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MODELING AND STABILIZATION OF LARGE FLEXIBLE SPACE STATIONS

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ABSTRACT

In this paper we present a preliminary formulation of a large space structure. The system consists of a (rigid) massive body, which may play the role of experimental modules located at the center of the space station and a flexible configuration, consisting of several beams, which is rigidly attached to the main body. The equations that govern the motion of the complete system consist of several partial differential equations with boundary conditions describing the vibration of flexible components coupled with six ordinary differential equations that describe the rotational and translational motion of the central body.

In our investigation we consider the problem of (feedback) stabilization of the system mentioned above.

This study is expected to provide an insight into the complexity of design and stabilization of actual space stations. Some numerical results will be presented.

Modeling and Stabilization of Space Stations

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OUTLINE

MOTIVATION

Modeling of Space Stations

- Motivation
- Reference Coordinate Systems
- Inertia Tensors
- Equations of motion

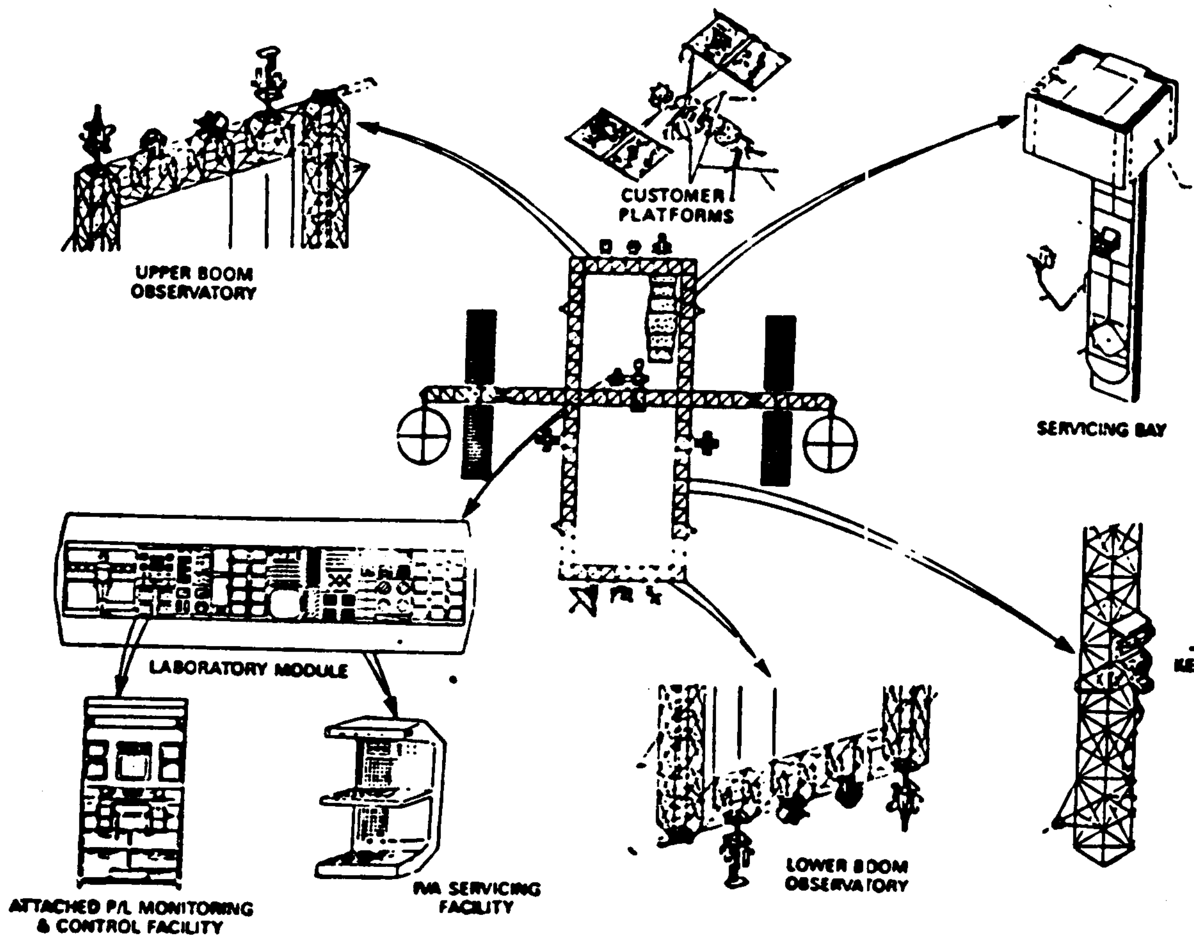
Stabilization

- Stability by feedback control
- Localized Control
- Other Control (B.B., D.Z.)

