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LASER PROPULSION TO EARTH ORBIT
HAS ITS TIME COME?

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Abstract

Recent developments in high energy lasers, adaptive optics, and atmospheric transmission bring laser propulsion much closer to realization. Perhaps more important, the need to transport much greater tonnages to orbit for commercial purposes, Space Station Freedom and for military purposes is now clear. A part (e.g. half the space station supplies) of this traffic could be orbited in small packages. Accordingly a workshop on this possibility was held at Livermore National Laboratory in July 1986 and this paper leans heavily on its results.

Proposed laser

~~This paper proposes~~ a reference vehicle for study which consists of payload and solid propellant (e.g. ice). A suitable laser pulse is proposed for using a Laser Supported Detonation wave to produce thrust efficiently.

It seems likely that a minimum system (10 Mw CO2 laser & 10m dia. mirror) could be constructed for about $150 M. This minimum system could launch payloads of about 13 kg to a 400 km orbit every 10 minutes. The annual launch capability would be about 683 tonnes times the duty factor. Laser propulsion would be an order of magnitude cheaper than chemical rockets if the duty factor was 20% (10,000 launches/yr.) and launches beyond that would be even cheaper.

The chief problem which needs to be addressed before these possibilities could be realized is the design of a propellant to turn laser energy into thrust efficiently and to withstand the launch environment.

INTRODUCTION

The key cost which determines the magnitude of realistic possibilities in space is the cost of transportation to low Earth orbit (LEO). One of the great disappointments in the utilization of space is that in the 29 years since Sputnik this key cost has not declined.

One opportunity for dramatic improvement is to transmit the orbital energy from a laser on the ground to the ascending vehicle. Lasers can easily vaporize any material and it is possible to transfer energies to the vapor which are large compared to chemical energies. The evaporated material produces a jet which propels the vehicle, and the kinetic energy of the propulsive jet can be a large fraction of the energy absorbed from the laser. If the vapor is heated to very high temperatures, correspondingly high jet velocities can be achieved so that the amount of propulsive material (and the lift-off weight) required to launch a given payload can be about an order of magnitude less than that required for a chemical rocket. The laser which is the dominant component remains on the ground so that laser propulsion systems are, in principle capable of launching a payload every few minutes.

When this system was first proposed fifteen years ago (Refs. 1 and 2) four major extrapolations of existing technology were required for its implementation:

1. Laser average powers had to be extended by several orders of magnitude.

2. Atmospheric transmission problems needed to be explored.

3. Collimating mirrors larger than conventional dimensions needed to be developed.

4. Technology for efficiently converting laser energy into the kinetic energy of a jet with speeds up to about 10^6 cm/sec (specific impulse 1000) and with thrust vectors considerably off the laser beam axis needed to be developed.

While none of these extrapolations seemed difficult enough to prevent development of laser propulsion, taken together at a time when the decision had been made to develop and to depend on the Shuttle for the nation's space transportation needs, it is not surprising that no major program was undertaken in the early seventies.

At this time the first three extrapolations are being vigorously pursued largely under SDI programs. There are strong indications that lasers of any required power can be built. Combining modules of molecular e.g. CO2 lasers or constructing very large free electron lasers are two avenues which now seem open. The problems involved in transmitting many megawatts through the atmosphere are being addressed. While some of these may be somewhat different for laser propulsion than they are for other SDI* purposes, there are persuasive indications that these beams can be transmitted through the atmosphere with the aid of adaptive optics. Adaptive optics also has made it possible to build very large mirrors, e.g., the 10 meter Keck telescope.

The development of thruster technology has not been vigorously pursued and such work as has been carried out seems more adapted to the task of changing the orbit of a satellite (which needs much smaller laser power).

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*Strategic Defense Initiative

