

N 9 0 - 1 0 1 2 1**ACTIVE VIBRATION MITIGATION OF DISTRIBUTED PARAMETER, SMART-TYPE
STRUCTURES USING PSEUDO-FEEDBACK OPTIMAL CONTROL (PFOC)**

By

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ABSTRACT

A new, near-optimal feedback control technique is introduced that is shown to provide excellent vibration attenuation for those distributed parameter systems that are often encountered in the areas of aeroservoelasticity and large space systems. The technique relies on a novel solution methodology for the classical optimal control problem. Specifically, the quadratic regulator control problem for a flexible vibrating structure is first cast in a weak functional form that admits an approximate solution. The necessary conditions (first-order) are then solved via a time finite-element method. The procedure produces a low dimensional, algebraic parameterization of the optimal control problem that provides a rigorous basis for a discrete controller with a first-order "like" hold output.

Simulation has shown that the algorithm can successfully control a wide variety of plant forms including multi-input/multi-output systems and systems exhibiting significant nonlinearities. In order to firmly establish the efficacy of the algorithm, a laboratory control experiment was implemented to provide planar (bending) vibration attenuation of a highly flexible beam (with a first clamped-free mode of approximately 0.5 Hz). Base actuation for the cantilever was accomplished using a three degree-of-freedom active bay (variable geometry truss) actuator. On-line processing was accomplished with a 14 mhz "AT" type microcomputer with data acquisition capability. The results of the tests corroborate the utility of the method.

957

ACTIVE VIBRATION MITIGATION OF
DISTRIBUTED PARAMETER SYSTEMS
USING
PSEUDO-FEEDBACK OPTIMAL CONTROL

958

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VA TECH

ORGANIZATION OF PAPER

- **OVERVIEW OF METHOD (PFCA)**
- **EXPERIMENTAL RESULTS**
- **DISCUSSION**

VARIATIONAL FORMULATION OPTIMAL CONTROL PROBLEM

$$\text{Min } J_0 [\bar{u}] \equiv C(\bar{x}(T)) + \int_T g(\bar{x}, \bar{u}, t) dt$$

$$\dot{\bar{x}} = \bar{f}(\bar{x}, \bar{u}, t) + \bar{B}(\bar{x}, t) \bar{u} \quad \bar{x}(0) = \bar{x}_0$$

$$H = g + \langle \bar{Y}, (\bar{f} + \bar{B} \bar{u}) \rangle$$

$$J_1[\bar{u}] = \int_T \{ H - \langle \bar{Y}, \dot{\bar{x}} \rangle \} dt$$

NECESSARY CONDITIONS

$$0 = \int_{\Gamma} \{ \langle \bar{e}_1, \delta \bar{x} \rangle + \langle \bar{e}_2, \delta \bar{y} \rangle + \langle \bar{e}_3, \delta \bar{u} \rangle \} dt$$

$$0 = \langle (c, \bar{x}^-, \bar{y}), \delta \bar{x} \rangle \Big|_{\Gamma}$$

$$\bar{0} = \bar{e}_1 = \bar{H}_{x,x} + \bar{\gamma}$$

$$\bar{0} = \bar{e}_3 = \bar{H}_{u,u}$$

$$\bar{0} = \bar{e}_2 = \bar{H}_{y,y} - \bar{x}$$

CLASSICAL SOLUTION TECHNIQUES

- LINEAR SYSTEMS

 - CLOSED FORM SOLUTIONS

 - RICCATI METHODS

- NONLINEAR SYSTEMS

 - GRADIENT METHODS

 - SHOOTING METHODS

 - QUASILINEARIZATION

WEAK (VARIATIONAL) FORMULATION

WHY?

- **TECHNIQUE FAMILIAR TO COMMUNITY**
- **EASE OF IMPLEMENTATION**
- **USING FINITE ELEMENT PARADIGMS AND CODES**
- **PROVIDES AUTOMATED BASIS FOR FEEDBACK CONTROL LAWS**

FOR LINEAR AND NONLINEAR SYSTEMS

WHAT IS

PSEUDO

FEEDBACK CONTROL

- DRIVER -IN-THE-LOOP CONTROL
(OHIO STATE --LATE 60'S)

