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A New State Reconstructor for Digital Control Systems Using Weighted-Averaged Measurements

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TECHNICAL PAPER

A NEW STATE RECONSTRUCTOR FOR DIGITAL CONTROL SYSTEMS USING WEIGHTED-AVERAGED MEASUREMENTS

I. INTRODUCTION

Consider the usual linear continuous-time plant driven by a zero-order-hold with sampled output, as shown in Figure 1. The most common approach to reconstructing the state of this system is undoubtedly the state observer [1]. However, it is well known that the state observer has some undesirable characteristics. Firstly, it is a dynamical system in itself, and, hence, adds additional states and eigenvalues to the system. Secondly, as a consequence, the reconstructed state is normally an approximation to the true state and is usually not a good one early in the state reconstruction process, unless the initial state of the system is well known.

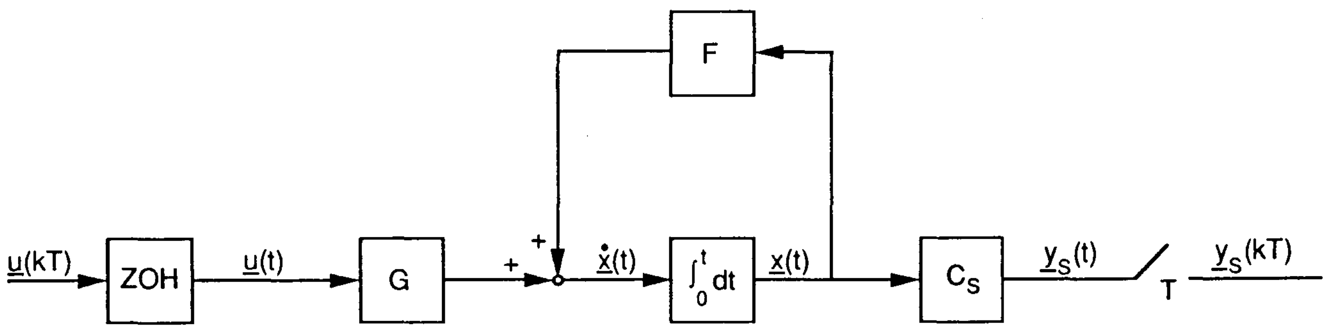

Figure 1. Linear continuous-time plant driven by a zero-order-hold with a sampled output.

The work in References 2 to 7 focuses on a relatively new state reconstructor that has neither of these problems. If the plant is noise-free and can be modeled precisely, the output of this state reconstructor exactly equals the true state of the plant and accomplishes this without any knowledge of the plant's initial state. Besides, it adds no new states or eigenvalues to the plant equation in the discrete time model of the system. In fact, it does not affect the plant equation in the discrete time model at all; it affects the measurement equation only. It is characterized by the fact that discrete measurements are generated every T/N seconds and input into a multi-input/multi-output moving average process. The output of this process is sampled every T seconds and utilized in reconstructing the state of the system. Should process noise, measurement noise, or modeling errors be a problem, it can be catenated with an observer or a Kalman filter for a synergistic effect.

In this paper, another new state reconstructor is presented. This one has the same advantages, compared to the state observer, as the one developed in References 2 to 7. However, it is unique in the way it achieves state reconstruction. A continuous-time output vector from the plant is convoluted with a weighting-function matrix whose elements are time dependent. This result is integrated over T second intervals to generate weighted-averaged measurements, every T seconds, that are used in the state reconstruction process. In actuality, the weighting-function matrix is the impulse-response matrix for a multi-input/multi-output system and the weighted-averaged measurements are the forced responses of this system to the plant output over T second intervals. As a result, an equally valid way of generating the weighted-averaged measurements is as follows. The continuous-time output vector from the plant is input into a continuous-time multi-input/multi-output system that is initially at rest. After T seconds, the output of this system is the weighted-averaged measurement vector. This process is repeated every T seconds.

This new state reconstructor is presented in the remainder of this paper. In Section II, some preliminary mathematical formulations pertinent to the development of it are derived. In Section III, the new state reconstructor is presented. An example of it is given in Section IV. Some final comments about it and its implementation are made in Section V.