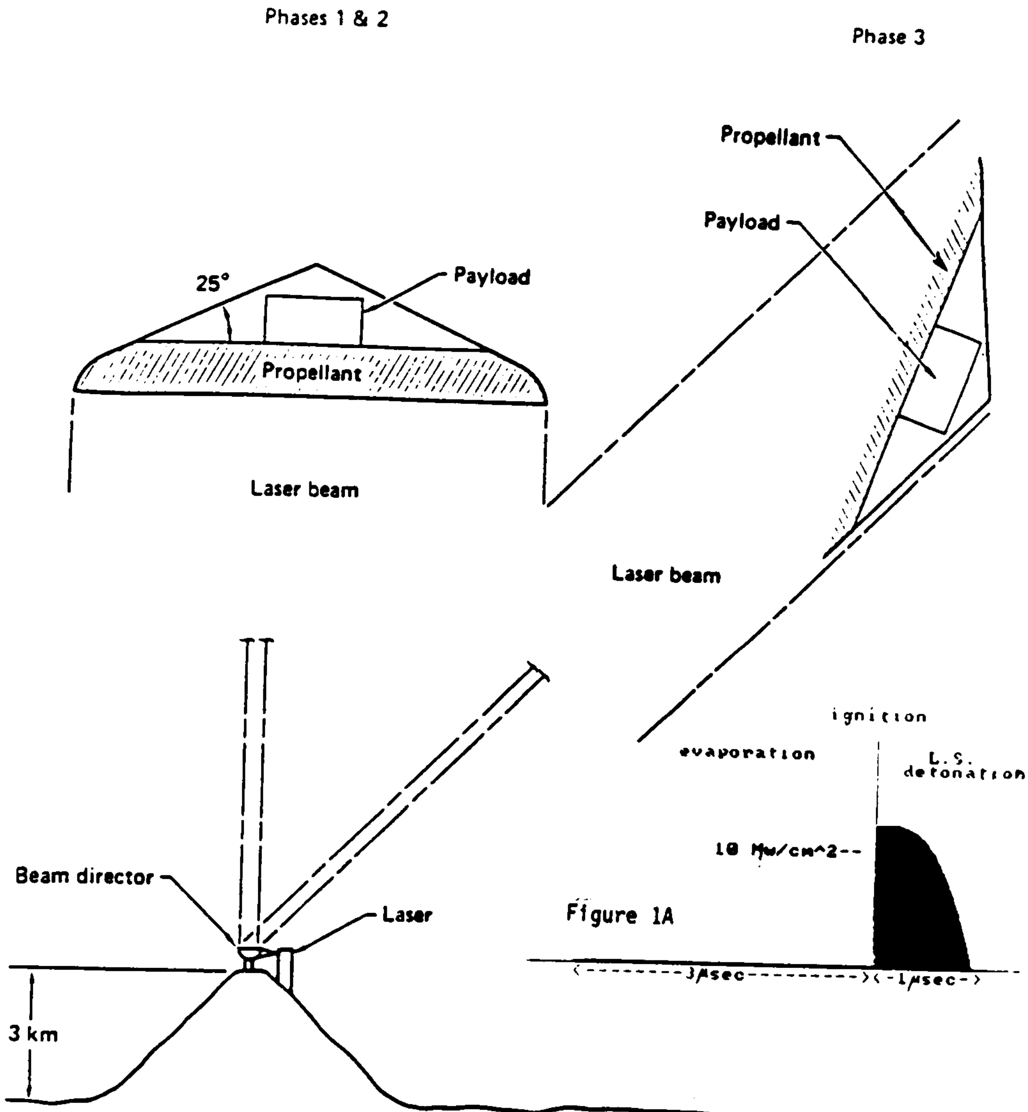

Figure 1. A very schematic rendition of the principle of laser propulsion and the reference vehicle. The ground-based laser generates a double pulse: the first evaporates a designed amount of propellant, and the second heats the vapor to a temperature high enough to produce the desired specific impulse. The propellant (about 7.7 times the payload weight at lift-off) is a solid of low molecular weight. The conic shape allows the laser to illuminate the base at an angle of incidence up to 1 radian without damaging the payload. The reference vehicle is shown at launch and in the exoatmospheric phase 3. The thrust is vertical for the ascent through the atmosphere (phases 1 & 2), and the propellant can be cylindrical. For phase 3, the large angle of incidence of the laser necessary to produce a thrust component perpendicular to the laser beam requires a conical payload bay to keep the payload in the shadow of the propellant. A large part of the propellant is consumed in the ascent through the atmosphere.

Figure 1A. The three phase pulse. The evaporation phase controls the density distribution which is acted upon by the high power Laser Supported Detonation (LSD) phase. The ignition phase uses the highest instantaneous power available from the laser system to ignite the gas close to the surface as rapidly as possible. The plasma formed will then shield the propellant from the LSD phase which follows.

I would like to propose the modification of this two pulse system illustrated in Fig. 1A. Here the pulse is divided into three phases with the addition of an "ignition spike." The evaporation energy is typically one to two orders of magnitude smaller than the energy needed to drive the LSD. The specific energy deposited in an element of gas by the LSD is proportional to the 2/3 power of the ratio of the flux to the density (Ref. 3). Thus the deposited energy can be controlled either by controlling the density or the flux during the LSD phase. It is clearly more economical to control the density distribution which depends only on the flux as a function of time during the evaporation phase. It is therefore proposed that the flux during the evaporation be an adjustable function of time chosen to produce a density distribution designed to achieve the desired specific impulse with maximum efficiency.

The ignition phase consists of a spike, perhaps a gain switched spike, intended to ionize the vapor close to the propellant surface sufficiently to make it opaque to the laser radiation. It is important that this process be accomplished rapidly to avoid evaporating too much propellant at this stage. It will be seen that the highest attainable flux will lead to minimum evaporation (see reference 4). It is therefore proposed that the flux in the ignition spike be as large as can be delivered. The limitation will probably be imposed by surface breakdown in the laser. Required duration of the ignition spike will depend on the nature of the propellant e.g. for lithium Hyde calculated that, at a flux of 25 Mw/cm^2, the time required was 7.5 nsec and thus a fluence of less than .2 joules/cm^2 to achieve unit optical depth.

