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INPUT-OUTPUT ORIENTED COMPUTATION ALGORITHMS FOR THE CONTROL OF LARGE FLEXIBLE STRUCTURES

By

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ABSTRACT

This presentation will overview work in progress aimed at developing computational algorithms addressing two important aspects in the control of large flexible space structures; namely, the selection and placement of sensors and actuators, and the resulting multivariable control law design problem.

The issue of sensor/actuator set selection is particularly crucial to obtaining a satisfactory control design, as clearly a poor choice will inherently limit the degree to which "good" control can be achieved. Moreover, it is becoming increasingly clear that systematic methods are required for determining prior to the control law design phase whether a particular candidate sensor/actuator set will yield acceptable closed-loop performance, irrespective of the particular control system design methodology used.

With regard to control law design we are driven by concerns stemming from the practical issues associated with eventual implementation of multivariable control laws, such as reliability, limit protection, multimode operation, sampling rate selection, processor throughput, etc. Naturally, the burden imposed by dealing with these aspects of the problem can be reduced by ensuring that the complexity of the compensator is minimized.

Our approach to these problems is based on extensions to input/output oriented techniques that have proven useful in the design of multivariable control systems for aircraft engines. In particular, we are exploring the use of relative gain analysis and the condition number as a means of quantifying the process of sensor/actuator selection and placement for shape control of a large space platform. Complementing this activity is the development of a new multivariable design approach that allows the designer to precisely control the complexity of the resulting compensator. The technique incorporates input-output performance criteria such as the popular singular-value loop-shaping approach, yet without resorting to high-order compensators inherent to observer-based design approaches.

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OUTLINE

Motivation for research

Approach

- design philosophy
- focus: shape control

Results

- preliminary experiences with key aspect of design problem: S/A set selection

Motivation

- control of flexible structures recognized as a key emerging technology for GE

- leverage considerable design experience with MIMO design process for medium complexity problems (aircraft engines) to high complexity systems (LFSS, IFPSC)

- a particularly important unresolved issue: decoupling the process of sensor/actuator selection from control law design phase

GOALS:

- Quantitative, systematic approach to problem of decoupling S/A selection from control law design process
- Complexity reduction/management
  - Design process
  - Final product

Complexity clearly a major issue ...

(1) countably infinite number of S/A sets

- number
- placement
- types, etc.

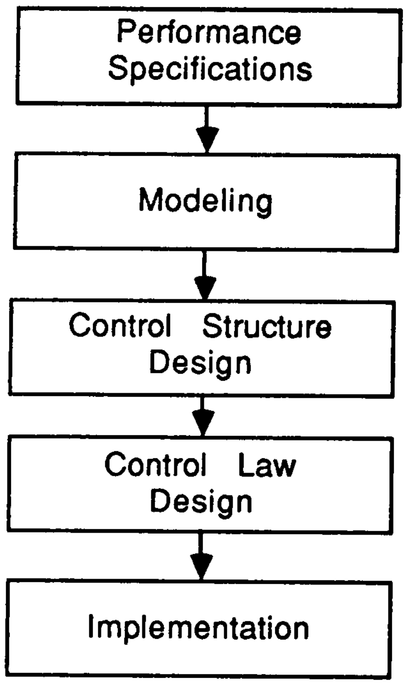
(2) large dynamic-order models (many flex. modes)

- order reduction a critical step, due primarily to limitations in traditional control law design approaches (observer-based compensators imply high-order)
- model accuracy/fidelity often sacrificied to accomodate these inherent computational limitations (spillover effects, etc.)
- conflict with S/A selection process, where numerical behaviour improves with model dynamic order

(3) Shape Control - very large I/O dimensionality

Approach

- traditional F.E.M. /MIMO design approach, based on the following cycle



Focus:

