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ABSTRACT

This paper extends the integrating matrix technique of computational mechanics to include the effects of concentrated masses. The stability of a flexible rotating beam with discrete masses is analyzed to determine the critical rotational speeds for buckling in the inplane and out-of-plane directions. In this problem, the beam is subjected to compressive centrifugal forces arising from steady rotation about an axis which does not pass through the clamped end of the beam. To determine the eigenvalues from which stability is assessed, the differential equations of motion are solved numerically by combining the extended integrating matrix method with an eigenanalysis. Stability boundaries for a discrete mass representation of a uniform beam are shown to asymptotically approach the stability boundaries for the corresponding continuous mass beam as the number of concentrated masses is increased. An error in the literature is also noted for the discrete mass problem concerning the behavior of the critical rotational speed for inplane buckling as the radius of rotation of the clamped end of the beam is reduced.

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NOTATION

$D_\xi(x)$	discontinuous function defined by Eq. 15
D_ξ	diagonal matrix obtained from $D_\xi(x)$
E	Young's modulus of elasticity
I_{yy}, I_{zz}	second moments of area about y and z axes, respectively
I_1	integrating matrix
L	length of beam
m	mass distribution of beam (continuous plus concentrated)
M	concentrated mass
n	number of intervals into which the beam is divided
p_x, p_y, p_z	distributed inertial loads in x, y, and z directions, respectively
R	radius of rotation of clamped end of beam
S_ξ	diagonal matrix defined by Eq. 24
T	tensile force in beam
t	time
T_ξ	diagonal matrix defined by Eq. 25
v, w	deformation of elastic axis in y and z directions, respectively
XYZ	coordinate system which rotates with beam such that X-axis lies along the initial or undeformed position of elastic axis and passes through the axis of rotation, which is parallel to the Z axis
x	running coordinate along X-axis measured from clamped end of beam
y, z	centroidal principal axes of beam cross section
Δ_ξ	interpolation matrix
$\delta(x)$	Dirac delta function
η	variable of integration

λ	eigenvalue
μ	continuous mass distribution of beam
ξ	position of discrete mass measured from clamped end of beam
Ω	rotational speed of beam
( )	time derivative, $\frac{\partial}{\partial t}$
$(\ )'$	spatial derivative, $\frac{\partial}{\partial x}$
[ ]	square matrix
[ ]	diagonal matrix
{ }	column matrix

INTRODUCTION

The dynamic behavior of flexible rotating beams continues to receive considerable attention in the literature as it constitutes a fundamental problem in applied mechanics. Further, beams comprise parts of many rotating structures of engineering significance. One area which is receiving attention is the problem of the buckling of beams due to rotation-induced compressive centrifugal forces. Such forces arise, for example, in a rotating beam whose axis of rotation does not pass through the clamped end of the beam. As the beam rotates, its clamped end describes a circle of nonzero radius about the axis of rotation. This geometrical arrangement can be specified by a dimensionless parameter involving the radius of this circle and the length of the beam. If such a rotating beam has one of its cross-sectional principal axes parallel to the axis of rotation, transverse buckling can occur either in the plane of rotation or out of the plane of rotation, depending, among other factors, on the value of the geometrical parameter and the rotational speed. The problem of determining the vibrations and stability of rotating beams subjected to compressive centrifugal forces has been treated by several investigators [1-9]. References 8 and 9 solved the differential equations of motion numerically using an integrating matrix (Refs. 10-12) in combination with an eigenanalysis to determine the eigenvalues from which stability is assessed. The method of solution was shown to be numerically exact and not to exhibit the convergence problems associated with the approximate methods of solution which have been applied by others to the study of rotation-induced buckling. The integrating matrix procedure has been extended to include both nonuniformly spaced grid points (Ref. 13) and concentrated masses (Ref. 14). The purpose of this paper is to employ the integrating matrix procedure extended to include concentrated masses in a more complete and rigorous examination of the buckling behavior of the discrete mass representation of the flexible rotating beam which has been treated in Ref. [6].