The third LSD phase involves most of the energy and the cost of creating it controls the system cost. The objective of this pulse shape design has been to prepare the vapor to efficiently utilize whatever pulse shape minimizes this laser cost.

ESTIMATES OF THE CAPABILITY OF THE REFERENCE SYSTEM

It will be useful to start by attempting an estimate of the losses which are foreseen for this process.

First there are the losses in the laser itself. Assuming electrically driven lasers, the "wall plug" efficiency EL gives the ratio of the power in the collimated laser beam to the power drawn from the utility lines. For example for a 10 micron free electron laser Briggs (LLNL) informed us that an appropriate EL would be about 20%. For the CO2 laser Daughtery (AVCO) suggested 16%.

Second there are losses in the atmosphere. Among these are scattering due to Thermal Blooming, Stimulated Raman Scattering, and Atmospheric Turbulence, and there is absorption chiefly due to water vapor. We assumed that at the intensities to be used (for the sample trajectory 10^5 w/sq cm at the collimator) and noting that we have a cooperative target, that the beam would be essentially diffraction limited. Starting from a mountain top about 10,000 ft high we took the transmission through the remainder of the atmosphere, EA, to be .9/cos(TH) where TH is the zenith angle.

The third set of losses occurs in converting the laser energy arriving at the vehicle into thrust. An ideal thruster would convert the laser energy arriving at the vehicle into kinetic energy MDOT*VJ^2/2 where MDOT is the propellant used per second and VJ is the designed jet velocity. (Note that in the double pulse system, the propellant mass and the propulsive energy can be chosen independently; VJ can be chosen to optimize overall performance.) Losses in this conversion which result in a thruster efficiency, ETH, include:

A. The latent heat of the evaporated propellant.

B. Chemical or internal molecular energy remaining in the jet following the one dimensional expansion. These losses will be reduced by the use of the longest duration pulses which still allow essentially one dimensional expansion (1 microsecond for a 1 meter dia vehicle).

C. Losses due to non-homogeneities in gas velocity in the jet.

These losses are estimated in reference 5 by Rod Hyde for lithium as a propellant. (Lithium was chosen simply for ease of calculation.)

The uncertainties in the value of ETH are presently the leading uncertainty in the efficiency of laser propulsion. At the present state of the art the workshop saw no reason to change the guess that ETH would be about 40% (which was made in Ref. 1).

Finally the kinetic and potential energy in the payload is of course smaller than the kinetic energy in the propulsive jet integrated over the trajectory. We will call the ratio of these energies the trajectory efficiency, ETR. For the sample trajectory ETR was 27%.

ESTIMATE OF THE MASS LAUNCHING CAPABILITY, Mo, OF A PULSED LASER

The range to orbit, D, will be dominated by the acceleration during the high velocity (V) portions of the trajectory. If we take this acceleration, VDOT, as constant, using the final acceleration and taking VJ = V,

$$VDOT*Mo = MDOT*VJ = 2*ETH*P'/V, \quad (1)$$

we get

$$D = \frac{V^2}{2*VDOT} \quad \frac{Mo*V^3}{4*ETH*P'}, \quad (2)$$

where P' is the average laser power at the vehicle, and Mo is the mass launched.

The radius, rv of a vehicle base which can be illuminated with a flux ϕ with peak power PP from the laser is

$$rv = (PP/pi*\phi)^{.5}.$$

We have assumed a "flat top" distribution of intensity. Actually D will be limited by diffraction to approximately

$$Dmax = rm*rv/.3*\lambda \quad . \tag{3}$$

where rm is the radius of the collimating mirror, λ is the laser wavelength and the constant .3 is chosen to correspond to a value of rv halfway to the first dark ring. For λ = 10 microns, rv = 50 cm, and rm = 5 m we get Dmax = 833 km.

Setting D = Dmax (eq. 2&3) we get

$$Mo = \frac{7.5*rm*ETH*P'*(PP/\phi)\hat{}.5}{\lambda * V\hat{}3} \tag{4}$$

Allowing for atmospheric absorption as discussed above we take for our model 10 MW laser, P' = 8 MW, PP = P'*10^4, ϕ = 10^7 watts/cm^2 and the above values of rm, rv, ETH, and 1, the mass which can be accelerated to orbital velocity is 18 kg. The agreement of this estimate with the sample trajectory result calculated below (13.79 kg.) is as good as might be expected since in that case the acceleration was not constant.

A SAMPLE TRAJECTORY*

When we consider the practical applications of laser propulsion an important consideration is the minimum** scale of an initial trial.