Numerical Simulation

- Finite Difference Method

Future Directions



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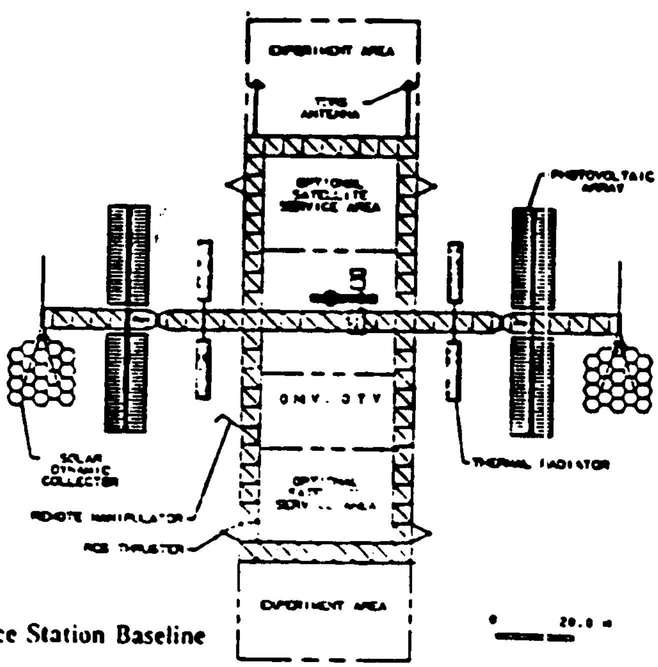

Space Station Baseline

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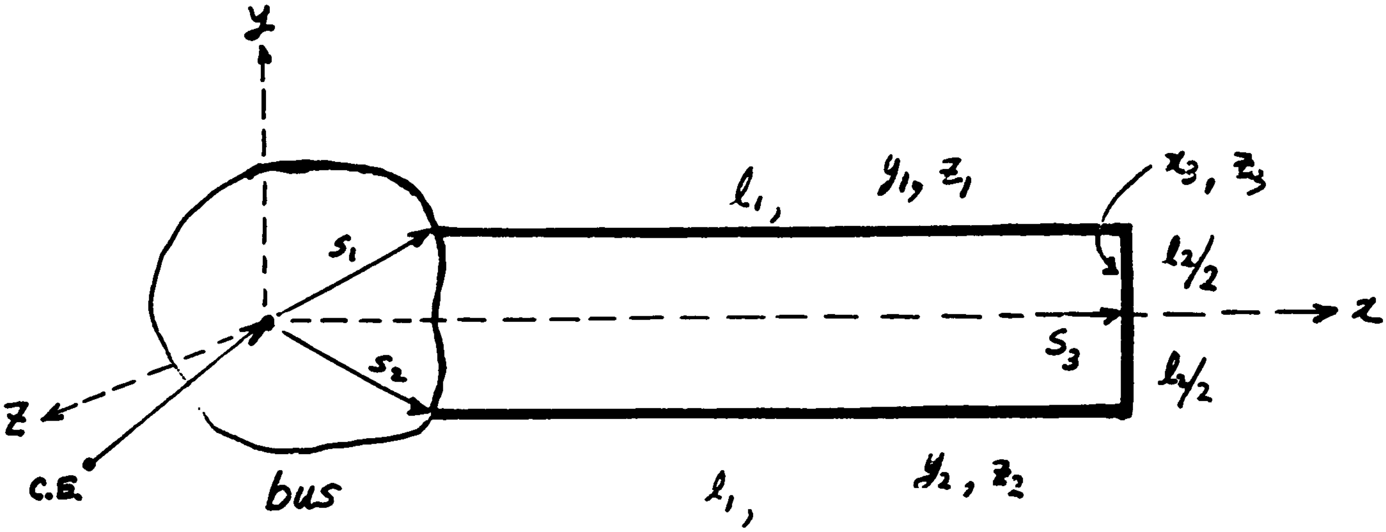


$$\varphi^1 = (0,\ y_1,\ z_1)',$$

$$\varphi^2 = (0,\ y_2,\ z_2)',$$

$$\varphi^3 = (x_3,\ 0,\ z_3)',$$

$$\tilde{r}_i = R + \underbrace{s_i + \varphi^i}_{r_i}, \qquad i = 1, 2, 3$$

Modeling of Space Station

Three types of motion:

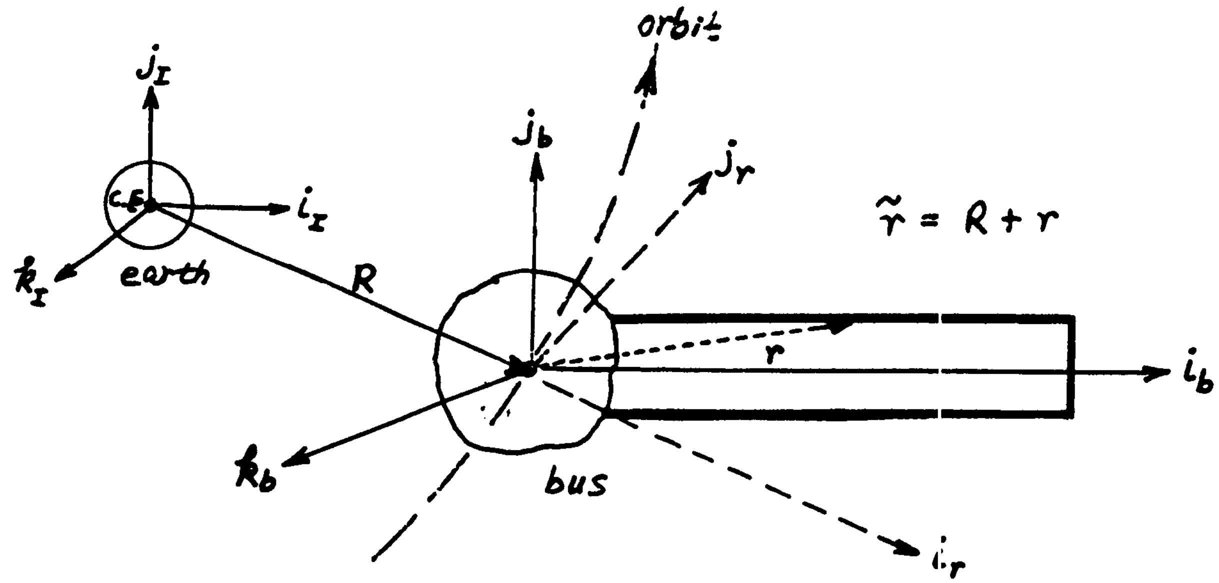
(a) Rigid body translation perturbing the orbit

(b) Rigid body rotation perturbing the orientation

(c) Vibration of elastic members

Reference Coordinate Systems

Body coord. : (i_b, j_b, k_b)
Orbit Coord. : (i_r, j_r, k_r)
Inertial coord. : (i_I, j_I, k_I)



Angular velocities

$$\omega_b = (\omega_x, \omega_y, \omega_z)'$$
$$\omega_r = (\omega_X, \omega_Y, \omega_Z)'$$
$$\omega = \omega_b + \omega_r = (\omega_1, \omega_2, \omega_3)'$$

Inertia Tensor

I_S = Inertia tensor of the bus

I_b = Inertia tensor of the elastic members

$$= \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{yx} & I_{yy} & -I_{yz} \\ -I_{zx} & -I_{zy} & I_{zz} \end{bmatrix}$$

where $I_{xx} = \int (y^2 + z^2)\, dm$,

$I_{yy} = \int (x^2 + z^2)\, dm$,

$I_{zz} = \int (x^2 + y^2)\, dm$,

$I_{xy} = \int x y \, dm$,

$I_{yz} = \int y z \, dm$,

$I_{zx} = \int z x \, dm$,

$r = (x, y, z)'$.