FORMULATION

A geometrical arrangement giving rise to rotation-induced radial centrifugal forces which can be compressive rather than tensile, and thus leading to the possibility of a buckling-type instability, is depicted in Fig. 1. A beam of length L is clamped to the inside of a ring of radius R which is rotating with constant angular velocity Ω about an axis perpendicular to the plane of the ring and passing through its center. If the principal bending planes of the beam are oriented perpendicular and parallel to the plane of the ring, transverse buckling can occur either in the plane of rotation or perpendicular to the plane of rotation, depending on the value of such parameters as the geometric ratio R/L and the rotational speed. The use of an integrating matrix procedure in the numerical solution of the buckling behavior of the uniform rotating beam depicted in Fig. 1 for $0 \leq R/L \leq 2.0$, and in the identification of the limiting values of R/L below which buckling cannot occur, has been treated in Ref. [9]. The integrating matrix formulation for the case of a beam with continuous properties described in Ref. [9] has been extended to include the case in which concentrated masses are also present [14]. The modifications required to account for discrete masses result in new matrices which simply have to be added to the matrices previously established in Ref. [9] for a beam with continuous mass. Because the formulation for the beam with continuous mass remains unchanged, for brevity here reference will be made to Ref. [9] for pertinent details and explicit expressions. The notation of Ref. [9] is maintained as much as possible.

Governing Equations

The equations of motion which describe the uncoupled out-of-plane and in-plane bending of the beam in Fig. 1 are, from Ref. [9],

$$m\ddot{w} - (Tw')' + (EI_{yy}w'')'' = 0 \tag{1}$$

$$m(\ddot{v} - \Omega^2 v) - (Tv')' + (EI_{zz}v'')'' = 0 \tag{2}$$

where the tensile force $T(x)$ is given by

$$T = -\int_x^L m\Omega^2(R - \eta)d\eta. \tag{3}$$

A positive value of T indicates tension. The boundary conditions corresponding to a clamped end at $x = 0$ and a free end at $x = L$ are

$$w(0,t) = w'(0,t) = v(0,t) = v'(0,t) = 0 \tag{4}$$

$$w''(L,t) = w'''(L,t) = v''(L,t) = v'''(L,t) = 0. \tag{5}$$

A procedure using an integrating matrix as an operator to eliminate the spatial dependence from the partial differential equations of motion, and the reduction of the resulting equations to matrix eigenvalue form for determination of the eigenvalues from which the stability of the beam can be assessed, is fully described in Ref. [9] for a beam with a continuous mass distribution.

The extension of the integrating matrix technique to include concentrated masses proceeds initially along the same lines employed previously in Ref. [9]. The governing equations are first formally integrated to isolate the derivatives w'' and v''. To this end, equations (1) and (2) are written in the form

$$(EI_{yy}w'')'' = p_z + (Tw')' \tag{6}$$

$$(EI_{zz}v'')'' = p_y + (Tv')' \tag{7}$$

where

$$\begin{aligned} T &= \int_x^L p_x(\eta)d\eta \\ p_x &= -m\Omega^2(R - x) \\ p_y &= -m(\ddot{v} - \Omega^2 v) \\ p_z &= -m\ddot{w}. \end{aligned} \tag{8}$$

Formally integrating equations (6) and (7) twice from x to L and applying the boundary conditions of zero shear at the free end of the beam leads to

$$EI_{yy}w''(x,t) = \int_x^L \{-p_x(\eta,t)[w(\eta,t) - w(x,t)] + p_z(\eta,t)(\eta - x)\}d\eta \tag{9}$$

$$EI_{zz}v''(x,t) = \int_x^L \{-p_x(\eta,t)[v(\eta,t) - v(x,t)] + p_y(\eta,t)(\eta - x)\}d\eta. \tag{10}$$

As these equations are linear, it is sufficient to consider extension of the analysis for the case of a single concentrated mass, M, at an arbitrary position $x = \xi$ $(0 < \xi < L)$. The aggregate effect of multiple concentrated masses may then be obtained by superposition.

