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TURBULENCE KINETIC ENERGY EQUATION FOR DILUTE SUSPENSIONS

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ABSTRACT

This paper presents a multiphase turbulence closure employing one transport equation, namely the turbulence kinetic energy equation. The proposed form of this equation is different from the earlier formulations in some aspects. The power spectrum of the carrier fluid is divided into two regions, which interact in different ways and at different rates with the suspended particles as a function of the particle-eddy size ratio and density ratio. The length scale is described algebraically. A mass/time averaging procedure for the momentum and kinetic energy equations is adopted. The resulting turbulence correlations are modeled under less restrictive assumptions comparative to the previous work. The closures for the momentum and kinetic energy equations are given. Comparisons of the predictions with experimental results on liquid-solid jet and gas-solid pipe flow show satisfactory agreement.

NOMENCLATURE

a	amplitude ratio
$\underline{b}$	body force
$C_1, C_{\phi1}, C_{\phi2}, C_{\phi3}, C_{\phi4}, C_{\phi5}, C_D, C_\mu$	= constants
d	particle diameter
D	turbulent diffusion coefficient for solid phase
$E(\kappa)$	energy spectrum
f	flow variable
$(\underline{I}_K)_{-K}$	interaction term
$\underline{J}_\phi$	flux vector for a variable ϕ
k	kinetic energy of turbulence
ℓ	length scale
N_S	Stokes number
$K(\underline{r},t)$	phase distribution function, Eq. (1)
p	pressure
$\underline{r}$	position vector
Re	Reynolds number based on the most energetic eddy size
$t, \Delta T, \Delta t$	time and averaging time intervals corresponding to the turbulence production and transfer range, respectively
u,v	Eulerian and Lagrangian velocities, respectively
$V, \Delta v$	volume and averaging volume, respectively
x_i, x_j, x_n	Cartesian coordinates
y	distance from the wall
α	concentration of volume
ϵ	dissipation rate of turbulence kinetic energy
η	Kolmogorov length scale
κ	wave number
μ	dynamic viscosity
ρ	density
σ	turbulent Schmidt/Prandtl number
τ	shear stress

Subscripts

i,j,n	denote Cartesian coordinates (= 1,2,3)
e	eddy
k	turbulence kinetic energy
K	flow component K
KP	phase K in the production range
KT	phase K in the transfer range
ℓ	laminar
L,S	denote liquid and solid, respectively
P	production range
t	turbulent
T	transfer range
Tot	total
Σ	average over area
ϵ	dissipation rate of turbulence kinetic energy

Superscripts

$\bar{f}$	mass average of f
f^t	turbulence fluctuation of f
f'	fluctuation of f at low wave number (production range)
f''	fluctuation of f at intermediate wave number (transfer range)
P	production
T	transfer
$\bar{f}^K, f_K$	volume/time and mass/time averages of f over K
$\langle f_K \rangle$	intrinsic space average of f over K

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1. INTRODUCTION

Multiphase flows are widely applied in engineering processes from chemical, petroleum, mining and other industries. Various theoretical and experimental techniques for the investigation of those flows are available. Some of them are a straightforward extension from the single phase flow models by introducing some ad hoc modifications. Other investigations originate from the gas-solid flow (Soo, 1983) or fluidized bed models (Wang et al., 1985).

Increasing concern for the prediction of turbulent multiphase flows have been noticed during the last twenty years (Danon et al. (1974), Al-Taweel and Landau (1977), Genchev and Karpuzov (1980) , Melville and Bray (1979), Crowe and Sharma (1978), Michaelides and Farmer (1983), and Shuen et al. (1983)). Two equation turbulence models have been proposed for dilute particulate flows by Elghobashi et al. (1982, 1983, 1984) and Crowder et al. (1984). Algebraic and one equation turbulence models have been suggested also for dense liquid-solid flows (Roco et al., 1983, 1985, 1986) in which the particle-particle interactions play an important role besides the fluid-fluid and fluid-solid interactions. Most of these studies as well as other earlier investigations have some limitations. In the above mentioned studies the response of solids to the turbulent fluctuations of the carrier fluid is obtained under restrictions similar to those refered by Hinze (1975, p. 460), which limit their use. In addition to that, empirical constants and empirical functions are usually introduced in these models.

The purpose of the present paper is

i) To propose a specific mass/time averaging approach for multiphase turbulent flows. Even if the approach is developed for incompressible flow, its application for other multiphase flows is foreseen: From the liquid-solid interaction forces only the drag force is considered in this paper.

ii) To improve the one-equation turbulence model reported in [28] by including the modulation of turbulence by particles as a function of particle size and density.

iii) To test the proposed model with other models and experimental data for various two-phase flows, without adopting any adjusting empirical coefficients.

2. MIXED AVERAGING APPROACH

The continuum transport equations for multiphase flows can be obtained by assuming a continuum medium with averaged field quatities by using either time, local volume, local mass or spectral averaging (see Buyevich (1971), Soo (1967), Vernier and Delhaye (1968), Hetsroni (1982)). The averaging for multiphase flow systems may be performed in various ways. Mass averaging technique was applied by Abou-Arab (1985) for turbulent incompressible and compressible flows. To express the spatial nonuniformities and interactions between the flow components Roco and Shook (1985) have developed a specific volume/time averaging technique for turbulent multicomponent systems, in which the size of the averaging volume Δv is related to the turbulence scale. Since the Eulerian description of the flow is more convenient than the Lagrangian description and there are more comprehensive mathematical schemes for such formulation, they transformed the volume/time averaging into double time averaging. For any flow function f and any component K in the mixture, at a position $\underline{r}$ and time t

$$\bar{f}^K(\underline{r},t) = \frac{1}{\Delta T}\int_{t-\Delta T/2}^{t+\Delta T/2} \langle f_K \rangle \, dt \qquad (1)$$

where

ΔT = time averaging interval corresponding to the turbulence production range ($\Delta T \to \infty$).

$$\langle f_K \rangle = \frac{1}{\Delta t}\int_{t-\Delta t/2}^{t+\Delta t/2} f(\underline{r},\tau)\, K(\underline{r},\tau)\, R(\tau-t)\, d\tau \qquad (2a)$$

= intrinsic averaging of f over Δt and the flow component K.

Δt = Eulerian time scale for the most energetic eddies $=\int_0^\infty R(\tau-t)d\tau$ (corresponding to the Taylor's length scale). Note that $\Delta t \ll \Delta T$, but much larger than particle residence time at $\underline{r}$.

$$K(\underline{r},\tau) = \begin{cases} 1 & \text{if the considered flow component resides at point } \underline{r} \text{ and time } \tau \\ 0 & \text{otherwise} \end{cases} \qquad (2b)$$

= phase distribution function

$$R(\tau-t) = \frac{\overline{V''(t)V''(t-\tau)}}{\overline{V''^2(t)}} \qquad (2c)$$

= auto-correlation coefficient of the velocity fluctuations of the most energetic eddies (Taylor's scale)

The time averaging over Δt corresponds to the averaging over local volume Δv. The dimension of Δv is given by the turbulence mixing length, and Δt = (length scale of Δv)/(mean velocity) [29]. By integrating over a flow component K its interfaces with other flow components become boundary of integration, and the interaction terms are derived in a straighforward manner in the differential formulation. According to (1), any instantaneous value differs from the mean value by a turbulence fluctuation with two components f'_K and f''_K:

$$f_K = \bar{f}_K + f^t = \bar{f}_K + f'_K + f''_K \qquad (3)$$

where

$$f''_K = f_K - \langle f_K \rangle \qquad (4)$$

= spatial nonuniformities within Δv (or temporal nonuniformities within Δt)

$$f'_K = \langle f_K \rangle - \bar{f}^K \qquad (5)$$

= temporal nonuniformities of $\langle f_K \rangle$ within ΔT.