II. PRELIMINARY

For the plant in Figure 1, $\underline{x}(t) \in R^n$ is the state, $\underline{u}(kT) \in R^r$ is the control input, $\underline{y}_S(kT) \in R^m$ is the standard output or measurement vector, $F \in R^{nxn}$ is the system matrix, $G \in R^{nxr}$ is the control matrix, and $C_S \in R^{mxn}$ is the standard output matrix. It is well known that this system can be modeled at the sampling instants kT by the discrete state equations [8]

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{1}$$

$$\underline{y}_S(k) = C_S \, \underline{x}(k) \quad , \tag{2}$$

where k is the usual shorthand notation for time kT,

$$\phi(t) = \mathcal{L}^{-1} \left[(sI-F)^{-1} \right] \quad , \tag{3}$$

$$A = \phi(T) \quad , \tag{4}$$

and

$$B = \left[\int_0^T \phi(\lambda) \, d\lambda \right] G \quad . \tag{5}$$

$\phi(t) \in R^{nxn}$ is the state transition matrix. $A \in R^{nxn}$ is the system matrix and $B \in R^{nxr}$ is the control matrix for the discrete state equations (1) and (2).

Now consider the plant in Figure 2, which is a generalization of the one in Figure 1. In addition to the standard output $\underline{y}_S(kT)$, it has the output $\underline{y}_A'(kT) \in R^q$ found by differencing the weighted-averaged measurement $\underline{y}_A(kT) \in R^q$ with the delayed-input term $H\underline{u}[(k-1)T]$ where $H \in R^{qxr}$. The vector $\underline{y}_A(kT)$ is generated by convoluting the continuous-time output $\underline{z}(t) \in R^p$ for $(k-1)T < t \leq kT$ with the weighting-function matrix $\theta(t) \in R^{qxp}$ and integrating this result over T second intervals. Observe that this is equivalent to inputting $\underline{z}(t)$ for $(k-1)T < t \leq kT$ into the multi-input/multi-output system defined by the transfer-function matrix

$$\theta(S) = \int_0^\infty \theta(t)\, e^{-st}\, dt \in R^{qxp}$$

where s is the Laplace transform variable. If the system is at rest at time (k–1)T, its output at time kT will be $\underline{y}_A(kT)$. It follows from the definitions of $\underline{x}(t)$ and $\underline{z}(t)$ that the output matrix $C_A \in R^{pxn}$.

For the system in Figure 2, equations 1 to 5 apply. Observe that

$$\underline{y}_T(kT) = \begin{bmatrix} \underline{y}_S(kT) \\ \underline{y}_A'(kT) \end{bmatrix} \qquad (6)$$

where

$$\underline{y}_A'(kT) = \underline{y}_A(kT) - H\underline{u}[(k-1)T] \qquad (7)$$

and

$$\underline{y}_A(kT) = \int_{(k-1)T}^{kT} \theta(kT-\xi)\, \underline{z}(\xi)\, d\xi = \int_{(k-1)T}^{kT} \theta(kT-\xi)\, C_A\, \underline{x}(\xi)\, d\xi \quad . \qquad (8)$$

It is known that [8]

$$\underline{x}(\xi) = \phi(\xi-t_o)\, \underline{x}(t_o) + \int_{t_0}^{\xi} \phi(\xi-\tau)\, G\underline{u}(\tau)\, d\tau \quad . \qquad (9)$$

Letting $t_o = kT$ in equation (9) and substituting this result into equation (8) yields

$$\underline{y}_A(kT) = \left[\int_{(k-1)T}^{kT} \theta(kT-\xi)\, C_A\, \phi(\xi-kT)\, d\xi\right] \underline{x}(kT) + \int_{(k-1)T}^{kT} \theta(kT-\xi)\, C_A \left[\int_{kT}^{\xi} \phi(\xi-\tau)\, G\underline{u}(\tau)\, d\tau\right] d\xi \quad . \qquad (10)$$