Let the continuous mass distribution of the beam be denoted by $\mu(x)$. Then, the mass distribution $m(x)$ of the beam (distributed plus concentrated) can be written as

$$m(x) = \mu(x) + M\,\delta(x - \xi) \tag{11}$$

where $\delta(x)$ is the usual Dirac delta function. Substituting (11) into (9) and (10) and assuming time solutions of the form

$$\begin{aligned} v(x,t) &= \bar{v}(x)e^{\lambda t} \\ w(x,t) &= \bar{w}(x)e^{\lambda t} \end{aligned} \tag{12}$$

equations (9) and (10) take the form

$$\begin{aligned} &EI_{yy}\bar{w}''(x) + \int_x^L \mu(\eta)\Omega^2\{(\eta - R)[\bar{w}(\eta) - \bar{w}(x)]\}d\eta \\ &\quad + D_\xi(x)M\Omega^2\{(\xi - R)[\bar{w}(\xi) - \bar{w}(x)]\} \\ &= \lambda^2\{\int_x^L \mu(\eta)[x - \eta]\bar{w}(\eta)d\eta + D_\xi(x)M[x - \xi]\bar{w}(\xi)\} \end{aligned} \tag{13}$$

$$EI_{zz}\bar{v}''(x) + \int_x^L \mu(\eta)\Omega^2\{(\eta - R)[\bar{v}(\eta) - \bar{v}(x)] - \bar{v}(\eta)[\eta - x]\}d\eta$$

$$+ D_\xi(x)M\Omega^2\{(\xi - R)[\bar{v}(\xi) - \bar{v}(x)] + \bar{v}(\xi)[x-\xi]\}$$

$$= \lambda^2\{\int_x^L \mu(\eta)[x - \eta]\bar{v}(\eta)d\eta + D_\xi(x)M[x - \xi]\bar{v}(\xi)\} \tag{14}$$

where, by the properties of the Dirac delta function, $D_\xi(x)$ is the discontinuous function given by

$$D_\xi(x) = \begin{cases} 1 & x \leq \xi < L \\ 0 & 0 < \xi < x \end{cases} . \tag{15}$$

Equations (13) and (14) are valid for all values of x along the beam. In particular, they are valid at $n + 1$ discrete grid points (stations) with ordering

$$0 = x_0 < x_1 < \quad < x_{n-1} < x_n = L. \tag{16}$$

The points need not have uniform spacing, and the location ξ of the concentrated mass need not coincide with a grid point. Writing equations (13) and (14) at $n + 1$ discrete points along the beam the resulting sets of equations may be cast into matrix form. The integrals appearing in these equations are associated with the continuous mass distribution and are evaluated by introducing an integrating matrix operator as described in Ref. [9]. The integrating matrix operator is also used to express the displacements $\bar{v}$ and $\bar{w}$ in terms of the curvatures $\bar{v}''$ and $\bar{w}''$ so that the curvatures appear as the dependent variables in the final matrix equations. Because the underlying differential equations are uncoupled, the resulting matrix equations are also uncoupled. However, following Ref. [9], for computational convenience the resulting equations are written in the combined form.

$$\begin{matrix} [\ _{11}] & [0] \\ [0] & [\ _{22}] \end{matrix} \begin{matrix} \{\overline{w}\ \} \\ \{\overline{v}\ \} \end{matrix} = \lambda^2 \begin{matrix} [\ _{11}] & [0] \\ [0] & [\ _{22}] \end{matrix} \begin{matrix} \{\overline{w}\ \} \\ \{\overline{v}\ \} \end{matrix} \qquad (17)$$

or, in condensed notation,

$$[\ ]\{\Phi\} = \lambda^2[\ ]\{\Phi\} \qquad (18)$$

where

$$\{\Phi\} = \begin{matrix} \{\overline{w}''\} \\ \{\overline{v}''\} \end{matrix}$$