Since the averaging domain (Δv or Δt) has the dimensions of the mixing length or Eulerian time scale, respectively, the turbulence fluctuation f_K'' corresponds to the turbulence transfer range. The temporal nonuniformities f_K' reflect the turbulence fluctuations in the production range.

By averaging with formula (1) the point instantaneous conservation equations, one obtains the double time averaged equations. The formulation is equivalent to the volume/time averaging. The momentum and kinetic energy equations are given in Appendix A.

The local mass average of f over a flow component K is denoted $\bar{f}_K$. It is obtained by applying (1), in which the phase distribution function $K(\underline{r},\tau)$ is weighted by the specific mass ρ_K.

In this paper we model the phase interaction by using spectral analysis and suggest a closure of the mass averaged equations for linear momentum and kinetic energy. The averaged equations are initially written with all the terms, and then simplified formulations for various flow conditions are suggested.

3. ENERGY SPECTRUM AND SOLIDS - EDDY INTERACTION

It is well accepted that turbulence is characterized by fluctuating motions defined by an energy spectrum (Tennekes and Lumely (1972)). Single time scale models, which are normally used for the prediction of turbulent flows, seems simplistic because different turbulent interactions are associated with different parts of the energy spectrum (Hanjalic' et al. (1979)). A typical energy spectrum can be divided into three regions. The first region is the production region of large eddies and low wave number. The third region is the dissipation region with small eddies and high wave number, in which the total kinetic energy produced at the lower wave number is dissipated. The intermediate range of wave numbers represents the Taylor's transfer range. The total kinetic energy k of turbulence may be divided into production range (k_P) and transfer range (k_T) because there is negligible kinetic energy in the dissipation range:

$$k \simeq k_P + k_T \tag{6}$$

where

$$k_P = \frac{1}{2} \overline{u_i'^2} , \tag{7a}$$

$$k_T = \frac{1}{2} \overline{u_i''^2} \tag{7b}$$

u_i', u_i'' = fluctuating velocities in the production and transfer range, respectively.

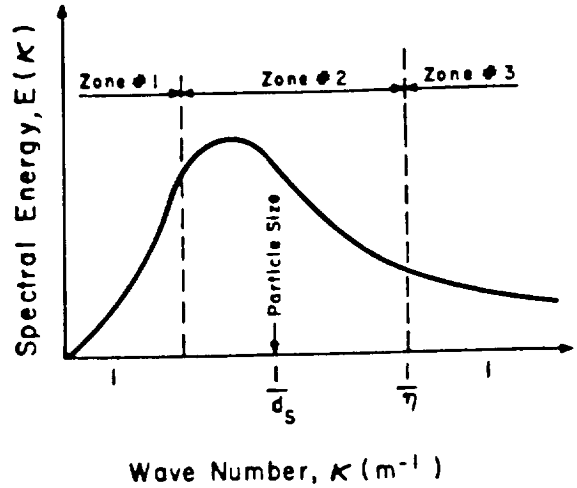


Figure 1. Schematic showing the relative particle size to different eddy sizes in the energy spectrum.

This partitioning of the energy spectrum was shown to be important for swirling flows (Chen (1986)), and hetrogeneous mixture flows such as two-phase jet (Al-Taweel and Landau (1977)).

By using spectral analysis in conjunction with mass/time averaging some additional turbulent correlations will result in the mixture flow equations comparative to homogenous flows. These correlations can be classified into <u>five categories</u>:

i) Eddy-eddy interaction
ii) Eddy-mean flow interaction
iii) Eddy-particle interaction
iv) Particle-mean flow interaction
v) Particle-particle interaction (for dense suspension flow).

These correlations have to be modeled. Since the suspended particles may be of different sizes and different materials, their response to the carrier fluid fluctuations will vary as a function of the mean and fluctuating properties of the flow. The present work will consider a <u>two-way interaction mechanism</u> between solid particles and fluid vorticies in dilute suspensions. This interaction mechanism depends on the ratio between the particle size d_S and the turbulent vortex (eddy) size l_e. These length scales are compared with the turbulence dissipation micro-scale (η).

To determine the <u>particle-eddy interaction</u> the energy spectrum for multiphase flow system is divided into three typical zones (Figure 1):

1. "Large vortex zone" (#1), where the turbulence energy is extracted from the mean flow by low frequency eddies. Here, the eddy length scale ℓ_{e1} is larger than the particle size d_S:

$$l_{e1} > d_S > \eta \tag{8a}$$

2. "Medium vortex zone" (#2), where the solid particles are about the same size with the vortex size, i.e.

$$l_{e2} \approx d_S > \eta \tag{8b}$$

3. "Small vortex zone" (#3), which would correspond to the Kolmogorov's length scale, i.e.

$$d_S > l_{e3} \approx \eta \tag{8c}$$

In zone #1 the solid particles generally follow the motion within a vortex, and have an energy dissipation effect. The particle response to the turbulent fluctuations (turbulence modulation) is fully determined (see Hinze, 1975). In the small vortex zone #3 the solid particles can not significantly affect the turbulence microstructure. For the intermediate zone #2 a linear variation of the particle response is considered. This partitioning allows for the particles-nonuniform size eddies interaction to be efficiently modeled.

The present closure formulation originates from the idea of subgrid scale modeling. If this idea is to be accepted, any flow quantity u, v, α, k, ..., etc. may be separated into three parts according to (3), where $f_K^{'}$ and $f_K^{''}$ define the fluctuations in the production and transfer ranges of the energy spectrum, respectively. By starting from the particle equation of motion in its general form, the relation between the particle motion and different fluid eddies can be determined, and from here the fluctuation components $f_K^{'}$ and $f_K^{''}$ (see section 6).

4. COMPOSED AVERAGED EQUATIONS

4.1 Mean Flow Governing Equations

The mass/time averaged momentum equation (Appendix A, Eq. A2) with $f_K^t = f_K^{'} + f_K^{''}$ yields:

$$\rho_K \frac{\partial}{\partial t} (\bar{\alpha}_K \bar{u}_{Ki} + \overline{\alpha_K^{'} u_K^{'}} + \overline{\alpha_K^{''} u_K^{''}} + \overline{\alpha^{'} u_{Ki}^{''}} + \overline{\alpha^{''} u_{Ki}^{'}})$$

Time rate change of the mean flow convection

$$+\rho_K \frac{\partial}{\partial x_j} [\bar{\alpha}_K \bar{u}_{Ki} \bar{u}_{Kj}]$$

Mean flow convection

$$+ \rho_K \frac{\partial}{\partial x_j} [\bar{\alpha}_K \overline{u_{Ki}^{'} u_{Kj}^{'}} + \bar{\alpha}_K \overline{u_{Ki}^{''} u_{Kj}^{''}}]$$

Inertial effect

$$+\rho_K \frac{\partial}{\partial x_j} ([\overline{\alpha_K^{'} u_{Ki}^{'} u_{Kj}^{'}} + \overline{\alpha_K^{''} u_{Ki}^{''} u_{Kj}^{''}}] + \bar{u}_{Kj}[\overline{\alpha_K^{'} u_{Ki}^{'}} + \overline{\alpha_K^{''} u_{Ki}^{''}}]$$