(1) Control Structure Design

- selection, pairing

(2) Control Law Design

- MIMO design w/ fixed order compensators

(3) Uncertainty Modeling

(4) Computations

- S/A placement
- Frequency-domain control-law design
- Stable Factorization (balancing, order reduction)

- demonstrate via shape control problem for LFSS

Control Structure Design:

" That portion of the control system design process which deals with the selection and pairing of measurement and manipulation variables "

- S.O.A.: typically Ad - hoc, often arbitrarily chosen

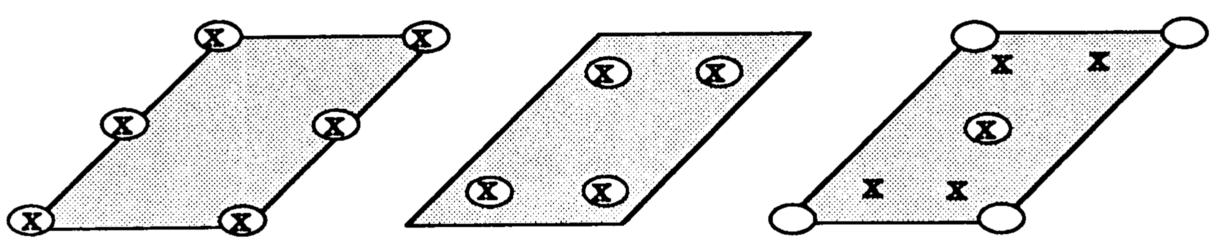
  much iteration, involving control law design phase

- probably the most critical step in entire design process (certainly true for shape control via MIMO techniques ...)

<u>(1) Sensor/Actuator Selection</u>

- how many?
- locations?
- types* (* ignor for the present ...)

colocated vs. non-colocated



x = sensor ○ = actuator

- systematic search for candidates that guarantee "good" closed-loop control

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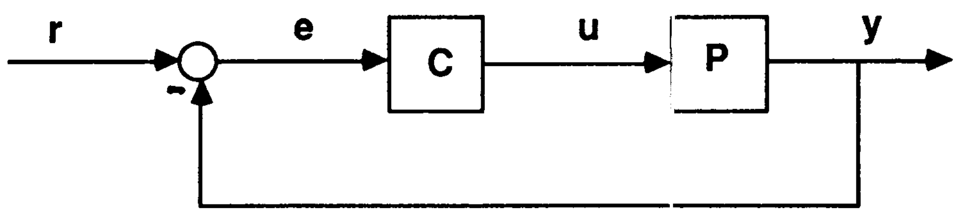
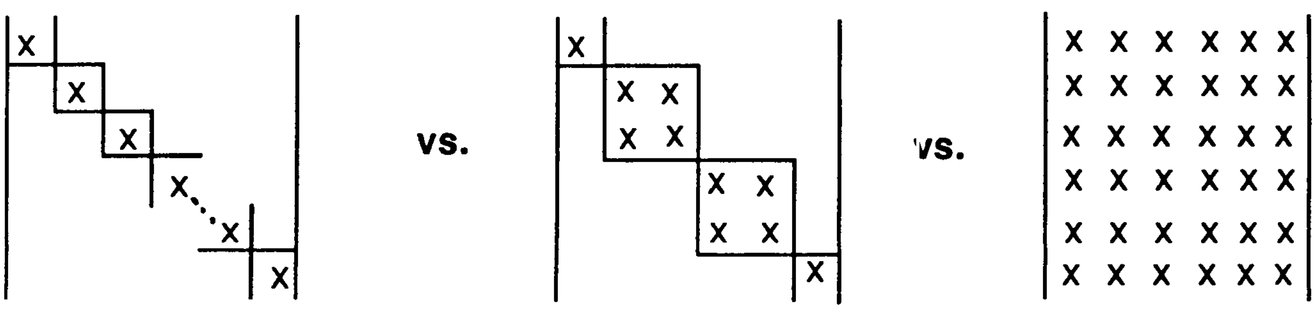
(2) Pairings and Decentralization

- given a S/A set, how do we interconnect for minimal closed-loop interaction?