OVERVIEW OF ANALYTICAL DEVELOPMENT

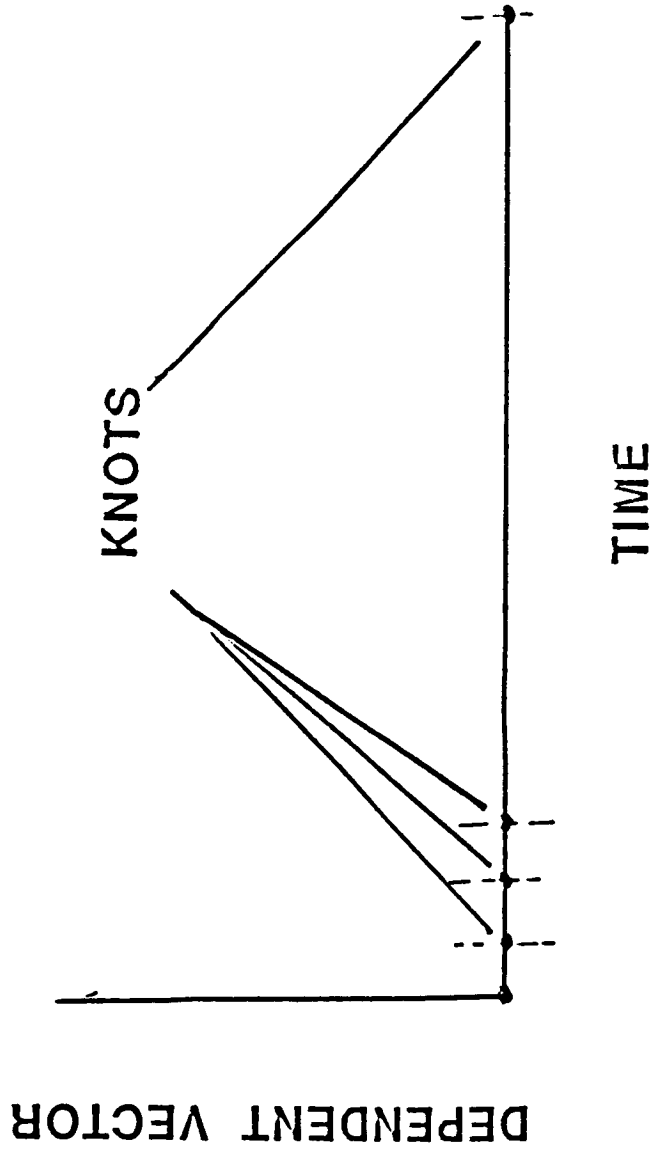
TWO-POINT BOUNDARY VALUE PROBLEM

WEAK FORM OF NECESSARY CONDITIONS

$$\bar{u} = -\bar{R}^T \cdot \bar{E}^T \cdot \bar{y}$$

$$0 = \int_{\tau} \langle \bar{e}_1, \delta \bar{x} \rangle dt \qquad 0 = \int_{\tau} \langle \bar{e}_2, \delta \bar{y} \rangle dt$$

DISCRETIZE DOMAIN OF OPERATOR (TIME)



996

SLIDING INTERVAL CONTROL

CONSTRUCT INTERPOLENTS TO APPROXIMATE
STATE, COSTATE AND CONTROL VECTORS
ON EACH OF THE SUBDOMAINS

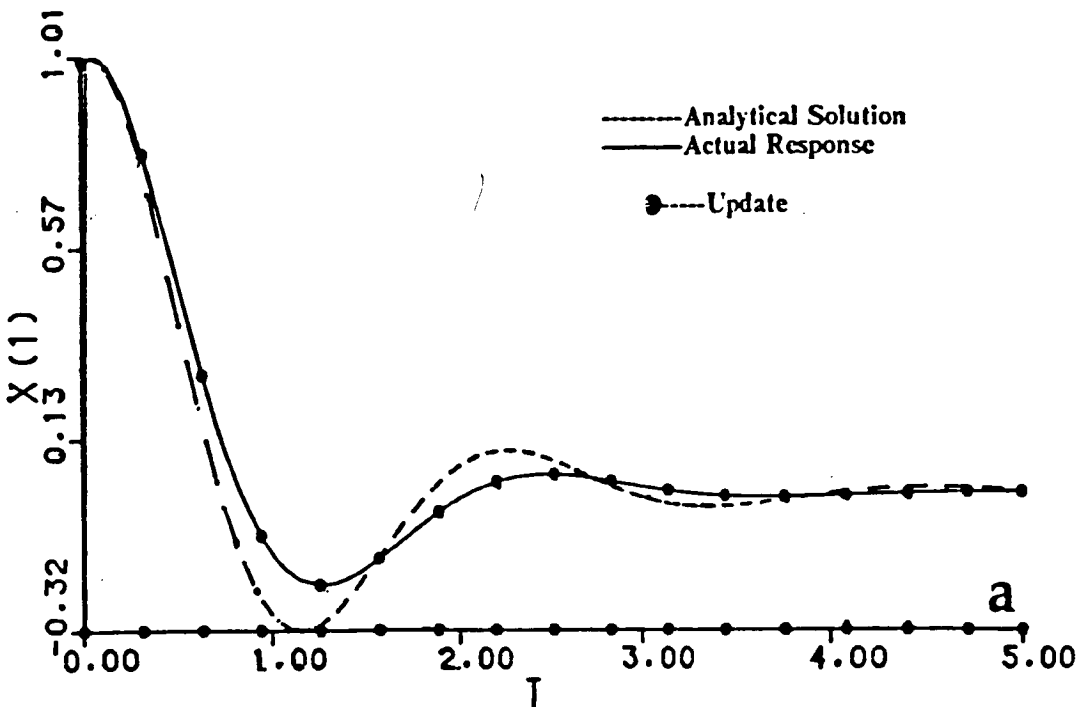
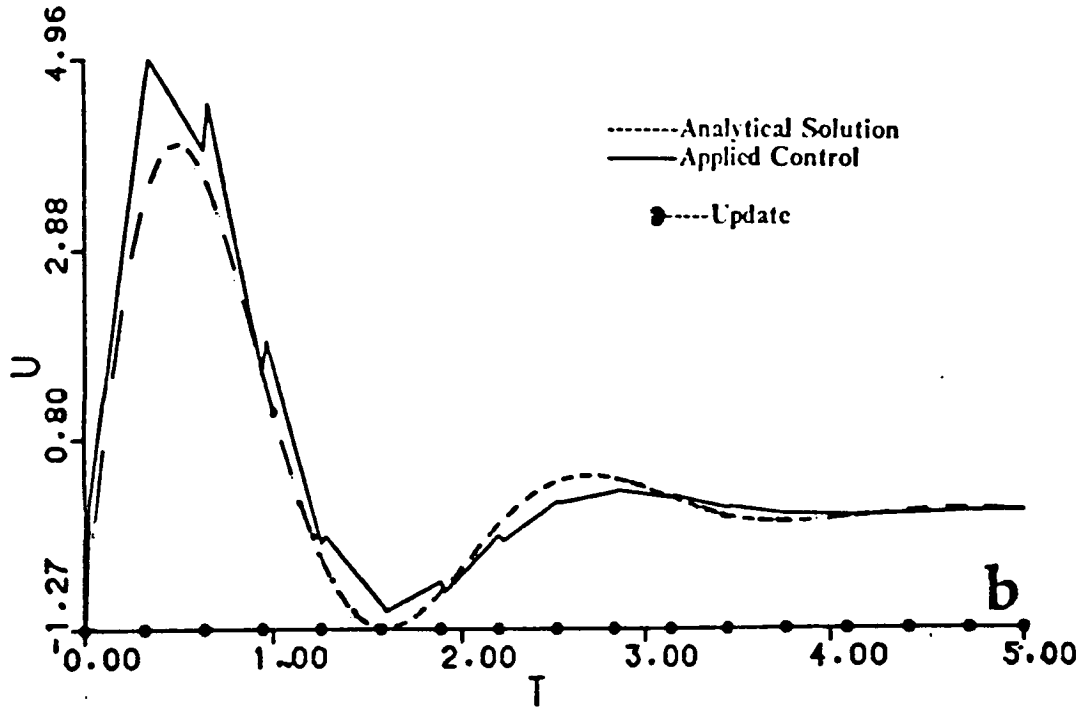
$$\|\bar{z}\|_{W_2^1}^2 \equiv \int_{\tau} [\dot{\bar{z}}^t \cdot \dot{\bar{z}} + \bar{z}^t \cdot \bar{z}] dt$$

