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Algorithms for Adaptive Control of Two-Arm Flexible Manipulators Under Uncertainty

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1. Abstract

~~The paper uses~~ a nonlinear extension of model reference adaptive control (MRAC) technique to guide a double arm nonlinearizable robot manipulator with flexible links, driven by actuators collocated with joints subject to uncertain payload and inertia. The objective is to track a given simple linear and rigid but compatible dynamical model in real, possibly stipulated time and within stipulated degree of accuracy of convergence while avoiding collision of the arms. The objective is attained by a specified signal adaptive feedback controller and by adaptive laws, both given in closed form. A case of 4 DOF manipulator illustrates the technique.

2. Introduction

The MRAC technique becomes popular proposition for guidance of recent robot manipulators, with demand for precision pointing in difficult conditions, under the action of full scale dynamic forces, and subject to uncertainty in parameters. Such manipulators, particularly these used on spacecraft are highly nonlinear and nonlinearizable structures (geometric nonlinearity of elastic links, large angle articulation, nonlinear coupling of DOF's, nonignorable gyro and Coriolis forces, several equilibria), while classical MRAC is linear and applicable to rigid bodies only. Thus the extension is needed for handling nonlinearity, see [1], and flexible links, see [2]. On the other hand many robotic objectives, again particularly these in difficult space conditions require at least two arm systems. Thus the tracking has to be a double MRAC (mutual reference adaptive control) which secures tracking the same model by two arms while avoiding mutual collision - cf. [3], [4]. If adaptive (self-organizing) control is intended, the tracking relates not to a given path but to a given dynamic target-model with prescribed target-parameters. We take the model simple thus rigid and linear, but locally compatible with the nonlinear arms regarding equilibria. Each arm is represented as an open chain with n DOF, nonlinear characteristics and coupling, elastic links, driven by actuators collocated with joints, under uncertain inertia parameters and uncertain payload. The tracking is done in real possibly stipulated time by a designed signal adaptive feedback controller and integrable adaptive laws in the state space, while avoiding collision between arms of all the joints (and elastic nodes) in Cartesian configuration space.

3. Motion Equations

Lagrange motion equations give the rigid dynamics of the arms in the general format

$$A^j(q^j,s^j)\ddot{q}^j + \Gamma^j(q^j,\dot{q}^j,\lambda^j) + \Pi^j(q^j,\lambda^j,s^j) = B^j(q^j,\dot{q}^j)u^j \ , \quad j = 1,2 \ , \tag{1}$$

where $q^j(t) \in \Delta_q \subset \mathbf{R}^n$, $t \geq t_o = 0$, is the configuration vector of the joint variables $q_1^j,\dots,q_n^j$ of the j-th arm varying in the known bounded work region Δ_q of the configuration space $\mathbf{R}^n$; $\dot{q}(t)$ is the corresponding vector of joint velocities in the specified bounded subset $\Delta_{\dot{q}}$ of the space tangent to $\mathbf{R}^n$; $u^j(t) \in U \subset \mathbf{R}^n$ are the control vectors in given compact set of constraints U ; $\lambda^j(t) \in \Lambda \subset \mathbf{R}^\ell$, $\ell \leq 2n$, are the vectors of adjustable system parameters in bounded bands of values Λ , and $s^j(t) \in S \subset \mathbf{R}^k$ is an uncertainty parameter within the known band S . Moreover $A^j(q^j,s^j)$ are the inertia n×n matrices obtained in the known way from the quadratic form of kinetic energy. The vectors $\Pi^j = (\Pi_1^j,\dots,\Pi_n^j)^T$ represent potential forces (gravity, spring) while $\Gamma^j = (\Gamma_1^j,\dots,\Gamma_n^j)^T$ represent the internal nonpotential acting forces (Coriolis, gyro, centrifugal, damping structural or viscous, etc.) and B^j is the actuator transmission (gear) nonsingular n×n matrix. The control vectors $u^j(t)$ are selected for the objectives of tracking and avoidance by adaptive feedback control programs $u^j(t) = p^j(q^1(t),q^2(t),\dot{q}^1(t),\dot{q}^2(t),\lambda^1(t),\lambda^2(t))$ on corresponding products of $\Delta_q \times \Delta_{\dot{q}} \times \Lambda$. For convenience the superscripts "j" will be dropped until they are needed to avoid ambiguity.