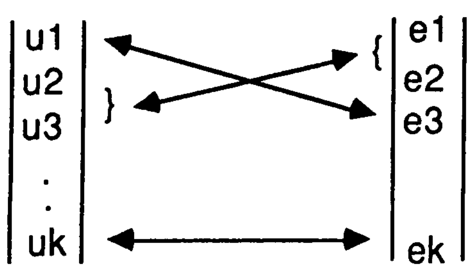
- assume standard unity-feedback configuration



Decentralization: Choose C as follows ...



I/O Pairing:



Example:

Colocated, fully decentralized:

ui <--> ei, for all i.

(1) S/A Selection

High level algorithm ...

- determine a large number of candidate sensor/actuator sets
- reduce to a manageable number of acceptable control structures (design - by - analysis)

Specifics ...

- develop a necessary condition for assessing a candidate control structure based on stability robustness
- uncertainty characterization: modified additive perturbations ...

$$\tilde{G} = G + \Delta G \qquad \frac{\bar{\sigma}(\tilde{G} - G)}{\bar{\sigma}(G)} < \delta$$

- connect control structure design process with stability robustness via following accepted fact: " plants with low condition number are 'easy' to control ... "

- *base selection process on condition number*

- we have the following result ...

Theorem: If C stabilizes P, then necessarily

$$K(P) < \frac{\delta^{-1}}{\underline{\sigma}(PC(I+PC)^{-1})} \cong \delta^{-1}, \quad \text{for} \quad \omega < \omega_c$$

- assumes "perfect" control at DC ...

<u>Selection Process:</u>

- compute condition number of candidate structures at DC
- discard those with large condition number ...

<u>Computational Aspects:</u>

- RGA (relative gain array) yields lower bounds on condition number, hence a necessary condition for viable control structures

- computational burden of RGA calculation small, but problem with exponential growth in complexity required to examine all possible combinations

- Example: > 3 x 10D10 ways to choose a 12 x 12 control structure from a set of 20 possible I/O pairs

Heuristic Solution:

- direct selection of inputs/outputs to minimize condition number

- based on SVD of plant DC-gain.

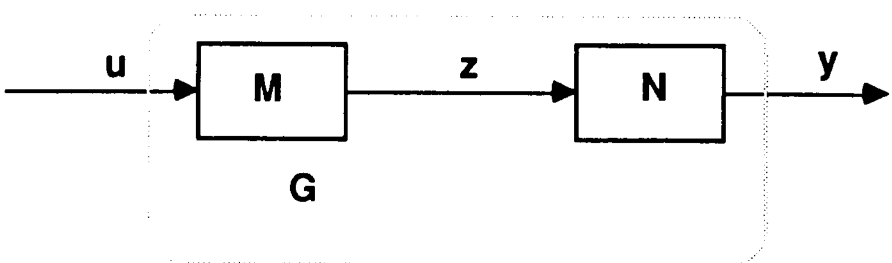
$$\kappa(G) := \frac{\bar{\sigma}(G)}{\underline{\sigma}(G)} \equiv \frac{\max \| G u \|}{\min \| G u \|}$$

- reduce condition number <-> (i) reduce max. sv
  (ii) increase min. sv

- DC gain: $G(0) = D + C(-A)^{-1}B$

- SVD: $G(0) = U S V'$

- introduce notion of input/output coupling operators ...



$$M = S^{\frac{1}{2}} V' \qquad N^{-1} = S^{\frac{-1}{2}} U'$$

Note: $z = M u$, $z = N^{-1} y$

- express u, y as sums of standard basis vectors, i.e.