$$\bar{x} \approx \sum_{e=1}^n \bar{x}^e = \sum_{e=1}^n \bar{N}^e \cdot \bar{q}^e = \sum_{e=1}^n \bar{N}^e \cdot \bar{T}^e \cdot \bar{g}_q$$

$$\bar{y} \approx \sum_{e=1}^n \bar{y}^e = \sum_{e=1}^n \bar{N}^e \cdot \bar{p}^e = \sum_{e=1}^n \bar{N}^e \cdot \bar{T}^e \cdot \bar{g}_p$$

EXAMPLE

LINEAR SECOND ORDER OSCILLATOR (UNDAMPED)



EXAMPLE
NONLINEAR SECOND ORDER MODEL OF
WINGROCK EXHIBITED BY
FREE-TO-ROLL MODEL OF

X-29

EQUATION OF MOTION

$$\ddot{\phi} + \omega^2 \phi = \mu_1 \dot{\phi} + b_1 \phi^3 + \mu_2 \phi^2 \dot{\phi} + b_2 \phi \dot{\phi}^2$$

where

$$\omega^2 = -c_1 a_1$$

$$\mu_1 = c_1 a_2 - c_2$$

$$b_1 = c_1 a_3$$

$$\mu_2 = c_1 a_4$$

$$b_2 = c_1 a_5$$

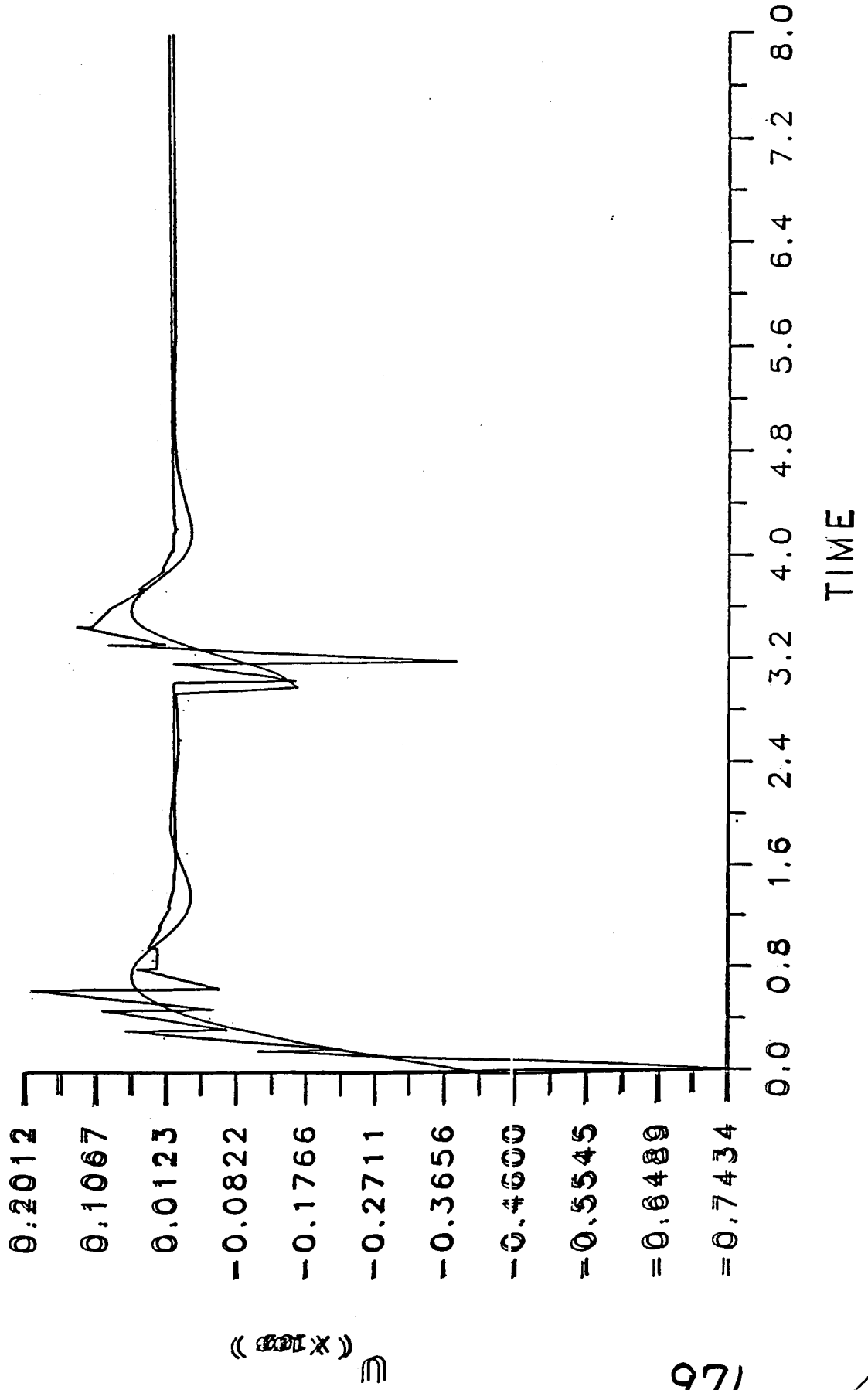
c_1 - CONSTANT

c_2 - CONSTANT

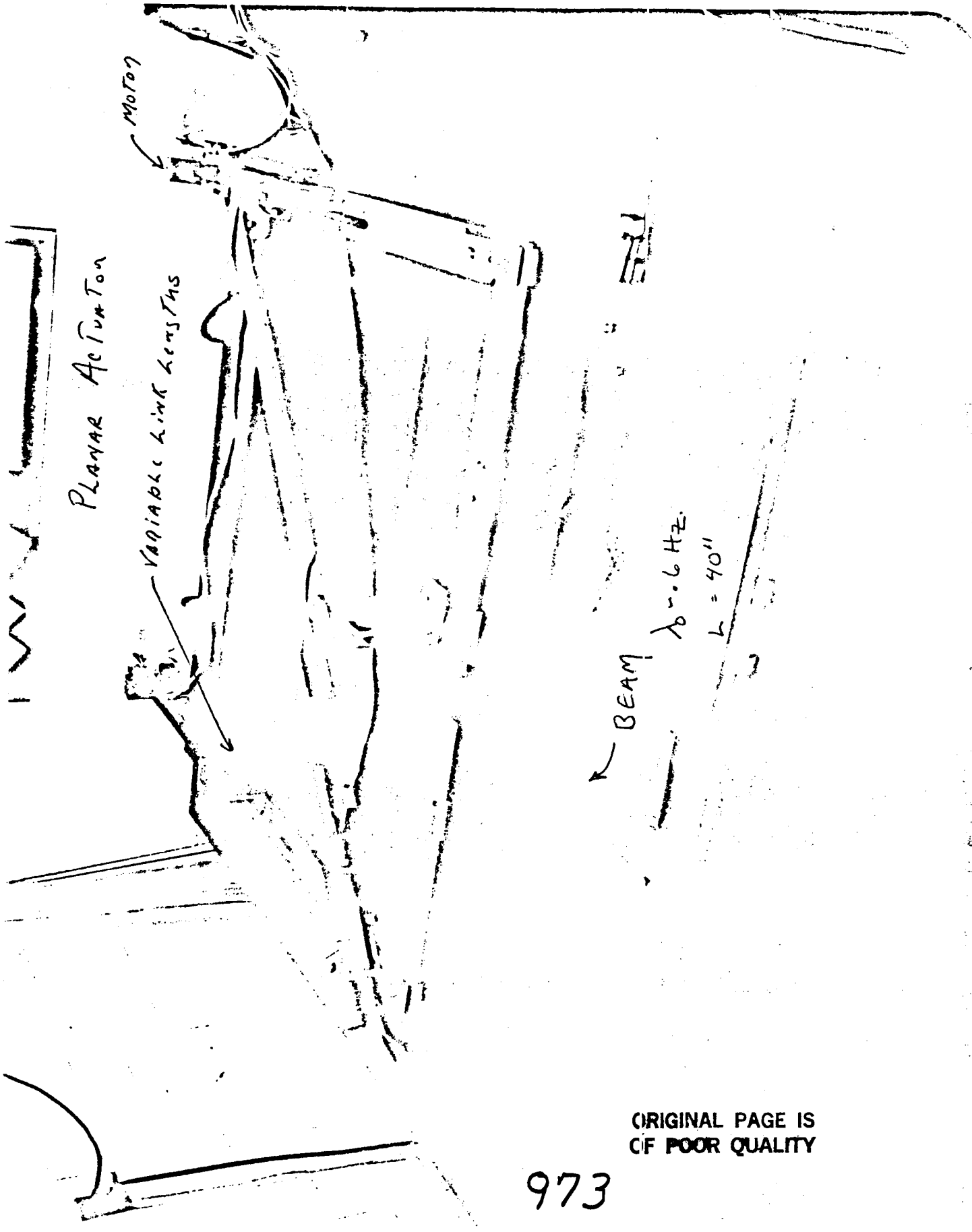
$a_i = f(\alpha) \quad i = 1, 5$

CONTROL-U .VS. TIME

Figure 6



EXPERIMENTAL VALIDATION
OF ALGORITHM
THE CONTROL OF
A FLEXIBLE BEAM USING
A PLANAR TRUSS ACTUATOR



ORIGINAL PAGE IS
OF POOR QUALITY

PROCEDURE

$$\dot{X} = AX + BU$$

X = [LINK STATES, BEAM STATES]