$$[\ _{11}] = [G_{11}] + [\hat{G}_{11}]$$

$$[\ _{22}] = [G_{22}] + [\hat{G}_{22}] \qquad (19)$$

$$[\ _{11}] = [H_{11}] + [\hat{H}_{11}]$$

$$[\ _{22}] = [H_{22}] + [\hat{H}_{22}].$$

The submatrices $[G_{11}]$, $[G_{22}]$, $[H_{11}]$, and $[H_{22}]$ are functions of the continuous properties of the beam, rotational speed, and integrating matrix operators. Explicit expressions for these submatrices are given in Ref. [9]. The submatrices $[\hat{G}_{11}]$, $[\hat{G}_{22}]$, $[\hat{H}_{11}]$ and $[\hat{H}_{22}]$ are associated with the concentrated mass and are functions of the magnitude and location of the mass, rotational speed, and integrating matrix operator. Explicit expressions for these submatrices are derived below.

It should be noted that the inclusion of a concentrated mass leads to an eigenvalue problem of the same form as for a beam with only continuous mass. Further, the effects of a concentrated mass enter only through terms which are additive to terms obtained previously in Ref. [9] for a beam with no concentrated masses. Thus, the size of the eigenvalue problem given by equation (18) is not increased when concentrated mass effects are included. The eigenvalue problem given by equation (18) can be solved using standard eigensolution techniques to yield the eigenvalues and eigenvectors which characterize the dynamic behavior and stability of both the inplane and out-of-plane motions of the beam. Since this is a kinetic approach [15] for assessing stability, the solution of equation (18) will identify all the instabilities of the system, both dynamic and static. However, because of the absence of both Coriolis forces and damping forces in the present equations, only static (buckling) instabilities can occur.

Explicit Form of Additive Matrices for Concentrated Mass

Explicit expressions for the submatrices $[\hat{G}_{11}]$, $[\hat{G}_{22}]$, $[\hat{H}_{11}]$, and $[\hat{H}_{22}]$ which arise from the concentrated mass terms in equations (13) and (14) are rather straightforward to derive following the procedures outlined in Ref. [14]. However, special considerations are required for the case where the mass is not at one of the grid points established along the beam and in the treatment of the discontinuous function $D_\xi(x)$.

Equations (13) and (14) involve $\bar{v}$ and $\bar{w}$ evaluated at the concentrated mass position ξ. If the mass M is not at a grid point, an interpolation polynomial (e.g., Lagrange interpolation) must be employed to express $\bar{v}(\xi)$ and $\bar{w}(\xi)$ in terms of the values of $\bar{v}$ and $\bar{w}$ at the grid points. Because the integrating matrix has elements which depend only on the grid point locations x_o, $,x_n$, and not on the values of the function at the grid points, the interpolation polynomial employed for the concentrated mass must have coefficients which depend only on ξ and the grid point locations. Consistent with this requirement, consider an interpolation of the form

$$\bar{u}(\xi) = a_0(\xi)\bar{u}(x_0) + a_1(\xi)\bar{u}(x_1) + \quad + a_n(\xi)\bar{u}(x_n) \ . \tag{20}$$

If equation (20) is an mth order interpolant $(m \leq n)$ and ξ is not a grid point, then in general $a_k(\xi) \equiv 0$ for $k < \gamma$ and/or $k > \gamma + m$. Here, γ is an appropriate reference integer and $x_0 \leq x_\gamma < \xi < x_{\gamma+m} \leq x_n$. By choosing m sufficiently large (e.g., Lagrange interpolation with $m = 7$) the resulting interpolating polynomial gives a high degree of accuracy without the need to cluster grid points about ξ. To use the interpolation polynomial given by equation (20) in the present context, let $[\Delta_\xi]$ be the square matrix of order $n + 1$ with the ijth element

