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# Stability Robustness Improvement of Direct Eigenspace Assignment Based Feedback Systems Using Singular Value Sensitivities

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**Stability Robustness Improvement of Direct Eigenspace Assignment  
Based Feedback Systems Using Singular Value Sensitivities**

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**Abstract**

A methodology to improve the stability robustness of feedback control systems designed using direct eigenspace assignment techniques is presented. The method consists of considering the sensitivity of the minimum singular value of the return difference transfer matrix at the plant input to small changes in the desired closed-loop eigenvalues and the specified elements of the desired closed-loop eigenvectors. Closed-form expressions for the gradient of the minimum return difference singular value with respect to desired closed-loop eigenvalue and eigenvector parameters are derived. Closed-form expressions for the gradients of the control feedback gains with respect to the specified eigenspace parameters are obtained as an intermediate step. The use of the gradient information to improve the guaranteed gain and phase margins in eigenspace assignment based designs is demonstrated by application to an advanced fighter aircraft.

**Introduction**

A fundamental objective in the design of flight control systems is to change the transient response of the flight vehicle to a desirable one using feedback control. As discussed in Ref. [1], the so called direct eigenstructure (or eigenspace) assignment techniques are really well suited to designing feedback control systems to meet this objective. Various applications of the eigenspace assignment techniques have appeared in

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the literature in the recent past. Some of these are – design of pitch pointing flight control systems [2], design of flutter and gust load alleviation systems [3], and transient response shaping for flexible vehicles [4]. All these applications were multivariable in nature and demonstrated that direct eigenstructure assignment is a viable multi–input multi–output (MIMO) control system design technique.

One of the drawbacks of direct eigenspace techniques, as compared to some other multivariable techniques – specially the Linear Quadratic Gaussian/Loop Transfer Recovery (LQG/LTR) approach [5], is that the synthesis procedure does not guarantee stability robustness with respect to variations in plant dynamics. Even using the Linear Quadratic Regulator (LQR) based methodology to asymptotically approach the desired eigenspace [6] does not guarantee the well known stability margins of LQ Regulators [7] as the procedure results in non–diagonal control weighting matrices which violates the conditions under which the LQ Regulator stability margins are guaranteed. Also, as discussed in Ref. [4], the direct eigenspace assignment techniques are preferable to the LQR based approach because the LQR approach requires very high actuator bandwidths for the desired eigenstructure to be achieved.

In direct eigenspace assignment techniques the design parameters are the desired closed–loop eigenvalues and specified elements of the closed–loop eigenvectors. Once the design parameters are specified, the feedback control gains are uniquely determined (provided enough parameters are specified – see Ref. [8] for discussion of limits on achievable eigenspace using direct eigenspace assignment). So, given a set of specifications, the feedback control gains will provide the desired closed–loop transient response (or come as close to it as possible within the system constraints), but they might result in a system with poor stability robustness, i.e. a small change in the plant dynamics may cause the closed–loop system to go unstable. The designer is then faced with the dilemma of how to change the design specifications such that the resulting feedback system will also provide adequate stability robustness. Note that in general the designer *does* have a certain amount

of freedom in choosing the design specifications – rarely does he want an exact value for a closed-loop eigenvalue or exact shape for a corresponding eigenvector, the specifications are rather in terms of desired *regions* for the closed-loop eigenvalues and acceptable *sets* of eigenvector shapes. The objective of this paper then is to develop a methodology which will provide adequate information to the designer to change the design specifications in a systematic step-wise manner such that at each step the guaranteed stability robustness of the feedback system is improved while the eigenstructure is within the desirable *regions*.

In multivariable feedback systems, a reliable (but sometimes conservative [7]) measure of stability robustness is the minimum singular value of the return difference matrix at the plant input evaluated as a function of frequency. The methodology presented in this paper is based on sensitivities of the minimum singular value of the return difference matrix to the design parameters, which in this case are the desired closed-loop eigenvalues and eigenvectors. Note that the notion of using return difference singular value sensitivities to design robust controllers is not new. Singular value sensitivities to compensator parameters were used in Ref. [9] to directly design robust reduced-order compensators, and singular value sensitivities to plant parameters (elements of the plant system matrices) were used in Ref. [10] to determine which elements need to be modeled more accurately for the feedback system to guarantee stability.

In the following a technique for solving the feedback gains for direct eigenspace assignment is first briefly described and the mathematical problem formulation for deriving analytical expressions for the return difference singular value sensitivities is presented. Closed-form expressions for the singular value gradients with respect to closed-loop eigenvalues and elements of the closed-loop eigenvectors are then developed by first considering the singular value gradients w.r.t. the feedback gains and then deriving the closed-form expressions for the gradients of the feedback gains to the closed-loop eigenspace parameters. Finally, the use of the gradient information to improve guaranteed stability robustness is demonstrated by application to a modern fighter aircraft.

After the completion of the present manuscript, it was brought to the author's notice that somewhat similar research has been reported earlier in Ref. [11]. However, as pointed out in a later section, the research reported herein differs from that reported in Ref. [11] in the important detail of the choice of independent parameters with respect to which the singular value sensitivities are calculated.

## Problem Formulation

### Direct Eigenspace Assignment Gain Synthesis

In the direct eigenspace assignment technique, the control objectives are stated in terms of a desired eigenstructure for the augmented system. For the full-state feedback case, the synthesis problem is as follows :

Given a linear, time-invariant dynamical system with the state-space representation

$$\dot{\bar{x}} = A\bar{x} + B\bar{u} \quad (1)$$

with  $\bar{x} \in \mathbb{R}^n$  and  $\bar{u} \in \mathbb{R}^m$ , find a control law of the form

$$\bar{u} = -K\bar{x} \quad (2)$$

to achieve some desired eigenspace for the augmented system

$$\dot{\bar{x}} = (A-BK)\bar{x} \quad (3)$$

To determine the feedback gains  $K$ , note that the augmented (closed-loop) system eigenvalues and eigenvectors are related by

$$(A-BK)\bar{v}_{c_i} = \lambda_i \bar{v}_{c_i}, \quad i = 1, \dots, n \quad (4)$$

where  $\lambda_i$  is the  $i^{\text{th}}$  closed-loop eigenvalue and  $\bar{v}_{c_i}$  is the corresponding closed-loop eigenvector.

For full-state feedback, the limitation on the achievable eigenspace is that all the desired closed-loop eigenvalues can be exactly placed while only "m" elements of their associated eigenvectors can be exactly achieved [8] (here,  $m$  = dimension of  $\bar{u}$  and it is

assumed that the system given by (1) is controllable). Since in general  $m < n$ , we cannot exactly obtain all elements of the desired eigenvector for each closed-loop mode. One approach for determining the feedback gains is to obtain the "best" achievable eigenvectors, for each of the closed-loop modes, so as to minimize the mode's cost function  $J_i$  given by

$$J_i = \frac{1}{2} (\bar{v}_{a_i} - \bar{v}_{d_i})^* Q_i (\bar{v}_{a_i} - \bar{v}_{d_i}) , \quad i = 1, \dots, n \quad (5)$$

where

$\bar{v}_{a_i}$  =  $i^{\text{th}}$  achievable eigenvector associated with eigenvalue  $\lambda_i$

$\bar{v}_{d_i}$  =  $i^{\text{th}}$  desired eigenvector

$Q_i$  =  $i^{\text{th}}$   $n$ -by- $n$  symmetric positive semi-definite weighting matrix on eigenvector error elements,

and  $[\cdot]^*$  denotes complex-conjugate transpose of  $[\cdot]$ .

Equation (4) can be rewritten as

$$(\lambda_i I - A) \bar{v}_{a_i} = -B K \bar{v}_{a_i} \quad (6)$$

Defining the vector  $\bar{w}_i \triangleq -K \bar{v}_{a_i}$  and using eqn. (6), the solution to minimizing the cost in eqn. (5) is obtained as (see Ref. [4] for a complete derivation) :

$$\bar{w}_i = [L_i^* Q_i L_i]^{-1} L_i^* Q_i \bar{v}_{d_i} \quad (7)$$

where  $L_i = (\lambda_i I - A)^{-1} B$ .

Once  $\bar{w}_i$  are obtained, the achievable eigenvectors are given by

$$\bar{v}_{a_i} = L_i \bar{w}_i , \quad i = 1, \dots, n \quad (8)$$

and the feedback gains are obtained as

$$K = -WV^{-1} \quad (9)$$

where  $W = [\bar{w}_1 \ \bar{w}_2 \ \dots \ \bar{w}_n]$  and  $V = [\bar{v}_{a_1} \ \bar{v}_{a_2} \ \dots \ \bar{v}_{a_n}]$ .