Considering the links elastic we introduce the deformation coordinates for the i-th link as shown in Fig. 1, while using the Ritz-Kantorovitch series expansion

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$$r_i(y_i,t) = \sum_{\nu=1}^{m} r_i^{\nu}(y_i) r_i^{\nu}(t) = r_i(y_i) r_i(t) \tag{2}$$

and for $v_i(y_i,t)$, $w_i(y_i,t)$ analogously, with the exact solution expected for $m \to \infty$. We take m large

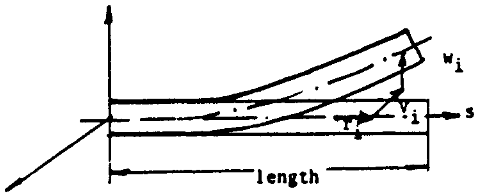

Figure 1. Flexible link

enough so that the Kantorovitch linearization is physically justified. The technical way about it is to stepwise subdividing the links between grid as long as the difference of results for successive m's becomes small. Having specified (2) we form the vector $\eta(t) \overset{\Delta}{=} (\eta_1(t),\dots,\eta_n(t))^T$, where $\eta_i(t) \overset{\Delta}{=} (r_i(t),v_i(t),w_i(t))^T$ and following [5] write the hybrid system as

$$\begin{pmatrix} A & A_c \\ A_c^T & A \end{pmatrix}\begin{pmatrix} \ddot{q} \\ \ddot{\eta} \end{pmatrix} + \begin{pmatrix} 0 & D_c \\ 0 & D \end{pmatrix}\begin{pmatrix} \dot{q} \\ \dot{\eta} \end{pmatrix} + \begin{pmatrix} 0 & P_c \\ 0 & P \end{pmatrix}\begin{pmatrix} q \\ \eta \end{pmatrix} + \begin{pmatrix} \Gamma(q,\dot{q}) \\ \Gamma_\eta(\eta,\dot{\eta}) \end{pmatrix} + \begin{pmatrix} \Pi(q,\lambda,s) \\ \Pi_\eta(\eta,s) \end{pmatrix} = \begin{pmatrix} B(q,\dot{q}) \\ 0 \end{pmatrix} u \tag{3}$$

where $A_\eta(\eta,s)$, $\Gamma_\eta(\eta,\dot{\eta})$, $\Pi_\eta(\eta,s)$ are the elastic correspondents of A, Γ, Π while $A_c(q,\eta)$, $D_c(q,\dot{q},\eta,\dot{\eta})$, $P_c(q,\eta)$ and the internal damping $D(q,q,\eta,\eta)$ as well as the hybrid restoring coefficients $P(q,\eta)$ are matrices coupling the elastic and joint coordinates. These matrices are formed by integrals over the shape functions, see [5]. Letting

$$A(q,\eta,s) = \begin{pmatrix} A & A_c \\ A_c^T & A_\eta \end{pmatrix}$$

to be the hybrid inertia matrix which is nonsingular positive definite,we inertially decouple (3):

$$(\ddot{q},\ddot{\eta})^T + \mathcal{D}(q,\dot{q},\eta,\dot{\eta},\lambda,s) + \mathcal{P}(q,\eta,\lambda,s) = \mathcal{B}(q,\dot{q},s)u \tag{4}$$

where $\mathcal{D} \overset{\Delta}{=} A^{-1}(D_c\dot{\eta}+\Gamma,D\dot{\eta}+\Gamma_\eta)^T$ and $\mathcal{P} \overset{\Delta}{=} A^{-1}(P_c\eta+\Pi,P\eta+\Pi_\eta)^T$ are successively vectors of nonpotential and potential forces and the meaning of the matrix $\mathcal{B}$ is obvious. The vectors q, $\dot{q}$, η, $\dot{\eta}$ form the state vector $x(t) = (x_1(t),\dots,x_N(t))^T \overset{\Delta}{=} (q(t),\eta(t),\dot{q}(t),\dot{\eta}(t))^T \in \Delta_q \times \Delta_\eta \times \Delta_{\dot{q}} \times \Delta_{\dot{\eta}} \overset{\Delta}{=} \Delta \subset \mathbf{R}^N$, $N = 4n$, for each arm. For convenience (4) may be then written in the general state form

$$\dot{x} = f(x,u,\lambda,s) \tag{5}$$

with $f = (f_1,\dots,f_N)$ of the shape specified by (4) in an obvious way. Formally (5) may be written in the contingent form:

$$\dot{x} \in \{f(x,u,\lambda,s) \mid s \in S\} \tag{5'}$$

which for suitable $f(\cdot)$, $p(\cdot)$, $\lambda(\cdot)$ has solutions $x(t) = k(x^o,t)$, $t \geq 0$, absolutely continuous curves through each $x^o = x(0)$ in Δ . We shall consider the class of such solutions $K(x^o)$ by exhausting all values of $s(t)$ in (5) at each t .