$$(\Delta_\xi)_{ij} = a_{j-1}(\xi), \qquad j = 1,2, \quad , n + 1 \tag{21}$$

i.e., $[\Delta_\xi]$ has identical rows and each row has as elements the successive coefficients in the interpolation polynomial. Premultiplying $\{\bar{u}\}$ by $[\Delta_\xi]$ and using equation (20) leads to the relation

$$[\Delta_\xi]\{\bar{u}\} = \{\bar{u}(\xi)\}. \tag{22}$$

The matrix $[\Delta_\xi]$ may be regarded as an interpolation matrix. If the mass is at a grid point, equation (20) shows that $[\Delta_\xi]$ is a matrix with a single nonzero column consisting of ones.

If the mass is not at a grid point, some care must also be taken in defining a diagonal matrix $[D_\xi]$ that gives the effect at the grid points of multiplying by the discontinuous function $D_\xi(x)$ which changes from unity to zero across $x = \xi$. Suppose that ξ lies in the subinterval $x_{j-1} < \xi \leq x_j$, $j = 1,2, \quad , n$. Then if $[D_\xi]$ is the diagonal matrix of order $n + 1$ with the diagonal elements

$$(D_\xi)_{ii} = \begin{matrix} 1 & i \leq j \\ 0 & i > j \end{matrix} \tag{23}$$

$[D_\xi]\{f\}$ gives the vector of values of the function $D_\xi(x)f(x)$ at the grid points.

Using the matrices $[\Delta_\xi]$ and $[D_\xi]$ introduced above and introducing the diagonal matrices $[S_\xi]$ and $[T_\xi]$ defined by

$$[S_\xi] = M[x - \xi] \tag{24}$$

$$[T_\xi] = M\Omega^2(\xi - R)[I] \tag{25}$$

the concentrated mass terms in equations (13) and (14) lead to the explicit matrix expressions

$$[\hat{G}_{11}] = [D_\xi][T_\xi][[\Delta_\xi] - [I]][I_1]^2$$

$$[\hat{G}_{22}] = [D_\xi][[T_\xi][[\Delta_\xi] - [I]] + \Omega^2[S_\xi][\Delta_\xi]][I_1]^2 \qquad (26)$$

$$[\hat{H}_{11}] = [\hat{H}_{22}] = [D_\xi][S_\xi][\Delta_\xi][I_1]^2$$

where [I] is the identity matrix and $[I_1]$ is the integrating matrix of Refs. [9] and [14]. To arrive at equation (26) use has been made of the relations

$$\{\bar{v}\} = [I_1]\{\bar{v}'\} = [I_1]^2\{\bar{v}''\}$$

$$\{\bar{w}\} = [I_1]\{\bar{w}'\} = [I_1]^2\{\bar{w}''\} \qquad (27)$$

to express the displacements $\bar{v}$ and $\bar{w}$ in terms of the curvatures $\bar{v}''$ and $\bar{w}''$ as was done in Ref. [9]. The effect of several concentrated masses is obtained by superposition.

If the concentrated mass, M, is placed at the free end of the rotating beam, (i.e., $\xi = L$), the boundary conditions $\bar{v}'''(L) = 0$ and $\bar{w}'''(L) = 0$ must be replaced by the differential equations for the inplane and out-of-plane motion of the concentrated mass. However, it is shown in Ref. [14] that the application of an appropriate limiting procedure to the governing equations for $\bar{v}$ and $\bar{w}$ given by equations (13) and (14) for an interior concentrated mass as ξ tends to L produces the proper boundary conditions for a concentrated mass at $\xi = L$. Hence, equations (13) and (14), which have been derived assuming that the concentrated mass is not at the free end of the beam, remain valid for the case in which the mass is at the free end of the beam. Thus, for computation there is no need for any different treatment if the mass is at the free end.