Note that this algorithm requires that the specified closed-loop eigenvalues  $\lambda_i$  be distinct and different from the open-loop eigenvalues (eigenvalues of plant system matrix

A). Also, we are considering full-state feedback rather than reduced-order output feedback because reduced-order output feedback further restricts the achievable eigenspace [8], and full-state feedback can always be implemented with state estimation without any significant loss in stability robustness by using either the loop recovery procedure of the LQG/LTR approach or an eigenspace assignment based loop recovery procedure discussed in Ref. [3].

### Problem Statement

Given a multivariable system as in eqn. (1) with a control law of the form (2) and the state feedback gains given by eqn. (9) such that the closed-loop eigenvalues are

$$\begin{aligned} \lambda_{2i-1,2i} &= -\gamma_i \pm j\delta_i, \quad i = 1, \dots, k, \quad j \triangleq \sqrt{-1} \text{ and } \delta_i > 0 \\ \text{and } \lambda_i &= -\eta_i, \quad i = 2k+1, \dots, n \end{aligned} \quad (10)$$

and the desired eigenvectors are

$$\begin{aligned} \bar{v}_{d_{2i-1,2i}} &= \text{col.}[\mu_{ij} \pm j\rho_{ij}] \quad \text{for } \lambda_{2i-1,2i} = -\gamma_i \pm j\delta_i \\ \text{and } \bar{v}_{d_i} &= \text{col.}[\nu_{ij}] \quad \text{for } \lambda_i = -\eta_i, \end{aligned} \quad (11)$$

we wish to derive analytical expressions for the sensitivities of the minimum singular value of the return difference matrix at the plant input to the specified eigenstructure elements, i.e. closed-form expressions for the partial derivatives

$$\frac{\partial \underline{\sigma}[I+KG(s)]}{\partial \xi} \quad (12)$$

where  $G(s) = (sI - A)^{-1}B$  with  $s$  being the Laplace operator,  $\underline{\sigma}[\cdot]$  denotes minimum singular value of  $[\cdot]$ , and  $\xi$  represents the eigenspace parameters  $\gamma_i$ ,  $\delta_i$ ,  $\eta_i$ ,  $\mu_{ij}$ ,  $\rho_{ij}$  and  $\nu_{ij}$  defined in (10) and (11).

In Ref. [11], the parameters with respect to which the singular value sensitivities are calculated are the desired closed-loop eigenvalues  $\lambda_i$  (and hence  $\gamma_i$ ,  $\delta_i$  and  $\eta_i$  as above), and the elements of the vectors  $\bar{w}_i$  given by Eqn. 7. ( $\bar{w}_i$  corresponds to the  $t_i$  of Ref. [11]). As seen from Eqn. 7,  $\bar{w}_i$  depend on the choice of  $\lambda_i$ , and hence it is inappropriate to choose  $\bar{w}_i$  and  $\lambda_i$  as independent parameters for calculating the return difference singular value

sensitivities. It is important to note that the design specifications available to the control designer are in terms of the desired eigenvalues and eigenvectors,  $\lambda_i$  and  $\bar{v}_{d_i}$  respectively, and *not* in terms of  $\lambda_i$  and  $\bar{w}_i(t_i)$ . Therefore, it is more meaningful to choose  $\lambda_i$  and  $\bar{v}_{d_i}$  as the independent parameters with respect to which the stability robustness singular value sensitivities are obtained.

The discussion in the present paper is limited to singular value sensitivity analysis for the return difference matrix at the plant input. In general, it is important to guarantee the stability robustness at the plant output also. As will be apparent from later discussions, the ideas presented in this paper can be extended to deriving sensitivities of the minimum singular value of the return difference matrix at the output simply by replacing  $I+KG(s)$  with  $I+G(s)K$ . Also note that the singular value sensitivities to the natural frequency and damping,  $\omega_n$  and  $\zeta$  respectively, of a complex closed-loop mode can be obtained by using

$$\frac{\partial \sigma}{\partial \omega_{n_i}} = \frac{\partial \sigma}{\partial \gamma_i} \frac{\partial \gamma_i}{\partial \omega_{n_i}} + \frac{\partial \sigma}{\partial \delta_i} \frac{\partial \delta_i}{\partial \omega_{n_i}}$$

and

$$\frac{\partial \sigma}{\partial \zeta_i} = \frac{\partial \sigma}{\partial \gamma_i} \frac{\partial \gamma_i}{\partial \zeta_i} + \frac{\partial \sigma}{\partial \delta_i} \frac{\partial \delta_i}{\partial \zeta_i}$$

with

$$\frac{\partial \gamma_i}{\partial \omega_{n_i}} = \zeta_i ; \quad \frac{\partial \gamma_i}{\partial \zeta_i} = \omega_{n_i} ; \quad \frac{\partial \delta_i}{\partial \omega_{n_i}} = \sqrt{1-\zeta_i^2} ; \quad \frac{\partial \delta_i}{\partial \zeta_i} = -\frac{\zeta_i \omega_{n_i}}{\sqrt{1-\zeta_i^2}}$$

where we have used the equalities  $\gamma_i = \zeta_i \omega_{n_i}$ , and  $\delta_i = \omega_{n_i} \sqrt{1-\zeta_i^2}$ .

Lehtomaki et al. [7] have shown that if

$$\underline{\sigma}[I+KG(j\omega)] \geq a_0 , \quad 0 < \omega < \infty \quad (13)$$

for some constant  $a_0 \leq 1$ , then simultaneously in each loop of the feedback system there is guaranteed gain margin (GM) given by



$$GM = \frac{1}{1 \pm a_0} \quad (14)$$

and also a guaranteed phase margin (PM) given by

$$PM = \pm \cos^{-1} \left[ 1 - \frac{a_0^2}{2} \right] \quad (15)$$

Therefore the gradient information provided by expressions of the form (12) can be used to move the eigenspace parameters within the desirable regions in a systematic, step-wise manner such that the lowest value of the minimum singular value of the return difference matrix, and hence the guaranteed gain and phase margins, are improved at each step.

### Singular Value Sensitivity Derivation

In Ref. [12] it has been shown that for a general complex matrix  $H$  of rank  $m$  which has distinct singular values  $\sigma_i$ ,  $i = 1, \dots, m$ , the sensitivity of the singular value  $\sigma_i$  with respect to a real parameter  $p$  is given by

$$\frac{\partial \sigma_i [H]}{\partial p} = \text{Real} \left[ \bar{u}_i^* \frac{\partial H}{\partial p} \bar{v}_i \right] \quad (16)$$

where  $\bar{v}_i$  and  $\bar{u}_i$  are the right and left singular vectors [13], respectively, corresponding to the singular value  $\sigma_i$ .

The eigenspace parameters defined in (10) and (11) are all real and the open-loop state frequency response matrix ( $G(j\omega)$ ) is independent of these closed-loop eigenspace parameters. Therefore, using (16), we get

$$\frac{\partial \underline{\sigma} [I + KG(j\omega)]}{\partial \xi} = \text{Real} \left[ \bar{u}^* \frac{\partial K}{\partial \xi} G(j\omega) \bar{v} \right] \quad (17)$$

where  $\bar{v}$  and  $\bar{u}$  are the right and left singular vectors, respectively, of  $[I + KG(j\omega)]$  corresponding to the minimum singular value  $\underline{\sigma}$ . With  $K$  given by (9), we have

$$\frac{\partial K}{\partial \xi} = - \left[ \frac{\partial W}{\partial \xi} + K \frac{\partial V}{\partial \xi} \right] V^{-1} \quad (18)$$

The expressions for the feedback gain sensitivities to eigenspace parameters are fully expanded in the Appendix by first considering the eigenvalue parameters ( $\eta_i$ ,  $\gamma_i$  and  $\delta_i$ ) and

then the eigenvector parameters ( $\nu_{ij}$ ,  $\mu_{ij}$  and  $\rho_{ij}$ ). An algorithm which uses the sensitivity information for stability robustness improvement is briefly discussed in the following section and finally an example application is presented.

