[\frac{1}{2} M\left(q_{1}, q_{2}\right) \Delta q_{2}-c\left(q_{1}, q_{2}\right) q_{2}\right\}
$$

Uaing the above notation, the following identities are quoted from [1] without proof. In these icenticies, $x, y$ and $z$ are used to denote arbitrary vector: of appropriate dimension,

## Identity 1

$$
\dot{M}\left(q_{1}, q_{2}\right) z-M_{D}\left(q_{1}, z\right) q_{2} \quad \text { where vector } z \text { is arbitrary }
$$

Identity 2

$$
C\left(q_{1}, z\right) z=\frac{1}{2}\left(M_{D}\left(q_{1}, z\right)-J\left(q_{1}, z\right)\right) z
$$

Identity 3

$$
J\left(q_{1}, z\right)=M_{D}^{T}\left(q_{1}, z\right)-M_{D}\left(q_{1}, z\right)
$$

### 2.3 Important Leman

In this section, useful lema is raviewed, quated directiy without proof fron [1]. For convenience, thie reault will be alternatively reforred to as the b-ball Leman due to the method used to prove it.

Lesian 2-1 (B-Ball Lema)
Given a dynanical syates

$$
\dot{x}_{1}=f_{1}\left(x_{1}, \ldots x_{1}, t\right), x_{1} \in R^{n_{1}}, t \geq 0
$$

Let $f_{f}$ 's be locally Lipechitz with reapect to $x_{2} \ldots, x_{k}$ uniforaly in $t$ on bounded intervals and continuous

$V\left(x_{1}, \ldots, x_{1}, t\right)=\sum_{1, j=1}^{N} x_{1}^{T} P_{1 j}\left(x_{1}, \ldots, x_{j}, t\right) x_{j}$,
$V$ is positive definite in $x_{1}, \ldots, x_{N}$
$\forall\left(x_{1}, \ldots, x_{N}, t\right) \leq \sum_{i \in I_{1}}\left(\alpha_{1}-\sum_{j \in I_{21}} \gamma_{1 j}\left\|x_{j}(t)\right\|^{k_{1 j}}\right)\left\|x_{1}(t)\right\|^{2}$
where $a_{i}, Y_{1 j}, k_{1 j}>0, I_{2_{1}}<I_{1} \subset(1, \ldots, N)$
Let $\xi_{1}>0$ be such that,

$$
\begin{equation*}
\xi_{i}\left\|x_{i}\right\|^{2} \leq v\left(x_{1}, \cdots, x_{N}, t\right) \tag{2.5}
\end{equation*}
$$

Let $v_{0} \triangleq v\left(x,(0), \ldots, x_{N}(0), 0\right)$
If VicI $\quad$, $a_{1}>\sum_{j \in L_{21}} Y_{1 j}\left(\frac{v_{0}}{\varepsilon_{1}}\right)^{\frac{k_{11}}{2}}$
then $v \lambda_{1} c\left(0, a_{1}-\sum_{j \in I_{2 i}} \gamma_{1 j}\left(\frac{v_{0}}{\varepsilon_{j}}\right)^{\frac{k_{11}}{2}}\right)$,
$\dot{v}\left(x_{1}, \ldots, x_{N}, t\right) \leq-\sum_{i \varepsilon I_{1}} \lambda_{1}\left\|x_{i}\right\|^{2} \quad v t \geq 0$

### 2.4 Exponentially Stabilizing Control Laws

In [1], various new exponentially stabilizing compengators were introduced for both the set-poine and tracking concrol problems. For the purposes of adaptive control, it is of interest to consider the subset of chis class summarized in Table I. In addition, the well-known computed torque control has also been included in Table I for comparison purposes. It ia noted that the deaired potential field $U *\left(\Delta q_{1}\right)$ used in [1] has been chosen here simply as,

$$
\begin{equation*}
U *\left(\Delta q_{1}\right)=\frac{1}{2} \Delta q_{1}^{T} K_{p} \dot{q_{1}} \tag{2.7}
\end{equation*}
$$

so as not to obscure the presentation with additional obstacle avoidance objectives. Nevertheless, many of the adaptive control results presented herein are easily extended to the more general case.

It is useful to observe that all Control have 1-7 diffar from the computed torque method in that the mass matrix $M\left(q_{1}\right)$ does not premultiply the position and velocity feedback gaing $K_{p}$ and $K_{y}$ reapectively. This property is critical since it renders this entire class of control laws amenable to simple sdaptation schemes (i.e., certainty equivalence adaptation) which can be shown to lead to desired asmptotic stability propertiea. The presence of the mass matrix premultiplier othervise prevents simple cancellations in the Lyapunov function derivative, hindering most attempts to apply adaptive control directly to the nonlinear dynamic aanipulator equations. A recent exception to this can be found in the work of Craig. Hsu and Sastry [9].
However, the resulting adaptation law requires that the estimated mass matrix be invertible for all values of estimated parameters. This in turn requires on-line projections of parameter estimates inco prespecified bounded regions of parameter apace where $\hat{M}\left(q_{1}\right)$ is not only invertible, but where the true parameters are certain to lie. This approach not only requires tighe bounde on parameter uncertainty, but involves a very difficult (al beit off-line) determination of the proper parameter prajection domains. This problem is further exacerbated by the fact that the adaptation law is not parameterized by physical parameters and is of the form where the tranaformation back to physical parameters is neither straightforward or unique. These problems are overcome in this paper by using the exponentially stabilizing controi laws of Table $I$, which do not involve premultiplying mass matrix on the feedback gaina.

CONDITIONS FOR STABILITY ${ }^{\boldsymbol{+}}$
$u=-n\left(q_{1}\right)\left(K_{p} \Delta q_{1}+K_{v} \Delta q_{2}\right)+K\left(q_{1}\right)+B\left(q_{1}\right) \dot{q}_{2 d}+C\left(q_{2}, q_{2}\right) q_{2}$
cross-refrexice to [1] AND (20)
new exponentially stable control laws

$$
\begin{align*}
& \text { 1. } u-K_{p} \Delta q_{1}-K_{v} \Delta q_{2}+k\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}-\frac{1}{2} J\left(q_{1}, q_{2}\right) q_{2 d}+\frac{1}{2} n_{D}\left(q_{1}, q_{2 d}\right) q_{2} \quad \text { None required }  \tag{4.2a}\\
& \text { 2. } u=-x_{p} \Delta_{1}-K_{v} \Delta q_{2}+K\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}-\frac{1}{2} J\left(q_{1} \cdot q_{2 d}\right) q_{2}+\frac{1}{2} K_{0}\left(q_{1} \cdot q_{2}\right) q_{2 d} \quad \text { None Required }  \tag{4.2b}\\
& \text { 3. } u-K_{p} \Delta q_{1}-K_{v} \Delta q_{2}+k\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}-\frac{1}{2} J\left(q_{1} \cdot q_{2 d}\right) q_{2 d}+\frac{1}{2} n_{0}\left(q_{1} \cdot q_{2}\right) q_{2 d} \quad \text { None Required }  \tag{6.2s}\\
& \text { 4. } u=-x_{p} \Delta q_{1}-k_{v} \Delta q_{2}+k\left(q_{1}\right)+m\left(q_{1}\right) \dot{q}_{2 d}-\frac{1}{2} J\left(q_{1}, q_{2}\right) q_{2}+\frac{1}{2} n_{0}\left(q_{1}, q_{2 d}\right) q_{2} \quad \text { None Required }  \tag{4.2d}\\
& \text { s. } u=-k_{p} \Delta q_{1}-\sum_{1} \Delta q_{2}+k\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}+c\left(q_{1} \cdot q_{2 d}\right) q_{2 d}  \tag{4.6}\\
& 0_{\min }\left(x_{v}\right)>\frac{n_{2}}{2} \\
& \text { 6. } u-K_{p} \Delta q_{1}-K_{v} s q_{2}+k\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}+C\left(q_{1}, q_{2}\right) q_{2}  \tag{4.7}\\
& \sigma_{\text {ain }}\left(X_{v}\right) \text { sufficiantly large } \\
& \text { 7. } u-K_{p} \Delta q_{1}-K_{v} s q_{2}+k\left(q_{1 d}\right)+M\left(q_{1 d}\right) \dot{q}_{2 d}+C\left(q_{1 d} \cdot a_{2 d}\right) q_{2 d}  \tag{4.8}\\
& \text { W.r.t. Initial condition }
\end{align*}
$$

$$
\begin{aligned}
& \text { *Let } U *\left(\Delta q_{1}\right) \Delta \frac{1}{2} \Delta q_{1}{ }^{T} K_{p} \Delta q_{1} \text { in ( } 1 \text { ) }
\end{aligned}
$$

In the nonadaptive case, comparisons between the new control lawa of Table I and the coaputed torque method can be found in [I]. Nevertheleas, a brief account is in order here. In particular, Control lave 1 , 2,3,4 are roughly "on par" with the computed torque method in the nonadaptive case, guaranteeing exponeatial stability with no conditions on $K_{p}$ or $X_{v}$. Unlike the computed corque method, however, they are not in a form suitable for application of the recuraive Newton-Euler compucation cechnique. Thie presently appears co be their major disadvantage. In order to overcome this difficulty, Control Lave 5, 6 and 7 were developed in a form suitable for recursive Newton-Euler computation. Relative to the conputed torque aethod, Control Law 5 utilizes the desired velocity signal $q_{2 d}$ in place of the measured velocity $q_{2}$ in the nonlinear ceras of the controller. This "cleans up" the feedback signal in the sense that nonidealities due to sensor dynalics and measurement noise in $q_{2}$ are avoided in the nonlinear feedback teras. Control Law further replaces 71 in $K, M$ and $C$ by $q_{1 d}$. This decouples the nonlinear terma from real-time measurements, which completely removes the requirement for on-line computation of noulinear ceras in the controller implementacion. Control Law 6 is exactly the computed torque method without the premultiplying mase matrix terw described earlier. The advantages of these controllers are off-set silightly by the conditions imposed on $\mathrm{K}_{\mathrm{p}}$ and $\mathrm{X}_{\mathrm{v}}$ for guaranteeias asymptotic stability i.e., that $K_{v}$ be chosen sufficiencly large for Control Lava 1, 2, 3, 4, 5,6 and that both $K_{v}$ and $K_{p}$ be chosen sufficiently large for Control Lav 7 . It will be seen in the adaptive case that theae requirements can be removed by adapting these feedback gains appropriately.

The use of $q_{2 d}$ rather than $q_{2}$ in many of the new control laws offera additional advantages. In particular, in the set-point control application $\mathrm{q}_{2 \mathrm{~d}}{ }^{-\dot{q}} \mathrm{~g}_{2} \mathrm{~d}^{-0}$. Hence, there is conadderable aimplification in the control laws relative to the computed torque method, i.e., the nonlinear terme vanish from the control law. This simplification carries over directly to the adaptive case and provides substantlal simplification in set-point control relative to the recent adaptive control laws of slotine and Li [11] and Paden [10].

## 3. A New Class of Asymptotically Stable Adaptive Control Laws

All of the new exponentially stabilizing control laws sumarized in Table 1 have the unique property that can be adapted in real-time so as to yield asympotically atable adaptive control systems. Furthermore, the adaptation is done in a certainty equivalence fashion, i.e., by simply replacing unknown quancities in the control laws by their estimates - as generated by an appropriate parameter adaptation algorithm. In this section, asymptotic atability for the various control lave will be proved, and the proper mechanisms for parameter adaptation will be derived.

The eimplicity in itructure of the edapeive control scheme prasented bere in largely due to a "Incar in the parametera" formulation of the problem. This particular parameterisation is beconing inereasingly popular in recent licarature (cf.. (9)\{10](18)(19D and will be diecuased la more detall belov.

### 3.1 Linear in the Parmaters Formulation

A useful paremeterization of the nonlinear dyamical equations arisen by noting the following relacions ( $x, y$ and $z$ arbitrary vectors).

$$
\begin{aligned}
& C(x, y,) y=G_{C}(x, y) \theta_{C} \\
& M(x)=-G_{M}(x, z) \theta_{M} \\
& K(x)=G_{X}(x) \theta_{k} \\
& K_{D}(x, y) y=H_{D}(x, y) \theta_{D}
\end{aligned}
$$

where $H_{C}, H_{H}, H_{k}$ and $H_{D}$ are known matix valued functions of $x, y$ and $m$, and where $\theta^{0}$, $\theta_{M}, \theta_{k}$, and $\theta_{D}$ are vectors of conatant pareaters related directly to true physical parameters (masen, inertias, link lengchs, center of gravities, etc.). It is emphaized that this parameterisation does not contain any hidden "slowly varying" gtatas in the parametar vector definition asd does not require any linearization of the dyneaical equations of motion.

### 3.2 Clobal Asymptotic Stability for Adaptation of Control Lawe 1. 2, 3. 4

In this section, slobal amptotic atability is proved for adaptation of Control Lave 1, 2, 3 and 4. In order to avoid redundant analyala, the detalla of the proof will be conaldered only for control Lav 1 . and the extension to the other coatrol laws vill follow fmediately by taking advantage of the unifiad treatment of these control lawe given in [1:.

### 3.2.1 Asymptotic Stability

Conalder Control Lay 1 ,

$$
\begin{equation*}
u^{\circ}=-K_{P} \Delta q_{1}-K_{V} \Delta q_{2}+k\left(q_{1}\right)+H\left(q_{1}\right) \dot{q}_{2 d}-\frac{1}{2} J\left(q_{1} \cdot q_{2}\right) q_{2 d}+\frac{1}{2} R_{D}\left(q_{1}, q_{2 d}\right) q_{2} \tag{3.1}
\end{equation*}
$$

Here, superscript "o" is uned to denote the idenl nonadaptive control law. 1.e., the completely "tuned" control law wich would be used if the parameters were know axactly. Using the ifnear in the parametere formulation discussed in Sec. 3.1 there existe ematix $H_{1}(91,92,92 d$, $92 d$ and a veccor of parametere o such that.

$$
\begin{equation*}
u^{\circ}=-K_{p} \Delta Q_{1}-K_{v} \Delta q_{2}+H_{1}^{\theta} \tag{3.2}
\end{equation*}
$$

where

$$
\begin{equation*}
H_{1}^{g} \stackrel{\vdots}{=}\left(q_{1}\right) \dot{q}_{2 d}-\frac{1}{2} J\left(q_{1} \cdot q_{2}\right) q_{2 d}+\frac{1}{2} M_{D}\left(q_{1} \cdot q_{2 d}\right) q_{2} \tag{3.3}
\end{equation*}
$$

Here, the parameters in $\theta$ are conatant with time and are related directly to phyical link and payload parameters. When these parameters are unknow, the parameter vector $\theta$ is replaced by its eatimate $H(t)$ in real-time to give the following adaptive concrol law,

$$
\begin{equation*}
u=-K_{p} \Delta q_{1}-K_{v} \Delta q_{2}+H_{1} \dot{\theta} \tag{3.4}
\end{equation*}
$$

Subtrarting (3.2) from (3.4) and rearranging gives

$$
\begin{equation*}
\left.u=u^{0}+H_{1} \dot{\theta}-\theta\right) \Delta u^{0}+H_{1} \tag{3.5}
\end{equation*}
$$

This is an important relation aince it shows that the adaptiye control ia equal to the nonadaptive control plus an expression which is inear in the parameter error $\dot{\theta}_{\dot{\theta}-\theta}$.

The proof of stability then follows by choosing the following lyapunov function,

$$
\begin{equation*}
v=v^{0}+\frac{1}{2} r^{T} \quad r=r^{T}>0 \tag{3.6a}
\end{equation*}
$$

where $v^{0}$ is the Lyapunov function for the nonadaptive control law used in [1], and where ${ }^{T}$ rp is a positive definite function in the parameter error $\rightarrow$. For completeness, $v$ is rewritten here ( $\mathbb{f}$.. [1], (4.4) where $\left.U *\left(\Delta q_{1}\right) \geqslant \frac{1}{2} \Delta q_{1}^{T} K_{n}, s q_{1}\right)$,

$$
\begin{equation*}
v^{0}=\frac{1}{2} \Delta q_{2}^{T} n\left(q_{1}\right) \Delta Q_{2}+\frac{1}{2} \Delta Q_{2}^{T}\left(R_{p}+c X_{\nabla}\right) \Delta_{q_{1}}+c \Delta_{q_{1}}^{T} M\left(q_{1}\right) \Delta q_{2} \tag{3.6b}
\end{equation*}
$$

Takiag the derivative of $V$ eloas eystem trajectorien and eubatituting coatrol lav (3.5) givee upoo reartanging.

$$
\begin{equation*}
\dot{v}=\dot{v}^{0}+\left(\Delta_{q_{2}}+c A_{q_{2}}\right)^{T} R_{2}+\dot{j}^{T} r \tag{3.7}
\end{equation*}
$$

whare $\dot{j}^{\circ}$ is the Lyapunov function derivative for the nonadaptive caee, and-where the addicional terne involviag on the right hand alde of (3.7) arice directly from the additional cerme favolviag in the control law (3.3) and the Zyapunay function (3.6) reapectively.

The eecond and third terme of (3.7) are cancelled exectly by the choice of edaptation lav,
$i=: \dot{\theta}=-r^{-1}{m_{1}}^{T}\left(\Delta q_{2}+c \Delta q_{1}\right)$
 alac note that Control Lay 1 correapondi to care ( 6.2 zb ) for which a $\frac{3}{2}$
$\dot{v}=\dot{v}^{0}$

$$
\begin{equation*}
\cdots a_{1}\left\|\Delta_{a_{2}}\right\|^{2}-a_{2}\left\|\Delta a_{2}\right\|^{2}+r_{21}\left\|\Delta_{a_{1}}\right\|\left\|\Delta{q_{2}}_{2}\right\|^{2} \tag{3.9}
\end{equation*}
$$

where

$$
\begin{aligned}
& a_{1}=c\left(\sigma_{\ln }\left(X_{p}\right)-\frac{3}{4} \eta_{2} D^{2}\right) \\
& a_{2}-a_{1 n}\left(K_{p}\right)-c\left(\mu+\frac{3}{4} \frac{n_{2}}{p^{2}}\right) \\
& r_{21}=\frac{c}{2} n_{1} \\
& n_{2}=\max _{q_{2 d}}\left\|a_{2 d}\right\| n_{1} \\
& n_{1}=\operatorname{gax}_{1}\left(\sum_{1}\left\|M_{1}\left(q_{1}\right)\right\|\right) \\
& u=\max _{q_{1}} \mid\left\|m\left(q_{2}\right)\right\| \\
& 0<c<\ell^{2} \text { arbicrary } \\
& 0^{2} \text { arbitrary } \\
& e^{2} \text { arbicrary }
\end{aligned}
$$

Applying the B-ball armuent of Lemen 2.1 to (3.9) using the valuea of $a_{1}, a_{2}$, and $r_{21}$ given in
0 ) it follows that if, (3.10), it followe that if,

$$
\begin{align*}
& \sigma_{\text {min }}\left(R_{p}\right)>\frac{3}{4} n_{2} D^{2}  \tag{3.12a}\\
& \sigma_{\min }\left(K_{v}\right)>c\left(u+\frac{3}{4} \frac{n_{2}}{D^{2}}+\frac{n_{1}}{2}\left(\frac{v_{n}}{E_{1}}\right)^{\frac{2}{2}}\right) \tag{3.12b}
\end{align*}
$$

Then,

$$
\begin{equation*}
\dot{v} \leq-\lambda_{1}\left\|\Delta a_{1}\right\|^{2}-\lambda_{2}\left\|\Delta a_{2}\right\|^{2} \tag{3.13}
\end{equation*}
$$

for any $\lambda_{1}$ and $\lambda_{2}$ auch that,

$$
\begin{align*}
& \lambda_{1} c\left(0, c\left(\sigma_{m 1 n}\left(x_{p}\right)-\frac{3}{4} n_{2} \rho^{2}\right)\right)  \tag{3.16a}\\
& \lambda_{2}=\left(0, \sigma_{n 1 n}\left(n_{v}\right)-c\left(u+\frac{3}{4} \frac{n_{2}}{\rho}+\frac{\eta_{1}}{2}\left(\frac{v_{1}}{\varepsilon_{1}}\right)^{\frac{1}{2}}\right)\right.
\end{align*}
$$

where

$$
\begin{align*}
& v_{0}=\left.v\right|_{t=0}  \tag{3.15}\\
& \varepsilon_{1}=\frac{1}{2}\left[\sigma_{\min }\left(K_{p}+\operatorname{cK}_{v}\right)-2^{2} c \underline{g}(M)\right]  \tag{3.16a}\\
& \varepsilon_{2}=\frac{1}{2}\left(1-\frac{c}{e^{2}}\right) \subseteq(M)  \tag{3.16b}\\
& \underline{o}(M) \Delta_{q_{1}} a_{\min }\left(M\left(q_{1}\right)\right)
\end{align*}
$$

since $\rho^{2}$ is arbitrary, it can be chosen sufficiantly andi so that (3.12a) is aetiofied. With this chotce of $D^{2}$. the value of $c$ in ( 3.12 b ) can oe chosen sufficiently amall so that inequality (3.12b) is satisfiod. Bence (3.13) follove. This is essentislly the eme proof of stability as in the nooedaptive case (c.f., (1) Theorem 4-1) with the following exceptione,

1) The value of $V_{0}$ tn (3.12b) and (3.14b) now includee the faitial parmetar error $\frac{1}{2} \phi^{T}(0)$ ro $(0)$.
2) The value of c is now required for impleantation of the parametar adaptation law (3.8).
3) In (3.13) Is now oniy negative seaidafinite in the atate since the full state vector in the edaptive cace is augmented by 4.

It is moted that proporty 3 destroye the simple exposential atability argumat uaed earliar in the somedaptive cace (cf., (1), Theorem 4-1) to insure ampptotic convergence of $\mid$ |Aq || and $|\mid \Delta q$ | $|$. In addition
 priociplen are not applicabie. Alternatively, weake use of a leam due originally to Rerbalat, quoted vithout proof from Popov (14) (ps. 211).
Lemer 3-1 (Barbalat)
If $w$ is a real function of the real variable $t$, defined and uniformly continuous for $t \geq$ and if the 1init of the integral

$$
\lim _{t \rightarrow \infty} \int_{0}^{t} W\left(t^{\prime}\right) d t^{\prime} .
$$

exists and is a finite number, than

$$
\lim _{c \rightarrow \infty} W(t)=0 .
$$

For our purposes let,

$$
W(t) \Delta \lambda_{1}\left\|\Delta q_{1}(t)\right\|^{2}+\lambda_{2}\left\|\Delta q_{2}(t)\right\|^{2}
$$

so that

$$
\begin{equation*}
\dot{v} \leq-w \tag{3.17}
\end{equation*}
$$

Integrating both sides of (3.17) from 0 to $t$, yields upon rearranging,

$$
\begin{equation*}
\int_{0}^{t} W d t^{\prime} \leq v_{0}-V(t) \tag{3.18}
\end{equation*}
$$

Since $V_{0}$ is bounded, and $V(t)$ is nonincreasing and bounded below, it follows that

$$
\lim _{t \rightarrow \infty}^{t} \int_{0}^{t} W d t^{\prime}<\infty
$$

Also, fince $\dot{W}$ is bounded, $W(t)$ ia uniforaly continuous. Hence, application of Barbalat's Lemen gives,

$$
\begin{equation*}
\lim _{\mathrm{c} \rightarrow \infty} W=0 \tag{3.19}
\end{equation*}
$$

or equivalentiy $\left\|\Delta q_{1}\right\| \rightarrow 0$ and $\left\|\Delta q_{2}\right\|+0$.
This complates the proof of esymptotic stability. The proof, hovever, is not a global one due to property 2, 1.e., the value of $c$ which vas not required in the nonadaptive case now appeare in the parameter adaptation law (3.8). Hence, one is comitted to choosing a particular value of $c$ in the adaptive faplementarion. Of course, $c$ can alwas be choson aufficiently eall to eatiafy the requiresent. however, the poaition tracking performance determined by the magnitude $0: \lambda_{1}$ in $i^{3} .14 a$ ) must be compromieed as a result. Hence in practice, the initial choice of $c$ can be ade uaing whatever bounde on $\mathrm{n}_{1}$, $\mathrm{n}_{2}$; $\mu_{\text {. }}$ $g(K)$ and $\nabla_{0}$ are available a-priori, and the value of $c$ can be improved (incrased) on-line as more faformation becomes available. It is noted that (3.16a) and (3.16b) impose additional conatrainta on how large can become, since ic is required that $\xi_{1}>0$ and $\boldsymbol{G}_{2}>0$ for a positive defiaita $V$ (theae conditions can be shown suiflcient).

The daymptotic stability proof presented above for adaptation of Control Lev 1 , is easily extended to sdaptation of Concrol Lavs 2, 3 and 4 , since the corresponding nonadaptive lyapunov tunction derivatives for thee control lave are of exactly the same form as in (3.25) (see [1], Theorea 4-1 for details). For convenience, all asyaptotically stable adaptive control. lave discussed thus far, and cheir appropriate parameter adaptation lavs are sumarized in rable II, corresponding to casea la, 2.a, 3.a, and 4.a, respectively.

An alternative to choosing $c$ sufficiently sanll in the above asymptotic stability argument is to choose $X_{y}$ sufficiently large. In this case, the condition on $c$ above can be removed completely by adapting $K_{y}$ on-line. This modification insures global asymptotic stability of the adaptive control system (i.e., choice of $c$ independent of the inicial condition $V_{0}$ ) and ia diacussed in more detail belov.

### 3.2.2 Global Aayaptotic Stability-Adapting $\mathrm{K}_{\mathrm{y}}$

since the velocity gain $\mathrm{K}_{\mathrm{y}}$ entera inearly in the control law, it can be adapted as if it vere an unknova parmeter using the seme formulacion of Sec. 3.2.1. It will be show that thia approach removes the dependence of the choice of $c$ on the initial condition $V_{0}$ and chis coupletes the proof of global asyaptotic atability for the edaptive case.

Conader Control Lav 1 written in adaptive form, where both 0 and $k$ are adapted in real time $1 . e$. .

$$
\begin{equation*}
u=-X_{p} \Delta q_{1}-\dot{\dot{F}_{v}} \Delta q_{2}+R_{1} \dot{\theta} \tag{3.20}
\end{equation*}
$$

Here, $\dot{\phi_{\sim}}$ is a tine-varying quantity which remaine to be specified, and $H_{1}$ is as defined aarlier in (3.3). The nonadaptive control lay $\mathrm{u}^{\circ}$ in (3.1) is ubtracted from (3.20) to give the following expresaion,

$$
\begin{equation*}
u-u^{0}-\Delta z_{v} \Delta q_{2}+g_{1} \tag{3.21}
\end{equation*}
$$


The Lyapunov function for the stability anelyais is given as

$$
\begin{equation*}
V=\nabla^{0}+\frac{1}{2} \nabla_{V}+\frac{1}{2} \delta T R\left(\Delta X_{v}^{T} \Delta X_{v}\right\}, \delta>0, r-r^{T} \geqslant 0 \tag{3.22}
\end{equation*}
$$

whare a nev tern has been added relative to (3.6a), quadratic in the error $A \mathcal{O}_{\text {, }}$. Taking the derivative of $V$ along system trajectories and substituting control lav (3.21) gives upon rearranging

$$
\begin{align*}
\dot{v}=\dot{v}^{0} & \left.+\left(\Delta q_{2}+c \Delta q_{1}\right)^{T} H_{2}\right\rangle+\dot{\phi}^{T} r \\
& +\operatorname{TR}^{T}\left(\left[\delta \Delta \dot{q}_{v}^{T}-\Delta q_{2}\left(\Delta q_{2}+c \Delta q_{1}\right)^{T}\right\} \Delta F_{v}\right\} \tag{3.23}
\end{align*}
$$

The latter terme are cancelled exactly by the choice of parametar adaptation lawe,

$$
\begin{align*}
& :=-r^{-1} \underline{q}_{1}^{T}\left(\Delta q_{2}+c \Delta q_{1}\right)  \tag{3.24a}\\
& \Delta \dot{x}_{v}=\dot{x}_{v}=+\frac{1}{\delta}\left(\Delta q_{2}+c \Delta q_{1}\right) \Delta q_{2} T \tag{3.24b}
\end{align*}
$$

The choice leavee $\dot{V}$ exactly of the form (3.9) 1.e., applying the $8-8, B l l$ Leman 2.1,

$$
\begin{equation*}
\dot{v}=\dot{v}^{0} \leq-\lambda_{1}\left\|\Delta q_{1}\right\|^{2}-\lambda_{2}\left\|\Delta q_{2}\right\|^{2} \tag{3.25}
\end{equation*}
$$

1f.

$$
\begin{align*}
& \sigma_{m i n}\left(R_{p}\right)>\frac{3}{4} n_{2} o^{2}  \tag{3.26}\\
& \sigma_{m=1 n}\left(R_{v}\right)>c\left(u+\frac{3}{4} \frac{n_{2}}{\rho^{2}}+\frac{n_{1}}{2}\left(\frac{v_{0}}{\varepsilon_{1}}\right)^{\frac{1}{2}}\right) \tag{3.27}
\end{align*}
$$

In (3.26) and (3.27), all quantities are defined exactly as in (3.12a) and (3.12b) respectively, except for $V_{0}$ which is presentiy the inicial value of $V$ in (3.22). Furthermore, the valuea of $\xi_{1}$ and $\xi_{2}$ are once again given as

$$
\begin{align*}
& \varepsilon_{1}=\frac{1}{2}\left\{a_{\min }\left(K_{p}+c K_{v}\right)-2^{2} c_{g}(M)\right]  \tag{3.28}\\
& \varepsilon_{2}=\frac{1}{2}\left(1-\frac{c}{2^{2}}\right) \underline{g}(M) \tag{3.29}
\end{align*}
$$

An important observation is that,

$$
\begin{align*}
& v_{0}=\alpha\left\|x_{v}\right\|^{2},\left\|x_{v}\right\| \rightarrow \infty  \tag{3.30a}\\
& \left.\varepsilon_{1}=\alpha\left\|x_{v}\right\|\right),\left\|\mathbf{r}_{v}\right\| \rightarrow- \tag{3.305}
\end{align*}
$$

Hence, for any choice of $\delta>0, c>0$ and $R_{p}-R_{p} T_{>0}$, there exist values of $\rho^{2}, \mathcal{L}^{2}$, and $K_{V}-R_{v} T_{>0}$ (with $\sigma_{m i n}$ ( $K_{v}$ ) aufficiently large) such that inequalities ${ }^{P}(3.26)$ and (3.27) are satisfied, and $\xi_{1}>0, \xi_{2}>0$ in ( 3.28 ) and (3.29), respectively. Global asymptotic stability of this adaptive control scheme then followa lamediately by application of Barbalat's Leman to the Lyapunov function derivative (3.25), as was done earlier in equations (3.17) through (3.19).

The slobal asyaptotic stability of adaptive controllere based on Control Lave 2, 3 and 4 (where K, is adapted on-line) follow from an identical argument, since fo corresponding to the nonadaptive Lyapunov function derivativen for these control lave are of exactly the ane form aa fo in this analyals (see [1], Theore 4-I for decails). For convenience, these adaptive control lave involvint adaptation of tha are oumarized in table II, corresponding to cases l.b, 2.b, 3.b, and 4.b, respectively.

### 3.3 Global Aoyptotic Stability for Adaptation of Control Lave 5, 6 and 7

Global ammptotic atability for adaptation of Control Lave 5, 6 and 7 can be proved ualng exactly the sage techniqued at applied in Sec. 3.2. The oniy difference lies in alight variations in the nonadaptive lyapunov function derivative vo which ariaes in each adaptive control analyeis

Due to epace lisitations, these proofs have been onitted, but the results are sumarized in Table II corresponding to casea $5 . a, 5 . b, 6 . a, 6 . b$, and $7 . a, 7 . b$, respectively. Details can be found in [21], to which the equation numert in Table II are referenced.

## 4. Adaptive Computed Torque Method

It was mentioned earlier that in the computed torque gethod (i.e., control iaw 0 ) the presence of the M(q1) tere premultiplying the $K_{p}$ and $K_{v}$ geine coaplicates the Lyapunov analysis and hinders most simple attempta to make it adaptive. Nevertheless, the coaputed corque controller is a well-known control lay in the literature and is widely applied in practice. Hence, it is uaeful to inveatigate under what conditions it can be made adaptive, and to what extent adaptive stability can be guaranteed. For this purpose, we conaider apecial case of the computed torque control law which has scalar gains $k_{p}$ and $k_{v}$, $1 . e .$,

$$
\begin{equation*}
u^{0}=-M\left(q_{1}\right)\left(k_{p} \Delta q_{1}+k_{v} \Delta q_{2}\right)+k\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}+C\left(q_{1}, q_{2}\right) q_{2} \tag{4.1}
\end{equation*}
$$

This ia written in adapeive form as,

$$
\begin{equation*}
u=u^{\circ}+H_{8} \tag{4.2}
\end{equation*}
$$

where $0 \hat{\theta}-\theta$, and the linear in the parameters part has been chosen as,

$$
\begin{equation*}
H_{8}^{\theta} \Delta-M\left(q_{1}\right)\left(k_{p} \Delta r_{1}+k_{v} \Delta q_{2}\right)+k\left(q_{1}\right)+M\left(q_{1}\right) \dot{q}_{2 d}+C\left(q_{1}, q_{2}\right) q_{2} \tag{4.3}
\end{equation*}
$$

Let a Lyapunov function be defined as,

$$
\begin{equation*}
v=v^{o}+\frac{1}{2} \phi^{T} r \phi \quad, \quad r=r^{T} \geqslant 0 \tag{4.48}
\end{equation*}
$$

where

$$
\begin{equation*}
v=\frac{1}{2} \Delta q_{2}^{T} M\left(q_{1}\right) \Delta q_{2}+\frac{1}{2}\left(k_{p}+c k_{v}\right) \Delta q_{1}^{T} M\left(q_{1}\right) \Delta q_{1}+c \Delta q_{1}^{T} M\left(q_{1}\right) \Delta q_{2} \tag{4.4b}
\end{equation*}
$$

Then, the derivative of (4.4) along syatem trajectories induced by control (4.2) is given by

$$
\begin{align*}
\dot{V} & =\Delta q_{2}^{T}\left[-k_{V} M\left(q_{1}\right) \Delta q_{2}+\frac{1}{2} M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}\right] \\
& +c \Delta q_{1}^{T}\left[-k_{p} M\left(q_{1}\right) \Delta q_{1}+M_{D}\left(q_{1}, \Delta q_{2}\right) q_{2}+M_{D}\left(q_{1}, \Delta q_{1}\right) q_{2}\right]  \tag{4.5}\\
& +\left(\Delta q_{2}+c \Delta q_{1}\right)^{T} H_{8}+\dot{p}^{T} \Gamma  \tag{4.5}\\
\dot{ } & =\theta=-r^{-1} H_{8}^{T}\left(\Delta q_{2}+c \Delta q_{1}\right) \tag{4.6}
\end{align*}
$$

Let,

Then,

$$
\begin{align*}
\dot{v} \leq & -\alpha_{1}\left\|\Delta q_{1}\right\|^{2}-a_{2}\left\|\Delta q_{2}\right\|^{2}+\gamma_{12}\left\|\Delta q_{2}\right\|\left\|\Delta q_{1}\right\|^{2} \\
& +\left(\gamma_{21}\left\|\Delta q_{1}\right\|+\gamma_{22}\left\|\Delta q_{2}\right\|\right)\left\|\Delta q_{2}\right\| \|^{2} \tag{4.7}
\end{align*}
$$

where

$$
\begin{align*}
& a_{1}=c\left(k_{p} g(M)-n_{2}\left(1+\rho^{2}\right)\right)  \tag{4.8}\\
& a_{2}=k_{v} \sigma(M)-\frac{1}{2} n_{1}-\frac{c n_{2}}{\rho^{2}}  \tag{4.9}\\
& r_{12}=c n_{1}  \tag{4.10}\\
& r_{21}=c n_{1} ; r_{22}=\frac{1}{2} n_{1} \tag{4.11}
\end{align*}
$$

Applying the 8-Ball Leman, it follows thet,

$$
\begin{equation*}
\dot{v} \leq-\lambda_{1}\left\|\Delta q_{1}\right\|^{2}-\lambda_{2}\left\|\Delta q_{2}\right\|^{2} \tag{4.12}
\end{equation*}
$$

1f.

$$
\begin{align*}
& \left.c k_{p} \underline{\sigma}(M)>n_{2}\left(1+\rho^{2}\right)\right)+c n_{2}\left(\frac{v_{0}}{\xi_{1}}\right)^{\frac{1}{2}}  \tag{4.13}\\
& k_{v} \underline{\sigma}(M)>\frac{1}{2} n_{1}+\frac{c n_{2}}{\rho^{2}}+\frac{1}{2} n_{1}\left(\frac{v_{0}}{\xi_{2}}\right)^{\frac{1}{2}}+c n_{1}\left(\frac{\nabla_{0}}{\xi_{1}}\right)^{\frac{1}{2}}  \tag{4.14}\\
& \varepsilon_{1}=\frac{1}{2}\left(k_{p}+c k_{v}-c l^{2}\right]_{g}(M),  \tag{4.15}\\
& \varepsilon_{2}=\frac{1}{2}\left(1-\frac{c}{i^{2}}\right) \underline{\sigma}(M) ; i^{2}>0 \text { arbitrary } \tag{4.16}
\end{align*}
$$

It is noted that for any $c>0$, both $k p$ and $k y$ can always be chomen sufficiently large so that $\xi_{1}>0$ and $5_{2}>0$ (for appropriate choice of $t^{2}>0$ in (4.15), (4.16)), and inequalities (4.13) and (4.14) are satisilied. Hence, the adaptive computed torque control law given by (4.2), (4.3) with parameter adaptation (4.6) is asymptotically stable when $k_{p}$ and $k_{v}$ are chosen sufficiently large.

Since $k_{p}$ and $k_{v}$ must be chosen sufficiently large with respect to the initial condition $V_{0}$ (c.f., (4.13), (4.14)) this proof of asymptotic stability is not global (1.e., for fixed kp and ky there will alwaya exist aome $V_{0}$ such that $\lambda_{1}$ and/or $\lambda_{2}$ are not positive). For this particular algoritha, it is presently not clear hou to adapt $k_{p}$ and $k_{v}$ to insure global asamptotic stability alace the control $u$ in ( 4.1 ) is not linear in the parameters ( $\theta, k_{p}, k_{y}$ ).

## 5. Summary and Remarke

The adaptive control laws derived herain, along with the sufficient conditions for stability and appropriate parameter adaptation laws are summarized in Table II. Several remarkmare in order at this point in the discussion.

Remark 5-1 All adaptive control laws in this peper were derived for the general tracking control law. However, significant simplification occurs in many of these designs for the special case of set-point control (i.e., $9_{2 d}=\dot{q}_{2 d}-0$ ).

Remark 5-2 The adaptive robustness issue remains open. Certainly for parameter adaptation laws of the form given in Table II, there will be senaitivities to noise disturbances and unmodelled dynamics directly analogous to those which arise in the linear adaptive control case. It presently appears that many of the robustness techniques developed in the inear adaptive control literature will carry over to the nonlinear adaptive control application. This conjecture, however, remains to be investigated.

Remark 5-3 In the nonadaptive case, wany of the control lawa in Tabla II are in a form appropriate for application of the recursive Newton-Euler computational algorithm. However, the Newton-Euler algorithm requires knowledge of all physical parameters-more parameters than, are actually needed to control the systea adaptively and more than are actually adapted on-line in the vector $\theta$ of Table II. Hence, the transformation from $\theta$ back to physical parameters is required in order to salvage use of the Newton-Euler algorithm in the adaptive case. However, the transformation ia generally nonlinear and will not lead to a unique solution unleas further constraints are imposed. One typical set of constraints arisea when only the payload mase is unknown. In the more general adaptive case, it is useful to note that all linear in the parameters expressions can be taplemented directly, since representations of the form He are assumed to be avillable in symbolic fora.

Remark 5-4 The control laws of Table I were derived in [1] for the general deaired potential energy function. This feature was dropped in the adaptive case in order to simplify the analysis. However, it appears that the adaptive control laws developed herein can be extended to the more general case and this line of research presently under invegtigation.

Remark 5-5 A brief comparison with the recent results Paden [10] and Slotine and Li [11] is useful. In [10] [11], adaptive control laws are derived by choosing to cancel various terms in the lyapunov function derivative, rather chan overbounding then (via Lemma 2.1) as was done here. This approach has the advantage of providing global asymptotic convergence without adapting gains $\mathbb{K}_{v}$ and $K_{p}$. The control laws, hovever, are by necessity more complex than those designs considered here, and do not simplify in the set-point control case.

## 6. Conclusions

A new class of asymptotically stable adaptive control laws is defined by adapting the control lavs of [1] in a certainty equivalence fashion. These algorithms are proved to be asymptotically stable without approximations, inearizations or ad-hoc assumptions concerning the nonlinear manipulator dynamics. Furthermore, the asymptotic convergence properties can be made global by appropriate adaptation of feedback gains. On-going research efforts are directed at adaptive robustness, computation, and obstacle avoidance problems.

 noee required



$\therefore \cdots r^{-1} n_{1}{ }^{1}\left(\Delta_{2}+c \Delta A_{1}\right)$
$x_{v}=\frac{1}{4}\left(\Delta_{2}+c \Delta A_{1}\right) \Delta A_{2}^{T}$
$\therefore \cdot r^{-1} \|_{6}^{\mathrm{T}}\left(\mathrm{saq}_{2}+\mathrm{taq}_{1}\right)$
$\mathbf{u}_{2} 0-k\left(q_{1}\right)+\mu\left(q_{1}\right) i_{2 d}-\frac{1}{2} J\left(q_{1} \cdot q_{2 d}\right) q_{2 d}+\frac{1}{2} M_{b}\left(q_{1} \cdot q_{2}\right) q_{2 d}$


 Lumen in panatrias expaission

$H_{2}-k\left(q_{1}\right)+H\left(q_{1}\right) \dot{q}_{2 d}-\frac{1}{2} J\left(q_{1} \cdot q_{2 d}\right) q_{2}+\frac{1}{2} \mu_{b}\left(q_{1} \cdot q_{2}\right) q_{2 d}$ $\mathrm{H}_{2^{\circ}}=$ ane as 2.0 above
3.4 u- $-x_{p \Delta q_{1}}-x_{v} \Delta q_{2}+n_{3} i$


 2.b $u=-x_{p} \Delta a_{1}-\dot{x}_{v} \Delta_{2}+y_{2} \dot{i}$
$4.4-x_{p} \Delta a_{1}-x_{Q} \Delta a_{2}+H_{4} \dot{\theta}$

-

H10-ase ac 1.4 above

$H_{2} 0-k\left(q_{1}\right)+n\left(q_{1}\right) \dot{q}_{24}-\frac{1}{2}$ -
noures
*Clobal anympeotic scablisty
*asmploctic stability
${ }^{4}$ Aemprotic seability

-clobel deymetactc stabilicy -


## Acknowledgementa

The authors are Indebted to Dr. K. Kreutz for eany helpful technical diecussions. This work was done et the Jet Propulsion Laboratory, California Inatitute of Technology under contract with the National Aeronautice and Space Administration.

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# Algorithms for Adaptive Control of Two-Arm Flexible Manipulators Under Uncertainty 

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## 1. Abstract



The paper-wer a nonl年ear extension of model reference adaptive control (MRAC) tech $\overline{1 T q} u e / t i$ guide a double arm nonlinearizable robot manipulator with flexible links, driven by actuators collocated with joints subject to uncertain payload and inertia. The objective is to track a given simple linear and rigid but compatible dynamical model in real, possibly stipulated time and within stipulated degree of accuracy of convergence while avoiding collision of the arms. The objective is attained by a specified signal adaptive feedback controller and ty adaptive laws, beth given in closed form. A case of 4 oOF manipulator illustrates the technique.

## 2. Introduction

The MRAC technique becomes popular proposition for guidance of recent robot manipulators, with demand for precision pointing in difficult conditions, under the action of full scale dynamic forces, and subject to uncertainty in parameters. Such manipulators, particularly these used on spacecraft are highly nonlinear and nonlinearizable structures (geometric nonlinearity of clastic links, large angle articulation, nonlinear coupling of DOF's, nonignorable gyro and Coriolis forces, several equilibrial). while classical MRAC is linear and applicable to rigid bodies only. Thus the extension is needed for handling nonlinearity, see [1], and flexible links, see [2]. On the other hand many robotic objectives, again particularly these in difficult space conditions require at least two arm systems. Thus the tracking has to be a double MRAC (mutual reference adaptive control) which secures tracking the same model by two arms while avoiding mutual collision - cf. [3], [4]. If adaptive (self-organizing) control is intended, the tracking relates not to a given path but to a given dynamic targetmodel with prescribed target-parameters. We take the model simple thus rigid and linear, but locally compatible with the nonlinear arms regarding equilibrial. Each arm is represented as an open chain with n DOF, nonlinear characteristics and coupling, elastic links, driven by actuators collocated with joints, under uncertain inertia parameters and uncertain payload. The tracking is done in real possibly stipulated time by a designed signal adaptive feedback controller and integrable adaptive laws in the state space, while avoiding collision between arms of all the joints (and elastic nodes) in Cartesian configuration space.
3. Motion Equations
lagrange motion equations give the rigid dynamics of the arms in the general format

$$
\begin{equation*}
A^{j}\left(q^{j}, s^{J}\right) \ddot{q}^{j}+\Gamma^{J}\left(q^{J}, \dot{q}^{J}, \lambda^{j}\right)+\Pi^{j}\left(q^{j}, \lambda^{j}, s^{j}\right)=B^{j}\left(q^{j}, \dot{q}^{j}\right) u^{j}, \quad j=1,2, \tag{1}
\end{equation*}
$$

where $q^{j}(t): J_{q}: R^{n}, t: t_{0}=0$, is the configuration vector of the joint variables $q_{1}^{j}, \ldots, q_{n}^{j}$ of the $j$-th arm varying in the known bounded work region $\therefore_{q}$ of the configuration space $R^{n}$; $\dot{q}(t)$ is the corresponding vector of joint velocities in the specified bounded subset $A_{\dot{q}}$ of the space tangent to $\mathbf{R}^{n}$; $u^{j}(t): U: R^{n}$ are the control vectors in given compact set of constraints $u ; \lambda^{j}(t) \in \Lambda \subset R^{2}$, $i \leq 2 n$, are the vectors of adjustable system parameters in bounded bands of values $A$, and $s^{j}(t) \in S \in R^{k}$ is an uncertainty parameter within the known band $S$. Moreover $\lambda^{j}\left(q^{j}, S^{j}\right)$ are the inertia nan matrices obtained in the known way from the quadratic form of kinetic energy. The vectors $\|^{j}=\left(\pi_{1}^{j}, \ldots, \eta_{n}^{j}\right)^{T}$ represent potential forces (gravity, spring) while. $r^{j}=\left(i j_{1}^{j}, \ldots, j_{n}^{j}\right)^{T}$ represent the internal nonpotential acting forces (Coriolis. gyro, centrifugal, damping structural or viscous, etc.) and $B$ is the actuator transmission (gear) nonsingular $n m$ matrix. The control vectors $u^{j}(t)$ are selected for the objectives of tracking and avoidance by adaptive feedback control programs $u^{j}(t)=p^{j}\left(0^{1}(t), q^{2}(t), \dot{q}^{l}(t), \dot{q}^{2}(t), 1^{1}(t), i^{2}(t)\right)$ on corresponding products of $\Delta_{q} \times \lambda_{q} \times \lambda$. For convenience the superscripts $" j "$ will be dropped until they are needed to avoid ambiguity.

Considering the links elastic we introduce the deformation coordinates for the isth link as shown in fig. 1 , while using the Ritz-Kantorovitch series :xpansion

$$
\begin{equation*}
r_{i}\left(y_{i}, t\right)=\sum_{v=1}^{m} r_{i}^{v}\left(y_{i}\right) r_{i}^{v}(t)=r_{i}\left(y_{i}\right) r_{i}(t) \tag{2}
\end{equation*}
$$

and for $v_{i}\left(y_{i}, t\right), w_{i}\left(y_{i}, t\right)$ analogously, with the exact solution expected for $m \rightarrow \infty$. We take $m$ large


Figure 1. Flexible link
enough so that the Kantorovitch linearization is physically justified. The technical way about it is to stepwise subdividing the links between grid as long as the ditference of results for successive m's becomes small. Having specified (2) we form the vector $n(t) \Delta\left(\eta_{1}(t), \ldots, \eta_{n}(t)\right)^{T}$, where $\eta_{i}(t) \Delta\left(r_{i}(t), v_{i}(t), w_{i}(t)\right)^{T}$ and following [5] write the hybrid system as

$$
\left(\begin{array}{ll}
A_{1} & A_{c}  \tag{3}\\
A_{c}^{T} & A
\end{array}\right)\binom{\ddot{u}}{i i}+\left(\begin{array}{ll}
0 & D_{c} \\
0 & 0
\end{array}\right)\binom{\dot{q}}{\dot{\eta}}+\left(\begin{array}{ll}
0 & p \\
0 & p
\end{array}\right)\binom{q}{\eta}+\binom{\Gamma(q, \dot{q})}{\Gamma_{\eta}(\eta, \dot{\eta})}+\binom{\Pi(q, 1, s)}{\eta_{\eta}(\eta, s)}=\binom{B(q, \dot{q})}{0} u
$$

where $A_{\eta}(\eta, s), \Gamma_{\eta}(\eta, \dot{\eta}), \eta_{\eta}(\eta, s)$ are the elastic correspondents of $A, \Gamma$, ll. while $A_{c}(q, \eta)$, $D_{c}(q, \dot{q}, \eta, \dot{n}), P_{c}(4, \eta)$ and the internal damping $D(4, q, \eta, \eta)$ as well as the hybrid restoring coefficients $P(q, \eta)$ are matrices coupling the elastic and joint coordinates. These matrices are formed by integrals over the shape functions, see $[5]$. Letting

$$
A(q, \eta, s)=\left(\begin{array}{ll}
A_{1} & A_{c} \\
A_{c}^{T} & A_{\eta}
\end{array}\right)
$$

to be the hybrid inertia matrix which is nonsingular positivi definite, we inertially decouple (3):

$$
(\ddot{4}, i i)^{r}+D(4, \dot{4}, r, \dot{n}, t, s)+P(4, \pi, \lambda, s)=B(4, \dot{q}, s) u
$$

 forces and the meaning of the matrix $B$ is obvious. The vectors $4, \dot{4}, \eta, \dot{\eta}$ form the state vector
 convenience (4) may be then written in the general state form

$$
\begin{equation*}
\dot{\mathbf{x}}=\mathbf{f}(x, u, f, s) \tag{5}
\end{equation*}
$$

with $f=\left(f_{1}, \ldots, f_{N}\right)$ of the shape specified by (4) in an obvious way. formally (5) may be written in the contingent form:

$$
\dot{x}=|f(x, u, t, s)| s \in s
$$

which for suitable $f(\cdot), f(\cdot), i(\cdot)$ has solutions $x(t)=k\left(x^{0}, t\right), t=0$, absolutely continuous curves through each $x^{0}=x(0)$ in $\Delta$. We shall consider the class of such solutions $K\left(x^{0}\right)$ by exhausting all values of $s(t)$ in (5) at each $t$.

## 4. The Reference Model

We let the given Cartesian "world" coordinates representation of the reference nodel in general terms

$$
\begin{equation*}
\vdots_{m}=F\left(\lambda_{m}\right) \vdots \tag{b}
\end{equation*}
$$

with in DOF, $\bar{s}(t) \in R^{3.2 n}$, and $F\left(t_{m}\right)$ suitable matrix, be off-line recalculated to the joint coordinate format of the rigid linear system

$$
\begin{equation*}
\ddot{u}_{m}+D_{m}\left(i_{m}\right) \dot{u}_{m}+P_{m}\left(\Lambda_{m}\right) q_{m}=0 \tag{}
\end{equation*}
$$

with the $2 n$-vectors $q_{m}, \dot{4}_{m}$ of joint coordinates and velocitics, state $x_{m}(t)=\left(4_{m}(t), \dot{4}_{m}(t)\right)^{T}$. $=R^{T}$, and
$D_{m}, P_{n}$ suitable matrices, while $\lambda_{m}=\left(\lambda_{m 1}, \ldots, \lambda_{m}\right)-$ const $\in \Lambda \subset R^{l}, l=n$. Moreover

$$
\begin{equation*}
P_{m}\left(\lambda_{m}\right)\left(q^{e}, n^{0}\right)=0 \tag{8}
\end{equation*}
$$

with $\left(q^{a}, \eta^{e}\right)$ denoting the equilibria of (3) on the surface $\dot{q}=0, \dot{\eta}=0$. The total energy of the model wlll be denoted by $E_{m}\left(\xi_{m}, \xi_{m}\right)$ in the world coordinates and $E_{m}\left(q_{m}, \phi_{m}\right)$ in the joint coordinates, obviously equal to one another. Then

$$
\begin{equation*}
E_{m}\left(q_{m}, \dot{Q}_{m}\right)=1 \dot{q}_{m} \dot{q}_{m}+\int_{\dot{q}_{m}}^{q_{m}} P_{m}\left(\lambda_{m}\right) d \sigma \tag{9}
\end{equation*}
$$

and substituting (7),

$$
\begin{equation*}
\dot{E}_{m}\left(q_{m}, \dot{q}_{m}\right)=-D_{m}\left(\lambda_{m}\right)\left(\dot{q}_{m}\right)^{2} \tag{10}
\end{equation*}
$$

The model is selected such as to allow achieving of a stipulated target behavior in the state space. To focus attention on something specific and yet general enough, let it be stability of the origin, guaranteed by the nonaccummulation of the total energy i.e. non-negative damping

$$
\begin{equation*}
\dot{E}_{m}\left(q_{m}, q_{m}\right) \leq 0, \quad \forall \dot{q}_{m} \neq 0 \tag{1}
\end{equation*}
$$

while

$$
\begin{equation*}
\nabla E_{m}\left(q_{m}, \dot{q}_{m}\right)>0 \tag{12}
\end{equation*}
$$

in-the-large i.e. on same $C \Delta_{L}=\Delta-\Delta_{L}$, where $\Delta_{L}$ is the set in $R^{N}$ enclosing all the equilibria.
5. Objectives

Now we consider both arms $j=1,2$ and the model together. The block scheme of the system is shown in Fig. 2.


Figure 2. Block scheme of the system
 which vary in $\Delta^{2} x_{i}$ generating the product trajectories $\left(X^{j}\left(X^{j 0}, t\right), x^{j}\left(x^{j 0}, t\right)\right), t \geq 0, x^{j 0}=X^{j}(0)$, $\alpha^{j o}=x^{j}(0)$. Then we define the "diagonal" sets

$$
M^{j}=\left\{\left(x^{j}, x^{j}\right) \cdot j^{2} x_{i} \mid x^{j}=x_{m}, x^{j}=0\right\}, j=1,2,
$$

and given stipulated $\mu^{j}>0$, their neighbourhoods

$$
M^{j}=\left(x^{j}, x^{j}\right), j \sum_{x_{i}}: j x^{j}-x_{m} \quad \cdots, A^{j}, \mu^{j} ; j=2
$$

Moreover we let $\lambda_{0}$ be a desired subset of $\lambda$ where we want the tracking to occur, and let $t_{c}$ be the
stipulated time after which the tracking is attained with accuracy $\mu^{\mathbf{j}}$.
First objective: The manipulator arms (1) are mutuaily $\mu$-tracking the target (7) on $\Delta_{0}$ if thero is a pair of controllers $p^{j}(\cdot), j=1,2$ such that for each solution $k^{j}\left(x^{j 0}, t\right)$, $t \geq 0$ of (4) in $K\left(x^{j 0}\right)$, the set $\Delta_{9}^{2} \times \Lambda$ is positively invariant: $\left(X^{j o}{ }_{,} a^{j o}\right)$ e $\Delta_{0}^{2} \times \Lambda \oplus\left(X^{j}(t), \alpha^{j}(t)\right)$ c $\Delta_{0}^{2} \times \Lambda$ and given $t_{c}$, for each ${ }^{\prime}{ }^{\boldsymbol{j}}(\cdot)$ © $K\left(x^{\text {jo }}\right)$ the product trajectories satisfy

$$
\begin{equation*}
\left(x^{j}(t), a^{j}(t)\right) \in m_{\mu}^{j}, v t \geq t_{c} \tag{13}
\end{equation*}
$$

The convergence is illustrated in Fig. 3.