APPLY FEM APPROXIMATION

ALGEBRAIC EVOLUTION OPERATOR

$$Z = K Z_0$$

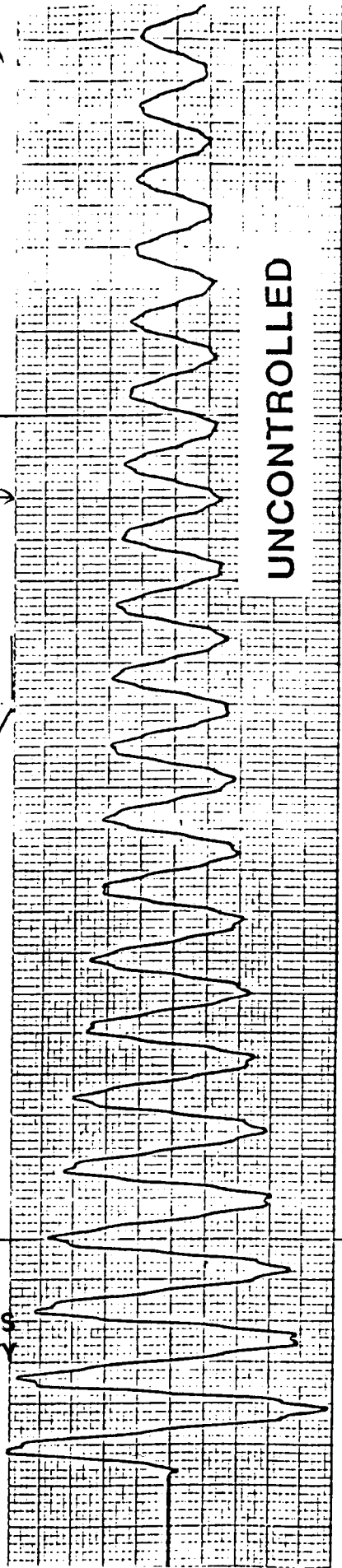
Z = [STATE VECTORS, COSTATE VECTORS]

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$\lambda_0 \approx 64z$
 $\xi_0 \approx 0.02$

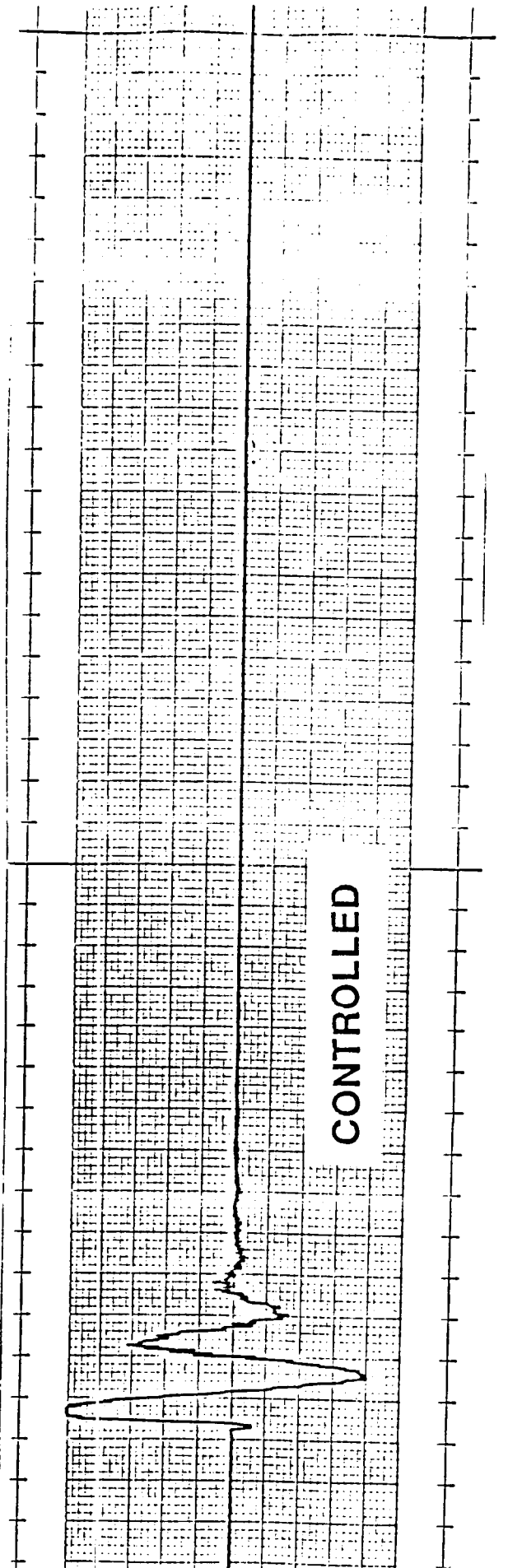
40"

$\delta(T_0)$

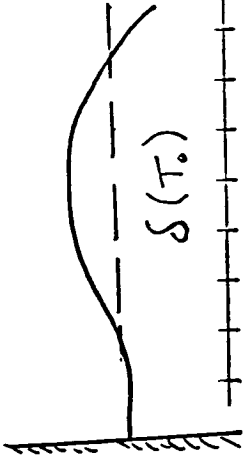


1 SEC.

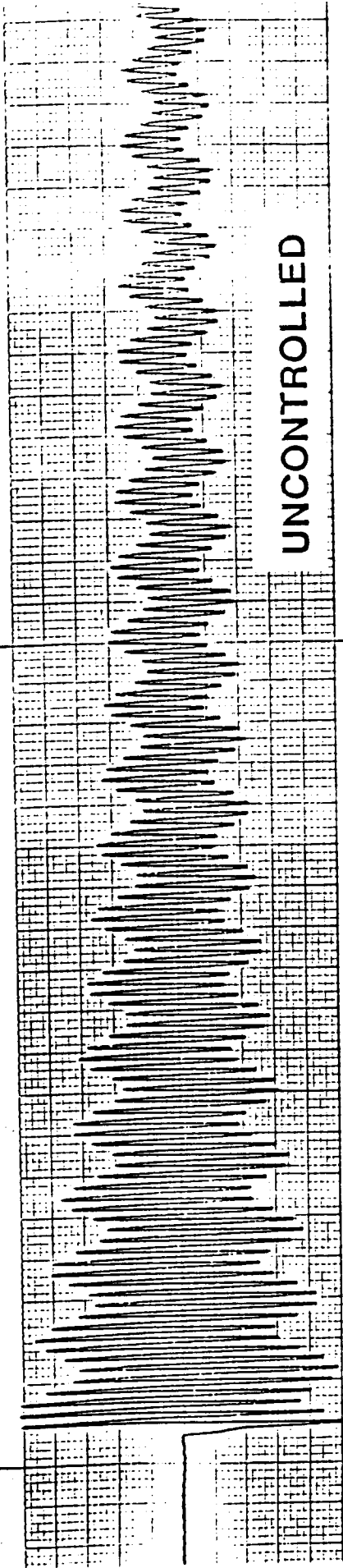
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"SECOND MODE LITE
INITIAL CONDITION"