### Stability Robustness Improvement Algorithm

Based on the derivation of the return difference singular value sensitivities to eigenspace parameters in the previous section and the Appendix, a step-by-step algorithm for improving the guaranteed stability robustness of direct eigenspace assignment feedback control designs is as follows :

- Step 1 : Formulate the eigenspace requirements; the desired closed-loop eigenvalues and closed-loop eigenvectors, and identify the design freedom available in the closed-loop eigenspace specification.
- Step 2 : Solve for the control feedback gains  $K$ , and check for the guaranteed stability robustness using the minimum singular value of the return difference matrix at the input,  $\underline{\sigma}[I+KG(j\omega)]$ . If the guaranteed stability margins are acceptable, then stop, otherwise go to step 3.
- Step 3 : Calculate  $\frac{\partial W}{\partial \xi}$  and  $\frac{\partial V}{\partial \xi}$  for the eigenspace parameters  $\xi$ , identified in Step 1, for which some design freedom is available. The calculation procedure is as discussed in the Appendix for each case of eigenspace parameter.
- Step 4 : Using the results of Step 3, calculate  $\frac{\partial K}{\partial \xi}$  from (18) and then  $\frac{\partial \underline{\sigma}[I+KG(j\omega)]}{\partial \xi}$  from (17).
- Step 5 : Using the information from Step 4, make small changes in the eigenspace parameters such that  $\underline{\sigma}[I+KG(j\omega)]$  will increase in the desired frequency region while making sure that the changed eigenspace parameters are within the bounds specified in Step 1. Go to Step 2.

The above algorithm can be implemented in the form of a constrained optimization

technique with the objective of minimizing the area under a specified value for the minimum return difference singular value frequency response within the constraint that the design parameters stay within certain specified bounds. Such an optimization approach is discussed in Ref. [9]. However, the choice of changing the eigenspace parameters in step 5 requires a considerable amount of "engineering intuition" which cannot be easily put in the form of a computer program. Therefore, for the present study, a computer program was developed just to implement steps 2 to 4 and the selection in step 5 was based on knowledge about the particular dynamics being controlled.

### Example

The flight vehicle model considered is the short period approximation for the AFTI/F-16 aircraft as discussed in Ref. [2]. The model is for a flight condition corresponding to an altitude  $h = 3000$  ft and Mach number  $M = 0.6$ . The equations of motion are given in the form of (1) with

$$\bar{x} = [\gamma, q, \alpha, \delta_e, \delta_f]^T$$

where

$\gamma$  = flight path angle

$q$  = pitch rate

$\alpha$  = angle of attack

$\delta_e$  = elevator deflection

$\delta_f$  = flaperon deflection,

and

$$\bar{u} = [\delta_{ec}, \delta_{fc}]^T$$

where

$\delta_{ec}$  = elevator deflection command

$\delta_{fc}$  = flaperon deflection command.

The system matrix A and the control distribution matrix B have the following numerical

values :

$$A = \begin{bmatrix} 0 & 0.00665 & 1.3411 & 0.16897 & 0.25183 \\ 0 & -0.86939 & 43.223 & -17.251 & -1.5766 \\ 0 & 0.99335 & -1.3411 & -0.16897 & -0.25183 \\ 0 & 0 & 0 & -20 & 0 \\ 0 & 0 & 0 & 0 & -20 \end{bmatrix} ; B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 20 & 0 \\ 0 & 20 \end{bmatrix}$$

The eigenvalues of the open-loop system are given by

$$\left. \begin{array}{l} \lambda_1 = -7.662 \\ \lambda_2 = 5.452 \end{array} \right\} \text{unstable short period mode}$$

$$\lambda_3 = 0.0 \quad \text{pitch attitude mode}$$

$$\lambda_4 = -20 \quad \text{elevator actuator mode}$$

$$\lambda_5 = -20 \quad \text{flaperon actuator mode}$$

The control design objective is to provide decoupled tracking of flight path and pitch attitude commands with a well damped response and zero steady-state error to step commands. The control law is of the form

$$\bar{u} = F\bar{y}_c - K\bar{x} \quad (19)$$

with  $\bar{y}_c = [\gamma_c, \theta_c]^T$ .  $F$  is the feed-forward gain matrix and  $K$  is the feedback gain matrix. The feedback gains are to be obtained using direct eigenspace assignment techniques with the objective of providing decoupled flight path and pitch response modes. The feedback gains should also be such as to guarantee gain margins of at least  $\pm 3.5$  dB for simultaneous gain changes in each control loop at the plant input and phase margins of at least  $\pm 30$  deg for simultaneous phase changes in each control loop. These stability margin specifications translate into the requirement that  $\underline{\sigma}[I+KG(j\omega)] \geq 0.51$ . Once a set of feedback gains that satisfy the transient response and stability robustness requirements are obtained, the feedforward gains ( $F$ ) will then be obtained using a special case of Broussard's command generator tracker [14]. As discussed in Ref. [2], a set of feedforward gains that provide command following with zero steady-state error is given by

$$F = \Omega_{22} + K\Omega_{12} \quad (20)$$

with  $\Omega_{ij}$  given by

$$\Omega = \begin{bmatrix} \Omega_{11} & \Omega_{12} \\ \Omega_{21} & \Omega_{22} \end{bmatrix} = \begin{bmatrix} A & B \\ H & 0 \end{bmatrix}^{-1} \quad (21)$$

where H is defined by

$$y = H\bar{x} \quad (22)$$

with  $\bar{y}$  being the controlled variables of the plant. For the present example,  $\bar{y} = [\gamma, \theta]^T$ .

A set of feedback gains was obtained initially to achieve the desired eigenspace listed in Table 1. In Table 1, the short period frequency and damping (corresponding to  $\lambda_{1,2}$ ) were chosen to be  $\omega_n = 7$  rad/s and  $\zeta = 0.8$ , respectively, so as to meet the MIL-F-8785C [15] specifications for Category A, Level I flight, and the flight path mode ( $\lambda_3$ ) was chosen to provide adequate bandwidth for flight path control. The eigenvectors corresponding to the short period mode and the flight path mode were chosen to minimize the coupling between the pitch rate and the flight path angle. The actuator mode eigenvalues were chosen to be close to their open-loop values and the corresponding eigenvectors were chosen to minimize actuator cross-feed. Note that we are only specifying two elements for each desired eigenvector ('X' in Table 1 denotes arbitrary), and since we have two control inputs the specified eigenspace can be exactly achieved as seen from the achieved eigenvectors listed in Table 1. The control feedback gains for this initial design are also listed in Table 1. The minimum singular value of the return difference frequency response at the plant input is shown in Fig. 1. We note from Fig. 1 that the minimum singular value at low frequency is much lower than that required to guarantee the desired stability margins. Therefore, although this initial design will meet the performance requirements, a control law redesign is necessary for the stability robustness specifications to be met.

The singular value sensitivity procedure was then applied to the initial feedback design. The eigenspace parameters for which there is design freedom are :

- (a) Short period frequency and damping (and hence  $\lambda_{1,2}$ ). Based on MIL-F-8785C, the allowable regions for Level I flight response, at the given flight condition, are  $0.35 \leq$

$\zeta \leq 1.35$  and  $2.5 \leq \omega_n \leq 8.5$  rads/sec.

- (b) Flight path mode ( $\lambda_3$ ). Any value  $\geq 1$  rad/s will provide adequate flight path control, however there will be an upper limit due to maximum allowable flap deflection.
- (c) The desired eigenvector elements  $v_{d_{11}}$  (and corresponding element  $v_{d_{21}}$ ) and  $v_{d_{32}}$ , i.e. the short period contribution to the flight path and the flight path mode contribution to pitch rate. Although these elements were chosen to be 0 (zero) in the initial design, the only requirement is that they be "small" ( $\ll 1$ ) in order to keep the coupling of the modes to a low level.

There is no design freedom available in placing the actuator modes as it is desirable to keep these close to their open-loop values.

The predicted change in the minimum singular value of the return difference matrix for a 10 % (percent) increase in the short period frequency and damping, obtained using  $\frac{\partial \sigma}{\partial \omega_n}$  and  $\frac{\partial \sigma}{\partial \zeta}$  respectively, is shown in Fig. 2 and that for a 10 % increase in the flight path mode ( $|\lambda_3|$ ) is shown in Fig. 3. The return difference singular value sensitivities to the real and imaginary parts of the desired eigenvector element  $v_{d_{11}}$ ,  $\mu_{11}$  and  $\rho_{11}$  respectively, are shown in Fig. 4 and the sensitivity to eigenvector element  $v_{d_{32}}$  ( $\nu_{32}$ ) is shown in Fig. 5. From these figures we note that changing the eigenvector element  $v_{d_{11}}$ , decreasing  $\mu_{11}$  and increasing  $\rho_{11}$ , will be most effective in increasing the minimum singular value of the return difference matrix.

