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SIMULATION AND CONTROL PROBLEMS IN ELASTIC ROBOTS

By

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ABSTRACT

Computational issues associated with modeling and control of robots with revolute joints and elastic arms are considered. A manipulator with one arm and pinned at one end is considered to investigate various aspects of the modeling procedure and the model, and the effect of coupling between the rigid-body and the elastic motions.

The rigid-body motion of a manipulator arm is described by means of a reference frame attached to the "shadow beam," and the linear elastic operator denoting flexibility is defined with respect to this reference frame. The small elastic motion assumption coupled with the method of assumed modes is used to model the elasticity in the arm. The complete model coupling the rigid-body and the elastic motion is highly nonlinear, and contains terms up to quartic in powers of the amplitudes of the assumed modes. It is shown that only terms up to quadratic in these model amplitudes need to be retained.

An important aspect of the coupling between the rigid-body and the elastic motion is the centrifugal stiffening effect. This effect stiffens the elastic structure, as to be expected on physical grounds, gives rise to a time-varying inertia term for the rigid-body motion, and, in general, results in an effective inertia term smaller than the rigid-body inertia term. In fact, this reduction in inertia determines the limitation of the small motion assumption. If the elastic behavior is excited sufficiently so as to cause a vanishing effective rigid-body motion inertia term, one should either modify the manipulator model, or consider the forcing profiles that excite the elastic motion least. The Fourier series expansion of a few such profiles is examined to provide insight in this regard.

Simulation results are presented for an elastic beam pinned at one end and free at the other, and rotating in a horizontal plane, and control issues such as the order of the model, number of sensors, and modal extraction are examined within this context. It is shown that the effect of centrifugal stiffening is pronounced on the rigid-body motion during transition, and ignoring it in the control model leads to gross inaccuracies in response. The effect of including varying amounts of flexibility on the response is studied.

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OBJECTIVE

To investigate modeling, control, and computational issues associated with elastic manipulators

SCOPE

Revolute joints

Actuators at joints only

Shadow beam approach

Small elastic motion, and limit of such an assumption

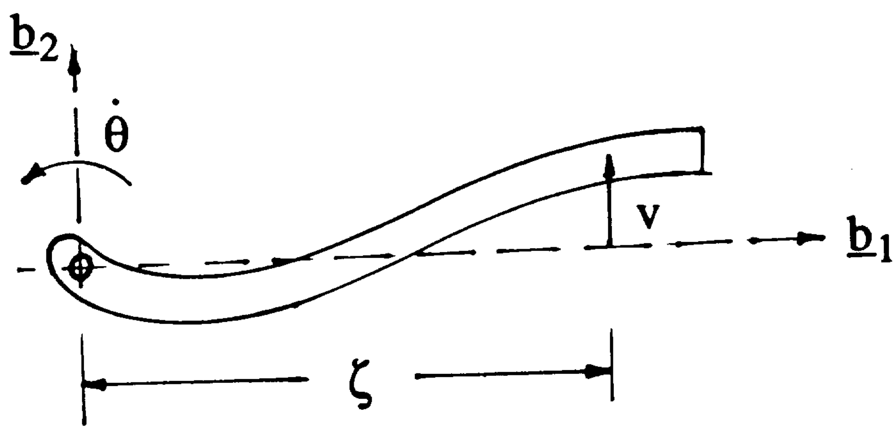
Nonlinear model

Control issues

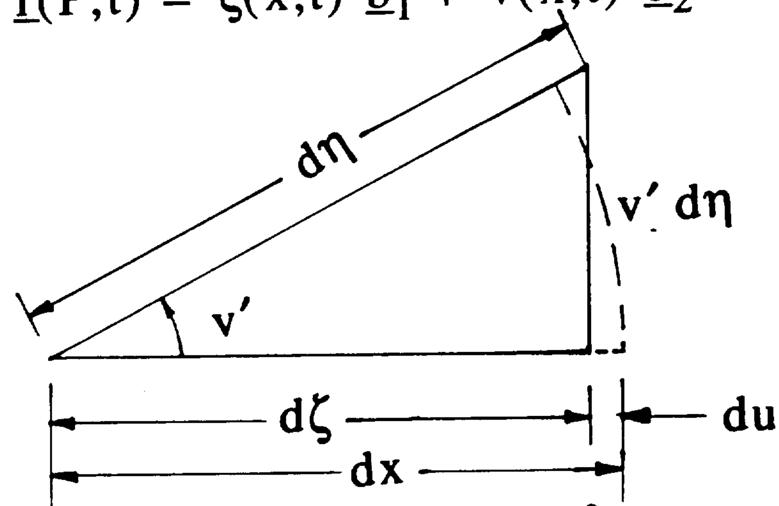
Illustrative example

Pinned - free link

Reference frame located at the pin joint; describes rigid-body motion. Elastic motion is defined with respect to this frame