$I_T = I_S + I_b$

Equations of Motion

$$\delta \int_{t_1}^{t_2} (T + W)\, dt = 0$$

where T = Kinetic energy of the system,
W = Total work done by the forces.

$$T = T_S + T_b$$

$$T_S = \frac{1}{2}\int_{bus} \frac{d\tilde{r}}{dt}\cdot\frac{d\tilde{r}}{dt}\, dm, \qquad \tilde{r} = R + r$$

$$T_b = \frac{1}{2}\int_{beam} \frac{d\tilde{r}}{dt}\cdot\frac{d\tilde{r}}{dt}\, dm$$

$V = V_e + V_g$, Total potential energy;

$$V_e = \frac{1}{2}\int_0^l EI\, \varphi_{\xi\xi}\cdot\varphi_{\xi\xi}\, d\xi$$

$$V_g = -\frac{G m_E m_T}{|R|}$$

where m_T = Total mass of the system
m_E = Mass of the earth
G = Universal gravitational constant

Notations:

$\frac{d}{dt}(\cdot)$ = time derivative of $(\cdot)$ w.r.t. Inertial frame

$\dot{(\;)}$ = time derivative of ( ) w.r.t. body frame

$A\cdot B$ = Scalar product of vectors A and B.
$A\times B$ = vector product of vectors A and B.

$$\delta\tilde{r} = \delta R + \delta\theta \times r + \delta r$$

$$\delta\left(\frac{d\tilde{r}}{dt}\right) = \delta\left(\frac{dR}{dt}\right) + \delta\omega \times r + \delta\theta \times (\omega \times r + \dot{r}) + \frac{d}{dt}(\delta r)$$

$$\delta\int_{t_1}^{t_2} T_S \, dt = -\int_{t_1}^{t_2}\left\{ m_s \delta R \cdot \frac{d^2R}{dt^2} + \delta\theta \cdot \frac{d}{dt}\left(I_S\,\omega\right)\right\} dt,$$

$$\delta\int_{t_1}^{t_2} T_b \, dt = \delta \sum_{i=1}^{3} \int_{t_1}^{t_2}\int_{\ell_i} \frac{1}{2}\, \frac{d\tilde{r}}{dt}\cdot\frac{d\tilde{r}}{dt}\, dm\, dt$$

$$= -\sum_{i=1}^{3}\int_{t_1}^{t_2}\int_{\ell_i}\left\{ \delta R \cdot \frac{d^2\tilde{r}_i}{dt^2} + \delta\theta\cdot\left(r_i \times \frac{d^2\tilde{r}_i}{dt^2}\right) + \delta r_i \cdot \frac{d^2}{dt^2}\tilde{r}_i \right\} dm\,dt,$$

$$\delta\int_{t_1}^{t_2} V_e \, dt = -\sum_{i=1}^{3}\delta\int_{t_1}^{t_2}\int_{\ell_i}\frac{1}{2}\,\varphi^i_{\xi\xi}\cdot EI_i\,\varphi^i_{\xi\xi}\, d\xi\, dt$$

$$= \sum_{i=1}^{3}\left\{ \int_{t_1}^{t_2} \delta\varphi^i_{\xi}\cdot EI_i\,\varphi^i_{\xi\xi}\Big]_{\ell_i} dt - \int_{t_1}^{t_2}\delta\varphi^i\cdot\left(EI_i\,\varphi^i_{\xi\xi}\right)_{\xi}\Big]_{\ell_i} dt + \int_{t_1}^{t_2}\int_{\ell_i}\delta\varphi^i\cdot\left(EI_i\,\varphi^i_{\xi\xi}\right)_{\xi\xi}\, d\xi\, dt \right\},$$

$$\delta\int_{t_1}^{t_2} V_g \, dt = \int_{t_1}^{t_2}\frac{G\,m_E\,m_T}{|R|^2}\,1_R\cdot\delta R\, dt, \qquad 1_R = \frac{R}{|R|}$$

$$\delta\int_{t_1}^{t_2} W\, dt = -\int_{t_1}^{t_2} V\,dt + \int_{t_1}^{t_2}\left(\delta R\cdot F_S + \delta\theta\cdot\tau + \sum_{i=1}^{3}\int_{\ell_i}\delta r_i\cdot f_i\, d\xi_i\right)dt.$$