Collisional/Inertial Effects

$$+ \bar{u}_{Ki} [\overline{\alpha_K^{'} u_{Kj}^{'}} + \overline{\alpha_K^{''} u_{Kj}^{''}}] + \bar{\alpha}_K[\overline{u_{Ki}^{'} u_{Kj}^{''}} + \overline{u_{Ki}^{''} u_{Kj}^{'}}]$$

$$+ \bar{u}_{Ki} \overline{\alpha_K^{'} u_{Kj}^{''}} + \bar{u}_{Kj} \overline{\alpha_K^{'} u_{Ki}^{''}} + \overline{\alpha_K^{'} u_{Ki}^{''} u_{Kj}^{'}} + \overline{\alpha_K^{'} u_{Ki}^{''} u_{Kj}^{''}}$$

$$+ \overline{\alpha_K^{''} u_{Ki}^{'}} \bar{u}_{Ki} + \overline{\alpha_K^{''} u_{Ki}^{'}} \bar{u}_{Kj} + \overline{\alpha_K^{''} u_{Kj}^{'} u_{Ki}^{'}} + \overline{\alpha_K^{''} u_{Ki}^{''} u_{Kj}^{'}})$$

$$= \rho_K[\bar{\alpha}_K \bar{b}_{Ki} + \overline{\alpha_K^{'} b_{Ki}^{'}} + \overline{\alpha_K^{''} b_{Ki}^{''}} + \overline{\alpha_K^{'} b_{Ki}^{''}} + \overline{\alpha^{''} b_{Ki}^{'}}]$$

Body force

$$- \frac{\partial}{\partial x_i} [\bar{\alpha}_K \bar{p}_K + \overline{\alpha_K^{'} p_K^{'}} + \overline{\alpha_K^{''} p_K^{''}} + \overline{\alpha_K^{'} p_K^{''}} + \overline{\alpha_K^{''} p_K^{'}}]$$

Pressure effect

$$+ [\bar{p}_K \frac{\partial \bar{\alpha}_K}{\partial x_i} + \overline{p_K^{'} \frac{\partial \alpha_K^{'}}{\partial x_i}} + \overline{p_K^{''} \frac{\partial \alpha_K^{''}}{\partial x_i}} + \overline{p_K^{''} \frac{\partial \alpha_K^{'}}{\partial x_i}} + \ldots]$$

$$+ \frac{\partial}{\partial x_j} [\bar{\alpha}_K \bar{\tau}_K + \overline{\alpha_K^{'} \tau_{K\Sigma ji}^{'}} + \overline{\alpha_K^{''} \tau_{K\Sigma ji}^{''}}$$

Frictional effect

$$+ \overline{\alpha_K^{''} \tau_{K\Sigma ji}^{'}} + \overline{\alpha_K^{'} \tau_{K\Sigma ji}^{''}}] + (\overline{I_{Ki}})_K \tag{9}$$

Phase interaction

where K is a flow component, b_{Ki} is the body force in the i-th direction, and $(I_{Ki})_{-K}$ is the projection in the i-th direction of the interaction vector $(\underline{I}_K)_{-K}$

Equation (9) contains correlations which are related to the production and transfer wave number ranges of the turbulence spectrum. It contains also mixed correlations.

4.2 The Kinetic Energy Equation

Similar to the mean flow governing equation the mass/time averaging form of the turbulent kinetic energy equation can be derived for a flow component K. The exact form of this equation for steady state turbulent flow is given in Appendix A (Eq. A5). It contains more than one hundred correlations which are related to the eddies in the production and transfer ranges as well as some mixed correlations. However, only some of these are predominant in a given flow situation as a function of relative particle-vortex size and density ratio.

4.3 Some Modeling Principles and Assumptions

By analysing the derived mean momentum and kinetic energy conservation equations, it can be easily recognized that some modeling assumptions must be made, based on the physical interpretation and the nature of each term. Previous experimental and theoretical findings can help in modeling "collectively" similar terms with minimum number of empirical constants. Secondly, carrying out an order of magnitude analysis for different correlations which appear in the governing equations some terms may be neglected. Thirdly, the micromechanics which control the ability of the flow variables to correlate with each other and the factors affecting the magnitude of these correlations should be considered. With the previous remarks in mind, one can assume for the sake of simplicity that:

1) The correlations between the large eddies from the production range and the small eddies from the transfer range (mixed correlations e.g. $\overline{u'u''}$) can be neglected as they originate differently and they are related to different ranges of the power spectrum. Similar assumptions are accepted in the classical single fluid turbulence theory.

2) The void fraction fluctuations occur mainly at low frequencies i.e.

$$\alpha = \bar{\alpha} + \alpha' + \alpha'' \tag{10}$$

with $\alpha' \gg \alpha''$ (11)

This is a simplifying assumption which is acceptable for such complicated problems. If the particles are of small diameter their concentration is relatively uniform distributed in the Taylor length scale. High void fractions are mostly associated with large size and high density particulate flows. These large size particles are mainly fluctuating at low frequencies due to its high inertia, hence they in turn correlate weakly at high frequencies.

3) The correlations of higher order than three, for instance $\overline{\alpha' u' \partial p' / \partial x_i}$, $\overline{u_i' u_j' \frac{\partial p'}{\partial x_j}}$ ... etc., are neglected. These are at least an order magnitude smaller than those of the third order (see Hanjalic' and Launder (1972)).

4) Pressure diffusion contribution to the total turbulent diffusion in the kinetic energy equation will be neglected because of its relatively small magnitude (Hanjalic' and Launder (1972)).

5) The Boussinesq gradient type approximation is adopted for modeling of different fluxes and triple correlations, with assumptions similar to Elghobashi and Abou-Arab (1983) and Roco and Mahadevan (1986)

6) The following constitutive relations are employed for the shear stress of carrier fluid:

$$-\rho\overline{u_i'u_j'} = \mu_{LP}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) - \frac{2}{3} k_{LP}\delta_{ij} - \frac{2}{3}\mu_{LP}\delta_{ij}\bar{u}_{n,n} \tag{12}$$

$$-\rho\overline{u_i''u_j''} = \mu_{LT}\left(\frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i}\right) - \frac{2}{3} k_{LT}\delta_{ij} - \frac{2}{3}\mu_{LT}\delta_{ij}\bar{u}_{n,n} \tag{13}$$

while the total shear stress $-\rho\,\overline{u_i^t u_j^t}$ can be given by

$$\rho\,\overline{u_i^t u_j^t} = \rho\,\overline{u_i' u_j'} + \rho\,\overline{u_i'' u_j''}\ . \tag{14}$$

where

$$\mu_{LP} = C_{\mu P}\,\rho_L k_{LP}^{\ 1/2}\, l_P. \tag{15}$$

$$\mu_{LT} = C_{\mu T}\,\rho_L k_{LT}^{\ 1/2}\, l_T \quad \text{, and} \tag{16}$$

$$\mu_{Lt} = \mu_{LP}^{P} + \mu_{LT}^{T} = C_\mu \rho_L k_L^{\ 1/2}\, l_L \tag{17}$$

Similar relations can be written for the viscosity of the dispersed phase μ_{St} (see Roco and Balakrishnan (1985)). However, in the present work we choose to define the eddy viscosity of solids as follows:

$$\nu_{St} = \nu_{Lt} / \sigma \tag{18}$$

where

$$\sigma = \sigma_{\alpha S} / \sigma_S \tag{19}$$

and

$$\sigma_{\alpha S} = \nu_{Lt} / D_S \tag{20}$$

$$\sigma_S = \nu_{St} / D_S \tag{21}$$

Appropriate expressions for $\sigma_{\alpha S}$ are cited in many articles such as Peskin (1971), Picart et al. (1986), and Hetsroni (1982). σ_S is a Schmidt number and its value is about 1.5 (Abou-Ellail and Abou-Arab (1985)).