4. The Reference Model

We let the given Cartesian "world" coordinates representation of the reference model in general terms

$$\dot{\xi}_m = F(\lambda_m)\dot{\xi} \tag{6}$$

with 2n DOF, $\xi(t) \in \mathbf{R}^{3.2n}$, and $F(\lambda_m)$ suitable matrix, be off-line recalculated to the joint coordinate format of the rigid linear system

$$\ddot{q}_m + \mathcal{D}_m(\lambda_m)\dot{q}_m + \mathcal{P}_m(\lambda_m)q_m = 0 \tag{7}$$

with the 2n-vectors q_m, $\dot{q}_m$ of joint coordinates and velocities, state $x_m(t) = (q_m(t),\dot{q}_m(t))^T \in \Delta \subset \mathbf{R}^N$, and

$\mathcal{D}_m$, P_m suitable matrices, while $\lambda_m = (\lambda_{m1},\ldots,\lambda_{m\ell}) = \text{const} \in \Lambda \subset R^\ell$, $\ell = n$. Moreover

$$P_m(\lambda_m)(q^e,\eta^e) = 0 \tag{8}$$

with (q^e,η^e) denoting the equilibria of (3) on the surface $\dot{q} = 0$, $\dot{\eta} = 0$. The total energy of the model will be denoted by $E_m(\xi_m,\dot{\xi}_m)$ in the world coordinates and $E_m(q_m,\dot{q}_m)$ in the joint coordinates, obviously equal to one another. Then

$$E_m(q_m,\dot{q}_m) = \tfrac{1}{2}\dot{q}_m^T\dot{q}_m + \int_{q_m^o}^{q_m} P_m(\lambda_m)\, d\sigma \tag{9}$$

and substituting (7),

$$\dot{E}_m(q_m,\dot{q}_m) = -\mathcal{D}_m(\lambda_m)(\dot{q}_m)^2 . \tag{10}$$

The model is selected such as to allow achieving of a stipulated target behavior in the state space. To focus attention on something specific and yet general enough, let it be stability of the origin, guaranteed by the nonaccummulation of the total energy i.e. non-negative damping

$$\dot{E}_m(q_m,\dot{q}_m) \le 0 , \quad \forall \dot{q}_m \neq 0 \tag{11}$$

while

$$\nabla E_m(q_m,\dot{q}_m) > 0 \tag{12}$$

in-the-large i.e. on same $C\Delta_L = \Delta - \Delta_L$, where Δ_L is the set in R^N enclosing all the equilibria.

5. Objectives

Now we consider both arms $j = 1,2$ and the model together. The block scheme of the system is shown in Fig. 2.

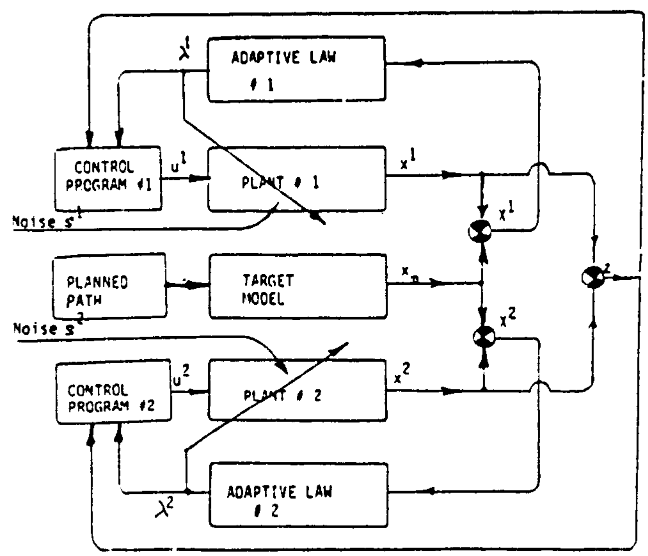

Figure 2. Block scheme of the system

Define two product 2N-vectors $X^j(t) = (x^j(t),x_m(t))^T \in \Delta \times \Delta \triangleq \Delta^2$ and two ℓ-vectors $\alpha^j(t) = \lambda^j(t) - \lambda_m$, which vary in $\Delta^2\times\Lambda$ generating the product trajectories $(X^j(X^{jo},t),\ \alpha^j(\alpha^{jo},t))$, $t \ge 0$, $X^{jo} = X^j(0)$, $\alpha^{jo} = \alpha^j(0)$. Then we define the "diagonal" sets

$$M^j = \{(X^j,\alpha^j) \in \Delta^2\times\Lambda \mid x^j = x_m,\ \alpha^j = 0\} , \quad j = 1,2 ,$$

and given stipulated $\mu^j > 0$, their neighbourhoods

$$M^j = \{(X^j,\alpha^j) \in \Delta^2\times\Lambda \mid |x^j-x_m| < \mu^j,\ |\alpha^j| < \mu^j\} , \quad j = 2 .$$

Moreover we let Δ_o be a desired subset of Δ where we want the tracking to occur, and let t_c be the stipulated time after which the tracking is attained with accuracy μ^j .

First Objective: The manipulator arms (1) are mutually μ-tracking the target (7) on Δ_o if there is a pair of controllers $p^j(\cdot)$, $j = 1,2$ such that for each solution $k^j(x^{jo},t)$, $t \geq 0$ of (4) in $K(x^{jo})$, the set $\Delta_o^2 \times \Lambda$ is positively invariant: $(x^{jo},\alpha^{jo}) \in \Delta_o^2 \times \Lambda \Rightarrow (x^j(t),\alpha^j(t)) \in \Delta_o^2 \times \Lambda$ and given t_c, for each $k^j(\cdot) \in K(x^{jo})$ the product trajectories satisfy

$$(x^j(t),\alpha^j(t)) \in M_\mu^j \ , \ \forall t \geq t_c \ . \tag{13}$$

The convergence is illustrated in Fig. 3.