To illuminate the choice for this minimum scale we will attempt to calculate the payload which can be launched with a 10 MW laser, making the guess that this will be within one order of magnitude of the practical minimum. It was assumed that the pulse duration was 10^(-4) of the time between pulses so that the flux on the 1 sq meter vehicle base would be 10 MW/sq cm. Note that the mass which can be launched varies inversely with the square root of the minimum flux sufficient to sustain an efficient LSD. The achievement LSDs at low flux will be one of the most important objectives of propellant research.

After several trials it was found possible to launch 13.79 kg into a 411 km circular orbit making the assumptions listed in Table 1.

* In Ref. 5 Jordin Kare gives a more complete modelling of the laser launching. Close agreement between his results and those presented here provides some confidence that the remaining bugs are not too important.

** We will not consider here the utility of small satellites other than to note that Freeman Dyson* proposes that satellites as small as 1 kg would be useful for space science purposes.

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*Dyson F., see his March 26, 1986 talk at Analog Devices, Norwood, MA.

TABLE 1

ASSUMPTIONS FOR SAMPLE 10 MW LASER LAUNCH MODELLING

A. Initial mass (propellant + payload) = 120 kg.
B. The base area of the propellant = 1 sq. meter
C. The coefficient of atmospheric drag = .4
(note that this assumes that the vehicle will be streamlined as well as a sphere.)
D. The trajectory starts at the laser which is on a mountaintop 3 km above sea level.
E. The jet velocity, VJ, can be adjusted in magnitude between 3.6 and 10 km/sec by varying the energy ratio in the two pulses.
F. The thruster efficiency will be ETH = 40% for any VJ in this range. This assumes that a propellant can be found which will perform as well as the Lithium in Hydes calculation while avoiding environmental and cost impacts of Lithium.
G. The direction of the thrust, which is normal to the vehicle base, can be adjusted by tilting the vehicle. It was assumed that the vehicle attitude would be continuously measured from the ground and controlled by moving the laser pulses off center. A short simulation indicated that, if the vehicle was spinning at a few rps, it would be possible to control the thrust axis to within about 5 degrees.
H. Vehicle design was assumed to allow an angle of incidence between the laser and the base up to 65 degrees without exposing the payload to damaging laser radiation (see Fig. 1).
I. It was assumed that the beam director mirror would be 10 meters in dia. allowing the 10 micron beam to be focussed on the 1 meter día. vehicle base out to a range of about 800 km. This implies performance not very far from the diffraction limit. More work is needed to specify tolerances on optical performance.

The program used to calculate the trajectory is to be found in Appendix 1. A sample result is shown in Fig. 2 and the numerical results are given in Table 2. The ascent to orbit is divided into four phases.

1. Phase 1 starts with a liftoff close to the laser and with a vehicle weight (propellant plus payload) of 120 kg which was close to the largest load the laser could lift with VJ=3.6 km/sec. VJ was varied to make the best compromise between gravity and drag so as to minimize the mass loss per unit altitude gain. Phase 1 was terminated (somewhat arbitrarily) when the acceleration reached 1 g. At the end of phase 1 the mass was 57 kg.

2. In phase 2, continuing the vertical ascent to 130 km, the vertical acceleration was maintained at 1 g and VJ varied from 5 to 10 km/sec, the mass ended up at 38 kg and the vertical velocity was 1.45 km/sec.

3. At the beginning of the extraatmospheric acceleration, phase 3, the vehicle was tilted so that the angle of incidence of the laser on the vehicle base was 1 radian. The thrust, which is normal to the base, was at the beginning of phase 3, 33° up from the horizontal. The vehicle was maintained at this angle of incidence to the laser as it