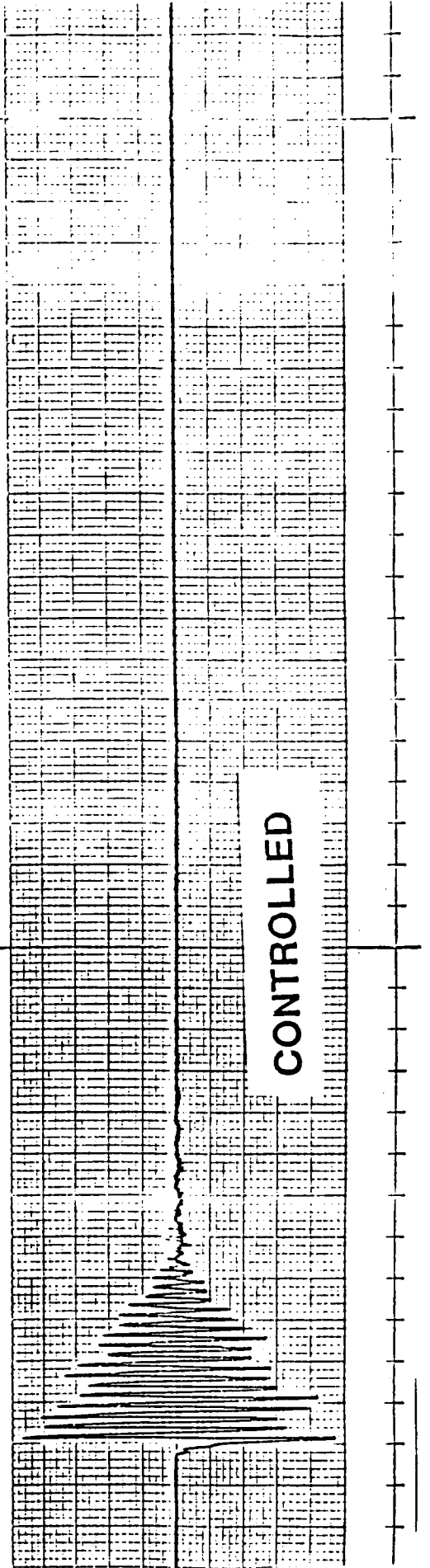
$S(T)$



UNCONTROLLED

→ 1 SEC.

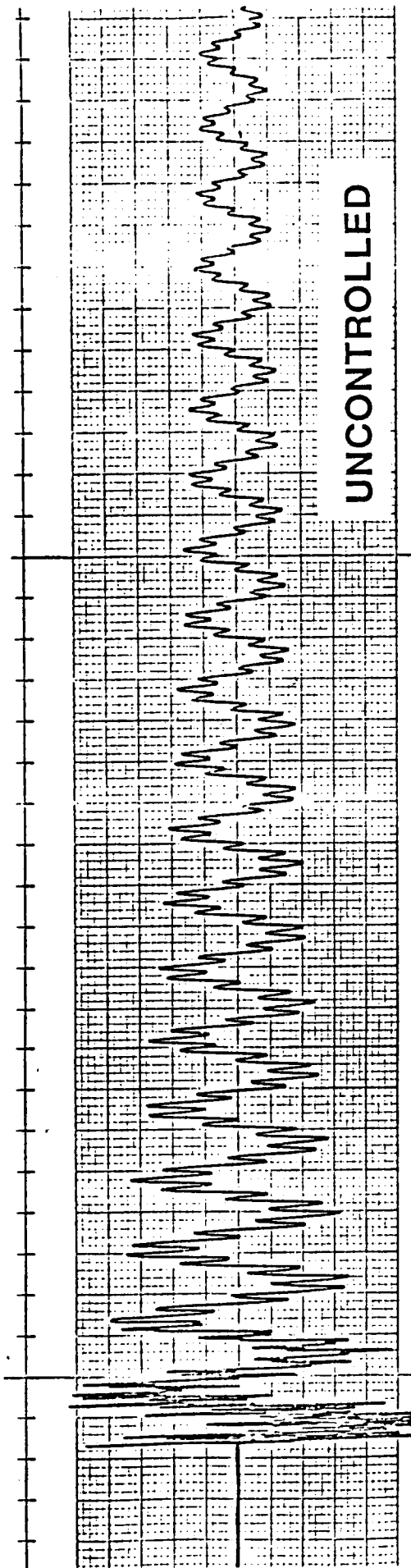
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CONTROLLED