$$\underline{r}(P,t) = \zeta(x,t)\,\underline{b}_1 + v(x,t)\,\underline{b}_2$$



$$d\eta^2 = d\zeta^2 + (v'\,d\eta)^2$$

Notes: x is the position of the point in the undeformed configuration
The beam rotates in a horizontal plane

u(x,t) is obtained by integrating

$$d\zeta = [\ 1 - (\frac{\partial v}{\partial x})^2\ ]^{1/2}\ d\eta$$

where

$$\eta(x,t) = x + s(x,t)$$

$$\zeta(x,t) = x - u(x,t)$$

On integration,

$$\zeta(x,t) \approx \eta - 1/2\int_0^{\eta}[\ (\frac{\partial v}{\partial \sigma})^2\ ]\ d\sigma$$

or

$$u(x,t) \approx -\ s(x,t) + 1/2\int\ [(\frac{\partial v}{\partial \sigma})^2\ ]\ d\sigma$$

s(x,t) : axial vibration term

Integral : results in centrifugal stiffening term

Neglect axial vibration

Elastic Displacement

$$\underline{u}(x,t) = -\ u(x,t)\ \underline{b}_1 + v(x,t)\ \underline{b}_2$$

Position

$$\underline{r} = \zeta(x,t)\ \underline{b}_1 + v(x,t)\ \underline{b}_2$$

Velocity

$$\dot{\underline{r}} = -\ \frac{\partial u}{\partial t}\ \underline{b}_1 + \frac{\partial v}{\partial t}\ \underline{b}_2 + \dot{\underline{\theta}}\ X\ \underline{r}$$

Kinetic Energy

$$K = 1/2 \int \dot{\underline{r}} \cdot \dot{\underline{r}}\ dm$$

Potential Energy

$$V = 1/2 \int_0^L EI(x)\ (v'')^2\ dx$$

Lagrangian

$$L = K - V$$

Notes : ($\dot{}$) corresponds to partial derivative with respect to time, ( )$'$ corresponds to spatial derivative, ρ is the mass per unit length, and EI(x) is the flexural rigidity

$$L = 1/2 \int_0^L \rho \, (\dot{v}^2 + \dot{\theta}^2 v^2 + 2 x \dot{v} \dot{\theta} + x^2 \dot{\theta}^2) \, dx$$

$$- 1/2 \int_0^L EI(v'')^2 dx$$

$$- 1/2 \, \dot{\theta}^2 \int_0^L \rho \, x \int_0^x (v')^2 d\sigma \, dx$$

$$+ 1/2 \int_0^L [- 1/2 \int_0^x \frac{d}{dt}((v')^2) d\sigma]^2 \rho \, dx$$

$$+ 1/2 \, \dot{\theta} \int_0^L \rho \, v \int_0^x \frac{d}{dt}((v')^2) \, d\sigma \, dx$$