Figure 3. Convergence of product trajectories
Suppose the transformation from joint to world coordinates (forward kinematics) is given by

$$
\begin{equation*}
\xi_{\mathrm{J}}^{\mathrm{j}}=\overbrace{\sigma}^{j}\left(q^{j}, r^{j}\right), \quad c=1, \ldots, 3 \cdot 2 n \tag{14}
\end{equation*}
$$

and denote $z(t) \Theta\left(x^{2}(t), x^{2}(t)\right)$. Then we let the set

$$
A \quad A\left\{z=\Delta^{2}| | \xi_{3}^{4}-\zeta_{v}^{2} \mid=d, v \sigma, v=1, \ldots, 3 \cdot 2 n\right\}
$$

be the collision set between arms to be avoided. We define $C A=A_{0}^{2}-A, \operatorname{spec} i f i e d$ by $\left|f_{0}^{f}-\xi_{1}^{2}\right|>d$, and let

$$
\Delta_{A} \triangleq\left\{2 \cdot \Delta^{2}\left|d<\left|\xi_{0}^{4}-\xi_{0}^{2}\right|<2\right\}\right.
$$

be the "slow down" safery zone, with $i>0$ suitable constant.
second objective: The tracking arms (1) avoid collision iff there is $A_{A}$ such that for any $z^{0}$. CA and any pair $k^{J}(\cdot) \in K^{j}\left(x^{j o}\right)$ the corresponding product trajectory

$$
\begin{equation*}
z\left(z^{0}, t\right) \cdot C A, \quad V t \geqslant 0 \tag{15}
\end{equation*}
$$

## 6. Sufficient Conditions

We return now to the first objective and specify by $N\left[f\left(S_{0}^{2} \times 1\right)\right]$ a neighborhood of the boundary $\quad\left(\therefore a_{0}^{2} N\right)$

 with the positive constants

$$
\begin{aligned}
& v_{s}^{j}=v_{s}^{j}\left(X^{j}, 1^{j}\right),\left(x^{j}, x^{j}\right), \quad\left(i x_{\Lambda}\right) \\
& v_{\mu}^{j-}=\inf v_{d}^{j}\left(x^{j}, x^{j}\right) \mid\left(x^{j}, x^{j}\right) \cdot M_{\mu}^{j} \cap \overline{C M}_{j}^{j} \\
& v_{\mu}^{j+}=\sup v_{,}^{j}\left(x^{j}, x^{j}\right) \mid\left(x^{j}, x^{j}\right) \cdot i\left(i^{2} \times, N\right): C M^{j}
\end{aligned}
$$

The first relation obviously requires forming $V_{S}^{J}(\cdot)$ from suitable $\left(j_{0} \times N\right.$ ) taken asits level, or conversely, forming $3 A_{0}$, fi from levels of suitable $V_{S}(\cdot)$. In the latter case $\mathrm{a}^{\circ} \mathrm{A}_{\mathrm{o}}$, $A$ smaller than these desired will be the secule choice.
THEOREM $i$ : Objective 1 is attained if, given $A_{o}, A, u$ thereare prog-ams $p^{j}(\cdot)$ and functions $v_{S}^{j}(\cdot), v_{\mu}^{j}(\cdot)$ such that for all $\left(x^{j}, i^{j}\right), A_{0}^{2} \times i$,

$$
\begin{equation*}
v_{s}^{j}\left(x^{j}, a^{j}\right) \leq v_{s}^{j}, v\left(x^{j}, a^{j}\right) \in N_{\varepsilon}, j \in 1,2 \tag{i}
\end{equation*}
$$

(ii) for each $u^{j} \in p^{j}\left(x^{4}, x^{2}\right)$;

$$
\begin{equation*}
v_{s}^{j}\left(x^{j}(t), a^{j}(t)\right)<0, v_{s}^{j} \in s \tag{17}
\end{equation*}
$$

along the product trajectories $\left(x^{j}\left(x^{j 0}, t\right) \alpha^{j}\left(\alpha^{j 0}, t\right)\right), t 20, j=1,2$;
(iii) $\quad 0<v_{\mu}^{j}\left(x^{j}, a^{j}\right) \leq v_{\mu}^{j+}, v\left(x^{j}, a^{j}\right), \overline{O H}_{\mu}^{j}, j=1,2$;
(iv) $\quad v_{\mu}^{j}\left(x^{j}, \alpha^{j}\right) \leq v_{\mu}^{j-}, v\left(x^{j}, a^{j}\right) \in D^{j} \cap M_{\mu}^{j}, j=1,2$;
(v) for each $u^{j}=p^{j}\left(x^{4}, x^{2}\right)$ there is a constant $c_{j}>0$ such that

$$
\begin{equation*}
\dot{v}_{\mu}^{j}\left(x^{j}(t), a^{j}(t)\right) \leq-c_{j}, v_{s}^{j} \in s \tag{18}
\end{equation*}
$$

along the product trajectories $\left(x^{j}\left(x^{j 0}, t\right), a^{j}\left(a^{j 0}, t\right)\right), t \geq 0, j=1,2$.
Remark 1: Tie objective 1 holds after a stipulated $t_{c}<\infty$ if Theorem 1 is satisfied with $c_{j}$ selected by

$$
\begin{equation*}
c_{j} \Delta \frac{\Delta_{\mu}^{v_{c}^{j+}}}{t_{c}^{j}}, j=1,2 . \tag{19}
\end{equation*}
$$

THEUREM 2: Objective 2 is attained if Theorem 1 holds and given $d$ there is a $c^{1}$-function $\quad V_{A}(\cdot): \dot{L}_{A} \rightarrow R$ such that for the tracking pair $\mathrm{p}^{\mathrm{j}}(\cdot)$, for all $2 \cdot \mathrm{CA}$,
(vi) $\quad V_{A}(z)>V_{A}(z), V_{z} \in i A$;
(vii) for each $u^{j} \in p^{j}(z)$,

$$
\begin{equation*}
\dot{v}_{A}\left(z\left(z^{0}, t\right)\right)=0, z^{0} \in \Delta_{A}, v_{s}^{j} \in s \tag{20}
\end{equation*}
$$

along product trajectories $z\left(z^{\circ}, t\right), \tau \geq 0$.
 which contradicts (vil).
7. Controllers and Adaptive Laws
leet us set up

$$
\begin{align*}
& v_{S}^{j}=E_{i x}\left(x^{j}\right)+E_{m}\left(x_{m}\right)+a^{j} x^{j} ;  \tag{21}\\
& v_{\mu}^{j}:\left\{\begin{array}{l}
\left|E_{m}\left(x^{j}\right)-E_{m}\left(x_{m}\right)\right| \cdot a^{j} a^{j} \cdot\left(x^{j}, x^{j}\right), C M_{\mu}^{j} . \\
a^{j} x^{j},\left(x^{j}, x^{j}\right) \cdot M_{\mu}^{j}:
\end{array}\right.  \tag{22}\\
& v_{A}=\mid i t_{m^{\prime}}\left(x^{4}\right)-E_{m}\left(x^{2}\right) i \tag{23}
\end{align*}
$$

where $a^{j}=\left(\operatorname{sign} x_{1}^{j}, \ldots, \operatorname{sign} x_{n}^{j}\right), j=1,2$, and $E_{m}\left(x^{j}\right)$ is $E_{m}(\cdot)$ with $x_{m}$ exchanged for $x^{j}$. Choosing $N$ of $\mathrm{C}_{\mathrm{l}}$. , the character of $E_{m}(\cdot)$ specified additionally by (12), satisfies (i), (iii) and (iv).

To see that ( $V_{1}$ ) holds, observe that $E_{m}\left(x^{j}\right)=E_{m}\left(F^{j} i^{j}\right)$ of (6) and that increasing the distance $i 5_{\nu}-\tilde{j}_{j}^{2}=0$ for at least one $\mathcal{O}$ from its iA value increases the value of $V_{A}$.

To check upon conditions (ii), (v), (vii) we differentiate (21) - (23) with respect to time

$$
\begin{align*}
& \dot{v}_{S}^{j}(t)=\dot{E}_{a}\left(x^{j}\right)+E_{a}\left(x_{n}\right)+a_{i}^{j} ; \tag{24}
\end{align*}
$$

$$
\begin{align*}
& \dot{V}_{A}(t)=\left[E_{m}\left(x^{4}\right)-E_{m}\left(x^{2}\right)\right] \cdot\left[\dot{E}_{m}\left(x^{4}\right)-\dot{E}_{m}\left(x^{2}\right)\right] \text {. } \tag{26}
\end{align*}
$$

where

The brackets of the functions B, D, P dropped for clarity. Moreover $C^{+}{ }_{\mu}^{j}$ are subsets of $O H_{\mu}^{j}$ defined by

$$
\begin{array}{ll}
C^{*} M_{\mu}^{j}: & E_{m}\left(x^{j}\right) 2 \dot{c}_{m}\left(x_{m}\right) \\
C^{-} M_{\mu}^{j}: & E_{m}\left(x^{j}\right)<E_{m}\left(x_{m}\right)
\end{array}
$$

With a suitable choice of initial states the following set of conditions iaples (ii), (v) and (vii):
(a) $\min _{u^{j} \max _{j} \dot{E}_{m}\left(x^{j}\right) \& \dot{E}_{m}\left(x_{m}\right), V\left(x^{j}, a^{L}\right), C^{+} M_{\mu}^{j}, ~}^{m}$,

$$
\max _{u^{j} \min _{j}^{j} \dot{E}_{m}\left(x^{j}\right) \geq \dot{E}_{m}\left(x_{m}\right), Y\left(x^{j}, a^{j}\right) \in C^{-} M_{\mu}^{j} ; ~}^{x}
$$

(b) $\quad \max _{u^{4}} \min _{s^{4}} \dot{E}_{m}\left(x^{4}\right)>\min _{u^{2}} \max _{s^{2}} \dot{E}_{m}\left(x^{2}\right), V Z, C^{4} A$, $\min _{u^{4}}^{\max } \dot{s}_{s^{4}}\left(x^{4}\right)<\max _{u^{2}}^{\min }{s^{2}}_{\dot{E}_{m}}\left(x^{2}\right), V z \cdot C^{-A}$.
for $\dot{q} \neq 0, \dot{\eta} \neq j=1,2$. In the above $C^{2} A$ are subsets of $C A$ defined by:

$$
\begin{array}{ll}
C^{+} A: & E_{m}\left(x^{4}\right) \geq E_{m}\left(x^{2}\right) \\
C^{-} A: & E_{m}\left(x^{4}\right)=E_{m}\left(x^{2}\right)
\end{array}
$$

(c) $\quad a^{j} \dot{i}^{j}=\dot{E}_{m}\left(x_{m}\right) \cdot c_{j}, \quad i^{j} \neq 0, j=1,2$.

Observe that for $x^{j}=0$ there is no need for adaptation and that the system (4) crosses the surface $\dot{4}$. 0 . il 0 time instancenously (vertically) so there is no need for controf in view of the smoothness of trajectories. Conditions (a), (b) are called control conditions helping to design $p(\cdot)$, condition (c) is called adaptive, helping to design adaptive laws. Let us check that (a), (b), (c) indeed imply (ii), (v), (vii). Consider first the case $E_{m}\left(x^{j}\right) \notin E_{m}\left(x_{m}\right)$. Substituting (c) into (23) in view of (ii) we obrain $\dot{V}_{s}$ a negative terms - $\dot{E}_{\mathbf{m}}\left(x^{j}\right)$. Boundedness of the work space necessitates the power: $\dot{E}_{w^{\prime}}\left(x^{J}\right) \leqslant 0$ thus (1b). Substituting (a), (c), and (11) Into (24) with (18, we satisfy (v) in stipulated time $t_{c}$. Note that this holds for any initial states. The case $E_{m}\left(x^{j}\right)=E_{m}\left(x_{m}\right)$ is trivial as then $\dot{V}_{S}^{J}$ - $3 \dot{E}_{m}\left(x_{m}\right)=0, \dot{V}_{j}=\dot{E}_{m}\left(x_{m}\right)-c,-c_{j}$, Finally we check (vit). Again first let $E_{m}\left(x^{4}\right) \neq E_{m}\left(x^{2}\right)$ and observe that (b) substituted to (20) implies (vii). The casc $\dot{E}_{m}\left(x^{0}\right)=\dot{E}_{m}\left(x^{2}\right)$ is obviousily trivial.

Observe that, with (10), (c) is implied by the following adaptive laws

$$
\begin{equation*}
i_{i}^{j}=-\operatorname{sign}!_{1}^{1}\left(D_{m 1} \dot{q}_{m i}^{2}-\frac{c}{n}\right) \tag{28}
\end{equation*}
$$

for $t_{i}^{j} \neq 0,1=1, \ldots, n$. Phystally the solutions $i^{j}\left(a^{j 0}, t\right)$ represent the model energy flux which become positite or negative depending upon where fo is located (below or above the surface al a ) thus regulating the increment of $j$ to sero irom anywhere outside the surface t. 0 .
8. Modular Double RP-Manipulator

Our technique is illustrated below on the case study of the four nof manipulator with two arms shown in Fig. 4.


Figure 4. the modular aRf zinipulator

The Lagrange equetions of motion for each arm result in the following motion equations

$$
\begin{gather*}
\left(m_{1} r^{2}+m_{2} q_{2}^{2}\right) \ddot{q}_{1}+2 q_{2} \dot{q}_{1} \dot{q}_{2}+\lambda_{3}\left|\dot{q}_{1}\right| \dot{q}_{1}+B\left(m_{1} r+m_{2} q_{2}\right) \cos q_{1}-m_{1} g r+\lambda_{1} q_{1}+{ }_{2}^{q} q_{1}^{3}=u_{1}  \tag{29}\\
m_{2} \ddot{q}_{2}=m_{2} q_{2} q_{1}^{2}+\lambda_{4} q_{2}+m_{2}^{g} \sin q_{1}=u_{2} .
\end{gather*}
$$

Here $\lambda_{3}, \lambda_{4}$ are daping coefficients, $\lambda_{1}, \lambda_{2}$ spring cofficients, s-gravity acceleration, the reainder of notations shown in Fig. 4. The superscripts "j", $j=1,2$, are ignored for the time being. ive take the possible payload on the grippers as unknow but within known bounds which askes $\mathbf{m}_{2}$ specified by

$$
\underline{m} m_{2} \leq \bar{m}
$$

where $\quad$, m positive constants. Allowing $\sin q_{1}=q_{1}-\frac{1}{6} q_{1}^{3}, \cos q_{1}=1-1 q_{1}^{2}$, and subdividing the equatiuns (29) by corresponding inertia coefficients we obtain:

$$
\begin{equation*}
\ddot{q}_{i}+r_{i}+\|_{i}=u_{i}, i=1,2 \tag{30}
\end{equation*}
$$

where

$$
\begin{align*}
& r_{1}=\frac{2 m_{2} q_{2} \dot{q}_{1} \dot{q}_{2}+\lambda_{3}\left|\dot{q}_{1}\right| \dot{q}_{1}}{m_{1} r^{2}+m_{2} q_{2}^{r}} \\
& r_{2}=-q_{2} \dot{q}_{1}^{2}+1 / m_{2} \lambda_{4} \dot{q}_{2}, \\
& n_{1}=\frac{\lambda_{1} q_{1}-\lg m_{1} r q_{1}^{2}+\lambda_{2} q_{1}^{3}-18 m_{2} q_{2} q_{1}^{2}+g m_{2} q_{2}}{-m_{1} r^{2}+m_{2} q_{2}^{2}}  \tag{31}\\
& \pi_{2}=8 q_{1}-\frac{1}{6^{2} q_{1}^{3}}, \\
& B_{1}=\frac{1}{m_{1} r^{2}+m_{2} q_{2}^{2}}, B_{2}=\frac{1}{m_{2}}
\end{align*}
$$

The reference model is taken as

$$
\left.\begin{array}{l}
\ddot{q}_{m 1}+\operatorname{m}_{n 3} \dot{q}_{m 1}+1_{m 1} q_{m 1}+g q_{m 2}=0  \tag{32}\\
\ddot{q}_{m 2}+\lambda_{m 4} \dot{q}_{m 2}+y_{m 1}=0 .
\end{array}\right\}
$$

The total encrgy uf the model is

$$
\begin{equation*}
E_{m}\left(q_{m}, \dot{q}_{m}\right)=1\left(\dot{u}_{m 1}^{2}+\dot{q}_{m 2}^{2}\right)+i \lambda_{m l} u_{m 1}^{2}+28 q_{m 1} u_{m 2} . \tag{33}
\end{equation*}
$$

Differentiating it with respect to time and substituting (32),

$$
\dot{E}_{m}\left(q_{m} \cdot \dot{q}_{m}\right)=-i_{m 3} \dot{q}_{m 1}^{2}-\lambda_{m 4} \dot{q}_{m 2}^{2}
$$

Accordingly,

$$
\dot{\varepsilon}_{n}(q, \dot{4})=\left(B_{1} u_{1}-\dot{r}_{1}\right) \dot{u}_{1} \cdot\left(B_{2} u_{2}-r_{2}\right) \dot{u}_{2}
$$

Choose $\left(x^{J 0}, x^{j 0}\right), C^{+} M^{J}, j=1,2$ and $z^{0}, C^{+} A$. Then the control conditions (a), (b) hold if successively
and

(35)

Thus we choose $u_{1}^{4}$ such that for $\dot{q}_{i}^{4} \neq 0$.
$\min _{u_{1}}^{\max }\left[\left(B_{1}^{4} u_{1}^{4}-[1)\right) \dot{q}_{1}^{4}\right]=-\lambda_{m 3}\left(\dot{q}_{m 1}\right)^{2}$
and for such $u_{i}^{4}$, we choose $u_{i}^{2}$ satisfying

$$
\min _{u_{1}^{2} \max _{l}^{2}}\left[\left(B_{1}^{2} u_{1}^{2}-\Gamma_{1}^{2}\right) \dot{q}_{1}^{2}\right]<\max _{u!} \min _{m_{2}^{4}}\left[\left(B_{1}^{4} u_{1}^{4}-\Gamma_{1}^{4}\right) \dot{q}_{1}^{4}\right]
$$

The procedure for $u_{2}^{4}$ and $u_{2}^{2}$ is identical utilizing the second inequalities of (34), (35). Assuming
 $r_{i}^{j}, \mathbb{Z}_{i}^{j}, B_{i}^{j}, i, j=1,2$, we obtain the tracking controllers
and the collision avoidance controllers
which imply the control conditions (a), (b) for our example. The adaptive laws (It) are

$$
\begin{aligned}
& i_{i}^{j} 0, i_{2}^{j}-0 \\
& \cdot i_{3}^{j}=-\left(\operatorname{sign} i_{3}^{j}\right) y_{m} \dot{4}_{m 1}^{2}-L_{2}^{c},
\end{aligned}
$$

 simulation of our modular case, with the data $m_{l}=70 \mathrm{~kg}, \underline{m}=30 \mathrm{~kg}, \bar{m}=10 \mathrm{~kb}, \mathrm{r}=10.60 \mathrm{~m}, \mathrm{ml}=20,1 \mathrm{~m}=20$, $\mathrm{m}_{\mathrm{m}}=5$, $\mathrm{m}_{4}=2$, is shown in Fig .5 , and confirms the convergence-ivoidance required.


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# Problems and Research Issues Associated With the Hybrid Control of Force and Displacement* 

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## 1 Introduction

Robot e working in a perfectly structured world do not need sensing: a structured world in which the dimensions of all parts are within tolerance and in which careful planring has taken place to ensure that such parts can be assembled, a world in which everything is precisely located and everything functions as planned, a world in which all necessary jiga[1] have been designed and provided. Such a world is the production engineer's dream and will probably never exist. Even in today'a most automated and structured factories there are still operators present to "un-jam" the perfect machines when the struclure gets a little out of line. These machines, which we call robots, are of course only programmable universal transfer devices, machines which can be programmed to move tirelesaly from position to position as perfectly as the parts and machines they work with. Real robots don't belong in factories any more than people do; what is needed in factories is well designed automation tended by operators.

Of course there is a limit to the number of well designed pieces of automation we can have. In the home, for instance, 2 sewing machine and a food-processor do their jobs much better than humans[ 1 ], but the modern kitchen is slowly beginning to fill up with such special purpose devices, which the displaced humans now spend their "leisure" time fixing. Humans are needed to provide the structure required by these devices. The dish-washer functions we!! in its own environment [1], but who puts the dishes in and take them out? Further, the automation of many tasks such as dishwashing requires the substituion of massive quantities of energy and natural resources (The Regular Cycle uses 20 gallons of water which must be heated to at least $180^{\circ}$ ) in place of intelligence ("That plate's o.k.,"he didn't use it, just brush off the crumbs,").

When any lack of atructure occurs, however, we must rely on sensors. If something is misplaced, or if something will not fit, relying on a sensor-less, geometry controlled approach would be a disaster. Sensors have two roles, to monitor task execution and to establish the state of the world. Both theme task require the use of a world model. Both tanks also require reasoning and planning. Establashing the state of the world requires a sensor strategy and the interpretation of sensor data in terms of the world model. Monitoring task execution also requires sensor strategies and interpretation of sensor readings in terms of the world model. Errors, when they occur, are detected by the interpretation of sensor data, once again in terms of the world model. If this is to ba done reliably a number of independent sensors is needed (sensors fail also). Once an error state is determined, appropriate recovery action must be planned. If a part is dropped on the floor, then it may be left, but if it is dropped inside a mechanism where it could prevent functioning, then it must either be retrieved or the mechanism replaced. Error recovery is not simple.

Of all the sensors that a robot might have, force sensing in the most fundamental. Blind people function quite well in the world but people who have no kinesthetic feedtrek are totally helpless. Consider the well structured world of manufacturing with a task fully under position control: the detection of any unexpected force is a clear indication that something has gone wrong. Force sensing can provide this vital information. In any situation where complete structure is absent, force sensing becomes mrimary in the sequencing of a task. Consider teleoperation where tasks have some structure 2 ).


The general-purpose manipulator may be used for moving objects, moving levers and knobe, asembling parta, and manipulating wrenches. In all these operations the manipulator must come into physical contact with the object before the desised force and movement can be unade on it. A collision occurs when the man nipulator makes this contact. General-purpose manipulation consiats easentially of a series of collisione with unwanted forces, the application of wanted forces, and the application of desired motions. The collision forces should be low, and any other unwanted forces should aleo be amall.

## Goertz identifies three clear atates:

1. Motion in free space.

## 3. The exertion of a force.

This work of Goertz influenced the use of force sequencing in the manipulator control system WAVE[3] and later that of Inoue[4]. Two types of commands were included in a language WAVE deacribing a sequence of manipulator motions.

STOP terminate the current motion when a force equal to the argument in detected, also known as a "guarded move" [5].

FORCE during the next motion, the force in a give: direction, is to be controlled to the value given as an argument. If the force is specified to be zero then the manipulator is "free" to move in the direction specified.

A further command allowed for a force to be applied by the manipulator, of course, the manipulator would have to be free in the direction In the "Force Vector Assembler Concept [ 6$]^{\text {" }}$ commanded manipulator Cartesian velocitiea cculd be modified by measured end-effector forces and moments

$$
\begin{equation*}
\mathbf{v}=\mathbf{v}_{0}-[\mathbf{M} \mid \mathbf{f} \tag{1}
\end{equation*}
$$

Off-diagonal elements of the matrix $M$ allowed for motion to be specified in directions orthogonal to an appiied force. A curious side effect of this produced a switching phenomenon similar to that described above-a continuous control system with two states. The end effector would trace along an edge until a corner was reached and then proceed to trace along the next edge. Unfortunately it was not possible to continue in this fashion along the following edge. A similar "switching" phenomenon occurs in a special device for making chamfer-less insertions. The pin is brought into the hole at an angle, on contact a linkage rotates the pin to align it with the hole axis whereupon insertion occura. These phenomena are, however, limited to only two states and do not generalize further. Recently this type of control has formed the basis of more complex insertion strategiea $[7$ ] in the form of a "generalized damper" in which the force expected is proportional to the velocity error along some direction. Both of these strategies are limited to two-state systems. A task to insert a key into a lock, turn the lock 180 degrees, and then
withdraw the key, cannot be characterized by such a continuous syatem. It is, however, simple to deacribe auch a task in terms of force/displacement transitions ind control mode switches such as used in WAVE [8].

We may then characterize manipulator control into two basic statee and transitiona between them:

- When a manipuiator is moving in free apace it controle diaplacement and monitora force. The detection of any unpredicted forces indicates a serious error atate.
- When a manipulator in constrained by the environment it controls force and monitors displacement. The detection of any unpredicted displacement indicater a serious error.
- On contact, or on breaking contact, the control and monitoring modes switch. As contact is made the reaction forces rise, indicating contact. When the desired contact force in obtained the control mode awitches from displacement control to force control and the contact force is maintained at the required value. As contact is broken the reaction forces go to zero and the control mode switches to displacement control with the contact force maintained at zero.

The detection of contact is a problem for a rigid manipulator of finite inertia. When contact is detected the manipulator is brought to reat discontinuously - it is stopped. The kinetic energy is dissipated by various mechanisms, some potentially destructive. Given the stiffness of the manipulator and of the environment there is a clearly defined maximum apeed at which contact may be safely detected and controlled.

## 2 Force and Position Controlled Degrees of Freedom

When a manipulator is constrained by the environment, force is controlled. There are, however, six environment constraints, three of translation and thren of rotation. For each of these six degrees-of-freedom either force control or poaition control may be specified[3].

A robot manipulator closing a door by grasping the handle firmly has only one degree of rotational freedom - the rotation of the door about the hinge axis. In this situation force control is required along ail three translation axes and force control is required about the two rotation axes perpendicular to the hinge axis. Rotational position control is required about the hinge axis. Note that one doesn't simply push on the door handle to close the door but one controls the angle of closing as a function of time - how fast the door is closed - "Don't slam the door!" All the remaining axes are in a iz...econtrol mode with a desired force of zero along and about all other dxes. Notice also in the above example that the forces and displacement control modes may be simply described in some orthogonal coordinate frame. In the example given, the origin of the cuordinate frame would be along the hinge axis with one of the axes aligned with the hinge axis. If the $z$ axis were aligned with the hinge axis then
we could apecify the compliance needed as shown in Figure 1. Notice the motion request ROTATE and the motion WITH

PORCE $X=0$.
PORCE $Y=0$.
FORCE $z=0$.
TORQUE $X=0$.
TORqUE Y - O
ROTATE ABOUT 2

## UXTIL

TORqUE 2 - 100;

Figure 1: A Program to Close a Door
termination specification UATIL TORQUE $Z=100$.
Manipulators are controlled by actuators located at their joints. To provide for the control of the six Cartesian environment variables, position and rotation, six joints are required. Ii a degree of freedom of the manipulator is constrained then attempting to control all six joints wilt result in an over-constrained system; large internal forces can result. If one of the joints which contributes to motion in the constrained direction or axis is controlled in force in place of displacement, the overconstraint disappears and the system is controllable. This approach was first used by Inoue in turning a crank|9] and later formed the basis of the compliance used in WAVE[3]. If more than one degree-of-freedom of constraint exists then additional joints must be force controlled to provide for each constraint. In the door closing example given above, five joints of a six-degree-of-freedom manipulator would be force controlled at zero force, and one joint, whose principal motion was in the door closing direction, would be displacement controlled.

If the motion of the joint selected to provide a degree-of-freedom does not correspond completely with the constrained direction then the the position of the manipulator will be modified in the unconstrained directions In turning the crank, the crank would be either slightly ahead or behind its correct position. If this matters it may be compensated for by modifying the commanded Cartesian position[10,11]. The major problem with compliance provided in this manner is in selecting the appropriate joints to provide the compliance. While it is always obvious which joints should be controlled in any given situation, there is as yet no formal algorithm to select these joints automatically. Another drawback is that in certain motions the joint to provide the compliance changes as the motion is made. Consider turning a crank: with the crank at the top of its motion, a joint which controls vertical motion would be appropriate to provide the necessary radial compliance, but as the crank is turned the radial direction requires a joint which controls horizontal motion. Switching between joints can be done[3] but it is difficult.

This form of compliance is very simple to implement in manipulators whose actuators are powered by electric motors as motor current is directly proportional to torque[3]. Joint friction and gearing, however, detract from this simple form of control and various attempts have been made to close a torque control ioop around the joint[12]. These
methods have met with only moderate success a the control coupled two rigid aystema of comparable frequency response[13].

In 1981 Raibert and Craig developed a control method called "Hybrid Poaition/Force Control| 14 |," bued on the theoretical formulation of the above compliance methods by Mason|[5]. In this method not only was the compliance apecifled in an appropriate Carteaian coordinate frame but the control exparation between poaition and force was also performed in Cartesian coordinatea. The observed joint ponition of the manipulator was converted into Cartesian coordinates and subtracted from the dorired Carteaian coordinate poaition yielding Cartesian position errors. Any poaition errors in a complying or force control direction were then set to zero and the remaining errore were tranaformed back into joint coordinates using the Jacobian inverse. These errors wert then fed to a PID controller to reduce errors in position controlled directions to zero. Note that no position feedback is applied in any complying direction. Similarly, force errors were compared to the desi.ed forca to yield force errors in the Cartesian control frame. Any errors in a nor.-compliant or position controlled direction were then set to zero before these force errors were transformed into joint torque errors by the Jacobian transpose. Note that no forces were specified in any position controlled direction. In the system implemented by Raibert and Craig|14|a force and torque sensor was mounted at the wrist of the manipulator to provide feedback for the force loop. Stabilization of the force loop was, however, marginal with resort to ad hoc control methods necessary. Once again we have two rigid systems of comparable frequency response, the manipulator and the force sensor, such a system is very dificult to stabilize \{13|. A similar system naking use of the relationship between motor currents and joint torques has also been implemented(11). This system, with open-loop torque control, does not suffer from the stability problems but does suffer from frictional and gearing disturbances.

In 1983, Khatib, at Stanford University went one step farther and resolved the manipulator joint inertias into effective Cartesian Inertias seen from the end-effector of the manipulator|16|. Once Cartesian position errors were detected, using the hybrid position/force control scheme, a PID controller was implemented in Cartesian end-effector coordinates to produce corrective accelerations which were then transformed into corrective forces by the effective Cartesian inertias. The resulting forces were transformed into joint torques in order to control the manipulator, again using the Jacobian transpose relationship between forros and joint torques. Unfortunately, as the manipulator configuration changes, the rate of change of effective Cartesian joint inertia varies much more rapidly than does the corresponding joint inertias. This is a considerable computational burden. A second problem is that feedback gains are applied in Cartesian coordinates while the manipulator is actuated in joint coordinates, and while it is possible to set constant high gains in joint coordinates it is not clear if similarly high gains are possible in Cartesian coordinates. No comparison between control methods for the same manipulator has been made.

## 3 Stability

In the previous Section we deacribed the hybrid control of position and force. This represent the two states described by Goertz|2]. In this Section we will consider the stability of these two modes and the problems of transitions between them. In the position control of manipulators high atiffess is desired so that the manipulator will be unaffected by disturbances. We would like the manipulator to move swiftly from position to position, stopping as quickly as possible with no overshoot. When the manipulator is begin controlled at some position, we would like it to be unaffected by the application or any external forces of moments. It should act like a very stiff damped spring, very hard to deflect and dead-beat in its response to external disturbances. This is achieved by the application of feedback. Position feedback is required to provide stiffness, velocity feedback to provide damping, and integral feedback to provide for the removal of any bias forces. Feedback gains are limited by the stiffness of the manipulator itself. The setting of gains and the design of a manipulator for a given stiffness are a difficult engineering problems. The result is a system which has a well behaved basic response with a number of high frequency modes which decay slowly when excited. Such systems behave adequately in position mode but perform poorly in force control. The force sensor and environment are, unfortunately, both sys:ems with natural frequency responsen of the same order of magnitude as that of the manipulator. When these are coupled by contact of the manipulator with the environment then the resulting system is very difficult, if not impossible, to stabilize [ $17,13,18,19,12,14$ ]. Whitney and Eppinger : $n$ their papers both indicate that stability may only be obtained when the sensor is stiff and the environment soft or when the sensor is soft and the environment stiff. Unfortunately, a soft sensor completely negates the stiffess required for position control.

The remaining problem is the implementation of the transitions between position and force control. This occurs when the manipulator makes contact with the environment. Contact between a rigid manipulator and a rigid environment is not well defined-the manipulator is moving at some velocity and then it is stopped. Where does the energy go? It is absorbed by the compliances in the system and, hopefully, dissipated. This can be destructive of many mechanical components such as, precision gears, shafts, actuators, etc. To run any commercially available robot into a brick wall would result in considerable damage! The use of any form of force sensing aggravates this problem: as the furce sensor is typically the least stiff member of the system, the most iragile, and absorbs all the energy. The design problem of Scheinman's "Maltese C: 2 ss" wrist force sensor was not the sensor itself but the force overload mechanism nreded to protect it from damage. No form of force sensor based feedback changes this problem as the time constant of the interaction is much shorter than that of the regulator. On
contact, the force sencor mane mapldy increnaing force and the seneor output goes immodiately offecale. The time acale of thin interaction is of the order of a few microseconde. This aignal in processed by a regulator which has a well defined minimum time response of the order of milliseconds. Contact is long since over before the regulator can reapond and any damage to oceur han already occurred. The contact problem is unsolved for rigid manipulator, rigid sensor, rigid environment problems.

## 4 Mechanical Compliance

Based on a careful analysia of a peg-in-hole insertion [20] and the force-vector steering method $\left\{6_{i}\right\}^{2}$ a mechanical implementation of an insertion algorithm was developed at Draper Laboratories, the "remote center compliance - RCC" $|21|$. This device provided the necessary compliance to make peg insertions into low clearance holes from a vertica! direction. The compliance was provided passively by springs. ${ }^{1}$

In the initial version of the remote center compliance no displacement sensing was provided, making the device very susceptible to damage if the displacement capacity of the device was exceeded. However, a later version, "the Instrumented Remote Center Compliance - IRCC" also provided displacement sensing which could be monitored to prevent damage. Both devices could be locked for position control to provide the two necessary control modes. The device was low inertia with high bandwidth so that contact could be made at high speed by the manipulator with the small energy of contact (due to the low inertia of the RCC) absorbed by the passive compliance. The use of passive compliance solves the contact probien ${ }^{2}$ although the device must be locked to provide for position control and the stiffness $k$ is defined mechanically and may not be programmed.

In order to overcome the locking problem Roberts $\left[23^{\circ}\right.$ investigated an instruinented single compliant link. The displacement of the link was used to stiffen the link for position control and to soften the link for force contrcl. In the position control mode any displacement of the end of the terminal link caused the manipulator to move in the opposite direction so as to restore the initial position. In the force control mode any displacement of the terminai link would cause the manipulator to move so as to restore the initial displacement. Contact could be detected by the deflection of the terminal link and the resuiting motion, while the manipulator was brought to rest, absorbed by the compliant link as in the IRCC. It was shown thas both modes were stable. We are currently worixing on a six-degree-of-freedom version of the device and hope to show stability and function.

[^11]
## 5 Conclusions

The hybrid control of force and position is basic to the science of robotics but is only poorly understood. Bofore much progress can be made in robotics, this problem need to be solved in a robust manner. However, the use of hybrid control implies the existence of a model of the environment, not an exact model (as the function of hybrid control in to accommodate these errora), but a model appropriate for planning and reasoning. The monitored forces in position control are interpreted in terms of a model of the task as arr the monitored displacernents in force control. "he reaction forcen of the task of "writing" are far different from those of "hammering." The programming of actions in such a modeled world becomes more complicated and systems of "task level" programming need to be developed.

Sensor iased robotics, of which force sensing is the moat basic, implies an entirely new level of technology. Indeed, robot force sensors, no matter how compliant they may be, must be protected from accidental collisions. This implies other sensors to monitor task execution and again the use of a world model. This new level of technology is the "task level," in which task actions are specified, not the actions of individual sensors and manipulators.

## 6 Research Issues

We may identify the following research issues in position and force control:

- Matching individual joints to Cartesian degrees-offreedom.
- Control of the force of all the links of a manipulator not simply control of the force exerted at the endeffector.
- The hybrid position/force control of redundant manipulators.
- Robust rigid manipulator, rigid environment force and contact control.
- Contact transitions
- Compliant end-effector control of robot manipulators to provide for both position and force control.
- Compliant manipulator control to provide for both position and force contral.
- Task level systems to provide for the protection of sensors.
- Motion modeling.

[^12]
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# Adaptive Hybrid Control of Manipulators 

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Abstract

5 controllers fur poirot manipulators Within the hybrid control architecture, The force controliot is composed of an adaptive PID feedback controller, an
 desired fores setpoints in the constraint directions. The position controller consists of adaptive feedback and fecdformardeontroliers and an miliary final; and it accomplishes tracking of desifedpositiontrajectories in the free directions. The controllers are capable of compensating for dynamic crosscouplings that exist between the position and force control loops in the by rid control architecture. The adaptive controllers do not require knowledge of the complex dyenelc model or parameter values of the manipulator or the environment.
 implementation in on-ife control with high aneling rates.

## 1. Introduction

Although control of robot manipulators has been studied extensively fa recent years. this study hes been focused primarily os position control of manipulators in fret motion lithia an unconstrained onvirongert. In may practical applications. the manipulator is constrained by the environment end certain degress-of-freodran
 the contact forces mist be controlled in the constralat directions, vile the positions are controlled simultaneously in the free directions.




 architecture, based on the analysis of Mason [fl. for "hybrid control" which alloys forces to be controlled in
 directions by position controller. Rabert and Craig. however. do not prescribe teal end systematic method for the design of position and force controllers. Nevertheless. hybrid control has gained considerable popularity over the other two alterative for almultaneons position and force control (6-:3).

The present paper pats forth aystemetcesthods for the design of adaptive force and position controllers Within the by bid control architecture. The fore e controller achieves tracking of desired force setpotats, while the position controller accomplishes tracking of desired position trajectories. The force and position
 controllers are computationally fast and satiable for online implementation with high agility rates. The
 manipulator or the environment.

The paper is structured as follows. Ia Section 2, the hybrid control architecture is outlined and the
 control (MRAC) theory. The design of position control system is discussed briefly in Section 4 . In Section S. the force and position controllers are integrated in the hybrid control architecture. Finally. Section 6 discusses the results of the paper and draws some conclusions.

## 2. Problem Statement

 position trackiag coatrol problems ere stated.


 directions of notion ead force depend on the ature of the partionler task to be performed sad are refleceed la

 position and force vagtabies in the dometralat frame defined vith respeot to the task geonetgy [4]. Note that







In the bybrid force/position control problen nddressed in this paper, we coasider the "virtmal Cartetias force $F$ acting on the end-effector ta the meipulated variable and the poition or force of the end-iffector at the controlied variables [14]. The hybid control arohitecture is besed on two indepeadent and nom-isteractiag
 controller which acts in (Z). The position controller generates the Cartesian end-affector force fy required co cace the end-affector motlon to track a desired position trajectory in (Y). The force controlior produces the Cartesian ind-effector force $\mathrm{E}_{\mathrm{z}}$ aeeded to ensure that the ead-effector force follows desired force setpotat in (Z). Since we cemaot physically epply Cartesian forces to the end-affector, we initead competand implement the equivalent jolat torques needed to effectively canse these forces. The required joiat torques are obtained from the Cartesias forces by matas of the Jacobian mitin $J(\underline{\theta})$ of the mantpulator, vhere $Q$ is the joint angle vector.

We shall now addrest the problems of force ad postion control soparately in sectione asd 4 ase thea integrate the results in Section 5 .

## 3. Design of Force Control Systen

 adaptive force control schene is developed.

### 3.1 Dypaeis Force Model

 complex [13]. Bovever, the dyanic behavior of this systen can be modelied approximately by atas-apriagdamper in each degree-of-fresdom as sbovi in figure 2 ated described by tbe differential equation

$$
\begin{equation*}
m i(t)+d i(t)+z z(t)=f(t) \tag{1}
\end{equation*}
$$

 (Z) cao be expressed by the differeatial equetion

$$
\begin{equation*}
n_{0} \ddot{Z}(t)+D_{0} \dot{Z}(t)+E_{0} Z(t)=E_{2}(t) \tag{2}
\end{equation*}
$$


 metrix ad $\mathrm{F}_{2}$ is the mil force vector applide to the ead-affector in the force anbsace (Z). The element of

 forceltorque vector $P(t)$ eierted by tht end-effector oathe eviroaneat tis ralited to $Z(t)$ by the geacralization of 8ooke's lav is

$$
\begin{equation*}
\underline{P}(t)=\mathbf{I}_{0} \underline{Z}(t) \tag{3}
\end{equation*}
$$

Fron equations (2) and (3). Te obiain

$$
\begin{equation*}
A \underline{P}(t)+B \underline{\dot{P}}(t) \bullet \underline{P}(t)=E_{z}(t) \tag{4}
\end{equation*}
$$



 parameters such as the equivalent stiffass and the payload nase. whichars represented by the paramer vector
 included in equetion (4) to represent the dyasic conplise from the poiftion loop into the force loop. -bere I

[^13]


### 3.2 Eeren Control Elamp



$E_{1}(t)=E_{g}(t)+E_{p}(t) E(t)+E_{I}(t) \int_{0}^{t}(t) d t+E_{D}(t) \dot{B}(t)+I(t)$







 veryias bohavioz of the eystoe model (3).

On applying the lianer control lav (6) to the ayoten aodel (3), ve obita

 (7) cam be vittcen as
$\ddot{B}(t)+A^{-1}\left(B+E_{D}\right) E(t)+A^{-1}\left(I+E_{p}\right) E(t)+A^{-1} E_{I} E^{0}(t)=A^{-1}\left[g_{P}-\mathbb{E}(t)\right]$
 space form as

vhere $Z_{\theta}(t)=\left(\begin{array}{l}E^{\theta}(t) \\ E(t) \\ E(t)\end{array}\right)$

 differeatial equetion

$$
\begin{equation*}
\ddot{E}_{m}(t) \cdot D_{3} \dot{E}_{m}(t) \cdot D_{2} E_{m}(t) \cdot D_{1} E_{m}^{t}(t)-Q \tag{10}
\end{equation*}
$$

 the desifed performance of the force control system. Dy chooing $D_{1}$. $D_{2}$ and $D_{3}$ as diagonal matrices. the force efrors vill be decorpled: for iastance

$$
\begin{equation*}
\tilde{E}_{1 i}(t) \cdot d_{3 i} \dot{E}_{m_{i}}(t) \cdot d_{2 i} E_{1 i}(t) \cdot d_{1 i} E_{i i}^{*}(t)=0 \tag{11}
\end{equation*}
$$

 destred behavior ead $d_{2 i} d_{3 i}$ ) $d_{1 i}$ to ensere stability. Bqation (10) can be vitien at

$$
i_{m}(t)=\left(\begin{array}{ccc}
0 & I_{m} & 0  \tag{12}\\
0 & 0 & I_{m} \\
-D_{1} & -D_{2} & -D_{3}
\end{array}\right) I_{m}(t)=0 I_{m}(t)
$$

 the context of MRAC theory. Simee ehe initial values of the actual asd desired forcet are often the same, the




$$
\left(\begin{array}{c}
0  \tag{13a}\\
0 \\
a^{-1}
\end{array}\right)=\sigma_{0}^{1} \quad n_{z_{0}}+\theta_{0}^{0} \mu_{i_{0}}
$$



 cymetric goitivo-dofinite Jange matrix

$$
n=\left(\begin{array}{lll}
m_{1} & m_{2} & n_{3} \\
m_{2} & m_{4} & n_{3} \\
n_{3} & m_{3} & n_{4}
\end{array}\right)
$$

Is the colution of the Lyapanov equation for the referenet model (12). baeoly

$$
\begin{equation*}
M D+D^{\prime} M=-M \tag{14}
\end{equation*}
$$






$$
\begin{equation*}
a_{0}=\frac{1}{\delta_{1}} A^{*}: a_{1}-A^{*}: a_{0}^{\bullet}-b_{2}\left|A^{*}\right|^{-1} ; a_{1}^{*}=[A \cdot]^{-1} \tag{15}
\end{equation*}
$$

where $\left(\delta_{1}, \delta_{2}\right)$ are poititio and sero or poitive scelari, and



$$
\begin{align*}
& \dot{d}(t)=\delta_{1} g(t)+\delta_{2} \dot{g}(t)  \tag{16}\\
& \dot{i}_{1}(t)=t_{1}[g(t) E \cdot(t)] \cdot L_{2} \frac{d}{d t}[g(t) E \cdot(t)]  \tag{17}\\
& \dot{i}_{p}(t)-\beta_{1}[g(t) g \cdot(t)] \cdot \beta_{2} \frac{L}{d t}\left[g(t) E^{\prime}(t)\right]  \tag{18}\\
& \dot{u}_{D}(t)=Y_{1}[g(t) \dot{s} \cdot(t) L]+Y_{2} L_{t}[g(t) \dot{B}(t) L] \tag{19}
\end{align*}
$$



$$
\begin{equation*}
g(t)=u_{3} g^{-}(t)+u_{g} E(t)+u_{6} \dot{B}(t) \tag{20}
\end{equation*}
$$

 are obtaised as

$$
\begin{align*}
& d(t)=g(0)+s_{1} \int_{0}^{t} g(t) d t+g_{2 g}(t)  \tag{21}\\
& \mathbf{I}_{I}(t)=I_{I}(0)+w_{1} \int_{0}^{t} q(t) g \theta(t) d t+G_{2} g(t) \underline{q} \cdot(t) \tag{22}
\end{align*}
$$

$$
\begin{aligned}
& E_{5}(t)=E_{0}(0) \cdot H_{i} \int_{0}^{t}(t) E^{\prime}(t) \Delta_{t} \cdot \|_{2}(t) E(t) \\
& E_{D}(t)=E_{p}(0)+y_{2} \int_{0}^{t}(t) E \cdot(t) L t+T_{2}(t) \text { 童 (t)L }
\end{aligned}
$$

The ferce centrel ing is iken given iy




$+\left\{B_{2} m_{s}+B_{2} m_{3}\right] \int_{0}^{t} E(t) d t+\left[s_{2} m_{3}\right] \int_{0}^{t}\left\{\int_{0}^{t} B(t) d t\right\} d t$


 efing equation (3). This yiolits the limear slaptive forse comitol lay
$E_{g}(t)-E_{F}(t)+I(t)+E_{I}(t) \int_{0}^{t} E(t) d t+E_{p}(t) E(t)-E_{V}(t) \dot{Z}(t)$



$$
\begin{align*}
& 1(t)=1(0)+t_{1} \int_{0}^{t} t(t) d t+\delta_{2} t(t)  \tag{27}\\
& g_{I}(t)=Z_{I}(0)+a_{1} \int_{0}^{t}(t) E \cdot(t) d t+a_{2}(t) \sum_{0} \cdot(t)  \tag{20}\\
& E_{p}(t)=E_{p}(0)+E_{1} \int_{0}^{t}(t) E^{\prime}(t) d t+p_{2}(t) t \cdot(t)  \tag{29}\\
& Z_{V}(t)=E_{v}(0)-r_{1} \int_{0}^{t}(t) \dot{z}^{\prime}(t) d t-r_{2 t}(t) \dot{z} \cdot(t) \tag{30}
\end{align*}
$$

-here

$$
\begin{equation*}
s(t)=m_{3} E(t)+m_{5} E(t)-n_{4} i(t) \tag{31}
\end{equation*}
$$


 respectively, sad hanee equation (31) besomes

$$
\begin{equation*}
S(t)=T_{P}(t)+\nabla_{P}(t)-W_{1}(t) \tag{32}
\end{equation*}
$$



 the term $(t) y(t)$ in the costrol laf (26) cas be written es

$$
\begin{equation*}
\mathrm{S}(t)=\mathrm{L}(0)+\mu_{1} \int_{0}^{t} \underline{(t) Y^{\prime}(t) d t+\mu_{2} g(t) Y^{\prime}(t), ~} \tag{33}
\end{equation*}
$$



 persilel comection of tour suel aodelea.

The force control seleme developed ia this section is ertramely simplo, eface the adaptation lavi (27)(30) generate the controller geise by aeses of siaple lategration asiag. for instace. the traperoldal rale la equaton (33) can be Implemented as
$\left[(i)=\left[\left.(1-1)+\mu_{1} \cdot \frac{I_{1}}{2}\left[g(1) z^{\prime}(1)+g(1-1) z^{\prime}(1-1)\right]+\mu_{2} \right\rvert\, g(1) I^{\prime}(1)\right]\right.$








## 4. Desiga of Posieion Conerol System

 position control cehene ts ioriefty expleised.



 equation (35) can be pritica as [17]

 represeati the dyanic cupling effect from the force loop lato the poition loop which is a faction of the
 Equetion (36) is etet of highly complez noalimes and coupled secosd-order differestal equations.

### 4.2 Pesitifor_conicol Schene

 the results art sumerised in thit section.

The ligeat edaptive position control lev ie given by


 the contribulions due to the feedbeck and feedforwerd controllers respectively. The requifed atilliery aigal and controller gains ari adapted accordiag to the follovias lave:

$$
\begin{align*}
& f(t)=f(0)+\delta_{1} \int_{0}^{t}\left[(t) d t+8_{2} I(t)\right.  \tag{38}\\
& \mathbf{L}_{p}(t)=E_{p}(0)+v_{1} \int_{0}^{t} f(t) E_{p}(t) d t+v_{2} \underline{I}(t) E_{p}(t)  \tag{39}\\
& I_{v}(t)=E_{v}(0)+\eta_{1} \int_{0}^{t}\left[(t) \dot{E}_{\dot{p}}(t) d t+\eta_{2} \underline{f}(t) \dot{E}_{p}(t)\right.  \tag{40}\\
& C(t)=C(0)+\mu_{1} \int_{0}^{t} t(t) \mathbf{c}^{\prime}(t) d t \cdot \mu_{2} I(t) E^{\prime}(t)  \tag{41}\\
& B(t)=B(0)+\gamma_{1} \int_{0}^{t} I(t) g^{\prime}(t) d t+r_{2} I(t) t(t)  \tag{42}\\
& A(t)=A(0)+\lambda_{1} \int_{0}^{t}(t) \ddot{B}(t) d t \cdot \lambda_{2} I(t) \ddot{B}(t) \tag{43}
\end{align*}
$$

Where the 2 al "weighted" position error vector $\mathrm{C}(\mathrm{t})$ it defined as

$$
\begin{equation*}
g(t)=\left\|_{p} E_{p}(t) \cdot\right\|_{v} \dot{E}_{p}(t) \tag{44}
\end{equation*}
$$










## 5. Dyrid Force/Position Control Systan


 cencot be apliod to the ead-affostor le preetice, these end-offector forces mist be mapod into the equivalent
 is given by (19)

$$
\begin{equation*}
I(t)=J^{\prime}(\ell)\binom{E_{5}(t)}{E_{7}(t)} \tag{45}
\end{equation*}
$$

 the matipulasor. Fith appropriate roorderiag of the columas of $f$ if mecusiary.

It is iaportant to control architecture, there eifiste dyamio orosecoupliag from the force coatrol loop fato the position control loop and vice veria. This coupliag is due to the fact that the efd-offector dyeatios ia the Cartitian sace (X) is strongly crosecospledi lic. the applicition of end-effegtor force in asy direction affects the end-


 the hybid control architecture le an importat fatura of the adaptive control scheas of sections ind 4 .

## 6. Discrasion and Conclusions

Simple sdaptive force and position control themes for anipolators in aybid control architecture are described in thia paper. The control achemes are compatetionaliy fast and do mot require the complez dynamic

 design.

There are certala differesces betreea the proposed epproach and the cosveational hybrid control of Reibert and Crais [4]. Firstiy, ia the prasent approach. the force ind posilion control problems are formulated in the
 problems are formalated in the joint space. The proposed formiation resalts in computational improvement since iaverse Jacobians are not ieededia the control loops. Secoadiy, tatheproposedapprosit, thentask astrim operates on the measured variables $t 0$ at to produce the position add force variables that aeed to be controlied; fhereas in [4]. a selection antiz and lis complement are meed after formetion of tracking-arrors. The present epproch sefns eore etralitheformard and appositiag thea the coaventionel appromeh.

An ettractive fentare of the adeptive controllers designed ia this paper is their abilities to conpensate
 architectura. Furthermore, the adaptive force and position controllers bave "learning capabilities" to cope
 fact that the controller gaitos are adipted rapidiy on the basis of the mapilator perforance. The low
 coatgol vith high sanpliag ratas.

## 7. Acknoviedgenent

The research described in this paper was performed at the Jet Propulaion Laboratory. California lastitute of Tachnolosy, under contrect fith the Nationel Aeronatics and Spece Adminiztration.

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Figure 1. Hybrid Control Architecture


Figure 2. Mass-Spring-Damper Model


Figure 3. Adaptive Force Control System


Figure 4. Structure of the Basic Adaptation Module


Figure 5. Adaptive Position Control System

# Adaptive Hybrid Position/Force Control of Robotic Manipulators 

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1. Abstract

Inmethepeper, the problem of position and force control for the compliant motion of the manipulators is considered. The external force and the position of the end-effector are related by a second order impedance function. The force control problem is then translated ito a position control problem. For that, an adaptive controller is de. ged to achieve the compliant motion. The design uses the Liapunov's direct method to derive the adaptation law. The stability of the process is guaranteed from the blapunov's stability theory. The controller does not require the knowledge of the system parameters for the implementation, and hence is easy for app :actions.

## 2. Introduction

While position control is appropriate when a manipulator is following a trajectory through space, when any contact is made between the end-effector and the manipulator's environment, position control may not suffice. Precise control of manipulators. In the face of uncertainties and variations in their environments, is a prerequisite to reasiule application of robot manipulators to complex handing and assembly problems, In industry and space. An important step toward achieving such control may be taken by providing manipulator hands with sensors that provide information about the progress of interactions with the environment. Properly applied force control can reduce the positioning accuracy necessary to perform a given task accurately, and in fact make possible assembly tasks which would be otherwise impossible.

The problem of position/force control has attracted many researchers in the recent past years $[1-5]$. Among these works one can distinguish two different approaches. The first approach is aimed at providing the user with a means of specifying and controlling forces and positions in a non-conflicting way. [1-3]. This involves specification of a set of position. controlled axes and an orthogonal set of force controlled axes. The second approach is aimed at developing a relationship between Interaction forces and manipulator positions, [4.5]. This way, by controlling the manipulator position and specifying its relationship to the interaction forces, a designer can ensure that the manipulator will be able to maneuver in a constrained environment while malntalning appropriate contact forces.

In the first group. Paul and Shimano [1] partition che cartesian space and find the best joints to force servo to approximate the desired force and position commands. Ralbert and Craig [2] involve all joints in satisfying the cartesian position and force commands simultaneously. Whitney [3] arrives at a single loop velocity control scheme with the net effect of controlling the contact force. In that paper. the impedance matrix approach establishes a connection between the two different approaches mentioned above. In all the above works, the structure of the controller depends on the kinematics and dynamics of the manipulator and of the environment. That is, if the end-effector of a manipulator in motion encounters a point with new constraint, then the controller structure must be changed. In the second group, Salisbury [4] defines a il near static function that relates interaction forces to end-effector posit ton. by a stiffness matrix $\ln$ a cartesian coordinate frame. Monitoring this relationship ensures that the manipulator will be able to maneuver successfully in a constrained environment. Kazeroonl. et. al. [5] extend the previous work [4] and define a generalized mechanical impedance for the manipulator which is used for the compliant motion control. Their approach is an extended frequency somali approach of Sallsoury's stiffness control. Also. their testing ts stable and shu.is robustness in the face of bounded uncertainties. In the second group approach, the controller's structure does not depend on the kinematics and dynamics of the manipulator and that of the environment. However, in both groups, the controller requires the knowledge of the parameters of the system.

In this work, the concept of mechanical impedance, $[4,5]$ is used in order to relate the external forces to the position and orientation of the end-efrector. Hence, the prodem of force control is recasted in the position control problem. The objective is to design a controller for the manipulator, so that the perturbed dynamic relationship for the overall system is given by a second order impedance function. For that, a model reference adaptive controller is designed $[6,7]$, where the desired impedance function is used to select the adaptive control model. The direct method of Llapunov is used for the derivation of adaptation laws. This guarantees the stability of the overall system.