7) In the present approach for dilute particulate flows the turbulence kinetic energy equation is written only for the carrier flow. The solid phase turbulence kinetic energy and turbulence correlations are evaluated from the available solution of the linearized equation of motion of a solid particle after its transformation from the real time to the frequency domain.

8) Terms which are of similar nature i.e. convection, diffusion, dissipation, etc. can be modeled collectively. The length-, velocity- and time-scale which are appropriate for the description of their rate of change must be identified from the physical interpretation of these terms.

9) The response function which shows the ability of solid particles to follow the eddies is obtained from the equation of motion of particles for different local dimensionless parameter, d_S/l_e and ρ_S/ρ_L.

10) To establish the degree of generality of the proposed model validation tests were carried out for air-laden and water-laden jet, and air-solid pipe flow.

5. CLOSURE FOR THE MEAN FLOW EQUATIONS

With the modeling assumptions given in the previous section, the steady state mean flow momentum equations for any flow component, reads

$$\rho_K \frac{\partial}{\partial x_j}(\bar{\alpha}_K \bar{u}_{Ki}\bar{u}_{Kj}) + \frac{\partial}{\partial x_j}\rho_K(\bar{\alpha}_K\overline{u_{Ki}'u_{Kj}'} + \bar{\alpha}_K\overline{u_{Ki}''u_{Kj}''}) +$$

$$+ \frac{\partial}{\partial x_j}\{\rho_K[(\overline{\alpha_K^{\cdot}u_{Ki}^{\cdot}u_{Kj}^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}u_{Ki}^{\cdot\cdot}u_{Kj}^{\cdot\cdot}}) + \bar{u}_{Ki}(\overline{\alpha_K^{\cdot}u_{Kj}^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}u_{Kj}^{\cdot\cdot}})$$

$$+ \bar{u}_{Kj}(\overline{\alpha_K^{\cdot}u_{Ki}^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}u_{Ki}^{\cdot\cdot}})] + (\overline{\alpha_K^{\cdot}\tau_{K\Sigma ji}^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}\tau_{K\Sigma ji}^{\cdot\cdot}})\}$$

$$= \rho_K(\bar{\alpha}_K\bar{b}_{Ki} + \overline{\alpha_K^{\cdot}b_{Ki}^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}b_{Ki}^{\cdot\cdot}}) - \frac{\partial}{\partial x_i}(\bar{\alpha}_K\bar{p}_K + \overline{\alpha_K^{\cdot}p_K^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}p_K^{\cdot\cdot}})$$

$$+ (\bar{p}_K\frac{\partial\bar{\alpha}_K}{\partial x_i} + \overline{p_K^{\cdot}\frac{\partial\alpha_K^{\cdot}}{\partial x_i}} + \overline{p_K^{\cdot\cdot}\frac{\partial\alpha_K^{\cdot\cdot}}{\partial x_i}})$$

$$+ \frac{\partial}{\partial x_j}(\bar{\alpha}_K\bar{\tau}_{K\Sigma ji}) + (\overline{I_{Ki}})_{-K} \qquad (22)$$

In Eq. (22) there are 16 correlations, half of them in the production "large eddy" range of the spectrum. The terms are modeled following the criteria: i) Physically correct behavior, ii) Minimum number of empirical functions and constants, and iii) Comparisons against experimental data over a wide range of conditions are required to check the validity of the model.

The turbulent stresses caused by the large and small size energetic eddies, $-\rho_L\overline{u_i^{\cdot}u_j^{\cdot}}$ and $-\rho_L\overline{u_i^{\cdot\cdot}u_j^{\cdot\cdot}}$, are defined by Eqs. (12) and (13). The correlation between $u_i^{\cdot\cdot}$ and $u_j^{\cdot\cdot}$ is weak when $i \neq j$. This finding will be explained in the next section.

The collisional effect correlations are modeled after Launder (1976):

$$\overline{\alpha_K^{\cdot}u_{Ki}^{\cdot}u_{Kj}^{\cdot}} = - C_{\phi5}^{P}(\frac{k_{KP}}{\epsilon_{KP}})(\overline{u_{Ki}^{\cdot}u_{Kl}^{\cdot}}\frac{\partial}{\partial x_l}\overline{u_{Kj}^{\cdot}\alpha_K^{\cdot}}$$

$$+ \overline{u_{Kj}^{\cdot}u_{Kl}^{\cdot}}\frac{\partial}{\partial x_l}\overline{u_{Ki}^{\cdot}\alpha_K^{\cdot}}) \qquad (23)$$

where $C_{\phi5}^{P}$ is a constant of a value of about 0.1. Similarly

$$\overline{\alpha_K^{\cdot\cdot}u_{Ki}^{\cdot\cdot}u_{Kj}^{\cdot\cdot}} = - C_{\phi5}^{T}(\frac{k_{KT}}{\epsilon_{KT}})(\overline{u_{Ki}^{\cdot\cdot}u_{Kl}^{\cdot\cdot}}\frac{\partial}{\partial x_l}\overline{u_{Kj}^{\cdot\cdot}\alpha_K^{\cdot\cdot}}$$

$$+ \overline{u_{Kj}^{\cdot\cdot}u_{Kl}^{\cdot\cdot}}\frac{\partial}{\partial x_l}\overline{u_{Ki}^{\cdot\cdot}\alpha_K^{\cdot\cdot}}) \qquad (24)$$

where $C_{\phi5}^{T}$ must be optimized by comparison with experimental data. The fluxes in (23) and (24) can also be collectively modeled as:

$$\overline{\alpha_K^{\cdot}u_{Ki}^{\cdot}u_{Kj}^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}u_{Ki}^{\cdot\cdot}u_{Kj}^{\cdot\cdot}} =$$

$$- C_{\phi5}\frac{k_K}{\epsilon_K}(\overline{u_{Ki}^{t}u_{Kl}^{t}}\cdot\frac{\partial}{\partial x_l}\overline{u_{Kj}^{t}\alpha_K^{t}}$$

$$+ \overline{u_{Kj}^{t}u_{Kl}^{t}}\cdot\frac{\partial}{\partial x_l}\overline{u_{Ki}^{t}\alpha_K^{t}}) \qquad (25)$$

The diffusion fluxes namely $\overline{\alpha_K^{\cdot}u_{Ki}^{\cdot}}$, $\overline{\alpha_K^{\cdot\cdot}u_{Ki}^{\cdot\cdot}}$ .... etc. are modeled using Boussinesq approximation as:

$$- \overline{\alpha_K^{\cdot}u_{Ki}^{\cdot}} = \frac{\nu_{KP}}{\sigma_{\alpha K}^{P}}\frac{\partial\bar{\alpha}_K}{\partial x_i} \qquad (26)$$

while

$$- \overline{\alpha_K^{\cdot\cdot}u_{Ki}^{\cdot\cdot}} = \frac{\nu_{KT}}{\sigma_{\alpha K}^{T}}\frac{\partial\bar{\alpha}_K}{\partial x_i} \qquad (27)$$

Similar expression can be written for $\overline{\alpha_K^{\cdot}u_{Kj}^{\cdot}}$ and $\overline{\alpha_K^{\cdot\cdot}u_{Kj}^{\cdot\cdot}}$.... etc. However, according to our assumptions, especially those concerning the void fraction fluctuation at high frequency α'', the fluxes in Eqs. (26) and (27) can be modeled collectively as:

$$- \overline{\alpha_K^{t}u_{Ki}^{t}} = \frac{\nu_{Kt}}{\sigma_{\alpha K}}\frac{\partial\bar{\alpha}_K}{\partial x_i} \qquad (28)$$

The solution of the two transport equations (for k_P and k_T) would require also a description for the length scales. This point will be explained in more details in the next section.