$$u = \sum_i \alpha_i e_i, \quad y = \sum_i \beta_i e_i$$

- then we have

$$z = \sum_i \alpha_i M e_i = \sum_i \alpha_i M_i$$

$$z = \sum_i \beta_i N^{-1} e_i = \sum_i \beta_i N_i^{-1}$$

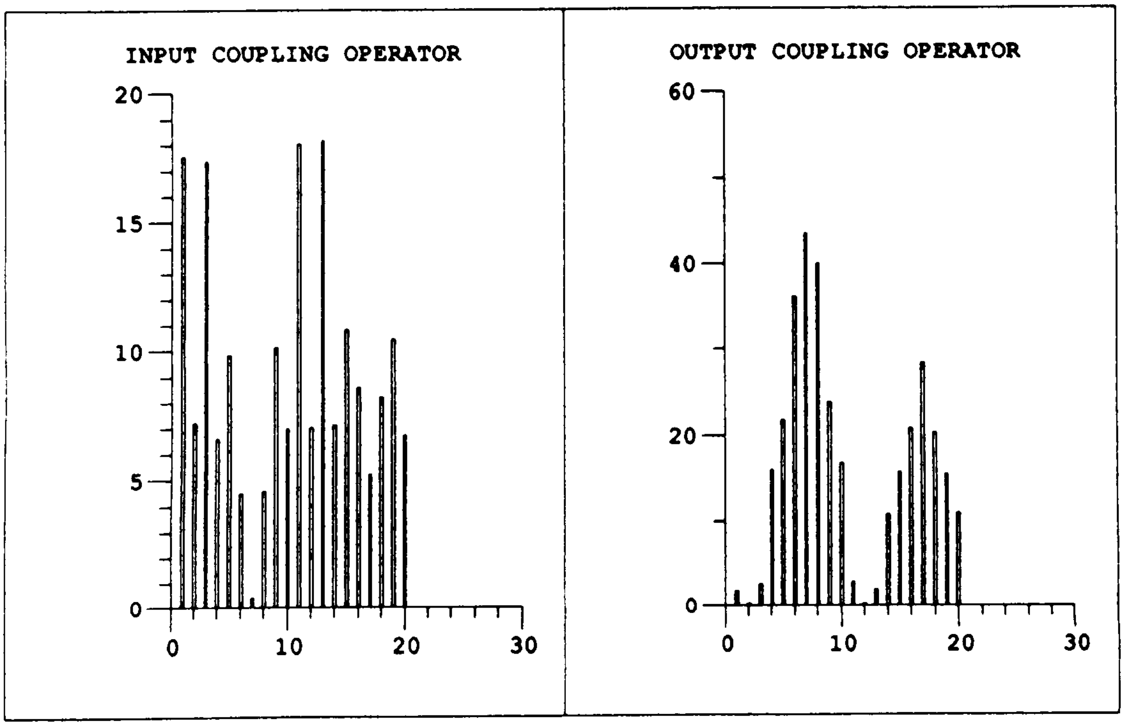
Design Heuristic:

- drop those inputs (outputs) corresponding to the maximum and minimum gains from u to z (y to z), i.e.