Based on the results in Fig. 5, letting  $v_{d_{11}} = -0.01 + j0.01$ , and keeping the rest of the desired eigenspace parameters the same as in Table 1, should result in an increase of  $\approx 0.115$  in the lowest value of the minimum singular value of the return difference matrix while still keeping the coupling between flight path and pitch rate, for the short period mode, to be very low. The minimum return difference singular value with the feedback

gains corresponding to this choice of eigenspace parameters, referred to as trial 1 from here onwards, is shown in Fig.6. Also shown in Fig. 6 is the minimum singular value predicted using the singular value sensitivities, i.e.  $\underline{\sigma}[I+K_0G(j\omega)] - 0.01 \frac{\partial \underline{\sigma}}{\partial \mu_{11}} + 0.01 \frac{\partial \underline{\sigma}}{\partial \rho_{11}}$  where  $K_0$  are the feedback gains corresponding to the initial design. Note that although there is not very good agreement between the actual and predicted values at low frequencies, the sensitivities *did* accurately predict the direction of the change (increase) in the singular value. Also note that the actual feedback gains for trial 1 and those predicted using  $\frac{\partial K}{\partial \mu_{11}}$  and  $\frac{\partial K}{\partial \rho_{11}}$ , both listed in Table 2, are virtually identical. From Fig. 6 we get  $\underline{\sigma}[I+KG(j\omega)] \geq 0.4$  for trial 1, which is much improved over the initial design but still not high enough to meet the stated stability robustness requirement. Therefore we need to further change the eigenspace design parameters in order to improve the guaranteed stability margins.

For trial 1 the singular value sensitivity calculations showed that the most effective way to increase the minimum singular value of the return difference matrix was to further decrease  $\mu_{11}$  and increase  $\rho_{11}$ . However, doing so will result in increased coupling in the flight path and pitch rate response for the short period mode which will be undesirable. Next to the desired eigenvector element  $v_{d_{11}}$ , the return difference singular value was most sensitive to changes in the short period mode. The changes in  $\underline{\sigma}[I+KG(j\omega)]$  from that for trial 1, using singular value sensitivities for a 10 % increase in short period frequency and a 10 % increase in damping are shown in Fig. 7. From Fig. 7 we note that increasing both the short period frequency and damping by 10 % each over that for trial 1 will result in an increase of 0.114 in the minimum singular value of the return difference matrix. We will then have  $\underline{\sigma}[I+KG(j\omega)] \geq 0.51$  which will satisfy the design requirement.

The short period frequency and damping for a 10 % increase over that for trial 1 are  $\omega_n = 7.7$  rad/s and  $\zeta = 0.88$  which are within the region for Level I flight requirements.

The desired and the achieved eigenspace for this change are summarized in Table 3 and the corresponding control feedback gains are also listed there. The resulting minimum singular value of the return difference matrix is shown in Fig. 8. From Fig. 8 we note good agreement between the actual singular value response and that predicted using singular value sensitivities. For this case also there was excellent agreement between the actual feedback gains and those predicted using feedback gain sensitivities for trial 1. The plot in Fig. 8 shows that this set of feedback gains satisfy the stability robustness design requirement, therefore this case will be referred to as the final design. The closed-loop performance of the initial and the final designs is compared in the following.

The feed-forward gains for the initial and final designs, calculated using eqns. (37) and (38), are listed in Table 4. The response of the initial design to a unit step gamma command ( $\gamma_c(t) = 1$  deg) is shown in Fig. 9. Shown in Fig. 9 are the time histories of the controlled variables  $\gamma$  and  $\theta$ , and also the control input deflections  $\delta_e$  and  $\delta_f$ . The response of the final design to a unit step gamma command was identical to that of the initial design, so those time histories are not shown here. From Fig. 9, then, we note that both the designs provide well-damped tracking of flight path commands with a reasonably fast rise time and without any perturbations in the pitch attitude. The responses of the initial and final designs to a unit step pitch attitude command ( $\theta_c(t) = 1$  deg) are shown in Figs. 10 and 11 respectively. Note that both the designs provide fast tracking of pitch attitude commands, however the initial design does so with no perturbation in the flight path angle while the final design does result in a small initial perturbation in the flight path angle.. This coupling of the flight path with the pitch attitude is due to small contribution of the short period mode to the flight path that was allowed in order to improve the guaranteed stability robustness. Therefore, in just two iterations the judicious use of the singular value gradient information led to a feedback control law design with much improved stability robustness while still maintaining acceptable performance.



## Summary and Conclusion

A methodology to improve the stability robustness of feedback control systems designed using direct eigenspace assignment techniques was presented. A full-state feedback gain synthesis technique for direct eigenspace assignment was briefly discussed and closed-form expressions for the sensitivity of the minimum singular value of the return difference matrix, at plant input, to changes in desired closed-loop eigenvalues and specified elements of the desired closed-loop eigenvectors were derived. The closed-form expressions for the sensitivity of the control feedback gains were obtained as an intermediate step. An algorithm discussing the steps involved in calculating the return difference singular value gradients was presented and the use of the gradient information to improve the guaranteed gain and phase margins was demonstrated by application to an advanced fighter aircraft. The aircraft example consisted of the short period approximation of the longitudinal dynamics with the design objective of providing decoupled tracking of flight path and pitch attitude commands using the elevator and flaperon as control effectors. The nominal feedback control design which provides ideal decoupling between flight path and pitch rate was found to have very low guaranteed stability robustness. Using the singular value gradient information it was shown that by allowing the short period mode contribution to the flight path angle to be non-zero, but still small ( $\ll$  contribution to pitch rate), and by increasing the short period mode frequency and damping by 10 percent from that for the nominal design, the guaranteed stability margins could be increased significantly while still maintaining acceptable performance.

## Appendix

### Feedback Gain Sensitivity to Eigenvalue Parameters

$\frac{\partial K}{\partial \eta_i}$  : We first consider the  $n-2k$  real closed-loop eigenvalues,  $-\eta_i$ ,  $i = 2k+1, \dots, n$ . Then

$$\frac{\partial W}{\partial \eta_i} = \begin{bmatrix} 0 & | & \frac{\partial \bar{w}_i}{\partial \eta_i} & | & 0 \\ & & \underbrace{\phantom{\frac{\partial \bar{w}_i}{\partial \eta_i}}}_{i^{\text{th}} \text{ col.}} & & \end{bmatrix} \quad (\text{A.1})$$

because, as seen from (7),  $\bar{w}_j$ ,  $j \neq i$ , does not depend on  $\eta_i$ . Using (7) we further have

$$\frac{\partial \bar{w}_i}{\partial \eta_i} = \frac{\partial [L_i^T Q_i L_i]^{-1}}{\partial \eta_i} L_i^T Q_i \bar{v}_{d_i} + [L_i^T Q_i L_i]^{-1} \frac{\partial L_i^T}{\partial \eta_i} Q_i \bar{v}_{d_i} \quad (\text{A.2})$$

which can be expanded to give

$$\frac{\partial \bar{w}_i}{\partial \eta_i} = [L_i^T Q_i L_i]^{-1} \left\{ - \left[ \frac{\partial L_i^T}{\partial \eta_i} Q_i L_i + L_i^T Q_i \frac{\partial L_i}{\partial \eta_i} \right] [L_i^T Q_i L_i]^{-1} L_i^T + \frac{\partial L_i^T}{\partial \eta_i} \right\} Q_i \bar{v}_{d_i} \quad (\text{A.3})$$

Furthermore, using the definition of  $L_i$ , we have

$$\frac{\partial L_i}{\partial \eta_i} = (-\eta_i I - A)^{-1} L_i \quad (\text{A.4})$$

Substituting (A.4) into (A.3) we will get an expression for  $\frac{\partial \bar{w}_i}{\partial \eta_i}$  in terms of known quantities, and substituting the result in (A.1) we will get  $\frac{\partial W}{\partial \eta_i}$ .

Next we determine  $\frac{\partial V}{\partial \eta_i}$  where  $V$  is the matrix of achievable eigenvectors as defined

earlier. Noting that  $\bar{v}_{a_j}$ ,  $j \neq i$ , (as defined in (8)), does not depend on  $\eta_i$ , we have

$$\frac{\partial V}{\partial \eta_i} = \begin{bmatrix} | & \frac{\partial \bar{v}_{a_i}}{\partial \eta_i} & | \\ 0 & | & \frac{\partial \bar{v}_{a_i}}{\partial \eta_i} & | & 0 \\ | & & & & | \end{bmatrix} \quad (\text{A.5})$$

and

$$\frac{\partial \bar{v}_{a_i}}{\partial \eta_i} = \frac{\partial L_i}{\partial \eta_i} \bar{w}_i + L_i \frac{\partial \bar{w}_i}{\partial \eta_i} \quad (\text{A.6})$$

Knowing  $\frac{\partial L_i}{\partial \eta_i}$  from (A.4) and  $\frac{\partial \bar{w}_i}{\partial \eta_i}$  from (A.3), we can determine  $\frac{\partial \bar{v}_{a_i}}{\partial \eta_i}$  from (A.6) and then substitute in (A.5) to get  $\frac{\partial V}{\partial \eta_i}$ . Once  $\frac{\partial W}{\partial \eta_i}$  and  $\frac{\partial V}{\partial \eta_i}$  are known,  $\frac{\partial K}{\partial \eta_i}$  are determined from eqn.