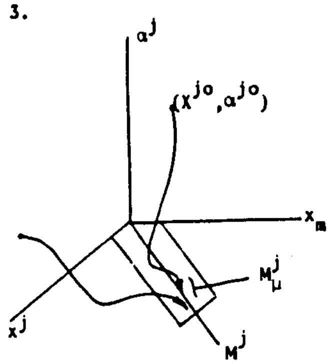

Figure 3. Convergence of product trajectories

Suppose the transformation from joint to world coordinates (forward kinematics) is given by

$$\xi_\sigma^j = \xi_\sigma^j(q^j,\eta^j) \ , \quad \sigma = 1,\ldots,3\cdot 2n \tag{14}$$

and denote $Z(t) \triangleq (x^1(t),x^2(t))$. Then we let the set

$$A \triangleq \{Z \in \Delta^2 \mid |\xi_\sigma^1-\xi_\nu^2| \leq d, \ \forall \sigma,\nu = 1,\ldots,3\cdot 2n\}$$

be the collision set between arms to be avoided. We define $CA \triangleq \Delta_o^2 - A$, specified by $|\xi_\sigma^1-\xi_1^2| > d$, and let

$$\Delta_A \triangleq \{Z \in \Delta^2 \mid d < |\xi_\sigma^1-\xi_\sigma^2| < \varepsilon\}$$

be the "slow down" safety zone, with $\varepsilon > 0$ suitable constant.

Second Objective: The tracking arms (1) avoid collision iff there is Δ_A such that for any $Z^o \in CA$, and any pair $k^j(\cdot) \in K^j(x^{jo})$ the corresponding product trajectory

$$Z(Z^o,t) \in CA \ , \quad \forall t \geq 0. \tag{15}$$

6. Sufficient Conditions

We return now to the first objective and specify by $N[\partial(\Delta_o^2\times\Lambda)]$ a neighborhood of the boundary $\partial(\Delta_o^2\times\Lambda)$ of the region $\Delta_o^2\times\Lambda$. Then let $N_\varepsilon \triangleq [\partial(\Delta_o^2\times\Lambda) \cap \overline{\Delta_o\times\Lambda}]$, $CM_\mu^j \triangleq (\Delta_o^2\times\Lambda) - M_\mu^j$ and introduce open $D^j \supset \overline{CM}_\mu^j$ such that $D^j \cap M^j = \emptyset$. Further we consider four C^1-functions $V_s^j(\cdot)$: $\bar{N}_\varepsilon \to R$, $V_\mu^j(\cdot)$: $D^j \to R$, $j = 1,2$ with the positive constants

$$\left.\begin{aligned} v_s^j &= V_S^j(x^j,\alpha^j) \ , \ (x^j,\alpha^j) \in \partial(\Delta^2\times\Lambda) \\ v_\mu^{j-} &= \inf V_\mu^j(x^j,\alpha^j) \mid (x^j,\alpha^j) \in \partial M_\mu^j \cap \overline{CM}_\mu^j \\ v_\mu^{j+} &= \sup V_\mu^j(x^j,\alpha^j) \mid (x^j,\alpha^j) \in \partial(\Delta^2\times\Lambda) \cap CM_\mu^j \end{aligned}\right\} \tag{16}$$

The first relation obviously requires forming $V_S^j(\cdot)$ from suitable $\partial(\Delta_o\times\Lambda)$ taken as its level, or conversely, forming $\partial\Delta_o$, $\partial\Lambda$ from levels of suitable $V_S(\cdot)$. In the latter case a Δ_o, Λ smaller than these desired will be the secure choice.

THEOREM 1: Objective 1 is attained if, given Δ_o, Λ, μ there are programs $p^j(\cdot)$ and functions $V_S^j(\cdot)$, $V_\mu^j(\cdot)$ such that for all $(x^j,\alpha^j) \in \Delta_o^2 \times \Lambda$,

(i) $\quad V_S^j(X^j,\alpha^j) \le v_s^j \, , \; \forall (X^j,\alpha^j) \in N_\varepsilon \, , \; j = 1,2$

(ii) $\quad$ for each $u^j \in p^j(X^1,X^2)$;

$$\dot{V}_S^j(X^j(t),\alpha^j(t)) < 0 \; , \quad \forall s^j \in S \tag{17}$$

along the product trajectories $(X^j(X^{jo},t)\alpha^j(\alpha^{jo},t))$, $t \ge 0$, $j = 1,2$;

(iii) $\quad 0 < V_\mu^j(X^j,\alpha^j) \le v_\mu^{j+} \, , \quad \forall (X^j,\alpha^j) \in \overline{CM}_\mu^j \, , \quad j = 1,2$;