The fourth group of terms with the void fraction-shear stress turbulent correlations contains the laminar viscosity as a multiplier, and will be neglected due to its smaller order of magnitude.

The pressure effect contribution to the mean flow equation consists of two groups. Each contains three terms. The first of these terms is the mean pressure-void fraction. The second and the third terms are the pressure-void fraction correlations which can be modeled after Elghobashi and Abou-Arab (1983) by

$$- (\overline{\alpha_K^{\cdot}p_K^{\cdot}} + \overline{\alpha_K^{\cdot\cdot}p_K^{\cdot\cdot}}) = \psi_1 + \psi_2 \qquad (29)$$

where

$$\psi_1 = - C_{\phi3}\rho_K k_K^{1/2}\cdot\overline{u_{Km}^{t}\alpha_K^{t}}$$

and

$$\psi_2 = - C_{\phi4}\rho_K k^{1/2}\cdot u_{Km}^{t}\alpha_K^{t}$$

The values of the constants $C_{\phi3}$ and $C_{\phi4}$ are about unity.

The second correlation in the second group of terms can be also modeled following Launder (1976). The final form is

$$\overline{p_K^{\cdot}\frac{\partial\alpha_K^{\cdot}}{\partial x_i}} + \overline{p_K^{\cdot\cdot}\frac{\partial\alpha_K^{\cdot\cdot}}{\partial x_i}} = \rho_K(\frac{\epsilon}{k})_K[C_{\phi1}\cdot\overline{u_{Ki}^{t}\alpha_K^{t}}$$

$$+ C_{\phi 2}(\frac{\overline{u^t_{Ki}u^t_{Kl}}}{k_K} - \frac{2}{3}\delta_{il})\cdot\overline{u^t_{Kl}\alpha^t_K}]$$

$$+ \rho_K(0.8\ \overline{u^t_{Kl}\alpha^t_K}\cdot\frac{\partial\bar{u}_{Ki}}{\partial x_i} - 0.2\ \overline{u^t_{Kl}\alpha^t_K}\cdot\frac{\partial\bar{u}_{Kl}}{\partial x_i}) \quad (30)$$

The values of the constants $C_{\phi 1}$ and $C_{\phi 2}$ are 4.3 and -3.2, respectively. The modeling of these correlations suffers from the embodied assumptions concerning the velocity and length scale description. It would require a large number of transport equations to model accurately each of the above correlations.

The interaction term $(\overline{I_{Ki}})_{-K}$ for K = L (liquid) and -K = S (solid) is modeled for dilute suspensions with particle Reynolds numbers less than unity:

$$\overline{(I_{Li})_S} = -(18\mu_L/d_S^2)\ [(\bar{u}_{Li} - \bar{u}_{Si})\ \bar{\alpha}_S$$

$$+ (\frac{\nu_{LP}}{\sigma^P_{\alpha S}} + \frac{\nu_{LT}}{\sigma^T_{\alpha S}})\frac{\partial\bar{\alpha}_L}{\partial x_i} + (\frac{\nu_{LP}}{\sigma\sigma^P_{\alpha S}} + \frac{\nu_{LT}}{\sigma\sigma^T_{\alpha S}})\frac{\partial\bar{\alpha}_S}{\partial x_i}] \quad (31)$$

where the first term is the drag interaction for particles in the Stokes range. The second and third terms are the turbulent fluxes due to the relative motion between the particles and fluid. The gradient transport model with the exchange coefficients ν_{LP} and ν_{LT} corresponding to the production and transfer ranges is adopted for these fluxes ($\overline{\alpha'_L u'_{Li}}$, $\overline{\alpha''_L u''_{Li}}$, $\overline{\alpha'_S u'_{Si}}$, and $\overline{\alpha''_S u''_{Si}}$).

If only single velocity scale is chosen for the whole energy spectrum, k_L, there will be only one momentum exchange coefficient ν_{Lt} instead of ν_{LP} and ν_{LT}

$$\nu_{Lt} = \frac{\nu_{LP}}{\sigma\sigma^P_{\alpha S}} + \frac{\nu_{LT}}{\sigma\sigma^T_{\alpha S}} \quad (32)$$

6. CLOSURE FOR THE KINETIC ENERGY EQUATION

In the present work, the turbulence kinetic energy for the liquid phase "k_L" (turbulence velocity scale) is obtained from an exact transport equation (Eq. A5 in Appendix A), and the length scale "l" is described algebraically. The kinetic energy equation A5 contains a large number of turbulence correlations. In order to obtain an engineering turbulence model, it is sufficient to consider the principles and the assumptions given in Section 4.3. By engineering turbulence model, it is meant, a physically correct model with minimum number of empirical coefficients.

The first group of terms in the k-equation (Group #1) is the <u>convection</u> of the total specific kinetic energy, where

$$\frac{\partial k_L}{\partial x_j} = \frac{\partial k_{LP}}{\partial x_j} + \frac{\partial k_{LT}}{\partial x_j} \quad (33)$$

<u>Diffusion transport</u> of k is composed of two main parts. The first part (Group #2) is the velocity diffusion and it contains the 3rd order velocity correlations, while the second part (Group #3) is the pressure diffusion with the pressure-velocity correlations. The modeling of the velocity diffusion part is obtained as follows:

$$\rho_L\ \bar{\alpha}_L\ \overline{u'_{Lj}u'_{Li}u'_{Li}} = -\rho_L\bar{\alpha}_L\ \frac{\nu_{LP}}{\sigma^P_k}\frac{\partial k_{LP}}{\partial x_j} \quad (34)$$

and

$$\rho_L\ \bar{\alpha}_L\ \overline{u''_{Lj}u''_{Li}u''_{Li}} = -\rho_L\bar{\alpha}_L\ \frac{\nu_{LT}}{\sigma^T_k}\frac{\partial k_{LT}}{\partial x_j} \quad (35)$$

where σ^P_k and σ^T_k are the turbulent Prandtl/Schmidt numbers for the kinetic energy in the production and transfer ranges. To reduce the number of empirical constants and the number of governing equations the above two correlations are modeled collectively as follows

$$-\rho_L\overline{u^t_{Lj}u^t_{Li}u^t_{Li}} = -\rho_L\bar{\alpha}_L\ \frac{\nu_{Lt}}{\sigma_k}\frac{\partial k_L}{\partial x_j} \quad (36)$$

where σ_k is of order of unity.

The <u>presure diffusion</u> term is negligible relative to the velocity diffusion (Hanjalic' and Launder (1972)).

Mixed and higher order correlations (Groups #3 and #5) can be neglected according to the modeling assumptions stated and discussed in section 4.3 of the present paper.