(18).

$\frac{\partial K}{\partial \gamma_i}$  : Next we consider the  $k$  complex modes,  $-\gamma_i \pm j\delta_i$ ,  $i=1, \dots, k$ , by first deriving the feedback gain sensitivities to the real part of the complex eigenvalues. If we arrange the columns of  $W$  such that

$$W = [\bar{w}_1, \bar{w}_2, \dots, \bar{w}_{2i-1}, \bar{w}_{2i}, \dots, \bar{w}_{2l-1}, \bar{w}_{2l}, \bar{w}_j \text{ (} j=2k+1, \dots, n)]$$

where  $\bar{w}_{2i-1}$  corresponds to  $\lambda_{2i-1} = -\gamma_i + j\delta_i$  and  $\bar{w}_{2i}$  corresponds to  $\lambda_{2i} = -\gamma_i - j\delta_i$ , then  $\bar{w}_{2i} = \text{conj}(\bar{w}_{2i-1})$  where  $\text{conj}(\cdot)$  denotes complex conjugate of  $(\cdot)$  and we have

$$\frac{\partial W}{\partial \gamma_i} = \begin{bmatrix} 0 & \frac{\partial \bar{w}_{2i-1}}{\partial \gamma_i} & \frac{\partial \bar{w}_{2i}}{\partial \gamma_i} & 0 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (\text{A.7})$$

Proceeding just as in the case of  $\frac{\partial W}{\partial \eta_i}$  above, we get

$$\begin{aligned} \frac{\partial \bar{w}_{2i-1}}{\partial \gamma_i} &= [L_{2i-1}^* Q_{2i-1} L_{2i-1}]^{-1} \left\{ - \left[ \frac{\partial L_{2i-1}^*}{\partial \gamma_i} Q_{2i-1} L_{2i-1} + L_{2i-1}^* Q_{2i-1} \frac{\partial L_{2i-1}}{\partial \gamma_i} \right] \right. \\ &\quad \left. [L_{2i-1}^* Q_{2i-1} L_{2i-1}]^{-1} L_{2i-1}^* + \frac{\partial L_{2i-1}^*}{\partial \gamma_i} \right\} Q_{2i-1} \bar{v}_{d_{2i-1}} \end{aligned} \quad (\text{A.8})$$

and

$$\frac{\partial L_{2i-1}}{\partial \gamma_i} = \{(-\gamma_i + j\delta_i)I - A\}^{-1} L_{2i-1} \quad (\text{A.9})$$

Using the relationship  $\frac{\partial \bar{w}_{2i}}{\partial \gamma_i} = \frac{\partial[\text{conj}(\bar{w}_{2i-1})]}{\partial \gamma_i} = \text{conj} \left[ \frac{\partial \bar{w}_{2i-1}}{\partial \gamma_i} \right]$ , we can get  $\frac{\partial W}{\partial \gamma_i}$  by making

use of (A.9), (A.8) and (A.7).

Next, just as in (A.7), we have

$$\frac{\partial V}{\partial \gamma_i} = \begin{bmatrix} 0 & \frac{\partial \bar{v}_{a_{2i-1}}}{\partial \gamma_i} & \frac{\partial \bar{v}_{a_{2i}}}{\partial \gamma_i} & 0 \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix} \quad (\text{A.10})$$

Also  $\bar{v}_{a_{2i}} = \text{conj}(\bar{v}_{a_{2i-1}})$  and from the definition of  $\bar{v}_{a_i}$  in (8) we have

$$\frac{\partial \bar{v}_{a_{2i-1}}}{\partial \gamma_i} = \frac{\partial L_{2i-1}}{\partial \gamma_i} \bar{w}_{2i-1} + L_{2i-1} \frac{\partial \bar{w}_{2i-1}}{\partial \gamma_i} \quad (\text{A.11})$$

Using (A.9), (A.8) and (A.11), we can determine  $\frac{\partial V}{\partial \gamma_i}$  and hence determine an expression

for  $\frac{\partial K}{\partial \gamma_i}$ .

$\frac{\partial K}{\partial \delta_i}$  : The procedure for determining the control feedback gain sensitivities to the

imaginary part of the complex closed-loop eigenvalues is similar to that for determining  $\frac{\partial K}{\partial \gamma_i}$  with  $\gamma_i$  replaced by  $\delta_i$ . The only difference is in calculating  $\frac{\partial L}{\partial \delta_i}$ , which is now

$$\frac{\partial L_{2i-1}}{\partial \delta_i} = -j\{(-\gamma_i + j\delta_i)I - A\}^{-1} = -j \frac{\partial L_{2i-1}}{\partial \gamma_i} \quad (\text{A.12})$$

Using (A.12) ,instead of (A.9), in (A.8) and (A.11), we can get analytical

expressions for  $\frac{\partial \bar{w}_{2i-1}}{\partial \delta_i}$  and  $\frac{\partial \bar{v}_{a_{2i-1}}}{\partial \delta_i}$ , respectively, and then we can determine  $\frac{\partial K}{\partial \delta_i}$  using

(A.7) and (A.10) with  $\gamma_i$  replaced by  $\delta_i$ .

### Feedback Gain Sensitivity to Eigenvector Parameters

$\frac{\partial K}{\partial \nu_{ij}}$  : We first consider the feedback gain sensitivities to the eigenvector elements  $\nu_{ij}$

corresponding to the real eigenvalues  $-\eta_i$  for  $i = 2k+1, \dots, n$ . The sensitivities  $\frac{\partial W}{\partial \nu_{ij}}$  and  $\frac{\partial V}{\partial \nu_{ij}}$

are as given by eqns. (A.1) and (A.5), respectively, with  $\eta_i$  replaced by  $\nu_{ij}$ . Furthermore,

noting that  $L_i$  does not depend on elements of the desired closed-loop eigenvectors, we

have

$$\frac{\partial \bar{w}_i}{\partial \nu_{ij}} = [L_i^T Q_i L_i]^{-1} L_i^T Q_i \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \leftarrow j^{\text{th}} \text{ element} \quad (\text{A.13})$$

and

$$\frac{\partial \bar{v}_{a_i}}{\partial \nu_{ij}} = L_i \frac{\partial \bar{w}_i}{\partial \nu_{ij}} \quad (\text{A.14})$$

The expressions (A.13) and (A.14) can be used to obtain  $\frac{\partial W}{\partial \nu_{ij}}$  and  $\frac{\partial V}{\partial \nu_{ij}}$ , and then  $\frac{\partial K}{\partial \nu_{ij}}$  can

be obtained using (18).

$\frac{\partial K}{\partial \mu_{ij}}$  : Next we consider the eigenvector elements corresponding to the  $k$  complex modes,

$-\gamma_i \neq j\delta_i$ ,  $i=1, \dots, k$ , by first deriving the feedback gain sensitivities to the real part of the eigenvectors ( $\mu_{ij}$ ). The forms for  $\frac{\partial W}{\partial \mu_{ij}}$  and  $\frac{\partial V}{\partial \mu_{ij}}$  are obtained from (A.7) and (A.10),

respectively, with  $\gamma_i$  replaced by  $\mu_{ij}$ . Furthermore, we have

$$\frac{\partial \bar{w}_{2i-1}}{\partial \mu_{ij}} = [L_i^* Q_i L_i]^{-1} L_i^* Q_i \begin{bmatrix} 0 \\ \vdots \\ 1 \\ \vdots \\ 0 \end{bmatrix} \leftarrow j^{\text{th}} \text{ element} \quad (\text{A.15})$$

and

$$\frac{\partial \bar{v}_{a_{2i-1}}}{\partial \mu_{ij}} = L_{2i-1} \frac{\partial \bar{w}_{2i-1}}{\partial \mu_{ij}} \quad (\text{A.16})$$

Using (A.15) and (A.16) along with the equality  $\frac{\partial \bar{w}_{2i}}{\partial \mu_{ij}} = \text{conj} \left[ \frac{\partial \bar{w}_{2i-1}}{\partial \mu_{ij}} \right]$  (and similarly for

$\frac{\partial \bar{v}_{a_{2i}}}{\partial \mu_{ij}}$ ), we can obtain  $\frac{\partial W}{\partial \mu_{ij}}$  and  $\frac{\partial V}{\partial \mu_{ij}}$ , and then determine  $\frac{\partial K}{\partial \mu_{ij}}$ .