(iv) $\quad V_\mu^j(X^j,\alpha^j) \le v_\mu^{j-} , \quad \forall (X^j,\alpha^j) \in D^j \cap M_\mu^j \, , \; j = 1,2$;

(v) $\quad$ for each $u^j = p^j(X^1,X^2)$ there is a constant $c_j > 0$ such that

$$\dot{V}_\mu^j(X^j(t),\alpha^j(t)) \le -c_j \; , \quad \forall s^j \in S \tag{18}$$

along the product trajectories $(X^j(X^{jo},t),\alpha^j(\alpha^{jo},t))$, $t \ge 0$, $j = 1,2$.

Remark 1: The Objective 1 holds after a stipulated $t_c < \infty$ if Theorem 1 is satisfied with c_j selected by

$$c_j \triangleq \frac{v_\mu^{j+}}{t_c} \, , \quad j = 1,2 \, . \tag{19}$$

THEOREM 2: Objective 2 is attained if Theorem 1 holds and given d there is a C^1-function $V_A(\cdot): \Delta_A \to R$ such that for the tracking pair $p^j(\cdot)$, for all $Z \in CA$,

(vi) $\quad V_A(Z) > V_A(z) \; , \quad \forall z \in \partial A$;

(vii) $\quad$ for each $u^j \in p^j(Z)$,

$$\dot{V}_A(Z(Z^o,t)) \ge 0 \; , \quad Z^o \in \Delta_A \; , \quad \forall s^j \in S \tag{20}$$

along product trajectories $Z(Z^o,t)$, $t \ge 0$.

PROOF. Suppose some $Z(Z^o,t)$, $t \ge 0$, $Z^o \in \Delta_A$ crosses ∂A at $t_1 > 0$. Then by (vi) , $V_A(Z(t_1)) < V_A(Z^o)$ which contradicts (vii).

7. Controllers and Adaptive Laws

Let us set up

$$V_S^j \triangleq E_m(x^j) + E_m(x_m) + a^j x^j \; ; \tag{21}$$

$$V_\mu^j \triangleq \begin{cases} |E_m(x^j) - E_m(x_m)| + a^j x^j \; , & (X^j,x^j) \in CM_\mu^j \, , \\ a^j x^j , & (X^j,x^j) \in M_\mu^j \; ; \end{cases} \tag{22}$$

$$V_A = \tfrac{1}{2}|E_m(x^1)-E_m(x^2)| \tag{23}$$

where $a^j = (\text{sign } x_1^j,\ldots,\text{sign } x_n^j)$, $j = 1,2$, and $E_m(x^j)$ is $E_m(\cdot)$ with x_m exchanged for x^j . Choosing N_ε in $C\Delta_L$, the character of $E_m(\cdot)$ specified additionally by (12), satisfies (i), (iii) and (iv).

To see that (vi) holds, observe that $E_m(x^j) = E_m(\xi^j,\dot{\xi}^j)$ of (6) and that increasing the distance $|\xi_\upsilon - \xi_\upsilon^2| \ne 0$ for at least one υ from its ∂A value increases the value of V_A .

To check upon conditions (ii), (v), (vii) we differentiate (21) - (23) with respect to time

$$\dot{V}_S^j(t) = \dot{E}_m(x^j) + \dot{E}_m(x_m) + a^j \dot{x}^j \; ; \tag{24}$$

$$\dot{V}_\mu^j(t) = \begin{cases} \dot{E}_m(x^j) - \dot{E}_m(x_m) + a^j \dot{x}^j \; , & (X^j,x^j) \in C^+M_\mu^j \, , \\ \dot{E}_m(x_m) - \dot{E}_m(x^j) + a^j \dot{x}^j \; , & (X^j,x^j) \in C^-M_\mu^j \, , \\ a^j \dot{x}^j \; , & (X^j,x^j) \in M_\mu^j \; ; \end{cases} \tag{25}$$

$$\dot{V}_A(t) = [E_m(x^1)-E_m(x^2)]\cdot[\dot{E}_m(x^1)-\dot{E}_m(x^2)] \; , \tag{26}$$

where

$$\dot{E}_m(x^j) = \nabla E_m(x^j)\cdot f^j(x^j,u^j,\lambda^j) = (Bu-D-P+P_m q_m)(\dot{q},\dot{q}) \; . \tag{27}$$

The brackets of the functions $\mathcal{B}$, $\mathcal{D}$, $\mathcal{P}$ dropped for clarity. Moreover $C^{\pm}M_{\mu}^{j}$ are subsets of CM_{μ}^{j} defined by

$$C^{+}M_{\mu}^{j}: \quad E_m(x^j) \geq E_m(x_m)$$
$$C^{-}M_{\mu}^{j}: \quad E_m(x^j) < E_m(x_m) \ .$$