$$- 1/2 \, \dot{\theta} \int_0^L \rho \, \dot{v} \int_0^x (v')^2 d\sigma \, dx$$

$$+ 1/2 \, \dot{\theta}^2 \int_0^L \rho/4 \, [\int_0^x (v')^2 d\sigma]^2 \, dx$$

Assumed Modes

$$v(x,t) = \sum_{i=1}^{N_1} \phi_i(x) \, a_i(t)$$

$\phi_i(x)$: Admissible functions

Define

$$m_{ij} = \int_0^L \rho \phi_i(x)\ \phi_j(x)\ dx$$

$$k_{ij} = \int_0^L EI(x)\ \phi_i''(x)\ \phi_j''(x)\ dx$$

$$s_{ij}(x) = \int_0^x \phi_i'(x)\ \phi_j'(x)\ dx$$

$$p_{ij} = \int_0^L \rho\ x\ s_{ij}(x)\ dx$$

$$s_{ijkl} = \int_0^L s_{ij}(x)\ s_{kl}(x)\ dx$$

$$q_{ijk} = \int_0^L \rho\ \phi_k(x)\ s_{ij}(x)\ dx$$

$$r_i = \int_0^L \rho\ x\ \phi_i(x)\ dx$$

Then,

$$\int_0^x (v')^2\ dx = s_{ij}(x)\ a_i\ a_j$$

$$\int_0^L \rho\ x\ [\ \int_0^x (v')^2\ d\sigma\ ]\ dx = p_{ij}\ a_i\ a_j$$

$$\int_0^L [\int_0^x \frac{d}{dt}((v')^2)\ d\sigma]^2 \rho\ dx = 4\ s_{ijkl}\ a_i\ \dot{a}_j\ a_k\ \dot{a}_l$$

$$\int_0^L \rho\ v\ [\ \int_0^x \frac{d}{dt}((v')^2)\ d\sigma\ ]\ dx = 2\ q_{ijk}\ a_i\ \dot{a}_j\ a_k$$

Substituting in the Lagrangian,

$$L = 1/2\, I_o\, \dot{\theta}^2 + 1/2\, m_{ij}\, \dot{a}_i\, \dot{a}_j + \dot{\theta}\, r_i\, \dot{a}_i$$

$$- 1/2[k_{ij} + (p_{ij} - m_{ij})\dot{\theta}^2]\, a_i\, a_j$$

$$+ 1/2\, s_{ijkl}\, a_i\, \dot{a}_j\, a_k\, \dot{a}_l + \dot{\theta}^2/8\, s_{ijkl}\, a_i\, a_j\, a_k\, a_l$$

$$+ \dot{\theta}\, q_{ijk}\, a_i\, \dot{a}_j\, a_k - \dot{\theta}/2\, q_{ijk}\, a_i\, a_j\, \dot{a}_k$$

Example :

Beam parameters

Cross-section : 6 in x 3/8 in
Length = 3.6576 m (12 ft)
ρ = 4.015 kg/m
EI = 756.65 N . m

Admissible functions : Normalized eigenfunctions of a pinned-free beam

$$m_{ij} = \delta_{ij}\ ; \quad k_{ij} = \omega_i^2\, \delta_{ij}$$

where δ_{ij} is the Kronecker delta

Notes : The summation convention, $\sum\sum m_{ij}\, a_i\, a_j = m_{ij}\, a_i\, a_j$, etc., will be employed for conciseness - i.e., repeated indices in an expression indicate summation over appropriate range.

Natural frequencies and Centrifugal stiffening coefficients

i	ω_i	p_{ij}		
		j = 1	j = 2	j = 3
1	15.82	6.397	1.861	-0.366
2	51.282	1.861	17.905	6.195
3	106.983	-0.366	6.195	35.999

Coriolis terms, q_{ijk}

i	j	k = 1	k = 2	k = 3
1	1	-0.152	0.143	0.008
1	2	0.415	-0.144	0.169
1	3	0.077	0.347	-0.143
2	1	0.415	-0.144	0.169
2	2	-0.175	0.152	-0.117
2	3	0.883	-0.196	0.145
3	1	0.077	0.347	-0.143
3	2	0.883	-0.196	0.145
3	3	-0.178	0.171	-0.152

Other coupling terms s_{ijkl}

Note: $s_{ijkl} = s_{jikl} = s_{ijlk} = s_{jilk}$

i	j	k	l = 1	l = 2	l = 3
1	1	1	0.669	0.099	0.001
1	1	2	0.099	1.800	0.444
1	1	3	0.001	0.444	3.570

1 1 1	2 2 2	1 2 3	0.099 0.275 -0.050	0.275 0.397 0.594	-0.050 0.594 0.901
1 1 1	3 3 3	1 2 3	0.001 -0.050 0.183	-0.050 -0.005 -0.074	0.183 -0.074 -0.113
2 2 2	1 1 1	1 2 3	0.099 0.275 -0.050	0.275 0.397 0.594	-0.005 0.594 0.901
2 2 2	2 2 2	1 2 3	1.800 0.397 -0.005	0.397 5.010 0.147	-0.005 0.147 9.94
2 2 2	3 3 3	1 2 3	0.444 0.594 -0.074	0.594 1.470 1.500	-0.074 1.500 3.160
3 3 3	1 1 1	1 2 3	0.001 -0.050 0.183	-0.050 -0.005 -0.074	.183 -0.074 -0.113
3 3 3	2 2 2	1 2 3	0.444 0.594 -0.074	0.594 1.470 1.500	-0.074 1.500 3.160
3 3 3	3 3 3	1 2 3	3.57 0.901 -0.113	0.901 9.94 3.160	-0.113 3.160 20.000

Notes : The magnitudes of the terms q_{ijk} and s_{ijkl} are small. In addition, they are multiplied by the cubic and quartic powers of modal amplitudes. Hence they will be dropped from further development.