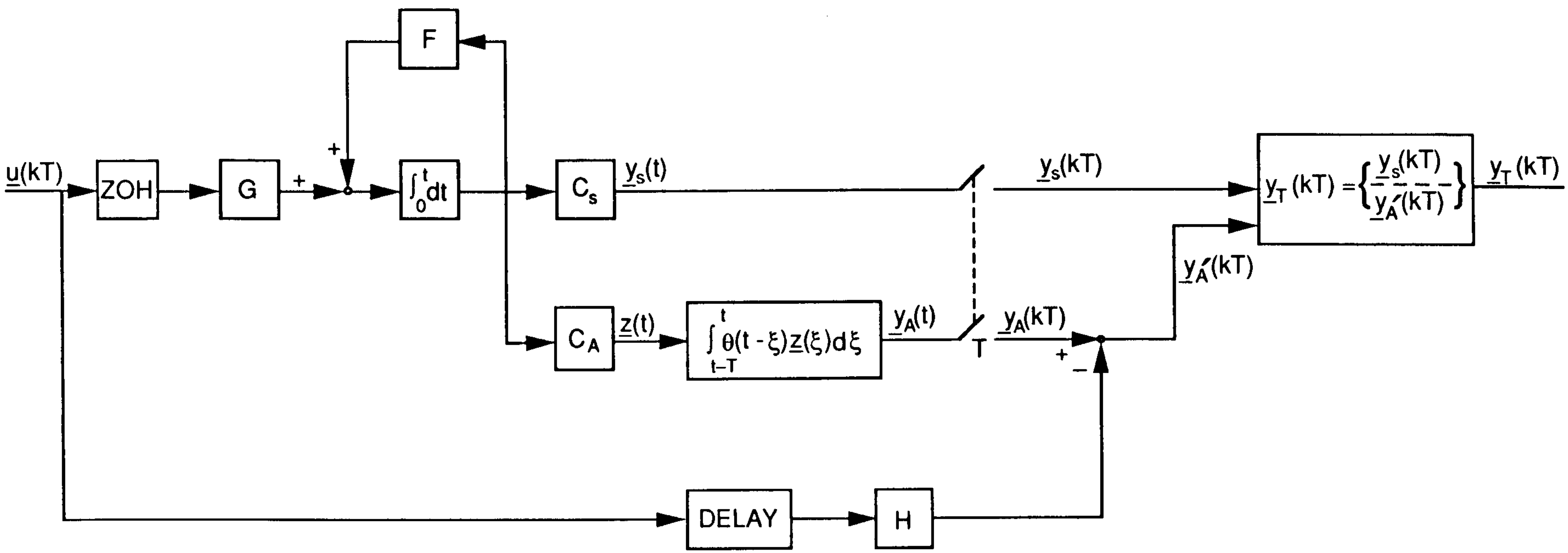

Figure 2. Linear continuous-time plant driven by a zero-order-hold with standard and weighted-averaged measurements.

Since $\underline{u}(t) = \underline{u}[(k-1)T]$ for $(k-1)T \leq t < kT$, equation (10) can be written as

$$\underline{y}_A(kT) = D\,\underline{x}(kT) + E\,\underline{u}[(k-1)T] \tag{11}$$

where

$$D = \int_{(k-1)T}^{T} \theta(kT-\xi)\, C_A\, \phi(\xi-kT)\, d\xi \tag{12}$$

$$E = \int_{(k-1)T}^{kT} \theta(kT-\xi)\, C_A \left[\int_{kT}^{\xi} \phi(\xi-\tau)\, d\tau\right] G \quad . \tag{13}$$

the transformation $\lambda = \xi - kT$ in equation (12) produces

$$D = \int_{-T}^{0} \theta(-\lambda)\, C_A\, A(\lambda)\, d\lambda \tag{14}$$

$$A(\lambda) = \phi(\lambda) \quad . \tag{15}$$

he transformations $\lambda = \xi - \tau$ and $\rho = \xi - kT$ successively in equation (13) yields

$$E = \int_{-T}^{0} \theta(-\rho)\, C_A\, B(\rho)\, d\rho$$

$$3(\rho) = \left[\int_{0}^{\rho} \phi(\lambda)\, d\lambda\right] G \quad .$$

From equations (2), (6), (7), and (11),

$$\underline{y}_T(kT) = \begin{bmatrix} C_S \\ D \end{bmatrix} \underline{x}(kT) + \begin{bmatrix} 0 \\ E-H \end{bmatrix} \underline{u}[(k-1)T] \quad . \tag{16}$$