With a suitable choice of initial states the following set of conditions imples (ii), (v) and (vii):

$$\text{(a)} \quad \min_{u^j} \max_{s^j} \dot{E}_m(x^j) \leq \dot{E}_m(x_m) \ , \ \forall (x^j,\alpha^j) \in C^{+}M_{\mu}^{j} \ ,$$
$$\max_{u^j} \min_{s^j} \dot{E}_m(x^j) \geq \dot{E}_m(x_m) \ , \ \forall (x^j,\alpha^j) \in C^{-}M_{\mu}^{j} \ ;$$

$$\text{(b)} \quad \max_{u^1} \min_{s^1} \dot{E}_m(x^1) > \min_{u^2} \max_{s^2} \dot{E}_m(x^2) \ , \ \forall Z \in C^{+}A \ ,$$
$$\min_{u^1} \max_{s^1} \dot{E}_m(x^1) < \max_{u^2} \min_{s^2} \dot{E}_m(x^2) \ , \ \forall Z \in C^{-}A \ ,$$

for $\dot{q} \neq 0$, $\dot{n} \neq 0$, $j = 1,2$. In the above $C^{\pm}A$ are subsets of CA defined by:

$$C^{+}A: \quad E_m(x^1) \geq E_m(x^2) \ ,$$
$$C^{-}A: \quad E_m(x^1) < E_m(x^2) \ .$$

$$\text{(c)} \quad a^j \dot{\alpha}^j = \dot{E}_m(x_m) - c_j \ , \quad \alpha^j \neq 0 \ , \quad j = 1,2 \ .$$

Observe that for $\alpha^j = 0$ there is no need for adaptation and that the system (4) crosses the surface $\dot{q} = 0$, $\dot{\eta} = 0$ time instantenously (vertically) so there is no need for control in view of the smoothness of trajectories. Conditions (a), (b) are called control conditions helping to design $p^j(\cdot)$, condition (c) is called adaptive, helping to design adaptive laws. Let us check that (a), (b), (c) indeed imply (ii), (v), (vii). Consider first the case $E_m(x^j) \neq E_m(x_m)$. Substituting (c) into (23) in view of (ii) we obtain $\dot{V}_S^j$ = negative terms + $\dot{E}_m(x^j)$. Boundedness of the work space necessitates the power; $\dot{E}_{\mu}(x^j) \leq 0$ thus (ii). Substituting (a), (c), and (ii) into (24) with (18), we satisfy (v) in stipulated time t_c . Note that this holds for any initial states. The case $E_m(x^j) = E_m(x_m)$ is trivial as then $\dot{V}_S^j = 3\dot{E}_m(x_m) < 0$, $\dot{V}_{\omega}^j = \dot{E}_m(x_m) - c_j \leq -c_j$. Finally we check (vii). Again first let $E_m(x^1) \neq E_m(x^2)$ and observe that (b) substituted to (26) implies (vii). The case $\dot{E}_m(x^1) = \dot{E}_m(x^2)$ is obviously trivial.

Observe that, with (10), (c) is implied by the following adaptive laws

$$\dot{\alpha}_i^j = -\text{sign}\ \alpha_i^j \left(\mathcal{D}_{mi}\dot{q}_{mi}^2 - \frac{c_j}{n}\right), \tag{28}$$

for $\alpha_i^j \neq 0$, $i = 1,\ldots,n$. Physically the solutions $\dot{\alpha}^j(\alpha^{jo},t)$ represent the model energy flux which become positive or negative depending upon where α^{jo} is located (below or above the surface $\alpha^j = 0$) thus regulating the increment of α^j to zero from anywhere outside the surface $\alpha^j = 0$.

8. Modular Double RP-Manipulator

Our technique is illustrated below on the case study of the four DOF manipulator with two arms shown in Fig. 4.

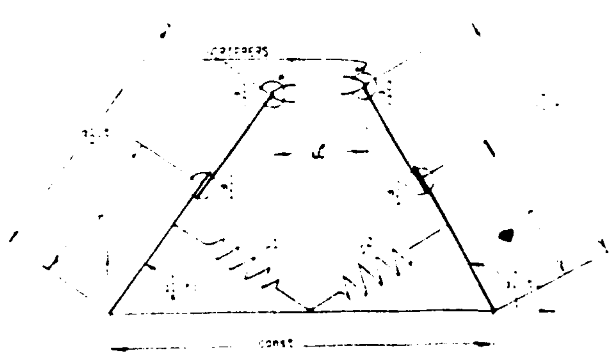

Figure 4. The modular 2-RP manipulator

The Lagrange equations of motion for each arm result in the following motion equations

$$(m_1 r^2 + m_2 q_2^2)\ddot{q}_1 + 2m_2 q_2 \dot{q}_1 \dot{q}_2 + \lambda_3 |\dot{q}_1| \dot{q}_1 + g(m_1 r + m_2 q_2)\cos q_1 - m_1 g r + \lambda_1 q_1 + \lambda_2 q_1^3 = u_1$$
$$m_2 \ddot{q}_2 - m_2 q_2 \dot{q}_1^2 + \lambda_4 \dot{q}_2 + m_2 g \sin q_1 = u_2 \,. \tag{29}$$