The <u>production terms</u> are divided into two groups. The first group (Group #6) is common for single and multiphase incompressible flows,

$\rho_L\bar{\alpha}_L\ \overline{u'_{Li}u'_{Lj}}\cdot\frac{\partial\bar{u}_{Li}}{\partial x_j} + \rho_L\bar{\alpha}_L\ \overline{u''_{Li}u''_{Lj}}\cdot\frac{\partial\bar{u}_{Li}}{\partial x_j}$, and can be modeled collectively as follows:

$$\rho_L\bar{\alpha}_L\ \overline{u^t_{Li}u^t_{Lj}}\cdot\frac{\partial\bar{u}_{Li}}{\partial x_j} = \bar{\alpha}_L\mu_{Lt}(\frac{\partial\bar{u}_{Li}}{\partial x_j})^2 \quad (37)$$

The physics of turbulence and the consideration of the spectral energy transfer assume that the production is only due to the interaction between the mean flow and the large eddy. Since the $\overline{u''_{Li}u''_{Lj}}$ correlation is for medium size eddies which have almost no direct interaction with the mean flow, it results that the contribution of these small eddies to the turbulence production via the mean field is smaller than that of the large eddies. This means also that $\overline{u''_{Li} u''_{Lj}}$ correlation is weak if $i \neq j$, i.e. it is only of significant value if the turbulent normal stress components ($\overline{u''^2_{Li}}$, $i = 1, 2, 3$) are considered. According to Hanjalic' et al. (1979) multiple scale model the turbulent viscosity is defined as follows:

$$\mu_{Lt}/\rho_L = C_\mu \, k_L \, (k_{LP}/\epsilon_{LP}) \tag{38}$$

$$\text{where } k_L = k_{LP} + k_{LT} \tag{39a}$$

$$\text{and } C_\mu = 0.09 \tag{39b}$$

This equation can be rewritten as

$$\mu_{Lt}/\rho_L = C_\mu \left(\frac{k^2_{LP}}{\epsilon_{LP}} + \frac{k_{LT}\,k_{LP}}{\epsilon_{LP}}\right)$$

$$= C_{\mu P} k^{0.5}_{LP} l_{LP} + C_{\mu T} k^{0.5}_{LT} l_{LT} = (\mu_{LP} + \mu_{LT})/\rho_L \tag{40}$$

where $C_{\mu P}$ and $C_{\mu T}$ are two additional constants and l_{LP} and l_{LT} are also two additional length scales for the large and medium size eddies. The length scale can be related with the following relations (from Eqs. (38) and (40))

$$l_{LT} = C_1 \, l_{LP} \, (k_{LT}/k_{LP}) \tag{41}$$

Since the ratio k_{LT}/k_{LP} is of the order of unity and $l_{LP} > l_{LT}$, the constant C_1 should be smaller than unity. Thus if the multiple time scale model is not recommended (due to its large number of additional constants) an alternative approach is to consider a multiple velocity scale model. In this model only two differential equations for k_{LP} and k_{LT} have to be solved. The length scales can be obtained by using the previous relation Eq. (41) and any expression for the length scale of the large eddies l_{LP}, for example that used by Roco and Shook (1983) for cylindrical pipes.

The modeling of the <u>additional production</u> terms (Group #7) is achieved as follows:

$$-\overline{\alpha'_L u'_{Li}} \frac{\partial \bar{p}_L}{\partial x_i} = \frac{\nu_{LP}}{\sigma^P_{\alpha S}} \frac{\partial \bar{\alpha}_L}{\partial x_i} \frac{\partial \bar{p}_L}{\partial x_i} \tag{42}$$

$$-\overline{\alpha''_L u''_{Li}} \frac{\partial \bar{p}_L}{\partial x_i} = \frac{\nu_{LT}}{\sigma^T_{\alpha S}} \frac{\partial \bar{\alpha}_L}{\partial x_i} \frac{\partial \bar{p}_L}{\partial x_i} \tag{43}$$

and both collectively as

$$-(\overline{\alpha'_L u'_{Li}} + \overline{\alpha''_L u''_{Li}}) \frac{\partial \bar{p}_L}{\partial x_i} = \frac{\nu_{Lt}}{\sigma_{\alpha S}} \frac{\partial \bar{p}_L}{\partial x_i} \frac{\partial \bar{\alpha}_L}{\partial x_i} \tag{44}$$

Terms like $\overline{\alpha'_L u'_{Li}} \frac{\partial \bar{u}_{Li}}{\partial x_i}$ and $\overline{\alpha''_L u''_{Li}} \frac{\partial \bar{u}_{Li}}{\partial x_i}$ are modeled in a similar manner to that of the above terms i.e.

$$-(\overline{\alpha'_L u'_{Lj}} + \overline{\alpha''_L u''_{Lj}}) \frac{\partial \bar{u}_{Li}}{\partial x_i} = \frac{\nu_{Lt}}{\sigma_{\alpha S}} \frac{\partial \bar{u}_{Li}}{\partial x_i} \frac{\partial \bar{\alpha}_L}{\partial x_j} \tag{45}$$

The <u>extra production</u> terms (Group #7) can be written in the following form

$$\text{Extra production} = \frac{\nu_{Lt}}{\sigma_{\alpha S}} \frac{\partial \bar{\alpha}_L}{\partial x_i} \left(\frac{\partial \bar{p}_L}{\partial x_i} + \bar{u}_{Lj} \frac{\partial \bar{u}_{Li}}{\partial x_j}\right)$$

$$+ \frac{\nu_{Lt}}{\sigma_{\alpha S}} \frac{\partial \bar{\alpha}_L}{\partial x_j} \left(\bar{u}_{Li} \frac{\partial \bar{u}_{Li}}{\partial x_j}\right) \tag{46}$$

The terms $\rho_L \bar{\alpha}_L \mu_L \overline{\frac{\partial u'_{Li}}{\partial x_i} \frac{\partial u_{Li}}{\partial x_i}}$ and $\rho_L \bar{\alpha}_L \nu_L \overline{\frac{\partial u''_{Li}}{\partial x_j} \frac{\partial u''_{Li}}{\partial x_j}}$ in Group #8 represents the <u>dissipation rate</u> from large eddies ϵ_{LP} and transfer eddies ϵ_{LT}. Since viscous dissipation is mainly confined to small scale eddies and to simplify the mathematical form of the multiple time scale turbulence models, Hanjalic' et al. (1979) have assumed that there is an equilibrium spectrum energy transfer between the dissipation and transfer region i.e., $\epsilon_{total} = \epsilon_L$, where ϵ_L is the dissipation rate in the single scale scheme. Thus

$$\rho_L \bar{\alpha}_L \nu_L \overline{\frac{\partial u'_{Li}}{\partial x_j} \frac{\partial u'_{Li}}{\partial x_j}} = -\rho_L \bar{\alpha}_L \epsilon_{LP} = \bar{\alpha}_L \mu_{LT} \left(\frac{\partial \bar{u}_{Li}}{\partial x_j}\right)^2 \tag{47}$$

and

$$\rho_L \bar{\alpha}_L \nu_L \overline{\frac{\partial u''_{Li}}{\partial x_j} \frac{\partial u''_{Li}}{\partial x_j}} = -\rho_L \bar{\alpha}_L \epsilon_{LT} \tag{48}$$

where

$$\epsilon_{LP} = C_{DP} \, k^{1.5}_{LP} / l_{LP} \qquad \text{and}$$

$$\epsilon_{LT} = C_{DT} \, k^{1.5}_{LT} / l_{LT}$$

In equation (47) a spectral cascading between the production and transfer eddies is considered (Hanjalic' et al. (1979)). This equation gives an additional relation between the spectrum scales.