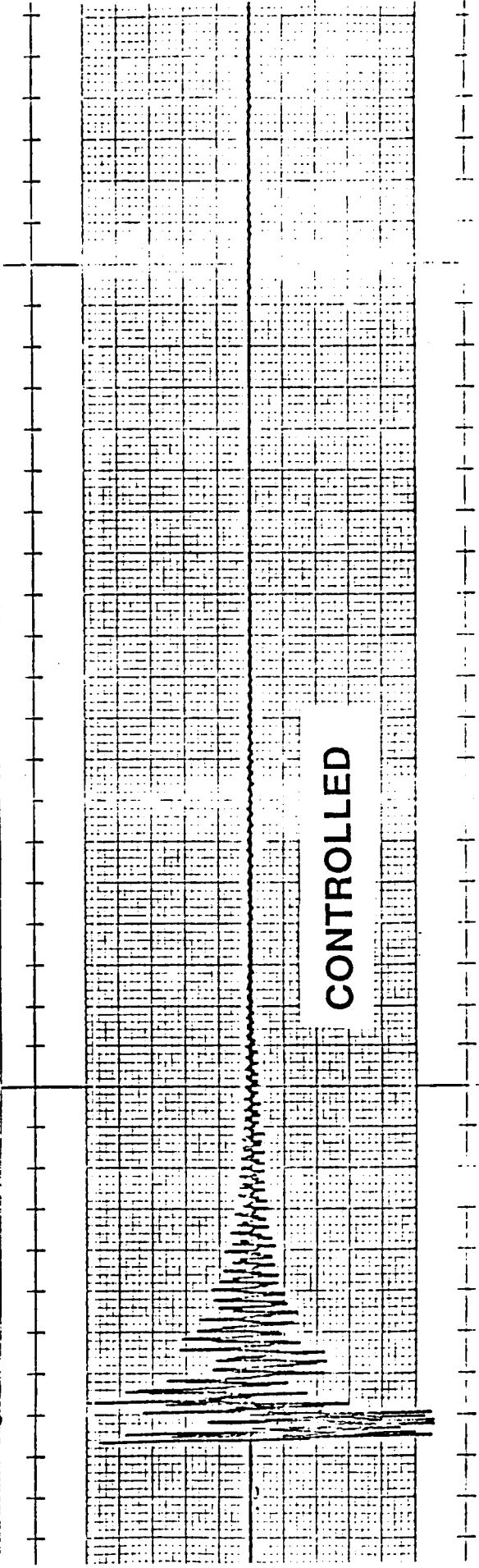
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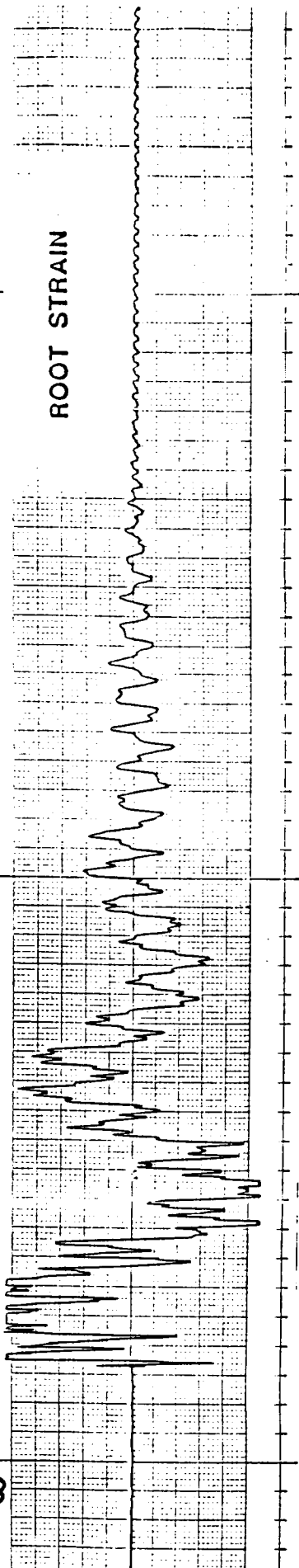
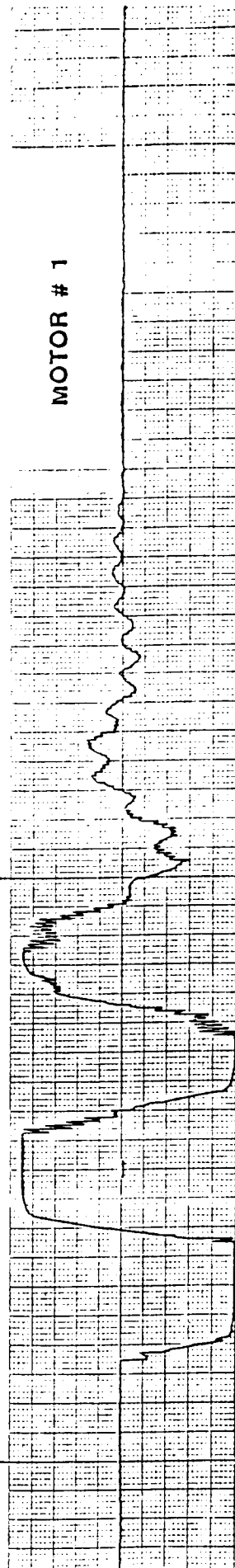
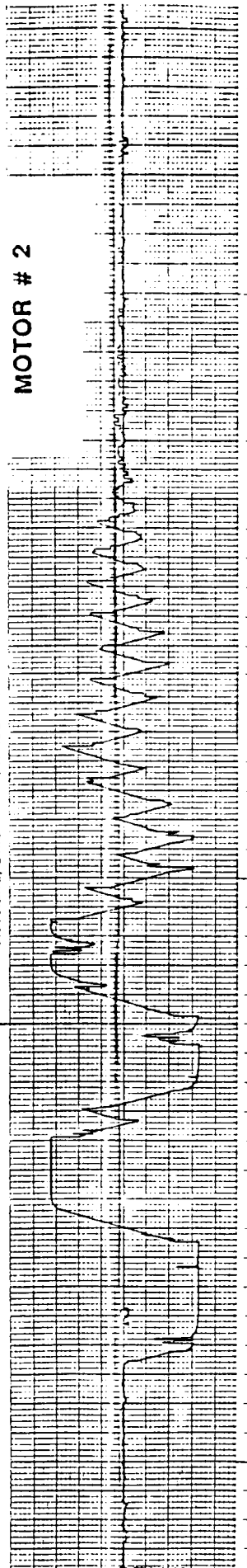
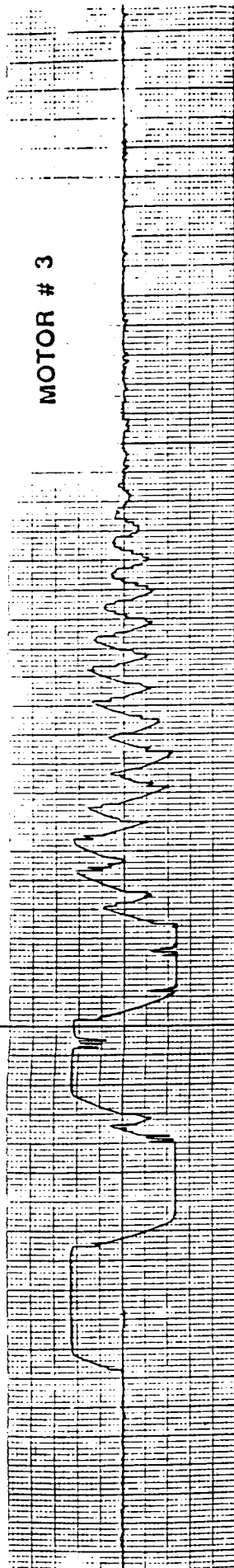
IMPULSE RESPONSE