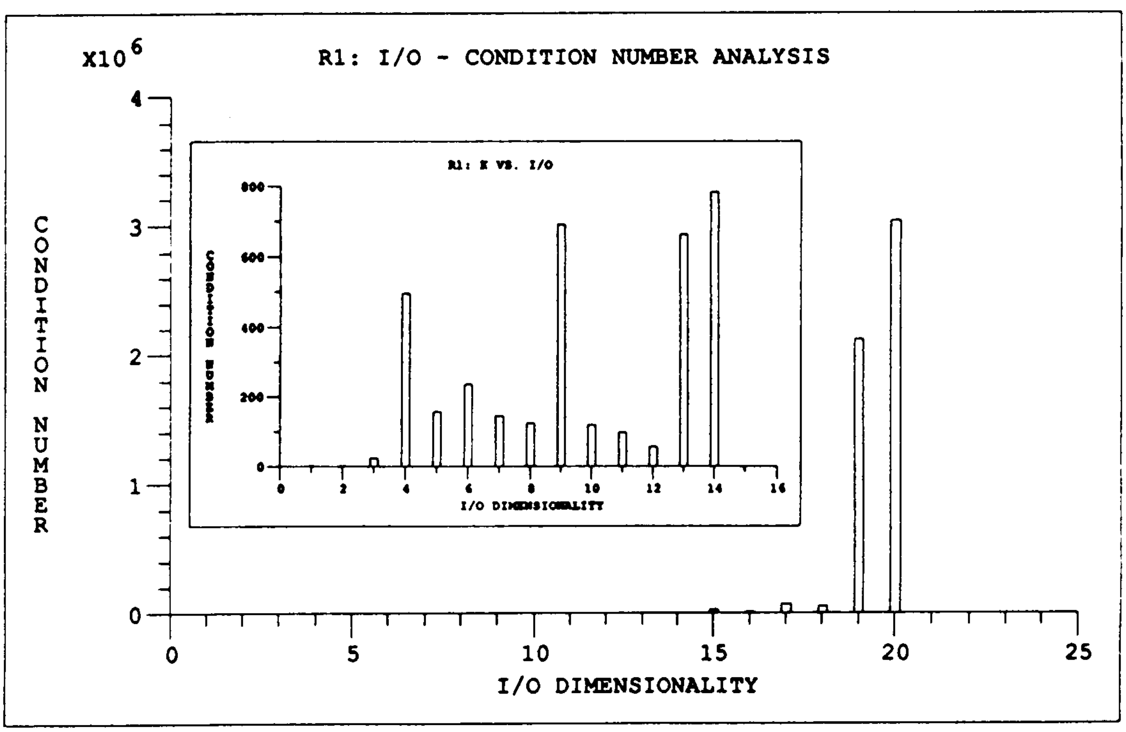
$$\| z \|_2 <= \sum_i | \alpha_i | \, \| M_i \|_2$$

- encouraging results for shape control application ...

Coupling Operators: 20 x 20 I/O, 25 flexible modes



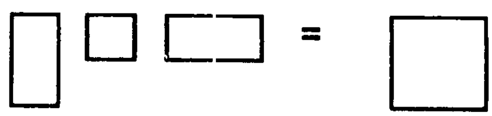
S/A selection: Condition number vs. I/O dimension

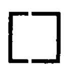


Some Observations :

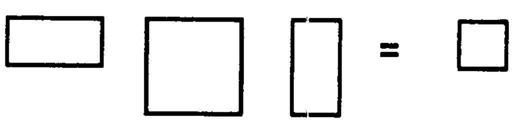
- systems characterized by large condition numbers (in general), and relatively large uncertainty
    - extremely difficult to control
    - severe performance limitations
- tendency towards partially colocated feedback structures (as a result of S/A selection process)
- lower condition numbers with non-colocated, and in particular, non-square control structures (non-square systems a challenge for MIMO control law design)
- numerical conditioning improves (generically) with increasing information content

RGA: $G(0) = C(-A)^{-1}B$

= 

Rank(G(0)) $\leq$ dim()

K(G(0)) $\downarrow$ as dim(A) $\uparrow$



Generically full-rank ...

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(2) Pairings/Cross-feed Degradation