NUMERICAL RESULTS

The stability of a beam which is rotating in a plane perpendicular to the axis of rotation and clamped off this axis as indicated in Fig. 1 has been analyzed to determine the critical rotational speeds for buckling in the inplane and out-of-plane directions. An application of the integrating matrix technique to treat the case of a beam with a continuous mass distribution is described in Ref. [9]. In the present paper, attention is directed to the case in which the mass distribution is approximated by concentrated masses at discrete points along the length of the beam. There are several well-established discrete mass approximations which can be used in dynamic analyses of beams. However, the less usual concentrated mass representation employed in Ref. [6] is adopted here to allow a rigorous comparison of the present numerical results with the analytical results given in Ref. [6]. It should be emphasized that this choice is not restrictive as all the observations made and the conclusions reached based on the numerical studies using the mass model of Ref. [6] are equally valid for other discrete mass representations.

In the present studies, three concentrated mass approximations to a uniform beam are considered, as indicated in Fig. 2. In each case, the sum of the concentrated masses is maintained equal to the total mass of the uniform beam. As in Ref. [9], numerical results here are obtained by dividing the beam into ten equal segments and using an integrating matrix $[I_1]$ which expresses the integrand as a seventh-degree polynomial in the form of Newton's forward-difference interpolation formula. The computer program which is employed to solve the eigenvalue problem given in equation (17) uses the double shift QR algorithm (Ref. [16]) and is written to take advantage of any uncoupling which may be present in the combined matrix equation. Stability analyses were made for each of the three concentrated mass representations depicted in Fig. 2 over a range of values of the ratio R/L for a fixed value of the beam length. The resulting stability boundaries are shown in Fig. 3, where the nondimensional critical rotational speeds for buckling in the inplane and out-of-plane directions are presented as a function of R/L. Also shown in Fig. 3 for comparison with the present results are the corresponding boundaries for the beam with a uniform mass distribution obtained in Ref. [9]. The nondimensional rotational speed in Fig. 3 involves the inplane bending stiffness EI_{zz}. The numerical results are for the particular case in which the inplane and out-of-plane bending stiffnesses are equal, that is, $EI_{zz} = EI_{yy}$.

For the continuous mass case, it is seen that instability is first encountered in the plane of rotation for all values of R/L. Although the entire beam is in tension for $R/L < 0.5$, the term $m\Omega^2 v$ which appears in the inplane equation of motion represents a component of centrifugal force normal to the beam which decreases the inplane natural frequency with increasing rotational speed. Thus, for sufficiently large rotational speeds, the possibility of buckling in the plane of rotation can not be precluded even when the beam is entirely under tension. The results (which are numerically exact) indicate that the limiting value of the ratio R/L below which buckling cannot occur is R/L = 0.0. The results also show that the stability boundary is asymptotic to the vertical line given by R/L = 0.0, so that the beam is stable for R/L = 0.0 for all finite values of the rotational speed. The numerical results for the case of buckling out of the plane of rotation indicate that the limiting value of the ratio R/L below which buckling cannot occur is R/L = 0.50. As in the case of inplane buckling, the stability boundary is asymptotic to the vertical line through the limiting value of R/L, so that the beam is stable for R/L = 0.50 for all finite values of the rotational speed. This limiting value is consistent with the fact that for R/L < 0.50 the beam is entirely in tension and no other destabilizing centrifugal forces exist as in the case of the inplane direction. The reader is referred to Ref. [9] for further discussion of this case and comparison with the results of similar work by others which has appeared in the literature.

The stability boundaries for the beam with concentrated masses are seen to exhibit a behavior which is similar to that obtained for the beam with a continuous mass distribution. Again, because of the destabilizing term $m\Omega^2 v$ in the inplane equation of motion, instability is first encountered in the plane of rotation for all values of R/L. For values of R/L greater than about 1.2, both the inplane and out-of-plane stability boundaries approach the corresponding continuous mass boundaries from below as the number of concentrated masses is increased. As R/L is reduced, the limiting values of