## 3. Manlpulator Dynamice

Consider asnipulator with $n$ joints, providing $n$ degrees of freedom. The dynamic equation of auch manipulator ia given by

$$
\begin{equation*}
M(q) q+h(q, q) \cdot g(q)-T \tag{1}
\end{equation*}
$$

where $q$ is the $n$-dimensional veotor of joint angular positions, $q$ and $q$ are, respectively, the veotors of joint angular velocities and joint angular acoelorations, m(q) is the nxn aymetric. positive definite inertia matrix of the manlpulator, $h(q, q)$ is the $n-d i m e n s i o n a l$ vector of Corlolis and centrifugal forces, $g(q)$ is the $n-d i m e n s i o n a l$ vector of gravitational forces, and $T$ is the $n$-dimensional vector of torque inputs, applied to the manipulator.

Let 89 be the perturbation of the jolnt angular position vector $q_{\text {, from }} q_{0}$, and $6 T$ be the perturbation of the input torques, from $\mathrm{T}_{\mathrm{a}}$. Then the linearized dynamic equation is given by

$$
\begin{equation*}
M\left(q_{1}\right) 8 \ddot{q} \cdot G\left(q_{0}\right)=6 T \tag{2}
\end{equation*}
$$

where, $G\left(q_{0}\right)=\left[\partial g / \partial q_{,} \ldots \partial g / \partial q_{n}\right]$ for $q=q_{0}$.
The joint input torques applied to the manipulator, the external forces on the endeffector, and the actuator torques are related by

$$
\begin{equation*}
\delta T=L \delta T_{a}+J_{c}^{T_{\delta}} \delta F \tag{3}
\end{equation*}
$$

where $6 T$, $\delta T$ and $6 F$ are n-dimensional perturbations of the joint input torques, the actuator torques and the end-erfector external forces, and $J_{c}$ is the jacobian matrix which transform Joint angle coordinates to end-effector position and orientation. Also, , the dynamic equation of actuators are approximately given by

$$
\begin{equation*}
\delta \dot{T}_{a}=A_{a} \delta T_{a} \cdot B_{a} \delta U \tag{4}
\end{equation*}
$$

where

$$
\begin{aligned}
& \left.A_{a}=\operatorname{diag}\left[-\lambda_{a l} \ldots \ldots\right)^{-\lambda_{a n}}\right] \\
& B_{a}=\operatorname{diag}\left[b_{1}, \ldots, b_{n}\right]
\end{aligned}
$$

and $6 U$ is the $n$-dimensional vector of actuator Inputs, [5].
From equations (2), (3,) and (4), the dynamic equation of the manipulator and the actuator is given by

$$
\begin{align*}
& \delta \dot{x}-A \delta x+B \delta U+D \delta F  \tag{5}\\
& \delta q=C \delta x
\end{align*}
$$

where
$\delta X=\left[6 q^{T}, \delta q^{\top}, \delta T_{a}^{T}\right]^{T} \in R^{3 n}$

$$
A=\left[\begin{array}{ccl}
0 & 1 & 0^{-1} \\
M^{-1} C & 0 & M^{-1} L \\
0 & 0 & A_{a}
\end{array}\right]
$$

B $=\left[\begin{array}{l}0 \\ 0 \\ B_{a}\end{array}\right]$
$D=\left[\begin{array}{ccc}0 & & \\ M^{-1} & & T \\ 0 & & c\end{array}\right]$
$C=\left[\begin{array}{lll}I & 0 & 0\end{array}\right]$.
and the pairs ( $A, B$ ) and ( $A, C$ ) are respectively controllable and observable.

## 4. Model Reforence Adaptive Control

In this section, controller is designed so that the dynamio perturbation equation of the overall closed-laop manlpulator eystes is given according to the inverse of the desired impedance. To achieve this, a model reference adaptive control strategy is employed. The reference model is chosen such that its transfer matrix is identioal to the inverse of the desired mechanicel lmpedanoe. Hence, the dynanio equation of the reference model is given by

$$
\delta \dot{x}_{m}-A_{m} \delta x_{m}+B_{m} \delta F
$$

$$
\begin{equation*}
8 a_{m}=c_{m} 6 x_{m} \tag{6}
\end{equation*}
$$

where $\begin{array}{rl}A_{m} & =\left[\begin{array}{lll}0 & I & 0 \\ 0 & 0 & I \\ A_{m 0} & A_{m l} & A_{m 2}\end{array}\right] \quad . \quad B_{m}=\left[\begin{array}{l}0 \\ 0 \\ B_{m 0}\end{array}\right] \\ C_{m} & =[I \\ 0 & 0\end{array}$
such that $\delta X_{\text {f }}$ is the $3 n-d m e n s i o n a l$ ineremental veator of model's joints and actuators values, $6 F$ is the $n$ fideensional vector of incremental external forces. The transfor function matrix of the model is given by

$$
C_{m}(s)-\delta q_{m}(s) / \delta F(s)-C_{m}\left[s 1-A_{m}\right]^{-1} B_{m}
$$

such that the two dominant poles of the model are given by

$$
\begin{equation*}
J_{c} C_{m}(s)=\left(J s^{2} \cdot k_{1} s \cdot k_{0}\right)^{-1} \tag{7}
\end{equation*}
$$

where $J_{0}$ is the Jacobian matrix, and $J, k_{0}$, and $k_{1}$ are respectively the inertia matrix, stifrness matrix, and damping matrix of the desired mechanlcal lapedance, given by

$$
\delta F / \delta Y_{m}-\left(J^{2} \cdot k_{1} s \bullet k_{0}\right), \quad \delta Y_{m}=\text { motel'u spatiai displacement. }
$$

Let us define the state error to be

$$
\begin{equation*}
=8 x_{m}-8 x \tag{8}
\end{equation*}
$$

Subtracting equation (6) from (5), we get the dynamic equation of the state error as

$$
\begin{equation*}
-A_{m} e \cdot\left(A_{m}-A\right) 6 X+\left(B_{m}-D\right) 6 F-B 6 U . \tag{9}
\end{equation*}
$$

Let us now choose the input torque to be

$$
\begin{equation*}
\delta U=K_{x} \delta X \cdot K_{F} \delta F \cdot K_{8} \theta \tag{10}
\end{equation*}
$$

where $K_{x}, K_{e}, K_{F}$ are variable gain matrices with appropriate dimensions. Plugging sU from (10) into (9) ${ }^{x}$ we get

$$
\begin{equation*}
\dot{\theta}=\left(A_{m}-B K_{e}\right) e+\left(A_{m}-A-B K_{x}\right) \delta X \cdot\left(B_{m}-D-B K_{F}\right) \delta F . \tag{11}
\end{equation*}
$$

The problem, now, is how to vary the feedback and the reedforward gain matrics. $K_{x}$. K $\mathrm{K}_{\mathrm{F}}$ and $\mathrm{K}_{\mathrm{e}}$ : such that equation (IT) is stable and the state error e approaches zero, according to a prespecifled transient behavior.

To achieve perfect model following, the state error and its derivative should become zero, that is e e. 0. The conditions for perfect model rollowing are given by

$$
\begin{align*}
& B_{M}-D-B K_{F}=0  \tag{12}\\
& A_{m}-A-B K_{X}=0 \\
& A_{A}-A_{m}+B K_{e}=0
\end{align*}
$$

Furthermore, under perfect mocel rollowing conditions in (i2) the error equation (11) will become

$$
\begin{equation*}
\bar{e}-\overline{\mathrm{A}} \mathrm{e} \tag{13}
\end{equation*}
$$

that is, the transient behavior of the state error is determined by the constant matrix $\bar{A}$, which is derined by the desizner and is Hurwitz.

The controller gains of the adaptive syatea should de adjusted such that the overall olosed-100p systen is stable and follows the reference model. The direct method of Liapunov may te chosen for determining the adaptation law, $[6,7]$.

Let the corresponding Liapunov function for adaptation be given by

$$
\begin{equation*}
v=\|e\|\left\|_{P}+\right\| B_{m}-D-B K_{F}\| \|_{R}+\left\|A_{M}-A-B K_{x}\right\|_{S}+\left\|\bar{A}-A_{m}+B K_{P}\right\|_{M} \tag{14}
\end{equation*}
$$

where $P, R$, and $S$ are $3 n \times 3 n$ aroitrary positive definite symetric constant matrices. Also, the quadratio norm for any matrix $F$ and any positive definite symetrio constant matrix $G$ is defined by
$\|F\|_{0}=\operatorname{tr}\left[F^{T} G F\right]$, where $\operatorname{tr}$ - trace.
The function $V$ is positive definite, exoept when there is a perfect model matching it becomes zero. Differentiating $V$, we get

$$
\begin{align*}
\dot{V}= & e^{T}\left(P \bar{A}+A \overline{A P}_{e}\right.  \tag{15}\\
& +2 \operatorname{tr}\left(\left(B_{m}-D-B K_{F}\right)^{T}\left[P \in \delta F^{T}+R \frac{d}{d t}\left(B_{m}-D-B K_{F}\right)\right]\right. \\
& +2 \operatorname{tr}\left(\left(A_{m}-A-B K_{x}\right)^{T}\left[P \in \delta X^{T}+S \frac{d}{d t}\left(A_{m}-A-B K_{x}\right)\right]\right. \\
& +2 \operatorname{tr}\left(\left(A-A_{m}+B K_{e}\right)^{T}\left[P e e^{T}+M \frac{d}{d t}\left(A_{m}-B K_{e}-A\right)\right] .\right.
\end{align*}
$$

Also notice that, since matrix $\bar{A}$ is Hurwitz, then for any given positive definite symatric matrix $Q$ there exists a positive definite symotric matrix $P$ such that
$P \bar{A} \cdot \bar{A}^{T_{P}}=-0$
Now, for the stablilty, $\dot{V}$ should be negative. One way to satisfy this is to choose $\dot{K}_{F}=B^{\dagger} R^{-1} P \in \delta F^{T}$
$\dot{K}_{x}=B^{\dagger} S^{-1} \operatorname{Pe\delta } x^{T}$
$\dot{K}_{e}=-B^{\dagger} M^{-1} \mathrm{Pe} e^{T}$
where $B^{\dagger}$ - $\left[0,0, B^{-1}\right]$ is the pseudo-inverse of $B$. However, since $R, S$ and $M$ are arbitrary
 scalars. Then denoting E - [0,0,I], the adaptation aws can be fiven by

$$
\begin{align*}
& \dot{\mathrm{K}}_{\mathrm{F}}=a E P e \delta F^{T} \\
& \dot{\mathrm{~K}}_{x}=\text { BEPe } x^{T}  \tag{17}\\
& \dot{\mathrm{~K}}_{\mathrm{e}}=- \text { YEPee }^{T}
\end{align*}
$$

With these adaptation laws, the derivative of the Llapunov function, $V$, is given by

$$
\dot{v}=-\|e\|_{Q}=-e^{T} e_{e<0}
$$

which ls negative for non-zero state frror, (i.e. ea). This guaranteas the asymtotic stablilty of the equiliorium polnt, e $=0$.

The proposed design adaptively controls both the position and the end-effector force, and 1s appropriate for compliant motion or the robotic manipulators. The proposed adaptive controller is shown in figure 1.

Moreover, if the spatial displacement and velocity can be directly measured, then the knowledge of $J_{c}$ is not necessary for the implementation of the adaptive controller.

## 6. Conolualons

In this paper, the definition of mechanicel impedance used in [4,5], is employed. The external force and the position of the end-effector are related by a second order lapedance function. The force control problem ta then translated to position control problef. An adaptive controlier is designed for the latter problem to achieve the compliant motion for the manipulator. The design uses the Llapunov'a direot method to derive the adaptation law. The stability of the process is guaranteed from the liapunov's atablifty theory. the alajor advantage of thls method is that the controlier does not depend on the knowledge of the asatect parameters and those of the environment. It uses the measured forces at the end-effector and the position and velocity of the end-effector in the joint space. The controller is siaple and can be easily impleaented by small computers.

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Figure 1. Adaplive Position/Force Controller for Robot

# Flexible Manipulator Control Experiments and Analysis 

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#### Abstract

This-pormotation-deacribet modeling and control design for flexible manipulators, both from all experimental and analytical viewpoint From the application perspective, mana on ${ }^{\prime} \mathrm{n}$ olugoing effort within the laboratory environment at The Ohio State University. where experimentation on a single link flexible arm is underway, Several unique features of this study are ilescribed here. First, the manipulator arm is slewed by a direct drive de motor and hat a rigid counterbalance appendage. Current experimentation is from two viewpoints: 1) rigid body slewing and vibration control via actuation with the hub motor, and 21 vibration suppression through the use of structure-mounted proof-masa actuation at the tip. Such an application to manipulator control is of interest particularly in design of space-hamedt telecoloatic control systems. but has received little attention to date. From an analytical viewpoint, wedicuen parameter entimation techniques within the closedloop for elf tuning adaptive control approaches - Ats tifitioduced is a control approach based on output feriliack and frequency weighting to counteract effects of spillover in reduced -refer model design. A model of the flexible manipulator based on experimental measurements is evaluated for such estimation and control approaches.


## 1. Introduction

Traditionally, robotic manipulator arms have been modeled an being composed of rigid links, with co-located actuators and sensors. towards the goal of ensuring stable and reliable control. In order for typical manipulator arms to maintain this rigid pmperty as modeled while carrying payloads, the mechanical design requires large and massive links. This in turn dictates that the torques applied by the joint actuators be large. and heavy, usually geared motors are needed for actuation. Moreover, the controller for such a system is forced to move the arm slowly and deliberately so as to prevent any swaying or vibrations.
In rent years there has been much. interest in using light-weight, higher performance arms for both commercial and space-baned applications. leading to the study o! Hexible manipulator control. The advantages of flexible robotic manipulators are many. including faster system response and lower energy consumption, smaller actuators and overall trimmer mechanical design, reduced nonlinearity effects due to elimination of gearing, less overall mass and generally less cost. Obvious tradeoffs. however. complicate the issue of tirxibie manipulator control, primarily centering on the design of controllers to compensate for, or to be robust in the presence of flexure effects. With the advent of advanced computational resources, strides are currently being made towards solution of the many problems associated with control design.
Control Impi for lightweight triable manipulator arms has gained the attention of control theorists only recently, and several approacho.s have emerged. Most prominent are approaches which either linearize and truncate for controller design, or solve the nonlinear robotics problem for rigid link motion control and treat the flexible dynamics separately. For example, the problem of observation spillover and truncation error effects is treated in 1 !, where in simulation studies a linear fred back scheme around a reduced order model is introduced for a songle-link manipulator. In 2 control of the rigid motion is accomplished via state feedback $k$ linearization whereas vibrational dynanues are treated as dist urbaner effects. Several other analyses have appeared along these bani. lines. withe various approaches 3.4 .5 .6 .5 . From an applications viewpoint. however. only a few studies have bern documenter hor parameter estimation, system dentiticatom, and control. Most promanamt among these are the works of Book. it al. W. 9.110 .11 for time optimal slew experiments, related studies at JPL in flexible bean control 12.131 , Schmitz and Canon 14

In this presentation we report on progress made to date on modeling and control design for flexible manipulators, both from an experimental and analytical viewpoint. Specifically, we discuss the ongoing effort within the Control Research Laboratory at The Ohio State l'miversity. where experimentation on a single link flexible arm is underway. The manipulator arm is slewed by a direct drive de inter and has a rigid counterbalance appendage. Current experimentation is from two viewpoints: 11 rigid body slewing and vibration control via actuation with the hub motor, and 2) vibration suppression through the use of structure-mounted proofmass actuation at the tip. Real-time parameter estimation techniques, within the closed-lomp for self-tuning adaptive control. is nader investigation and is described briefly here. In these initial studies, a model of the flexible manipulator based on experimental measurements is evaluated.

## 2. Experimental Sotup

### 2.1 Apparetma

Within tise Control Research Laboratory at The Obio Stinte University, everal experimental configurations are uader audy for system identifieation and slewing and vibration contrel for Bexible mechanical structures. In this prementation we foeme on experimeatation and mimulation aanlynis with a single link Aexible arm, depicted in Figure 1 . The arm is made of 0.0625 iach


Figure 1: OSU Flexible Arm
thick aluninum and is counterbalanced with a rigid aluminum appendage with mase equal to that of the arm. Hab actuation is accomplished by a $3 .+\mathrm{ft} \cdot \mathrm{lb}$ direct drive de motor which has an optical encoder with a quadrature digital output to rease motor shaft pocition, and a tachometer to measure motor shaft speed. This, then, allows both hub pooition and velucity feedback for coatrol. Strain gauges (for monitoring and parameter eatimation) and an accelerometer are placed along the arm, and a 512 -elemena CID linear array camera with RS-422 interface is used for sensing the tip position by observing the lamp fixed to the tip of arm. With such a scheme, the tip sensing nsechanism (camera) is utilized in verification and tuning of the predicted endpoint position. A related objective for this setup is to achieve control without camera information feedback, with for example rate and acceleration sensing feedback, for application in space-based manipulator systems where off-structure reference for sensing is impractical.

Sume characteristics of the arm are given below.
Table 1: Arm Characteristic:

| Material |  | 6061-T6 Aluminum |
| :---: | :---: | :---: |
| Modulous of Elasticity |  | $68.944 \times 10^{0} \mathrm{~N} / \mathrm{m}^{2}$ |
| Cross Sectional Area Moment of Inertia |  |  |
|  | Flexible Arm | $3.350 \times 10^{-11} \mathrm{~m}^{4}$ |
|  | Rigid Appendage | $2.427 \times 10^{-6} \mathrm{~m}^{4}$ |
| Lengths |  |  |
|  | Flexible Arm | 1.10 m |
|  | Rigid Appendage | 0.381 m |

The unique features of the structure are the direct drive mechaniam, rhosen to minimize effects of backlash and other nonlinearities due to gearing, and the counterbalance appendage, which provides a more realistic model of application-oriented structures. Such a hybrid structure does, on the other hand, pose unique prisblems for analytical modeling.

Two computing environments are avilable in the laboratory for real-time control and data acquisition. The first such system is an IBM AT which uses different combinations of several custom-built cards in addition to the $A / D$ equipment. These cards inclade a controller for the slewing motor with electronics utilized in processing data received from the linear-array camera. Another card, designed in-house. processes strain gauge and accelerometer data. and includes a low-noise, high-gain amplifier with a low pass

Gilter and excitation to the bridge circuitry. A third custom board is used to drive the proof-manasctuators (discusced in $|17|$ and later in this presentation). The board receives an analog voltage from the $D / A$ and amplifies it to a current which in adequate to drive the actuators. The linear array camera is interfaced to the computer using a cuatom board which converta the camern's nerial data stream to a number corresponding to the position of the beam endpoint. The second computing aystem is the MicroVax II, equipped with commercially available A/D-D/A boards and real-time operating system software. For the study presented here, the data acquisition is carried out using the IBM AT due to the availability of the camera interface electronics.

## :.: Modeling and Frequency Response

For purposes of finite element modeling studies, the arm is enalysed in two separate components, thove being the Alexible arm itself and the rigid counter balance. The cylindrical masa at the end of the rigid component is modeled as a point mase, and the two components are connected at the pinned joint (motor shaft). For the FEM analyain, each component is modeled an a two-dimensional elaatic beam, and the software package ANSYS (18) was used to generate the first five moden of the system shown in Table 2 below. We note that torsional modes were asumed to be insignificant, and were therefore neglected in the analysis. The principle advantage to modeling the arm in this manner is that the effects of the counter balance in the atatic rharacteristirs arr included. Several other approachen were utilized, auch as conaidering the joint at the motor shaft to be a fixed point (clamped free). negating any effect the counter balance may have on the beam dynamics. Experimental reaulta (described belowi) inlicate that the former approach. described above, given the dowest match to meanured responves.

For purponev of comparison, several experiments were conducted in teating response characteristics of the apparatus. An open lowp frequency response was found by applying a sinumsidal system input torque (varying the unotor current), and recording areasurenirut of the tip position: the procodure is similar to that employrd in $1+1$. Data was taken over the range 0.2 Hz to 13.0 Hz . in , trpn , 0.1 Hz . and the results are shown in Table 3. The system poles and zeros were found by noting the frequencies which producrd maxamum and minimum tip deflection. respectively. An inherent aswumption in this technique is that the damping of the bram is very small (this fact was expermentally veritied in an independent study 19|). The damping ratio calculations representel in the talite are based on the asumption that excitation near a modal frequency will result in the reaponse showing primarily omly that particular modal frequency.

## Table 2: FEM Results

| Shode | Firquency (Hertz) |
| :---: | :---: |
| 1 | 2.0091 |
| $\geq$ | +.2509 |
| 1 | 23.1197 |
| 1 | 16.5637 |
| ; | -9.534 |

Table 3: Frequency Response Data

| Minaman Tip Response | Maximum Tip Respotise Damping Hatho |  |
| :---: | :---: | :---: |
| 3.0 Hz | 1.2 Hz | 0.139 |
| 10.3 Hz | 7.6 Hz | $0.0 \% 0$ |
| 11.3 Hz | 12.0 Hz | $0.00 \times$ |

The open lowp itep response (in posstion) of the arm was found by rotating the motor shaft through an angle of 10 degrees and measuring the tip detlection from its nominal value (initial point). After this maneuver the motor holds the new position (that is. is servoingl since the local feedback loosp is active. Figure 2 illustrates a plot of the step response. Note that while the torque is applird at the hib at the $t=0$, the tip defiertion response is delayed by approximately 30 nulliseconds and. in fact. initinily moves in the direction upposite that of the hub rotation. The step responses indicates a setting time of about one minute. A fast Finirier transform of the data allows. iear identitication of the first two modal frequencies; these oncur at 1.18 Hz and 7.5 Hz. respecturly. Figure 3 , hows the result of the FFT for the tip position in the step response test. We note that the rigid body mode , if component) due to the pinned joint has been subtracted ont of the FFT plot for clarity.
3. Control Analysis
\& f Prabic on Firmulation

Comsier axan the ongle link Hexible manipulator system described above, redrawn in Figure $\&$. The displacement of any point
 vibration in present and that the deftertion due to this vibration is small. Let $L$ be the arm length so that in keneral terms

$$
\begin{align*}
& y(x . t)=x(x . t)+r \theta(f) . \tag{111}
\end{align*}
$$



Figure 2

FFT QF PIP POSITION


Figure 3
where $E /(r)$ is (lir elastic stiffness, $A$ is the rrosn-sectional area, and $m$, in the mass denaity. For the mechanical configuration under rominideration, i:) must satisfy the boundary conditions
where $T$ is the turflue at the hub and $I_{H}$ is the actuator inertia. Accordiagly, (2) may be put into the familiar form for the generalized noodal cosordinates q(I) as

$$
\begin{equation*}
M \overline{\mathbf{q}}+D \dot{\mathbf{q}}-K \dot{\mathbf{q}}=B f \quad . \quad \dot{\mathbf{q}}=\left|\dot{q}_{1}, \dot{q}_{2}, \ldots, \dot{\boldsymbol{q}}_{n}\right|^{\boldsymbol{r}} . \tag{4}
\end{equation*}
$$

where.$/ /$ is the mass matrix. $K$ is the xtiffess matrix, and $D$ rontains terma associated with the damping. For position and velority inrasurenirnta lia the $y$ direction) the solution $t o(t)$ in approximated by

$$
\begin{equation*}
y|x .1|=\sum_{t=0}^{n} b_{b}\left(f \mid c_{b}(f)\right. \tag{5}
\end{equation*}
$$



 14.1.

$$
\left[\begin{array}{l}
q  \tag{6}\\
\dot{q}
\end{array}\right]-\left[\begin{array}{cc}
0 & l \\
1 \Omega^{2} & -\phi^{r} D \phi
\end{array}\right]\left[\begin{array}{l}
q \\
q
\end{array}\right]-\left[\begin{array}{c}
0 \\
\phi_{B}
\end{array}\right] f \quad . \quad r(f)=c \cdot\left[\begin{array}{l}
q \\
q
\end{array}\right] \text {. }
$$





We note that in 151 the ignd body mode has bern incluied. Since the only control input for this example is the torque. tinen $f \quad r$ Finally, the matrices ${ }^{\prime}$ and $\phi^{T} G$ are quen by

The fuadameatal ineve in the mathematical formulation of Berible mechanical struct ures lies in the tact that such distributed parameter ayateme must be identified (controlled) with oaly a limited aumber of sencors (actuatora). Incin-d, for the analysis of the siagle-liak Bexible arm we typically consider hub actuation oaly, and tip position and/or hub relocity meanurements to be employed in modeling and feedback control. Moreover, without reliable models for the control denign the analysis beeomet even more difficult. Philonophically, there are several differeat view to take in the coatrol denign. One approach is to conatruct a controller that is reasonably robust in the presence of modeling uncertainties and apillover, and yet simple enough in structure to be easily implementable (for exnmple the variable atructure control approsch [20|). Another approach is to perform aystem identification exercises to model the system an accurately ae poasible prior to coatrol design. A third approach is a combination of the first iwo: eatimate the system parameters on-line (in the cloced loop) and bace the control design on the reanlting model. This late viewpoint is often refered to as Self-Tuning Adaptive Control (STAC).
In the STAC approarh, the manipulator dynamics are represeated by linear discrete-time models, affording the primary advantage that the controller design is inherently digital in nature. In the application to fexible atructurea, tuning parametera include combinations of the damping and modal frequencies, of nome combination of other free parameters which make up the manipulator model. Our approach to the parameter eatimation problem involves recursive leant squares methods with covariance reselting. That is, in order to maintain a fast overall convergence rate, the covariance of the estimates is reset at regular intervals in the algorithm. Such a acheme is particularly attractive for the manipulator control problem due to the time-varying aature of the tuning paranueters during slew maneuvers and varying payload exercises. Experimental atudies of the parameter estimation and STAC approach for the arm described above are presently underway. In the following we present simulation resulta which indicate avenues io pursue regarding implementation.


Fixure 4: Deflection $0(x, 1)$, Slew angle $\theta(t)$


Figure 5: Parameter Estimation Simulation Results

The elements of the $C$ matrix can be found according to (14)

$$
\begin{equation*}
\psi_{i}(L)=\left[\frac{d \psi_{i}}{d x}(0)\right] \omega_{i}\left[\frac{y}{d \psi_{i} / d t}\right] \tag{9}
\end{equation*}
$$

and the $\left(d \psi_{i} / d t\right)(0)$ are solved from a syitem of noalinear equations. The model frequencies can be computed a priori, or identified as discussed in the previous section. We model the system as a stochastic ARMA process and excite a fourth-order model of the arm with a white noise input. Such a representation allows a delay (in tip ponition response, as observed in the actual aystem) to be inserted into the model. Using a arro-order hold circuit in the model and aampling the simulated FEM model, the AR and MA parameters converge to their nominal values an depicted in the sample plota of Figure 5, which shows time histories of one AR parameter and one MA parameter. Values for damping coefficients are then calculated from these parameters. These resulta are not useful in closed-loop control however, due to the length of time for convergence to the true parameters. Note also that a primary difficulty results because of the approximate pole-zero cancellation in the ayatem model (indicated by the spike at about 0.7 se.onds). Slightly better results are obtained if the rigid mode is removed from the model, which corresponds to exciting the unforced system with an initial disturbance. Such an exercise is possible since the motor inertia is considerably greater than that of the arm.

Prior to actual experimentation on the arm, several modifications must be investigated. For example, simulation studies for this and c.,her example systems have indicated improvement for different resetting intervals; for detaila, the reader may wish to consult [21|. Also. simple digital low-pass filtering of the measured variablea has produced improved performance of the parameter eatimator. For control purposes the simulations have shown that an algorithm which turns on the control after allowing the estimator to run for a short period of time (for example. as illustrated in the simulations, about 1 to 1.5 seconds) will achieve the control objective. However, we are presently pursuing ways of improving the time to convergence in the closed loop with approarhes using state fredback.

## J. 5 Outpul Fecdburl. and Frequency Shaping

Generally speaking, hixh dimensionality and multiplicity of inputa in large-scale syatems such as flexible mechanical structures leads to complex crutralized controller schemes. One solution to this problem is to simplify the structure of the model ria decomposition intos subsystrms with associated subcontrollers in a decentralized output feedback formulation. Moreover, centralized or decentralized sutput feedback is one of the more straightforward algorithms, from the viewpoint of implementation, for the control of Hexible mechanical structures; see, for example, $\{22,23,20 \mid$.
For the problem of single-link fiexjble manipulator control, where only hub actuation is employed in the control action, the output feedback approach to controller implementation is centralized in nature. The problem of spillover is, however, a critical issue to consider in the design. In order to minimize the effects of spillover, we consider a frequency-shaped cost functional [24], where penalties ate assigned to the truncated modes and high penalies are assigned to the high harmonics at the input in order to minimize the efferts due to excitation of the residual modes.
We consider the cost functional $J$ to be minimized as formulated in the frequency domain utilizing Parseval's Theorem. With infinite time horizon, such a cost is written in the manner

$$
\begin{equation*}
J=\int_{-\infty}^{\infty} \cdot X \cdot(j \omega) Q(j w) \cdot X(j w)+\Gamma^{\bullet}(j w) R(j w) T(j w) \mid d \omega, \tag{10}
\end{equation*}
$$

 a schemp, onsider the diagram of Figure 6, where the parameters $K_{1}, K_{2}, K_{3}$ are solved for in the minimization oi ( 10 ), and the filter pole lucation (?) is dependent on the system dynamics. In the example uader consideration. $Q(j \omega)$ is the system matrix and


Vider this furmulation. the open-loop state variable representation of the system has the form


Figure 6: Output Feedback Scheme

$$
\dot{v}(t)=\left[\begin{array}{cccccccc}
L & 0 & \psi_{1}(L) & 0 & \cdots & \psi_{n}(L) & 0 & 0  \tag{12}\\
0 & 1 & 0 & \frac{\operatorname{dn}}{\alpha_{1}(0)} & \cdots & 0 & \frac{d x_{1}(0)}{d_{i}(0)} & 0 \\
0 & 0 & 0 & 0 & \cdots & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
q \\
\dot{q} \\
T
\end{array}\right],
$$

where the new measurement $\hat{v}(t)$ now includes the torque $T$. Incorporating the system of (11)-(12) into the output feedback ntructure of Figure 6 (and subsequent solution of the corresponding Lyapunor equation) allows the off-line calculation of the feedback gains from minimization of (10) by an appropriate nonlinear optimization routine.
We consider now a simulation of the flexible arm system, using a five-mode model from the FEM as the "truth modei", from which measurements are taken and fed back in the output feedback scheme. The controller design for this example is based on the reduced system of rigid mode plus first flexible mode, and the resulting control is then tested against the full-order truth model to illustrate the effects of the frequency weighting approach in reducing apillover. A conjugate gradient method is used in the optimization portion of the design, and the final values obtained for the control law (with $\gamma=4$ ) are $K_{1}=-110.09, K_{2}=-111.53, K_{3}=-88.83$ (feasible values for the system under cousideration) for the cost which reached a minimum after apprcximately 3000 iterations. The results using this cuntroller are illustrated in Figure if for a step input torque; this applied input is such that the tip rotates through a small angle (of less than $5^{\circ}$, in terms of the rigid position). The values for torque begin at zero, that is, the de component is subtracted out. In the simulation, the response settles in about ten secondn, whereas the free response decays after about one minute due to damping included in the model.

## 4. Structure-Mounted Proof-Mass Arzuation

Sirce the large-angle slewing problem is complicated due to the flexibility effects inherent in the structure to be slewed, one is naturally led to investigate the possibility of relegating, at least partially, the task of vibration damping to a separate sensoractuator pair and associated feedback loop. To this end, we liave been investigating utilization of a structure-mounted momentumexchange device mounted near the tip of the single-link manipulator. The practicality of such active vibration damping in a robotic environurent is closely coupled to the availability of lightweight and effective devices. The device we have considered in our preliminary stulims is a proof-mass actuator developed in our labs to study active vibration in space based flexible mechanical structures.
Non-ground referencil linear actuators are not yet widely available on the market, and this fact led to an in-house development; a general view of the device as mounted on the arm is shown in Figure 8. The device is built around a linear motor manufactured by the Kimco division of BEI Motion Systems which has a total mass of 25 grams, and can deliver a peak force of 2.2 N . The coil (solenoid) is rigidly mounted to a beam clamp which fixes the actuator to the arm. Also connected to the clamp is a rigid aluminum bracket which supports the springs. The proof mass is coupled to the framework through the springs, which are in turn coupled to the framework with adjusting screws so that their rest tension and proof mass rest position can be controlled. There is sufficient adjustment so that springs of different length and stiffness constant can be acconodated. The springs provide a restoring force for the proof mass and transfer force to the structure. A hanger was also mounted to provide strain relief for the feed wires.
The proof mass consists of a rectangular steel ring with a central steel nember. This central member passes through the coil and restricts motion to a single axis. Samarium cobalt magnets are fixed to the top and bottom inside edges of the ring (adjacent to the coil) ow that the interaction of the permanent magnetic field with a current in the coil results in a force on the proof mass.
Details of the development of the dynamic model of the actuator may be found in [17]. The net force applied to the tip of the arm (where the actuator is mounted) may be given as

$$
\begin{equation*}
f=2 k:-K_{F} i_{a}+B \vdots, \tag{13}
\end{equation*}
$$



Figure 7: Output Feedback Example


Figure 8: OSU Proof-Mass Actuator

$$
\begin{equation*}
m \bar{z}+B \vdots+2 k==K_{r} i+m \bar{j}, \tag{14}
\end{equation*}
$$

where $m$ is the proof mass, $y$ the displacement of the structure at the point of actuator attachment, $f$ the force acting at that point, $=$ the relative displacement of the proof mass, $B$ the viscous damping coefficient, $K_{r}$ the motor force constant, and $i_{\text {, }} i_{4}$ are the input and armature currents, respectively. The actuator constanta taken from the data sheets which accompany the individual componeats, are $m=0.0207 \mathrm{~kg}, \mathrm{i}=262.7 \mathrm{~N}-\mathrm{m}^{-1}, K_{r}=1.112 \mathrm{~N}$-ampere ${ }^{-1}$. The incorporation of the above actuator creates a second feedback loop to which the task of vibration damping is relegated. The two control loops (for slewing and for vibration damping) can be buth designed and implemented in a decentralized manner. Note that $y$ includes the displacement due to both the rigid body mode and the flexibility (see (1)). The principle of relegation implies that we design the feedback control only for the latter portion. To, this end consider a vibration damping loop for only the first mode, such that the relevant expreasion is

$$
\begin{equation*}
\vec{q}_{1}+w_{1}^{2} q_{1}=b_{1} f, \tag{15}
\end{equation*}
$$

where $\omega_{1}$ is the natural frequency of the first mode and $b_{1}$ is an influence factor determined from the mode shape at the point of interest. Acceleration feedback can be used from the co-located accelerometer and a simple PI controller has been designed. It is evident, however, that the STAC approach or the frequency weighted control approach outlined earlier, can also be used here. The incorporation of the more sophisticated design approaches resulting in more complicated controllers will aid in handing more than the first vibrational mode. Studies along this direction are presently continuing.

## 5. Conclusion

In this workshop presentation we have described work in progress on modeling, parameter estimation, and control studies for an experimental, one-meter single-link flexible manipulator arm. Models have been developed for the apparatus based on finite element analysis and experimental verification. These, with the closed-loop parameter estimation procedures described here, and subsequent STAC approach for control, are being evaluated on the laboratory arn.
Unier insestigation is experimentation involving local proof-mass actuation for vibration control at the tip of the arm, using a device ir. $\cdot$ rloped in the Control Research Laboratory at Ohio State for flexible structures control work. The output feedback fregnency shaping approach described here may be easily extended to this application, where the formulation is decentralized in nature: resinlts on this technique for general Hexible structure vibration control will appear in [25]. Finally, various other centralized (for tine rase of '.nls actuation only) and decentralized approaches are currently being evaluated in the laboratory.

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# Dual Arm Robotic System With Sensory Input 

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#### Abstract

1. Abstract

The need for dual arm robots in space station assembly and satellite maintainance is of increasing significance. Such robots will be in greater demand in the future when numerous tasks will be assigned to then to relieve the direct intervention of humans in space. Technological demands from these robots will be high. They will be expected to perform high speed tasks with a certain degree of autonomy. Various levels of sensing will have to be used in a sophisticated control scheme.


 time software to produce a two-arm robotic' system that can accomplish generic assembly tasks. The paper will_concanoale-moally_on the control hierarchy, the specific control ore approach selected being the Variable Structure (Sliding Mode) Control approach mill commenter a decentralized implementation of model-reference adoptive control using Variable Structure" controllers and the incorporation of tactile feedback into its con sided.
2. Introduction

The need for dual arm robots in space station assembly and satellite maintenance is of increasing significance. Such robots will be in greater demand in the future when numerous tasks will be assigned to them to relieve the direct intervention of humans in space. Technological demands from these robots will be high. They will be expected to perform high speed tasks with a certain degree of autonomy. Various levels of sensing will have to be used in a sophisticated control scheme.
In this presentation we will briefly describe ongoing research in control, sensing and real-time software to produce a two -arm robotic system that can accomplish generic assembly tasks. The paper will concentrate mostly on the $\cdot$ astrol hierarchy, the specific control approach selected being the Variable Structure (Sliding Mode) Control approach. We will consider a decentralized implementation of model-reference adaptive control using Variable Structure controllers and the incorporation of tactile feedback into it.

We assume that multi-arm robotic operations have a hierarchical/decentralized control structure. However, the appropriate control algorithms have to be chosen for feedback to properly fit the special hierarchy of multiple robots with dextrous end effectors. A specific control approach has to be selected, and its requirements can be clearly specified:

- It must easily decompose into a hierarchy.
- It must be ameanable for modular implementation.
- It must poses low real-time computation requirements.
- It minus be able to receive changes from sensor data.
- It must be insensitive to modeling errors and load variations.

It is expected that robotic systems will become an important part of future space missions. Orbital Maneuvering Vehicles have been proposed with dual arm systerns for space station assembly, satellite servicing, etc.. Although the importance of dual arm robotic systems have been recognized for some time, little work of a general nature has been done in controlling such systems.
Early multi-processor robotic controllers were based on the principle of a simpler low-level processor and a more sophisticated
high-lovel computer. This made interfacing fairly difficult and expansion almost imponsible. With today's procescors and appsopriate coftware load distribution, tanke at all levela can be handled by processors of the same family. Coordination of data tranafers are extremely aimplified. It is apparent that certain improvementa will have to be made over couventional control atructures (ar used say, in the PUMA) if there is hope of accomplishing sophisticated assembly type operations using multiple manipulators. For versatile performance the control hierarchy will oxhibit a finer tack decomposition. Tanke will have to be relegated to a large number of procensors. Sensory inputa will have to be appropriately ascigned.

The control of robots in a precise, reliable and repentable manaer is by liself a hard problem. The problem becomes nomewhat more complicated when considering the control of coordinated robot arms. Limited work has been done in the asen of multi-arm robot syatems $[1][2],[3],[4],[3],[6]$ and $[7]$.
A mothod developed for controlling manipulator arma by Young [8], Ögüner and co workers [9], [10], and orbers [11] utilising variable atructure control theory is particularly ameanable to extencion to multiple arm aystems controlled within a hierarchical framework. Initial work along these lines have already been performed. In this paper we will be reporting on seceat developments in the above approach and especially tactile sensing feedback from the end effector an included in the hierarchical control structure.
There appear to be certain generic task that are imbedded in many assembly and maintainance operatioca. There include:

- Pick and place type tasks.
- Pin in the hole type tanks.
- Combined rotation-translation type tacks.

Many complex operations can be partitioned into combinations of these generic taks. Thus the control algorithm design and related software will coucentrate on the above tasks.
Figure 1 summarizes the control hiefurchy to be used. At Level $I$, parsing interpreting and decoding user commanda and high level censory input are accomplished. Error messages to the user are also generated at this level. Level II includes trajectory planning, associated coordinate aystem transforma and analyais of bounds of the workspace. Joint-level coordination and tranafer of information required by control algorithma is carried out at Level III. At Level IV, generation of the feedback control and I/O with actuators and force senaing is accomplished. The control algorithm selected has to be strongly coupled to the information atructure selected. The algorithma must be decomposable into the hierarchy impored and inherently adaptive to load and trajectory variations. The algorithm/control approach we are utilizing is the Decentralized Model Reference Adaptive approach using Variable Structure (sliding mode) controllers. It appears that this agorithm with appropriate modifications to accomodate sensory input and user commands can be mapped onto a multiproceson syst-m.


Figure 1: The General Hierarchy for the Control of Two Manipulators

Various resulta have been recently reported in the utilisation of sencory information from the end-effector in the foedbeck control structure. In the present work we will be using the force feedback approach (tactile sensing) as reported in (12). The incorporation of force leedback into the control algorithm is not straight-forward, and in the next section we will introduce the concept of interaction compensation which will add in the analysis.

## a. The Concept of Interaction Compenation

It has been previoualy claimed that the levels of a hierarchy (a in the multi-manipulator aystem of Figure 1), can be clasified no that one identifies "increacing intelligence with decreasing precinion", a one moves up [13]. As with mont labeling achemes, this may be an over-zeneralisation and there may be numerous cases where proper relegation of (a) Control Authority, aad (b) Information Distribution, may reault in preferrable operation of the over-all aystem.

We will conaider the regulation of an interconnected system to introduce the coacept of Interaction Compeasation, which we will subsequently apply to the apecific case of multi-manipulator control; under a faxed Control Authority atructure and control algorithm.
Consider a large-scale syatem consiating of $N$ interconneited subsystems each defined by

$$
\begin{align*}
& \dot{x}_{1}=A_{1}\left(x_{i}\right) x_{i}+B_{1}\left(x_{i}\right) u_{i}+E_{i}(x)  \tag{1}\\
& y_{i}=D_{i} x_{1}, \tag{2}
\end{align*}
$$

 and the matrices are of compatible dimension. $E_{1}(x)$ denotee the totality of interaction effecta from the remaining subaystems to aubaystem $i$. Note that $E_{1}(x)$ may also include modeling errors.

Let us first define what is meant by insensitivity to interaci:Jn. Consider again (1)-(2), rewritten for brevity as

$$
\begin{equation*}
\dot{x}_{i}=f\left(x_{1}, u_{i}\right)+\Delta(x, \ell), \tag{3}
\end{equation*}
$$

for the atate tranaition mapping $f: \Re^{n} \times \Re \longrightarrow \Re^{n}$ and the total interaction term $\Delta(x, t)$. The system (3) is sid to be insensitive to interaction effecta if the solution $x_{1}(t)$ may be expressed as

$$
x_{1}(t)=\dot{z}_{1}(t)+\mathcal{O}\left(c_{1}\right),
$$

where $\overline{\dot{x}}_{i}(t)$ solves $\dot{\boldsymbol{r}}_{\mathbf{i}}=f\left(x_{i}, u_{i}\right)$, for all $\epsilon_{i}>0$ and all $t>T$, for some finite time $T$, and $\mathcal{O}\left(\varepsilon_{i}\right)$ represents termas of degree two or higher in $\epsilon_{1}$.

In reference to measurements available for use at the control inputa of subsystem i we can now consider three possibilities:

1. Full (real-time) interaction information.
2. Partial interaction information.
3. Interaction modeling.

It can be shown that interaction information provides the opportunity of directly negating all effects within the range apace of $B_{1}$. The more interesting cases are when partial information is available or can be generated through dynamic modeling of the interactions.
Given the model, the decentralized control problem is to design a controller to feedback locally available real-time information such that the states of each local subsystem are regulated to zero or track the states of a local reference model.
Let the local reference model for the $i$-th subsystem be given as

$$
\begin{align*}
\cdot \dot{\dot{x}}_{1} & =\dot{A}_{1} \dot{x}_{1}+\dot{B}_{1} r_{1}  \tag{4}\\
\dot{y}_{1} & =\dot{r}_{1}, \tag{5}
\end{align*}
$$

 and its local reference model in the manner

$$
\begin{equation*}
e_{1}=\dot{x}_{1}-x_{1} \tag{6}
\end{equation*}
$$

so that the local error system dynamics may be written in the form

$$
\begin{equation*}
\dot{e}_{1}=\dot{A}_{i} e_{i}+\left(\dot{A}_{i}-A_{i}\right) x_{i}+\dot{B}_{i} r_{i}-B_{i} u_{i}-E_{i}(x) . \tag{7}
\end{equation*}
$$

Within this framework, aname that

- Each local controller denign is dependent only on the local model.
- The syatems (1)-(2) and (4)-(3) are controllable.
- The states $x_{i}$ and $\dot{x}_{i}$ are measureable locally for feedback to the $i$ ith input.
- Reference trajectory information may be fed to a subaystem from a higher level coordinator but interaction information is only partially avilable in real-time, and the subaystem is not allowed to communicate with the other local controllers


## 4. Variable Structure Controllers

In this presentation we are going to asoume that the basicn of Variable Structure Control are known. An important leature of Variable Structure Controllers is the fact that, for the decentralised ease, the local subsystem is made insensitive not ouly to local parameter changes but alno to dynamical interactions with neighboring subaystems once the aliding surface is reached.
We define the sliding surface corresponding to the system (1)-(2) as

$$
\begin{equation*}
\sigma_{i}=\Gamma_{i} e_{i}, \tag{8}
\end{equation*}
$$

 the control law is formulated in the manner

$$
\begin{equation*}
u_{i}=K_{r,} e_{i}+K_{2, x_{i}}+K_{r, r_{i}}+\delta_{i}, \tag{9}
\end{equation*}
$$

 a constant picked according to the norm of the interactions. The elements $\boldsymbol{K}_{r, 1}, K_{e, 1}, K_{r, 1}$, and $\delta_{1}$ are diacontinuous functions of the sliding surface and the coefficients of syatem and reference model state equations. We furthermore claim that estimater for $\delta_{\text {, }}$ can he refined with knowledge on interactions. In the following we derive the appropriate forms based on the reaching condition which, in view of (7), (8) and (9), becomes

$$
\begin{align*}
\sigma_{1} \dot{\sigma}_{1} & =C_{1}\left|\dot{A}_{1}-B_{1} K_{r_{i}}\right| e_{1} \sigma_{i}+C_{1}\left\{\left(\hat{A}_{i}-A_{1}\right\}-B_{i} K_{a_{1}}\right\} x_{i} \sigma_{i} \\
& +C_{1}\left|\dot{B},-h_{r_{i}} B_{i}\right| r_{1} \sigma_{i}-C_{i}\left\{B_{1} \delta_{i}+E_{i}(x)\right\} \sigma_{i} \\
& <0 \tag{10}
\end{align*}
$$

The condition (10) is satisfied provided that the gain parameters and $\delta$, are chosen so as to make each term negative. Thus,

$$
\begin{align*}
& \left(K_{r_{i}}\right)_{\ell}=\alpha_{i l}\left(C_{1} B_{i}\right)^{-1}\left\{\sum_{j=1}^{n_{i}} c_{j}^{i} a_{j \ell}^{j}\right\} \operatorname{sgn}\left(\left(e_{i}\right), \sigma_{i}\right) ;  \tag{11}\\
& \left(K_{z,}\right)_{l}=\beta_{l e}\left(C, B_{i}\right)^{-1}\left\{\sum_{j=1}^{n_{1}} c_{j}^{\prime}\left(\dot{a}_{j l}^{i}-a_{k l}^{i}\right)\right\} \operatorname{sgn}\left(\left(x_{i}\right) \sigma_{i}\right) ;  \tag{12}\\
& \kappa_{r_{0}}=\left\{\gamma_{1}\left(C, B_{1}\right)^{-1} C_{1} \dot{B}_{1}\right\} \operatorname{sgn}\left[r_{1} \sigma_{1}\right] ; \tag{13}
\end{align*}
$$

where $\alpha_{i}, \beta_{i,}, \gamma_{\text {, }}$ are positive constants, and ( $)$, represents the $\boldsymbol{C}$ h element of the indicated vector. Let

$$
\begin{equation*}
\Delta=\max \|E(x)\| \tag{14}
\end{equation*}
$$

and assuming that the only locally available information is $\Delta_{1}$, we can pick

$$
\begin{equation*}
\delta_{1}=\lambda_{1}\left(C_{1} B_{i}\right)^{-1}\left\|C_{1}\right\| \operatorname{sgn}\left|\sigma_{i}\right| \tag{15}
\end{equation*}
$$

On the other hand, if the interaction effects are sflit into two portions; namely an unknown (but with known bound) portion, and a measurable portion, as given below:

$$
\begin{equation*}
E_{\mathrm{r}}(f)=E_{\mathrm{iu}}(x)+E_{\mathrm{tm}}(x), \tag{16}
\end{equation*}
$$

the concept of interaction compensation cas be utilised to directly negate the effects of the measured portion. Furthermore; If for apecific applications, the meamurable interactions are to aseume desired rives, or follow prespecified trajectories in time, they can be included caily into the moded-reforence framework above.

## 5. Robotic Syatem Conifguration and Modeling

The ayatern under connideration consiats of two diferent robotic arms, each a pianar three-link manipulator (Figuse 2). The parameters of each link are shown is the figure where


Figure 2: Two three-link, planar manipulators.
$L_{1 j} \triangleq$ Length of each link $i=1,2 ; j=1,2,3$
$\ell_{i j} \triangleq$ Loiation of center of gravity with respect to the end of the previous link
$m_{i j} \triangleq$ Mass of $i j$-th link
$J_{1 j} \triangleq$ Moment of inertia about the corresponding center of gravity
$\theta_{i j} \triangleq$ Angular position measured counterclock wise
$T_{i j} \triangleq$ Torque actuating the $i j$-th joint
d $\triangleq$ Distance separating both arms (on the base)
The overall task can be divided into three phases: approaching phase, grasping phase, and lifting (coordination) phase. In the approaching phase each arm movea toward the object to be picked. Speed and position control are applied according to the characteristics of each arm. The grasping phase design, although not considered in our study, depends on the sennory system assumed to be avilable. Force sensing can be utilized to implement a controller using force feedback. The last phase is the lifting phase during which the two-arm robotic system forms a closed-chain mechanical manipulator. Again, tactile feedback can be applied at this phase and we will discuss incorporation of such interaction information below. Lagrange's method han been used in finding the dynamical equations of this robotic system, where the equations are written in terms of the total kinetic energy ( $K$ ) of the system, the total potential energy ( $P$ ) of the system, and a set of independent coordinates $\left(g_{i}\right)$ chosen to describe the configuration of the system. Furthermore, these equations may include dissipation functions for non-conservative systems. We will not present these equations here but junt briefly analyse the conditions while the two arma are in contact. For the first arm, let

$$
\begin{align*}
& \theta_{1}=\left(\theta_{11} \theta_{12} \theta_{13}\right)^{4} \\
& T_{1}=\left(T_{11} T_{12} T_{13}\right)^{4}, \tag{17}
\end{align*}
$$

It ean then be shown that, in the approching phace the dynamical equations of the first arm ean be written as

$$
\begin{equation*}
M_{1} X_{1}+F_{1}\left(O_{1}, \partial_{1} H_{i}+G_{2} O_{2}=T_{1}\right. \tag{18}
\end{equation*}
$$


On the other hand, during the graping and holding phases, a cloned chain robot in formed through the continuoun cometact of the end effectors of both arms with the object. This constraint defines a frictionless manifold which can be expreseed in cloeed form. It is asumed that an additional torque (to be denoted a $T_{1}$ ) can be definod to maintain the tip of the and effector on the manifold. The dynamical equations of the clowed chain may then be written as

$$
\begin{equation*}
M_{1} \ddot{\theta}_{1}+F_{1}\left(\theta, \dot{\theta}_{1}\right) \dot{\theta}_{1}+C_{1} \theta_{1}=T_{1}+T_{1} \tag{10}
\end{equation*}
$$

To find the equation for $T_{1}$, consider a vertical displacement (ds). The corresponding work done by $r_{2}$ is reso; that in,

$$
\begin{align*}
\delta W & =r_{1} \delta_{2} \\
& =-F_{14}^{0} \delta x+F_{14}^{\varphi} \delta y \tag{20}
\end{align*}
$$

where $F_{14}$ is a generalized force due to the contact. The above equation can then be expressed in terme of the joint anglea mince

$$
\begin{aligned}
x & =L_{11} \sin \theta_{11}+L_{12} \sin \theta_{12}+L_{13} \sin \theta_{12} \\
y & =-L_{11} \cos \theta_{11}-L_{12} \cos \theta_{12}-L_{18} \cos \theta_{13} \\
\delta x & =L_{11} \cos \theta_{11} \delta \theta_{11}+L_{12} \cos \theta_{12} \delta \theta_{12}+L_{19} \cos \theta_{12} \delta \theta_{12} \\
\delta y & =L_{11} \sin \theta_{11} \delta \theta_{11}+L_{12} \sin \theta_{12} \delta \theta_{12}+L_{12} \sin \theta_{12} \delta \theta_{13} .
\end{aligned}
$$

Subatituting the above into (20) reaules in

$$
T_{1}=\frac{\delta_{w}}{\delta \theta}=\left[\begin{array}{l}
\left(F_{14}^{u} \cos \theta_{13}+F_{14}^{\omega} \sin \theta_{13}\right) L_{11}  \tag{21}\\
\left(F_{14}^{\mu} \cos \theta_{12}+F_{14}^{\mu} \sin \theta_{12}\right) L_{12} \\
\left(F_{14}^{4} \cos \theta_{13}+F_{14}^{4} \sin \theta_{13}\right) L_{13}
\end{array}\right]=\left[\begin{array}{l}
r_{11} \\
r_{12} \\
r_{13}
\end{array}\right] .
$$

Furthernore, since $F_{14}$ and $F_{14}^{y}$ are along the direction of motion and normal to it, they are related by $F_{14}^{W}=\mu F_{14}$ where $\mu \leq 1$ is the coefficient of friction.

Following the same procedure used for deriving the equation of the firat arm, one can easily find the dynamical equations of the second arm. Furthermore, a similar equation to (21) can be found for the second arm to satisfy the coordination movement constraint. The decentralized model-reference adaptive controller is now utilised to control the robotic system. To this end, each link is considered an independent subsystem with coupling forces and/or torques being the interactions. Thus,

| $S$ | Link \#1 of the first arm with $X_{1}=\left(x_{11} x_{12}\right)^{t}=\left(\theta_{11} \dot{\theta}_{11}\right)^{\text {d }}$ |
| :---: | :---: |
| $S_{2}$ | Link \#2 of the first arm with $X_{2}=\left(x_{21} x_{22}\right)^{4}=\left(\theta_{12} \dot{\theta}_{12}\right)^{4}$ |
| $S_{3}$ | Link \#3 of the first arm with $X_{2}=\left(x_{31} x_{32}\right)^{\prime}=\left(\theta_{13} \dot{\theta}_{13}\right)^{\prime \prime}$ |
| $S_{4} \triangleq$ | Link 41 of the second arm with $X_{4}=\left(x_{41} x_{42}\right)^{2}=\left(\theta_{31} \dot{\theta}_{11}\right)^{2}$ |
| $S_{8}$ | Link \#2 of the second arm with $\mathrm{X}_{5}=\left(x_{51} x_{32}\right)^{\prime}=\left(\theta_{22} \dot{\theta}_{32}\right)^{2}$ |
| S ${ }_{\text {c }}$ ¢ | Link \#3 of the second arm with $X_{4}=\left(x_{61} x_{02}\right)^{\prime}=\left(\theta_{33} \dot{\theta}_{33}\right)^{2}$ |

Each subsystem of the above has the following general form:

$$
\begin{equation*}
\dot{X}_{1}=A_{1}\left(. X_{1}, 1\right) . X_{i}-B_{i} U_{i}+\sum_{\substack{j=1 \\ j=1}}^{\infty} A_{i,}(X, t), X_{j} \tag{22}
\end{equation*}
$$

where $U_{i}=T_{i}$. Detailed derivations of these equations in the above form may be found elsewhere [16]. One can note that any measured torque or force betwean the links can be incorporated into the control structure discussec., ireviously.