→ ←
1 SEC.

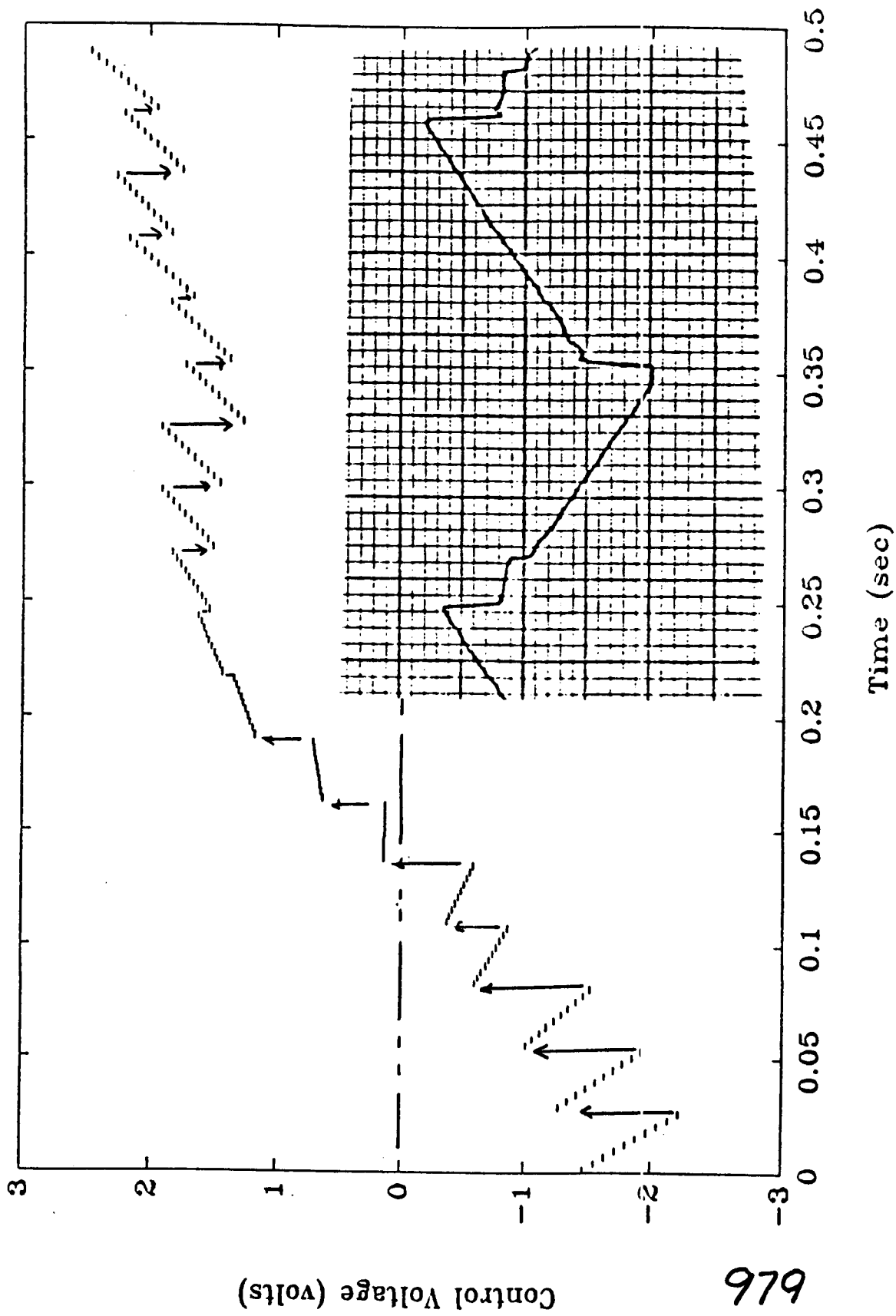
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CONCLUSIONS

- ALGORITHM VALIDATED
- REAL TIME IMPLEMENTATION POSSIBLE
ON LOW-LEVEL PC ARCHITECTURE
(80286 WITH 27.5 KHZ D/A)

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