The correlations between the fluctuating velocity component and the fluctuating friction forces (<u>interaction terms</u> in Groups #8 and #10) are due to fluid-fluid and fluid-solid drag force in dilute flows. The friction interaction terms due to molecular collision (fluid-fluid interaction, Group #8) are given above by Eq. (47) and (48). The form of the correlation between the fluctuating drag force and the velocity fluctuations depends on the expression adopted for the drag force. The viscous drag correlation (VDC) in Group #10 for Stokes flow over particles in dilute suspensions reads

$$VDC = -\frac{18\mu_L}{d_S^2}(\bar{u}_{Li} - \bar{u}_{Si})\cdot\Delta_1$$

$$-\frac{18\mu_L}{d_S^2}(\bar{\alpha}_S\cdot\Delta_2 + \Delta_3) \quad (49a)$$

where:

$$\Delta_1 = \overline{\alpha_S^{'}u_i^{'}} + \overline{\alpha_S^{''}u_i^{''}} \quad (49b)$$

$$\Delta_2 = \overline{u_{Li}^{'}(u_{Li}^{'} - u_{Si}^{'})} + \overline{u_{Li}^{''}(u_{Li}^{''} - u_{Si}^{''})} \quad (49c)$$

$$\Delta_3 = \overline{\alpha_S^{'}u_{Li}^{'}(u_{Li}^{'} - u_{Si}^{'})} + \overline{\alpha_S^{''}u_{Li}^{''}(u_{Li}^{''} - u_{Si}^{''})} \quad (49d)$$

The first correlation group Δ_1 (Eq. 49a) can be approximated using the gradient type assumption. The second correlation in this expression (49b) is that due to the relative slip fluctuating motion $\overline{u_{Li}^{'}(u_{Li}^{'} - u_{Si}^{'})}$ and $\overline{u_{Li}^{''}(u_{Li}^{''} - u_{Si}^{''})}$. These can be modeled using similar approach to that of Elghobashi and Abou-Arab (1983) but with some modifications which allow for different particle-eddy interaction according to their relative size. These modifications are based on the spectral analysis carried out by Dingguo (1987) for the response of the particles to the turbulent fluctuations of the carrier fluid. For very large eddies $\kappa << \kappa_S$.

$$u_S^{'} = (v_S^{'})_\kappa = (v_L^{'})_\kappa\ a\cdot \exp[-i(\kappa\bar{u}_L t-\beta)] \quad (50)$$

where κ is the wave number; κ_S is the Basset-Boussinesq-Ossen wave number defined as $1/d_S$; $(v_S^{'})_\kappa$ and $(v_L^{'})_\kappa$ are the solid and liquid velocity components with the wave number κ; a is the amplitude ratio of oscillations

$$a = [(1 + q_2)^2 + q_1^2]^{0.5} \quad (51a)$$

and β is the phase angle of oscillation

$$\beta = tg^{-1}[q_2/(1 + q_1)] \quad (51b)$$

The expressions of q_1 and q_2 are

$$q_1 = \frac{(1 + \frac{9N_S}{\sqrt{2}(s+0.5)})(\frac{1-s}{s+0.5})}{\frac{81}{(s+0.5)^2}(2N_S^2 + \frac{N_S}{\sqrt{2}})^2 + (1 + \frac{9N_S}{\sqrt{2}(s+0.5)})^2} \quad (52a)$$

$$q_2 = \frac{\frac{9(1-s)}{(s+0.5)^2}(2N_S^2 + \frac{N_S}{2})}{\frac{81}{(s+0.5)^2}(2N_S^2 + \frac{N_S}{\sqrt{2}})^2 + (1 + \frac{9N_S}{\sqrt{2}(s+0.5)})^2} \quad (52b)$$

with the following dimensionless parameters:

$$s = \rho_S/\rho_L$$

$$N_S = \sqrt{\frac{\upsilon_L}{u_L\kappa d_S^2}} = \frac{\kappa}{\kappa_T}\sqrt{\frac{\kappa Re}{\kappa_T}}$$

where

κ_T = is the wavenumber of the most energetic eddies.

Re = Reynolds number based on l_T, and

N_S = Stokes number.

For small eddies with wave numbers $\kappa >> \kappa_S$, the particle response can be described by

$$u_S^{'} = (v_S^{'})_\kappa = (v_L^{'})_\kappa (\frac{\kappa_S}{\kappa})^3\frac{\rho_L}{\rho_S} \quad (53)$$

For the intermediate size eddies $\kappa \simeq \kappa_S$ one can use (50) with

$$a = \frac{1}{s} \quad \text{and} \quad (54a)$$

$$\beta = \tan^{-1}(s) \quad (54b)$$

If the fluctuating slip velocity $w_i^{'}$ is defined as

$$w_i^{'} = u_{Si}^{'} - u_{Li}^{'} \quad (55)$$

then the ratio of the mean square $\overline{w_i^{'2}}$ and $\overline{u_{Li}^{'2}}$ becomes

$$\overline{w_i^{'2}}/\overline{u_{Li}^{'2}} = (\overline{u_{Si}^{'2}} - 2\overline{u_{Li}^{'}u_{Si}^{'}} + \overline{u_{Li}^{'2}}) / \overline{u_{Li}^{'2}} \quad (56)$$

with

$$\overline{u_{Li}^{'}u_{Si}^{'}} = \frac{1}{2}\overline{u_{Li}^{'2}}(1 + \overline{u_{Si}^{'2}}/\overline{u_{Li}^{'2}} - \overline{w_i^{'2}}/\overline{u_{Li}^{'2}})$$

$$= \frac{1}{2}\overline{u_{Li}^{'2}}(1 + \Gamma_{SL} - \Gamma_{RL}) \quad (57)$$

The values of Γ_{SL} and Γ_{RL} can be obtained by using the preceding solution (Eq. 50):

$$\Gamma_{SL} = [\int_0^{\kappa_S} a^2 E_L(\kappa)d\kappa + \int_{\kappa_S}^{\infty}(\frac{\kappa_S}{\kappa})^6 s^{-2}E_L(\kappa)d\kappa]/\int_0^{\infty}E_L(\kappa)d\kappa \quad (58)$$

or

$$\Gamma_{SL} = \frac{2h}{\pi(h+1)}\tan^{-1}(\kappa_S/\kappa_T) \quad (59)$$

where

$$h = \frac{A}{\kappa_T u_L} = \frac{18}{(s+0.5)}(\kappa_S/\kappa_T)^2/Re,\quad A = \frac{18\upsilon_L}{(s+0.5)d_s^2}$$