## 6. Controller Denign

The use of Variable Structure Controlless for robotic system control was introduced by Young [8). Among more recent work in the area one can cite thowe of Morgan and Oagüner $[9]$ where decentralized controllers were employed, of Slotine and Santry [11] who dwell on the reduction of chatloring, and of Young [14] who introducen the design of variable atructure modetfollowing control ayateme. The present work differs from the above in that it uses a Deceatralised, Model Reference Adaptive approach and apecifically addreseen the multiple manipulator coatrol problem. The controller devised for this robotic aystem is organised in a hierarchical framework. The levele of the hierarchy are divided into two, and the information processed at each level is not directly avilable to the other level. Figure 3 ahowa the levele and information fow of the hierarchy, indicatiag that there are two pathe of information Bow. Downward moving data presents the flow of commands while upward moving dats presenta the flow of feedback information. As shown in Figure 4 , three tasks ase defined at upper level: planaing the motion of the end effectors of both arms, defining the local reference inpute for each link, and finding the upper bound of the dynamical interactions betwern aubaystemn. Figure 4 achematically depicta the operations done at this level.
The ead effectors of both arma are required to move in the work-space in a apecific way. A path is a continuous curve in the ayatem workspace connecting the tip initial configuration to the final configuration through all intermediate configurations. On the other hand, a trajectory is a continuous curve in the state apace of every link joining the initial atate to the final state. In other worda, the trajectory contains all the information about the time history of porition, velocity, and acceleration for each link. Therefore, a trajectory includes not only a path but aleo velocity and acceleration at every point at the path.
The first atep in generating the two arm robot motion is to characterise the path in some manner, typically by applying physical intuition to some extent. Esentially, the arm should atart and stop alowly with a smooth moiion. A number of different trajectories unay be proposed to satisfy the requirements, such as exponential and polynomial trajectories.

In the present study, the following equations were used

$$
\begin{align*}
& x(t)=x\left(t_{f}\right)+\left|x\left(t_{0}\right)-x\left(t_{f}\right)\right| e^{-\theta\left(t-t_{0}\right)^{n}}  \tag{23}\\
& y(t)=y\left(t_{f}\right)+\left|y\left(t_{0}\right)-y\left(t_{f}\right)\right| e^{-x\left(t-t_{0}\right)^{m}} \tag{24}
\end{align*}
$$



Figure 3: Control Hierarchy


Figure 4: Bigh Level Controlier
where $a, b, m$, and $n$ are real numbers. In finding the reference input $r_{i}$ for every link of each arm, the inverse dynamics approach is utilized. Note that, in the coordination (actual lifting) phave, both end effectors move on the same path. During this phase, if a desired coatact force profile is required, this can also be cast in the framewort of reference generation in the given formulation.
The low level controller consists of individual controllers for each link. Esech controller is designed following the model reference adaptive, rariable structure system approach. In this level, each subeystem is controlled separaiely to follow the reference model. This is to be done uxing only local information such as position and speed information of both the subaystem and the corresponding local reference model. Furthermore, the local controller required to force the local atates to follow the states of the corresponding reference model is designed using the VSS approsch presented earlier. Furtber details and simulation studies of cases without tactile feedback may be found in Ref. [16].

## 7. Conclusion

In this presentation we have briefly described ongoing research in control, sensing and real-time software to produce a two-armi robotic aystem that can accomplish generic assembly takks. We have concentrated mostly on the control hierarchy, the specific control approsch selected being the Variable Structure (Sliding Mode) Control approsch. The decentralised implementation of mudel-reference adaptive control using Variable Structure controllers was shown to be particularly suitable for such an application and the incorporation of tactile feedback was possible. Research is presently continuing on adjustments in the feedback gains when a deaired torque/force profile is given for end-effectors in contact with each other or other external surfaces.

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# Geometric Foundations of the Theory of Feedback Equivalence 

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## 1. INTRODUCTION

For the past ten years, a group of resoarchers-mathematicians and theoretical engineers, centered at, and partially supported by, the Flight Control group at NASA-AMES--have attempted to push beyond what was done in the 1960 's for 2 incr control theory, and develop effective methods for controlling systems whose dynamical equations are fundamentally nonlinear. Our applied focus has been the practical problem a encountered in designing aircraft and helicontess, but our methodology--based as it is on fundamental mathematical principles--is adaptable to robotic systems.

Conversely, we hope that use of the new ideas under development in the computer science and AI community will help us use computer technology in a more effective way to handle types of control problems--particularly of a "discrete event" nature--that have been difficult to include in a differential-eçuations based methodology.

Taking a historical view of progress in engineering and engineering-related mathematics, the situation becomes clarified. The breakthroughs of the 1960 's in control theory were closely linked to the development of computers, which could solve differential equations very efficiently. Mathematically, assumptions on linearity worked well because of the nature of the engineering problems that needed to be solved, especially in the Apollo Program, where the space craft could be treated satisfactorily as point particles, or at worst as rigid bodies. In the 1970's we attempted to adapt the mathematical techniques developed in the 1960's to the more difficult problems of control of aircraft and helicopters in circumstances where the assumptions of linearity of the dynamics could no longer be realistically justified. Recently, there has been a change in computer technology--such as LISP logic-based symbolic computation and greatly increased potentialities for parallelism--that has not yet been fully integrated into the main body of control theory. Further, computer science has achieved greater maturity and substance, and I believe that there are great scientific and technological possibilities in combining the talents and insights in the two communities. What control theory has to offer is a mature, mathematically based overview of a certain class of engineering problems, based on concepts of differential equations and dynamics, while the youthful vigor of the computer science discipline is generating a lot of energy, but exhibiting the need (in my opinion, at least) for more scientific and mathematical direction.

For the past two years, I have been trying-with George Meyer's advice and support on the engineering questions--to push in two directions. First, to understand how the control techniques of feedback linearization--developed as a useful control algorithm by Hunt, Meyer and $S u$ at NASA-AMES [1,2]--can be integrated into the mainstream of differential geometry and extended in the direction of understanding the relation between global and local feedback linearization. Second, I have tried to familiarize myself with the LISP and logic-based computer technology and algorithms, and help in the job of introducing it into control theory. Since tine first part of this program is further along-a major mathematical paper is now completed [3] and awaits publication-I will describe some of the ideas it contains fere, and leave my ideas bout developing relations between computer science and control theory to another occasion.
2. A VIEW OF FEEDBACK CONTROL IN THE CONTEXT OF DIFFERENTIAL EQUATIONS, DIFFERENTIAL GEOMETRY, AND LIE THEORY

A feedback control system can be taken as an underdetermined system of ordinary differentrial equations of the following general form:

$$
\begin{aligned}
& f\left(x, \frac{d x}{d t}, u\right)=0 \\
& x \in R^{n} ; u \in R^{m} \\
& f \text { is a map: } R^{2 n+m} \rightarrow R^{p}
\end{aligned}
$$

53 Jordan Road, Brookline, MA 02146
This work was begun while the author was a National Reseda. Council Senior Research Associate at the Ames Research Center, and continued under grant iNAG2406.
" $x^{\prime \prime}$ is a vector of $\mathrm{R}^{n}$ describing components of the syatem (airciraft, helicopter, spacecraft, robot,...) that are fixed in value, such as velocities, positions, angular or inear momenta, etc. "u" are the control variables, which we must choose in some way to achieve a prescribed gosl.

Feedbaok control can be described as follows. A feedback map or law in a map

$$
\begin{align*}
& x \rightarrow P(x)=u  \tag{2.2}\\
& 0 \rightarrow R^{m}
\end{align*}
$$

from an open subset 0 of $R^{n}$ to the control apace $R^{m}$. A trafeotory of the feedback control law (2.2) is a curve

$$
\begin{equation*}
t \rightarrow x(t) \tag{2,3}
\end{equation*}
$$

in $R^{n}$ that satisfies the following ordinary differential equation:

$$
\begin{equation*}
f\left(x(t), \frac{d x}{d t}, F(x(t))\right)=0 \tag{2.4}
\end{equation*}
$$

In engineering practice, we will want to choose the feedbock law (2.2), so that the family of trajectories defined by (2.3) and (2.4) will have certain stability, robustness, and design properties. (For example, for the latter one might want the trajectory (2.3) to start off at time $t=0$ at a point $x_{0}$ and end up exactly or approximately at a point $x_{1}$ at $t=t_{1}$.) Stabilisation is the property that is best understood mathematically, hence I will use it as a toucistone here.

Much of the work in control theory of the 1960 's--which was very successful on both the mathematical and practical fronts--was oriented toward linear control systems, i.e., those of the form:

$$
\begin{equation*}
\frac{d x}{d t}-A x-B u=0 \tag{2.5}
\end{equation*}
$$

where $A$ and $B$ are constant matrices of appropriate size. Here, it is natural to require that the feedback (2.1) preserve this linearity. This can be accumplished by specifying that the feedback map (2.2) be of the following form:

$$
\begin{equation*}
u=K x \tag{2.6}
\end{equation*}
$$

where $K$ is an $m \times n$ real matrix. The trajectory equations (2.2) are then of the following form:

$$
\begin{equation*}
\frac{d x}{d t}=(A+B K) \tag{2.7}
\end{equation*}
$$

One may then require that these trajectories have a prescribed degree of stability. Because (2.7) is a system of differential equations that can be handled with well-known mathematical tecnniques, we know that this behavior can be specified by imposing conditions on the eigenvalues of the $n \times n$ matrix

$$
\begin{equation*}
A+B K \tag{2.8}
\end{equation*}
$$

In turn, this "pole-placement" problem can be handled with well-known mathematical techniques (matrix Riccati equations or Kronecker pencil theory) that were applied in the 1960 's, but that of course go back many years in the mathematical literature.

It is especially interesting that the useful sufficient conditions for stabilization of (2.5) via linear feedback (2.6), i.e., "pole-placement" in the engineering jargon, involve controllability of the control system (2.5) and is a mathematical concept that is--as i showed many years ago (4]--essentially differential-geometric and Lie-theoretic in nature. Thus, it should be no surprise that the problem of stabilization and feedback control of a more general nonlinear system of type (2.1) also involves differential geometry and lie theory.

Indeed, the work of Hunt, Meyer, and Su [1,2] (preceeded by work of Krener [5], Brockett [6], Sommer [7], Jakubzy' and Respondek (8)) demonstrate this in a decisive way. Their work only dealt with feedback control of a certain class of systems (the feadback linearizable ones, with the functions $f($, , ) occurring in (2.1) satisfying certain conditions) if the trajectory stayed within a small neighborhood $\mathbf{R}^{n}$, whose size could not be specified in advance. This posed the question of finding conditions for global feedback equivalence. There has been important partial work on this problem by Boothby, Dayawansa, and Elliot [9-11] using the tools of differential topology and foliation theory. In my paper [3] I have begun to develop ways of applying the Ehresmann-Haefliger [12] theory of pseudogroup cohomology to this problem, but there is a long way to go before the results that are useful in practical situations will come forth.

The mathematical heart of the mathoda I have developed in [3] is the theory of vector field ayatean (or distributiona) on manifold and their equivalence. I will now sketch some of this basic differential-geometric theory, then return to the control aituation.
3. VECTOR FIELD SYSTEMS AND FEEDBACK EQUIVALENCE

I will now use the formaliem "calculus on manifolds." particularly the theory of voctor Eields (1.e.. first-order linear partial differential operators) and the Jacobi-Lie bracket [ J (i.e., commatar) of auch vector fields. See Isidori's book (13) for an engineer's introduction to these concepts.
(2i) Let $z$ be a manifold, with $V(Z)$ the space of vector fields. In terma of coordinates
 form:

$$
\begin{equation*}
v=A^{1}(z) \frac{\partial}{\partial z^{1}} \tag{3.1}
\end{equation*}
$$

(sumation convention in force)
If

$$
\begin{equation*}
V^{\prime}=s^{i} \frac{\partial}{\partial z^{I}} \tag{3.2}
\end{equation*}
$$

then

$$
\begin{equation*}
\left[V, V^{\prime}\right]=\left(A^{i} \frac{\partial\left(B^{j}\right)}{\partial z^{i}}-B^{i} \frac{\partial A^{j}}{\partial z^{I}}\right) \frac{\partial}{\partial z^{j}} \tag{3.3}
\end{equation*}
$$

Let $\mathrm{F}(\mathrm{Z})$ be the ring of $\mathrm{C}^{\infty}$, real-valued functions on $\mathrm{Z}, \underline{\mathrm{V}}(\mathrm{z})$ is module over P(2), sincé vector fields can be multiplied by functions:

$$
\begin{equation*}
(E, V)+E A^{i} \frac{\partial}{\partial z^{I}} \tag{3.4}
\end{equation*}
$$

Nefinition. A veotor field system $\underline{W}$ on 2 is a subspace of $\underline{V}(z)$ satisfying the following condition:

$$
\begin{aligned}
& f V \in \underline{W} \quad \text { for } V \in \underline{W} \\
& v_{1}+V_{2} \in \underline{W} \text { for } V_{1}, v_{2} \in \underline{W}
\end{aligned}
$$

i.e., W is a submodule of $\underline{V}(2)$.

Let $W$ be such a vector field system. For $z \in Z$, set
$\underline{W}(z)=\{V(z): V \in \underline{W}\}$
$W(z)$ is a linear subspace of the space of tangent vectors at $z$. Its dimension is called the rank of $W$ at 2 . W is said to be nonsingular if the rank is constant as $z$ rangea over 2 . In this paper we will assume that all vector field systems considered have constant rank, unless specified otherwise. The concept defined next will play a basic role in this work.

Definition. Let $\underline{w} \subset \underline{V}$ be a vector field system. Set
$\mathbf{C}(\underline{W}):\{V \in \underline{W}:\{V, \underline{W}] \subset \underline{W}\}$
$C(\underline{W})$ is called the Cauchy Characteristic system associated with w.
Theorem 3.1. $C(W)$ is another vector field system on 2 with the following properties:
$C(\underline{W}) \subset \underline{W}$

i.e., $C(\underline{W})$ is Frobenius integrable as a vector field system
$[\mathbf{C}(\underline{W}), \underline{W}] \subset \underline{W}$
Proof. Follows from (3.6).

Definition. A curve $t \cdot x(t)$ in $z$ is called an orbit oupve of the vector lield eyatem In If Ehe Iollowing condition is atisesied:

There is a vector tield $V$

$$
\begin{align*}
& V=A^{1} \frac{\partial}{\partial z^{1}} \\
& \text { in } \underline{v} \text { auch that } t \rightarrow z(t) \text { is an orbit curve of } V, \text { i.e... if } \\
& \frac{d z}{d t}=V(z(t)) \tag{3.10}
\end{align*}
$$

or, in coordinata terme:

$$
\begin{equation*}
\frac{d z}{d t}=\lambda(z(t)) \tag{3.11}
\end{equation*}
$$

In this way, a vector field system defincs a family of curves on $Z$. It is this geometric property that if the key to the usefulness of vector field eystem in control theory, an we have seen, control system are also defined by families of curves, namely solutions of the control quations:

$$
\begin{align*}
& f\left(x(t), \frac{d z}{d t}, u(t)\right)=0  \tag{3.12}\\
& x \in R^{n}, u \in R^{m}
\end{align*}
$$

Set:

$$
\begin{aligned}
& 2=R^{n} \times R^{m} \\
& z=(x, u)
\end{aligned}
$$

We can then define a vector field system $W$ on $z$ as the mallest ubmodule of $V(z)$ whose orbit curves are solitions of the control equation (3.12).

Let $X$ and $X^{\prime}$ be manifolds. Let $W$ and $W^{\prime}$ be nonsingular vector field ysteme on $x$ and $X^{\prime}$, respectively. Let

$$
a: X+X^{\prime}
$$

be a diffeomorphism.

Definition. a is called an equivalenoe from the vector field ystem $\underset{\sim}{\mathrm{w}}$ to the vector field yystem Wi if the following condition is satisfied:

$$
\begin{align*}
& \alpha_{*}(W(x))=W^{\prime}(\alpha(x))  \tag{3.13}\\
& \text { for all } x \in x
\end{align*}
$$

i.e., if $a$ maps an orbit curve of $W$ into an orbit curve of $W^{\prime}$. Our problem is to describe numbers attached to vector field systems that are invariant under equivalence. We shail cite (without proofs) some of the theorems from (3) that do provide ach invarianta.

Theorem 3.2. Let $a: X \rightarrow X^{\prime}$ be a diffeomorphism from $X$ to $X^{\prime}$ that is an equivalence of vector field system $\underline{W}$ to vector field system $W^{\prime}$. Let $C(W)$ and $C\left(W^{\prime}\right)$ be the Cauchy characteristic systems of $W$ and $W^{\prime}$, respectively. Then, the following condition is satisfied:

$$
\begin{equation*}
a_{*}(C(\underline{W}))=C\left(\underline{W}^{\prime}\right) \tag{3.14}
\end{equation*}
$$

i.e.. $\alpha$ is an equivalence between the Cauchy characteristic systems of the given vector field sytems.

Definition. For the vector field system W. set:

$$
\begin{equation*}
\underline{W}^{1}=\underline{W}+[\underline{W}, \underline{W}] \tag{3.15}
\end{equation*}
$$

It is called a derived aystem of $W$.
Theorem 3.3. Let $\alpha$ be ap isomorphism from $W$ to $W^{\prime}$. Then, it is an isomorphism of the

We can now iterate. set

$$
\begin{equation*}
\left(\underline{x}^{2}\right)^{2}=\underline{x}^{2}, \ldots \tag{3.16}
\end{equation*}
$$

to define the sucosseive derived eysteme of the given vector field system $W$, denoted as $\mathrm{H}_{\mathrm{O}}, \mathrm{W}_{\mathrm{X}}^{2}, \ldots$ We obtain an increasing filtration of submodules of the module of all vector fielda

$$
\begin{equation*}
\underline{w} \in \underline{w}^{1} \subset \underline{w}^{2} \in \ldots \tag{3.17}
\end{equation*}
$$

Thoorem 3.4. We have:

$$
\begin{equation*}
c(\underline{w}) \in c\left(\underline{w}^{1}\right) \in c\left(\underline{w}^{2}\right) \in \ldots \tag{3.18}
\end{equation*}
$$

In words, this mays that the Cauchy characteristics of the darived aystame also form an ascending, filtered sequence of submodules of the module of all vector fields on $X$.

We assume that all the modules (3.17) and (3.18) are of constant rank. set

$$
\begin{align*}
r & =\operatorname{rank} \underline{\underline{u}} \\
c & =\operatorname{rank} \underline{C}(\underline{w}) \\
r_{1} & =\operatorname{rank} \underline{\underline{w}}^{1}  \tag{3.19}\\
c_{1} & =\operatorname{rank} C\left(\underline{w}^{2}\right)
\end{align*}
$$

and so on.
Theorem 3.5. The sequence of integers

$$
\begin{equation*}
r \leq r_{2} \leq r_{2} \leq \cdots \tag{3.20}
\end{equation*}
$$

attached to the vector field system W are numarical equivalence invariante.
Let us now apply these results to control systems in state space form.
4. FEECBACK INVARIANTS FOR CONTROL SYSTEMS IN STATE SPACE FORM

Let us now specialize the feedback control system to consider those of the foliowing tate opace form:

$$
\begin{align*}
& \frac{d x}{d t}=f(x, u)  \tag{4.1}\\
& y \bullet R^{n}, \quad u \in R^{m} .
\end{align*}
$$

Theorem 4.1. Let

$$
\begin{aligned}
2 & =R^{n} \times R^{m} \\
& =\left\{(x, u): x \in R^{n}, u \in R^{m}\right\}
\end{aligned}
$$

Let $W$ be the vector field system $z$ generated by the components of the following vectorvalued vector fields on $z$ :

$$
\begin{equation*}
\underline{w}=\left\{f(x, u) \frac{\partial}{\partial x}, \frac{\partial}{\partial u}\right\} \tag{4.2}
\end{equation*}
$$

Then, the orbit curves on $W$ are precisely the curves $t \rightarrow(x(t), u(t))$ that satisfy the control equation (4.1).

Theorem 4.2. Let $d x / d t=f(x, u)$, and $d z / d t=h(z, v)$ be two feedback control systems with tne same number of states and controls. Let

$$
\underline{X}=\left\{f(x, u) \frac{\partial}{\partial x}, \frac{\partial}{\partial u}\right\}
$$

and

$$
\underline{w}^{\prime}=\left\{h(y, v) \frac{\partial}{\partial y}, \frac{\partial}{\partial v}\right\}
$$

be the vector field systems assigned to these control systems. Let

$$
\begin{equation*}
T: R^{n+m} \rightarrow R^{n+m} \tag{4.3}
\end{equation*}
$$

be a $c^{\text {© }}$ map of the following form:

$$
\begin{equation*}
T(x, u) \rightarrow(y, v) \tag{4.4}
\end{equation*}
$$

with

$$
\begin{align*}
& y=\alpha(x) \\
& v=\beta(x, u) \tag{4.5}
\end{align*}
$$

Than $T$ maps the control system $\{d x / d t m f(x, u)\}$ into the control system
$\{d y / d t=n(y, v)\}$, in the sense that it maps solution curves of tiae first system of ordinary differential equations into solution curves of the second, if and only if $t$ is an equivalence from the vector field system $W$ to the vector field system $W^{\prime}$. In particular, the integera r, r_,..; c, $c_{1} \ldots \ldots$ assigned by (3.3) to w are invariant under feedback equivalence. In the case of a linear control system, these integers can be computed in terms of the oontrollability indiceo.

Let us now consider the vector field systems sssociated wioh a linear, scalar input, control system, i.e.. one of the following form:

$$
\begin{align*}
& \frac{d x}{d t}=A x+b u  \tag{4.6}\\
& x \in R^{n}, u \in R, \quad b \in R^{n}
\end{align*}
$$

$$
\begin{equation*}
v_{0}=b \frac{\partial}{\partial x} \tag{4.7}
\end{equation*}
$$

Let $W$ be the vector field systems on $R^{n}$ spanned by these two vector fields and $\partial / \partial u$. Set:

$$
\begin{align*}
V_{i} & ={A d^{i}(V)\left(V_{0}\right)}
\end{align*}
$$

Theorem 4.3. The following commutation relations hold among these vector fields on $R^{n}$ :

$$
\begin{array}{ll}
\left\lfloor v, v_{i}\right] & =v_{i+1},  \tag{4.9}\\
{\left[v_{i}, v_{j}\right]} & =0,
\end{array} \quad \text { for } i=0,1,2 \ldots,
$$

The Jerived system $\underline{W}^{j}$ is the vector Eield system generated by

$$
\begin{equation*}
\left|\frac{\partial}{\partial u}, v, v_{i} ; i=i, \ldots, j\right| \tag{4.10}
\end{equation*}
$$

Theorem 4.4. If the system (4.6) is controllable, then

$$
\begin{equation*}
C\left(\underline{W}_{j+1}\right)=\left\{\frac{\partial}{\partial u}, v_{0}, v_{1}, \ldots, v_{j}\right\} \tag{4.21}
\end{equation*}
$$

As I show in [3], Theorem 4.4 is the geometric heart of the sufficient conditions that Hunt, Meyer, and Su [1,2] provided in their work on feedback linearization, namely:

Theorem 4.5. Let $V_{0}, V_{1}$ ive vector fields on $R^{n}$ generating a single input, controllable control system of the rollowing form

$$
\begin{equation*}
\frac{d x}{d t}=v(x)+u V_{0}(x) \tag{4.12}
\end{equation*}
$$

Suppose the following condition is satisfied:
The vector field systems

$$
\begin{equation*}
\left\{v_{0}, v_{1}=\left[v, v_{0}\right], \ldots, v_{j}=\left[v, v_{j-1}\right]\right\} \tag{4.13}
\end{equation*}
$$

are Frobenius integrable for all j .
Then, the system (4.12) is locally feedback equivalent to a chosen system.
In the Hunt-Meyer-Su work, the transformation $T$, which establishes the feedback equivalence of (4.12) with a linear system, is obtained as a solution of a system of first order, partial differential equations, and we can only prove existence of such linearizing transformations locally. A basic question is:

How to piece together such local feedback equivalences to find a global one?

The answer can be described in terms of cohomology theory [12]. Indeed, this is a typical problem of global differential geometry:

Find conditions for the existence (and computational feasibility!) of a global solution of a system of partial differential equations when the conditions for existence of local solutions are satisfied.

What complicates the analysis of the conditions for existence of a global solution is that the cohomology theory one must use involves an algebraic object--the groupoid of feedback automorphisms of the linear control systems--that is infinite dimensional, so that standard topological techniques are not very helpful. It is interesting to note that elementary particle physicists at the frontiers--in the so-called string theory-are involved with mathematical monstrocities that are very similar to these! Work on this question is in progress.

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# Reducing Model Uncertainty Effects in Flexible Manipulators Through the Addition of Passive Damping 

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## 1. Abstract

An important issue in the control of practical systems is the effect of model uncertainty on closed loop performance. This is of particular concern when flexible structures are to be controlled, due to the fact that states associated with higher frequency vibration modes are truncated in order to make the control problem tractable. Ia-*ht-paper-memptoy digital simulations of a single-link manipulator system to demonstrate that passive damping added to the flexible member reduces adverse effects dssdcidted with ore emplcyecl model uncertainty. A controller was designed bases on a model including only one flexible mode. This controller was applied to larger order systems to evaluate the effects of modal truncation. Simulations using an LQR design assuming full state feedback illustrate the effect of control spillover. Simulations of a system using output feedback illustrate the destabilizing effect of observation spillover. The simulations reveal that the system with passive damping is less susceptible to these effects than the untreated case.

## 2. Introduction

Many in-space robotic operations will require arms capable of very long reach. while like other space structures, they must be lightweight. Because such arms are likely to be highly compliant (as is the space shut te RMS arm). control strategies designed to accommodate structural flexibility must be considered. Controlling flexible structures through purely active measures can be cumbersome in terms of hardware and computation time requirements. Moreover, active controllers for flexible structures are subject to instability and other problems associated with model uncertainty. The burden of active control can be reduced by augmenting active control with passive damping. This enhances system stability and reduces the adverse affects of model uncertainty, thereby providing justification for the use of low order dynamic models and controllers.

In this paper we consider a single-link, single-axis arm which rotates in the horizontal plane about a pinned hub in response to d control torque $r(t)$. The system, illustrated in figure 1 , and the models employed in this investigation are based upon a laboratory version of the arm that has been used in experimental investigations [1-4] at Georgia Tech. The flexible member is a long slender beam that is assumed infinitely stiff in vertical tending but flexible in horizontal bending.


Figure 1. System Configuration
 menipulazors sometimes carry paylonds that have significant rotary inertia. the effect of this inertia ${ }^{11}$ qualitatively similar to that of a point mass for the configuration considered, and hence payload inertia il not included here. Friction in the pinned joint is represented by rotary viscous dashpot. Thil conflguration is viewed as being representutive of lightweight, large payload capacity manipulators. Parameris and dimensions for the arm that the system considered in this paper are tabulated in Appendix A.

Damping augmentation is provided oy a constrained viscoelastic layer damping ercatment [2.5]. Im approsen involves bonding a thin film of viscoelastic meterial to the flexible maber's surface. Thit viscoelastic layer in turn has a stiff elastic constraining layer bonded to ies surface. The combinen system forms a sandwich-like structure illustrated in figure 2. When elastic deflection of the structuri occurs. shear induced plastic deformetion is imposed in the viscoelastic layer. The energy dissipation associated with the plastic deformation provides the desired mechanical damping. The damping ratio for thm untreated beam mas approximately constant for all modes at.007. The treatment increased the damping ratio: associated with the modes of Interest (say the first stx modes) by about an order of magnitude. The treatei beam had adamping ratio of .03 for the first mode and the values for the 2 nd through 6 th modes ranged frot .052 to . 06. Addizionsl damping is introduced by joint friction.


Figure 2. Treated Beam Element Under Flexure

The first step of controller design is usually the development of a "design model" that is a simplifie representation of the actual plant dynamics. The design mocel serves as the basis for controller design In the case of flextble mechanical systems, the destign model is often a truncated representation of th actual plant, retaining only a few critical modes. This implies the assumption that a model based upon small number of vioration modes provides adequate representation of the much larger order actual plant, fo controller design purposes. The modeling error associated with the neglected modes, adversely affect closed loop system performance. In this paper, simulation results are presented to illustrate that the il effects associated with modeling error are reduced somewhat through the addition of passive damping to th system.

We consider a multivariable control system, designed according to the steady state linear quadrati regulator (LQR) approach. A four state model including only one flexible mode and the rigid body nod represents the design model. We consider the consequences of controlling larger order plants representative of the actual system, with a controller derived for the design model.

The regulator is formulated to penalize tip position and control effort. Two cases are considered The first assumes that full state feedback is available. The second case uses output feedback of ti position ( $v_{L}$ ), tip velocity, hub angle ( $\theta$ ) and hub angular velocity. The controller designs are kept simpl to facilitate comparisons between the damped and undamped systems. ${ }^{1}$

## 3. Dynamic Model

Linear transfer function models for the system of interest were developed based on the assumption ( small bending deflections and small hub angles. Transfer function modeling for similar systems has bet discussed by several authors $[2,4,6-8$ ] and we will not repeat the procedure here. Details on development, the model employed here may be found in [2]. The transfer function poles and zeros used in int investigation are tabulated in Appendix 3.

[^14]In the interest of obraining aste space realization from the transfer functions it is convenient to work with the pertial fraction expension form given by Equation 1. The w's are the model frequencies. the $y^{\prime} s$ are the cumped modal frequencies $y+1-5^{2}$. the $C^{\prime} s$ are modal damping ratios and $n$ is the number of fiexible moces represented. Dumping due 20 joini friction has not been accounted for in these transfer funcelons but will introduced later as a form of feectack. ${ }^{2}$ the rasidues $\lambda_{0}$ and $\mu_{0}$ forrespond to the rigid body mode.

$$
\begin{equation*}
\epsilon_{x}(s)=\frac{\lambda_{0}}{s}+\frac{\mu_{0}}{s^{2}}+\frac{\lambda_{1} s+\mu_{1}}{s^{2}+2 c_{1} \omega_{1} s+y_{1}^{2}}+\cdots \cdot \frac{\lambda_{n} s+\mu_{n}}{s^{2}+2 c_{n} \mu_{n} s+\varphi_{n}^{2}} \tag{1}
\end{equation*}
$$

The subscript $x$ on $G_{x}(s)$ represents the output variable of interest. For the present study four transfer functions were required. Equation 2 summerizes these and defines the notation used mere.

$$
\left[\begin{array}{l}
\text { nub angular position }  \tag{2}\\
\text { mub angular rate } \\
\text { beam tip position } \\
\text { beam tip rate }
\end{array}\right]=\left[\begin{array}{c}
\theta(s) \\
s \theta(s) \\
V_{L}(s) \\
s V_{L}(s)
\end{array}\right]=\left[\begin{array}{c}
G_{\theta}(s) \\
G_{\theta}(s) \\
G_{V L}(s) \\
G_{V L}(s)
\end{array}\right] T(s)
$$

Here $s$ is the Laplace operator and $T(s), \theta(s)$ and $Y_{L}(s)$ denote the Laplace transforms of the input torque. thub angle and tip position variables, respectively.

Figure 3 is a block diagram representation of the transfer function.


Figure 3. Block Diagram Equivalent of Equation 1

The system metices corresponding to Figure 3 are as follows:

$$
A=\left[\begin{array}{lllllll}
0 & 1 & & & & & \\
0 & 0 & 0 & 1 & & & \\
& & -y_{2}^{2} & -2 \omega_{1}^{c} & & & \\
& & & & \ddots & 0 & \\
& & & & & -y_{n}^{2} & -2 u_{h} C_{n}
\end{array}\right]
$$

$$
\begin{align*}
& =\left[\begin{array}{llllllll}
0 & 1 & 0 & 1 & \cdots & 0 & 1
\end{array}\right]^{\top}  \tag{3}\\
& c_{x}=\left[\begin{array}{lllllll}
\mu_{0} & \lambda_{0} & \mu_{1} & \lambda_{2} & \cdots & \mu_{n} & \lambda_{n}
\end{array}\right]
\end{align*}
$$

The leeding principle $2 \times 2$ submetrix of $A$ represents the rigid body mode in the tip and mid position transfer functions.

For each of the output variables $x$ there is unique outpus mitix $C_{x}$. will cenote these as $C_{\theta}, C_{g}, C_{v L}$ and $C_{i L}$ with the notation carrying the obvious maning. these are assembled to form a measurement metrix representing four outputs as indicated in the masurement equation (4) given by:

$$
y=\left[\begin{array}{c}
\theta  \tag{4}\\
\dot{\theta} \\
v_{L} \\
\dot{v}_{L}
\end{array}\right]=\left[\begin{array}{l}
c_{\theta} \\
c_{\dot{\theta}} \\
c_{V L} \\
c_{\dot{v}}
\end{array}\right] x \quad-g_{g x}
$$

In order to account for viscous joint demping we consider the feedback system tliustrated in Figure A.


Figure 4. Block Diagram Illustrating Feedback of Joint Damping

When the effect of joint damping is introduced through boundary condition eeedback, 1 new $A$ matix is formed:

$$
\begin{equation*}
A=A-B C_{g} b \tag{5}
\end{equation*}
$$

Where $b$ is the joint damping coefficient. The amalysis that follows is besed upon astem model of the form (A, B, C).

## 4. System Representation

We wish to design a controller for a plant with $2(n+1)$ states, using a design model including only one vibration mode. Here $n$ represents the number of flextble vibration modes required to provide accurate representation of the actual plant, and the additional two states represent rigid body motion. Controllers developed for the single mode system are applied to a model including three vibration modes ( 8 states), and one including six vibration modes ( 14 states). These are referred to as the plant models in the text that follows because they are intended to represent actul plants in the simulations presented here.

The system equations for the plant models are given by:

$$
\begin{align*}
& i_{n}(t)=A_{n} n_{n}(t) \cdot i_{n} u(t)  \tag{6}\\
& y(t)=\varepsilon_{n} n_{n}(t)
\end{align*}
$$

Mere $A_{n}$ is the 1 merix cefimed in Eqution 5 , In this case for an $n$ anderoximition of the plane. Sieliarly the $\mathcal{C}_{\mathrm{n}}$ merin is the $\mathcal{\xi}$ metrik of Equition 1 for an $n$ mode apponimition.

The systen equations for the design motels are given by:

$$
\begin{align*}
& \dot{x}_{1}(t)-A_{1} n_{1}(t)-t_{1} w(t)  \tag{7}\\
& y(t)=g_{1} n_{1}(t)
\end{align*}
$$

The systen ourput vector (y) considered is of the form

$$
y=\left[\begin{array}{llll}
0 & 8 & v_{L} & i_{L} \tag{8}
\end{array}\right]^{\top}
$$

## 5. Dogulater Design

The standard form of limear quadratic cost function is

$$
\begin{equation*}
J_{c}=\int_{0}^{\infty}\left(x^{\top} 0 x+n^{\top} R u\right) d x \tag{9}
\end{equation*}
$$

where $Q$ is asmmetric. positive semi-definite state meighting matrix and $R$ is a symmetric. positive definite control effort weighting metrix. Because we seek wo regulate tip position. a performince incex that penalizes the tid position output variable and control effort mas chosen. A cost function for thp position ouzput weighting is expressed as follows:

$$
\begin{equation*}
J_{c}=\int_{0}^{\infty}\left(v_{L}^{2}+r_{2}^{2}\right) d \tau \tag{10}
\end{equation*}
$$

The output melghted performance index (10) is equivalent to the standard state meighted version (9) with weighting matrices given by:

$$
0=c_{n}^{\top}\left[\begin{array}{llll}
0 & & &  \tag{11}\\
& 0 & & \\
& & 1 & 0
\end{array}\right] \quad c_{n} \quad, \quad R=[r]
$$

The system ( $A_{n}$, $\theta_{n}$, $G_{n}$ ) represents an actual plant with dynamics that are elther incompletely known or too cumbersome to permit the use of the full model in controller design. The four state design model $\left(\mathcal{~}_{1}, B_{1}, \mathcal{G}_{1}\right)$ will serve as an approximation to the actum) plant for controiler design purposes. In this case $G_{1}$ replaces $\mathcal{G}_{n}$ in the state mighting metrix 0 (11)

Two atractive features of the tip position wighted cost function (10) are that it has only one parameter ( $r$ ) to vary, and that a given value of $r$ can be expectad to fimpose stallar performance demands on both systems (damped and undamped). Reducing the value of $r$ decreases the penelty on control effort and is therefore equivalent to demonding higher performance at the expense of increased control energy.

## 6. State Feetback

In a system with decoupled modes, such as the Jordan canonical realization of Equation 3 , a state feedback law

$$
\begin{equation*}
u=-x_{1} x_{1} \tag{12}
\end{equation*}
$$

designed to stabilize the reduced order system (7) will stabilize the actual system (6), provided that the truncated modes are asymptotically stable. The neglected modes can, however, be excited at their natural frequencies in response to the applifed control input. This effect is called control spillover [9.10] in the literature related to controlling flexible spacecraft. The system me are considering mas a smili amount of modal coupling, due to the introduction of viscous joint damping using Equation 5. Since we normally do not
expect viscous daming to destabilize system, it is reasonable so expect that the state feedbeck law (12) will stabilize the plant models of interest.

In this section we cesign a controller besed on the output wighted performance facex (10). We asgume that the first four elemants of the state vector are somatom avallable for feedback. Control is appliet to these four geate elements according to Equition 12. The time varying gain $K_{1}(t)$ that minimizes the cost function (9) is given by

$$
\begin{equation*}
R_{1}(t)=R^{-1} L_{1} T_{S}(t) \tag{13}
\end{equation*}
$$

where $S(t)$ is the solution of the essociated metrix Riccati equation. An of ren used substitution fop the opeinal gif $K_{1}(8)$ is its constant stecdy state value $K_{1} \cdot K_{1}(-)$. The steady state vilue af $S(t)$ is the solution of the algebratc Riccatel equation:

$$
\begin{equation*}
-s(-)=s(-) A_{1}+A_{1}^{T_{s}} s(-)-s(-) t_{1} R^{-t_{1}} T_{s}(-) \cdot 0=0 \tag{14}
\end{equation*}
$$

The steady state gain solution is used ine simulations presenced mere.

## 7. Simalatieas Mth Seate Fecunct

Figure 5 illustrates the simulated response of the cesign model (ij to a 4.8 inch step command, for various values of $r$. The simulations indicate that both the dampad and undamped systems are capable of almost arbitrarily good performance is $r$ is decreased. In practice, the limit on the response tiat is dictated by the strength of the beat and the torque limit of the ectut tor. A theoretical lassuming ini inite beam strength and motor torque capacity) lialt on the speed of response is discussed by schmitz [6]. Inis limit is related to the non-minimu phase character of the tip position transfer function. Notice tnat the wrong way start phenomenon typical to systems with non-minimum zeros is Indicated in the plots. Schaitz interprets the thenretical response limit as being roughly equivalent to a pure delay associated with the initial period durim, which the t!p moves in the direction opoosite to the control commend.

When the state feedbeck law (12) is applied to the larger order systens (Figures 6 and 71, the excitation of the second mode of vibration is readily apparent when $r=10^{-5}$. in the undamped system (Figures 60 and 7a) the oscillation takes more than two seconds to dic out. Thus, in the case of the undamped system. we find that designing for higher performance (by reducing r) actually results in slower response. The excitation of the second mode also occurs in the damped system (Figure 60 and 7b), hovever. it dies out in about 0.8 seconds. Although the performance of the actual piant is not as good as tiat of the destign model (Figure 50 ). the simulation indicates that the response time for $r=10^{-6}$ is silghtly better than the lower levels of demended performace llarger values of r) considered. This is in sharp contrast with the results of Figures 60 and 70 for the undamped system. This example clearly findicates thet the damped system is less susceptible to control spillover than the undamped case.

It should be noted that the peak control torque comand, when $r=10^{-6}$. is about 4800 inch pounds. This value is well above the Deam's maximm bending moment capacity (~ 175 in . lbf. based on yield) and is about 60 times greater than the rated torque capacity ( 85 in . Ibf.) of the experimental system's motor. in lignt of these figures, one might argue that control spillover is not realistic concern for the system of incerest. The author concedes to the somewht articicial nature of this example, however, further consideration of the results adds to their significance. Suppose the initial step comend is scaled down by a factor of 50 to about 0.1 inches. Because the system model is linear. we know that the corresponding peak torque is about 100 inch.ibf. This is a realistic figure for the system of interest. In Figures fa and 7 a . peak tip position oscillation amplitude is about 1.5 inches. Scaling this figure down by factor of 50 gives 30 thousandths of an inch - stgnificant value in the context of robot accuracy.

## 8. Output Feedback

The staulations presented in the previous section were dased on the assumed avallability of states. Practical control systems must depend upon measured outputs for feedback. Frequently the outputs are different entities than the states. In contrast to systems using state feedback. output feedback systems are subject to instability as a consequence of model reduction [9-11] even when the neglected modes are asymptotically stable. This effect is sometimes called odservation spillover.

In this section we follow the steady state $L Q R$ controller design spproach employed in the previous section, however, we implement the controller using output feedback. The design model ( $\Lambda_{1}$. $\boldsymbol{B}_{1}$. $C_{1}$ ) has four states and four outputs and the measurement mitix $G_{1}$ is invertible. This allows us to calculate the state $x_{1}$ of the desing model from the output vector $y$ according to:

$$
\begin{equation*}
x_{1}=\subseteq_{1}^{-1} y \tag{15}
\end{equation*}
$$



Figure 5. Closed Loop Step Response Using State Feedback, One Vibrazion Mode Plant Model

a) Undamped Case

a) Unoamped Case
D) Damped Case


Figure 6. Closed Loop Step Response Using State Feedback. Three Vibration Mode Plant Model

b) Damped Case

Figure 7. Closed Loop Step Response Using State Feedback, Six Vibration Mode Plant Model

In this case the output feedback control 1t:i is given by:

$$
\begin{equation*}
u=-x_{1} \varepsilon_{1}^{-1} y \tag{16}
\end{equation*}
$$

The ourpus fredback law (16), when applied to the design model is equivalent to state feedback (12). Ven applied to controlling the actual model, the output feedback control law (16), expressed in the form of a suate feedback law is given by:

$$
\begin{equation*}
\mu=-k_{1} \varepsilon_{1}^{-1} \varepsilon_{n} x_{n} \tag{17}
\end{equation*}
$$

Morice that men applfed to the plant model, the output feedback law receives informetion from the states that were meglected in design. This is unwanted input (spillover), and can be viewed as a fort of measurement corruption.

## 9. Simulations with Output Feedback

Simulation results obtained using output feedback are presented in Figures 8 through 10. Figures 8 a and 80 are simulations of the design model using the output feedback law (16). When applied to the design model the outpur feedback lam considered is equivalent to state feedback (Figure 5). This case is preseated here as a basis for comparison. When the low order output feedback lam (16) is applifed to the actual systam models (Figure 9 and 10) we observe that the performance is limited by the onset of instability. In the undamped system, the first vibration mode is unstable for $r=0.01$ and $r=0.005$. The damped system remalns stable under the same conditions, however, some first mode oscillatory behavior becomes evident as we attempt to design for higher performance. The damped system is not immune to the instability experienced by the undamped case, however, due to 1 ts more favorable open loop pole placement it is more robust.

Upon comparison of the six mode and three mode systems, we find that the stable responses of the six mode plants do not differ noticeably from those of the three mode plants. On the other hand, the divergence rate of the unstable oscillations is greater in the six mode plant (Figure 10a) than in the three mode plant (Figure ga). This indicates that the presence of the higher, neglected modes ( 4 th , 5 th and 6 th ) do affect system stability slightly.

This example illustrates that the passive damping treatment considered reduces the flexible systea's susceptibility to observation splllover induced instability. The peak torque commanded at the higrest performance (when the system is stable) was about 80 in .lbs.. Indicating that the performance demanded was reasonable for the system under consideration. The example employs perhaps the most simplistic of all Dossible output feedback schemes. Systems employing state estimators also rely on measured data for feedback. and they too are subject to instability due to modeling error.

## 10. Conclusion

One form of modeling error that is relevant for control of flexible structures results from ignoring high order vibration modes in the process of deriving a design model. The effects of this type of modeling error are mantfested as control and observation splllover. We have presented simulations of multivartable control a particular flexible arm to illustrate that the adatition of passive damping yields a system that is less susceptible to these undesirable effects.

To some degree these results follow intuition, in that one naturally expects that increasing the damping terms of a system's elgenvalues will provide a more stable system with fmproved performance. The results presented are intended to demonstrate the concept of passive damping on an example that is representative of practical ilghtweight mantpulators.

## 11. Acknowledgements

The research described was performed at the Georgia lnstitute of Technology under the joint adviserent of Dr. Hayne J. Book and Dr. Stephen L. Dickerson. The author was supported through a Research Assistantship provided by the Georgia Tech Material Handing Research Center (MHRC).


Figure 5. Closed Loop Step Response Using Output Feedback, One Vibration Mode Plant Model


Figure 6. Closed Loop Step Response Using Output Feedback, ThreeVibration Mode Plant Model

a) Undamped Case

b) Damped Case

Figure 7. Closed Loop Step Response Using Output Feedback, Six Vibration Mode .Plant Model

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APPENOIX A - System Parameters and Dimensions

| Joint Inertia | $\mathrm{J}=30.2 \mathrm{in}^{2} .1 \mathrm{bm}$. |
| :---: | :---: |
| Payload Mass | $\mathrm{m}=0.091 \mathrm{dm}$. |
| Joint Damping Coefficient | $\mathrm{b}=0.10 \mathrm{in} .1 \mathrm{Df}$. $\mathrm{s}^{2}$ |
| Beam Oimensions | $48^{\prime \prime} \times 3 / 4^{\prime \prime} \times 3 / 16^{\prime \prime}$ |
| Material | 6065-T6 Aluminum |

APPENDIX B - Transfer Function Poles and Zeros
Table B-1 Undamped System

| Mode | System Poles |  | Hub Angle T.F. Zeros |  | Tip Position T.F. Zeros |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | -0.0541 | $\pm 37.726$ | -0.0149 | $\pm 32.1129$ | $\pm 8.3410$ |
| 2 | -0.1284 | *J18.3456 | -0.0985 | *514.0768 | t45.0741 |
| 3 | -0.2957 | * $\ddagger 42.2446$ | -0.2853 | $\pm$ 540.7557 | 1111.3047 |
| 4 | -0.5769 | $\pm$ j 82.4188 | -0.5719 | $\pm 381.6939$ | \$206.9715 |
| 5 | -0.9633 | * 5137.6207 | -0.9603 | $\pm$ \$137.1836 | $\pm 332.0743$ |
| 6 | -1.4532 | $\pm$ j207.5965 | -1.4511 | * j 207.2946 | 2486.6130 |



# Parallel Processing Architecture for Computing Inverse Differential Kinematic Equations of the PUMA Arm 

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## 1. Abstract

In advanced robot control problems, on-line computation of Inverse Jacobian solution is frequently required. Parallel processing architecture is an effective way to reduce computation time. fintintspaper, a parallel processing architecture is developed for the Inverse Jacobian (inverse differential kinematic equation) of PUMA arm. fol. The proposed pipeline/parallel algorithm can be implemented on IC chip using systolic linear arrays. This implementation requires 27 processing cells and 25 time units. Computation time is thus significantly reduced.

## 2. Introduction

In many advanced robot control problems, such as with sensor guided manipulations, it is essential that the end effector be appropriately controlled in Cartesian coordinates so that the robot can adapt to a changing environment. This means that we need to compute the inverse jacobian in real time to provide the required differential change in joint variables for a desired differential change in position and orientation. The speed of this computation directly affects the speed of robot operation. Thus efficient algorithms for computing the inverse Jacobian are needed.

There have been efforts made recently in developing computationally efficient algorthims to solve the Jacobian problem suitable for serial computer implementation [1,2]. In addition some work has been reported in algorithm development for implementation on pipeline or parallel computer [3]. These results show that such parallel algorithms can reduce computation time significantly.

A more important requirement in robot manipulation is the computing of the inverse jacobian solution. This is generally a troublesome problem when we try to invert the Jacobian numerically. A more direct approach is to derive an explicit solution of the inverse Jacobian for a given robot. Paul. Shimano, and Mayer [2] have shown that such solutions can be obtained by differentiating the kinematic equations. This approach has shown to result simpler inverse Jacobian solutions with regard to manipulator degeneracies and joint constraints. The inverse Jacobian of the PUMA arm has been solved specifically in [2].

In this paper, we present a pipeline/parallel algorithm and architecture for computing the puma arm inverse Jacobian derived tin [2]. With rapid advances in VLSI technology, this type of algorithm can be readily implemented on IC chips. These special purpose chips can be connected to a host computer system to achieve real-time Cartesian space control at sufficiently high sample rate. it is noted that a study has beta made recently to implement direct kinematic solution on VLSI chips to speed up computation time [4]. The goal here is to further exploit the advantages of VLSI technology for the design of customized chips dedicated to the computing of the inverse Jocobian of PUMA arm.

## 3. Differential Kinematic Solution of PUMA Arm

Differential changes in joint variables di can be related to the different changes in translation and rotation $d x, d y, d z, \delta_{x}, \delta_{y}$, and $\delta_{z}$ of the end effector by the relationship

$$
\begin{equation*}
\left[d_{x}, d_{y}, d_{z}, \delta_{x}, \delta_{y}, \delta_{z},\right]^{\top}=J\left[d q_{1}, d q_{2}, \ldots ., d_{n}\right]^{\top} \tag{1}
\end{equation*}
$$

In which $n$ is the number of joints, and $J$ is the Jacobian matrix. But in advanced robot control problems, we need the solution of day given the desired differential change $d_{x}, d_{y}, d_{z}, \delta_{x}, \delta_{y}, \delta_{z}$. That is we need to compute the inverse problem

$$
\begin{equation*}
\left[d q_{1}, d q_{2}, \ldots, d q_{n}\right]^{\top}=J^{-1}\left[d_{x}, d_{y}, d_{z}, \delta x, \delta y, \delta z\right]^{\top} \tag{2}
\end{equation*}
$$

This represents the inverse differential kinematic solution (inverse Jacobian) of the robot arm.
Instead of relying on the direct computing of the inverse Jacobian matrix $\mathrm{J}^{-1}$, an analytical solution of the inverse Jacobian problem can be frequently obtained, and such a solution for the puma arm is given in [2]. For the PUMA arm, the joint variables are the six rotational joint angles $\theta_{2}, \theta_{2}, \ldots, \theta_{6}$. Furthermore, the
differential changes in translation and rotation can be related to the differential change of the end e? fector homogeons matrix f [2]:

$$
\begin{equation*}
d T=T \cdot \Delta T \tag{3}
\end{equation*}
$$

where

$$
\begin{aligned}
& T=\left[\begin{array}{llll}
n & 0 & d & p \\
0 & 0 & 0 & 1
\end{array}\right] \\
& d T=\left[\begin{array}{llll}
d n_{x} & d o_{x} & d d_{x} & d p_{x} \\
d n_{y} & d o_{y} & d d y & d p_{y} \\
d n_{z} & d o_{z} & d d_{z} & d p_{z} \\
0 & 0 & 0 & 0
\end{array}\right] \\
& \Delta T=\left[\begin{array}{llll}
0 & -\delta_{z} & \delta y & d x \\
\delta_{z} & 0 & -\delta_{x} & d y \\
-\delta y & \delta_{x} & 0 & d_{z} \\
0 & 0 & 0 & 0
\end{array}\right]
\end{aligned}
$$

Therefore differential changes in transiation and rotation can also be specified in terms of the $x, y, z$ elements of dp, do, and da (dn vector is redundant). The desired solutions of doy in terms of dp, do, dint da for the PUMA arm obtained in [2] are given in the appendix. A pipeline/parallel processing architecture fer computing these equations is now developed below.

## 4. Systollc Array Processing

VLSI technology has created a new architecture horizon in implementing parallel algorithms directly on hardware. Central to this architecture is the use of systolic linear arrays which consist of interconnected simple and mostly identical processing cells. Algorithms that can be executed using iaentical operations simultaneously can take advantage of the systolic array architecture to reduce computation time.

The processing cell etructure we whil employ is the "inner product step processor" which performs matrix-vector multiplication using one-way pipeline algorithms. For example, computing
$A b=p$
where $A$ is mam and $b$ is $m x l$, can be carried out in the following recurrence manner:

$$
\begin{aligned}
& p_{i}^{(0)}=0 \\
& p_{i}^{(k+1)}=p_{i}^{(k)}+a_{i k} b_{k} \quad k=1, m, \quad i=1, n \\
& p_{i}=p_{i}^{(m)}
\end{aligned}
$$

This operation can be implemented by a linear array of $m$ inner product step processors shown in figure 1.
In the following section, we will reformulate the inverse differential kinematic equation given in the appendix in terms of a set of matrix-vector multiplications which can be computed in paraliel and pipelining fashton.
5. Algorithm Development

In this section, we present the matrix-vector multiplication processing schemes for computing the differentials doi, $1=1,2, \ldots, 6$. Here we assime that the trigonometric functions required are available. Typically these functions can be generated by employing gom look-up techniques [5,6]. The algorftha is broken down into 15 steps as described below. The notation $S_{i}=\operatorname{Sino}_{i} C_{i} \geqslant \operatorname{Cos} O_{i}$ are used.
(1) $A_{1} b_{1}=P_{1}$

$$
A_{1}^{\top}=\left[\begin{array}{ccccccccccc}
d p_{y} & d p_{x} & p_{x} & p_{y} & a_{y} & a_{x} & d d_{x} & -d_{x} & d d_{y} & o_{y} & o_{x}
\end{array} d o_{x} \quad d o_{y}\right]
$$

$$
\begin{aligned}
& b_{1}^{T}=\left[\begin{array}{ll}
c_{1} s_{1}
\end{array}\right] \quad p_{1}^{T}=\left[p_{11} \ldots p_{19} p_{110} \ldots p_{113}\right] \\
& \text { output: do } d o_{1}=p_{11} / p_{13}
\end{aligned}
$$

(2) $p_{1} m_{1}+g_{1}=n_{1}$
$P_{1}^{T}=\left[P_{14} \rho_{15} \rho_{18} \rho_{110}-\rho_{111}\right] \quad m_{1}=d o_{1}$
$g_{1}^{T}=\left[\begin{array}{lllll}P_{12} & P_{17} & P_{14} & P_{112} & P_{113}\end{array}\right]$
$h_{1}^{\top}=\left[h_{11} \ldots h_{15}\right]$
(3) $A_{2} b_{2}=P_{2}$
$A_{2}^{T}=\left[\begin{array}{cccc}-d_{3} d_{4} & P_{2} & -P_{13} \\ \sigma_{4} & a_{3} & P_{13} & P_{2}\end{array}\right] \quad b_{2}=\left[\begin{array}{l}s_{3} \\ C_{3}\end{array}\right]$
$P_{2}^{\top}=\left[\rho_{21} \cdots P_{24}\right]$
(4) $f_{2} m_{2}+g_{2}=h_{2}$
$p_{2}^{T}=\left[\begin{array}{lllll}C_{3} & S_{3} & p_{21} & p_{23} & p_{24}\end{array}\right]$
$g_{2}^{\top}=\left[\left.\begin{array}{llll}d_{3} d_{4} & 0 & 0 & 0\end{array} \right\rvert\,\right.$
$m_{2}=d_{2}, h_{2}^{\top}=\left[h_{21} \ldots h_{25}\right]$
(5) $A_{3} b_{3}=P_{3}$
$\left.A_{3}=\left[\begin{array}{lll}-p_{2} & \rho_{13}\end{array}\right] \quad b_{3}^{\top}=\left[d p_{2} n_{11}\right] \quad p_{3}=p_{31}\right]$
output: $\mathrm{dO}_{3}=\mathrm{P}_{31} / \mathrm{h}_{23}$
(6) $\begin{array}{rlr}A_{4} D_{4} & =D_{4} \\ A_{4} & =\left[\begin{array}{cc}p_{2} & -p_{13} \\ p_{13} & D_{2}\end{array}\right] \quad D_{4}=\left[\begin{array}{l}n_{21} \\ n_{22}\end{array}\right] \quad p_{4}=\left[\begin{array}{l}p_{41} \\ \rho_{42}\end{array}\right]\end{array}$
(1) $A_{5} b_{5}=P_{5}$

$$
\begin{aligned}
& d_{5}=\left[\begin{array}{lll}
-n_{21} & n_{22} & -n_{24} \\
n_{22} & n_{21} & n_{25}
\end{array}\right] \quad D_{5}^{T}=\left|d p_{2} n_{11} d O_{3}\right| \\
& P_{5}^{T}=\left\{p_{51} p_{52} \mid\right.
\end{aligned}
$$

(8) $A_{6} D_{6}=P 6$

$$
\begin{aligned}
& A_{6}^{T}=\left[\begin{array}{cccccccccc}
\rho_{42} & -d_{z} & \rho_{16} & -\rho_{51} & h_{12} & d d_{z} & \rho_{111} & -o_{z} & h_{14} & -d o_{2} \\
-\rho_{41} & -\rho_{16} & -d_{z} & -\rho_{52} & -d d_{z} & h_{12} & -o_{2} & -\rho_{111} & -d 0_{z} & -h_{14}
\end{array}\right] \\
& b_{6}^{\top}=\left\lfloor c_{23} s_{23}\right\rfloor \quad p_{6}^{\top}=\left\{p_{61} \ldots p_{610}!\right. \\
& \text { outputs: }{d O_{23}}=P_{64} / P_{51} \quad d O_{2}=d O_{23}=d O_{3}
\end{aligned}
$$

(9) $f_{3} m_{3}+g_{3}=h_{3}$
$f_{3}^{T}=\left[\begin{array}{llll}P_{3} & P_{63} & P_{68} & -P_{67}\end{array}\right] \quad m_{3}=d \theta_{23}$
$g_{3}^{\top}=\left[\begin{array}{llll}P_{65} & P_{66} & P_{69} & P_{610}\end{array}\right]$
$n_{3}^{\top}=\left[n_{31} \ldots n_{34}\right]$
(10) $A_{7} b_{7}=D_{7}$
$A_{7}=\left[\begin{array}{cc}p_{15} & p_{63} \\ p_{32}-h_{13}\end{array}\right] \quad D_{7}=\left[\begin{array}{l}p_{15} \\ \rho_{63}\end{array}\right] \quad \rho_{7}=\left[\begin{array}{l}\rho_{71} \\ p_{72}\end{array}\right]$
output: $\mathrm{dO}_{4}=\mathrm{P}_{72} / \mathrm{P}_{11}$
(11) $A_{8} b_{8}=P_{8}$
$A_{8}^{\top}=\left[\begin{array}{lllllll}\rho_{15} & \rho_{31} & \rho_{67} & \rho_{110} & h_{33} & -\rho_{67} & h_{15} \\ -\rho_{63} & h_{13} & \rho_{116} & -\rho_{67} & h_{15} & -\rho_{110} & -h_{33}\end{array}\right]$
$b_{8}^{\top}=\left[\begin{array}{ll}C_{4} & S_{4}\end{array}\right] \quad P_{8}^{\top}=\left[p_{81} \ldots P_{87}\right]$
(12) $f_{4} m_{4}+g_{4}=h_{4}$
$f_{3}^{\top}=\left[\begin{array}{lll}\rho_{81} & p_{84} & p_{86}\end{array}\right] \quad m_{4}=d \theta_{4}$
$g_{4}^{\top}=\left[\begin{array}{lll}\rho_{82} & \rho_{85} & \rho_{87}\end{array}\right] \quad n_{4}^{\top}=\left[\begin{array}{lll}n_{41} & h_{42} & h_{43}\end{array}\right]$
(13) $A_{g} b_{g}=P_{9}$
$A_{9}=\left[\begin{array}{rr}n_{41} & -h_{32} \\ -p_{68} & p_{83} \\ -h_{42} & -h_{34}\end{array}\right] \quad b_{9}=\left[\begin{array}{l}c_{5} \\ s_{5}\end{array}\right] \quad \rho_{9}=\left[\begin{array}{l}p_{91} \\ p_{92} \\ p_{93}\end{array}\right]$
output: $\mathrm{dO}_{5}=\mathrm{P}_{91}$
(14) $f_{5} m_{5}+g_{5}=h_{5}$

$$
f_{5}=\left[p_{92}\right] \quad g_{5}=\left[p_{93}\right] \quad m_{5}=d O_{5} \quad h_{5}=\left[h_{51}\right]
$$

(15) $A_{10} b_{10}=p_{10}$
$A_{10}=\left[\begin{array}{ll}h_{51} & -h_{43}\end{array}\right] \quad b_{10}^{\top}=\left[\begin{array}{ll}c_{6} & s_{0}\end{array}\right] \quad \rho_{10}=\left[\rho_{101}\right]$
output: $\mathrm{dO}_{6}=\mathrm{P}_{101}$

The data flow timing table for these computations are given in Tables 1 and 2 . It is shown that the solution requires 25 time units and 27 processing cells.