$\frac{\partial K}{\partial \rho_{ij}}$  : The procedure for determining the feedback gain sensitivities to the imaginary part

of the desired complex closed-loop eigenvectors is similar to that for determining the sensitivities to the real part with  $\mu_{ij}$  replaced by  $\rho_{ij}$ . The only difference is in calculating

$\frac{\partial \bar{w}_{2i-1}}{\partial \rho_{ij}}$ , which is now

$$\frac{\partial \bar{w}_{2i-1}}{\partial \rho_{ij}} = [L_i^* Q_i L_i]^{-1} L_i^* Q_i \begin{bmatrix} 0 \\ \vdots \\ j \\ \vdots \\ 0 \end{bmatrix} \leftarrow j^{\text{th}} \text{ element} \quad (\text{A.17})$$

Using (A.17), instead of (A.15), and proceeding as for the previous case, we can get  $\frac{\partial W}{\partial \rho_{ij}}$  and  $\frac{\partial V}{\partial \rho_{ij}}$  and hence obtain  $\frac{\partial K}{\partial \rho_{ij}}$ .

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Table 1 Initial Design Eigenspace Summary

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State :  $\bar{x} = [\gamma, q, \alpha, \delta_e, \delta_f]^T$

Eigenvalues :  $\lambda_{1,2} = -5.6 \pm j4.2, \lambda_3 = -1.0, \lambda_4 = -19.0, \lambda_5 = -19.5$

Eigenvectors :

Desired				Achieved			
$\bar{v}_{d_{1,2}}$	$\bar{v}_{d_3}$	$\bar{v}_{d_4}$	$\bar{v}_{d_5}$	$\bar{v}_{a_{1,2}}$	$\bar{v}_{a_3}$	$\bar{v}_{a_4}$	$\bar{v}_{a_5}$
$\begin{bmatrix} 0 \\ 1 \\ X \\ X \\ X \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ X \\ X \\ X \end{bmatrix}$	$\begin{bmatrix} X \\ X \\ X \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} X \\ X \\ X \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} 0 \\ 1 \\ -0.114 \mp j0.086 \\ -0.070 \mp j0.533 \\ 0.629 \mp j0.814 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ -1 \\ -2.801 \\ 3.234 \end{bmatrix}$	$\begin{bmatrix} -0.006 \\ 1.072 \\ -0.051 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -0.014 \\ 0.060 \\ 0.011 \\ 0 \\ 1 \end{bmatrix}$

Feedback Gains :  $K = \begin{bmatrix} -3.250 & -0.891 & -7.112 & 0.526 & 0.084 \\ 6.101 & 0.898 & 10.02 & -0.420 & -0.102 \end{bmatrix}$

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Table 2 Feedback Gains for Trial 1

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Actual :  $K = \begin{bmatrix} -2.865 & -0.827 & -6.624 & 0.485 & 0.081 \\ 1.833 & 0.196 & 4.612 & 0.035 & -0.061 \end{bmatrix}$

Predicted :  $K = \begin{bmatrix} -2.865 & -0.827 & -6.624 & 0.485 & 0.081 \\ 1.830 & 0.195 & 4.609 & 0.035 & -0.061 \end{bmatrix}$

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Table 3 Final Design Eigenspace Summary

$$\text{State : } \bar{x} = [\gamma, q, \alpha, \delta_e, \delta_f]^T$$

$$\text{Eigenvalues : } \lambda_{1,2} = -6.78 \pm j 3.66, \quad \lambda_3 = -1.0, \quad \lambda_4 = -19.0, \quad \lambda_5 = -19.5$$

Eigenvectors :

Desired				Achieved			
$\underline{v}_{d_{1,2}}$	$\underline{\bar{v}}_{d_3}$	$\underline{\bar{v}}_{d_4}$	$\underline{\bar{v}}_{d_5}$	$\underline{\bar{v}}_{a_{1,2}}$	$\underline{\bar{v}}_{a_3}$	$\underline{\bar{v}}_{a_4}$	$\underline{\bar{v}}_{a_5}$
$\begin{bmatrix} -0.01 + j0.01 \\ 1 \\ X \\ X \\ X \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ X \\ X \\ X \end{bmatrix}$	$\begin{bmatrix} X \\ X \\ X \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} X \\ X \\ X \\ 0 \\ 1 \end{bmatrix}$	$\begin{bmatrix} -0.010 \pm j0.010 \\ 1 \\ -0.104 \pm j0.072 \\ 0.023 \pm j0.414 \\ 0.637 \pm j0.245 \end{bmatrix}$	$\begin{bmatrix} 1 \\ 0 \\ -1 \\ -2.801 \\ 3.234 \end{bmatrix}$	$\begin{bmatrix} -0.006 \\ 1.072 \\ -0.051 \\ 1 \\ 0 \end{bmatrix}$	$\begin{bmatrix} -0.014 \\ 0.060 \\ 0.011 \\ 0 \\ 1 \end{bmatrix}$

$$\text{Feedback Gains : } K = \begin{bmatrix} -3.280 & -0.954 & -7.289 & 0.585 & 0.090 \\ 0.138 & -0.101 & 2.423 & 0.232 & -0.043 \end{bmatrix}$$

Table 4 Feedforward Gains

$$\text{Initial Design : } F = \begin{bmatrix} -0.373 & -2.877 \\ 4.124 & 1.976 \end{bmatrix}$$

$$\text{Final Design : } F = \begin{bmatrix} -0.373 & -2.906 \\ 4.122 & -3.984 \end{bmatrix}$$

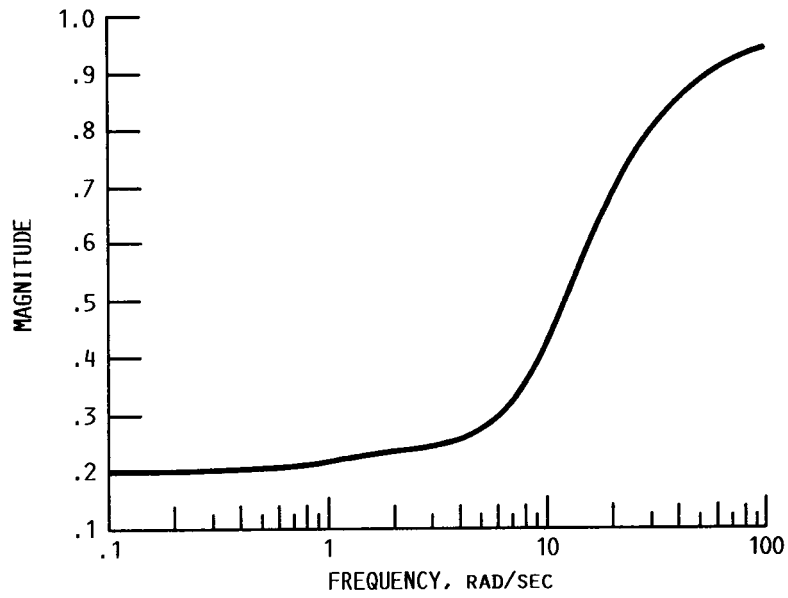


FIGURE 1. - MINIMUM RETURN DIFFERENCE SINGULAR VALUE ( $\underline{\sigma}[I+KG(j\omega)]$ ) FOR INITIAL DESIGN.

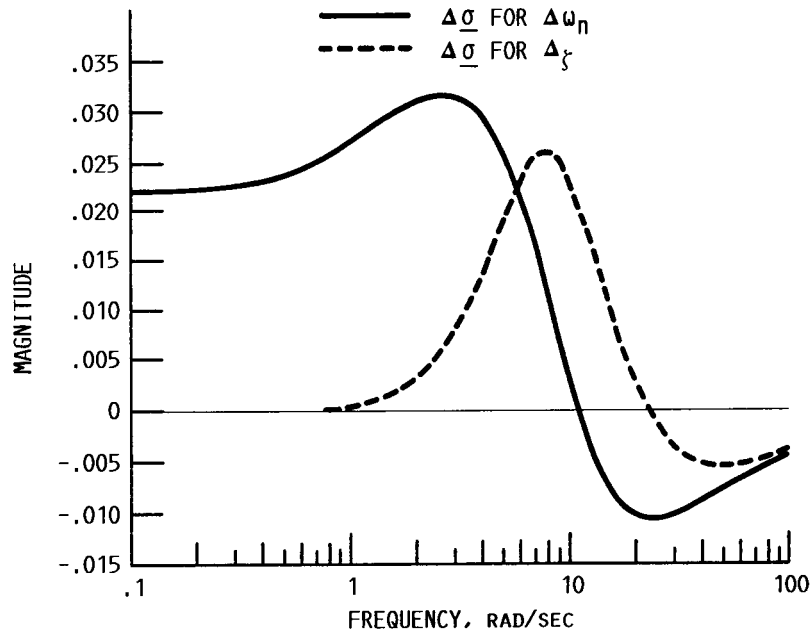


FIGURE 2. - CHANGE IN  $\underline{\sigma}[I+KG(j\omega)]$ , USING SINGULAR VALUE GRADIENTS, FOR 10 PERCENT INCREASE IN SHORT PERIOD  $\omega_n$  AND  $\zeta$  FROM THE INITIAL DESIGN.