$$\delta \int_{t_1}^{t_2} (T + W)\, dt = 0$$

$$\Rightarrow -\int_{t_1}^{t_2} \delta R \cdot \underbrace{\left\{ m_s \frac{d^2 R}{dt^2} + \sum_{i=1}^{3} \int_{\Omega_i} \frac{d^2 \tilde{r}_i}{dt^2}\, dm_i + \frac{G m_E m_T}{|R|^2} 1_R - F_s \right\}}_{I} dt$$

$$-\int_{t_1}^{t_2} \delta \theta \cdot \underbrace{\left\{ \frac{d}{dt}(I_s \cdot \omega) + \sum_{i=1}^{3} \int_{\Omega_i} r_i \times \frac{d^2}{dt^2} \tilde{r}_i\, dm_i - \tau \right\}}_{II} dt$$

$$-\int_{t_1}^{t_2} \sum_{i=1}^{3} \underbrace{\left\{ \int_{\Omega_i} \delta r_i \cdot \frac{d^2}{dt^2} \tilde{r}_i\, dm_i + \int_{\Omega_i} \delta \varphi^i \cdot (EI_i \varphi^i_{\xi\xi})_{\xi\xi}\, d\xi - \int_{\Omega_i} F_i \cdot \delta r_i\, d\xi \right\}}_{III} dt$$

$$-\int_{t_1}^{t_2} \sum_{i=1}^{3} \underbrace{\left[\delta \varphi^i_\xi \cdot EI_i \varphi^i_{\xi\xi} - \delta \varphi^i \cdot (EI_i \varphi^i_{\xi\xi})_\xi \right]_{\Omega_i}}_{IV} dt = 0$$

Since δR, $\delta\theta$, δr_i (or $\delta\varphi^i$) are arbitrary, we have

$I = 0$, $II = 0$, $III = 0$, $IV = 0$;

$$m_s \frac{d^2}{dt^2} R + \sum_{i=1}^{3} \int_{\Omega_i} \frac{d^2 \tilde{r}_i}{dt^2}\, dm_i + \frac{G m_E m_T}{|R|^2} 1_R = F_s \quad \text{(Translational eq. of the bus)}$$

$$\frac{d}{dt}(I_s \omega) + \sum_{i=1}^{3} \int_{\Omega_i} r_i \times \frac{d^2 \tilde{r}_i}{dt^2}\, dm_i = \tau \quad \text{(Rotational eq. of the bus)}$$

$$\sum_{i=1}^{3} \int_{\Omega_i} \delta r_i \cdot \frac{d^2 \tilde{r}_i}{dt^2}\, dm_i + \sum_{i=1}^{3} \int_{\Omega_i} \delta \varphi^i \cdot (EI_i \varphi^i_{\xi\xi})_{\xi\xi}\, d\xi_i = \sum_{i=1}^{3} \int_{\Omega_i} \delta r_i \cdot F_i\, d\xi_i \quad \text{(Beam eqs.)}$$

$$\sum_{i=1}^{3} \left[\delta \varphi^i_\xi \cdot (EI_i \varphi^i_{\xi\xi}) - \delta \varphi^i \cdot (EI_i \varphi^i_{\xi\xi})_\xi \right]_{\Omega_i} = 0 \quad \text{(Boundary conditions)}$$

Bus dynamics (Translation)

$$m_T \frac{d^2R}{dt^2} + \sum_{i=1}^{3} \int_{\Omega_i} \frac{d^2}{dt^2} r_i \, dm_i + \frac{G m_E m_T}{|R|^2} 1_R = F_S$$

Bus dynamics (Rotation)

$$I_T \dot{\omega} + \omega \times (I_T \omega) + \dot{I}_b \omega + \sum_{i=1}^{3} \int_{\Omega_i} \left\{ r_i \times \left(\frac{d^2R}{dt^2} + \ddot{r}_i \right) + \omega \times (r_i \times \dot{r}_i) \right\} dm_i = \tau$$