Retaining terms only up to quadratic in modal amplitudes,

$$L = 1/2\,[I_o - (p_{ij} - m_{ij})\, a_i\, a_j]\, \dot{\theta}^2 + 1/2\, m_{ij}\, \dot{a}_i\, \dot{a}_j$$

$$-\ 1/2\, k_{ij}\, a_i\, a_j$$

The equation for rigid-body motion is

$$\frac{d}{dt}[(I_o - (p_{ij} - m_{ij})\, a_i\, a_j)\, \dot{\theta}\,] = T$$

And the elastic motion is described by

$$m_{ij}\, \ddot{a}_j + [k_{ij} + (p_{ij} - m_{ij})\, \dot{\theta}^2]\, a_j = T\, \phi_i'(0),$$

$$i = 1, 2, \ldots.., N_1$$

Measurements at x = 0

$$\theta_1 = \theta + v'(0,t)$$

$$\dot{\theta}_1 = \dot{\theta} + \dot{v}'(0,t)$$

Choices for the control model

1. Ignore elastic effects completely

   Control model : $I_o \ddot{\theta} = T$

   $$\theta = \theta_1 \qquad \dot{\theta} = \dot{\theta}_1$$

2. Rigid-body model, with the shadow frame angle properly extracted

   Control model : $I_o \ddot{\theta} = T$

   $$\theta = \theta_1 - v'(0,t)$$
   $$\dot{\theta} = \dot{\theta}_1 - \dot{v}'(0,t)$$

3. A few elastic modes are included, and the modal coordinates are approximated

   Control model:

   $$\frac{d}{dt}[(I_o - (p_{ij} - m_{ij})\, a_i\, a_j)\, \dot{\theta}\,] = T$$
   $$i,j \leq N_2, \quad N_2 < N_1$$

Notes : N_1 is the number of modeled modes. N_2 is the number of modes used for controller design. $N_1 = 3$ for the following simulation results.