The results of 25 time units is a significant reduction of computation time in comparison with that when a serial computer is used to compute the original solution. The total number of multiplications of that solution is about 150. This is equivalent to 150 time units in the systoic array processing system as opposed to the 25 time units we have achieved by exploiting parallelism.

Table 1. Data flow timing table for steps 1 through 8 which compute dol dez and dej. Numbers on top row indicate time units.


Table 2. Data flow timing table for steps 9 through 15 which compute dea, des, de6.

| $10 \quad 11$ | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\mathrm{P}_{64}$ |  |  |  |  |  |  |  |  |  |  |  |  | $\mathrm{P}_{65}$ |
|  |  |  | $\mathrm{P}_{63}$ |  |  |  |  |  |  |  |  |  |  |  | $\mathrm{P}_{66}$ |
|  |  |  |  |  |  | $P_{68}$ |  |  |  |  |  |  |  |  | $\mathrm{P}_{69}$ |
|  |  |  |  |  |  |  | $-D_{67}$ |  |  |  |  |  |  |  | $\mathrm{P}_{610}$ |
| $P_{15} P_{63}$ |  |  |  |  |  |  |  |  |  |  |  |  |  |  | $\mathrm{P}_{15}$ |
|  |  |  | $h_{31}$ | $-h_{13}$ |  |  |  |  |  |  |  |  |  |  | $\mathrm{P}_{63}$ |
|  |  |  | $\mathrm{P}_{15}$ | $-p_{63}$ |  |  |  |  |  |  |  |  |  |  | $\mathrm{C}_{4}$ |
|  |  |  |  | h31 | $\mathrm{h}_{13}$ |  |  |  |  |  |  |  |  |  | $\mathrm{S}_{4}$ |
|  |  |  |  |  | $\mathrm{P}_{67}$ | $\mathrm{P}_{110}$ |  |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  | $\mathrm{P}_{110}$ | $-P_{67}$ |  |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  | $\mathrm{h}_{3}$ | n/ |  |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | $-P_{6}$ | -P |  |  |  |  |  |  |
|  |  |  |  |  |  |  |  | $\mathrm{H}_{1}$ | 5 -h |  |  |  |  |  |  |



## 6. Conclusion

It has been demonstrated in this paper that parallel computing architecture can be developed for the inverse differential kinematic equation of the FUMA arm. By using systolic linear arrays employed in VLSI chio design, the computation can be completed with 27 processing cells in 25 time units.

The differential kinematic equation in its original form requires about 150 multplications to compute. If one multiplication is counted as one time unit, the parallel architecture definitely provides a substantial reduction in computation time. A customized IC chip dedicated to this algorithm can be fabricated.
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$$
\begin{aligned}
& d \theta_{1}=\frac{C_{1} d P_{y}-S_{1} d P_{x}}{C_{1} P_{x}+S_{1} P_{y}} \\
& d \theta_{3}=\frac{d_{1}}{a_{2}\left(d_{4} c_{3}-d_{3}{ }_{3}\right)} \\
& d_{1}=f_{11}(p) d f_{11}(p)-f_{12}(p) d f_{12}(p) \\
& f_{11}(p)=C_{1} P_{x}=S_{1} P_{y} \\
& d f_{11}(p)=-S_{1} p_{x} d \theta_{1}+C_{1} d p_{x}+C_{1} p_{y} d \theta_{1}+S_{1} d p_{y} \\
& f_{12}(p)=-d p_{2} \\
& d \theta_{23}=\frac{-S_{23} d v_{2}-C_{23} d v_{1}}{C_{23 v_{2}}-S_{23} v_{1}} \\
& v_{1}=-\omega_{2}{ }_{11}(p)+\omega_{1} p_{2} \\
& v_{2}=\omega_{1} f_{11}(p)+\omega_{2} p_{2} \\
& \omega_{1}=a_{2} C_{3}+d_{3} \\
& \omega_{2}-d_{4}+{ }_{2} S_{3} \\
& d v_{1}=-\omega_{2} d f_{11}(p)-A^{d} f_{11}(p) C_{3} d \theta_{3}+\omega_{1} d P_{2}-d_{2} S_{3} P_{2} d \theta_{3} \\
& d v_{2}=\omega_{1} d f_{11}(p)-d_{2} S_{3} f_{11}(p) d \theta_{3}+\omega_{2} d P_{2}+d_{2} C_{3} P_{3} d \theta_{3} \\
& \Delta \theta_{2}=d \theta_{23}-d \theta_{3} \\
& d \theta_{4}=\frac{\mathrm{NC}_{4} d\left(\mathrm{NS}_{4}\right)-\mathrm{NS}_{4} d\left(\mathrm{NC}_{4}\right)}{\left(\mathrm{NS}_{4}\right)^{2}+\left(\mathrm{NC}_{4}\right)^{2}} \\
& N C_{4}=C_{23} \mathrm{D}_{41}-S_{23} \mathrm{a}_{2} \\
& D_{41}=C_{1} d_{x}+S_{1} d_{y} \\
& d\left(\mathrm{NS}_{4}\right)=-C_{1} \mathrm{a}_{x} d d_{1}-S_{1} a_{y} d \theta_{1}+C_{1} d a_{z}-S_{1} d d_{x} \\
& N S_{4}=-S_{1}{ }^{a_{x}}+C_{1}{ }^{d} y \\
& d\left(N C_{4}\right)=-S_{23} D_{41}{ }^{d \theta}{ }_{23}+C_{23}{ }^{d D_{41}}-C_{23}{ }^{\mathrm{d}}{ }^{d \theta}{ }_{23}-S_{23}{ }^{d d} \\
& d D_{41}=-S_{1} a_{x} d \theta_{1}+C_{1} d a_{x}+C_{1} a_{y} d \theta_{1}+S_{1} d a_{z} \\
& d \theta_{5}=C_{5} d S_{5}-S_{5} d C_{5} \\
& d S_{5}=-S_{4} N C_{4} d \theta_{4}+C_{4} d\left(N C_{4}\right)+C_{4} d \theta_{4} N S_{4}+S_{4} d\left(N S_{4}\right) \\
& d C_{5}=C_{23}{ }^{d \theta} 23^{D_{41}}+S_{23}{ }^{d D_{41}}-S_{23}{ }^{d}{ }^{d \theta}{ }_{23}+C_{23}{ }^{d a_{2}} \\
& S_{5}=\mathrm{C}_{4} \mathrm{NC}_{4}+\mathrm{S}_{4} \mathrm{NS}_{4} \\
& c_{5}=S_{23}{ }^{J_{41}}+C_{23}{ }^{a_{2}}
\end{aligned}
$$

$$
\begin{aligned}
& d 0_{6}=C_{6} d S_{6}-S_{6} d C_{6} \\
& S_{6}=-C_{5} \mathrm{~N}_{61}-\mathrm{S}_{5} \mathrm{~N}_{612} \\
& c_{6}=-5_{4} N_{611}+c_{4} N_{6112} \\
& d S_{6}=S_{5} \mathrm{~N}_{61} \mathrm{dO}_{5}-\mathrm{C}_{5} \mathrm{dN}_{61}-\mathrm{C}_{5} \mathrm{~N}_{612} \mathrm{dog}_{5}-\mathrm{S}_{5} \mathrm{dN}_{612} \\
& d C_{6}=-d S_{4} N_{611}-S_{4} d N_{611}+d C_{4} N_{6112}+C_{4} d N_{6112} \\
& =-C_{4} N_{611} d_{4}-S_{4} d_{611}+S_{4} N_{6112} \mathrm{do}_{4}+C_{4} \mathrm{dN}_{6112} \\
& N_{61}=C_{4} N_{611}+S_{4} N_{6112} \\
& N_{611}=c_{23} \mathrm{~N}_{6111}-\mathrm{s}_{23} \mathrm{O}_{2} \\
& N_{6112}=-s_{1} 0_{x}+c_{1} O_{y} \\
& \mathrm{an}_{61}=-\mathrm{S}_{4} \mathrm{~N}_{611} \mathrm{~d} \mathrm{\theta}_{4}+\mathrm{C}_{4} d N_{611}+\mathrm{C}_{4} \mathrm{~N}_{6112} \mathrm{dg}_{4}+\mathrm{S}_{4} d \mathrm{~N}_{612} \\
& \mathrm{aN}_{611}=-\mathrm{S}_{23} 3^{\mathrm{N}} 61111^{\mathrm{d} \mathrm{\theta}} 23+\mathrm{C}_{23} \mathrm{aN}_{6111}-\mathrm{C}_{23} \mathrm{O}_{2} \mathrm{~d} \mathrm{\theta}_{23}-\mathrm{S}_{23} \mathrm{dO}_{2} \\
& \Delta N_{6112}=-c_{1} 0_{x} d \theta_{1}-s_{1} d 0_{x}-s_{1} 0_{y} d \theta_{1}+c_{1} d \theta_{y} \\
& n_{6111}=c_{1} 0_{x}+s_{1} 0_{y} \\
& \Delta N_{6111}=-S_{1} 0_{x} \mathrm{~d} \mathrm{\theta}_{1}+C_{1} d 0_{x}+c_{1} 0_{y} d \theta_{1}+S_{1} d 0_{y} \\
& \mathrm{~N}_{612}-\mathrm{S}_{23}{ }^{\mathrm{N}} \mathrm{~N}_{6111}-\mathrm{C}_{23} \mathrm{O}_{2} \\
& \mathrm{aN}_{612}=-\mathrm{C}_{23} \mathrm{~N}_{6111} \mathrm{~d} \mathrm{\theta}_{23}-\mathrm{S}_{23} \mathrm{dN}_{1111}+\mathrm{S}_{23} \mathrm{O}_{2} \mathrm{~d} \mathrm{\theta}_{23}-\mathrm{C}_{23} \mathrm{dO}_{2}
\end{aligned}
$$



Figure 1. Inner product step processor

# Manipulator Control by Exact Linearization 

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1. Abstract.

Comments on the application to rigid link manipulators of Geometric Control Theory, Resolved Acceleration Control, Operational Space Control, and Nonlinear Decoupling Theory are given, and the essential unity of these techniques for externally linearizing and decoupling end effector dynamics la discussed. Exploiting the fact that the mass matrix of a rigid link manipulator la positive definite - a consequence of rigid link manipulators belonging to the class of natural physical systems - it is shown that a necessary and sufficient condition for a locally externally linearizing and output decoupling feedback law to exist is that the end effector jacobian matrix be nonsingular. Furthermore, this linearizing feedback la easy to produce.

## 2. Introduction.

Because of the difficulty in controlling rigid link manipulators, along with primary concern in controlling end effector (EF) motions, It ia natural to ask if a nonlinear feedback law exists which will make an EF behave as if it has linear and decoupled dynamics. it has been known at least since the early l970s [1]-|S] that exact linearization of manipulators in joint apace is readily accomplished by the so-called Inverse or Computed Torque Technique. Efforts to accomplish decoupled linearization of mf motions directly in task apace began soon thereafter as is evident in the work of $\{6 \mid-\{14]$.

The work of $|6|$, although concerned only with controlifig the tip location of a three-link manipulator in the plane, is surprisingly prescient in its approach in that it proceeds by the three explicit steps of 1) decoupled linearization of tip behavior; 2) stabilization of the resulting tip dynamics; followed by 3) trajectory control of the now linearly behaving tip. Such clarity of approach will only be retrieved in the latter work of $|19|-[22 \mid$. The work $|6|$ also presages future work in its dealing with the problems of manipulator redundancy and actuator saturation.

With hindsight, the work [6] can also be viewed as a direct precursor to the development of the Resolved Acceleration Control (RAC) approach to the end effector tracking problem [7][8]. RAC essentially extends the work of (6) to the case of a full six dot manipulator yielding linearized EF positional error dynamics and almost linearized EF attitude error dynamics (the extent to which attitude error dynamics are "almost" linearized will be discussed below). The work of $\{7||8|$, however, did not make clear the three steps of \{6| and consequently appears to have not been appreciated as a technique for performing decoupled exact linearization of $E F$ motions, but rather as a technique for end effector tracking which has (almost) linear tracking error dynamics. The fact that the attitude error dynamics are not completely linearized also apparently obscured the appreciation of RAC as an exactly linearizing control technique.

The work of [9]-[11] applies Nonlinear Decoupling Theory (NDT) co provide decoupled linearization of a manipulator EF with simultaneous pole placement of the linearized EF dynamics. The abstruse formulation of this approach has apparently discouraged serious comparison with other approaches, the notable exception being $\therefore 231$ where correspondences to RAC and the Computed Torque technique have been noted. The simultaneous pole placement and linearization of $E f$ dynamics represents a blurring of the distinct steps 1 and 2 described above tor the approach [0].

In [12|-114|, manipulator dynamics are expressed in the task space, or Operational Space of the EF. The resulting nonlinear effective end effector dynamics are then linearized by the computed Torque method. Thus, the Operational Space Control (OCS) of $|12|-|14|$ can also be viewed as a Generalized Computed Torque technique. In [12] correspondences to RAC and the Computed Torque technique have been noted.

Recently, Geometric Control Theory (GCT) based techniques tor exactly externally linearizing and decoupling general affine-in-the-input nonlinear systems have ten developed [15]-[19]. These techniques provide constructive sufficient conditions for local decoupled external linearization which, if satisfied. produces the !inearizing feedback law. GCT has been applied to exactly linearizing end effector motions in \{19]-\{22]. The work of [19]-\{22] also provides a clear and mature control perspective which keeps the following steps distinct: 1) Exactly linearize d nd decouple end effector dynamics to a canonical decoupled double integrator form, ide. to Brunovsky Canonical fro (BCF); 2) Effect a stabilizing lop (pole placement step); () Perform feedforward precompensation to obtain nominal model following performance: if) (institute an LQR error correcting feedback loop. Unfortunately. tu understand the theoretical underpinnings ff geT requires
an exposure to differential geometry and Lie algebra/Lie group theory which most practicing angineera are unlikely to have.

It can be abown that all of the above secmingly quite different approachas lead to the ane linearising control lav for exact external linearisation and decoupling of EF motions [24]. (This equivalence is epecille to the nonlinear aystems considered here, via. ayatem, dynamically similar to rigid liniz manipulatora. mir and CCT appiy to a much latger clase tban this, and so the equivalence to lac and osc holds for syatem restricted to this clasa but not in gemeral). Recogniaing this equivalence enablea ue to give a almpe meceasary and aufficient condition for local decoupled external limearisation and to give afmple form for the limeariziag control wich is applicable to a broad clase of so-called matural physical dymaical syetens (25) [26] of mich a cerial link manipulator is but a apecial case. Por brevity we do not discuss actuated redundant arme - for discussion of these cases, see [24].
3. Dymanics of Finite Dimensional Natural Syateme.

Many phyaical systems have finite dimansional nonlinear dynatice of the form (25][26]:

$$
\begin{equation*}
M(q) \ddot{q}+V(q, q)-\tau ; q \subset x^{n}: q, \ddot{q} \in x^{n} \tag{11}
\end{equation*}
$$

$$
M(q) \in R^{n \times p} ; M(q)=M^{T}(q)>0, \forall q
$$

 a revolute manipulator.

Typically (1) arises as a solution to the Lagrange equations:

$$
\frac{d}{d t} \frac{\partial}{\partial q} L-\frac{\partial L}{\partial q}=Q^{T}
$$

where $L=T-U, T=1 / 2 \dot{q}^{T} M(q) \dot{q}$ is positive deflnite and autonomous, $U$ ls a conservative potential function, $Q=r+F a r e s e n e r s i l z e d f o r c e s$, and $F$ are dissipative or constraint forces. This is exactly how manipulator dynamics are obtained and hence manipulator dynasics are precisely of the form (l). Syateng which arise in this way are known.as natural systema (25)(26). It is known that fot natural syatema not only is m(q) positive definate, but $V(q, \dot{q})$ of (1) has terme which depend on $M(q)$ In a very apecial way $[27]-\{29 \mid$. In fact, natural syatema are nongeneric in the class of al: affine-in-the-input nonlinear systoms [38][39]. Although we shall only exploit the fact that $M(q)$ is poifive definite for any $q$, it is worth noting that the nongeneric structure of (1) has recently enabled important statements to he made on the exiatence of time optimal control laws [38]-[40], on the existence of globally atable control laws [27]-[33], on the existence of robust exponentially stable control lawa $\{34 \mid$, and on the exlatence of stable adaptive control laws \{35|-\{37] for the natural system ( 1 ).

Recognizing the special propertiea of the astem (1), it is not surprising that results yielding externall. linearizing behavior can be obtained much more easily than by application of NDT or GCT - theories which apply to the whole general class of smooth affine-in-the-input nonlinear aystems.
4. End Effector Kinematics and Control After Linearlaation.

The system ( 1 ) is assumed to have a read-out map of either the form

$$
\begin{equation*}
y=h(q) \subset R^{m}, v=\dot{y}=J(q) \dot{q} \in R^{m}, J(q)=\frac{\partial h}{\partial q} c R^{m \times n} \tag{2}
\end{equation*}
$$

or of the more general form

$$
y=h(q) \in M^{R}, v=J_{0}(q) \dot{q} \subset R^{\infty}, J_{0}(q) \in R^{q 0 \times n}
$$

 on $N^{n}, N^{2}$ is some dimensional output manifold, $J$ or $J_{0}$ is $c^{l}$, and in general $m$ and $n$ have different valives. ot ten $h(\cdot)$ is smooth (i.e. $C^{\infty}$ ) or even a diffeomorphism when the domain is suitably restricted. In subsequent discussion $\mathcal{\psi}=\dot{J} \dot{q}$ will mean that $J$ can be sither $J$ or $J_{0}$. Let the state of system (i) be ( $q$, $\dot{q}$ ). Then for $y=h(q), \mathcal{N}=\vec{J}(q) \dot{q}$ will be called the "velocity associated with the output $y$." Note that (2) is a special case of (3) where $\mathcal{N}$, the velocity agsociated with $y$, is just $\mathcal{V}=\dot{y}$ and $\boldsymbol{p}^{m} \mathrm{R}^{\mathrm{m}}$ giving $J=J=\partial h / \partial q$. Also :ote that for the case ( 3 ). since $h$ is $C^{2}$, it is still meaningful to talk about $\dot{y}=\mathrm{Jq}$ and $\mathrm{J}=\mathrm{zh} / \mathrm{dq}, \mathrm{J}(\mathrm{q})$ : $T_{q} N^{n}=R^{n}-T_{h}(q)$, but now the case where $\mathcal{d} \not \dot{y}$ is admitted as a possibility.
 location, the EF limear velocity, and $a \boldsymbol{m}^{3}$ the EF angular rate of change. It is well known that is not the time derivative of any minimi (1.e. 3 dimeneional) representation of attitude, so that $4=\left(\frac{2}{4}\right) J_{0}(q) \&$ as ia (3). In this case, we call $J_{g}(q)$ the "Standard Jacobian." it is also comon to represent tipleitude by a proper orthogonal matrix $A \in \mathrm{R}^{2 \times 3}$.

$$
A \text { c } \operatorname{so}(3)=|A| A^{T} A=A^{T}=I, \operatorname{det} A=+1 \mid
$$

where the colums of $A$ deternine $E f$ fixed body axes in the usual way. it ia well known that $A=$ an where Sv: = $\omega \times$ for all vc $\mathrm{R}^{3}$. Thus EF location and kinematics are often given by

Which should be compared to (3). Alternatively, we can take (cf. (2))

$$
\begin{equation*}
y=\binom{x}{a}=h(q) \subset R^{6}, N=y=\binom{\hat{y}}{\dot{B}}-J(q) \& \in R^{6}, B \in \Omega \subset R^{3} \tag{5}
\end{equation*}
$$

B $c Q^{3}$ la a mindeal representation of EF attitude (1.e. of the rotation group SO(3)). In general a a $f(A)$ for some function $f(\cdot)$ which is many-to-one or undefined if the domain of $f(\cdot)$ on $\operatorname{SO}(3)$ is not properly restricted. That is, because $S(3)$ cannot be covered by a aingle coordinate chart, $B$ is not valid for all posidble EF orientations and tinere vill be singularity of attitude repreaentation unless we rentrict Ef attitude to the region of $\operatorname{sO}(3)$ for which $B$ le valid $[25]$ [41]|42]; Thia restriction then forcea ito be
 In the case of Euler-Rodrlques parametsers where aingularity of attitude representation corresponds to
 and Euler-Rodriques parameters $\{25 \mid$, $[41]-\{43]$. The kinematical relationahip between $B$ and $w$ is given by

$$
\begin{equation*}
\dot{B}=\Pi(B)_{w} \tag{6}
\end{equation*}
$$

where $n^{c} \mathrm{R}^{3 \times 3}$ will lose, rank, i.e, become aingular, precisely when $B$ becomes a aingular representation of EF attitude. Note $f$ rom (3)-(6) that

$$
J=\left(\begin{array}{ll}
1 & 0 \\
0 & \Pi
\end{array}\right) J_{0} .
$$

Generally, the standard Jacobian matrix $J_{0}$ will become singular only at a manipulator kinematic aingularity, in which case $J$ will also be singular. Furthermore, $J$ will be aingular when $B=B(q)$ givea a ingularity of EF atcitude representation. This compounds the trajectory planning problem for EF motions, aince now we must plan trajectoriea which avoid manipulator kinematic angularitiea and also ensure that $B(q)$ $\varepsilon$.

Henceforth the syntem (1). (2) or (1), (3) will be sald to be exactly externally linearized and decoupled it

$$
\begin{equation*}
\dot{v}=u \in R^{6} \tag{7}
\end{equation*}
$$

This is somewhat of an abuse of notation as a consideration of the system (1), (4) shows. For $u=\binom{u_{1}}{u_{2}}$, $\dot{y}=u$ yielde

$$
\begin{equation*}
\ddot{x}=u_{1} c R^{3}, \quad \dot{\omega}=u_{2} \in R^{3}, \quad \dot{A}=\tilde{\omega} A \tag{8}
\end{equation*}
$$

Although EF positional dynamics are decoupled and linearized to $\ddot{x}=u$, attitude dynamics are nonlinear and Ifven by $\dot{\omega}=u_{2}, \dot{A}=\mathrm{KA}$. Eq. (8) Is precisely the sense in which Rac can be said to almost "exactly externally linearize and decouple" attitude error dynamics as was discuased in the introduction. In the case of the byatea (1), (5), $0=\left(u_{2}\right)$ sives

$$
\begin{equation*}
\ddot{x}=u_{1} c R^{3}, \quad \ddot{B}=u_{2} c R^{3}, \quad B \subset \Omega \subset R^{3} \tag{9}
\end{equation*}
$$

which can indeed be aid to te exactly externally linearized and decoupled. Drawbacks to using (9) are that 8 mat always be controlled to remain in $Q_{\text {, }}$ trajectories involving $B$ gay be difficult to visualize, and the generalised force, $u_{2}$, which drives 8 may be nonintuitive. On the other hand, it is obvious how to obtala stable attitude tracking from (9). The advantage to using (8) is that and a are esily viaualised entities, whlle $u_{2}$ is the or ilinary torque that we are all familiar with. Fortunately, deaplte the nonlinear attitude dymaica, it is possible to use (8) to perform EP attitude tracking vith asympotically vanishing attitude error [7] [8].

Note that once (8) is obtained, it is easy to get (9) by use of the relationship (6). If we have $\dot{G}$ a, $B=\Pi(B) \omega_{0}$ and $s \in \Omega$ so that $\Pi^{-1}(B)$ exists, then use of

$$
\begin{equation*}
\dot{\Delta}=u_{1} \quad u=a^{-1}(B)\left(\pi-\dot{\Pi}(B)_{\omega}\right) \tag{10}
\end{equation*}
$$

sivan

$$
\begin{equation*}
\ddot{B}=\tilde{n}(B) \dot{\omega}+\dot{n}(A) \omega=\bar{u} . \tag{11}
\end{equation*}
$$

 and then control.
5. Compartson of GCT, NDT, and OSC.

For brevity, we consider the non-redundant manipulator case, taking $n=6$ in ( 1 ), and we omit derivations. A more detalled discussion is given in [24].

Note that the syatem (1), (5) can be written as

$$
\begin{equation*}
\frac{d}{d t}\binom{q}{\dot{q}}=\binom{\dot{q}}{-M^{-1} v}+\binom{0}{M^{-1}} r_{0} \quad y=h(q) \tag{12}
\end{equation*}
$$

or. taking $2=\binom{q}{q}$.

$$
\begin{equation*}
\frac{d}{d t} Z=A(Z)+B(Z) \tau, y=H(Z) \tag{13}
\end{equation*}
$$

where the definitions of $A, B$, and $H$ are obvious. CCT asks: does there exist (i) a nonlinear feedback $t=Q(2)+B(2) u$ and (ii) a nonlinear change of basis $x=X(2)$ such that (12) is placed into BCF?:

$$
\frac{d}{d c}\binom{y}{\dot{y}}=\left(\begin{array}{ll}
0 & I  \tag{14}\\
0 & 0
\end{array}\right)\binom{y}{\dot{y}}+\binom{0}{I} u \Leftrightarrow \ddot{y}=u .
$$

The constructive sufficient conditions of [19]-[22] can be applied and give the following linearizing and decoupling feedback law:

$$
\tau=-M J^{-1} \partial J \dot{q}+V+M J^{-1} u
$$

where

$$
\partial J=\left[\sum_{k=1}^{n} \frac{\partial J_{i k}}{\partial q_{j}} \dot{q}_{k}\right] .
$$

Although $\partial J \neq J$, it is true that $\partial J \dot{q}=j \dot{q}$ giving

$$
\begin{equation*}
\tau=-M J^{-1} j \dot{q}+M J^{-1} u+V \tag{15}
\end{equation*}
$$

Note that $J$ must be nonsingular for (15) to exist. This is consistent with the theory of [19]-[12] which provides sufficient conditions for local linearization. Note also that to implement (15), explicit expressions for $M, J^{-1}, J$, and $V$ are required.

The NDT approach of [11], constructs the lineariaing feedback in the following way. For the system (13) define

$$
\begin{equation*}
G(z)=\frac{\partial}{\partial z}\left(\left[\frac{\partial}{\partial z} C(z)\right] \cdot A(Z)\right), D(z)=G(Z) \cdot B(z), C^{*}(Z)=G(A) \cdot A(z) \tag{16}
\end{equation*}
$$

The use of

$$
\begin{equation*}
\tau=-D^{\star-1}(2) C \star(2)+D^{\star-1}(2) u \tag{17}
\end{equation*}
$$

will transform (12), (13) to $\ddot{y}=u$, i.e. to (14).

It is atraightforward to ghow that, for $A, 8$, and $C$ as in (12) and (13), eq. (17) is precisely eq. (15). Note that in (l3) we take $2=\binom{9}{\dot{d}}$ and not $2=\left(91, \dot{q}_{1}, \ldots, q_{n}, \dot{q}_{n}\right)^{T}$. The latter cholce for 2 is taken in [il] and serves to obscure the find result - namely that (17) and (15) are equivalent.

Now consider the OSC approach of [12]-[14]. In (1) let $V=B-G$ where $B$ is the coriolis forces and $G$ the gravity forces. Restrict the domain of the system (l), (5) to ensure that $h(\cdot)$, ia a bijection (and consequently det $J(q) \neq 0$ on this restriction). This restriction means that, as for DCC and NDT, the following gives a local reault for external linearization. In $[12]-[14]$, the effective Ef dynamics are determined to be

$$
\begin{align*}
\Lambda(y) \ddot{y}+C(y, \dot{y}) & =F, \tau=J^{T} F, \quad \Lambda=J^{-T} M J, C=U-P,  \tag{18}\\
P=J^{-1} G, U & =J^{-T} B-\Lambda \dot{J} \dot{q}, \quad q=h^{-1}(y), \dot{q}=J^{-1} \dot{y}
\end{align*}
$$

Recall that for the system (1), $M \ddot{q}+V=\tau$, the Computed Torque technique is to take $\tau=M u+V$, yielding $M(\ddot{q}-u)=0 \Rightarrow \ddot{q}=u, \operatorname{since} M(q), 0, V q$. Similarly, in (18) $A(y)>0$ for every $y=h(q)$ where $q$ is on the restricted domain. Therefore a cholce of

$$
\begin{equation*}
F=\Lambda(y) u+C(y, \dot{y}), \quad \tau=J T_{F} \tag{19}
\end{equation*}
$$

In (18) yleids $\Lambda(y)(\ddot{y}-u)=0 \Rightarrow \ddot{y}=u$. In this sense the work in $[12]-\{14]$ can be viewed as a Generalized Computed Torque technique. From (18) and (19) it is stralght forward to determine that $\tau$ of (19) is exactly $\tau$ of eq. (15).
b. Derivation of a Feedback Law for Local Exact Decoupled External Linearization and Its Relationship to RAC and GCT.

Recall that the system (1), (7) or (1), (3) is of the form
-

$$
\begin{align*}
& M(q) \ddot{q}+V(q, \dot{q})=\tau ; q \in N^{n} ; \dot{q}, \ddot{q} \subset R^{n} \\
& y=h(q) c M^{n} ; h(\cdot) \text { is } c^{2} ; q=J(q) \dot{q} c R^{m} ; J(q) \text { is } c^{1} ;  \tag{20}\\
& M(q) \in R^{n \times n} ; M(q)=M(q)^{T}, 0, q \in N^{n}
\end{align*}
$$

where in general, it may be that $m \neq n, M^{\prime \prime} \neq R^{m}, v \neq \dot{y}$, and $J \neq J=\partial h / \partial q$.

It is assumed that a necessary and sufficient condition for $h(q)$ to be onto some neighborhood of $y=h(q)$ in $\boldsymbol{q}^{m}$ is that the mapping $\vec{J}(q)$ be onto $R^{m}$, f.e. we assume that $\vec{J}(q)$ is onto $R^{m}$ if and only if $J(q)=\partial h(q) / \partial q$ is onto $T_{h}(g)^{M^{m}}=R^{m}$. This is a reasonable assumption; for example, when $M^{m}=R^{m}, v=\dot{y}=J(q) q^{\circ} c R^{m}$, and $J=j=\partial h / g q$ this is trivially true. For the case $y=h(q)=(x, A) c M^{6}=R^{3} x$ So( 3 ) and $\quad=J_{0}$ where $\nu=\left(\begin{array}{l}\dot{x} \\ \omega\end{array}=J_{0}(q) \dot{q}\right.$, the fact that $\dot{x} \varepsilon T_{x} R^{3}=R^{3}$ and $\dot{A}=\tilde{W} A \in T_{A} S O(3)$ means that tor $J_{0}(q)$ onto, we can fill out a neighborhood of ( $x, A$ ) and otherwise we cannot. (A general element of $T_{A} S O(3)$ is precisely of the form $\tilde{W} A$


Definition LEL: The system (20) can be locally exactly linearized and decoupled (LEL) over an open



Note that for an EF to be LEL at $j^{\prime}$ it must be true that $y^{\prime}$ be in the range of $h(\cdot)$, i.e. $y^{\prime}$ must be a
 only one of the possible configurationa $h^{-1}\left(y^{\prime}\right)$. Thua we can interpret $q^{\prime} \in h^{-1}\left(y^{\prime}\right)$ to be the actual physical configuration of a manipulator. If the aystem (20) is not LeL at $y^{\prime}$ in the conflguration $q^{\prime} c h^{-1}\left(y^{\prime}\right)$ it may be LEL at a different conflguration $q^{4} c h^{-1}\left(y^{\prime}\right)$.

Theorem LEL: A necesaary and sufficient condition for (20) to be LEL at $y^{\prime} c h\left(\mathcal{N}^{\mathrm{m}}\right)$ in the configuration
 exactly linearizing and decoupling feedback is given by

$$
\begin{equation*}
\tau=M(q) \xi+V(q, \dot{q}) \tag{21}
\end{equation*}
$$

where $\xi$ is any solution to

$$
\begin{equation*}
\bar{J}(q) \xi=-\dot{J}(q) \dot{q}+u . \tag{22}
\end{equation*}
$$

When $m=n$ this gives

$$
\begin{equation*}
\tau=-M(q) \bar{J}(q)^{-1 \dot{J}}(q) \dot{q}+M(q) \bar{J}(q)^{-1} u+V(q, \dot{q}) . \tag{23}
\end{equation*}
$$

0
Proof. Necessity: Suppose that $\dot{\mathrm{V}}=\boldsymbol{J}\left(q^{\prime}\right) \ddot{q}{ }^{\prime}+\dot{J}\left(q^{\prime}\right) \dot{q}^{\prime}=u$ can be made to hold regardless of the value of $u \in R^{m}$. This means that there must exist $q \in R^{n}$ such that

$$
\begin{equation*}
\bar{J}\left(q^{\prime}\right) \ddot{q}^{\prime}=-\dot{J}\left(q^{\prime}\right) \dot{q}^{\prime}+u . \tag{24}
\end{equation*}
$$

If $\bar{J}\left(q^{\prime}\right)$ is not onto, then Im $\bar{J}\left(q^{\prime}\right) \subset R^{m}$ and Im $\bar{J}\left(q^{\prime}\right) \neq R^{m}$. Let $u$ be such that $-\dot{J}\left(q^{\prime}\right) \dot{q^{\prime}}+u \& I m \bar{J}\left(q^{\prime}\right)$. Then there is no "̈' for which (24) holda, yielding a contradiction. Sufficiency: By assumption $\bar{j}(q$ ') la full rank and onto ( $=)^{\prime} J\left(q^{\prime}\right)=\partial h\left(q^{\prime}\right) / \partial q$ is full rank and onto. Since $J$ and $J$ are $C^{\prime}$. there exists neighborhoods $B^{m}\left(y^{\prime}\right)$ and $\mathrm{B}^{n}\left(q^{\circ}\right), y^{\prime}=h\left(q^{\prime}\right)$, such that $\mathrm{Bm}^{m}\left(y^{\prime}\right)=\mathrm{h}\left(\mathrm{B}^{n}\left(q^{\prime}\right)\right)$ and such that $\bar{J}$ fe fuli rank and onto when restricted to


$$
\begin{equation*}
\dot{v}=\bar{J}(q) \ddot{q}+\dot{\bar{J}}(q) \dot{q} . \tag{25}
\end{equation*}
$$

Let $\xi$ be any solution to (22). E is guaranteed to exist since $\mathrm{Im} \overrightarrow{\mathrm{J}}(\mathrm{q})=\mathrm{R}^{\mathrm{L}}$. Take r to be (21), then

$$
\ddot{M}+v=\tau=M \xi+v \Rightarrow M(\ddot{q}-\xi)=0 \Rightarrow \ddot{q}=\xi_{1}
$$

which with (22) and (25) gives $\dot{v}=u$.

## Comsents:

1) Note that this result applies to all systems of the form (20), of which rigid link manipuiators are a spectal case.
2) Note that with $y c M^{m}$ and $\tau \in R^{n}$, the fact that we need $m \leq n$ can be interpreted to mean that there must be least as many inputs as outputs.
3) When $\bar{J}=J=\partial h / \partial q, v=\dot{y}$, and $m=n$ we have that $\tau=-M J J^{-1} \dot{j} \dot{q}+m J^{-1} u+V \Rightarrow \ddot{y}=u$ when det $J \neq 0$. This is the same result provided by GCT, NDT, and OCS as seen in the last section.
4) Note that in the proof we force $\ddot{q}=\xi$ precisely like $\ddot{q}=u$ is forced to happen in the Computed Torque method. In fact, for $y=q$ we have $J=1$ and $j=0$ giving $\xi$. Thus the exact linearizing control of (21). (22) is seen to be a generalization of the Computed Torque method in a somewhat different, and perhaps more illuminating, way than ocs.

Let us consider the case of EF control given by the syatem (1), (4). Here $J=J_{0}$ where $\mathcal{N}=\binom{\dot{x}}{\omega}=J_{0} \dot{q}$. In this case, when men, (23) is

$$
\begin{equation*}
\tau=-M J_{0}^{-1} j_{0} \dot{q}+M J_{0}^{-1} u+V \tag{26}
\end{equation*}
$$

When det $J_{0} \neq 0$, use of $t$ yields $\binom{\ddot{x}}{\dot{e}}=\binom{u_{1}}{u_{2}}$. This is precisely kAC [7], [8]. Theorem LEL can be interpreted as an extension of RAC to the redundant arm case which allowifor the use of ainimal representation of EF attitude [24]. The more general case $m \leq n$ is given by

$$
\begin{equation*}
\tau=M u+V, J_{0} \xi=-\dot{j}_{0} \dot{q}+u \tag{27}
\end{equation*}
$$

By using the indirect form (27), t can be obtained, after $\xi$ has been found, by uge of the Newton-Euler recursion (45). Furthermore $\xi$ can be obtained recuraively-either directiy (46), or by first recursively obtaining $J_{0}$ and $j_{0}$ and then solving for $\mathcal{F}$ by Gausian Elimination. The major point to be drawn here, is that (27) sinows us how to perform exact external linearization without the need for an explicit manipulator model. After exactly linearizing to $\binom{x}{\dot{\omega}}=\binom{u_{1}}{u_{2}}$ one can perform EF tracking at this stage [7][8], or one can continue to the form (11) by the use of ${ }^{\omega}(10)$.

When using (26) or (27), the only way that rank $J_{0}$ \& $m$ can occur for $m \leq i s$ when the manipulator is at a mechanically singular configuration. Recall (section 4) that in the case when a minimal representation of $E F$ attitude is used, the resulting Jacobian matrix $J$ wlll be rank deficient not just for a manipulator singularity, but at a configuration which leads to a singularity of attitude presentation. Thus rank deficiency of $J_{0}$ is kinematically cleaner to understand. The necessity that rank $J_{0}=m$ in order to use (26) or (27) allowa two obvious, but important statements to be made: i) For a manipulator with a workspace boundary (ignoring joint stops), as in the case of a PUMA-type manipulator, exact linearization at the boundary 1s faposibible; if) for a nonredundant ( 6 dof) manipulator with workapace interior singularities, there cannot be exact linearization throughout the workspace interior. For a redundant manipulator with workspace interior singularities, it may be possible to avold worksace interior configurationa which cannot be exactly linearized by the use of self motions as described in [48][49]. This is related to the multiplicity of solutions available for $\xi$ in (27).

It is interesting to ask just how the control (23) fulfills the aim of GCT as stated in (12)-(14). We have the nonlinear feedback (taking $N=\dot{y}$ aud $\left.J=\bar{J}) \tau=Q(Z)+B(2) u=(V-M J-1 \dot{j} \dot{q})+(M J)^{-1}\right) u$ which when applied to (12), (13) gives

$$
\frac{d}{d t}\binom{q}{\dot{q}}=\left(\begin{array}{cc}
0 & I  \tag{28}\\
0 & -j \\
-1 & j
\end{array}\right)\binom{q}{\dot{q}} \cdot\binom{0}{J^{-1}} u
$$

Consider the local nonlinear change of basis given by

$$
\binom{y}{\dot{y}}=\binom{h(q)}{j \dot{q}}:\binom{q}{\dot{q}}=\binom{h^{-1}(y)}{J^{-1} \dot{y}} .
$$

The fact that $\dot{y}=J \dot{q}$ and $\ddot{y}=J \ddot{q}+j \dot{q}$ gives

$$
\frac{d}{d t}\binom{y}{\dot{y}}=\left(\begin{array}{ll}
\mathrm{J} & 0 \\
j & j
\end{array}\right) \frac{d}{d t}\binom{q}{\dot{q}}
$$

Writing (28) as

$$
\left(\begin{array}{ll}
J & 0 \\
j & j
\end{array}\right) \frac{d}{d t}\binom{q}{\dot{q}}=\left(\begin{array}{ll}
j & 0 \\
j & j
\end{array}\right)\left(\begin{array}{ccc}
0 & I & \\
0 & -J^{-1} & j
\end{array}\right)\left(\begin{array}{cc}
h^{-1}(y) \\
J^{-1} & \dot{y}
\end{array}\right)+\left(\begin{array}{ll}
j & 0 \\
j & j
\end{array}\right)\binom{0}{J^{-i}} u
$$

we obtain the BCF

$$
\frac{d}{d t}\binom{y}{\dot{y}}=\left(\begin{array}{ll}
0 & 1 \\
0 & 0
\end{array}\right)\binom{y}{\dot{y}}+\binom{0}{1} u, \Leftrightarrow \ddot{y}=u
$$

Of course we are benefiting from the hindsight provided us by GCT [151-(19).

## 7. Concluding Remarks

Recognizing the fundamental unity of RAC, GCT, OSC, and NDT [7]-\{22\} for exact linearization of manipulators, we can focus on their true differences - namely differences in implementation detail and design philosophy. With the awareness that they all produce essentially the same linearizing feedback, we can ask why this particular feedback form is appropriate for manipulator-like systems.

OCS and RAC exploit the specific structure of auch systems. Not surprisingly, the solutions arrived at, reflecting the philosophies and implementation perspectives of the researchers involved, are quite distinct in thetr flavor and presentation. Yet, since the properties specific to manipulator dynamics ultimately forced the solution, they are fundamentally the same. (Actually, apparently only ocs worked with a perspective directed specif!cally towards decoupled EF motions. RAC is content to stop at a point just shy of the goal. It is also interesting that [12] apparently shows an awareness of the relationship between OCS and RAC, and che degree to which RAC can be said to decouple and linearize EF motions). The important point here is that researchers consciously exploited the specific properties of system of interest, but without pin-pointing precisely what these properties were which made the system amenable to linearizing control.

CCT and NDT provide techniques for exactly linearizing general smooth affine-in-the-input dynamical systems. These techniques ignore any specific nongeneric structural properties that a system might have and as a consequence the solutions obtained are much less transparent than those of OCS or RAC. The strength of these approaches, particularly GCT, is that they can provide necessary and sufficient conditions for a system to be exactly linearizable and constructive sufficient conditions which produce the linearizing feedback when satisfied. These techniques can be applied to systems which defy our abilities to intuit or comprehend - such as manipulators coupled to complex electromechanical actuation devices. Interestingly, when applied to the problem of manipulator exact linearization the solutions ohtained can be shown to be equivalent to those of RAC and OCS. Again the structural properties of the system forced the solution. Once a solution is known to exist, it is reasonable to attempt to produce it from more physical arguments knowing now that the search is not fruitless. This leads to a reexamination of OCS and RAC.

The work of \{17]-\{22| stresses a perspective which serves to enable a clearer comparison between sompeting techniques for external linearization: place the system in a standard linear canonical form before acditional control efforts are made - this ensures that the process of linearizing the system is not mixed up with, and confused with, the process of stabilizing and controlling it. This perspective greatly aided the comparison of GCT, OCS, RAC, and NDT which resulted in [24]. In turn, this comparison focuses attention on the structural properties of manipulators.

Much current research makes it apparent that systems dynamically similar to rigid link manipulators have important structural properties which can be exploited to achieve results which are quite strong when compared to those available for general smooth affine-in-the-inputs nonlinear systems [2S]-[40]. Here we have seen that exploiting the nongeneric second order form of system (l) with an everywhere positive definite mass matrix and d $\because$ lucally unto readout map enables a simple form for the linearizing teedback.

## 8. Acknowledgments

This work was done the fet Propulsion Laboratory, Califoraid Institue of rechology, ander contract with the National deronauties and Space Administration.

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# A Virtual Manipulator Model for Space Robotic Systems 

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 ackn

## 1. Abstract

Future robotic manipulators carried by a spacecraft will be required to perform complex tasks in space, like repairing satellites. Such applications of robotic manipulators will encounter a number of kinematic, dynamic and control problems due to the dynamic coupling between the manipulators and the spacecraft. This-peper presents a new analytical modeling method for studying the kinematics and dynamics of manipulators in apace. The problem ia treated by introducing the concept of a Virtual Manipulator (VM). The kinematic and dynamic motions of the manipulator, vehicle and payload, can be described relatively easily in terms of the Virtual Manipulator movements, which have a fixed base in inertial space at a point called a Virtual Ground. It is anticipated that the approach described inchiepapar-will aid in the design and development of future space manipulator systems.

## 2. Introduction

Robotic manipulators are potentially very useful for performing complex tasks in non-industrial hostile environments $\{1,2]$, such as in space. A number of studies have considered the potential applications of manipulators in space and the capabilities that these systems must have to achieve anticipated mission goals $|3-5|$. These applications include tasks such as repairing, servicing and constructing space stations in orbit. Currently, these tasks can only be performed by astronaut Extra Vehicular Activity (EVA). Eliminating the need for EVA would obviously reduce hazards to the astronauts and mission costs.

Unfortunately, the use of manipulators in space is complicated by the manipulator/spacecraft dynamic coupling. For example, movements of a manipulator will disturb the attitude of the spacecraft carrying it. This coupling will adversely affect the manipulator's precision, and reduce the on orbit life of the system by consuming excessive attitude control fuel. Also, any motions of the spacecraft, say due to the firing of attitute control jets, will disturb the manipulator. Therefore, new manipulator concepts, designs and control techniques will be required to minimise and compensate for the manipulator/spacecraft dynamic coupling.

Researchers working on the control of space manipulators have focused their attention on issues such as semsor requirements, path planning algorithms, teleoperator control [6-8]; the problems of vehicle/manipulator dynamic interactions remain unresolved.

This paper presents a new and effective analytical modeling method for studying the kinematics and dynamics of manipulators in space. The problem is treated by introducing the concepts of a Virtual Ground (VG) and Virtual Manipulator (VM). As discussed below the VG is located at the center of mass of the manipulator/spacecraft, , stem. and the VM is an ideal kinematic chain connecting the VG to any point on the real manipulator. Motions of a system, including a vehicle, manipulator and payload can be described easily by the VM. This model has proven to be effective in calculating the kinematic and dynamic properties of the system; such as its inverse kinematic solution and workspaces. This paper shows that the VM approach can also be used \&o plan the manipulator's motions in order to minimize the degrading consequences of the manipulator/spacecraft dynamic interactions.

## 3. A Model of Manipulators in Space

Future space manipulator systems will have one or more mechanical arms carried by a vehicle, as shown in Figure 1. The vehicle will be capable of motion in six degrees of freedoms, and will have reaction jet for position and attitude control. Although manipulators could be driven by photovoltaicly powered electric actuators, which use no
where

$$
\begin{equation*}
M_{\mathrm{tot}}=\sum_{\mathrm{p}=1}^{N} M_{\mathrm{i}} \tag{2}
\end{equation*}
$$

Since there are no external forces, the VG is stationary in the frame $N$ and the vector $\mathbf{V}$, is always constant.
In the following development the VM properties such as link dimensions and joint axes, for initial manipulator configuration are described. Then the rules for its joint movemente as a function of the real manipulator joint movements are presented. Referring to Figure 3, which shows the end effector VM for the manipulator shown in Figure 2, the $i^{\text {th }}$ link of the Virtual Manipulator is defined by the vector $\mathbf{V}_{i}$,

$$
\begin{align*}
\mathbf{V}_{1} & =\mathbf{D}_{1} \\
\mathbf{V}_{2} & =\mathbf{H}_{1}+\mathbf{D}_{2} \\
& \vdots  \tag{3}\\
\mathbf{V}_{N} & \doteq \mathbf{H}_{N-1}+\mathbf{D}_{N}
\end{align*}
$$

where

$$
\begin{equation*}
D_{i}=\mathbf{R}_{i} \sum_{i=1}^{i} M_{i} / M_{i o t} \quad(i=1,2, \ldots, N) \tag{4}
\end{equation*}
$$

$$
\begin{equation*}
H_{i}=L_{i} \sum_{i=1}^{i=1} M_{i} / M_{t o t} \quad(i=1,2, \ldots, N-1) \tag{5}
\end{equation*}
$$

and

The first VM link represents the vehicle's orientation. This link is attached io the VG by a spherical joint and ite motion is equal to the three vehicle rotations with respect to inertial space. The end of the Virtual Manipulator terminstes at the end effector, defined by a vector E , fixed in the $N^{\text {th }}$ VM link.

The $i^{\text {th }}$ VM joint is taken as a revolute or a prismatic joint depending upon whether the $i^{\text {th }}$ joint of the real manipulator is revolute or prismatic. The axis of rotation for a revolute VM joint, $\dot{j}$, is parallel to the axis of the real manipulator joint $A_{i}$. Similarly, the translational axes of prismatic VM jointe are parallel to the correaponding axes of the real manipulator prismatic joints. Equations (1) through (5) define the VM and its position correaponding to the initial position of the system, as shown in Figure 2. The VM links will all be parallel to the real manipulator links in cases where all the centers of mass for all manipulator links lie on a line connecting the manipulator joints on the corresponding link.

The VM will move as the joints of the real manipulator move. The angular rotations of the VM revolute jointe, from their initial position, are equal to the angular rotations of the corresponding revolute joints for the real manipulator. The prismatic virtual joint tranalations are ratios of the corresponding real prismatic joint translations. For an end effector VM, translation of the virtual joint, $P_{j}$, is given by:

$$
\begin{equation*}
\mathbf{P}_{j}=\mathrm{T}_{j} \sum_{q=1}^{j} M_{q} / M_{\mathrm{tot}} \tag{6}
\end{equation*}
$$

For the VM in its position of construction, its initial position, the values of $\mathbf{T}_{j}$ are taken as zero. Hence the initial magnitudes of $\mathbf{P}_{\boldsymbol{j}}$ are zero. The prismatic joint motions, $\mathbf{T}_{\boldsymbol{j}}$, are referenced to the initial position.

If a VM that is constructed according to Equations (1) through (5), moves with the real manipulator according to the above description, and its link shapes and lengths remain constant as the manipulator moves, then it can be shown (see Appendix A) that:

1 The axis of the $i^{\text {th }}$ virtual joint is always parallel to the $i^{\text {th }}$ axis of the real system joint, and
2 The Virtual Manipulator end point will always coincide with the real manipulator's end effector.

These properties enable the kinematic and dynamic motions of a free-floating manipulator system to be described by the motions of a much simpler Virtual Manipulator which has a fixed base in inertial space. The properties of the VM remain the same as long as the mass property of the system does not change. For example, when the manipulator grasps a free-loating pay load, the VM changes. According to Equations (1) through (5), the VM link lengths will be reduced for the addition of a payload. Virtual Manipulators constructed for points other than the end effector have different links than the links defined in Equation (1); and their joint movements maybe different than the ones described above, for example, prismatic joint translations may be the vector ( $\mathbf{P}_{\boldsymbol{i}}-\mathbf{T}_{\mathbf{i}}$ ), depending upon location of the point used to construct the VM [9].
recction fuel, manipulator motions could disturb a vehicle's position and attitude and result in the consumption of excesaive amounts of attitude fuel. The useful life of apacecraft syatema is often limited by the amount of reaction jet fuel they can carry.

Two approaches to solve this problem are: 1) permit the vehicle to move and compensate for the baee motions in the manipulator task planning; and 2) plan the manipulator motions so that they do not cause the vehicle to move excesaively. The first of these approsches requires the ability to perform inverse kinematic and workapace calculatioss for a free-floating system [10]. The eocond requires methode for planning manipulator motions that would self-correct the vehicle's orientation with little or no reaction jet adjustmenta. These appramehes and amociated imucs are addressed here through the Virtual Manipulator technique. Asaumed in this work is that the external forces/torques acting on the aystem are negligible, and that the ayatem is free floating. Also ascumed is that the ayotem elementa may be modeled as rigid bodiea. The later amumption may not be valid if a manipulator must perform high apeed motions.

## 4. Analytical Development of the Virtual Manipulator

The Virtual Manipulator (VM) is a masoless kinematic chain terminating at an arbitrary point on the real manipulator. Its base is the Virtual Ground (VG), which is an imaginary fixed point in inertial apace. It is proven below that for a given system the properties of the VM and location of the VG are fixed. VMa exiat for many different manipulator structures, auch as open or closed chains, aingle or many branch arma, revolute or priamatic joints $|9-11|$. The discussion in this paper will be limited to manipulators composed of spatial serial chains with revolute or prismatic connections. Although VMs exist for any point on the real manipulator, this paper deals with VMs whose end points coincide with the real manipulator end effector.