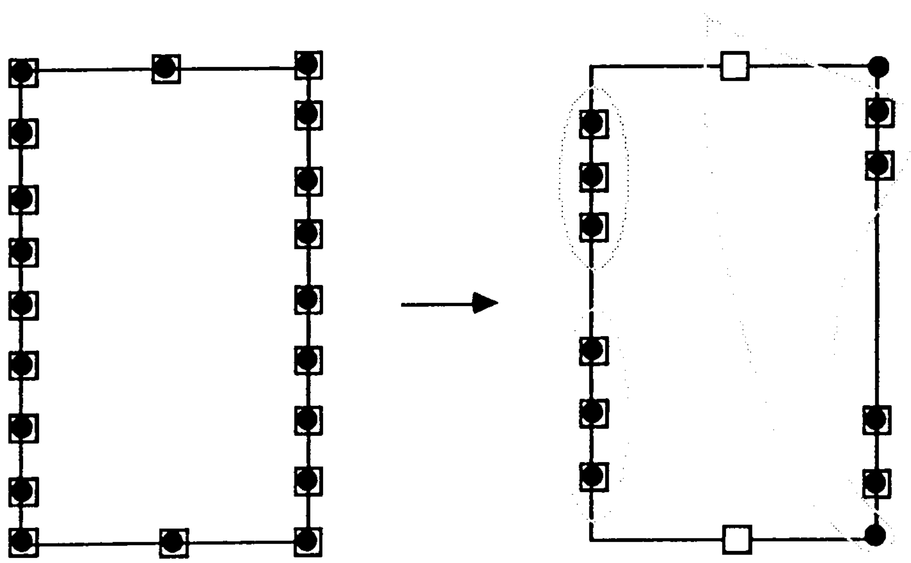
Tools:

- RGA, BRG (Bristol, Arkun, Maniousiouthakis)

Usage:

- Assess interactions between various feedback blocks
- Account for cross-feed degradation due to use of decentralized control structure

Preliminary result: 12 x 12 system of previous example ...



□ actuators ● sensors

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Control Law Design

- loop-shaping design philosophy
- account for practical implementation constraints
  - fault-tolerance and reliability
  - limit-protection and multimode operation
  - digital control aspects

  All severely impacted by complexity of compensator!

- 2 - stage design procedure allowing explicit constraints on compensator complexity

<u>1st Stage:</u> "Ideal" Compensator computation

- model-matching performance specification (sv loop-shaping basis)
- controller parameterization (IMC, SF)
- identify constraints on achievable performance (inner-outer factorization)
- compute "ideal" compensator, i.e. K* such that

$$\| H - H_D \|_2 = 0$$

2nd Stage: "Low-order" Compensator Computation

- analysis of "ideal" compensator (frequency response) to determine
  - (i) approx. required complexity (dynamic order)
  - (ii) values of compensator denominator terms, initial values of numerator terms

- parametric optimization to adjust compensator numerator terms (least-square approach) to minimize

$$\| H - H_D \|_2 = 0$$

Features:

- NOT an open-loop order-reduction procedure
- Closed-loop low (fixed) order design procedure, with a flavour of order reduction (Ideal comp. => lower-order parametric design)
- Frequency-domain oriented design, hence complexity proportional to I/O dimension, NOT state dimension
- Reduced emphasis/reliance on explicit order-reduction

Balancing & Order Reduction (SFPACK)

- balancing a popular method for model/controller order reduction

- also useful for avoiding large coefficients in state-space manipulations

Problems: Unstable or marginally stable systems

- decomposition solution: $G = G_+ + G_-$
- balance, reduce components, recombine
- Stable Factorization approach:

  $G = N D^{-1}$, N, D stable, right-coprime

  Form a composite system F, with state-space realization ...

$$F = \begin{bmatrix} D \\ N \end{bmatrix} \leftrightarrow \left[\begin{array}{c|c} A & B \\ \hline C_1 & E_1 \\ C_2 & E_2 \end{array}\right] \qquad \text{(as stable as we wish ...)}$$

  Order-reduction on F:

$$\hat{F} \leftrightarrow \left[\begin{array}{c|c} \hat{A} & \hat{B} \\ \hline \hat{C}_1 & E_1 \\ \hat{C}_2 & E_2 \end{array}\right] \leftrightarrow \begin{bmatrix} \hat{D} \\ \hat{N} \end{bmatrix}$$

  $\hat{G} = \hat{N} \hat{D}^{-1}$ - a reduced-order model of G