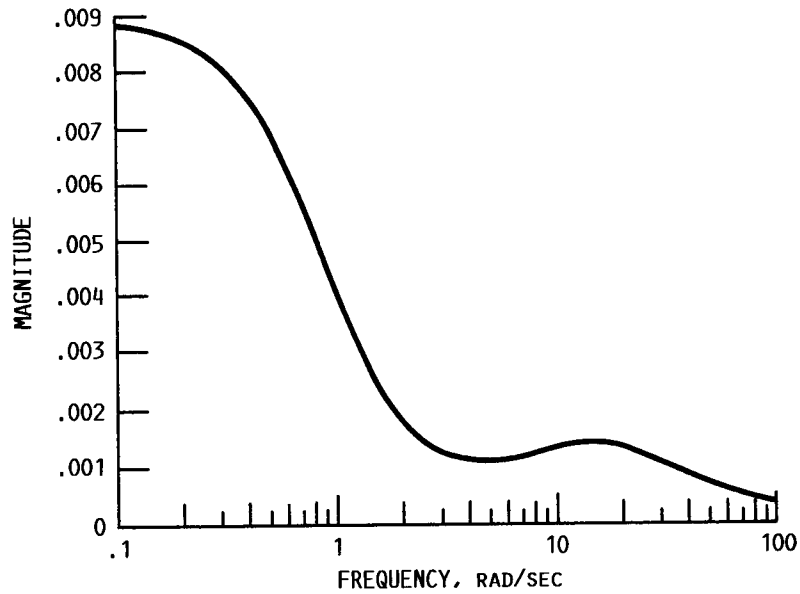


FIGURE 3. - CHANGE IN  $\sigma[I+KG(j\omega)]$ , FOR 10 PERCENT INCREASE IN  $|\lambda_3|$  (FLIGHT PATH MODE).

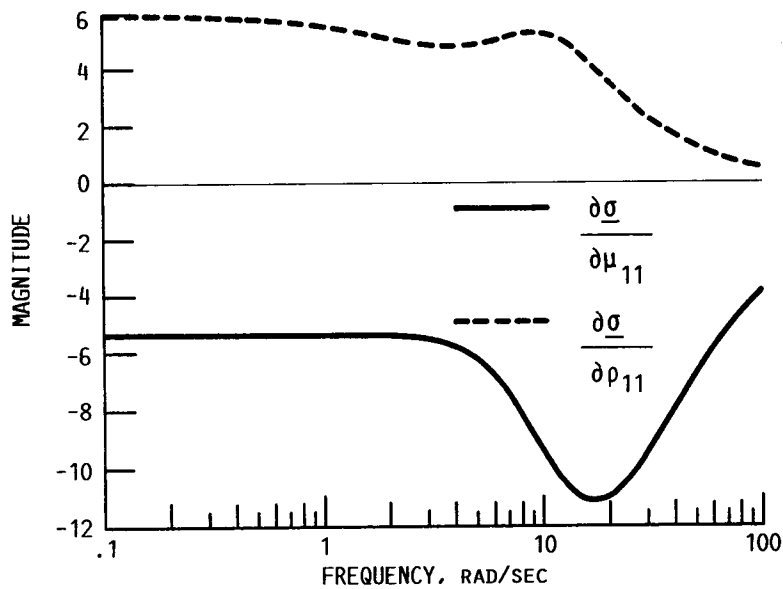


FIGURE 4. - MINIMUM RETURN DIFFERENCE SINGULAR VALUE SENSITIVITY TO REAL AND IMAGINARY PARTS OF THE DESIRED FLIGHT PATH ELEMENT OF THE SHORT PERIOD MODE

$$\text{EIGENVECTOR} \left( \frac{\partial \sigma}{\partial \mu_{11}} \text{ AND } \frac{\partial \sigma}{\partial \rho_{11}} \right).$$

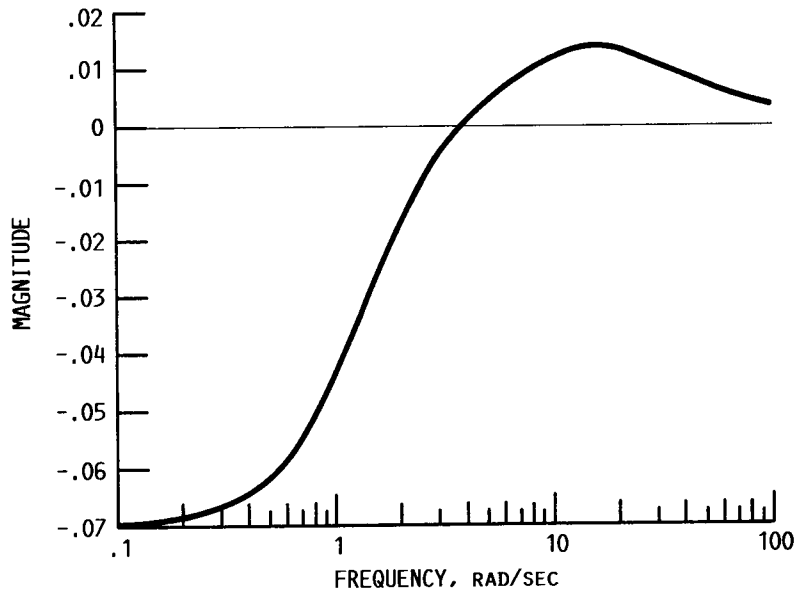


FIGURE 5. - MINIMUM RETURN DIFFERENCE SINGULAR VALUE SENSITIVITY TO DESIRED PITCH RATE ELEMENT OF THE FLIGHT PATH MODE EIGNEVECTOR  $\left( \frac{\partial \sigma}{\partial v_{32}} \right)$ .

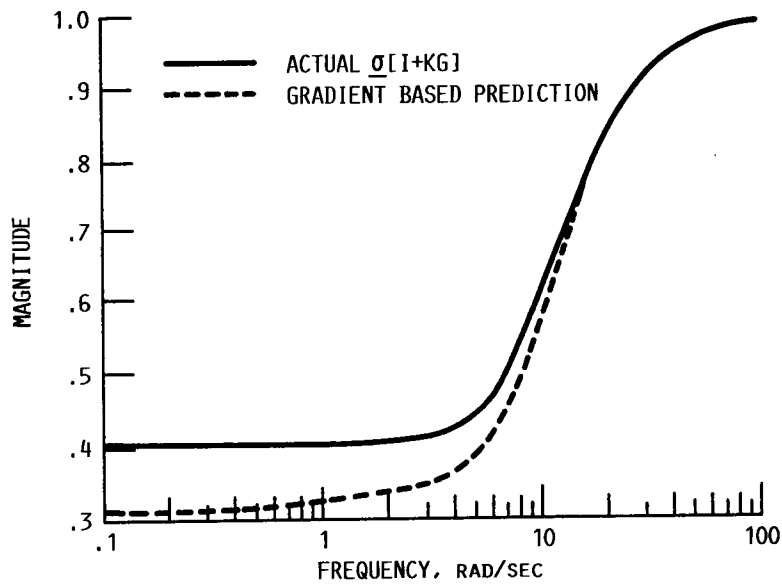


FIGURE 6. -  $\sigma[I+KG(j\omega)]$  FOR TRAIL 1 - ACTUAL AND PREDICTED (USING GRADIENT INFORMATION).