The VG is defined to be the center of mass of the manipulator aystem. From elementry mechanica, when there are no external forces, such as from reaction jets, the VG will be fixed in an inertial apace. It will not move due to any internal forces of the system such as joint torques, or due to any manipulator motions.

Figure 2 shows a schematic drawing of an N body spatial manipulator system. The first body in the chain represents the vehicle which carries the manipulator. The $N^{\text {th }}$ body is a combination of the payload and the last link. The $i^{i t h}$ joint is called $J_{i}$, and $C_{i}$ is the center of mase of the $i^{\text {th }}$ body. The vectora $\mathbf{R}_{i}$ and $\boldsymbol{L}_{i}$ connect $C_{i}$ to $J_{i}$ and $J_{i}$ to $C_{i+1}$, respectively. The vector $R_{N}$ connects $C_{N}$ to the end effector. The vectors $R_{i}$ and $L_{i-1}$ are fixed relative to the $i^{\text {th }}$ link, and hence the angle between these vectors is constant for all aystem configurations. If the $i^{\text {th }}$ manipulator joint is a revolute joint, the vector defining the axis of rotation of $J_{i}$ is called $A_{i}$, and the angle $\theta_{i}$ is the rotation of the $i^{\text {th }}$ joint. If the $i^{i^{t h}}$ manipulator joint is a priamatic joint, the vector $T_{i}$ ia defined to be the tranalation along the translational axis. If the $i^{\text {th }}$ joint is revolute, then the magnitude $T_{i}$ is equal to zero.


Figure 1: A Space Manipulator System.


Figure 2: N Body System in Space.

The location of the VG for this system in inertial space, the center of mass of the system, can be found by knowing some initial position of the system. The vector $\mathbf{S}(0)$ defines the initial known location of the end effector with respect to an inertial reference frame $N$. Then the location of the VG, the vector $\mathrm{V}_{\mathrm{g}}$, can be obtained from conservation of linear momentum by:

$$
\begin{equation*}
\mathbf{V}_{g}=\sum_{i=1}^{N}\left[\mathbf{S}(0)-\sum_{j=1}^{i-1}\left(\mathbf{R}_{j}+\mathbf{L}_{j}+\mathbf{T},\right)\right] M_{i} / M_{\ell 0 t} \tag{1}
\end{equation*}
$$



Figure 3: N Body Syatem and its VM.

Table 1 givea the properties of a very simple planar manipulator and ita Virtual Manipulator, shown in Figure 4. It should be remembered that the method is not restricted to planar aystems.

## 5. Applications of Virtual Manipulators

The Virtual Manipulator approach hat a number of poseible applications. VMa can be used to simplify the inverse kinematics of apace manipulators, calculate their workapaces, plan their motions and formulate the equations of motion $\{9-11 \mid$. It should be noted that using conventional methods, these problema are far more difficult for space manipulators than for industrial manipulatora with fixed bases. In the sections below, the use of the VM is shown for workspace analysis and path planning.

## A. Workapace Analysis

Since the vehicle and manipulator'dynamica are coupled, the manipulator's motions will cause the vehicle to move and this in turn makes it difficult to fint the manipulator workapace. In fact several different types of workapaces exist. In this section, a workspace called the conatrained workspace, for a manipulator in apace is defined, for a more complete discuseion of space manipulator workspaces refer to references $[9,10]$. For the constrained workspace it is asaumed that the attitude, but not the location, of the vehicle is controlled. This can be achieved without the use of attitude control fuel by employing reaction wheele, or by using the self correcting maneuvera discussed later in this paper.

To find the constrained workspace, a Virtual Manipulator is constructed to the end effector of the real manipulator. The joint limits of the real manipulator are transformed into VM joint limits. The workspace of the Virtual Manipulator is then found using conventional workapace analysia methods [12]. The real manipulator workspace will be equal to the VM workspace because of the following reasons. The VM end point coincides with the real manipulator end effector, and it is assumed that it is possible to control the orientation of the firat VM link, representing the vehicle, with respect to inertial space. The other joints are controlled with their actuators. This workspace will always be a spherical shell, asouming there are no limita on the vehicle orientations. Figure 5 shows the constrained workspace for the simple two link manipulator shown in Figure 4, it was found using its VM.

## B. Path Planning

In certain cases, the magnitude of the rotations of the vehicle caused by the manipulator's motion may not be acceptable. For example, vehicle rotations may cause communication devices to loose their signals. Vehicle rotations can be controlled using reaction wheels or reaction jets. However, these devices have the disadvantages of increased mechanical complexity and aystem weight or increased consumption of attitude control fuel.

It is shown below that the manipulator itself can be moved in such a way as to have the end effector follow a nominal specified path and yet have a prescribed vehicle orientation, with specified limita, without using attitude control fuel or requiring reaction wheels.

Table 1: Characteristics of Planar Manipulator and ita VM.

| Body <br> no | $M$ <br> $(\mathrm{Kg})$ | $R$ <br> $(\mathrm{~m})$ | $L$ <br> $(\mathrm{~m})$ | $D$ <br> $(\mathrm{~m})$ | $H$ <br> $(\mathrm{~m})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 50 | 1.0 | 1.0 | 0.33 | 0.33 |
| 2 | 50 | 0.75 | 0.75 | 0.5 | 0.5 |
| 3 | 50 | 0.5 | - | 0.5 | - |



Figure 5: Conatrained Workepece or Manipulator Shown in Pigure 4.

To find thie motion the principle of coneervation of agyular momentum is applied to the ayatem. For an $n$ degreee of freedom space manipulator the following equation can be writter.

$$
\begin{equation*}
\dot{\mathbf{X}}=\mathbf{F}(\boldsymbol{\Theta}, \mathbf{X}) \dot{\boldsymbol{\Theta}} \tag{7}
\end{equation*}
$$

$\mathbf{F}: 3$ by in matrix with elements $F_{i, j}$,
$\mathbf{X}: 3$ by 1 vector of vehicle inertial orientations with elemente, $X_{i}$,
$\Theta$ : $n$ by 1 vector of joint anglea with elemente $l_{i}$
In general, Equation (7) is non-integratable, that is:

$$
\begin{equation*}
\frac{\partial^{2} X_{i}}{\partial \theta_{k} \partial \theta_{j}} \neq \frac{\partial^{2} X_{i}}{\partial \theta_{j} \partial \theta_{k}} \tag{8}
\end{equation*}
$$

Therefore, the final vehicle orientation depends on path taken by the manipulator from one poevition to another. It follows that the final vehicle orientation will change if the manipulator moves along one path in joint apace and returne to ite initial position by another path. This is a similar notion to the one which permite astronaute to reorient their bodies by moving their limbe [13]. This leade to a atrategy for adjusting or correcting motions of the vehich's orientation. In this atrategy nominal trajectoriet are selected for the end effector and vehicle oriantation. Then the joint motions are executed asouming the vehicle follows ita trajectory. If at any point the vehicle orientation devietes from its desired path by more than a specific amount, a seriea of amall cyclic motions, selected to correct for the vehicle orientation are added to the joint motions.

To find the cyclical joint motions that achieve the deaired vehicle/base orientation corrections, it is aseumed that these motions are small enough that the end effector deviater only by amall amount from ite nominal trajectory. This small motion assumption permits the use of a nonlinear syatem model in which nonlinearities of order greater than 2 can be neglected.

First, let $X$ be a set of Euler angles defining the baee orientation with reapect to an inertial coordinate frame. The initial and desired final base orientations are $\mathbf{X}_{\mathbf{i}}$ and $\mathbf{X}_{\mathbf{d}}$, reapectively. The desired change in the Euler angles is defined by

$$
\begin{equation*}
\delta \mathbf{X}=\mathbf{X}_{\mathbf{t}}-\mathbf{X}_{\mathbf{d}} \tag{9}
\end{equation*}
$$

Let $\boldsymbol{\Theta}_{0}$ be the vector defining the initial and final joint positions at the beginning and end of the correction maneuver. Also let the vectors $\delta V$ and $\delta W$ define small joint movementa. The closed correction path is constructed by having the manipulator move along the straight lines, in joint apace defined by vectors $\delta \mathrm{V}$ and $\delta \mathbf{W}$ shown in Figure 6.

For small $\delta \mathrm{V}$ and $\delta \mathrm{W}$ the following equation can be obtained from Equation (7).

$$
\begin{equation*}
\delta X_{k}=\sum_{i=1}^{3} \sum_{j=1}^{3}\left[\left.\sum_{m=1}^{3}\left(\frac{\partial F_{k j}}{\partial X_{m}} F_{m i}-\frac{\partial F_{k i}}{\partial X_{m}} F_{m j}\right)+\frac{\partial F_{k j}}{\partial \theta_{i}}-\frac{\partial F_{k i}}{\partial \theta_{j}} \right\rvert\, \delta W_{j} \delta V_{i} \quad(k=1,2,3)\right. \tag{10}
\end{equation*}
$$

where $\delta X_{i}, \delta V_{i}$ and $\delta W_{i}$ are elements of the vectors $\delta X, \delta V$ and $\delta W$, reapectively. In the case of a three DOF apatial manipulator, Equation (10) will yield three equations with six unknowns. Three additional constrain equations are required to solve for $\delta \mathbf{V}$ and $\delta W$.

If vectors $\delta \mathrm{V}$ and $\delta \mathrm{W}$ are parallel, the cyclic motion will not produce any vehicle rotation. Therefore it is asumed that these vectors are perpendicular:

$$
\begin{equation*}
\delta \mathbf{V}^{\mathbf{T}} \cdot \boldsymbol{\delta} \mathbf{W}=\mathbf{0} . \tag{11}
\end{equation*}
$$

Further, the magnitudes of $\delta W$ and $\delta V$ are assumed to be equal:

$$
\begin{equation*}
\delta \mathbf{V}^{T} \cdot \delta V=\delta W^{T} \cdot \delta W, \tag{12}
\end{equation*}
$$

and one of the elements of $\delta \mathrm{V}$ is chosen to be a linear combination of the other two. Por example,

$$
\begin{equation*}
\delta V_{3}=\left(\delta V_{2}+\delta V_{1}\right) / 2 \tag{13}
\end{equation*}
$$

Equations (10) through (13) yield six scalar equations with six sealar unknowns, which can be solved for the denired joint trajectories, $\delta \mathrm{V}$ and $\delta \mathrm{W}$. If the required correction, $\delta \mathrm{X}$, is large, the values of $\delta \mathrm{V}$ and $\delta \mathrm{W}$ may violate the amall joint motion asumption. In this case the devired correction can be achieved by a soriee of meyclical correction maneuvers. It is shown below that at each cycle Equatione (10) through (13) do not have to be remolved and the final position can be achieved.

Referring to Figure 7, $\mathbf{T}\left(\mathbf{X}_{j}\right)$ is a 3 by 3 matrix which tranaforme a vector exprewed in vehicle body coordinatea ( $x, y, z$ ) into inertial or Newtonian coordinates $\left(N_{z}, N_{y}, N_{z}\right)$, when the body is at jth orientation. The tranaformation matrix for the initial vehicle orientation is $\mathbf{T}\left(\mathbf{X}_{i}\right)$. The traneformation matrix for the desired vahicie poaition to be achieved after $m$ cycles is $\mathbf{T}\left(\mathbf{X}_{\mathbf{d}}\right)$. After one correction cycle, the tranaformation matrix is $\mathbf{T}\left(\mathbf{X}_{\mathbf{i}}+\mathbf{\delta x}\right)$, where,

$$
\begin{equation*}
\mathbf{T}\left(\mathbf{X}_{i}+\delta \mathbf{x}\right)=\mathbf{T}\left(\mathbf{X}_{i}\right) \mathbf{A} \tag{14}
\end{equation*}
$$

and the matrix $\boldsymbol{A}$ is the transformation matrix from the vehicle position, one cycle from the initial vehicle position, back to the initial position. The A matrix will not change with each cycle because the total ayatem, vehicle and manipulator, have been aubject only to a rigid body rotation in inertial apace. Hence after $m$ cyclea the transformation matrix from the desired system position to inertial coordinates is simply:

$$
\begin{equation*}
T\left(X_{d}\right)=T\left(X_{i}\right) A^{m} \tag{15}
\end{equation*}
$$

Equation (15) can be solved for A:

$$
\begin{equation*}
\mathbf{A}=\mathbf{P} \mathbf{\Lambda}^{1 / m} \mathbf{P}^{-1} \tag{16}
\end{equation*}
$$

where $\Lambda$ is a diagonal matrix of the eigenvalues of $\mathbf{T}\left(\mathbf{X}_{\mathbf{i}}\right)^{-1} \mathbf{T}\left(\mathbf{X}_{d}\right)$ and $\mathbf{P}$ is a matrix of corresponding eigenvectors.
Using the A matrix obtained from Equation (16), the change in Euler anglea ( $\delta x$ ) are calculated from Equation (14). Then the joint correction motions for each cycle, $\delta V$ and $\delta W$, are obtained by solving Equationa (10) through (13). The manipulator should go through the derived joint transformations ( $6 \mathrm{~V}, \delta \mathrm{~W}$ ) m times to approach the desired vehicle orientation. However, the final vehicle orientation after $m$ cyclea will usually be slightly different than the desired orientation because of the neglected higher order nonlinearities. In order to achieve the desired vehicle orientation more precisely, the over all correction may need to be broken into several smaller corrections and the process repeated with a slightly different set of $\delta V$ and $\delta W$ for each subcorrection.


Figure 6: A Closed Path Correction in Joint Space for a Vehicle Rotation.


Figure 7: Vehicle Coordinate Rotation Due to Cyclic Manipulator Motion.

The above cechnique is now demonetrated for a spatial 3 DOP apace manipulator shown in Pigure 8. The propertiee of the manipulator are given in Table 2. It in decired to rotate the vehiche from ite initial orientation to ite Anal orientetion as ahown in Tmble 3. Lis thin example, it wae necemeary to solve for the joint trajectorive ( 5 V and 8W) 3 timee to precively obtain the deaired vehicle orientation. The joiat trajectories for theoe 3 cycle aete are ahown in Figurce 9 through 11. Pech eycle ie repented 30 timee to achiove the dexired vehicle oriontation. Table 8 ahowe the sydem agdee after cach eycle set. Duriag each eycle the vehiche oecillatee in aympathy to the manipelator's motion, ene Figure 12. However the mean oriontetion of the vehicle changen continuovaly and reaches $X^{4}$ at the anme time the jointe return to their initial positions. Pigure 18 chows the meen vehicle orientation during the joint cycles. The vehicle movemente during the joint motions are $\pm 0.1$ radiase from their mean trajoctory. Here ome can clearly see change in the base orientation an the manipulator jointe cycle through their motion.


Figure 8: Spatial 3 DOF Spece Menipulator.


Figure 10: Joint Angle Trajectories for Second Set of Cycles.

Table 3: Manipulator Angles at Different Instances.

| Angles (deq) | Inisial | devired | Alter one cycle ent | After escond cycie ser | Fine postion |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $x_{1}$ | 50. | 45. | 44.7 | 45.0 | 45.0 |
| $x_{2}$ | 40. | 45. | 43.9 | 45.0 | 45.0 |
| ${ }_{3}$ | 35. | 35. | 37.1 | 34.8 | 35.0 |
| ${ }^{1}$ | 45. | 45. | 45. | 45. | 45.0 |
| ${ }_{2}$ | 45 | 45. | 45. | 45. | 45.0 |
| ${ }_{3}$ | 45. | 45. | 45. | 45. | 45.0 |



Figure 9: Joint Angle Trajectories for First Set of Cycles.


Figure 11: Joint Angle Trajectorien for third Set of Cycles.

Table 2: Three DOF Manipulator Characteristics.

| Bocty no. | $\begin{gathered} \text { Mass } \\ (\mathrm{kg}) \end{gathered}$ | A in locad coord. (m) | 4 in local coord. (m) | inertia about prin. uns ( $\mathrm{Kg} \cdot \mathrm{m}{ }^{2}$ ) |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 20. | $-11+1]+2 x$ | 0.11 - 0.51 | 0.51 . 0.51 , 0.5k |
| 2 | 7. | 0.51 | 0.51 | 0.51 . 0.51 - 0.5k |
| 3 | 7. | 0.51 +0.1. | 0.31 +0.1k | 0.51, 0.11 $\cdot 0.5 \mathrm{k}$ |
| 4 | 5. | - | - | 0.5 [ 0.11 - 0.5k |



Figure 12: The $\boldsymbol{X}_{\mathbf{2}}$ Vehicle Coordinate for the First Set of Cycles.


Figure 13: Mean Vehicle Euler Anglea During Correction Cycle.

## 6. Summary and Concluaion

In this paper, the concepts of Virtual Manipulators and Virtual Grounds are discussed. The end effector YM characteristics and proof of its properties for serial link with revolute and prismatic joints were presented, and mone of ite applications were discussed. This is a new concept and further research is required to demonstrate ite full capabilities.

## 7. Acknowledgement

The support of this research by the Automation Branch of NASA Langley Research Center under Grant NAG-1489 is acknowledged.

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## Appendix A: Proof of Virtual Manipulator Properties

First it will be proven that for - VM constructed using the rules presented in section 4 of this paper the VM end point will coincide with the end effector. Then it will be proven that when the manipulator goes through o movement the VM joint motions described in section 4 will keep the VM end point on the end effector.

Firat, recognizing that the system center of mass is atationary in the inertial frame $N, V$, remaine atationary in thia frame and referring to Figure 2 yields:

$$
\begin{equation*}
M_{10 t} \mathbf{V}_{1}=M_{N} \mathbf{S}+M_{N-1}\left|\mathbf{S}-\mathbf{L}_{N-1}-\mathbf{T}_{N-1}-\mathbf{R}_{N-1}\right|+\ldots+M_{1}\left|\mathbf{S}-\sum_{i=1}^{N-1}\left(\mathbf{L}_{i}+\mathbf{T}_{i}+\mathbf{R}_{i}\right)\right| \tag{Al}
\end{equation*}
$$

Recall that if the $i^{\text {th }}$ joint is revolute $\mathbf{T}_{\boldsymbol{i}}=0$, otherwise, ite magnitude is equal to the priamatic joint translations from the initial manipulator configuration and ite direction is along the translational axis. Equation (A1) can be solved for $\mathbf{S ( t )}$ as follows,

$$
\begin{equation*}
\mathbf{S}(t)=\mathbf{V}_{t}+\frac{M_{1}}{M_{t o t}}\left(\mathbf{R}_{1}+\mathbf{T}_{1}+\mathbf{L}_{1}\right)+\ldots+\frac{M_{1}+M_{2}+\ldots+M_{N-1}}{M_{t o t}}\left(\mathbf{R}_{N-1}+\mathbf{T}_{N-1}+\mathbf{L}_{N-1}\right) \tag{A2}
\end{equation*}
$$

Equation (A2) can be written in terma of the vectora $\mathbf{D}_{\mathbf{i}}, \mathbf{H}_{\mathbf{i}}$ and $\mathbf{P}_{\mathbf{i}}$ by using Equations (4) through (6), to yield:

$$
\begin{equation*}
\mathbf{S}(t)=\mathbf{V}_{1}+\left(\mathbf{D}_{1}+\mathbf{H}_{1}+\mathbf{P}_{1}\right)+\ldots+\left(\mathbf{D}_{N-1}+\mathbf{H}_{N-1}+\mathbf{P}_{N-1}\right) \tag{A3}
\end{equation*}
$$

Using Equation set (3) and the fact that the end effector position is always equal to $\mathbf{S}(t)+\mathbf{R}_{N}$ gives:

$$
\begin{equation*}
\mathbf{E}(t)=\mathbf{V}_{1}+\mathbf{V}_{1}+\mathbf{P}_{1}+\ldots+\mathbf{P}_{N-1}+\mathbf{V}_{N} \tag{A4}
\end{equation*}
$$

It should be noted that this equation does not depend upon the existance of the Virtual Manipulator. The vector chain represented by Equation (A4) describes the end effector position relative to the N reference frame for all time.

For the initial manipulator position the VM conatucted according to the procedure outlined in section 4 has an end point described by the following vector chain,

$$
\begin{equation*}
\mathbf{v}_{g}+\mathbf{v}_{1}+\ldots+\mathbf{v}_{N} \tag{A5}
\end{equation*}
$$

Comparing Equations (A4) and (A5) it follows that in the initial position, when the $\mathbf{P}_{\mathbf{i}}$ 's are equal to zero, the end effector coincides with the VM end point.

Now it will be proven that as the real manipulator moves the VM joint motions described in section 4 will keep the VM end point on the real end effector. Say the manipulator goes through some joint movement, from section 4, the following vector chain describes the VM end point, where the $\mathbf{P}_{i}$ 's are no longer zero,

$$
\begin{equation*}
V_{i}+V_{i}+P_{i}+\ldots+P_{N-1}^{i}+V_{N}^{i} \tag{A6}
\end{equation*}
$$

In the following paragraphs it will be proven that the vectors $V_{i}, V_{i}$ and $P_{i}$ in Equation (A6) are the same as $V_{i}$, $V_{\text {, and }} P$, in Equation (A4), respectively, therefore, the VM end point will coincide with the real end effector. The vector $V_{\text {, }}$ is always constant, therefore,

$$
\begin{equation*}
V_{i}=V_{i} \tag{A7}
\end{equation*}
$$

The initial real manipulator links are composed of vectors $\mathbf{I}_{\mathbf{i - 1}}+\mathbf{R}_{\mathbf{i}}$ and since the manipulator links are rigid, the magnitude of the vectors $\mathbf{I}_{\mathbf{i - 1}}$ and $\mathbf{R}_{\mathbf{1}}$ and the anglee between them are always constant. Since the magnituden of $\mathbf{I}_{i-1}$ and $\mathbf{R}_{4}$ are constant, then from Equations (4) and (5) the magnitudes of $\mathbf{H}_{i-1}$ and $\mathbf{D}_{i}$ will also be constanta. It can also be seen that the angles between $\mathbf{H}_{i-1}$ and $\mathbf{D}_{\mathbf{i}}$ are constant. Then

$$
\begin{equation*}
\left|\mathbf{V}_{i}\right|=\left|\mathbf{H}_{i-1}+\mathbf{D}_{i}\right| \quad \forall t_{i, i} \tag{A8}
\end{equation*}
$$

The Virtual Manipulator links are composed of the $\mathbf{V}_{i}$ vectors. These linke don't change their shapen and lengths 0 a function of time and since magnitudes of $V_{i}$ are initially equal to magnitudes of $V_{i}$, and magnitudea of $V_{i}$ do not change with time it follow that:

$$
\begin{equation*}
\left|V_{i}^{*}\right|=\left|\mathbf{H}_{i-1}+D_{i}\right|=\left|V_{i}\right| \quad V_{t, i} \tag{A9}
\end{equation*}
$$

The magnitude of the vectors $\mathbf{P}_{i}$ in Equation (A4) and the $P_{i}$ vectora in Equation (A6) are both obtained from the real manipulator prismatic joint translations, using Equation (6), therefore by definition,

$$
\begin{equation*}
\left|\mathbf{P}_{i}\right|=\left|\mathbf{P}_{\mathbf{i}}\right| \quad \forall \ell, i \tag{A10}
\end{equation*}
$$

Now it will be proven that the direction of the vectors $\mathbf{V}_{i}$ and $\mathbf{P}_{i}$ in Equation (A6) are parallel to vectors $\mathbf{V}_{i}$ and $\mathbf{P}_{\mathbf{i}}$ in Equation (A4), respectively.

First it can be established that the rotations of the first VM link are set equal to the vehicle rotations and hence the first VM link will always be parallel to the vehicle. Therefore,

$$
\mathbf{V}_{\mathbf{i}}=\mathbf{V}_{1} \quad \forall t
$$

Since axis of rotation or translation of the first real and virtual joints are fixed relative to their corresponding firat links, and the rotations of the first VM link is the same as the vehicle, and these axes are initially constructed to be parallel, then they will always be parallel.

Now consider the case when the first real manipulator joint is revolute. The elements of the second VM link $\mathrm{Hi}_{\mathrm{i}}$ and $D_{i}$ will be parallel to $\mathbf{L}_{1}$ and $\mathbf{R}_{\mathbf{2}}$ and in turn the vectors $\mathrm{V}_{\mathbf{2}}$ and $\mathbf{V}_{\mathbf{2}}$ will be parallel and

$$
\begin{equation*}
V_{i}=V_{2} \quad \forall t \tag{A12}
\end{equation*}
$$

because the rotational axis of the first VM and real manipulator joints are parallel, as shown above, and the magnitude of their rotations are equal by construction.

In the cases where the first joint is prismatic, the elements of the second VM link $\mathrm{H}_{\mathrm{i}}$ and $\mathrm{D}_{\mathbf{i}}$ will be parallel to $\mathbf{L}_{1}$ and $\mathbf{R}_{\mathbf{2}}$ and in turn the vectors $\mathrm{V}_{\mathbf{2}}$ and $\mathbf{V}_{\mathbf{2}}$ will be parallel and

$$
\begin{equation*}
\mathbf{V}_{\mathbf{i}}=\mathbf{V}_{2} \tag{A13}
\end{equation*}
$$

because the VM and real manipulator translational axis for this joint are parallel. In the same manner it is possible to show that

$$
\begin{equation*}
\mathbf{V}_{\mathbf{i}}^{*}=\mathbf{V}_{\mathbf{i}} \quad \forall t, i \tag{A14}
\end{equation*}
$$

Also, in a similar manner all the tranalational axis of the real manipulator and the VM are alwaya parallel, and from Equation (A11),

$$
\begin{equation*}
\mathbf{P}_{i}=\mathbf{P}_{i} \quad \forall t, i \tag{A15}
\end{equation*}
$$

Substituting Equations (A15), (A14) and (A7) into (A6), and comparing the result with Equation (A4) shows that the end effector will always coincide with the VM end point, and this completes the proof.

# Model Reduction for Discrete Bilinear Systems 

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## 1. Abstract

A model reduction method for discrete bilinear systems is developed which matches $q$ sets of Voltarra and covariance parameters. These parameters are shown to represent both deterministic and stochastic attributes of the discrete bilinear system. A reduced order model which watches these $q$ sets of parameters in defined to be a-Volterra covariance equivalent realization (qVolcerra COVER). An algorithm is presented which constructs a clams of q-Volterra COVERs parameterized by solutions to a Hermitian, quadratic, matrix equation. The algorithm is applied to a bilinear model of a robot manipulator.


## 2. Introduction

While model reduction of linear systems has been extensively researched over the past few years, little work has been done in the area of model reduction for nonlinear systems. One class of nonlinear aytens which is especially appealing are bilinear system ([l]-[4]). Bilinear systems are linear in the state variables, linear in the control variables, but nonlinear in the state and control. One reason that this class is of interest is that nonlinear systems which are linear in the control variables can be accurately approximated by bilinear models ([5],[6]). Bilinear approximations will in general have higher order than the original nonlinear system and effective means for reducing bilinear models are needed.

Most approaches to model reduction of linear system a have strive to preserve or approximate a certain characterizing property of the full order model. For deterministic system e this property is typically the impulse response sequence or the system Handel matrix (egg., [7]-[10]). Model reduction of linear stochastic systems usually involves the output covariance sequence or the corresponding Handel matrix (ag., [11] and [12]). A model reduction technique which considers both deterministic and stochastic properties has also been developed ([13]-[15]) and the resulting reduced order models have been called q Markov COVER: (covariance equivalent realizations). The model reduction problem for discrete bilinear system has recently received some attention. Han et al. [16] develop a method for deterministic, discrete, bilinear aystevs using a generalized Handel matrix. Desalt has proposed an approach to stochastic model reduction based on his realization theory ([17]).

In this paper we develop a model reduction algorithm analogous to the 9 Markov covariance equivalent realization approach for linear systems. The algorithm produces a class of reduced order models which exactly match a specified number of deterministic and stochastic parameters. This class of reduced order models is parameterized by the solutions to a Hermitian, quadratic, matrix equation, Section 3 presents the deterministic and stochastic attributes of bilinear system which we will preserve in our method and defines a g-Volcerra covariance equivalent realization. Next, in section 4 the model reduction algorithm is outlined. In section $S$ d parametrization of reduced order models which watch $q$ Volterra parameters and q covariance parameters is formulated. Section 6 contains an application of the proposed algorithm to two degree of freedom robot manipulator. The final sections are our concluding remarks, acknowledgements and references.

## 3. q-Volterra Covariance Equivalent Realizations

Consider the time invariant discrete bilinear system

$$
\begin{gather*}
x(k+1)=A x(k)+\sum_{i=1}^{n_{u}}\left(N_{1} x(k)+b_{i}\right) u_{i}(k) \\
y(k)=C x(k)
\end{gather*}
$$

where $A$ and $N_{1}, i-1, \ldots, n_{u}$ are $n_{x} \times n_{x}$ matrices, $b_{i}, i=1, \ldots, n_{u}$ are $n_{x} \times 1$ matrices and $C$ is an $n_{y} n_{x}$ matrix. The state vector $x($.$) is n_{x} \times 1$, the inputs $u_{i}(),. i=1, \ldots, n_{u}$ are scalar, zero mean, independent Guassian white noise processes with $E u_{i}(j) u_{i}(k)=\delta_{j k}$ and for $j \geqslant k, E x(k) u_{i}(j)=0$. The output $y\left(f_{0}\right)$ is an $n \times 1$, zero mean, stationary stochastic process. We assume that the bilinear system driven by unit intensity Guassian white noise is stable in the sense that the state covariance

$$
\begin{equation*}
x^{\wedge} \lim _{k \rightarrow \infty} E x(k) x^{*}(k)>0 \tag{2}
\end{equation*}
$$

1s finite. It can be show that for these input processes the state covariance will satisfy the bilinear liapusov equa:ion

We also assum that there are no redundant inputs or outputa (s has limearly indepandent colvans and $C$ 1inearly independenc rows).

Ruberti et al. (3) define the product for an axl vector and an $r \times 1$ vector $b$ and the product [] for an ment mateix $L$ and an nes matrix $M$ by

$$
a * b \&\left|\begin{array}{c}
a b_{1} \\
\vdots \\
a b_{r}
\end{array}\right|, L \square M=\left[L_{1} \ldots L_{r}\right] D M \Delta\left[L_{1} M \ldots L_{r} M\right] \text {. }
$$

They also establish the following identity,

$$
\begin{equation*}
L[(M a * b)]=(L \square M)(a * b) \tag{4}
\end{equation*}
$$

With these definitions (1) becomes

$$
\begin{gather*}
x(k+1)=A x(k)+N[x(k) * u(k)]+B u(k) \\
y(k)=C x(k) \tag{5}
\end{gather*}
$$

and (3) aay be expressed as

$$
\begin{equation*}
X=A X A^{\star}+(N \square X) N^{*}+B B^{\star}, N \triangleq\left[N_{1} \ldots N_{n_{u}}\right] \tag{6}
\end{equation*}
$$

The zero initial state response of the bilinear myten (5) is an infinite Volterra series [4]. This series in regular fora is found to be
where identity (4) has been used repeatedly. The atrix valued function in each of the sumations is called a Volterra kernel, the $j^{\text {th }}$ Volterra kernel in regular form is then

$$
\begin{equation*}
h_{j}\left(1_{j}, 1_{j-1}, \ldots .1_{1}\right)^{\Delta} \mathrm{CA}^{1_{j}^{-1}} N D A^{1_{j-1}} \mathrm{I}^{-1} \mathrm{~N}_{\mathrm{N}} \ldots \mathrm{NDA} \mathrm{~A}_{1^{-1}}^{B} \tag{7}
\end{equation*}
$$

where $1_{m}>1, m 1, \ldots, j$ and the matrix $N$ occurs exactly $j-1$ tiaes. The step response, $u(k)=i_{n}$ for all $k>0$, is

$$
y(k)=\sum_{j=1}^{k} \sum_{j=1}^{k} \sum_{j-1}^{k-1} \sum_{j} \ldots \sum_{1}^{k-1_{2} \ldots-i_{j}} \sum_{j}\left(1_{j}, 1_{j-1}, \ldots, i_{1}\right) i_{j} .
$$

where $l_{\text {d }}$ is column vector of ones with melesents. We shall call the coefficients in the step response the Volterra parameters of the bilinear systen (5). The Volterra parameters of the $j^{t h}$ volterra serre: are $n_{y} \times n^{j}$ matrices. We define the set of $q^{\text {th }}$ order Volterra parameters as those coefficients in which the astrices $A$ and $N$ occur a cotal of $q$ tines. For example,

$$
v_{2} \Delta\left\{C_{A}^{2} \mathrm{~B}, \mathrm{CANDB}, \mathrm{CNDAB}, \mathrm{CNDNDB}\right\}
$$

We now see that the atep response is completely characterized by the sets of Volterra parameters. We also observe that for each $k$ new set of Volterra parameters effects this response. That is, if a reduced order nodel athes the first $q$ sets of Volterra parameters of the full order model then it will also match the step response for $k=0,1, \ldots, q+1$.

In addition to Volterra parameters we are concerned with a covariance sequence for the bilinear systen. Desai [17] and Frazho [18] utilize a covariance sequence which includes both second moments of the $x$ tput and higher moments between the output and input processes in their bilinear stochastic realization theories. We also
use this type of sequence, in particular the sequence of concern is

$$
\begin{aligned}
& R_{0}(0) \Delta E y(k) y^{*}(k)=\operatorname{CXC}{ }^{\star} \\
& R_{1}(1) \Delta \varepsilon y(k+1) y^{*}(k)=\operatorname{CAXC} \\
& R_{1}(0,0) \Delta E y(k+1)[y(k) \bullet u(k)]^{\star}=C N \square X C^{\star} \\
& R_{2}(2) \Delta E_{y}(k+2) y^{*}(k)-C A^{2} x c^{( } \\
& R_{2}(0,1) \triangleq E y(k+2)|y(k) \bullet u(k+1)|^{*}=C N D A X C * \\
& R_{2}(1,0) \Delta E y(k+2)\{y(k) \bullet u(k)]^{\star}=\operatorname{CANDXC}{ }^{\star} \\
& R_{2}(0,0,0) \triangleq E y(k+2)[y(k) \bullet u(k) \bullet u(k+1)]^{\star}=C N \square N \square X C * \\
& \stackrel{-}{\bullet}
\end{aligned}
$$

repreaent the pours of $A$ from

$$
\begin{aligned}
& R_{j-1+1_{j}+1_{j-1}+\ldots+1_{1}}\left(1_{j}, 1_{j-1}, \ldots, 1_{1}\right)
\end{aligned}
$$

$$
\begin{align*}
& =C A^{1}{ }_{N D A^{1}}{ }^{1-1} N \ldots N D A^{1}{ }^{1} \times C^{*} \text {. } \tag{8}
\end{align*}
$$

As with the Volterra parametera we shall define the eet of $q^{t h}$ order covariance parameters, $A_{q}$, as those covariances in which the matricea $A$ and $N$ occur a cotal of $q$ times, that is the set of second order covariance parametera is

These sets of covariance parameters coapletely characterize the stochastic bilinear aysten. it is writh noting that if a reduced order model matches the first $q$ sets of covarisnce paraneters of the full order model it will also match exactly the mean square value of the output and all output and input correlations up to $q$ ateps in tine.

Consider now a reduced-order bilinear model

$$
\begin{gather*}
x_{R}(k+1)=A_{R} x_{R}(k)+N_{R}\left[x_{R}(k)=u(k)\right]+g_{R} u(k) \\
y_{R}(k)=C_{R} x_{R}(k) \tag{9}
\end{gather*}
$$

where $x_{R}($.$) is an n_{r} \times 1$ vector, $n_{r}<n_{x}, y_{R}($.$) is an n_{y} x_{l}$ vector, and $A_{R}, N_{R}, B_{R}, C_{R}$ are matices of appropriate dimensions. In addition, we assume that the state covariance $X_{R}$ of the reduced model driven by zero sean Guasilan white noise is the unique positive definite solution to

$$
\begin{equation*}
X_{R}=A_{R} X_{R} A_{R}^{*}+\left(N_{R} \square X_{R}\right) N_{R}^{*}+B_{R} B_{R}^{*} \tag{10}
\end{equation*}
$$

We now define a particular type of reduced order model for discrete bilinear systems.
Definition: The reduced order model (9), with state covariance $X_{R}$ satisfying (10) is a q-Volterra covariance Equivalent Realization ( $q$-Volterra COVER) of the bllinear system ( 5 ) whenever

$$
v_{R_{1}}=v_{1}, 1=0,1, \ldots, q-1
$$

and

$$
R_{R_{1}}=R_{1}, 1=0,1, \ldots, q-1
$$

where $V_{R_{i}}$ and $R_{R_{i}}$ denote the sets of $i^{t h}$ order Volterra and covariance parametery of the reduced order model.
reapectively.
An algorithm which constructs the q-Volterra COVERs of a full order model is our maln objective. One such algoritho is presented next.

## 4. A Model Reduction Algoritha

Suppose that afll order model (5) and atate covarlance satisfying (6) are piven. The gth observabllit: matrix ([3],[4],[16]) of chis model is

$$
o_{q} \Delta\left|\begin{array}{c}
Q_{0} \\
Q_{1} \\
\cdot \\
\vdots \\
Q_{q-1}
\end{array}\right|, Q_{0} \Delta c, Q_{1} \Delta\left|\begin{array}{c}
Q_{1-1} A \\
Q_{1-1} N_{1} \\
\vdots \\
Q_{1-1} N_{n}
\end{array}\right|, i=1, \ldots, q-1
$$

The aatrix partitions $Q_{i}$ have diaension $\left(n_{u}+1\right)^{1-1} n_{y} \times n_{x}, 1-0,1, \ldots, q-1$. We obeerve that the watices $Q_{q} B$ an $Q_{q} x c^{*}$. contain the same information as the set: $V_{q}$ and $R_{q}$, respectively. Uaing the full order model we construc ti.e following aterices

$$
\begin{align*}
& D_{q}=0_{q} \times 0_{q}^{*} \\
& \vec{D}_{q} \Delta_{O_{q}}\left(A X A^{\star}+(N D X) N^{*}\right) O_{q}^{\star}=O_{q}\left(A\left(N D X^{1 / 2}\right)\right]\left(A\left(N D X^{1 / 2}\right)\right]^{*} O_{q}^{*} . \tag{1}
\end{align*}
$$

As a consequence of the quadratic form and using the Liapunov equation (6) it immedtately follows that the rang apaces of these matrices are

$$
\begin{equation*}
R\left(D_{q}\right)=R\left(1 A X^{1 / 2}\left(N \square X^{1 / 2}\right) B l\right), R\left(\bar{D}_{q}\right)=R\left(\left[A X^{1 / 2}\left(N \square X^{1 / 2}\right)\right]\right) \tag{114}
\end{equation*}
$$

and it is obvious that $R\left(\bar{D}_{q}\right)$ is contained in $R\left(D_{q}\right)$.
We now compute a full rank factorization of $D_{q}$

$$
0_{q} \cdot \dot{P A P}
$$

where $\operatorname{rank}\left(D_{q}\right)=r<-n_{x}$. By virtue of the full rank factorization the columas of $P$ form a basis for the rans space of $D_{q}$. Introducing $P^{+}$, the Moore-Penrose inverse of $P$, then it is well known that PP ${ }^{+}$is an orthogom projector onto the range of $D_{q}([19 \|)$. We now partition $P$ into blocks whose row dimensions are compatible wil the partitions of $O_{q}$ (11)

$$
P=\left|\begin{array}{c}
P_{0} \\
P_{1} \\
\cdot \\
\cdot \\
\cdot \\
P_{q-1}
\end{array}\right|, P_{1}\left|\begin{array}{c}
P_{1}^{A} \\
N_{1} \\
P_{1} \\
\cdot \\
\cdot \\
N_{n} \\
P_{1}
\end{array}\right|, 1=1, \ldots, q-1
$$

and detine new matrices

The matrix $G$ is $\left(n_{u}+1\right)^{q-1} n_{y^{\prime}}\left(n_{u}+1\right)$ r and tit must be determined such that

$$
\begin{equation*}
\vec{D}=\Gamma \pi_{P}^{*}, \tau=\operatorname{diag}(\Lambda)_{n_{u}+1} \tag{1}
\end{equation*}
$$

where $\pi$ is a block diagonal matrix with $n_{u}+1$ blocks. Given these constructions we now state our main result.
Theorem !: Given a discrete bilinear syytem $\{A, N, B, C, X \mid$ and matrix $G$ in (17) such that (i8) is satisfied il the reduced order model $\left\{A_{R}, N_{R}, B_{R}, C_{R}, X_{K} \mid\right.$ of order $n_{r}$ deflined by

$$
\left|A_{R} N_{R}\right| \Delta P^{+} P, \theta_{R} \Delta P^{+} O_{q} B, C_{R} \Delta P_{0}, X_{R} \Delta A_{r}, n_{r} \Delta r
$$

Where $r, P, A$ are from the full rank decomposition of $D_{q}(15), P_{0}$ is from the partition of $P$ ( 16 ), and satisfies (18), is a $q$-Volterra COVER.
Proof: first we will show that $P$ is the $q^{t h}$ observablitty matrix of the reduced order model (19). Usink
decompositions (15), (18) and the range space descriptiona (14) we find that $F$ is in the range apace of $P$ so that (19) leads to

$$
P\left(A_{R} N_{R}\right]=\bar{P}
$$

which iaplies that the partitions of $P$ have the required etructure (1I)

$$
P_{0}=C_{R}, P_{i}=\left|\begin{array}{c}
P_{1-1} A_{R}  \tag{10}\\
P_{1-1} N_{R} \\
\cdot \\
\vdots \\
P_{1-1} \hat{N}_{R_{n}}
\end{array}\right|, 1=1, \ldots, q-1
$$

To show that the reduced order model satisfies the bilinear liapunov equation we firat substituce (19) into whlch leads to

$$
\Lambda=P^{+} P_{1 P^{*}} P^{+}+P^{+} 0_{q} B B^{\star} O_{q}^{*} P^{+}
$$

Using (12), (13), (15), (18), and by pre and post multiply by $P$ and $P^{*}$, respectively, we have

Now using the projection property of $\mathrm{PP}^{+}$we $f$ ind that

$$
o_{q}\left(X=A X A^{*}+(N D X) N^{*}+B B^{*}\right) 0_{q}^{*}
$$

which is known to be satiafied (6). To show that the model (19) natahes Volterra parameters we again use the projection property

$$
0_{q_{R}} B_{R}=P P^{+} O_{q} B=O_{Q}^{B}
$$

and the matching of covariance parameters followa directly from (12),(15)

$$
0_{q_{R}} X_{R} O_{q_{R}}^{*}=P A P^{*}=o_{q} \times 0_{q}^{*}
$$

Our remaining task is to determine the unknom matrix $G$ in (17) in order to satisfy (18). This is the topic of the next section.

## 5. Parameterization of q-Volcerra COVERA

To obtain a characterization of the matrix $G$ we first examine the structure of the matrices $\bar{D}_{q}$ and $\bar{P}$. We observe that $\bar{\delta}_{q}$ can be partitioned as

$$
\bar{D}_{q}=\left|\begin{array}{cc}
\overline{0}_{q-1} & \bar{d}_{q}  \tag{20}\\
{\overrightarrow{\mathbf{d}_{q}}}_{q} & \bar{d}_{q q}
\end{array}\right|
$$

and that the partitioned form of the constraint (18) leads to the three relations

$$
\begin{equation*}
F \pi F^{*}=\bar{n}_{q-1}, P \bar{A} G^{*}=\bar{d}_{q}, G \bar{A} G^{*}=\bar{d}_{q q} \tag{21}
\end{equation*}
$$

The ifist relation is satisfied by virtue of the constuction of of $\bar{p}$ (17). It is easily seen that $\bar{d}_{q}$ is contained in the range space of $F$ so that the second relation is consistent and $G$ may be expressed as ([19])

$$
\begin{equation*}
G^{*}=\pi^{1}\left(F^{+} d_{q}+\left(I-F^{+} F\right) Y\right) \tag{22}
\end{equation*}
$$

where $Y$ is an unknown matix with dimension $\left(n_{u}+1\right) r \times\left(n_{u}+1\right)^{q-1} n_{y}$. Substituting for $G$ in the last relation we find that $Y$ must satigfy the Hermitian, quadratic, matrix equation

$$
\begin{equation*}
Y^{\star} K Y+L^{*} Y+Y_{L}^{*}+Y=0 \tag{23}
\end{equation*}
$$

$$
\begin{equation*}
K \triangleq\left(I-F^{+} F\right) \bar{\Lambda}^{1}\left(I-F^{+} F\right)=K^{*}, L A\left(I-F^{*} F\right) \bar{A}^{1} F^{+}{\underset{d}{q}}^{A}, M \Delta \bar{d}_{q}^{*} F^{+} \pi^{1} F_{F}^{+} d_{q}-\bar{d}_{q Q}=M^{*} \tag{26}
\end{equation*}
$$

By inspection we see that the matrix $K$ is nonnegative definite, and that the columns of the matrix $t$ are contatied in the range space of $K$. Based on these observations we now state $\boldsymbol{i}$ theorem which is motivated by a result of Crone [20].

Theorem 2: Let $K$ be an mon nonnegative definite matrix with rank $t, i$ an man matrix whose columa are contaisex In the range space of $K$ and $M$ an $n \times n$ Heraitian matrix. Then the matrix equation

$$
Y^{\star} K Y+L^{*} Y+Y^{\star} L+M=0
$$

hae a solution if and only if

$$
L^{*} K^{+} L-M \geqslant 0 \text { and } \operatorname{rank}\left(L^{*} K^{+} L-M\right)=a \in \operatorname{rank}(K)
$$

When chese conditions hold $Y$ is a solution if and only if it has the fors

$$
Y=K^{+/ 2}\left(V \Sigma^{1 / 2} U^{\prime \prime}-K^{+/ 2} L\right)+\left(I-K^{+/ 2} K^{1 / 2}\right) Y
$$

where $K^{1 / 2}$ is the unique nonnegative definite equare root of $K$ and $K^{+/ 2}$ is the Moors-Panrose inverse of $K^{1 / 2}$. Tm matrix $V$ is an mes matrix, $\sum 1 s x_{0}$ and $U$ is $n \times s$ and chey aust catiafy

$$
V^{\star} V=I, R(V) \text { is concained } \ln R(K), U^{\oplus} U=I, \Sigma>0, U E U^{\star}=L^{\star} K^{+} L-M \text {. }
$$

Yis an arbitrary men matrix.
Proof: It is well known that if $K$. Wi** is afll rank singular value decoapoaltion (SVD), then

$$
\mathrm{K}^{1 / 2}=\mathrm{WR}^{1 / 2} \mathrm{~W}^{\star}, \mathrm{K}^{+/ 2}=\mathrm{Wr}^{+/ 2} \mathrm{~W}^{*}
$$

and it follows that $K, K^{1 / 2}, K^{+/ 2}$ all have the ame range opace which is apanned by the coluans of $W$, an ax column unitary matrix. By the hypothesis that the coluans of $L$ are in the range apace of $K$, equation (25) $i$ : satisfied if and only if

$$
\left(K^{1 / 2} Y+K^{+/ 2} L\right)^{*}\left(K^{1 / 2} Y+K^{+/ 2} L\right)=L^{*} K^{+} L-M
$$

which is consistent if and only if $L^{*} K^{+} L-M \geqslant 0$. All watrix tartors of ehis relacion are

$$
K^{1 / 2} y+x^{+/ 2} L-v i^{1 / 2} U
$$

 find a solution $Y$ we must solve the following linear equation

$$
\begin{equation*}
\mathrm{K}^{1 / 2} \mathrm{Y}=\mathrm{V} \mathrm{I}^{1 / 2} \mathrm{U}^{*}-\mathrm{X}^{+/ 2} \mathrm{~L} \tag{128}
\end{equation*}
$$

This equation is consistent if and only if $V$ is contalned in the range spare of $K$. Since $V$ is column unitary in range spare of $V$ may be any matmensional spare with rank s, solutions of (28) exsist if and only $i$ $\operatorname{rank}\left(L^{*} K^{+} L-M\right)=s \in \operatorname{rank}(K)$. Given that equation (28) is consistent then $Y$ is a solution if and only if $i$ has the following form

$$
Y=K^{+/ 2}\left(V \Sigma^{1 / 2} U^{*}-K^{+/ 2} L\right)+\left(I-K^{+/ 2} K^{1 / 2}\right) \bar{Y}
$$

where $\bar{Y}$ is an arbitrary man matrix.
The results of this cheorem show that the matrix $L^{*} K^{+} L-M$ is the key to solutions of the quadratic antil equation (23). Substituting for $K, L, M$ from equations (24), and using the rules for the Moore-Penrose inverse o a matrix product (l2II), we find that

$$
\begin{equation*}
L^{*} K^{+} L-M=\bar{d}_{q Q}-d_{q} F^{+} \pi^{1 / 2}\left(I-\left(\pi^{1 / 2}\left(I-F^{+} F\right)\right)\left(\pi^{1 / 2}\left(I-F^{+} F\right)\right)^{+}\right) \pi^{1 / 2} F^{+} d_{q} . \tag{129}
\end{equation*}
$$

Froin this equation we find an inceresting result on the Moore-Penrose inverse of a quadratic form which we stat wtehout proot.

Fart: The Moore-Pentose inverse of che quadrate form $F T F^{*}$ is

$$
\begin{equation*}
\left(F \bar{A} F^{\star}\right)^{+}=F^{+} \pi^{1 / 2}\left(I-\left(A^{1 / 2}\left(I-F_{F}^{*}\right)\right)\left(T^{1 / 2}\left(I-F^{+} F\right)\right)^{+}\right) \Lambda^{1 / 2} F^{+} \tag{10}
\end{equation*}
$$

where $T$ is a posicive definite matix and $f$ is thy matrix which is mulciplication compatible.
Using this result and equation (21) in (29) we find that

$$
L L^{*} L-M=\bar{d}_{q q}-\bar{d}_{q}^{*} \bar{J}_{q-1}^{+} \bar{d}_{q}
$$

which is guarantecd to be nonnegative deftnite ([22]). Thus the first part of the constralnt (26) of theoret will always be satisfied.

To show that the serona part of the ronstratint (26) will also be gatigiled de note that from the range sar description (1is).

$$
\begin{equation*}
r=\operatorname{rank}\left(D_{q}\right) \geqslant \operatorname{rank}\left(\bar{D}_{q}\right) \tag{32}
\end{equation*}
$$

and from the dafinition of $K(24)$, the relations (21) and the column dimension of $F$.

$$
\begin{equation*}
t^{A} \operatorname{rank}(K)=\operatorname{rank}\left(1-F^{+} F\right)=\left(n_{u}+1\right) r=\operatorname{rank}\left(\bar{D}_{q-1}\right) \text {. } \tag{33}
\end{equation*}
$$

Rohde [23] has show that the partitioning (20) of the nonnegative definite atitix $\mathrm{D}_{\mathrm{q}}$ implies

$$
\begin{equation*}
\operatorname{rank}\left(\bar{J}_{q}\right) \cdot \operatorname{rank}\left(\bar{D}_{q-1}\right)+\operatorname{rank}\left(\bar{d}_{q q}-{\overrightarrow{d_{q}} \bar{D}_{q-1}}_{\bar{d}_{q}}\right) \tag{34}
\end{equation*}
$$

and by using (31)

$$
\begin{equation*}
\text { . } \Delta \operatorname{rank}\left(L^{*} K_{L}^{+}-M\right)-\operatorname{rank}\left(\bar{D}_{q}\right)-\operatorname{rank}\left(\bar{D}_{q-1}\right) . \tag{35}
\end{equation*}
$$

Collecting equatione (32), (33) and (34) we find chat * $t$ and therefore the eecond part of the constraint (26) in theorem 2 will always be setisfied.

We have shown that solutions of (23),(26) slways exist and by theorea 2 chey will have the fors

$$
\begin{equation*}
Y=K^{+/ 2}\left(V \Sigma^{1 / 2} U^{*}-K^{+/ 2} L\right)+\left(I-K^{+/ 2} K^{1 / 2}\right) \bar{Y} \tag{36}
\end{equation*}
$$

where ULU is the full rank aingular value decomposition of $L^{*} K^{+} L-M, V$ ia any coluan unitary antix whose range space is contained in the range space of $K$ and $F$ is arbitrary. We observe that the acond tera of (36) is in the nuil space of $K$ which is also the range apace of $\mathrm{F}^{*}$. It follows that when ( $\mathbf{3 6}$ ) to substituted into che expression for $G^{*}(22)$ that chis cerm will be annihlated by ( $1-F^{+} p$ ) whirh represents a projection onco the null space of $F$ along the range space of $F^{\boldsymbol{m}}$. The first tern of (36) is in the range apace of $K$, or the null apaca of $F$, so that under the projection ( $I-F^{+} F$ ) it rematns unchanged. Therefore $G^{*}$ becomes

$$
G^{*}=\pi^{-1}\left(F^{+} d_{q}+K^{+/ 2}\left(V \Sigma^{1 / 2} U^{*}-K^{+/ 2} L\right)\right)
$$

or by using equation (24) and conjugate transposing

Equation (37) is an explicit expression for $G$ which was the ublective of this gection. All of the freedon in $G$ is contained in the colum unitary matrix $V$ whose range space is constralned to be in the null spare of f.

## 6. Application to a Robot Mantpulator

Consider the two degree of freedum manipulator illustrsted in figure 1 . The am has its center of mass at point $C$, and it may be translated thruugh or rotated about the fixed point oby the force fand torque $T$, respectively. The manipulator carries a load at the point $L$.


Figure 1. Two Degree of Freedon Mantpulator
Treating the load as a polnt mass and allowing for joint stiffness and damping, the equations of ation are

$$
\begin{aligned}
& (m+M) \ddot{r}+b_{r} \dot{r}+k_{r} r-(a+M) r \dot{g}^{2}-M \dot{b}^{2}-r \\
& \left(J+I \bullet M_{A}^{2}+2 M_{A r}+(a+M) r^{2}\right) \ddot{\theta}+b_{\theta} \dot{g}+k_{\theta} \theta+2(n+M) r r^{\dot{\theta}}+2 \text { Mar }^{r} \dot{\theta}=\tau
\end{aligned}
$$

where a is the distance from $C$ to $L, M$ is the mas of the loed, $J$ is the moment of inertia of the joint, ats the mass of the arm and $I$ is ite mome of inertia about $C$. Joinc atifiness and dempins are represented by $k_{p}$, $k_{0}$ as $b_{r}, b_{g}$, raspectively. Introducing the acate vector and the control

$$
2-\left\{\left.r \dot{r} \otimes \dot{\theta}\right|^{T}, u \in\{r T\}^{T}\right.
$$

then the equations of motion have the generic form

$$
z=f(z)+g(z) u .
$$

A blifnear model of the anipulator can be conatructed by expanding each of the functions f(.) and g(.) Into a
 firnc three ceme of the Taylor series expansions of $(\mathbf{(})$ ) and g(.) and letting
then we have $34^{\text {th }}$ order bilinear model and after discrecizacion it has the fora ( 1 ).
 - $1=100 \mathrm{~kg}-\mathrm{m}^{2}$, and $k_{r}=6 \mathrm{~N} / \mathrm{m}, k_{\theta}=2.5 \mathrm{~N} / \mathrm{m}, \mathrm{b}_{r}=3 \mathrm{~N}-\mathrm{sec} / \mathrm{m}, \mathrm{b}_{\theta}-5 \mathrm{~N}-\mathrm{ecc} / \mathrm{E}$. Pigure 2 shows the etep reapone of the nonlinear equations of motion and the full order bilinear eodel. The bilinear model provides a falr approxiaction to the true nonlinear byste. A more accurate approxiation could be mata by retaining higber order terms in the power series expansions.


Flgure : Step Responye ot vonlanear and full brder 3alinear mudels
Applying the model reduction algoritha with qu3 (astching three sets of Volterra and covariance parameters) a rlass of 3 -Volterra Coveks was obtained. These reduced models have 14 states which is a greater than fifty percent reduction in model order. Figure 3 shows the response of a reduced model from the rlass of 3 -Volterra COVERs and the response of the full order audel to a unit pulse input with a second duration. Figure 4 thovs the response of thes adels driven by anit incenalty Gaussian white noise process. In Figure 3 we sef that ibe response of the full and reduced order blinear adels are nearly identical for the first io seconds. Similarly, in figure 4 the reduced order model alaics the full order model initially. These observations are in accordame with the theory which states that the response of the reduced order model equals that of the full order systea for $q$ steps in time. However. In both cases the quallty of the response of the reduced order model deteriorates with cime and te eventually goes unstable. This instability is input dependent and possibly in a closed loop seting the model behavior would be acceptable for greater periods in time.