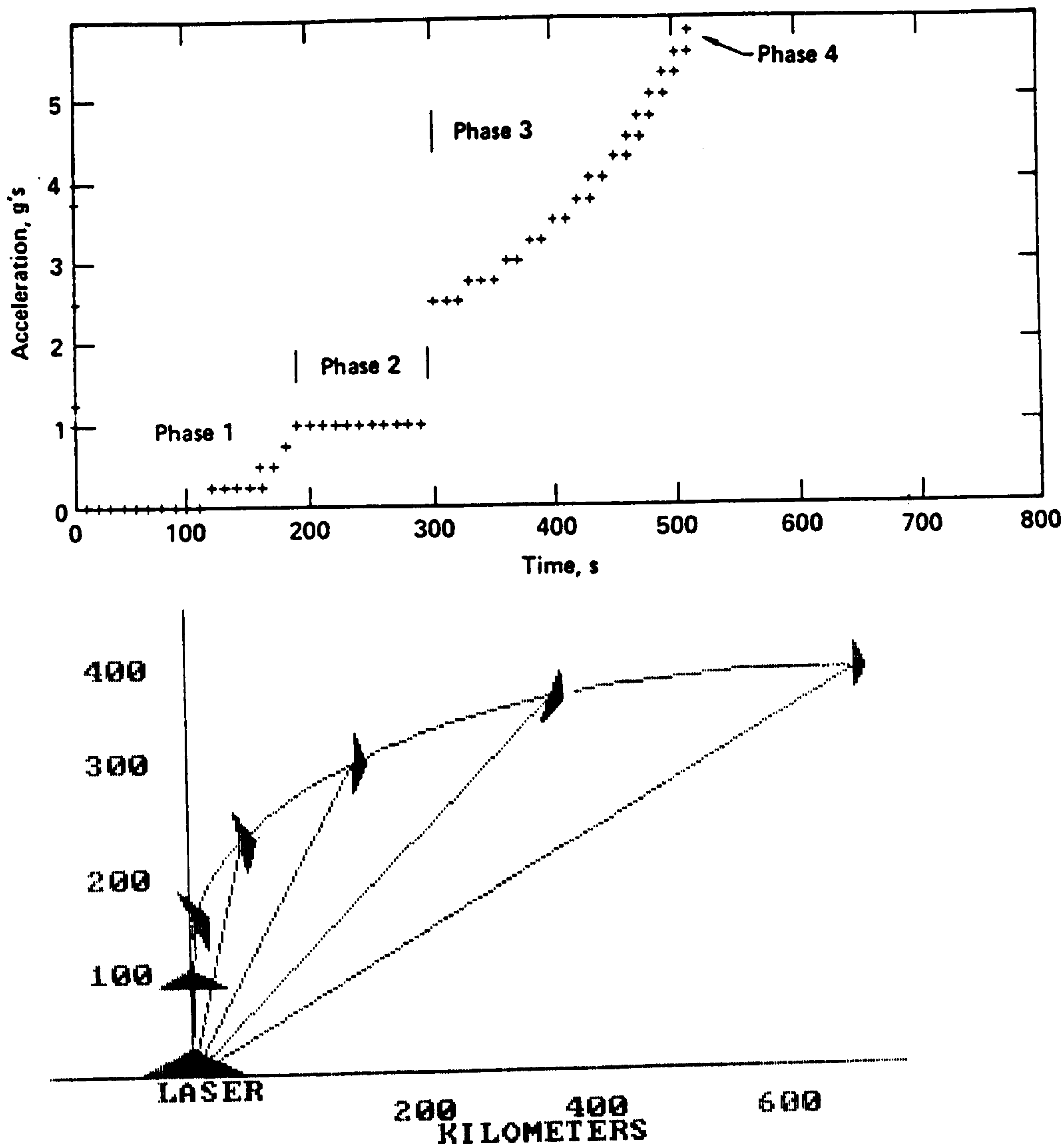

Figure 2. The sample trajectory. Beginning at a 3-km mountaintop, the launch is divided into four phases and reaches a 411-km orbit after 502 s. The vehicle coordinates and the angle (not scalable) to the laser are shown in the lower graph, and the vehicle acceleration is shown in the upper graph.

accelerated horizontally until after about 360 sec into the flight the zenith angle between the laser beam and the vertical reached .5 radians. Then the thrust became horizontal and afterward had a downward component which was continued until the vertical velocity was cancelled.

4. When the vertical velocity became negative near the end of acceleration the vehicle was tilted in phase 4 to maintain the vertical velocity close to 0. Phase 4 ended 502 seconds into the flight when orbital velocity was reached.

TABLE 2

SAMPLE 10 MW LAUNCH

TIME	MASS	HOR.DIST.	HEIGHT	VJ PHASE 1&2	VERT.VELOC.
		PHASE 1			
0	120	0	3	3.6	.085
20	107.09	0	4.7	3.648	.091
40	96.4	0	6.6	3.958	.099
60	87.45	0	8.7	4.268	.11
80	79.77	0	11.1	4.573	.124
100	73.13	0	13.8	4.862	.144
120	67.25	0	17	5.111	.173
140	61.87	0	20.9	5.261	.218
160	56.59	0	26	5.155	.3
173		PHASE 2			
180	50.62	0	33.6	4.973	.468
200	46.17	0	45	6.575	.664
220	43.53	0	60.4	8.055	.861
240	41.52	0	79.7	8.761	1.058
260	39.74	0	102.9	9.192	1.254
280	38.11	0	130.1	9.588	1.451
280		PHASE 3		HORIZONTAL VELOCITY	
300	35.86	4.2	159.8	.412	1.514
320	33.61	17.1	190.7	.863	1.575
340	31.36	39.6	222.7	1.36	1.619
360	29.12	72.7	255.3	1.924	1.633
380	26.89	117.6	287.8	2.545	1.605
400	24.66	175.6	319.1	3.231	1.523
430	22.46	248	348.2	3.982	1.377
440	2028	336.1	373.5	4.795	1.157
460	18.13	441.1	393.7	5.672	.855
480	16.03	564.3	406.8	6.608	.465
500	13.99	706.8	411.2	7.605	-.02
500		PHASE 4			
502	13.79	722.2	411.2	7.71	-.003