Let

$$H = E = \int_{-T}^{0} \theta(-\rho)\, C_A\, B(\rho)\, d\rho \quad . \tag{17}$$

In this case, the discrete state equations for the system in Figure 2 become

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{18}$$

$$\underline{y}_T(k) = C_T\, \underline{x}(k) \tag{19}$$

where the output matrix $C_T \epsilon R^{(m+q)xn}$ is

$$C_T = \begin{bmatrix} C_S \\ D \end{bmatrix} \quad . \tag{20}$$

This follows from equations (1), (16), and (17).

III. THE NEW STATE RECONSTRUCTOR

A block diagram of the new state reconstructor catenated with the plant is shown in Figure 3. Its output is $\underline{y}_T'(kT) \epsilon R^n$. Let the matrix $C_T^* \epsilon R^{nx(m+q)}$ be the pseudo-inverse of C_T. That is, let

$$C_T^* = (C_T^T\, C_T)^{-1}\, C_T^T \quad ,$$

as shown in Figure 3, where C_T is defined by equation (20) and $(C_T^T\, C_T)$ is assumed nonsingular. Then

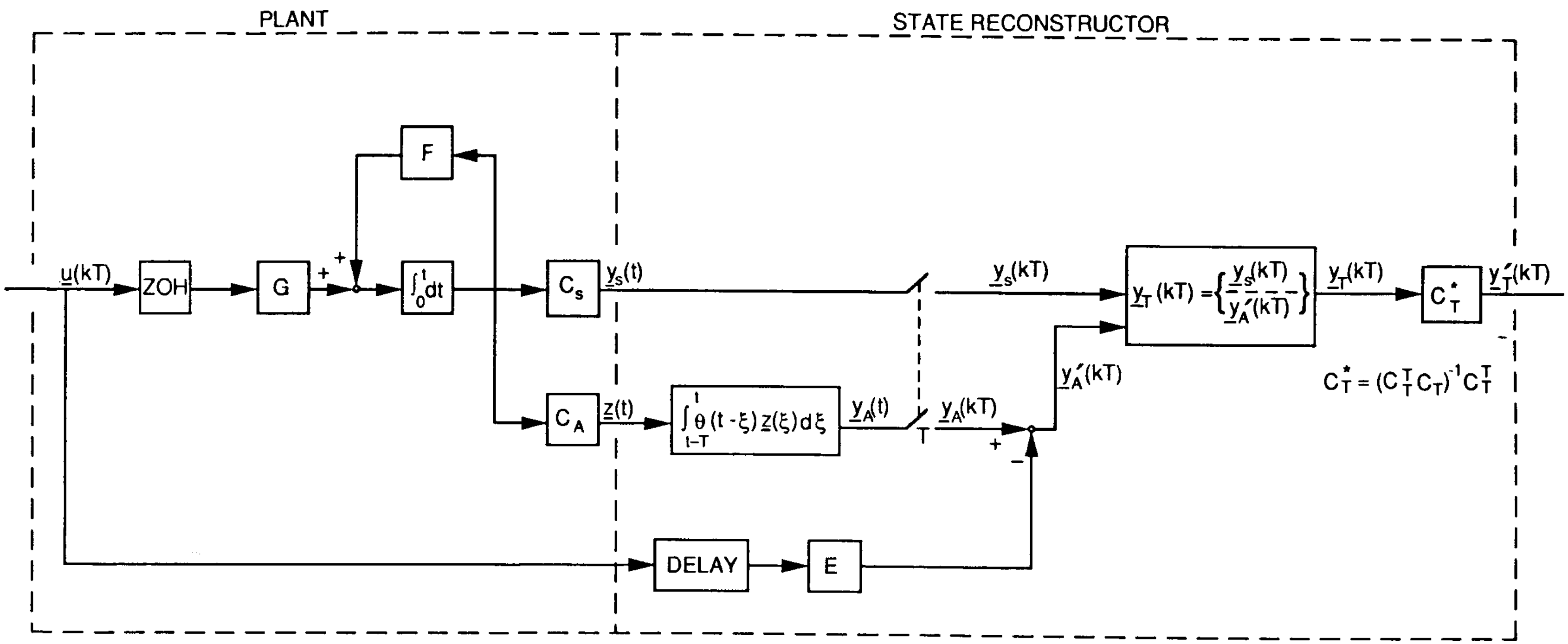

Figure 3. A block diagram of the new state reconstructor.

$$\underline{y}_T'(kT) = C_T^* \, \underline{y}_T(kT) = (C_T^T C_T)^{-1} C_T^T \underline{y}_T(kT) \quad . \tag{21}$$

It was shown in Section II that the discrete state equations for the system in Figure 3 from $\underline{u}(kT)$ to $\underline{y}_T(kT)$ are given by equations (18) and (19). Using these and equation (21), the discrete state equations for the system in Figure 3 become

$$\underline{x}(k+1) = A \, \underline{x}(k) + B \, \underline{u}(k)$$

$$\underline{y}_T'(k) = \underline{x}(k) \quad . \tag{22}$$

Hence, the state reconstructor exactly reconstructs the state of the system and does so without affecting the plant equation in the discrete time model of the system.