4. Appropriate number of sensors used to obtain accurate modal coordinates.

$$N_2 = N_1$$

Control synthesis

Computed torque method

Pointwise-optimal control method

Open-loop Maneuver

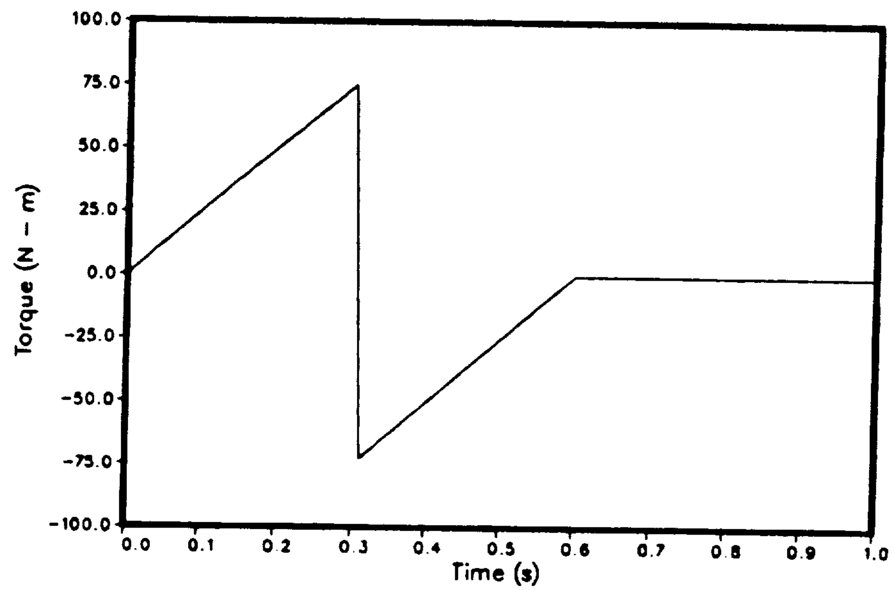


Fig. 1 : Torque Profile for Open-Loop Maneuver

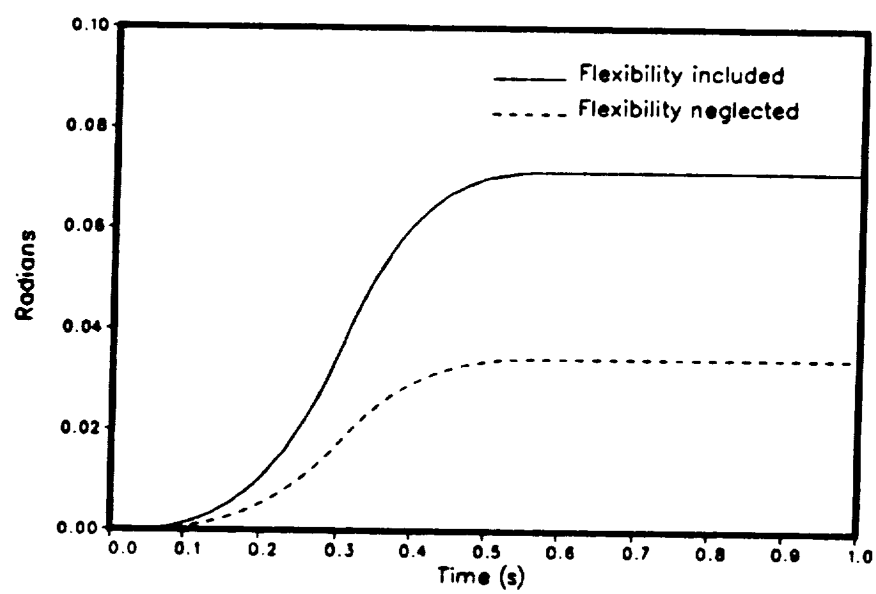


Fig. 2 : Position Response of the Beam for the Torque Above

Feedback Control

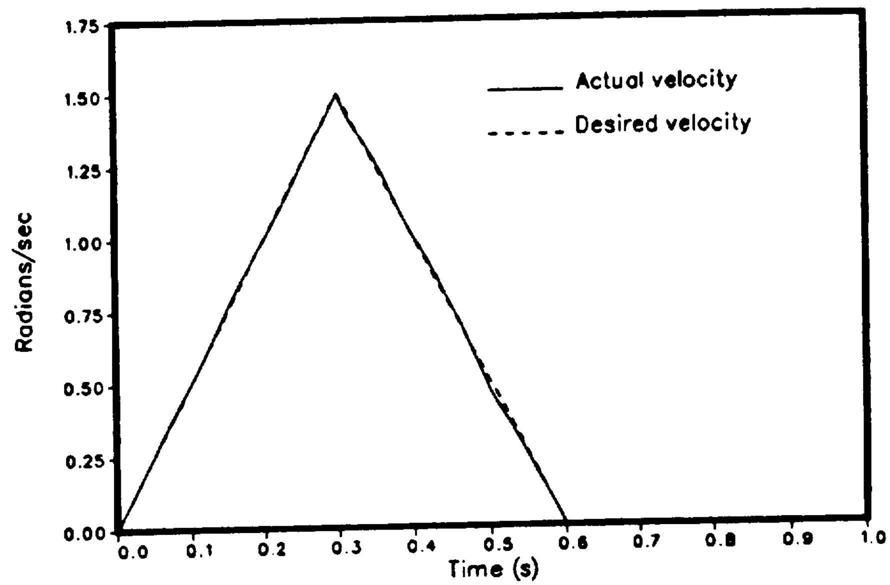


Fig. 3 : Rigid Model, Velocity response

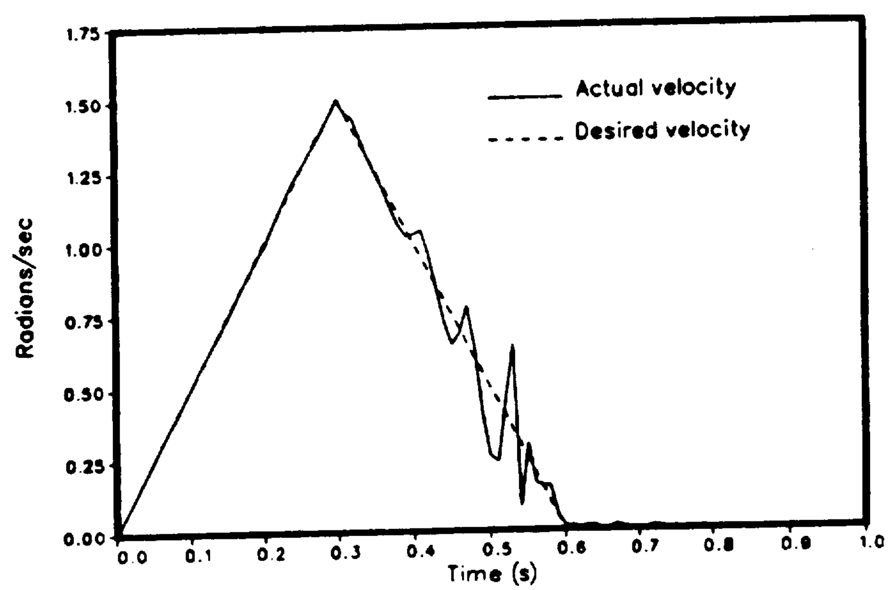


Fig. 4 : One Flexible Mode Included in the Model

Feedback Control, contd.