END RESULTS

INITIAL MASS, KG 120 FINAL MASS 13.79
RANGE= 831 FINAL ZENITH ANGLE = 60 ACC. = 5.75
ELEC. BILL/KG IN LEO = $ 10.1 PROPELLANT = $ 15.39

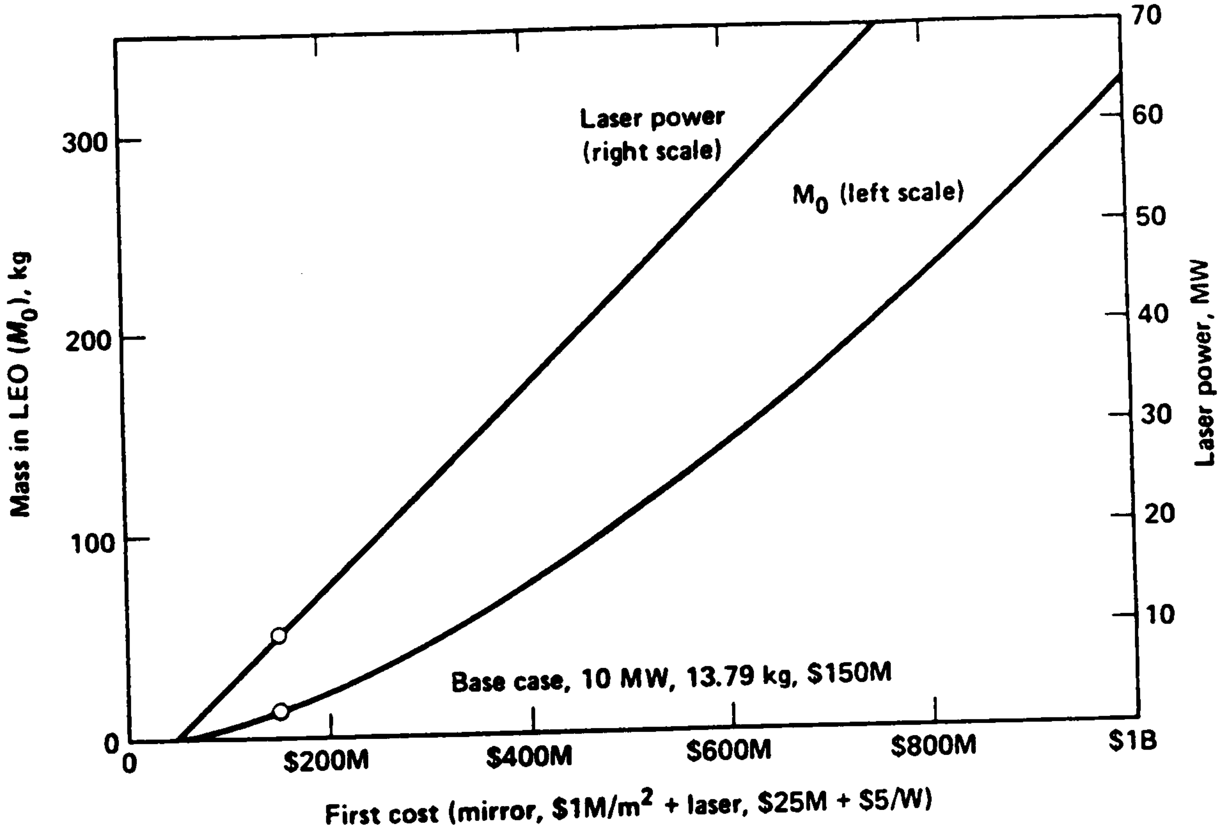

Figure 3. Estimates from Eq. (5) of laser launching capability to LEO (~400 km). Costs are based on estimates from Avco for CO_2 lasers and from Itek for adaptive optics.

ECONOMICS OF THE 10 MW LAUNCHER

What can we say now of the costs of transportation to LEO by this small scale laser propulsion?

The electricity used in the sample trajectory was about 505 kw hrs per kg of payload. Even though this is more than 50 times the ideal energy requirement, the cost of this electricity will not dominate transportation costs. This parallels the situation in chemical rockets where the fuel costs are also not dominant. For the calculations below and for Table 2, the U.S. Govt rate of $.02/kw hr was used.

It is harder to estimate propellant costs since a realistic propellant is still to be found. The mass of propellant, about 7.7 pounds per pound of payload, is small enough so that it would be expected that a propellant can be found which is cheap enough so that propellant costs probably will not have an important impact on overall launch costs. For the calculations below and for Table 2, $2/kg of propellant was assumed. It must be remembered that the choice of propellant will have a large impact on the thruster efficiency and thus a direct impact on the launching capability of a laser and on the economics of laser propulsion.