R/L below which buckling cannot occur approach the limiting values for the beam with continuous mass as the number of discrete masses is increased. For the inplane case, it should be noted that all the boundaries are asymptotic to the same vertical line given by $R/L = 0.0$, and that they approach the boundary for the continuous mass beam from the left as the number of concentrated masses is increased. For out-of-plane buckling the limiting values of R/L below which buckling cannot occur for the one, two and five-mass approximations are 1.0, 0.75, and 0.60, respectively. These limiting values of R/L correspond to values of R/L below which the beam is entirely in tension. The corresponding stability boundaries are asymptotic to the vertical lines through the limiting values of R/L and approach the boundary for the continuous mass beam from the right as the number of concentrated masses is increased. This is in contrast to the behavior for the inplane direction. The difference in behavior of the inplane and out-of-plane stability boundaries as R/L is reduced is associated with the term $m\Omega^2 v$ which appears in the inplane equation of motion. As mentioned earlier, this term represents a component of centrifugal force acting normal to the beam and decreases the inplane natural frequency with increasing rotational speed. As the number of concentrated masses representing the continuous mass beam increases, there are more and more masses being "shifted" toward the clamped end of the beam (where the displacements v in the buckling mode are smaller) thereby reducing the destabilizing effect associated with the term $m\Omega^2 v$.

The inplane and out-of-plane buckling of a rotating beam having both one and two concentrated masses has been treated in Ref. [6], which presents results from an exact solution of the differential equations obtained by applying Newton's law to the mass hypothetically cut from the beam and from equilibrium considerations of the massless flexible beam. The present numerical results for the out-of-plane buckling behavior are in excellent agreement with the results presented in Ref. [6]. However, the present results for the in-plane buckling behavior are not in agreement with the results presented in Ref. [6]. In particular, the inplane stability boundaries in Ref. [6] do not exhibit the correct limiting behavior as R/L is reduced to zero. It appears that the development in Ref. [6] has incorrectly accounted for the $m\Omega^2 v$ term.

CONCLUDING REMARKS

The stability of a flexible beam with discrete masses which is rotating in a plane perpendicular to the axis of rotation and clamped off this axis was analyzed to determine the critical rotational speeds for buckling in the in-plane and out-of-plane directions. The differential equations of motion were solved numerically using an extended integrating matrix method which includes the effect of concentrated masses in combination with an eigenanalysis to determine the eigenvalues from which stability was assessed. Extension of the integrating matrix method to include concentrated masses and its application appear to be new. It was shown that the extended integrating matrix procedure, when applied to the governing differential equations of motion for a beam which includes discrete masses, leads to a matrix eigenvalue problem in which the matrices associated with the discrete matrices are simply additive to the matrices previously derived for a beam with continuous mass. The stability boundaries obtained using the present method of solution were shown to asymptotically approach the corresponding boundaries for a beam with a continuous mass distribution as the number of concentrated masses is increased. As the ratio of the radius of the circle traced out by the clamped end of the rotating beam to the length of the beam is reduced, the inplane and out-of-plane behavior was shown to be different due to the presence of a destabilizing inertial term which appears in the inplane equation of motion. These results have identified what appears to be an error in the literature with respect to the limiting behavior of the critical rotational speed for inplane buckling as the radius of rotation of the clamped end of the beam is reduced.

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16. Abstract

This paper extends the integrating matrix technique of computational mechanics to include the effects of concentrated masses. The stability of a flexible rotating beam with discrete masses is analyzed to determine the critical rotational speeds for buckling in the inplane and out-of-plane directions. In this problem, the beam is subjected to compressive centrifugal forces arising from steady rotation about an axis which does not pass through the clamped end of the beam. To determine the eigenvalues from which stability is assessed, the differential equations of motion are solvdd numerically by combining the extended integrating matrix method with an eigenanalysis. Stability boundaries for a discrete mass representation of a uniform beam are shown to asymptotically approach the stability boundaries for the corresponding continuous mass beam as the number of concentrated masses is increased. An error in the literature is also noted for the discrete mass problem concerning the behavior of the critical rotational speed for inplane buckling as the radius of rotation of the clamped end of the beam is reduced.

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