It was assumed that $(C_T^T C_T)$ is nonsingular. This places certain restrictions on $C_T \epsilon R^{(m+q)xn}$ [9]. First, it is necessary that $(m+q) \geqslant n$, or equivalently, $q \geqslant (n-m)$. This is readily satisfied since q, the number of rows in $\theta(t) \epsilon R^{qxp}$, can be chosen arbitrarily. Secondly, it is necessary that rank $(C_T) = n$. By virtue of equations (14), (15), and (20), rank (C_T) is affected by $\phi(t)$, $\theta(t)$, C_S, and C_A. In most problems: $\phi(t)$ will be fixed; $\theta(t)$ can be chosen arbitrarily; C_S and C_A will lie somewhere in between. Hence, given $\phi(t)$, if $\theta(t)$ can be chosen, and perhaps C_S and C_A to some extent, so that rank $(C_T) = n$, the state of the plant can be reconstructed according to equation (22) for $k \geqslant 1$.

IV. AN EXAMPLE

Consider the double integrator plant driven by a zero-order-hold as shown in Figure 4. The output $z(t) = x_1(t)$ is to be used in generating the weighted-averaged measurements for state reconstruction; there are no standard measurements in this example. Manipulating this plant into the format of Figure 3 yields

$$F = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} ,$$

$$G = \begin{bmatrix} 0 \\ 1 \end{bmatrix} ,$$

$$C_A = [1 \quad 0] ,$$

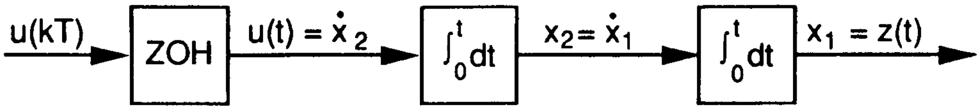

Figure 4. Double integrator plant in the example.

where C_S is a null matrix. It is straightforward to show that

$$A(t) = \phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} ,$$

$$\int_0^t \phi(\lambda)\, d\lambda = \begin{bmatrix} t & \frac{1}{2} t^2 \\ 0 & t \end{bmatrix} ,$$

and

$$B(t) = \left[\int_0^t \phi(\lambda)\, d\lambda \right] G = \begin{bmatrix} \frac{1}{2} t^2 \\ t \end{bmatrix} .$$