Pigure 3. Deterministic Response of Pull and Reduced Order Bilinear Mortela


Figure 4. Stochastir Reaponse of Full and Reduced Order Bilinear Modela

## 7. Conclusions

A sequence of sets of Volterra paraneters characterizes the deterninistif bilinear ayater, and a sequence of sets of covariance parameters describes the stochastic bilinear system. A model reduction terhnique vas developed for discrete bilinear systems which generates a rlass of reduced order models which exartly match che first $q$ setg of Volterra and covariance parameters of the full order model. These aodels are cherefore called q-Volterra covariance equivalent realizations, or q-Volterra CuVERs. Methods to choose sperific sodels from within the class to satisfy additional modelling considerations is a topic of future research.

## 8. Acknowledremants

Pars
arteof this research was sponsored by the National Science Poundation, grant RipA-8405133, and by the thighes Afrcraft Company through the Hughes Fellowship Progran. We also arknowledge Uday b. Dasal of Waehington State Univeralty whoe asistance was invaluable.

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# High Gain Feedback and Telerobotic Tracking 

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#### Abstract

Asymptotically stable linear time invariant systems are capable of tracking arbitrary reference signals with a bounded error proportional to the magnitude of the reference signal (and its derivatives). It is shown that a similar property holds for a general clans of nonlinear dynamical systems which includes all robots. As in the linear case, the error bound may be made "arbitrarily" small by increasing the magnitude of the feedback gains which stabilize the system.


$\qquad$

## 1 Introduction

Tracking is the archetypal pursuit of the control theorist. Given a dynamical system,

$$
\begin{aligned}
& \dot{x}=f(x, u), \\
& y=h(x)
\end{aligned}
$$

and a specified "reference signal", $r(t)$, it is required to find a control, $u^{*}(t)$ such that the forced system, $\dot{x}=f\left(x, u^{\bullet}\right)$ "tracks" $r$ in some sense - usually lima $\lim _{\rightarrow \infty} y=9$. Solutions to such problems generally involve pre-filtering the reference trajectory through a suitable "feedforward" algorithm, and then adding a compensating error driven "feedback" term to arrive at the input, $u^{\circ}$. If the reference signal is known i priori, then the feedforward algorithm may entail pure differentiation to "precompensate" for the lags introduced by the dynamical system itself. However, on-line differentiation of unknown and unpredictable signals has long been eschewed by control theorists as an unreliable technique for both theoretical an well as practical reasons.

This paper considers the problem of tracking in the context of telerobotic manipulators. It is shown that a general class of highly nonlinear control systems which includes all robot model admits tracking algorithms based upon high gain linear state variable feedback. The choice of a pure feedback based algorithm for tracking is surely not optimal in any sense of the word. However, the only other techniques which are known to guarantee tracking for this class of systerns make use of feedback algorithms which attempt exact cancellation [1,2,3|, (or "nearly" exact cancellation, egg. [4,5]) of intrinsic nonlinear dynamical terms via feedback, and pure differentiation of the reference trajectory in the feedforward path. In robot applications admitting the use of a "high level" planner it is plausible that the entire future strategy might be made available at once to the "low level" controller in which case tracking schemes requiring pure differentiation of the reference signal might be acceptable. In telerobotic applications the reference signal is, by definition, à priori unknown: it is generated as a record of the unpredicted arbitrary motion of a human agent of control. Schemes which require pure differentiation will probably not be useful in this context.

In a sense, the result reported here simply represents another example of the similarity between general mechanical systems and second order linear systems. It is well known that asymptotically stable linear time invariant systems are capable of tracking arbitrary reference signals with a bounded error proportional to the magnitude of the reference signal (and its derivatives). For - fixed bound on this magnitude, the asymptotic tracking error may be made "arbitrarily" small by increasing the magnitude of the eigenvalues in the left half of the complex plane. In practice, this is accomplished by increasing the gain of linear feedback compensators. In this paper it is shown that the analogous property holds true for the more general class of nonlinear mechanical systems.

As in the theory of linear servomechanisms, a practical obstacle to the systematic use of high gain feedback techniques in telerobotic applications is the inevitable presence of actuator torque limitations. Practical tracking strategies which address this problem while maintaining convergence guarantee are very much needed. This important consideration is entirely ignored here. The problem of characterizing the transient response of feedback compensated nonlinear mechanical systems is the topic of a paper currently in progress.

[^15]
## 2 Preliminary Discussion

### 2.1 Notation and Definitions

$\mathbb{I} f: \mathbb{R}^{n} \rightarrow \mathbb{R}^{m}$ has continuous first partial derivatives, denote ite $m \times n$ jacobian matrix an $D f$. When we require only a aubeet of derivativen, e.s. when $x=\left[\begin{array}{l}x_{1} \\ x_{2}\end{array}\right]$, and we denire the jacobian of $f$ with respect to the variables $x_{1} \in R^{n_{1}}$, as $x_{3}$ is held fixed, we may write

$$
D_{a_{1}} f \triangleq D S\left[\begin{array}{c}
I_{n_{1} \times n_{1}} \\
0
\end{array}\right]
$$

If $A: J \rightarrow \mathbf{R}^{n \times n}$ is asmooth map taking matrix values then let

$$
\mu(A) \triangleq \sup _{\bullet \in \mathcal{J}} \sup _{\| \| \|^{-1}}\left|x^{T} A x\right|
$$

and

$$
\nu(A) \triangleq \inf _{\in \in J} \inf _{\|=1}\left|x^{\top} A x\right| .
$$

If $J$ is compact, or the entriet of $A$ are bounded then both $\nu(A), \mu(A)$ are non-negative real numbera. For any constant matrix, $\mu(A)$ is the square root of the eigenvalue of greatest magnitude, while $\nu(A)$ is the square root of the eigenvalue of leant magnitude of $A^{\top} A$, from which it followe that

$$
\mu(A)=\sup _{q \in J}\|A(q)\| \quad 1 / \nu(A)=\sup _{q \in J}\left\|A^{-1}(q)\right\|,
$$

where $\|\cdot\|$ denotes the operator norm induced by the eulcidean norm of $\mathbf{R}^{n}$.
Given a set $P$, a smooth scalar valued map, $v: P \rightarrow R$ in said to be positive definite at a point $p \in P$ if $v(p)=0$, and $v>0$ in some open neighborhood of $p$. Given a smooth (time invariant) vector field, $f$, on some phase, space, $p$, we shall say that, $v$, a positive definite map at $p_{d} \in P$, constitutes a Lyapunov function for $f$ af $p_{d}$ if the time derivative along any motion of the vector field is non-positive,

$$
\dot{v}=D_{p} v f(p) \leq 0,
$$

in some neighborhood of $p_{d}$, and that it constitutes a atrict $L$ yapunov function for $f$ if the inequality is atrict $[6,7 \mid$. The domain of $u$ with respect to $p_{d}$ is the largest neighborhood around $p$ which is free of additional critical points and upon which the derivative is still non-positive.

The existence of a strict Lyapunov function at a point is a sufficient condition for asymptotic stability of that equilibrium state. If a strict Lyapunov function has not been found, aymptotic atability may, nevertheleas, be assured if a further condition on the possible limiting set holds. This is "LaSalle's Invariance Principle" [7]. It is possible, as well, to draw conciusions sbout the tracking capability of a forced dynamical system in consequence of of the stability properties of the unforced vector field at a particular equilibrium state. However, this seems to require the use of a strict Lyapunov function.

It has been known for quite some time that the total energy of a mechanical system may be interpreted as a Lyapunov function [8]. Unfortunately, this choice of Lyapunov function is never strict. The central contribution of this paper reats upon the construction of a strict Lyapunov function for the general class of nonlinear mechanical systems described below, (1). The tracking results follow as a standard consequence.

### 2.2 Dynamical Equations of Kinematic Chains

The equations of motion of a kinematic chain have been extensively discussed in the robotics literature, and this paper will rely upon the standard rigid body model of an open chain with revolute joints. Thus, we consider a robot to be a particular member of the class of mechanical syatems,

$$
\begin{equation*}
M|q| \bar{q}+B|\dot{q}, q| \dot{q}+k(q)=\tau \tag{1}
\end{equation*}
$$

where the generalized positions take values in a configuration space, $q \in J$, and $M$ is a positive definite invertible symmetric matrix for all $q \in J$. As shown in the appendix, in the case of kinematic chains, $M$, the "inertial" terms, $B$, the "coriolis and centrifugal" terms, and $k$, the gravitational disturbance vector, all vary in $q$ by polynomials of transcendental functions. It follows that $\nu(M)>0$ and $\mu(M)<\infty$.

This system may be rewritten in the form

$$
\begin{align*}
& \dot{q}_{1}=q_{2} \\
& \dot{q}_{2}=M^{-1}\left(B q_{2}+k-\tau\right] \tag{2}
\end{align*}
$$

where the generalized positions and velccities take values $p \triangleq\left[\begin{array}{l}q_{1} \\ q_{2}\end{array}\right] \in p \triangleq T J$ in phace space - the tangent bundle over $J$.

While $M, k$ are alway bounded, the coriolis and centripetal forces ase quadratic in the velocity - i.e. $B$ is linear in $\phi$ aad, therofore, may become unbounded. It in, however, bounded with reapect to q , as the following tachniced reault ahow.

Lermma 1 For any $k \in \mathbf{R}^{n}$,

$$
k^{T} B_{q_{2}}=g_{2}^{T} \dot{M}(k) g_{2}
$$

miere

$$
\dot{M}(k) \triangleq\left[\begin{array}{c}
k^{\top} D_{911} M \\
\vdots \\
k^{\top} D_{110} M
\end{array}\right]-\frac{1}{2} \sum_{i=1}^{n} k_{1} D_{f 16} M
$$

is bounded above.
Proof:

$$
\left.B q_{2}=i q_{2} \otimes I\right)^{\mathrm{T}} D_{1} M^{4} q_{2}-\frac{1}{2}\left[\left(q_{2} \otimes I\right)^{T} D_{1} M^{1}\right]^{\mathrm{T}} q_{2}
$$

and, hence,

$$
\begin{aligned}
& g_{i}^{\top} B^{\top} k=q_{2}^{T}\left[D_{8} M^{1}\right]^{\top}\left(q_{2} \otimes I\right) k-\frac{1}{2} q_{2}^{\mathrm{T}}\left(q_{2} \otimes I\right)^{\mathrm{T}} D_{3} M^{\boldsymbol{q}} k \\
& =q_{2}^{T}\left[D_{4} M^{4}\right]^{7}\left(k q_{2}^{T}\right)^{3}-\frac{1}{2} q_{2}^{T}\left(: \quad 3 n^{7} D_{8} M^{4} k\right.
\end{aligned}
$$

Since $M$ contains transcendental functions in $q$, all of its derivatives in $q$ must be bounded.

It follows that for some $\hat{\mu}<\infty$,

$$
\begin{equation*}
\|\dot{M}(k)\| \leq \hat{\mu}\|\boldsymbol{k}\| . \tag{3}
\end{equation*}
$$

Corollary 2 For all $p \in P$,

$$
q_{2}^{\tau}\left|\frac{1}{2} \dot{M}-B\right| q_{2} \equiv 0
$$

Proof:

$$
\begin{aligned}
\left.e_{2}^{\mathrm{T}}\right|_{2} ^{1} \dot{M}-B \mid e_{2} & =e_{2}^{\mathrm{T}}\left(\frac{1}{2} \dot{M}-\dot{M}\left(e_{3}\right)\right) e_{3} \\
& =\frac{1}{2} q_{2}^{\mathrm{T}}\left[\left[D_{\mathrm{r}} M^{\mathrm{s}}\right]^{\mathrm{T}}\left(q_{2} \otimes I\right)-\left(\boldsymbol{q}_{2} \otimes I\right)^{\mathrm{T}} D_{\mathrm{r}} M^{\mathrm{s}}\right] \boldsymbol{q}_{2} \\
& \equiv 0 .
\end{aligned}
$$

### 2.3 Stability Properties of "PD" Compensated Systems

Suppose we are presented with the mechanical system (2), and a desired point,

$$
p_{d} \triangleq\left[\begin{array}{c}
q_{\alpha} \\
0
\end{array}\right] \in P .
$$

Choose two positive definite matrices, $K_{1}, K_{1}>0$, and define the "PD" algorithm

$$
\begin{equation*}
r=k(q)-K_{1}\left|q_{4}-q\right|-K_{2} \dot{q} . \tag{4}
\end{equation*}
$$

In terms of the translated "error coordinate system" for $P$,

$$
e(p) \triangleq\left[\begin{array}{c}
q-q_{d}  \tag{5}\\
\dot{q}
\end{array}\right]
$$

the resulting closed loop system has the form

$$
\begin{align*}
\dot{e} & =\left[\begin{array}{cc}
0 & I \\
-M^{-1} K_{1} & -M^{-1}\left(B+K_{2}\right)
\end{array}\right] e  \tag{6}\\
& \triangleq A[\dot{q}, q] .
\end{align*}
$$

Proposition 3 For all $\boldsymbol{r}_{0}>0$,

$$
\hat{v}(e)=\frac{1}{2} e^{\tau} \bar{P}(q) e \triangleq \frac{1}{2} e^{\tau}\left[\begin{array}{cc}
\gamma_{0} K_{1} & 0 \\
0 & \gamma_{0} M(q)
\end{array}\right] e
$$

is a Lyapunou function for the closed loop ayatem (6).
Proof: It is clear that $\tilde{U}$ in positive definite at the origin of the error aystem. Taking the time derivatives along the colutions of the closed loop syatem, (6),

$$
\begin{aligned}
\dot{\hat{U}} & =\frac{1}{2} e^{T}\left|\dot{P} A+A^{T} \dot{P}+\dot{P}\right| e \\
& \left.=-e^{T}\left[\begin{array}{cc}
0 & 0 \\
0 & \gamma_{0} K_{2}
\end{array}\right] e+\left.\gamma_{0} e_{2}^{T}\right|_{2} ^{1} \dot{M}-B \right\rvert\, e_{2} .
\end{aligned}
$$

Noting that $e_{2} \equiv q_{2}$, it followa from Corolary 2 , that the second term in identically zero.

There follows the desirable result that proportional and derivative linear state feedback atabilizea a mechanical syatem, after the gravitational disturbance torques have been removed.
Theorem 1 ( $[9,10,11]$ ) The origin of the closed loop error coordinate syatem (0) is asymptotically stable.
Proof: The existence of a Lyapunov Function, $\dot{v}$, assures stability. According to LaSalle's invariance principle, the attracting set is the largest invariant set contained in $\left\{\left(e_{1}, e_{3}\right) \in P: \dot{v} \equiv 0\right\}$, which, evidently, is the origin, since the vector field is oriented away from $\left\{e_{2} \equiv 0\right\}$ everywhere else on that hyperplane.

Notice that the proof of attractivity requires an appeal to LaSalle's invariance principle in consequence of the fact that $\dot{u}$ is not a strict Lyapunov function. In order to obtain the desired extension to tracking problems it is necearary to construct one. Unfortunately, the constructions devised to date require the artificial limitation to decoupled PD feedback. Namely, in the sequel, it will be assumed that the gain matrices of (6) are specified as

$$
\begin{equation*}
K_{1} \triangleq \omega^{2} I ; \quad K_{2} \triangleq 2 \varsigma \omega I \tag{7}
\end{equation*}
$$

given two positive real numbers, $\omega, \varsigma$.

### 2.4 A Strict Lyapunov Function for Nonlinear Mechanical Systems

The following technical lemma will be of use in the main result, below.
Lemma 4 For $M(q)$ as in (1) and any positive scalars, $\alpha, \beta, \gamma \in \mathbf{R}^{+}$,

$$
\inf _{\mathbb{k} \cdot \|=1} e^{\top}\left[\begin{array}{cc}
\alpha I & \beta I \\
\beta I & \gamma M(q)
\end{array}\right] e \geq \nu(K)
$$

where

$$
K \triangleq\left[\begin{array}{cc}
\alpha & \beta \\
\beta & \gamma \nu(M)
\end{array}\right]
$$

In particular, the matrix is positive definite when

$$
\begin{equation*}
\alpha \gamma \nu(M)>\beta^{2} \tag{8}
\end{equation*}
$$

Proof: Since

$$
\left[\begin{array}{cc}
\alpha I & \beta I \\
\beta I & \gamma M(q)
\end{array}\right]>\left[\begin{array}{cc}
\alpha I & \beta I \\
\beta I & \gamma \nu(M)
\end{array}\right]=K \otimes I
$$

it will suffice to show that

$$
\nu(K \otimes I)=\nu(K)
$$

This follows since all eigenvalues of $K \otimes I$ are eigenvalues of $K$, according to Lemma 12 in the appendix. The particular conclusion obtains by taking the determinant of $K$.

Proposition 5 For all $p_{d} \in P$ and $\omega, \varsigma>0$, given any bounded set, $B \subset P$, containing $p_{d}$ there exists a scalar $\gamma_{0}>0$ such that

$$
v(e) \triangleq \frac{1}{2} e^{T} P(q) e=\frac{1}{2} e^{T}\left[\begin{array}{cc}
\omega^{2} \gamma_{0} I & \omega \varsigma I \\
\omega \varsigma I & \gamma_{0} M(q)
\end{array}\right] e
$$

is a strict Lyapunov Function for the closed loop system, (6) on the domain B, assuming the decoupled feedback gain matrices specified in (7).

Proof: Latting

$$
\beta \triangleq \sup _{\in \in \mathbb{S}}\|\in\|,
$$

find some $y_{0}$ satisfying

$$
\begin{equation*}
\gamma_{0}>\max \left\{\frac{s}{\sqrt{\nu(M)}}, \frac{1}{\nu(M)}, \frac{\mu(M) \beta}{\nu(M)}+1\right\} . \tag{9}
\end{equation*}
$$

According to Lemme 4 and the inequality involving the first entry of the inferior set in (9), it follows that $P$ in a positive definite matrix for all $q \in J$, hence $v$, in positive defnite at $p<$.
Taking time derivatives along the solutions of aystem (6), we have

$$
\dot{v}=\frac{1}{2} e^{\top}\left|P A+A^{T} P+\dot{P}\right| e,
$$

which may be expanded an

$$
\begin{gathered}
\dot{v}=-\omega \zeta e^{T}\left[\begin{array}{cc}
\omega^{2} M^{-1} & \omega M^{-1} \\
\omega M^{-1} & \gamma_{0} I
\end{array}\right] e \\
-\omega \varsigma\left(\gamma_{0}-1\right) e_{2}^{\top} e_{2}-\omega \varsigma e_{1}^{T} M^{-1} B e_{2} \\
+\gamma_{0} e_{2}^{T}\left(\left.\frac{1}{2} \dot{M}-B \right\rvert\, e_{2} .\right.
\end{gathered}
$$

The term in the last line vanishes according to Corollary 2 . Moreover, the block matrix in the first line is positive definite according to the inequality (9) and the result of Lemma 4 since

$$
\left[\begin{array}{cc}
\omega^{2} M^{-1} & \omega M^{-1} \\
\omega M^{-1} & \gamma_{0} I
\end{array}\right]=\left[\begin{array}{cc}
M^{-\frac{1}{2}} & 0 \\
0 & M^{-\frac{1}{2}}
\end{array}\right]\left[\begin{array}{cc}
\omega^{2} I & \omega I \\
\omega I & \gamma_{0} M
\end{array}\right]\left[\begin{array}{cc}
M^{-\frac{1}{2}} & 0 \\
0 & M^{-\frac{1}{2}}
\end{array}\right] .
$$

Finally, according to Lemma 1 , the term in the middle may be rewritten as

$$
\omega s\left[\left(\gamma_{0}-1\right) e_{2}^{\top} e_{2}+e_{1}^{\top} M^{-1} B e_{2}\right]=\omega s e_{2}^{\top}\left|\left(\gamma_{0}-1\right) I+\dot{M}\left(M^{-1} e_{1}\right)\right| e_{2}>0,
$$

where $k \triangleq M^{-1} e_{1}$, and the result follows from the inequality involving the last entry of the set in ( 9 ).

## 3 Consequences for Tracking Unknown Reference Signals

Now consider the decoupled "PD" compensated system forced by a continuously differentiable reference signal, qd $(t)$,

$$
\begin{equation*}
\left.r=k(q)-\omega^{2} \mid q_{\iota}(t)-q\right]-2 \omega \varsigma \dot{q} . \tag{10}
\end{equation*}
$$

Assume that the reference trajectory is "unpredictable" - i.e. its frst and second derivatives are unknown - but there is available an à priori bound on the maximum rate of change,

$$
\left\|\dot{q}_{d}\right\| \leq, \infty_{0} .
$$

Notice that the forced closed loop system may be written in the same error coordinates as (5), above,

$$
\begin{equation*}
\dot{e}=A|\dot{q}, q| e+d, \tag{11}
\end{equation*}
$$

where $d \triangleq\left[\begin{array}{cc}0 \\ 0 & (t) \\ 0\end{array}\right]$, is a "disturbance" input due to the unknown but non-zero reference derivative.
Theorem 2 The closed loop "disturbed" error system (11) has bounded trajectories which asymptotically approach the set

$$
\left\{e \in P:\|e\| \leq \frac{\mu(M)}{\nu(K)} 2 \rho_{0} \sqrt{\left(\gamma_{0} / 5\right)^{2}+1 / \omega^{2}}\right\}
$$

where

$$
K \triangleq\left[\begin{array}{cc}
\omega & 1 \\
1 & \gamma_{0} \nu(M) / \omega
\end{array}\right] .
$$

Proof: We have

$$
\begin{aligned}
\dot{v} & \left.\left.=\frac{1}{2} e^{\mathrm{T}} \right\rvert\, P A+A^{T} P+\dot{P}\right] e+e^{\mathrm{T}} P d, \\
& \leq-\omega^{2} s e^{\mathrm{T}}\left(\frac{1}{\mu(M)} K \otimes I\right) e+\omega^{2} \zeta\left[\begin{array}{c}
\gamma_{0} e_{1}^{\mathrm{T}} \dot{q}_{d} / \varsigma \\
e_{2}^{T} \dot{q}_{d} / \omega
\end{array}\right] . \\
& \leq-\frac{\omega^{\top} s}{2 \mu(M)}\|e\|\left[\|e\| \nu(K)-2 \rho_{0} \mu(M) \sqrt{\left.\left(\gamma_{0} / \varsigma\right)^{2}+1 / \omega^{2}\right] .}\right.
\end{aligned}
$$

This is negative whenever e is outside the set indicated in the statement of the theorem.

Corollary 0 The asymptotic tracking bound may be made arbitrarily small by increasing the magnitudes of the feallack gaine in (10)

Proof: Por a sufficiently large value of $\omega$ it is possible to choose two real numbers $\kappa_{1}, \kappa_{\mathbf{2}} \in(0,1)$ such that

$$
\varsigma=\kappa_{2} \gamma_{0} \sqrt{\nu(M)} \quad ; \gamma_{0}=\kappa_{1} \omega
$$

and the inequality (9) still holds. Using these definitions and the results of the theorem, the attracting regioa ia bounded by the magnitude

$$
\frac{\mu(M)}{\nu(K)} 2 \rho_{0} \sqrt{\left(\kappa_{2}^{2} / \nu(M)\right)+1 / \omega^{2}},
$$

Note that

$$
\begin{aligned}
\nu(K) & =\omega+\gamma_{0} \nu(M) / \omega-\sqrt{\left(\omega-\gamma_{0} \nu(M) / \omega\right)^{2}+4} \\
& =\omega+\kappa_{1} \nu(M)-\sqrt{\left(\omega-\kappa_{1} \nu(M)\right)^{2}+4},
\end{aligned}
$$

hence,

$$
\frac{d \nu(K)}{d \omega}=1-\frac{\omega-\kappa_{1} \nu(M)}{\sqrt{\left(\omega-\kappa_{1} \nu(M)\right)^{2}+4}}>0
$$

and $\nu(K)$ is bounded from below as $\omega$ increases. Since $\kappa_{2}$ may be made as amall as desired without violating (9), the resalt follows.

## A The Stack Representation

If $A \in \mathbf{R}^{n \times m}$, the "atack" representation of $A \in \mathbf{R}^{n m}$ formed by stacking each column below the previoua will be denoted $A^{8}$ [12].

If $B \in R^{p \times 4}$, and $A$ is as above then the kronecker product of $A$ and $B$ is

$$
A \otimes B \triangleq\left[\begin{array}{ccc}
a_{11} B & \ldots & a_{1 m} B \\
a_{21} B & \ldots & a_{2 m} B \\
& \vdots & \\
a_{n 1} B & \ldots & a_{n m} B
\end{array}\right] \in \mathbf{R}^{n \times \times m} .
$$

The kronecker product is not, in general, commutative. Note that while the transpose "distributea" over kronecker products,

$$
(A \otimes B)^{\mathrm{T}}=\left(A^{\top} \otimes B^{\mathrm{T}}\right),
$$

the stack operator, in general, does not.
Lemma 7 If $A \in \mathbf{R}^{n \times m}$ then there exists a nonsingular linear transformation of $\mathbf{R}^{n m}, T$, such that

$$
\left(A^{T}\right)^{3}=T A^{3}
$$

Proof:
For $p=n m$, let $B \triangleq\left\{b_{1}, \ldots, b_{p}\right\}$ cenote the canonical basis of $\boldsymbol{R}^{p}-$ i.e., $b_{i}$ is a column of $p$ entries with a sirgle - entry, 1 , in position $i$, and the other $n-1$ entries set equal to zero. The transpose operator is a reordering of the canonical basis elements, hence may be represented by the elementary matrix,

$$
T \triangleq\left[b_{1}, b_{n+1}, b_{2 n+1}, \ldots, b_{(m-1) n+1}, b_{1}, b_{n+2}, b_{2 n+2}, \ldots, b_{(m-1) n+2}, \ldots b_{n}, b_{2 n}, b_{3 n}, \ldots, b_{m n}\right] .
$$

$\square$
For $n=m$, if we define $P_{+} \triangleq I+T, P_{-} \triangleq I-T$ then both operators are projections onto the set of "skew-rymmetric" , "symmetric" operatora of $\mathbf{R}^{n}$, repsectively, since $P_{ \pm}^{2}=P_{ \pm}$. Note that Ker $P_{ \pm}=\operatorname{Im} P_{\mp}$.

The kronecker product doee "distribute" over ordinary matrix melliplication in the appropriate fashion.


$$
'(A \oplus B)(C \oplus D)=(A C \otimes B D)
$$

Lemman 9 ([12]) If $B \in \mathbf{R}^{m \times p}, A \in \mathbf{R}^{n \times m}$, and $C \in \mathbf{R}^{p \times 4}$ then

$$
|A B C|^{\boxed{B}}=\left(C^{\top} \odot A\right) B^{6}
$$

Noting that for any column, $c \in \mathbf{R}^{p n 1}$, we have

$$
c^{s}=\left[(c)^{T}\right]^{d}=c
$$

there followe the corollary
Corollary $10 \| B \in \mathbb{R}^{m n}, c \in \mathbb{R}^{p}$ then

$$
\begin{aligned}
B c & =B c^{d}=\left(d^{T} \otimes I\right) B^{d} \\
& =\left(|B c|^{T}\right)^{\mathrm{d}}=\left(c^{T} B^{T}\right)^{d}=\left(l \otimes c^{T}\right)\left(B^{T}\right)^{d} .
\end{aligned}
$$

Noting, moreover, that

$$
\operatorname{tr}\{A\}=\left(l^{8}\right)^{\top} A^{8}
$$

there follows the additional resuls
Corollary $11 \| A \in \mathbf{R}^{n \times m}, B \in \mathbb{R}^{m m}$ then

$$
\operatorname{tr}\left\{A B^{7}\right\}=\left(A^{0}\right)^{q} B^{B}
$$

Proof:

$$
\begin{aligned}
\operatorname{tr}\left\{A B^{T}\right\} & =\left(I^{d}\right)^{T}\left(A B^{T}\right)^{2} \\
& =\left(I^{s}\right)^{T}(B \otimes I) A^{s} \\
& =\left(A^{d}\right)^{T}\left(B^{T} \otimes I\right) I^{8} \\
& =\left(A^{d}\right)^{T} B^{s} .
\end{aligned}
$$

Lemma 12 For any square array, $A \in \mathbf{R}^{n \times n}$, if $I_{m}$ is the identity on $\mathbf{R}^{m}$ then the apectrum of $\left(A \otimes I_{m}\right)$ is contained in the apectrum of $A$.

Proof: Suppose $\lambda$ is an eigenvalue of $\left(A \otimes I_{m}\right)$. There muat be some non-zero vector, $x \in \mathbf{R}^{m n}$ in the kernel of $\lambda\left(I_{n} \otimes I_{m}\right)-(A \otimes I)$ Since $x=X^{\mathbb{A}} \in \mathbf{R}^{n \times m}$, it follows that

$$
\begin{aligned}
0 & =\left[\lambda\left(I_{n} \otimes I_{m}\right)-(A \otimes I) \mid x\right. \\
& =\left[\lambda X-X A^{\top}\right]^{8} \\
& =\left[X\left(\lambda I_{n}-A^{\top}\right)\right]^{,} .
\end{aligned}
$$

Thin implies that $\operatorname{lm} X^{\boldsymbol{\tau}} \subset \operatorname{Ker}^{\boldsymbol{\lambda}} I_{n}-A$, and since the former subapace has dimension at least 1 (according to the asoumption that $X \neq 0$ ), the latter muat as well. Thus, $\lambda$ in an eigenvalue of $A$.

0

## B General Robot Arm Dynamics

The rigid body model of robot arm dynamice may be most quickly derived by appeal to the lagrangian formulation of Newton'a Equations. If a acalar function, termed a lagrangian, $\lambda=k-v$, is defined as the difference between total kinetic energy, $\kappa$, and total potential energy, $v$, in a system, then the equations of motion obtain from

$$
\frac{d}{d t} D_{1} \lambda-D_{1} \lambda=r^{T},
$$

where r in a vector of external torques and forces [13,14].
First consider the kinetic energy contributed by a amall volume of masa $\delta m_{4}$ as position $p$ in link $\mathcal{L}_{\mathbf{i}}$,

$$
\delta c_{1}=\frac{1}{2} 0 \dot{p}_{i}^{T 0} \dot{p}_{1} \delta m_{4}
$$

where ${ }^{0} p_{i}={ }^{0} F_{i}{ }^{\prime} p$ is the matrix representation of the position $p$ in the bace frame of reference, ${ }^{0} F_{i}$ is the matrix representation of the frame of reference of link $\mathcal{L}_{i}$ in the base frame, and 'p is the matrix representation of the point in the link frame of reference, and, hence, ${ }^{1}$

$$
\dot{p}_{r}=\dot{F}_{\boldsymbol{f}}^{\prime}{ }^{\prime}
$$

since the position in the body is independent of the generalized coordinatea. The total kinetic energy contributed by this lisk may now be written

$$
\begin{aligned}
& \kappa_{i}=\int_{C_{c}} \frac{1}{2}\left[\dot{F}_{i}{ }^{\prime} p\right]^{\top} \dot{F}_{i}^{\prime} p d m_{i} \\
& =\int_{\mathcal{C}_{i}} \frac{1}{2} \text { trace }\left\{\dot{F}_{i}{ }^{\prime} p\left[\dot{F}_{1}{ }^{\prime} p\right]^{\top}\right\} d m_{1}, \\
& =\frac{1}{2} \text { trace }\left\{\dot{F}_{i} \int_{C_{i}}^{d} p^{\top} p^{\top} d m_{0}\left[\dot{F}_{1}\right]^{\top}\right\} \\
& =\frac{1}{2} \text { trace }\left\{\dot{F}_{i} \bar{P}_{i} \dot{F}_{i}^{\top}\right\} \text {, }
\end{aligned}
$$

(since the frame matrix is constant over the integration), where $\overline{\boldsymbol{P}}_{;}$is a symmetric matrix of dynamical parameters for the link. Explicitly, if the link has mass, $\overline{\mu_{i}}$, center of gravity (in the local link coordinate syatem) $\overline{\bar{p}_{i}}$, and inertia matrix, $\bar{J}_{i}$, then

$$
\bar{P}_{i} \triangleq\left[\begin{array}{cc}
\frac{J_{i}}{\overline{\mu_{i} p_{i}}} \mathrm{M} & \overline{\mu_{i} \bar{p}_{i}}
\end{array}\right] .
$$

Passing to the stack representation (refer to Appendix A)

$$
\begin{aligned}
2 \kappa_{i} & =\operatorname{trace}\left\{\dot{F}_{i} \bar{P}_{i} \dot{F}_{i}^{\mathrm{T}}\right\} \\
& =\left[\left(\dot{F}_{i} \bar{P}_{i}\right)^{\mathrm{T}}\right]^{\mathrm{T}} \dot{F}_{i}^{\mathrm{s}} \\
& =\left[\left(\bar{P}_{i} \otimes I\right)^{\mathrm{T}} \dot{F}_{i}^{\mathrm{s}}\right]^{\mathrm{T}} \dot{F}_{i}^{\mathrm{s}} \\
& =\left[\dot{F}_{i}^{B}\right]^{\mathrm{T}} \bar{P}_{i} \dot{F}_{i}^{\mathrm{s}} \\
& =\left[\left(D_{i} F_{i}^{\mathrm{s}}\right) \dot{q}^{\mathrm{T}} \dot{P}_{i}\left(D_{i} F_{i}^{\mathrm{s}}\right) \dot{q}\right. \\
& =\dot{q}^{\mathrm{T}} M_{i} \dot{q}_{1},
\end{aligned}
$$

where we have implicitly defined

$$
\mathcal{M}_{i}(q) \triangleq\left[D_{i} F_{i}^{s}\right]^{\top} \bar{P}_{i} D_{1} F_{i}^{s} ; \quad \bar{P}_{i} \triangleq \bar{P}_{i}^{\top} \otimes l .
$$

It follows that the total kinetic energy of the entire chain is given as

$$
\kappa=\frac{1}{2} \dot{q}^{\mathrm{T}} M(q) \dot{q} ; \quad M(q) \triangleq \sum_{i=1}^{n} M_{i}(q) .
$$

The potential energy contributed by $\delta m_{i}$ in $\mathcal{L}_{i}$ is

$$
\delta v_{i}=z_{0}^{\mathrm{T}} F_{i}{ }^{i} p g \delta m_{i}
$$

where $g$ is the acceleration of gravity, hence the potential energy contributed by the entire link is

$$
v_{i}=z_{0}^{\mathrm{T}} F_{i} \int_{\lambda_{i}}{ }^{\prime} p g d m_{i}=z_{0}^{\top} F_{i} \tilde{p_{i}} g,
$$

and $u=\sum_{i=1}^{n} z_{\mathrm{C}}^{\mathrm{T}} F_{\mathrm{i}} \overline{p_{i}} g .{ }^{2}$
To proceed with the computation, note that $D_{i} \lambda=D_{i} \kappa=\dot{\boldsymbol{q}}^{\mathrm{T}} M(\boldsymbol{q})$, hence,

$$
\frac{d}{d l} D_{i} \lambda=q^{\mathrm{T}} M(q)+\dot{q}^{\mathrm{T}} \dot{M}(q) .
$$

[^16]Moreover,

$$
\begin{aligned}
D_{1} x & \left.\left.=\frac{1}{1} \dot{q}^{\top} D_{p} \right\rvert\, M(q) \dot{q}\right] \\
& =\frac{1}{2} \dot{q}^{\top}\left[\dot{q}^{\mathrm{T}} \otimes I \mid D_{\imath} M^{\mathrm{a}}\right.
\end{aligned}
$$

hence, if all terms from Lagrange's equation involving the generalized velocity ase collected, we may express them in the form $\dot{\phi}^{\boldsymbol{T}} \boldsymbol{B}^{\boldsymbol{\top}}$, where

$$
B(q, q)^{T} \triangleq \dot{M}(q)-\frac{1}{2}\left|\phi^{T} \otimes I\right| D_{\mathrm{r}} M^{\mathrm{E}}
$$

Finally, by defining $k(q) \triangleq|D, v|^{\gamma}$, Lagrange's equation may be written in tha form (1)

$$
M(q) \ddot{q}+B(q, q) \dot{q}+k(q)=r .
$$

$M$, called the "inertia" matrix, may be shown to be positive definite over the entire workspace as well as bounded from above since it contains only polynomials involving transcendental functions of $q$. $B$ contains terms arining from "coriolis" and "ceatripetal" force, hence is linear in $\dot{q}$ (these forcee are quadratic in the generalised velocities), and bounded in $q$, aince it involves only polynomials of transcendental function in the generalised position. Finally, $k$ arisen from gravitational forces, is bousded, and may be obeerved to have much simpler structure (atill polynomial in transcendental terma involving $q$ ) than the other expreasions. An important study of the form of these terms was conducted by Bejesy [15].

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# Concept Development of a Tendon Arm Manipulator and Anthropomorphic Robotic Hand 

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#### Abstract

1. Abstract  development efforts leading toward "next-generation". robotic manipulator arm and endeffector technology/s summarize eat. Manipulator arm development has been directed toward a multiple-degree-of-freedom, flexible, tendon-driven concopt which we refer to as a Tendon Arm Manipulator (TAM). End-effector development has been directed toward a thrae-fingered, dextrous, tendon-driven, anthropomorphic configuration which member to as an anthropomorphic Robotic Hand (ARH). Key technology issues are identified for both concepts. is 


2. Introduction

The background, rationale, and requirements for a next-generation manipulator arm and end-effector are noted in order establish the foundational assumptions upon which the inhouse R GD program is based. In order to relate the context for development, this background includes a brief synopsis of the projected telerobotics evolutionary path which AMETEK/ORED has advocated. since its inception in 1977.

Over the past five years, AMETEK/ORED has pursued concept development of a Tendon Arr Manipulator (TAM) through a low-level-of-effort inhouse ReD program. This development has included three conceptual design configurations and two limited engineering development models. Results of this program to date are summarized. The latest tam design configuration is illustrated and discussed, including performance design goals. Technical issues and enabling technology development are noted.

The original R\&D for the tam included some preliminary work on dexterous three-fingered end-effector concept. About two years ago, in response to a planned NASA program, this work was formulated into a concept design for an Anthropomorphic Robotic Hand (ARH) The concept was further refined and some preliminary design performed in response to the proposed darpa Advanced Robotic Manipulator program. The baseline ARH concept design is illustrated and described. Technology issues and key enabling technology development are summarized.

## 3. Background

The first robotic manipulator arm and end-effector was adapted to a subsea remotely operated vehicle (ROV) in 1961 . Over the succeeding 25 years, increasingly capable manipulators have been designed and applied to subsea pRov's; in general, control of these manipulators has been 1 imited to master -slave teleoperation, but has included bilateral force feedback on the more sophisticated systems. AMETEK has been involved in this applications arena for many years.

In 1979, AMETEK/ORED initiated an inhouse study program to forecast next-generation manipulator technology.

Technology advancement of articulated (revjlute-coordinate) manipulator arms appeared to be well covered, but we identified operations in unstructured subsea environments, egg., around wellheads, where articulated arms were severely constrained in accessibility. in these cases, what was needed was a flexible "snakelike" multiple degree-of-freedom (DOF) configuration to work through and around a maze of obstructions. This need was not being addressed, and thus became a goal for further inhouse work. For end -effectors, other than specialized, task-specific end-effectors and tools, there appeared to be a driving need for a
general, dextrous end-effector with kinesthetic and haptic capabilities approsching that of the human operator. We elected to parallel research on dextrove end-effectors along with our manipulator arm research. progress to date on both the manipulator arm and end-effector is summarized in section 4.

In order to eatablish a context for this program, it is useful to brlefly note the differences in orientation and approach between robotics technology developeent directed toward teleoperation and that almed for autonomous applications. The issues, particularly with respect to control, are significantiy difierent. The most notable difference is in the nature of the pacing robotics technologies: for autonomous operation, higher levels of control [1] are the pacing item, and mechanical systems with dextrous capabilities cannot be fully utilized as yet; under teleoperation, the operator provides higher level contzol, so highly-capable mechanical systems can more readily be utilized. Hence the motivation to prioritize zuch advanced mechanical syatems development is greater for teleoperation.

AMETEK/ORED adrocater view of the evolution of robotics technologies from teleoperation toward fully-autonomous systems, through progressive implementation of supervised autonomous modes of operation, as sensing, control, and computational technologies mature. An informative technical paper on thia subject was written by J. Vertut, manager of the Advanced Teleoperation Program in Prench Advanced Robotica and Automation project $\{21$. Our views were expressed by ameterfored General manager jack stone in his article in ROV Magazine 13 . A more exhaustive treatment, including specific oxamples for apace telerobotica, was provided recently by NASA/Montemerlo (4).

## 4. Concept Developeent: taM and arh

AMETEK/ORED has separated the inhouse IR\&D program into two related concept development initiatives, the rendon arm Manipulator (tam) and the Anthropomorphic robotic hand (ARH). The tam concept is discussed flist, followed by a discussion of the ARif concept.

## Tendon Arm Manipulator (TAM):

Inspiration for the taM concept originated with fensor Arm Manipulator Design (pigure (a) by the Scripps institution of Oceanography (5). We also examined with interest the spine manipulator arm (Figure $1 b$ ) developed by Spine Robotics 161.


b) Spine Manipulator Arm

Eigure 1 , Elexible Manipuldtor Arm Configurations
Both of these configurations utilize a number of jointed discs, the planes of wich can be rotated in two dimensions with tespert to ano inother. Fioch ilisc is driven by four tendons. two for each degree of freedom. Thus the arm has a maximum of $2(n-1)$ Dof, where $n$ represents the number of discs (the first disc is fixed to the base or world frame, while the last disc serves as the base plate for the wrist). The number of independent dof can be ciduced, as desired, by establishing an angular relationship between disc rotatione, e.g.. the two sections of the Spine arm formonly circular arcsof varying radius, so that the arm has only four independent DOF.

The original $T A M$ design configuration (figure 2$)$ resembled the Sctipps design because of its adaptability to multiple vof and more arbitrary shapes. Four joint configurations were considered, varying the reldtive placement and connection of the joint with respect to the discs. A simple enginetring model was built in order to duplicate some of the results of the Scripps work. As a follow-on, darger engineering model wis constructed, specifically to emperically examine loading of tendons and joints and instabilities.


Figure 2, AMETEK/ORED TAM, Original Design Concept
Two major deflciencles of the basic scripps configuration were confiemed: (l) a buckilng instability (also noted by seripps) between discs; and (2) high torsional loading of the joints under certain ioading conditions.

The curtent TAM design, illustrated in figure 3 , draws on the lateat advances in "Serpentine Arm" technology, summarized in a recent lntelligent task Automation report (7), and addresses and coreects the deficiencies exhibited by the tam englneering models.


Figure 3, AMETEK/ORED Tendon Arm Manipulator (TAM) Baseline Concept Uesign

In order to eliminate the burkling instability, sheathed cables are used for the sendons, each sheath terminating at the disc preceding that being displaced by the tendon; this makes each displacement determinate and precludes the buckling exhibited by the previous TAM models. To reduce the high stresses yenerated by torsion, the joints were reconfigured to form large-diameter double-gimballed rings; this not only increases the effective radius for reacting torsional moments but also provides a conveniont center-arm space for routing of actuation cables.

Performance goals for the baseline TAM design include the following:

* Length: $\quad 6^{\prime \prime} \mathrm{frcm}$ shoulder to wrist base plates.
- Weight: 20 lbs incl structure and tendons.
- Payload: 50 lbs exci wrist and end-effector.
- Speed: 180 degrees/sec, Líload (all joints).
- Accurdcy: g.050" or better.
- Operational Envelope: approximately hemisphericai.

The curtent $T A M$ baseline design, with each joint 1 imited to 30 degrees angular deflection (as our research has indicated is practical deflection upper ifititior a tendondriven conflgurationi, requiref nine segments to achieve a hemispherical operational envelope -- actually somewhat more than hemispherical as shown in figure 3, closely coriesponding to an optimal operational envelope for an articulated manipulator arm.

Obviously, given the curtent state of the art, control of such an arm, with up to 18 dof for the baseline design, ls a major issue. We have generated control scenarios, however, to account for this limitation:

For teleoperation, each joint can be servo-controlled with a scaled replica master, with which the operator "ahapes" the spacialiy-correspondent TAM. If, after positioning the arm at work alte the operator absequentiy displaces the master arm such that the TAM contacts some obstruction, the bilateral control syatem compliantly reshapes the ram around the obstruction and simultaneously conformsthe master to the new shape. This represents a elmple extrapolation of curcent technology.

For autonomous operation, the TAM can be limited in independent dop by controiling groups of discsin telational manner, as with the spine arm. such groupris may be accomplished mechanicaliy or electronicaily. Initially, as few as four independent oop may be used (determinate), with increasing DOF and shape copabilities implemented as sensing, control, and computational technolngy advances. Ultimately, with the control loop closed around the end-point through sophisticated sensing and control. and with control stategies for indeterminate arm conflgurations le.g., world modeling with eensing updates and epacial distribution of allowable arm shapes and trajectories within the world modeli. the tam should be able to achieve accuracies and capabilities rivaling articulated arms.

## Anthropomorphic Robotic Hand (ARH):

Much relevant work has been done over the past thirty years on dextrous end-effectors. For the first twonty years, this work was almost exclusively in the ated of prosthetic devices. An interesting example is the belgrade hand lif. over the past ten years or so, there has been considerable infrest and effort directed toward dextrus end-effectors suitable for robotic (autonomous) or mixed-mode (teleoperation/autonomous) applications. "Teleoperation", in chis case, includes close-coupled prosthetic applications. The previousiy-noted report for the Intelligent task Automation program (7) includes a comprehensive sumary of dextrous end-effectors.

AMETEK/OREO's inatial work on dextrous end-effector concept for the tam focussed on the Multiple Prehension Manipulator System (MPMS) (9) design circa 1974. This hand, illustrated in figure 4a, has three fingers, each with base rotation and link curl (total of six independent $\operatorname{lof}$ ). It is able to simulate all six prehensile modes of the human las defined in the referenced article), but is not anthropomorphic.


Eigure 4, Dextrous End-Effector Designs

Using the prehensile modes analysis, along with an analysis of the conflgurations of existing dextrous end-effector designs, notably the jacobsen ilel, salisbury (ll], and Caporali/shahinpoor (12) hands (illustratad in figure 4 ), we derived a unique desfon, apecifically directed toward anthropomorphicity and simplicity. Because of our orientation toward teleoperation, we gave anthropomorphicity a high priority. AMETEK/ORED designated this design concept the Anthropomorphic Robotic Hand (ARH).

The baseline $A R H$ design concept, as illustratedinfigure 5, utilizes threefingers (conflgured at a thumb and two fingers) and tixed palm. in order to directiy inice the grasping modes of the human hand, of particiar advantage for teleoperation, the thumb has the capability to rotate trom opposition with the fingers to planarity with the pelm. In addition, the digit joints of the thumb independentiy gotate ev curl the thumb as does a human thumb. Each of the two flngers has thres joints; the knuckle and middie joints have Independent cotation, while the end joint rotation is ratioed to the rotation of the aiddie joint (approximately 2 sll. No lateral rotation is provided for the knuckles of the two fingers, but the base rotational axes are oriented with such that the tipe of the fingers converge during curl to meet at contact with the palm.


Figure 5, AriETEK/ORED Anthropomerphic Robotic Hand (ARH) Base'ine Concept Design

Thus, the ARH baseline configutation has a minimum number of independent Dor iseven, as compared to nine for the Salisbury hand and sixteen for the Jacobsen hand), and is able to achieve alt the prehensile modes of the human in a direct anthropomorphic manner.

The ability of the thumb to rotate to the plane of the palm uniquely provides a hook zrasping mode in an anthropomorphic manner. Spherical and cylindrical graspand closure are provided with thumb opposition and coordinated curl of thumb and fingers. direct pinch with both or either finger tip(s) is enabled by proper rotation and curl of the thumb with respect to the curl of each or both of the fingers, and coordination of these movementswill allow pinch tiansfer. Finally, lateral pinch is enaoled by rotating the thumb midway and closing onto the finger.

Key technology issues in the areas of actuation, sensing, and control are, in general, beini addressed through ongoing research throughout the community. Most of the citical elements currentlyexist commercialiy or are near transition from the laboratory. Acomplete review of the ARH baselifie design is beyond the scope of this paper, but some of the key technolouy issues for both the taM and ARH are noted in section 5 .

## 5. Technology Developeent

Preliminary iesign and development of the TAM and ARH have included research on pacing technoiojtes, actuataon, sensiny, and contiol. Key isisues, resilits, and recommendations follow:

Actuation for both the TAM and ARll is provided $[$ rom actuator mechanisms located in the base throurg sheathed cable tendons. The actuators could be electrical motors, shape memory alloy (SMA), hydraulic or pneumatic mechanisms. A particularly interesting actuation technoolory is that refer red to as "mechanical muscle" technology. SmA appears relatively less attractive because of the adverse relutionship of force visesponse, and high hysteresis.

In general, although "cleaner", pneumatics seem less suitable than hydraulics for actuators because of working fluid compressibility, resulting in compliance ("sponginess") and deflection rate ("station") characteristacs which are difficult to control. New hightorque rare-earth $U C$ motors offer a very competitive alternative for actuation.

Sultable sensing receptors for force/torqua are generally avallable, as well as position sensora, although curient technology advancea promise aignificantimprovements, ractile cenifing elements for proximity (stretching the definition of "tactilen), contect, force, imaging, surface and material characteristics, and alipare the subjects of much curient research.

Breakthroughs are in order to be realiy applicable and useful for the Aris, but very promising devices are on the horizon, e.g.. thin micromachined silicon ariays with both normal and shear measurement at each arcay ite on lmm $x$ immepacing. By comparison, curtentiy avallable commercial tactile sensoris have only normal force resolution capability at each site, and are approximately $1 / 2^{\prime \prime}$ thick (Lord tactile aensoris).

For teleoperation, sensing must include force/torque and tactile feedback itimulation for the operator. Force/torque feedback is state of the art for bilateral control systems, but tactile feedback is another matter. By comparison with tactile sansing receptors, relatively littlework is belng done in thiterea. An example of what might be done is to adapt the coleniod-actuated pin-matrix technology used for brallie readers to hand controller for the operator. Such a device, perhaps fitted into "glove" controller, could potentially provide the operator with simulated contact, imaging, and force tactile feedback. Another technique has been suggested by AMETEK/ORED: thermal simulation of contact, imaging, and possibly force tactile feedback uaing a Peletier junction array.

Control is $\varepsilon$ very complex issue, being addressed through many rosearch projects in the community. We are generally only tracking technology developments in theseceas for relevancy to the TAM and ARH. We have, however, developed short-and long-term strategies for control, focussing initially on teleoperation for the ghorteterm, and looking ahead for compatibility with likely future technological approaches for telerobotics and full autamation for the long-term. This has been noted in preceding discussion.

## 6. Conclusion

This paper has presented an overview of ameter/offshore Research and engineering Division linouse technology development effcets on an advanced manipulator arm (Tandon Arm Manipulator) and a dextrous end-effector concept (Anthropomorphic Robotic Hand). The curtent baseline design concepts for the $T A M$ and $A R H$ were prosented and discussed, and key enabling technological issues were summarized.

## 7. Acknowledgements

Much of the technical work reflected in this paper was contributed oy William Walker and steven Gates of AMETEK/ORED, and Mark Danna, curgently working witithe university of California at Santa barbara and the Center for Robotic systems in microelectronics, Santa Barbsra, California. In addition, Mark Danna provided a valuable critique of this technlcal paper.

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# Future Research Directions 

Moderator：G．A．Bekey<br>University of Southern California<br>Panel Members：R．Bajcsy，University of Pennsylvania， J．Y．S．Luh，Clemson University， A．Sanderson，Carnegie－Mellon University， G．Saridis，Rensselaer Polytechnic Institute， T．B．Sheridan，Massachusetts Institute of Technology， C．Weisbin，Oak Ridge Naticual Laboratories

The intent of this session was to provide views on the state of telerobotics research and to draw together a collection of suggested research opportunities to present to NASA． The panel members gave opening statements summarizing their views，the results of earlier sessions，and discussion periods．This was followed by a general discussion with the audience．The moderator concluded the session by stating a synthesized set of recommendations．

G．A．BBKBY：Welcome to the closing session of this symposium．［ suggest each of the panel members take about ten to fifteen minutes to tell you about the research directions that they see in their own field，based on their experience or on the topics that they heard at this symposium．Then $I$ suggest they take a ten－year leap into the future to give you a more distant extrapolation about where these research directions might or might not lead．After everyone has spoken，we will open the floor for comments，questions，contradictions，arguments and additional extrapolations．I am sure that some of you would like us to extrapolate an additional twenty years forward beyond the initial ten years．

C．WBISBIN：As most of you know，after being here for three days and hearing all of these talks，any attempt to summarize near－term and long－term research directions in fifteen minutes would be a rather formidable challenge，so I will respectfully defer and do something else．I propose to tell you about the research items that I think are interesting and not worked on very much，some things I have heard at this conference that I would express some cautions about，and some things that might surprise us．So， what I cannot do is to state all of the research that needs to be done in vision， multi－sensor integration，and so on．Many of these programs have been ongoing for some time．My comments will obviously reflect my very personal biases．In a meeting with parallel sessions such as this，one cannot be at every session，and therefore cannot always be fair to everything that was presented．So，three types of comments： high－priority，cautions，and surprises．

Migh Priority Issucs．The first one has to do with the Minnesota work which deals with unexpected events and self－understanding．We are working in this ares also．I happen to think that this is very important and it is an area which has not received a lot of attention．Most of us in this room realize thet systems are not necessarily going to work the way we expect them to every time．But if one counts the number of papers or articles that address the issues of coping with contingencies and understanding self－awareness，I think we would find they are very few．A second area has to do with three－dimensional world modeling and understanding the environment．We heard papers for example from Hans Moravac and others on characterizing the world in which the robot lives．There，I suspect that people may assume that it is an easy problem， whereas in reality it is quite difficult．A third kind of problem is the issue of man－machine cooperative problem solving which was alluded to in several talks．The
problem consists of having man and machine(s) act as a composite system to try to accomplish a specific task while equitably sharing the workload. One of the central problems is knowledge representation. When the man tells the machine: "you take over vision, I am going to assume the planning role," that assumes that each of them can communicate effectively at the same level. I do not think that we can do that today very well. Those are three topics (or topical areas) within the scope of the AI discipline that I regard as high priority issues. I do not think these areas are receiving adequate attention, although I feel they are very important.

Cautions. We heard a few papers which deal with the issue of reasoning with uncertainty. There has been a lot of time and effort spent on how to propagate uncertainty. One caution there, at least in my limited experience, is due to the potential difficulty in obtaining the uncertainties in the first place. We are not just talking about statistical uncertainties in data measurements. There are also uncertainties in rules and in heuristics which must be assessed. We must also be able to differentiate uncertainties from mistakes. There is much basic data that needs to be obtained. The limited efforts that we have made in that direction have indicated clearly that the problem of getting and characterizing the basic uncertainties data is just as important as that of propagating it and using it in a decision-making environment. We also heard some statements such as "this is an NP complete problem and we cannot solve $\mathrm{it}^{\prime \prime}$ and questions concerning the lack of formal methodologies to deal with the combinatorial explosion. But what if we are looking for a solution, not the best solution, just a satisifying solution. The warnings about combinatorial explosion are still appropriate and need to be heeded. But, in many cases, these problems might be made easier if we do not absolutely insist on getting the best solution and find an adequate solution instead. We also heard several papers which deal with qualitative physics. This has been very enigmatic to me. Sometimes when you try to form a hybrid of that type, (hard science ["physics"] tempered by heuristic or integral approaches ("qualitative"]), you can escape with the worst of both worlds. Qualitative physics has a very noble goal, but it can also be fraught with difficulties. Then we heard some papers on learning by discovery or by analogy. There 1 worry a little bit about the applicability to real-time problems. I think that is really hard, and lam nct really sure it is possible.