The important costs for laser propulsion are the capital and the operating costs of the ground laser installation. Eq. 4 can be used to optimize the distribution of costs between mirrors and lasers and to provide a rough estimate of the capital cost of a laser launching installation to launch Mo grams. If for example we take the cost of mirrors made with adaptive optics to be proportional to the mirror area ($1 M/m^2 was suggested by Itek) and we take laser costs to be proportional to average (not peak) power, we get that the costs should be distributed equally between laser and mirror. If also we take a laser cost $25M + $ 5 per watt (estimated by Jack Daugherty of Avco for CO2 lasers), and correcting eq. 4 by a factor 13.79/18 to agree with the result of the sample trajectory, we get

$$Mo = 11.5*(C - 50)*C^{.5} \qquad (5)$$

where C is the capital cost in millions of dollars.

Eq. 5 is plotted in fig. 3. The Department of Energy uses a rule of thumb for estimating the operating costs of large experimental installations of 20% of the capital cost per year. If we add amortization of the capital costs in 5 years then the costs of the ground installation comes to 40% per year. The 10 MW installation would have a capital cost of $150 M and an operating cost of $60 M/yr.

From the sample calculation, a 10 MW laser could launch 13.79 kg (30.4 lbs) in 502 secs. If the laser were used with a duty factor of 1 (62,821 launches/yr), it would then launch 866 tonnes/yr. Allotting the $60M/yr costs to the payload launched gives for the 10 MW laser

Launch cost/lb = $32/duty factor

\+ $12 (Electricity & Propellant) (6)

The \$1000/lb, estimated (Ref. 6) for the mid 1990s chemical rocket, would be bettered if the duty factory were greater than .032 (2000 launches/yr). The break-even point in eq. 6 will change quite rapidly with laser power. Neglecting the favorable variation of the launch time and the electricity and propellant costs with laser power, eq. 5 gives that a 20MW laser installation costing \$250 M would launch 36 kg. The break-even point would then occur at a duty factor of .02 (1300 launches/yr).

The primary uncertainty in these estimates comes from a lack of knowledge of what can be done to produce an efficient thruster without introducing too much flight hardware which has added so much to the cost of chemical rockets. In the same trajectory it was assumed that the 40% thruster efficiency would be maintained down to a flux of 10 MW/sq cm. Propellants will need to be developed to achieve high thruster efficiency at low flux to make laser propulsion a serious contender for space transportation to LEO. In view of the fact that almost no effort has been devoted to this requirement it should be evident that a great opportunity exists to creatively design materials.

It is a pleasure to acknowledge stimulating discussions of this subject with participants at the Livermore Workshop, especially Jordin Kare, Dennis Reilly and Rod Hyde. I am indebted to Freeman Dyson and Lowell Wood for the important suggestion that primary emphasis be placed on finding the minimum system for an initial trial of laser propulsion to orbit.

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6. Report of the National Commission on Space, Pioneering the Space Frontier, pg. 115. Bantam Books.