Here λ_3, λ_4 are damping coefficients, $\lambda_1, \lambda\lambda_2$ spring coefficients, g-gravity acceleration, the remainder of notations shown in Fig. 4. The superscripts "j", $j = 1,2$, are ignored for the time being. We take the possible payload on the grippers as unknown but within known bounds which makes m_2 specified by

$$\underline{m} \le m_2 \le \overline{m} \,,$$

where $\underline{m}$, $\overline{m}$ positive constants. Allowing $\sin q_1 = q_1 - \frac{1}{6}q_1^3$, $\cos q_1 = 1 - \frac{1}{2}q_1^2$, and subdividing the equations (29) by corresponding inertia coefficients we obtain:

$$\ddot{q}_i + \Gamma_i + \Pi_i = \beta_i u_i \,, \quad i = 1,2 \tag{30}$$

where

$$\left.\begin{aligned}
\Gamma_1 &= \frac{2m_2 q_2 \dot{q}_1 \dot{q}_2 + \lambda_3 |\dot{q}_1| \dot{q}_1}{m_1 r^2 + m_2 q_2^2} \,, \\
\Gamma_2 &= -q_2 \dot{q}_1^2 + 1/m_2 \, \lambda_4 \dot{q}_2 \,, \\
\Pi_1 &= \frac{\lambda_1 q_1 - \frac{1}{2} g m_1 r q_1^2 + \lambda_2 q_1^3 - \frac{1}{2} g m_2 q_2 q_1^2 + g m_2 q_2}{m_1 r^2 + m_2 q_2^2} \\
\Pi_2 &= g q_1 - \frac{1}{6} g q_1^3 \,, \\
\beta_1 &= \frac{1}{m_1 r^2 + m_2 q_2^2} \,, \quad \beta_2 = \frac{1}{m_2}
\end{aligned}\right\} \tag{31}$$

The reference model is taken as

$$\left.\begin{aligned}
&\ddot{q}_{m1} + \lambda_{m3}\dot{q}_{m1} + \lambda_{m1} q_{m1} + g q_{m2} = 0 \,, \\
&\ddot{q}_{m2} + \lambda_{m4}\dot{q}_{m2} + g q_{m1} = 0 \,.
\end{aligned}\right\} \tag{32}$$

The total energy of the model is

$$E_m(q_m, \dot{q}_m) = \tfrac{1}{2}(\dot{q}_{m1}^2 + \dot{q}_{m2}^2) + \tfrac{1}{2}\lambda_{m1} q_{m1}^2 + 2g q_{m1} q_{m2} \,. \tag{33}$$

Differentiating it with respect to time and substituting (32),

$$\dot{E}_m(q_m, \dot{q}_m) = -\lambda_{m3}\dot{q}_{m1}^2 - \lambda_{m4}\dot{q}_{m2}^2 \,.$$

Accordingly,

$$\dot{E}_m(q, \dot{q}) = (\beta_1 u_1 - \Gamma_1)\dot{q}_1 + (\beta_2 u_2 - \Gamma_2)\dot{q}_2 \,.$$

Choose $(X^{j0}, t^{j0}) \in M^j$, $j = 1,2$ and $Z^0 \in A$. Then the control conditions (a), (b) hold if successively

$$\left.\begin{aligned}
&\min_{u_1^j} \max_{m_2^j} [(\beta_1^j u_1^j - \Gamma_1^j)\dot{q}_1^j] \le -\lambda_{m3}(\dot{q}_{m1})^2 \,, \quad j = 1,2 \\
&\min_{u_2^j} \max_{m_2^j} [(\beta_2^j u_2^j - \Gamma_2^j)\dot{q}_2^j] \le -\lambda_{m4}(\dot{q}_{m2})^2 \,, \quad j = 1,2
\end{aligned}\right\} \tag{34}$$

and

$$
\left.\begin{array}{l}
\max\limits_{u_1^1}\min\limits_{m_2^1}[(B_1^1u_1^1-\Gamma_1^1)\dot{q}_1^1] > \min\limits_{u_1^2}\max\limits_{m_2^2}[(B_1^2u_1^2-\Gamma_1^2)\dot{q}_1^2] \\
\\
\max\limits_{u_2^1}\min\limits_{m_2^1}[(B_2^1u_2^1-\Gamma_2^1)\dot{q}_2^1] > \min\limits_{u_2^2}\max\limits_{m_2^2}[(B_2^2u_2^2-\Gamma_2^2)\dot{q}_2^2] \quad .
\end{array}\right\} \tag{35}
$$