Beam dynamics (Vibration)

$$\rho_i \ddot{r}_i + (EI_i \, r_{i\xi\xi})_{\xi\xi} + \rho_i \Big(\underbrace{2\omega \times \dot{r}_i}_{\text{Coriolis Acc.}} + \underbrace{\dot{\omega} \times r_i}_{\text{Euler Acc.}} + \underbrace{\omega \times (\omega \times r_i)}_{\text{Centrifugal Acc.}}$$

$$+ \frac{d^2}{dt^2} R \Big) = F_i, \qquad i = 1, 2, 3$$

Boundary Conditions

$$\delta \varphi^1_\xi \cdot (EI_1 \varphi^1_{\xi\xi}) \Big]_0^{l_1} - \delta\varphi^1 \cdot (EI_1 \varphi^1_{\xi\xi})_\xi \Big]_0^{l_1}$$

$$+ \delta \varphi^2_\xi \cdot (EI_2 \varphi^2_{\xi\xi}) \Big]_0^{l_1} - \delta\varphi^2 \cdot (EI_2 \varphi^2_{\xi\xi})_\xi \Big]_0^{l_1}$$

$$+ \delta \varphi^3_\xi \cdot (EI_3 \varphi^3_{\xi\xi}) \Big]_{-\frac{l_2}{2}}^{\frac{l_2}{2}} - \delta\varphi^3 \cdot (EI_3 \varphi^3_{\xi\xi})_\xi \Big]_{-\frac{l_2}{2}}^{\frac{l_2}{2}} = 0$$

Boundary Conditions

$$
\left.\begin{aligned}
y_1(0,t) &= z_1(0,t) = 0, \\
y_2(0,t) &= z_2(0,t) = 0, \\
y_{1,\xi}(0,t) &= z_{1,\xi}(0,t) = 0, \\
y_{2,\xi}(0,t) &= z_{2,\xi}(0,t) = 0,
\end{aligned}\right\}
$$

$$
\left.\begin{aligned}
x_3(l_2/2,t) &= x_3(-l_2/2,t) = 0, \\
z_1(l_1,t) &= z_3(l_2/2,t), \\
z_2(l_1,t) &= z_3(-l_2/2,t), \\
y_{1,\xi}(l_1,t) &= -x_{3,\eta}(l_2/2,t), \\
y_{2,\xi}(l_1,t) &= -x_{3,\eta}(-l_2/2,t),
\end{aligned}\right\}
$$

$$EI_1\, y_{1,\xi\xi}(l_1,t) = EI_3\, x_{3,\eta\eta}(l_2/2,t),$$

$$EI_2\, y_{2,\xi\xi}(l_1,t) = -EI_3\, x_{3,\eta\eta}(-l_2/2,t),$$

$$EI_1\, z_{1,\xi\xi\xi}(l_1,t) = -EI_3\, z_{3,\eta\eta\eta}(l_2/2,t),$$

$$EI_2\, z_{2,\xi\xi\xi}(l_1,t) = EI_3\, z_{3,\eta\eta\eta}(-l_2/2,t),$$

$$EI_1\, y_{1,\xi\xi\xi}(l_1,t) = EI_2\, y_{2,\xi\xi\xi}(l_1,t) = 0,$$

$$EI_1\, z_{1,\xi\xi}(l_1,t) = EI_2\, z_{2,\xi\xi}(l_1,t) = 0,$$

$$EI_3\, z_{3,\eta\eta}(l_2/2,t) = EI_3\, z_{3,\eta\eta}(-l_2/2,t) = 0,$$

STABILIZATION

$$\dot{\psi} = \mathcal{A}\psi + F(\psi, \dot{\psi})$$

$$\psi(0) = \psi_0$$

where $\psi = \left(R, \frac{dR}{dt}, \omega, \varphi_i, \dot{\varphi}_i\right)'$

Suppose that the velocity feedback controls are applied and given by

$$F_S = \left(-e_1\left[\frac{dR}{dt}\right]_i, -e_2\left[\frac{dR}{dt}\right]_j, -e_3\left[\frac{dR}{dt}\right]_k\right)', \qquad e_1, e_2, e_3 > 0$$

$$\tau = \left(-e_4\,\omega_1,\ -e_5\,\omega_2,\ -e_6\,\omega_3\right)', \qquad e_4, e_5, e_6 > 0$$

$$F_i = \left(-c_1(\xi)\left[\frac{\partial\varphi^i}{\partial t}\right]_1, -c_2(\xi)\left[\frac{\partial\varphi^i}{\partial t}\right]_2\right)', \qquad i=1,2,3 \quad c_1, c_2 > 0$$

Then the system is stable.