Appendix

A Sample Trajectory Using a 10-MW Laser

```
!LASERPROP TRAJECTORY (USING THRUSTER EFF THEFF = .4)
!  USE 10 MW LASER
! "LASPRP15" 12/31/86

! INITIALIZATION

 OPEN #1:PRINTER
 OPEN #2:NAME "OUTFILE"
 ERASE #2
 LET M = 120E3            ! MASS
 LET M0 = M
 LET CD = .4              ! ASSUMES DRAG LIKE A SPHERE
 LET B = 0                ! ACTIVATES PHASE 1
 LET AREA = 1E4           ! BASE AREA
 LET THEFF = .4
 LET Y = 3E5              ! ALTITUDE, MOUNTAIN
 LET Y0 = Y
 LET LPWR = 1E14*(1-.1* (1-EXP( (Y0-Y)/7E5) )/COS(TH) )    ! 10 MW, 10% VERT ATM LOSS
 LET VX = 0               ! HORIZONTAL VEL.
 LET RHO = (1.225E-3)*EXP(-Y/7E5)      ! EXPONENTIAL ATMOSPHERE
 LET VY = SQR(2*M*983/(RHO*AREA*CD))   ! INITIAL VY TO MINIMIZE MDOT/VY
 LET VJ = 3.6E5           ! INITIAL JET VELOCITY, PHASE 1
 PRINT #1: , "TABLE 2, SAMPLE 10 MW LAUNCH"
 PRINT #1:
 PRINT #1: "TIME"; "MASS", "HOR. DIST.", "HEIGHT", "VJ PHASE
 1&2", "VERT. VELOC."
 PRINT #1:
 PRINT #1: , "PHASE 1"

! TRAJECTORY

FOR T = 0 TO 2000
    IF T/20 = INT (T/20) THEN
        PRINT #1: T; INT (M/10)/100;,
        PRINT #1: INT (X/1E4)/10, INT (Y/1E4)/10,
        IF B = 0 OR C = 0 THEN
            PRINT #1: INT (VJ/100)/1E3,
        ELSE
            PRINT #1: INT (VX/100)/1E3,
    END IF
    PRINT #1: INT (VY/1E2)/1000
    END IF
    IF T/5 = INT (T/5) THEN          ! OUTPUT TO PLOTTER
        !  PRINT #1:   #2:INT (X/1E4)/10;",",INT (Y/1E4)/10
        ! IF T>1 THEN PRINT #1: #2: T;",", VDOT
    END IF
    IF (VX^2/(6.371E8 + Y))>983*(1 + Y/6.371E8)^(-2) THEN        ! ORBIT REACHED
        SOUND 500,1
        PRINT #1: T;INT (M/10)/100,
        PRINT #1: INT (X/1E4)/10,INT(Y/1E4)/10,INT (VX/1E3)/100,INT (VY/1E2)/1000
```

```
    EXIT FOR
END IF
!
IF Y<1.30E7 THEN          ! VERTICAL ASCENT THRU THE ATMOS.
    LET A = 0
  LET RHO = (1.225E-3) *EXP (-Y/7E5)
    LET DRAG = CD*.5*RHO*(VX^2 + VY^2)*AREA
    LET THRUST = (M*VDOT + M*983 + DRAG)
    LET VJ = 2* LPWR*THEFF/THRUST
    LET MDOT = THRUST/VJ
    ! PHASE 1 USE LOW VJ FOR HIGH THRUST
    IF B = 0 THEN                 ! TO MINIMIZE MDOT/VY
        LET VDOT = SQR(2*M*983/ (RHO*AREA*CD)) - VY
        IF VDOT>983 THEN              ! GO TO PHASE 2
            LET B = 1
            PRINT #1: T, "PHASE 2"
            EXIT IF
        END IF          SET VDOT = 983 PHASE 2
    ELSE IF B = 1 THEN            ! SET VDOT = 983 PHASE 2
        LET VDOT = 983
    END IF
    LET VY = VY + VDOT
    ! OUT OF THE ATMOSPHERE          PHASE 3
ELSE IF VY>0 THEN      ! TILT VEHICLE 1 RADIAN FOR HOR AND DOWN THRUST
    LET A = 1
    IF C=0 THEN
        PRINT #1: T,"PHASE 3                    HORIZONTAL VELOCITY"
        LET C=1
    END IF
    LET VJ = 8E5
    LET MDOT = LPWR*2*THEFF/VJ^2
    LET VDOT=MDOT*VJ/M
    LET VX = VX + VDOT*SIN(TH+1)
    LET VY = VY+VDOT*COS (TH+1)-983 + ((VX)^2)/6.371E8
ELSE IF VY<0 THEN             !              PHASE 4
    IF D = 0 THEN
        PRINT #1: T, "PHASE 4"
        LET D=1
    END IF
    LET A=2              ! HOLD VY NEAR 0
    LET VJ = 8E5
    LET MDOT = LPWR*2*THEFF/VJ^2
    LET VDOT = VJ*MDOT/M
    LET VYDOT = -983 + ((VX)^2) /6.371E8
    LET VX = VX+VDOT*SQR(1-(VYDOT/VDOT0^2)
    LET VY = VY-.983+((VX)^2)/6.371E8+VYDOT
END IF
LET X = X+VX
LET Y = Y+VY
LET M = M - MDOT
LET TH=ATN(X/Y)           ! ZENITH ANGLE

LET LPWR = 1E14*(1-.1*(1-EXP((YO-Y)/7E5))/COS(TH))         ! 10MW, 10% VERT ATM
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LOSS
NEXT T
LET D = 1E-5*SQR(X∧2 + Y∧2)            ! RANGE IN KM

PRINT #1:
PRINT #1:"           END RESULTS"
PRINT #1:
PRINT #1: "INITIAL MASS, KG"; INT (MO/1E3),
PRINT #1: "FINAL MASS"; INT (M/10)/100
LET AC = INT (.1*VDOT)/100
PRINT #1: "RANGE = ";INT(D),"FINAL ZENITH ANGLE = ";INT(57.3*TH);
PRINT #1: "        ACC. = ";AC
LET EB = INT(100*T*5E4*.02/( (3600)*(M/1E3) ) )/100    ! $.02/KWHR,LASER = 20% EFF
LET PB = INT(100*( (MO-M)/M*2)/100    ! $2/KG
PRINT #1: "ELEC. BILL/KG IN LEO    = $";EB,
PRINT #1: "PROPELLANT    = $";PB
END
```