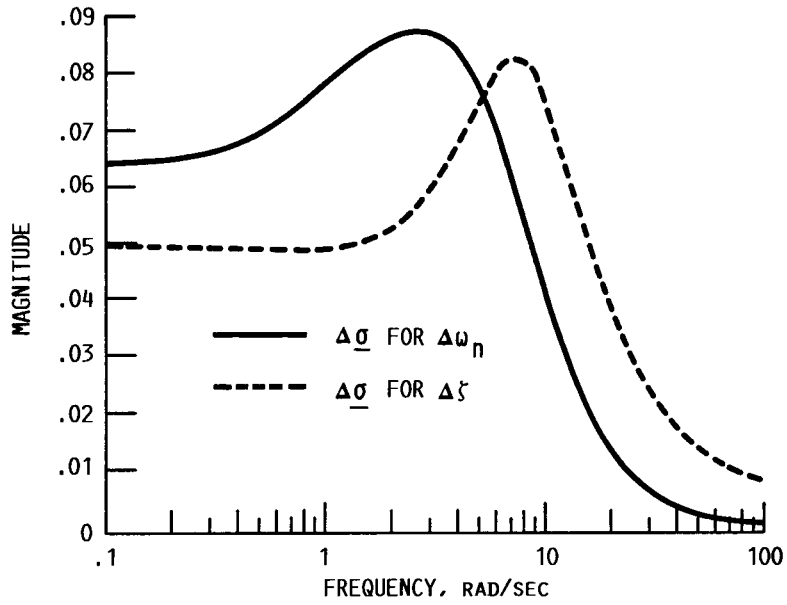


FIGURE 7. - CHANGE IN  $\sigma[I+KG(j\omega)]$ , FROM TRAIL 1, FOR 10 PERCENT INCREASE IN SHORT PERIOD  $\omega_n$  AND  $\zeta$ .

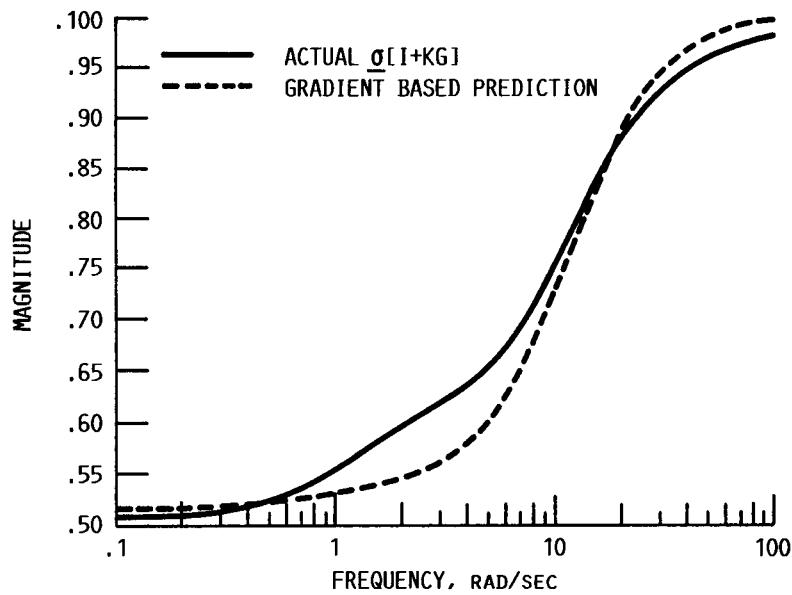


FIGURE 8. -  $\sigma[I+KG(j\omega)]$  FOR FINAL DESIGN - ACTUAL AND PREDICTED.

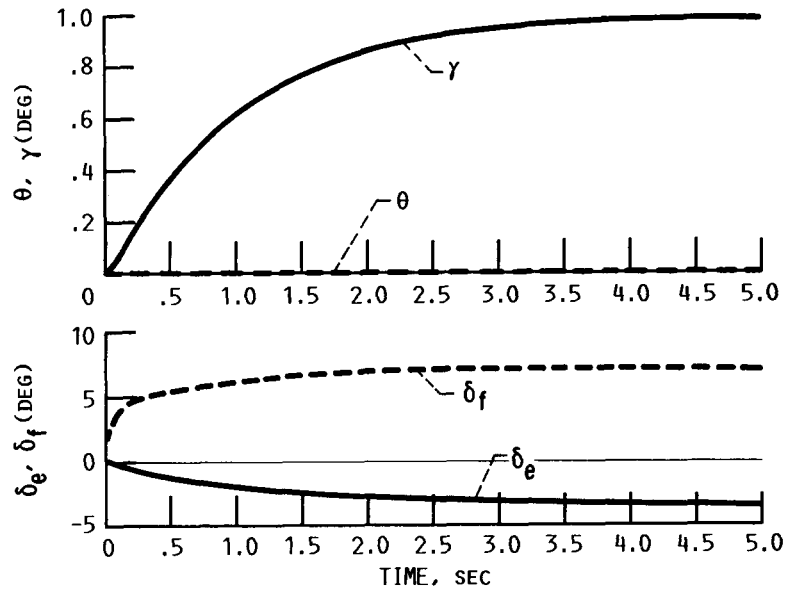


FIGURE 9. - INITIAL DESIGN RESPONSE TO STEP  $\gamma_c = 1$  DEG.

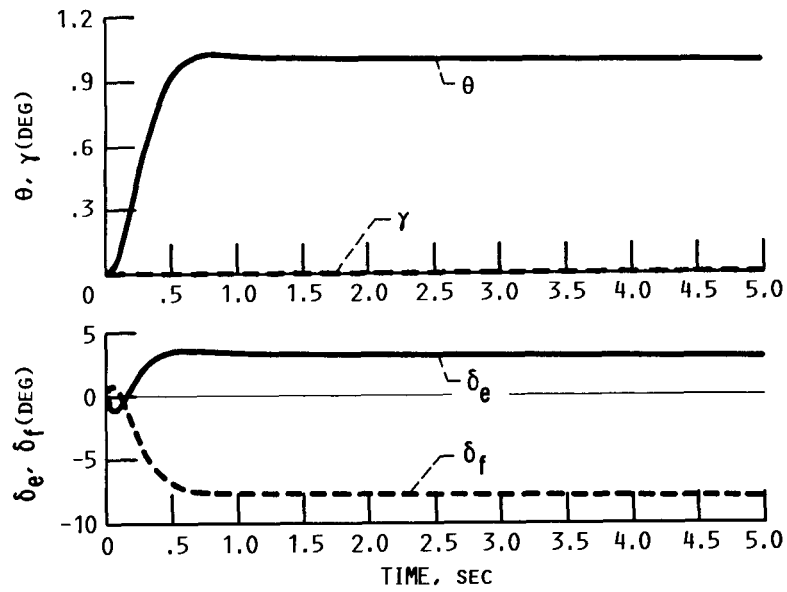


FIGURE 10. - INITIAL DESIGN RESPONSE TO STEP  $\theta_c = 1$  DEG.

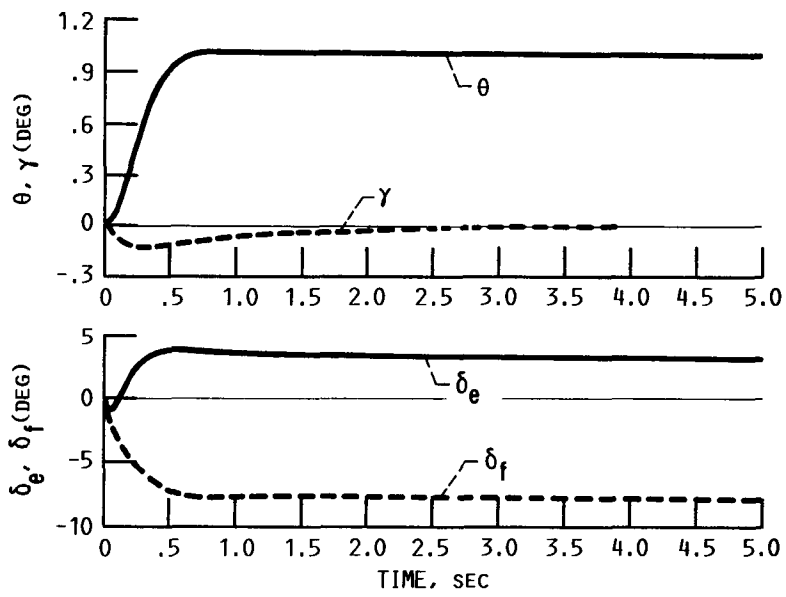


FIGURE 11. - FINAL DESIGN RESPONSE TO STEP  $\theta_c = 1$  DEG.



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16. Abstract A methodology to improve the stability robustness of feedback control systems designed using direct eigenspace assignment techniques is presented. The method consists of considering the sensitivity of the minimum singular value of the return difference transfer matrix at the plant input to small changes in the desired closed-loop eigenvalues and the specified elements of the desired closed-loop eigenvectors. Closed-form expressions for the gradient of the minimum return difference singular value with respect to desired closed-loop eigenvalue and eigenvector parameters are derived. Closed-form expressions for the gradients of the control feedback gains with respect to the specified eigenspace parameters are obtained as an intermediate step. The use of the gradient information to improve the guaranteed gain and phase margins in eigenspace assignment based designs is demonstrated by application to an advanced fighter aircraft.					
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