Thus we choose u_1^1 such that for $\dot{q}_1^1 \neq 0$,

$$
\min_{u_1^1}\max_{m_2^1}[(B_1^1u_1^1-\Gamma_1^1)\dot{q}_1^1] \leq -\lambda_{m3}(\dot{q}_{m1})^2
$$

and for such u_1^1 , we choose u_1^2 satisfying

$$
\min_{u_1^2}\max_{m_1^2}[(B_1^2u_1^2-\Gamma_1^2)\dot{q}_1^2] < \max_{u_1^1}\min_{m_2^1}[(B_1^1u_1^1-\Gamma_1^1)\dot{q}_1^1]
$$

The procedure for u_2^1 and u_2^2 is identical utilizing the second inequalities of (34), (35). Assuming symmetry of arms: $m_1^1 = m_1^2 = m_1$, $m_2^1 = m_2^2 = m_2 \in [\underline{m},\overline{m}]$, $r^1 = r^2 = r$, and substituting the expressions for Γ_i^j , Π_i^j , B_i^j , $i,j = 1,2$, we obtain the tracking controllers

$$
u_1^1(t) = \begin{cases} -\dfrac{\lambda_{m3}(\dot{q}_{m1})^2}{\dot{q}_1^1}[m_1r^2 + \overline{m}(q_2^1)^2 + 2\overline{m}q_1^1\dot{q}_2^1 + \lambda_3^1|\dot{q}_1|\dot{q}_1 \ , & \forall\dot{q}_i^j \neq 0 \\ \text{suitable constant}, & \forall\dot{q}_i^j = 0 \ , \end{cases}
$$

$$
u_1^2(t) = \begin{cases} \dfrac{-\lambda_{m3}(q_{m1})^2}{q_1^2}[m_1r^2 + \overline{m}(q_2^1)^2] + 2\underline{m}\dot{q}_2\dot{q}_1^2(\dot{q}_2^2) + \lambda_3^2|\dot{q}_1^2|(\dot{q}_1^2)^2 \ , & \forall\dot{q}_i^j \neq 0 \\ \text{suitable constant}, & \forall\dot{q}_i^j = 0 \ ; \end{cases}
$$

and the collision avoidance controllers

$$
u_2^1(t) = \begin{cases} \dfrac{-\lambda_{m4}(\dot{q}_{m2})^2\overline{m}}{\dot{q}_2^1} - \dfrac{\underline{m}}{q_2^1}q_2^1(\dot{q}_1^1)^2 + \lambda_4^1\dot{q}_2^1 \ , & \forall\dot{q}_2^1 \neq 0 \\ \text{suitable constant}, & \forall\dot{q}_2^1 = 0 \end{cases}
$$

$$
u_2^2(t) = \begin{cases} \dfrac{-\lambda_{m4}(\dot{q}_{m2})\underline{m}}{\dot{q}_1^2} - \overline{m}q_2^2\dot{q}_1^2\dot{q}_2^2 - \dfrac{\lambda_4^2(\dot{q}_2^2)^2}{q_1^2} \ , & \forall\dot{q}_1^j \neq 0 \ , \\ \text{suitable constant}, & \forall\dot{q}_i^j = 0 \ , \end{cases}
$$

which imply the control conditions (a), (b) for our example. The adaptive laws (24) are

$$
\begin{aligned}
&\dot{\lambda}_1^j = 0 \ , \quad \dot{\lambda}_2^j = 0 \\
&\dot{\lambda}_3^j = -(\text{sign}\ \lambda_3^j)\lambda_{m3}\dot{q}_{m1}^2 - \tfrac{1}{2}c_j \\
&\dot{\lambda}_4^j = -(\text{sign}\ \lambda_4^j)\lambda_{m4}\dot{q}_{m2}^2 - \tfrac{1}{2}c_j
\end{aligned}
$$

for $j = 1,2$. The first two laws vanish identically, since by design $\lambda_1^j = \lambda_{m1}$, $\lambda_2^j = \lambda_{m2}$. Numerical simulation of our modular case, with the data $m_1 = 70$kg, $\underline{m} = 30$kg, $\overline{m} = 40$kb, $r = 0.66$m, $\lambda_{m1} = 20$, $\lambda_{m2} = 20$, $\lambda_{m3} = 5$, $\lambda_{m4} = 2$, is shown in Fig. 5, and confirms the convergence-avoidance required.

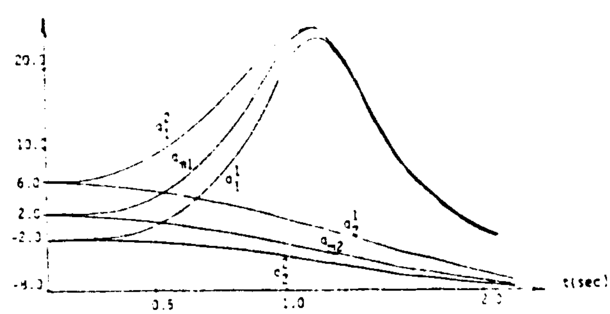

Figure 5. Numerical simulation

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