It is clear that n = 2, m = 0, and p = 1. The state reconstructor in Figure 3 requires that $q \geqslant (n-m)$ or $q \geqslant 2$. Choose q = 2. Hence, the weighting-function matrix $\theta(t) \in R^{q \times p}$ becomes $\theta(t) \in R^{2 \times 1}$. Choose

$$\theta(t) = \begin{bmatrix} \theta_1(t) \\ \theta_2(t) \end{bmatrix} = \begin{bmatrix} 1 \\ \frac{2}{T} e^{-(2/T)t} \end{bmatrix} . \tag{23}$$

Taking the Laplace transform of θ(t) produces

$$\theta(S) = \begin{bmatrix} \theta_1(S) \\ \theta_2(S) \end{bmatrix} = \begin{bmatrix} \frac{1}{S} \\ \frac{1}{\frac{T}{2} S + 1} \end{bmatrix} .$$

Observe that $\theta_1(S)$ is an integrator and $\theta_2(S)$ is a low-pass filter with a time constant of T/2 seconds. Hence, in implementing the state reconstructor for this example, the weighted-averaged measurements could be generated in either of two equally valid ways. One way is to determine the convolution integral shown in Figure 3 using equation (23). The other is to input z(t), $(k-1)T<t\leq kT$, into the system $\theta(S)$ with zero initial conditions at $t = (k-1)T$. At $t = kT$, the output of this system is $\underline{y}_A(kT)$. See how $\theta(S)$ can affect measurement noise in z(t) if it is present. Choosing $\theta(t)$, in general, to reduce the effects of measurement noise is an area for future study.

Having chosen $\theta(t)$ in this example, the other parameters in the state reconstructor can be found using the formulations in Section III. Doing this, it turns out that

$$C_T = D = \begin{bmatrix} T & -\frac{T^2}{2} \\ (1-e^{-2}) & \frac{T}{2}(3e^{-2}-1) \end{bmatrix},$$

$$C_T^* = \begin{bmatrix} \left(\frac{3-e^2}{2T}\right) & \left(\frac{e^2}{2}\right) \\ \left(\frac{1-e^2}{T^2}\right) & \left(\frac{e^2}{T}\right) \end{bmatrix},$$

and

$$E = \begin{bmatrix} \frac{T^3}{6} \\ \frac{T^2}{4}(1-5e^{-2}) \end{bmatrix}.$$

The new state reconstructor is now completely defined for this example.

V. FINAL COMMENTS

A new state reconstructor for a linear continuous-time plant driven by a zero-order-hold has been presented. It is unique in that it generates weighted-averaged measurements from a continuous-time output and utilizes these in reconstructing the state of the plant every T seconds. It has the same advantages, compared to the state observer, as the state reconstructor developed in References 2 to 7. These are enumerated in Section I. Should process noise, measurement noise, or modeling errors pose a problem for it, it could be catenated with an observer or a Kalman filter for a synergistic effect. Determining the effects of noise and modeling errors on the new state reconstructor is an area for future study. Reducing their effects by judiciously choosing the time-dependent elements of the weighting-function matrix is another area for future study.

With regard to implementing this new state reconstructor, it seems natural to generate a vector of weighted-averaged measurements wherever a scalar continuous-time output exists. Every T seconds, all measurements, weighted-averaged and standard, could be sent to a central processor. There, the weighted-averaged measurements could be properly combined and all further calculations and operations performed to reconstruct the state of the system. There are several ways of calculating the weighted-averaged measurements. They could be done in microprocessors using digital approximations or else in analog circuitry. In either case, a convolution integral, transfer function, or state equation approach is possible. This is also an area for future study.

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16. ABSTRACT

This paper presents a new state reconstructor for a linear continuous-time plant driven by a zero-order-hold. It takes a continuous-time output vector from the plant and convolutes it with a weighting-function matrix whose elements are time dependent. This result is integrated over T second intervals to generate weighted-averaged measurements, every T seconds, that are used in the state reconstruction process. If the plant is noise-free and can be modeled precisely, the output of this state reconstructor exactly equals the true state of the plant and accomplishes this without knowledge of the plant's initial state. If noise or modeling errors are a problem, it can be catenated with a state observer or a Kalman filter for a synergistic effect.

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