Surprises. The first surprise is that, out of the whole conference, I saw one or at most two papers in the area of concurrent computation and parallel computing. That surprised me because this is an area that is absolutely fundamental to getting anything done. There may have been many more papers in sessions that I did not attend. At least in those sessions that $I$ attended there was surprisingly little discussion of implementation of parallel algorithms. In a similar vein, you might be aware that there is a renewed interest i:: neural networks. The UCLA paper is an example of that. But, given the amount of interest that there is currently in neural networks, the number of papers on that topic that were discussed up at this conference was unusually few.

I have several remarks about the conference as a whole. First of all, I wanted to thank the organizers. I thought that it was a really interesting and well organized conference. You inevitably run into a difficulty when you do things too well. The large attendance forces you to go to parallel sessions. That is the normal problem. The accommodations, the people who were here, the subjects, the papers, were well worth coming. It was all very interesting as far as we were concerned. We would look forward to another meeting of this type.
T. B. SHBRIDAN: Each of us looks at the world with glasses colored in a different part of the spectrum. My part of the spectrum is supposed to be that of the man-machine interface, which $I$ have been interested in for a long time. From that
point of view, let me categorize my comments into several parts. One of the most human things is that we use words. In certain emerging fields, words sometimes get a bit fuzzy and imply things that we, if we are honest with ourselves, may not quite muster. But we persiat in using those words anyway. In a conference like this, and really for a long time, I have been a bit concerned about working toward using the Bnglish lenguage in such a way that we are not kidding ourselves and are meaning the same thing. A term like teleoperator, which has been around for a long time means to do something at a distance. It has come to mean doing something with more or less continuous human control at a distance, although I suppose you could have a computer teleoperating another computer. I think that term is more or less settled in terms of human control.

The word telerobot means controlling from a distance something which at least has some autonomy. It means human control, in presumably supervisory fashion, of a robot which to some extent is autonomous. But, it seems to me here NASA or somebody misht define what the term telerobot means. Let us not use it to mean all of teleoperation.

We come to the word telepresence, and there is an open opportunity for chaos and confusion. Telepresence can mean two things. It can mean feeling like you are present somewhere else. You could also use the word telepresence to mean you are operating at a distance just as though, and just as well, as if you were there. But, you can be telepresent and atill be doing supervisory telerobotic control. Or, you can be telepresent controlling and not feeling the present. What Iamsaying is that there are various interpretations of that term needing better definition. One can begin to name other terms. It may be time to draft a semi-official glossary defining such words. I think that is important, because people confuse each other with words.

Some things I would like to applaud. We have talked about end-effectors for a long time, but we are beginning to see a real experimental science emerge on the use of more complex end-effectors. We are beginning to understand what it means to have multiple degrees of freedom in an end-effector. The work of Ken Salisbury, Larry Leifer, and others I could name is beginning to give us some nice science. Some of David Aiken's work on the combined use of the body and the hands is providing data. We need data. We have not had much data in this area. We are beginning to get some microdata. Blake Hannaford, Larry Stark and others reported on microforce data. That is, within a task, there are different forces that do different things at different times. I really applaud getting instruments right into the active manipulation site and getting some good hard data. Also we are beginning to understand this whole problem of impedance control. Not just force control. Not just position control. It may be some kind of a hybrid that generalizes the whole story. These things all gotogether.

As a next category, let us look at vision. Some very exciting things are coming along with head-mounted virtual displaye. These help on the sensory side to achieve some telepresence. They also permit us to do other things that we have not been able to do before: wider fields of view, certain kinds of simulation experiences, use of computer graphics to help an operator see things that he or she normally could not see, superimposing video graphics on TV, rendering multiple views coming in part from a model, etc. These are all things that are relatively close in.

We jump a little bit farther and get the computer operating at a more profound level. We are beginning to see some nice work done in areas of linguistic interfaces with semi-automatic telerobotic systems. Here, we should not lose sight that we need both: what I would call symbolic language, which I am using now as I speak; and analogic language, which we use with steering wheels, joysticks, and our hands when we point, pull, push, etc. In our everyday life we use both of these languages when we
communicate with each other and with the physical world. I am sure we need to understand how to combine the use of both of those languages.

Computers for a number of years now have helped us resolve coordinate syatems. That kind of thing is very well in hand. They are going to begin helping coordinate two arms, helping us coordinate arms plus vehicles that have to be controlled at the same time, and indeed helping us coordinate any linkage or set of rigid interconnected bodies with mose than six degrees of freedom. There are some tough problems still to be resolved there.

Looking into the future a little bit more, we see computer aids for sensing and planning. Now that is a bigger package. But, 1 think that as long $a 8$ we consider that these are really, in large measure, aids to help the human sense and plan, we come to realize from looking at human anatomy that there are many more nerve cells for sens'g and planning than there are for motor control. Indeed, in telerobotic systems, that is going to be the same. We are going to have much more cost, expense and complexity in sensing, planning and decision than we are going to have in purely motor control.

Another topic which I believe has been neglected is finding good performance measures for teleoperators and telerobots. it is still very difficult to determine that any ziven telerobot is better than another. Of course, you have to be able to say that it is better at what. Now, you could say that there is no way of answering that question unless you are talking about a very specific task. That may be true in an ultimate sense. I would assert that there is, in-between, a possibility involving a battery of tests, intelligence tests or motor-skill performance tests. These tests would consider a whole range of different kinds of tasks that you can do with hands, eyes and sensors, automatic control system, or planning, etc. These might be a better foundation for comparing the performance of different telerobotic systems. We do not have that yet. In fact, if you look at the literature, very little research has been done on it. Everybody has a different task board, in effect.

Finally, I will touch on one of the problems that has been a concern to a lot of people. The way that AI researchers, and other researchers that work on heuristic programming and develop LISP programs, talk about models and control is somehow different than the way in which control engineers talk about modeling and control (which usually is in terms of differential equations and things like that). I think that it is time that we interconnected these two ways of thinking: the analog and the digital. There is a lot of hard work to be done there. I see that as a very long range problem.
G. SARIDIS: I will be trying to express my feelings, feelings is the right word, about the area that I am interested and concerned with. This is the theory of intellegent machines. In this meeting, as in several previous meetings that I have attended, the underlying idea is that of associating some kind of intelligence with the robots or machines. I think that there are three basic disciplines that contribute toward a theury of intelligent machines: artificial intelligence, operations research, and system theory. All of us probably agree that this is where we should be looking into, to develop such a theory. I was asked to express the present situation in my area and to discuss where we stand regarding intelligent machines. Let me be a little more specific. My interests are autonomous systems, in contrast to man-machine interactive systems. Autonomous systems, typically called robots, are systems that can perform autonomously without interaction with the human operator.

In terms of actual applications, I can say only one thing: there are very few. There are very few places where one can find a machine that has some kind of intelligence. There is a lot of research going on. For instance, I just came back from a visit to JPL, and I
was really impressed. I saw a "smart" hand, and I saw some other components of intelligent machines. For quite a while, JPL has been a pioneer in this area. In general, NASA has been interested because of the unique work in space that it is doing. This includes both manned and unmanned space exploration. I think we have been de-emphasizing unmanned space exploration. The funding seems to have decreased, although great work has been done. Instead it should have been increased. Industry is another area where these machines are applicable. We are talking about the factory of the future. It is going to be modularized. There will be no workers there doing things that the machine can do better. The human operators will be in a rote involving primarily monitoring and supervision. There might be some maintenance crews. The humans will be doing the more intellectual work of designing and building those machines. Underwater and underground exploration are other very important applications. Automation of mining operations is another application area in which there is much interest. Nuclear handling is another area. This has been explored for quite a while. There are, for example, interesting applications for smart robots in safeguarding nuclear plants. The problem of safety could be improved. There is also some role for automation and robotics in medicine. Other organizations fe.g. government agencies) have also established efforts in automation. Substantial research is also being conducted in universities. Most of the major universities have a strong program in automation. The automotive industry has also been pioneering in research to create an automated environment in the factory of the future. I would like to make a parenthetical remark. An intelligent machine can be any machine that can perform tasks (including anthropomorphic tasks) without interaction with a human operator. Some of the machines used in the automotive industry are examples of non-anthropomorphic machines that can perform anthropomorphic tasks. Work in japan has shown the first level of intelligent or semi-inteliigent (i.e., smart) machines that will remove the worker from the factory floor. Now they are working on the second level which will demonstrate more intelligence. They have different stations and robots that can move from one station to another and be reprogrammed.

I would like to now turn to the issue of how to build those machines. There exist, of course, various components. There is the motion hardware. There is sensing - different types of sensing such as vision, tactile and proximity sensing, etc. One issue is how to integrate them. How do you get the people involved in these areas to communicate with each other? How do you get what I call equalization of communications? How do you establish a common language so that there can be meaningful exchange among the various research groups? I am a strong believer in a mathematically structured intelligent machine. How is that going to be materialized? I proposed a probabilistic model. Other people have proposed fuzzy or possibilistic models. AI researchers want to have a purely heuristic model based on AI principles. Are we going to have a hierarchical structure or a distributed structure? What are we going to emphasize? Kinematics? Dynamics? Both? is control going to be adaptive? Is trajectory planning going to be done using simple kinematics? How are the actuators going to be incorporated? How is the hardware going to be improved? If there is some kind of heirarchy, how are the higher levels going to be constructed? is there some additional intelligence at the higher levels? How are tasks going to be executed autonomously? is the task to be decomposed or synthesized for comparison with the command?

Finally, I will make some brief comments about the future. What is the final goal of an intelligent machine? How can we tell that the machines that we are building are really intelligent? How can we measure that? In response to this question, I will offer the following remark. We will have succeeded in building machines that are really intelligent when we can build robots that can by themselves build more robots.
A. SANDBRSON: I will comment on the topics in artificial intelligence that have been discussed at the meeting. Those topics have focused on issues of task planning and
irajectory planning, as well as navigation of moblle robots. This is a subset of the broader spectrum of lssues in artificial intelligence, but it seems to be a subset that is particularly relevant to the kind of tasks that NASA has defined. In this area, there is no shortage of buzzwords and jargon. Por example, the discussions reflected the lesues of: what is scheduling vs real-time planning vs what is off-line planningi what is reactive planning. There is a whole set of terminology which reflects, in a sense, the rather nebulous view not only of the solutions but of the problems themselves. I think it also reflects the clear view that these kinds of rechniques are going to be the keys to the evolution of the technology, but in fact there are basic concepts and basic lasues that need to be worked out. It is not a case of taking existing tools and applying them appropriately. It is the case of working out some quite basic principles and understanding how in the case of particular domains and applications they can be made useful.

This background slide suggests a task domain where issues of task and trajectory planning are relevant to space applications.

## BACKGROUND

Space-based Diagnosis, Repair, and Assembly Tasks:

- Materials Handling
- Rault Diagnosis
- Reasoning about the Orisin of Raults
- Hypothesis Formarion and Testing
- Planning and Execution of Repairs
- Disassembly and Assembly
- Replacement of Parts

This slide suggests scme topics or tasks in diagnosis, repair and assembly of systems. The kind of issue which arises here is the mixture of complex tasks that need to be accomplished in a relatively isolated environment. There are problems of materials handling, reasoning about faults and diagnosis of faults, forming hypotheses and developing strategies to test systems, planning and execution of repairs of those systems (which in itself involves assembly and disassembly of parts and replacement of those parts). These are all subtasks which go along with the idea of having systems in isolated environments, and other systems with sufficient capability to keep those systems repaired, maintained and serviced.

You can look at this list and ask what is different about doing this in a space environment, as opposed to the problem of repairing your computer when it breaks down. I think there are several dimensions to that. One is that the physical environment in which you need to carry out these operations is quite different. The vacuum, zero-gravity, lighting conditions, the physics of sensing and manipulation are quite different. So you need to be able to do the manipulation and do the tasks in a different environment than might be done in other situations. Secondly, the situation itself tends to be more isolated and less interactive than in other cases. It really forces you into the issue of asking what it takes to do things in an autonomous or semi-autonomous way, where you do not have the fall-back position of someone walking in and interacting with the system. So it really forces the issue of how to accomplish useful work and tasks in an autonomous way. The third aspect is the nature of the
syaterst themelves. That ta, you are not typically deallag with a lune of computers or televiaion sets that you see the same things with the same faults every day. There are many unique systems which have to be addressed, unique designs, unique requirements and you have to be prepared to cope with those unique requirements. So that set of fasuas really defines these taske as quite different from any other related tasks. The collection of those poses a considerable burden on how you make systems, and sutonomous systems, to be able to cope with these issues. I could make a similar list for another area such as mavigetion, such as exploration, and with similar kinds of conatraints that are imposed that make the task in fact quite difficult and challenging. The technical challenge is, I think, apparent. It is a combination of reasoning and manipulation, and how to comblne those together.

I will use one other silide which summarizes some of the research issues which are posed by this kind of tesk.

## RESBARCH LSSUBS

## RBPRESBNTATION

- Geometrical, Physical, Punctional
- Uncertainty
- Dynamics

PLANNING

- Diagnosis and Repair
- Sequence Level
- Discrete Task Level
- Motion Level

CONTROL: MANIPULATION AND SENSING

- Adaptive Sensor-based Systems
- Learning Systems
- Multisensorz

This summarizes many of the issues which came up during technical sessions. I will not try to summarize all those in detail. Let me just make a few points. I think that there is a clear relationship between representation and planning and control. As someone suggested, planning and control are not as different as many people sometimes pretend. If you think about what even classical control attempts to do, it really largely has to do with resolving uncertainties. It has to do with being able to predict what those uncertainties are going to be, in a way that you can take timely actions to correct them. That has to do with the fact that, even in relatively simple control systems, you do not have a perfect model of the world, and you cannot devise your actions totally in advance in order to accomplish a task. So, you have to work with uncertainties, and you have to be able to develop predictions of the world in order to accomplish the task. This has a lot to do with planning, and planning, as it is used typically in AI, is a kind of reasoning in a symbolic world. It is a representation of the world which tends to be symbolic and relational. The kind of issues of representation includes geomeiry, physics,
functions of systems, temporsl aspects, etc. Representation of uncertainty is fundamental to that, in the same way that it is in control. It is uncertainty, again, not in the sense that there are things that you simply do not know. But, the more thet you can explicitly describe the nature of that uncertainty, the more knowledge that you have to incorporate into the systein. The dynamics are increasingly important as you look at the kinds of systems which are being discussed here. So, as you look at the strategies toward planning, you typically see a hierarchical approach. You typically see various levels of abstraction defined. In the case of manipulation, you can define requences of tasks. You can define sets of discrete operations. You can define explicit motions, in the sense of trajectory planning. It is easy in a sense to link those together in a block diagram. It is much more difficult to be able to make systems that work and accomplish those tasks. One of the issues, if you look at the work on planning, is that you do not see at this point complete working systems. You see pleces of systems and concepts loward working systems. At the lower levels of control, manipulation and sensing, you see a trend toward more adartive and learning systems with the ability to integrate information and resolve uncertainties among multiple sensors.

I have a few notes on specific topics, which I think are worth mentioning. One is the problem of validation and evaluation of systems. How do you know when a plan works? When it is working, how do you know when to go back and replant Again, in a dynamic and changing environment, that is important. The question of unexpected events and error recovery is implicit in this. Someone pointed out that, in many robot progrems. $90 \%$ of the program code deals with the unexpected events. The question of why is planning difficult, again, we could go on and on about. One of the basic issues is the computational and combinatorial barriers that you reach, given the complexities of the task. Trying to evaluate all the possible alternatives simply yields a combinatorial explosion. In practice, the solution to that is to look for efficient representations, and often domain-specific representations and control mechanisms.

I will finish with a couple of remarks. One is to reiterate that the payoff l see from this technology is really in the evolution of systems. That is, it is important to have the core technology which becomes embedded in the telerobot systems and evolves into autonomous systems. The final payoff as we expect is perhaps with autonomous systems, but issues such as representation, such as system architecture, we cannot simply wait and implement in autonomous systems. I think the concepts have to be embedded earlier in the evolution of the systems. The final remark is a rather philosophical view of robotics. One can think of robotics as a collection of devices (robots, hands, sensors, etc.), or one can think of it as a science which deals with principles. In fact, it is a mixture of those things. In many ways, I think of it as an experimental science, one in which you have to build systems and try to find out why they work and why they do not work in order to make progress. I think in this area in particular you have to build systems and use planners with real robots in order to make progress and corne up with useful results.
R. BAICSY: I will concentrate on stating my views on future research directions in sensing and perception. Here is the world which we live in. Here are the sensors, i.e. . hands, ears, eyes, etc. Here is the representation. To set the stage, I assume that I am interested only in the world of physics and geometry. I assume, for the purpose of this discussion, that we have only contact and non-contact sensors. This is not a severe limitation, since in fact there are no other types of sensors. I would like to impress upon you that sensors sense only partially through a window of the world and only some aspect of the world. So one sensor can never sense the full world. So, having set the stage, I proceed to the mission impossible. The job is to recover the world from the partial measurements. Here are my questions for the next iwo or three years. Then, 1 have another set of questions for the following two or three years. Then, I have another slide for the next ten years. Question number one, which is dear to my heart, what is the information that you lose through these transducers? From which follows, models
of hardware and/or sofiware. In my view, software is equivalent to hardware, i.e., an edge detector could be implemented eithar in hardware or in softwere. These models represent the processes that have to take place as you go from the world representation to the sensors and to the sensor data. So, one issue ts what is the information that is lost, and hence the modeling question. The second question is what are the rules, or principles, of recovery. In the first question, things are being taken apart, whereas in the second, things are being put together. The third question, which is very clear in isctile sensing and information processing, is that of dats acquisition. This is an essential par: of the recognition, or manipulation, or whatever use is made of the information. This issue, in my view, is just as important in vision, although it may not be as readily apparent. Hence, the control lasues are a critical part of the requirements for machine sensing. We must get away from an approach to machine sensing in which there are no global goals that involve how the information is to be used for the purpose of control (as an example). This is my view of the present state and of the important immediate issues.

I would like to share with you what I think is a plan for the next ten years. I really view space exploration and space robotics as representing a great opportunity as a research laboratory, or as a research environment, for discovering rules of evolution. Why? Well, space represents different physics and physical laws. Different radiation. Our sensors have been developed or have evolved to be sensitive to a certain spectral range. In space, different sensors will be needed perhaps. But, of more interest 1.0 me, is the different representations that will evolve from the different sensors and physical laws. Different rules for update of this knowledge will also evolve, as the sensorz iaie measurements and interact with the world. Hence, the rules of evolution.
J. Y. S. LUH: Being the last person in the panel has advantages (I will not say disadvantages). All of the important and interesting issues are well taken care of. That means that I do not have to say anything. On the other hand, I have noticed a few minor things which have not been mentioned by the previous panelists. These are down-to-earth, for instance, kinematics, dynamics and control. These are basic issues. Without kinematics, dynamics and control, there would be no robots, either in space or in industry. Of course, there has been a lot of research on these topics for mary years. Nonetheless, there are still open issues that need to be studied and investigated in order to improve space operations.

Por instance, one issue is that of robots with closed-kinematic-chains. These robots, it seems to me, would be very useful. This topic was studied many years ago in a very general sense. But, very few results were obtained that were specific to a robot and that were relevant to space applications. In order to investigate the closed-chain robot, extensive computations (for instance, to do inverse kinematics) are required quite often. To shorten the computations, parallel computation is one of the topics that needs to be looked into. Redundant robots is another topic which I think is important. I only judge from my experience. Bverybody knows that the human body has redundant kinematic chains. Without the redundancy, you could not do a lot of things that you normally like to do, such as scratching your back when it itches. If we look at dynamics, we need to do a lot of work on analysis of flexible joints and flexible links. So far, the most complicated flexible rokot that, to my knowledge, has been analyzed consists of two links. This is not really enough. There is a need to pursue that kind of research.

If we look at control, we use a lot of words and names. For instance, one of the oldest control schemes in robotics is that of computed torque. That was done ten or fifteen years ago. Then, later we developed resolved rate, resolved acceleration, nonlinear transfurmations, adaptive, force control, impedance control, etc. But, there is a result in a paper presented here by K. Kreutz of JPL in which he found a lot of equivalence
anong the various concepts. By chanetins the names and equations, a lot of control schemes which seem to be different are in fact equivalent.

But, whether the control schemes are equal or not, the most analyitical results are very seldom applied or implemented in hardware. As in several of the eariler comments made by the panel members, robotics is an experimental science. It is nice to do analyais to guide your experiments. But the final judgement really comez through experimentation. All the control schemes may look beautiful in analysis (and I expect to get a lot of comments from the audience on this), but we also need the implementation and experimentation to prove feasibility and performance based on our current hardware technology.

In space applications, I listened to a paper presented on a virtual robot. I think this type of work is particularly relevant to space applications. Similar or equivatent work should be promoted.

Now I move to teleoperation and telerobotics. I agree with comments made earlier about terminology. Telerobot implies remote control of robot. Teleoperation can involve two robots or even multiple robots. There are a lot of papers on that topic. There are two tems which are basic issues and which have not been mentioned. It is my opinion that the main issue in the coordination of two or more robots is how do you divide the work. This is the rask decomposition which is the main issue before we even start to do anything else. There is a Chinese story tha: is pertinent here. There is a monk in the monastery. If he wants to drink water, he carties the water from the well to the monastery in two buckets of water, one bucket at each end of a beam, and the beam is balanced on top of his shoulder. If there are two monks in the monastery, then the same beam can be used. Bach of the monks supports one end of the beam. Now, however, there is only one bucket at the mid-point of the beam. If you have three monks living in the monasiery, there will be no water. The point of the story is that in the monastery they must decompose the task to determine who is doing what. Otherwise, the three monks (or robots) will just sit there and argue who is doing what. A second topic which is also basic, in my view, and which has not been mentioned is that of limb coordination. We are talking now about two robots and about perhaps extending this to walking robots. There is a lot of interest in walking robots, instead of wheeled robots. Multiple arm robots with dexterous end-effectors are also being discussed for space applications. I beiieve that the coordination of arms with the legs and end-effectors is also 2 very important issue.
G. A. BBRBY: I would like to thank the paneliats for their contributions. We have about 45 minutes now for comments, rebutials, questions, suggestions, etc. If anyone would like to make a comment, I would like to suggest that he or she step to the microphone.
A. J. BEJCZY, JPL: I would like to make a comment on intelligent machines ard their relation to man. How do we communicate with intelligent machines? lassume that intelligence is also measured not only by the action that a machine can perform, but by the way that a machine can communicate with other machines, or in our case with an operator or with someone that is suppesed to use it. In a telerobot sense, an intelligent machine is for a user. I would like to ask a sharp question on the issues of how t: communicate with intelligent machines and how is intelligence defined from that point of view.
G. SARIDIS: This was a loaded question. Pirst, let me start with the intelligent machine itself. I do not think that we have built an intelligent machine. Therefore, we have not explored all the capabilities of our machines regarding intelligence. But, suppose for the momert that we could do that. A good idea is to use the same approach
that we are using in dealing with expert systems. With expert systems, we have some intelusence stored in the computer. The operator can, by means of if-then statements, communicate with that particular cornputer. This assumes that the computer in which the expert syatem resides has what i would call static intelligence. An intelligent machine would have what I would call dynamic intellisence. Ithink that it would be an improvement on an expert system to do thei job.
R. BAICSY: Communication with an intelligent mechine depends on the representation that the particular machine has. It was precisely this point that 1 was trying to promote in my preseatation. The world is out there. We have these umited sensors which reduce the data and store it in memory in some organized fashion. This we call representation. There are also additional rules and laws that control both how to gather the data and how to store the data. In space, we have this wonderful opportunity because we do not have a preconcelved a priori representation of that world. Therefore, we have an opportunity to gather this knowledge iteratively and to learn something about learning itself and also about evolution.
A. LOKSHIN, JPL: I would like to share my views on the problem of robot error recovery after what are referred to typically as unexpected events. I believe that they are quite close to the views that the panel expressed, but I would like to introduce additional ideas. I believe that the only unexpected event that could be properly dealt with is an event that is expected well in advance. If you look for example at our society, people (such as fire-fighters, commanders, etc.) that are supposed to deal with real unexpected events, have very rigorous training to be able to foresee any kind of possibility in advance. If a person behaves properly in real unexpected events, he usually gets a medal. I do not think that the PUMA 560 deserves a medal every time that it dues a simple task such as acquirins and grasping a tool. Therefore, a more precise term than unexpected events is that of abnormal events. If we use this term, then work that focuses on case studies in a particular task domain will be encouraged and more readily accepted. Such case studies that address the issue of dealing with abnormal events in particular lask domains are needed in robotics. For example, a robot could try a thousand times to unscrew a bolt and then tell us all of the problems that it encountered in all of those atlempts. This experimental approach would be very similar to the one used in the very early stages of the development of coding theory. There, a lot of very simple statistical experiments (such as counting the number of times a farticular letler of the alphabet was used in aver:ge speech) were conducted. I think this type of work and presentations should be encouraged in robotics also.
A. SANDERSON: I agree. There are levels of abstraction in terms of thinking about what is expected and about what is normal or abnormal. The key question is what is the capacity of the system to interpret any kind of event and how does it decide what to do about them. In some sen e what you are referring to with the PUMA is that, when you program it to read a montor that is tripped, and you have a fixed subroutine that it goes into, you can argue that it is error recovery, but it is not intelligent error recovery. The more general case that you are referring to is where you have a deeper representation of the system and where, instead of having a kind of subroutine, you have some capacity to reason about the nature of the event and its origin and to think in a deeper sense about the appropriate reaction, given all the constraints, the states of the system, and the goals.
C. WBISBIN: I think that it would be rather bold to suggest that we could deal with unknown unknowns, which is what I think is at issue. When you try to put a framework in place to deal with classes of potential events, as opposed to specific recovery modes, you need a framework for broad on-line dynamic replanning, which is more than simple error recovery.

I fully agree with your question. If I did not ever anticipate something to happen, it would be unilikely that our robot would cope with it properly. On the other hand, I do believe that you can anticipate broad classes of events that you can handle on-line through a replanner, without having prescribed in detail the particular event and the particular reaction to the event. No matter what we call that, whether it is error recovery, unanticipated events, robust planning, etc., the ability to perform this function is essential.
G. A. BEEBY: Let me add another comment to that from my personal expertence. Many years ago I worked on lazues of trying to model human performance in particular kinds of systems. Tom Sheridan has done similar work. One time, we were interested in modeling the way in which human pilots adapted to particular catastrophic fallures in aircraft, under situations where recovery was possible. There are obviously times, such as the extreme case in which the vertical stablizer falls off, when you cannot recover. There are situations where the stability sugmentation syatem fails, and recovery is possible, but the control strategy that the pilot has to follow is different. The issue here is very similar to that raised by the question. Is this an unknown unknown, or is it an unknown for which previous learning took place and there are strategias of recovery stored somewhere in the system? What we did was to try to model the recovery strategy, and it was obviously highly dependent on previous training. This raises very complex issues in Al and in the way in which knowledge is represented. It is a very interesting question with a lot of implications in many of the areas that we are dealing with.
R. SCOTTI, ORI: I would like to raise the lasue of what is the ultimate model; what sort of ultimate modei are we thinking about, when we think about robotics and artificial intelligence. The question came up in one of the sessions, where we discussed the prospect that perhaps it is an anthropological model that we are talking about if we follow the idea that man is the model. It is an issue, what is the model. As in our experiences with science in the past, solving a problem really revolves about understanding and having a clear idea about what the problem is. History has shown us sc many times that once we get to that stage, then the answer just comes out. The issue of what is the model is realiy important, and I would tike to hear something or that.

Angthar issue that has come up is that in many other fields of human activity, specifically sports, there is $2 n$ entirely different understanding about the relationship of thinking, acting and accomplishing. This is the concept of visualization. thave seen some really wonderful demonstrations of optical simulation which has been proposed as a method for doing things where you cannot sec. But, the whole issue keeps coming back to me of how we can investigate the relationship between what a person conceives and visualizes and what that person can actually accomplish. Or, turn it around in the other direction. I think that we have not yet begun to scratch the surface of this very interesting area.

A third area, which may be a bit sensitive, but which I feel strong enough and bold enough to mention: what is our responsibility as sincere dedicated people to unfolding these concepts and putting them before humanity in our evolution? What is our responsibility to the morality and to the integrity of these things to which we are dedicating our lives' energy. Specifically, what is our responsibility to the furure environment in which we have to work. it is definitely an economic, financial, environment. Space station for example is going to require funds and guidance coming down from the top. We are all of the age where we have seen these projects come off from a very sound position of integrity and morality. Then, for some other reasons, the scientific results are used to address other issues.
C. A. BEEBE: Let us respond to the three lasues one at time. Does anyone want to comment on what the right model for robots in Al isf is a human the right modal for a robot?
T. B. Skit RIDAN: We probably all have something to say about that. Bach of us cannot get out of his own skin. So, to some extent, our model of the world starts out being ourselves. There is no other way it can be, for starters. The better we deline and understand a problem, the less the thing we build to solve the problem looks like ourselves. This has been generally true in human technology as it has evolved over the centuries. It seems to me that we always start out anthropomorphically. But the better we understand it, the less it looks like a human because we refine the technology to serve that particuler need. At some point, it may stop looking like a human. In so far as it is so robust that it handles things that we cannot anticipate as human beings, we get back to our unexpected event, and there is no solution. My feeling about unexpected events, to come back to that for a moment, is that there is an expectation of zero on the density function. It involves infinite information, and there is zero apprehension on the part of the human. We are always in this sort of scale, starting from ourselves and what we totally expect as humans, and sliding toward specific technology for specific tasks as we understand them.
C. WEISBDN: I would take the position, as an engineer, that we are trying to get a job done. Alternatively, one could take the classical Al approach that aims to study how humans reason and think. I do not think that there is a right or a wrong answer. However, I would clearly come down on the side of getting the job done. I am not sure if you were asking whether the physical object that gets the job done should have anthropomorphic characteristics, or that the cognitive aspects should have anthropomorphic characteristics, or both. I think that we would be doing ourselves a disservice if we insisted that robots have only two arms, that they be able to lift only a certain amount weight, etc. - In order just to follow the human model. One could argue that, since most of our current plants are built for humans, we ought to build robots that look like humans because they would fit into current plants. This might be short-sighted. I would therefore come out against the view of physical anthropomorphism and the view of exclusively confining ourselves to paradigms of human reasoning. Where you can solve differential equations and accurately get the right answer, this should be the spproach. To take the view that we should reason with heuristics because people do, is not always the best approach, in terms of getting the job done. If you are trying to study human beings, then the context is different and it would make sense.
G. SARIDIS: I would like to bring up an example that I encountered when I was in West Germany last summer. There was a project about plugging some holes on the body of a Mercedes-Benz with rubber plugs. Humans used to do that, but it was to cumbersome and unhealthy. So, they decided to try to replace the human. They did a study to understand if a vision system could be used to center and align the arm to the hole and if an arm control system could then be used to push the arm into the hole. They were very unsuccessful. They replaced that with a magnetic scanner, and they were able to do the task with only one attempt. This proves the point that $C$. Weisbin made.
R. BAJCSY: If $I$ assume that in your question you are equating model with representation, then I wish to make a very strong statement. There is not one unique representation. There are multiple representations for anything that you wish to do. A second remark is that: I do not think that Homo sapiens should be so pretentious to think that they are always the best models for a given job. I think models should be rask-driver.
G. A. BEEBY: There is a cartoon that many of you may have seen. It shows a human beins whose function it was to push the emergency bution in case of a problem, and he was replaced with an anthropomorphic looking robot with its finger ready to push the same bution. This is clearly not the right kind of a model.

The second question had to do with the relationship between thinking and accomplishment. Since human beings can improve their tennis by playing inner tennis in the sense of using visualization to improve their performance), can robotics be improved by using a similar approach?
C. WBISBIN: Speaking as a rather poor athlete, who is uniformly poor in a variety of sports, l belleve that athletes improve performance through rote practice without necessarily visualizing each detatled step. When you throw a bowling ball 27 times, or 2000 times, after a while it becomes a routine. If you are trying to get a kill shot in racket-ball, you do not necessarily do that very well through visualization, you do it pretty much by rote after significant amounts of practice. I make no scientific claim that this is true. It would be quite reasonable to take some intelligent machine for example, and have it practice in a variety of related configurations and try to improve skills through perturbation and credit assignment reward, rather than differential model bullding. That would get us into the area of learning. I think the athlete analugy is more one where humans improve pretty much through repetition.
A. SANDERSON: Let me propose one possible scenario. You could think of a simulation as a kind of visuallzation in which the robot does not actually execute the command physically but could execute parts of it in conjunction with simulating other parts. In fact, it might be able to practice and learn without actually doing the execution itself. Some of the psychology that goes with the visualization approach says that humans may be able to do that.
G. A. BBEBY: ls any one willing to deal with the third and more difficult question, the one that deals with responsibility?
T. B. SHBRIDAN: Pirst, I wanted to commend you for asking such a question, because it is terribly important. It is the kind of question that is so embarrassing to talk about, that we do not talk sbout it at scientific meetings like this. I think however that it is ultimately very important in AI and robotics. One of the reasons that we behave ourselves in the world is that, when we start encroaching on the rights of other people, we put our own bodies, in effect (personal reputations, health, etc.), on the line. Once we are able, in a telerobotic sense, to guide or to initiate the actions of semi-autonomous devices that can go out and creep around the world, and make trouble in the backyards of others, it will be very easy to abandon responsibility and to claim that the driver is not us. In fact, responsiblity will be increasingly more difficult to trace back. We are getting into a territory here which is fraught wirh social problems. it is also my belief that SDI is the tip of that iceberg. I hope sincerely that NASA does not get in that business.
G. A. BBKBY: $I$ think it is also important to know, however, that much research in robotics, as G. Saridis pointed out earlier, is concerned precisely with removing humans from some of those jobs which are the most hazardous to them, whether it is in mines, or in the presence of hazardous chemicals, etc. I think that clearly is an issue in which rototics is contributing to human welfare.
G. SARIDIS: I think there are two questions involved in that. One is our responsibility in creating machines that might be used by others in detriment to humanity. Second, suppose that we create intelligent machines, and these machines get out of control. Will these machines have the built-in potential to create some kind of
chaos? Not because the humans ordered them so, but because of themselves. This type of hell is reminiscent of the movie 2001 Odyssey. We should heep this in mind as we aim toward intelligent machines. I think however, that education can solve the problem. It is a matter of developing social responsibility through education. We have, in the past, been very successful in creating instruments that can be used either to help humanity or to destroy humanity. Take the simplest of things, as an example. Take a glass, a small glass, that you can use to drink water. I can break that and cut somebody's throat. I can use that as a weapon. Unless I control myself, and I have to be educated to do that, I can use even the simple things as weapons. The second problem is much more serious, and I do not have an answer for it. But then,... I do not know either if we are going to ever create intelligent machines.
L. MARTIN, Telerobotica laternational: I would like to atate an alternative view on anthropomorphic kinematics. The thoughts behind anthropomorphic kinematics is that we are trying to replace a man in an BVA atmosphere trying to do a variety of tasks. No doubt there are machines that can do apecific tasks (such as punching buttons, plugging holes, etc..) in a much more simplistic manner. To be able to replace a man in that environment, with the design standards that are already in place for NASA to be used in satellites that will be put in place decades from now, we must look at machines that replicate the kinematic characteristics of the human (in size, lift capacity, etc.). I would like to make one observation in general. I think there is not a whole lot of hardware work being done at the present time. I would like to take you back about eighty year: by analogy. I would challenge this group, that if eighty years ago you had the challenge that the Wright brothers had, you would be designing the automatic pilot before designing the airplane. I would suggest the more pragmatic approach.
G. A. BBEBY: Thank you for your comment. That will not go unanswered here.
G. SARIDIS: Well, lam for parallel processing, instead of serial processing. The question was whether we were going to do that serially or in parallel. We do both research and experimentation, in parallel. If we were smart enough ten years ago, when the Japanese announced their automation program, and we were doing basic research as they did, our industries would not be in the bind that they are in now. We are trying to catch-up and establish a more competitive role with their industry.
R. BAICSY: I am sorry you got this impression. I am in academia and we do very hard hands- on experimentation. So do most of my colleagues in robotics. While we are doing hands- on experimentation, we have to look ahead. We have to look for generic problems, for problems that can be generalized. We have to look for principles. We have to make contributions to the general knowledge in robotics. While we are doing that, we are also testing on concrete examples. We are not avoiding doing hands-on experiments.
J. Y. S. LUH: That is a very good answer. I just want to add specific comments abou: the issue of kinematics. The problems of kinematics is well studied, and there are many results. The important topic currently is that of redundancy. The redundant kinematic robot has been studied by many researchers. There are to my knowledge anywhere betwees twenty to thirty papers, several of them even in journals. However, more experimentation is required.
G. A. BBEBY: I can also make one observation of my own. My colleague, Professor R. Tomovic of Yugoslavia, and l are in the process of not only studying but building a five-finger anthropomorphic hand for robotic applications. With any luck at all, you will see it at the forthcoming IEES Robotics Conference in Raleigh. But, at the same
time, I am very supportive of the efforts of my colleague Professor Lee, (who is sitling out here) who has designed a hand with two thumbs. Some of us may be all thumbs, but his is meant to be an altemative robotic hand which is not at all anthropomorphic. So, I think we need to keep the principles that various people have brought here all in mind. I think you would be surprised, back to the questioner, how much hardware work goes on. And, if we had the resources to do more prototyping and experimentation, we would do a lot more. Developing prototype hardware is very expensive, and most of us simply do not have the resources. But this is not from a lack of desire to do $s o$.
D. PIVIROTTO, JPL: I wanted to touch on the point on why there were 30 few papers on parallel processing. One of the things that 1 have been doing is to search for someone who is actually putting together flight computer systems that we might be able to use for space robots. There is no one, as far as I can determine. Do you have any knowledge about research in computer syateras for robotic applications in space? As probably everybody knows, space computers have to be reliable, fault-tolerant, radiation protected, etc. You cannot just take a PC and fly it. I also notice that there does not seem to be whole lot of work in computational systems for practical telerobotics applications in space. There was a paper by R. Travasos, at a recent robotics conference, who tried to address the problem of linking up different kinds of computers to solve the overall telerobotics problem. I have not seen very much of that going on. This work is interdisciplinery. I was wondering whether the panel could shed any light on this. Are there research programs of that nature?
A. SANDERSON: I cannot say that I know of a real effort at apace-hardened multi or parallel processing. The efforts at the research level tend to be at architectures for applications, and there are selected prototypes that have been built. At CMU, there is a WARP processor, a systolic array machine, which is being used on a mobile vehicle. It resides in the mobile vehicle. It is being used for low-level image processing. It is being used as part of the autonomous system. I do not know of an example which includes all of the aspects needed for space-flight applications.
G. A. BBKBY: It is very simple. Just get a space-hardened connection machine.
D. PIVIROITO: Actually, the DARPA machines appear to be based on the following approach. People have a theory of what makes a good AI processor. Somebody else then has a theory about how to link machines together to do processing. Actually, none appear to meet the anticipated performance. I have heard papers on several machines. None of them are designed from the point of view of defining a system that we are trying to make work, and putting together a complement of computers to do the job. I think that people get seduced into thinking that, if we have the connection machine, all of our problems will be solved. From the papers I have seen, that does not sound right.
C. WBISBIN: What I had in mind when I brought up that question was that, if you attended many of these sessions, you would see a myriad of boxes: vision, planning, control, etc.. Many of the boxes would tee running in parallel. The implication is that there is research on-going in which such an implementation is being considered. Without beginning to consider space-hardening or anything like that, our limited experience with a very narrow class of machines indicates to ine that there is significant potential out there in hardware. However, there is a major problem in software. If you had ten, a hundred, a thousand processors, the scheduling problem of how to allocate tasks to processors, and to determine how they would communicate, is a very legitimate research area. My comments were that I expected to see more papers here in this area. Regarding your question about whether anyone is to the stage of fielding something (and I am not even touching space now), there is one military project that I am aware of where that is being considered. Clearly, the conditions that you
would have in space would be quite different. In any event, I think it is premature. I do not think that we currently know how to effectively schedule and determine the communication modes in general for an arbitrary number of processors.
J. C. HORVATH, JPL: I am from the Hypercube group at JPL. I would lke to respond collectively to all of this. There has been a project underway for quite a few years at JPL and Caltech to do parallel processing. We have built hardware which is meant and designed specifically to be space-qualifiable, eventually. Sometimes we heve been criticized for being too simplistic fust because we are trying to stay simple, so that we can deal with real-world problems. We have a substantive effort ongoing in software and have solved quite a few real problems. I am here at this conference mostly to raise interest within the robotics community in using the Hypercube. So, we are ready if you are (Laughter).
C. WBISBIN: I would like 20 ask a question of the speaker. I applaud the effort in using Hypercubes. It is fully consistent with what we are doing. I just want to know whether you disagree with the statement I made that, as far as I knew, task allocation and scheduling of a variety of tasks (planning, manipulation, control, sensing, etc.), with real-time feedback, is something that is still in the research stage and not ready to go. Would you disagree?
J. HORVATH: I would hesitate in this forum to say yes or no. I know that there are other members of our group that want to do $i t$, but I do not really know how far along they are. I would have to refer you to them.
A. GOPORTH, NASA ARC: There is a program at Ames that is developing a apace-borne symbolic processor. You might see a paper in a year or so about its applicability to telerobotics. We considered it for the strawman design of the Plight Telerobotic Servicer. Yes, there is work on processors targetted for space. It is rather premature to present papers, when the details of the hardware have not yet been worked out. The big problem for a space-qualified system is the power consumption. If you want to use a parallel processor, the real problem is the bus or the interconnections. That uses a lot of power. So, a lot of power is required to support parallelism. I am not saying that it cannot be done, but that is the real tall pole. With regard to the issue of tasking spftware for parallel operations, it is true that no one knows how to $\therefore$ 'o that. But, for pure master-slave teleoperations, that is easy. It may be as easy as taking what is being done with VAX machines in laboratories and moving it into space-qualified equipment.
G. A. BBKBY: Let me suggest that it may be a good time to stop and perhaps continue on an individual basis. Let me summarize in a few sentences what I think has happened here. This has been an extremely enlightening discussion, at least for me. It is interesting to see how some of the issues came up again and again in the conversations.

- Many of the issues involve how we model the world. How do we extract information from it? Do we do it by using sensors? What kind of abstract models do we use for representation? Then, there is the issue of how knowledge is represented, transmitted and used for the design of systems.
- There is a question of where the intelligence resides, and the relation between building intelligent machines and, conversely, using machines which in some sense imitate intelligent behavior. Those two things may or may not be equivalent.
- The question of uncertalaty came up again and again - ways of modeling it, of detecting it, and incorporating it in systems.
- The question of evaluation and validation of systems, particularly man-machine systems, came up several times. It is clearly a very important one, if we are going to be able to assess our successes or our fatlures.
- The above issues of modeling and representation, intelligence, uncertainty, evaluation, and valldation are all very closely related to those of planning and scheduling.
- I appreciate the comment by Dr. Sanderson that robotics is an experinental science, and that there is a great deal to learn by experimentation.
- Pinally, we should not forget that we do in fact have a responsibility to humanity and to ourselves.

One of the panelists said that one of the important aress is to improve communication among researchers. This conference has contributed admirably to that gosl. I would like to thank Dr. Rodriguez for organizing it and bringing us all together.

# SPACE TELEROBOTICS WORKSHOP 

# FINAL PROGRAM (UPDATED TITLES FOR PROCEEDINGS) 

January 2.0-22, 1987 The Pasadena Center Pasadena, California

Jet Propulsion Laboratory California Institute of Technology

National Aeronautics and Space Administration

## PROGRAM COMMITTEE

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Program at a Glance*

|  | LOEBY | Registration |
| :---: | :---: | :---: |
|  | $\operatorname{LITILE}_{\text {THEATRE }}$ | Plenary Session |
| $8: 50$ | $\begin{aligned} & \mathrm{C} 101 \\ & \mathrm{C} 102 \\ & \mathrm{C} 103 \\ & \mathrm{C} 104 \end{aligned}$ | Concurrent Sessions <br> WAI: System Architecture III WA2: Manipulator Conerol III WA3: Planning \& Schcduling: Basic Research \& Tools 1 <br> WA4: Sensing \& Perception I |
| 1:15 |  | tunch |
|  | $\left\|\begin{array}{\|c\|c\|c\|} \text { LHEAREE } \end{array}\right\|$ | Plenary Session |
| 2:00 | C101 <br> C102 <br> C103 <br> Cl 104 <br> C105 | Concurrent Sessions <br> WP1: System Concepts <br> WP2: Manipulator Control N' <br> WP3: Planning a ScheJuling: Basic Research \& Tools II <br> WP4: Sensing \& Perception II WPS: Telerobots |
| 6:20 |  |  |
| 6:30 | $\begin{gathered} \text { HITON } \\ \text { INTL } \\ \text { BALIROOM } \end{gathered}$ | No-Host Reception |
| 9:00 | $\begin{array}{\|c\|} \hline \text { HIITON } \\ \text { INTI } \\ \text { BAIROOMA } \\ \hline \end{array}$ | Banquet |


TUESDAY
January 20, 198


[^17]Technical Sessions, Tuesday Morning, January 20


## SESS1ON TA MAN/MACHINE INTERFACE I

Chair: T. B. Sheridan. Meseechusetts Inalliute of Technolony

10:00-10:40

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K. Sellebury, Meseechumetia Indtitute of Tachnolowy

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C. Crangle and P. Suppes. Stanford Univaratity S. Michalowski. VA Medical Center. Palo Alto

12:00-12:20
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EVA Prichomere*
I. Letfier. I. Jemmeson. M. LeBlanc, D. Wilson. E. E. Sebelman. and D. Schwandi. Stanford Univeraity

## Technical Sessions, Tuesday Afternoon, January 20

| SESSION TP1 <br> SYSTEM ARCHITECTURE II | Room Ciol | SESSION TP2 <br> MANIPULATOR CONTROL II | Roum C102 | SESSION TP3 TRAJECTORY PLANNING FOR MANIPULATORS | $2 \mathrm{om} \mathrm{Cl}^{\text {cor }}$ |
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1:30 pm - 4:30 pm
Chatr: C.A. Bakey, Univeraliy of Southarn Callfornia
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Chalr: S. Dubuwaky, Masechumette Indilute of Technouluny
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O. Khatib, Slanfurd University

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Manipulators, Part II: Adaptive Cam
D.S. Bayard and f.T. Went, Jen Propulaiun Laboratory

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A.C. Sonderson. M.A. Pegtin. and
L.S. Homem-do-Mello. Carmegie-Mellon University

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P. U'tonmell. and P. Tuurnesmoud. Maseechusetis
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$5: 00 \mathrm{pm}-5: 30 \mathrm{pm} \quad$ Bus to JPL $\quad$ Visit to IPL
SESSION TP4 Room cIU SESSHON TPS Room CIOS
MAN/MACHINE INTERFACE II SIMULATION AND CONTROL

Cheir: T.B. Sheridan. Meceechueetts Imelitute of Technoloty
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G.C. Borchardt. Jer Propulsion Laboretory

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I.W. Jamemoa, Stanford Univemuly

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D.G. Bitha and T. C. Hale, Univeraily of California a Davis
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S. Hayali and B. Wikcon. If Propulaton Laboratory

1:30 - 4.30
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Technics: Sessions, Wednesday Morning, January 21

| 7:30 am - 8:00: am | Registration |
| :--- | :--- |
| 8:00 am - 8:45am | PLENARY SESSION - Lirte Theatre <br> Peter Friediand, Ames Research Center <br> Building Intelligent Systems - AI Research at NASA Ames |
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| sesshon wal <br> SYSTEM ARCHITECTURE III | anom C101 | SESSION WA2 <br> MANIPULATOR CONTROL III | noom C102 | SESSHON WA3 <br> PLANNING AND SCMEDULING EASIC RESEARCH AND TOOLS I | Moom Cios |
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SENSING AND PERCEPTION I

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11:30-12:10
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T. Kanede, Conmetomellon Univertity

## Technical Sessions, Wednesday Afternoon, January 21



6:30 pm - 7:00 pm
No-Host Reception, International Ballroom, Hilton Hotel

7:00 pm - 9:00 pm
Banquet, International Ballroom, Hilton Hotel

| SESSION WP4 | Room cion | SESSION WP5 | 200m Cios |
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| SENSING AND PEIICEPTION II |  | TELEROAOTS |  |

Chatr: T. Kagade, Carnegio-Mallon Uaiversity
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R. Belcsy and S.A. Scemefleld, Univerility of Pennalyanls
2:40-3:10

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P. Bech-y-是ta, J.C. Webeter, and W.J. Tomplina,

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T. Crabb. Astronaulice Corp. of America

3:10-3:40

U. Orginer. Ohio State University

3:40-4:00
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D. Cosemant, Carnegle-Mallon Univeraity

4:00-4:40
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Y. Hung, B. Cernuechi-Frins, and D.B. Cooper.

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K. Hoynus, and N. Murtay. Honnywell Symterne and

Research Center
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J.A. Sloan and S. Udomkemalee, Purkin-Elmor Corporation
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Chair: 1.E. Puanington. Langley Reoearch Cemter
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L.K. Barker and D. Soloway, LaRC

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D.R. White, MTS Syatema Corporation

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D.T. Lawtoa. T.S. Levili, C.C. McConnell. and
D.T. Lawton. T.S. Levila, C.C. McConnall. and
P.C. Nalson, Advanced Dectaion Syatems

5:00-5:20

A. Kumar N. and N. Permenwaran. Indian Inatitute of

Tochnology
5:20-6:20
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Technical Sessions, Thursday Morning, January 22

| 7:30 am - 8,00 am |  |  |
| :---: | :---: | :---: |
| 8:00 am - 8:45 am | N - Little Theatre <br> n, Langley Resaarch Center ley Space Telerobotics Program | - . |
| 8:45 am-9:00 am |  |  |
| SESSION THA1 <br> SYSTEM ARCHITECTURE IV | SESSION THA2 <br> Room C102 <br> MANIPULATOR CONTROL V | SESSION THA3 <br> Room C1Os <br> AUTONOMOUS SYSTEMS |
| 9:00 am - 12:00 noon |  |  |
| Chair: A. Meydel, Denxal University | Chair: A.f. Kotvo. Puadee Untworaty | Chatr: H. Montvec, Carnegr-Malloa Univeretity |
| 9:00-9:40 | 9:00-9:40 | $0: 00-0: 40$ |
|  C.N. Saridst, Resuaplear Polytechnic Inatitute K.P. Velavania, Noetheestern Ualveraity |  <br> Botween Two Indertilal mahere <br> J.Y.S. Luh and Y.F. Zheng. Clemeon Univerathy |  <br>  <br> M. Coldmeta. F.G. Pis, G. de Fevomurs and C. Weisbta Oak Ridge Nathonal leboratory |
|  | $9: 40-10: 00$ |  |
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|  |  | N.S.V. Reo. S.S. Iyongar and C.R. Weichin. Louiaina State Univeralty |
| Intrapetion <br> R. Brown, C. Coht and C. Sablonaki, Johneon Spece Center <br> D. Bochalof, LinCom Corpotation |  M.L. Agronin. Jot Propulstion Leboratory $10: 20-10: 40$ <br>  | $10: 20-10: 40$ <br>  Partielly Ocectedad Obiecte* <br> H. Wechaler, Univeralty of Minceede |
|  | Operselme | 10:40-11:00 |
| J.S. Kwhn and E.D. Sallo. ORNTEC | W.T. Miller. II, University of Now Hampahise $\begin{aligned} & \text { 10:40 - 12:00 } \\ & \text { Pamal Diecmelen } \end{aligned}$ |  M. Dudziak, Martin-Merietta Baitimone Aeroeppece 11:00-11:20 |
| 12:00-12:20 <br>  M. Thomas and W.J. Wolfo. Martin Marietta Denver Auroepece |  |  <br> C. Peanon, FMC Corpontion 11:20-12:00 <br>  |
| $\begin{aligned} & \text { 11:20-12:00 } \\ & \text { Pamel Dlecmelon } \end{aligned}$ |  | H. Moravec, Cennegio-Mellon University |

-No paper raceived for procsedinge

12:00 noon - 1:30 pm
Lunch Break

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SESSION THAA

SENSING AND PERCEPTION III

9:00 am - 12:00 noon
Chatr: B. Wilcex. It Propulation Lebormiony
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11:40-12:00

H.D. Chens and L.T. Kou. Univwily of Calitornie
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\section*{Technical Sessions, Thursday Afternoon, January 22}
\begin{tabular}{ll} 
1:30 pm - 3:30 pm & Panel Discussion - Little Theatre \\
& Future Research Directions
\end{tabular}

Future Research Directions
Moderator: G. A. Bekey, University of Southern California
The intent of this session is to provide views on the state of telerobotics research and to draw together d collection of suggested research opportunities to present to NASA. The panel members will give opening statements summarizing the results of earlier sessions and disrussion periods. This will be followed by a general discussion with the audience. The moderator will conclude the session by stating a synthesized set of recommendations.

3:30 pm - 5:30 pm Closing Reception, The Pasadena Center Lobby

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[^0]:    Research conducted under the McDonnell Douglas Independent Research and Development Program.

[^1]:    I We use the terms "action" and "even'" interchangezoly, as convenient.

[^2]:    - Also antiated with the Center for the Study of Language and Information, Stanford University. Stanford. California.

    This research has been mode prasible in part by a git from the System Development Foundation, the Office of Naval Research under contract no. Nocoitias-C-025i and FMC under contract no. FMC.147466.

[^3]:    ${ }^{1}$ The symbol $\epsilon$ is meant to denote an infinitesimal: a number greater than 0 and smaller than any positive number.

[^4]:    ${ }^{2}$ An alternative formulation deacribed in $[3]$ states that the antecedent conditions of a projection rule must be true throughout the trigger event rather than true just at the outset. Both formulations are aupported in the TDBMs, though we will only be discussing the true-at-the-outset formulation in this paper.

[^5]:    ${ }^{1}$ Research supported by Air Force Office of Scientific Research grans N2 025.5, National Science Foundation grams MCS-alo5032, and Defense Advanced Research Projertn Agency, RADC Contract
    
    

[^6]:    The research reported here in supported by Air Fore e Office of Scientinc Research Contract F49820-72-C.0188.

[^7]:    This research was sponsored, in part, by the National Science Foundation under Grant MCS-8306327, by the National Science Foundation under Support and Maintenance Grant DCK-831bïif, by the National Science Foundation under CER Grant DCR-8500.332, and by the Defense Advanced Research Projects Agency (DOD), monitored by the Office of Naval Research under Contract NR 040-041.

[^8]:    tin general terms, an intermediate goal in any interpretation task is to process a new piece of information and to integrate it into the current partial incerpretation.
    'If the predicted cost of satiafying in intermediate goal deviates substantially from the crude estimate based on the abstract view. the ordering of the intermediate goals may need to be revised.

[^9]:    'In fact the turn to $R$ : exceeds these :onstrants, as does the surn to $R_{?}^{\prime}$, so that the galy track that tatufies the constrants is $R_{1} R_{2} R_{3} R_{4} R_{5} R_{5}$.

[^10]:    ${ }^{4}$ In fact, the WORD-SEQ tnowledre source in the Hearsay-II speech enderstanding system easentially a a clustering mechanam: by applyiag meak arammatical coastranis about parwue sequencet of words. WORD-SEQ generated approximate word sequeaces sotely to controd the application of the more expensive PARSE KS that applied full grammatical conatraints aboat eequences of arbitrary leagth il

[^11]:    ${ }^{1}$ Hanafusa and Asada made use of a apring loaded band to provide compliznce between the workpiece, the manipstator, and the environment but did not directly address the contact problem:22).

[^12]:    ${ }^{2}$ This was graphically demonstrated by Dan Whitney at a conference in which he marched, arm rigidly outstretched, towards a an unknown wall. Without the compliance of a bent arm (to provide mechanical compliance) he would not have been able to react fast enough (regulator) to avoid hurting hinself on contact

[^13]:    - In this paper. "position" teplies position and orioatation and "forco" inplies force and torque.

[^14]:    1/ Although it possesses some light structural damping and is affected by joint friction, we shall designate the untreated arm as "undamped".

[^15]:    - This work is supported in part by the National Science foundation under grant no. DMC-6505 100

[^16]:    ${ }^{1}$ We will omit the prior superscript, 0 , when it in clear the the coordinate ayatem of reference is the base
    ${ }^{2}$ Assume that $x_{0}$ "points up" in a direction opposing the gravitational field.

[^17]:    
    플
    $5: 01$
    $5: 30$
    $7: 01$
    8:01)