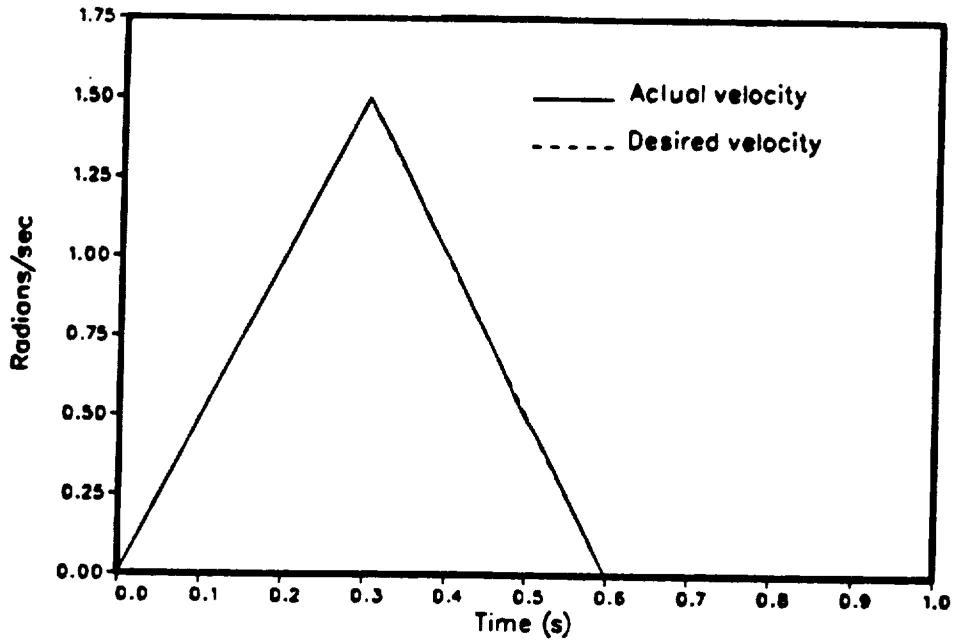


Fig. 5 : Three Flexible Modes Included in the Model

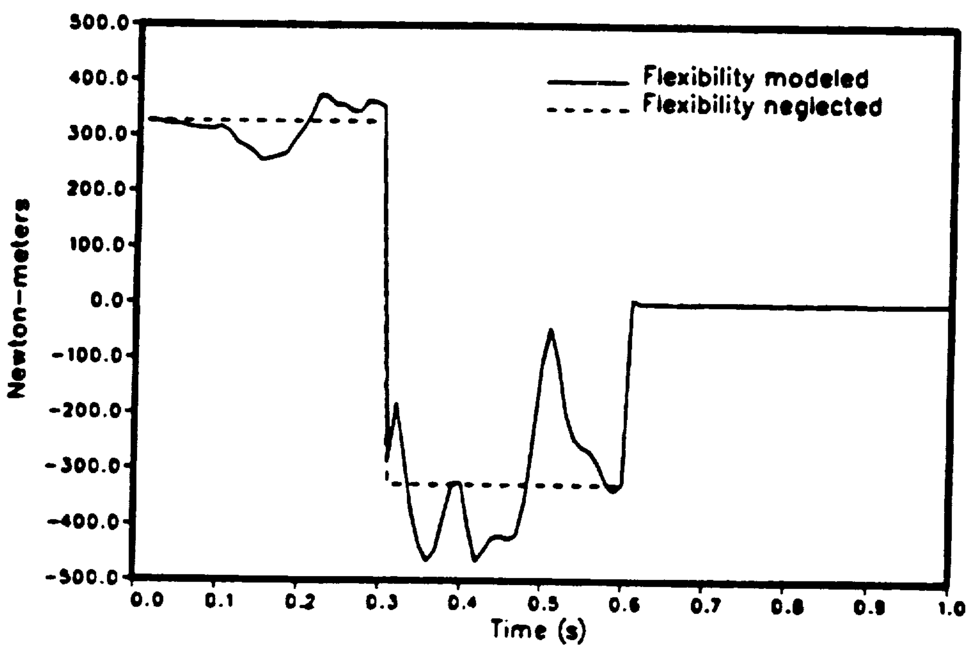


Fig. 6 : Comparison of Open- and Closed-Loop Torques

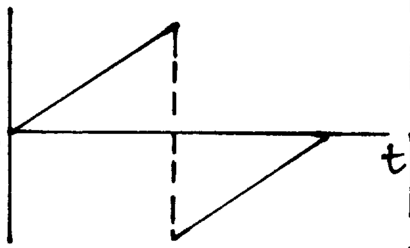
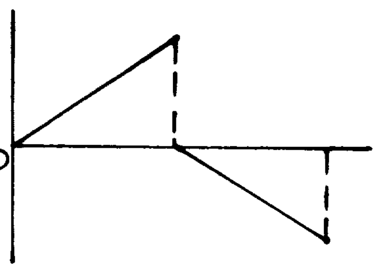
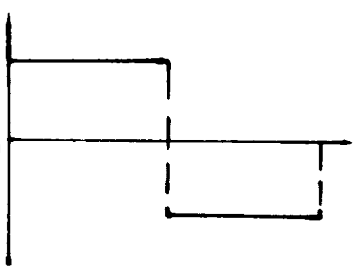
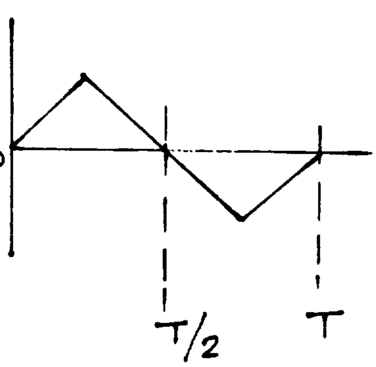
Effects of centrifugal stiffening

1. Provides a strong coupling between the rigid-body and elastic motion

2. Increases the stiffness of the structure

3. Reduces the effective rigid-body inertia term. Can cause it to vanish if the elastic motion is large. May have to modify the model, or vary the torque profiles.

Torque profiles and their Fourier coefficients

	Coefficients of $\sin(n\omega_0 t)$	Coefficients of $\cos(n\omega_0 t)$
	$\dfrac{(-1)^{n-1}}{n\omega_0}$	0
	$\dfrac{(1-(-1)^n)}{3n\omega_0}$	$\dfrac{2((-1)^n-1)}{3\pi\omega_0 n^2}$
	$\dfrac{(1-(-1)^n)}{n\omega_0}$	0
	$n = 2m+1$ $\dfrac{8}{3\pi\omega_0 n^2}$	0

$\int_0^T\int_0^T$ Torque . dt is the same for all cases

$$\omega_0 = \frac{2\pi}{T}$$

Computational Issues for Control of multi-link flexible robot arm

1. The dynamic model can be arrived at by modeling each link independently and imposing constraints at the joints

2. The link geometry may not be simple

3. s_{ijkl}, q_{ijk}, may not be negligible, and the control model may include all the terms

4. The choice of admissible functions for each of the links may be different

5. Sampling rates - should not excite elastic motion

6. Control input computation may pose formidable burden.

The above issues can be adequately addressed by selecting pointwise-optimal control law for control input computations, where, the inputs can be computed at least one time step ahead.

Conclusions

1. A complete model for control of a flexible link is developed

2. Modeling issues are examined within the context of an example

3. Several control issues are investigated

4. It is shown that centrifugal stiffening effect on rigid-body motion is significant

5. There is a strong coupling between rigid-body and elastic motions; ignoring this coupling results in gross inaccuracies in response.