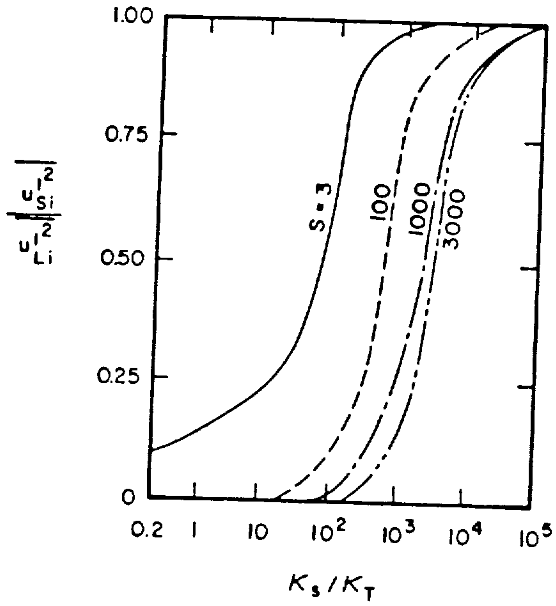


Figure 2. The effect of eddy-particle size ratio (κ_S/κ_T) on the particle response to the eddy fluctuations ($\overline{u'^2_{Si}}/\overline{u'^2_{Li}}$)

and

$$\Gamma_{RL} = [\int_0^{\kappa_S} (a^2-1)\ E_L\ (\kappa)\ d\kappa$$

$$+ \int_{\kappa_S}^{\infty} ((\frac{\kappa_S}{\kappa})^3\ s^{-1}-1)^2\ E_L(\kappa)d\kappa]\ /\ \int_0^{\infty} E(\kappa)\ d\kappa \qquad (60)$$

where $a_1 = (f_1^2 + f_2^2)^{0.5}$ and $E_L(\kappa)$ is the liquid energy spectrum function for which any arbitrary form can be adopted, for instance

$$E_L(\kappa) = \frac{2}{\pi} \frac{\overline{u'^2_{Li}}}{\kappa_T} \frac{1}{1+(\kappa/\kappa_T)^2} \qquad (61)$$

Figure 2 illustrates the effect of eddy-particle size ratio expressed as κ_S/κ_T on the ratio $\Gamma_{SL} = \overline{u'^2_{Si}}/\overline{u'^2_{Li}}$ which indicates the particle response to the eddying motion. It can be noticed that high values of κ_S/κ_T (i.e. small particle or large eddy) the particles follows quite well the eddy motion.

Substituting the expressions for Γ_{SL} and Γ_{RL} (Eqs. (58) or (59), and Eq. (60)) into Eq. (57), the correlation Δ_2 can be obtained

$$\Delta_2 = \frac{1}{2} \overline{u'^2_{Li}} (1 - \Gamma_{SL} + \Gamma_{RL}) \qquad (62)$$

The above analysis applies equally well to the large eddies as to small eddies. The energy spectrum function $E_L(\kappa)$ for a two-phase flow is given by Al-Taweel and Landau (1977). However, since its form is not essential (Dingguo (1987)) it is sufficient to adopt any simple form as that given above by Eq. (61). It is clear from the above analysis that the particle response to the carrier fluid fluctuations is a function of the density ratio ρ_S/ρ_L, size of interacting eddy relative to particle size κ_S/κ and Reynolds number based on the size of the most energetic eddies of the flow. An analogous expression to that given by Eq. (62) is that based on the Chao's solution (see Chao (1964)). This solution can be considered as a substitution of Dingguo's solution only for $\kappa << \kappa_S$ i.e. for fine particles.

The last term to be modeled in the VCD group, Δ_3, is separated into four correlations:

$$\Delta_3 = -\frac{18\mu_L}{d_S^2} [(\underbrace{\overline{\alpha'_S u'_{Li} u'_{Li}}}_{T1} + \underbrace{\overline{\alpha''_S u''_{Li} u''_{Li}}}_{T2})$$

$$- (\underbrace{\overline{\alpha'_S u'_{Li} u'_{Si}}}_{T3} + \underbrace{\overline{\alpha''_S u''_{Li} u''_{Si}}}_{T4})] \qquad (63)$$

where the triple correlations T3 and T4 are modeled in a similar manner to that used for the calculation of T1 and T2 by using Eqs. (23) and (24), thus

$$T1 = \overline{\alpha'_S u'_{Li} u'_{Li}} = -C^P_{\phi 5}\ (k_{LP}/\epsilon_{LP})\ (\overline{2u'_{Li}u'_{Ll}}\ \frac{\partial \overline{u'_{Li}\alpha'_S}}{\partial x_l})_P,$$

$$T2 = \overline{\alpha''_S u''_{Li} u''_{Li}} = -C^T_{\phi 5}\ (k_{LT}/\epsilon_{LT})\ (\overline{2u''_{Li}u''_{Ll}}\ \frac{\partial \overline{u''_{Li}\alpha''_S}}{\partial x_l})_T,$$

$$T3 = -C^P_{\phi 5}\ (k_{LP}/\epsilon_{LP})(\overline{u'_{Li}u'_{Si}}\frac{\partial \overline{u'_{Si}\alpha'_S}}{\partial x_l} + \overline{u'_{Si}u'_{Ll}}\ \frac{\partial \overline{u'_{Li}\alpha'_S}}{\partial x_l}\ )_P$$

$$T4 = -C^T_{\phi 5}\ (k_{LT}/\epsilon_{LT})(\overline{u''_{Li}u''_{Si}}\frac{\partial \overline{u''_{Si}\alpha''_S}}{\partial x_l} + \overline{u''_{Si}u''_{Ll}}\ \frac{\partial \overline{u''_{Li}\alpha''_S}}{\partial x_l}\ )_T \qquad (64)$$

These correlations can also be collectively modeled using single velocity and length-scale, and total k_L and ϵ_L. The first two and the last two terms yield, respectively:

$$T1 + T2 = C_{\phi 5}\ \frac{2k_L}{\epsilon_L}\ (\overline{u^t_{Li}u^t_{Ll}}\ \ \frac{\partial \overline{u^t_{Li}\alpha^t_S}}{\partial x_l}) \qquad (65a)$$

$$T3 + T4 = C_{\phi 5}\ \frac{2k_L}{\epsilon_L}\ (\overline{u^t_{Li}u^t_{Si}}\ \ \frac{\partial \overline{u^t_{Sl}\alpha^t_S}}{\partial x_l}$$

$$+ \overline{u^t_{Si} u^t_{Li}} \; \frac{\partial \overline{u^t_{Li} \alpha^t_s}}{\partial x_i} \;) \qquad (65b)$$

The terms in Group #9 of Eq. A5 are of diffusive and dissipative nature. The diffusion terms as they appeared in Group #9 are multiplied by the molecular viscosity and therefore will be neglected due to their relatively small magnitude. Other higher order correlations and mixed correlations in this group are also neglected according to the modeling principles stated previoulsy in section 4.3 and as they are also multiplied by the molecular viscosity.

By substituting all previously modeled terms into the exact form of the turbulence kinetic energy equation, and rearranging these terms, one obtains the simplified modeled form given in Appendix B (Eq. B2).

Since the present model is based on an exact equation, namely the turbulence kinetic energy equation, and the modeled form of this equation has no adjusting coefficients it is expected that the model will generally produce good results and have less limitations compared to other models. The only modeling assumption is that of the Boussinesq gradient type, which generally is accepted. The correlations that requires questionable semi-empirical modeling assumptions and introduction of empirical constant are (i) fewer in number (for $\overline{\alpha_L' u_{Li}' u_{Lj}'}$ and $\overline{\alpha_L' u_{Si}' u_{Lj}'}$), and (ii) for terms having an order of magnitude smaller (by ratio $\alpha'/\bar{\alpha}$) compared to other main terms in the k-equation. The only significant new correlation used in the present closure is that due to the relative motion between the phases. This is modeled with less restrictions and taking into consideration the effect of the particle diameter-eddy size ratio on the particle response to the eddying motion. The limitation of the present one-equation k model closure is the algebraic formulation for the length scale. Since there are many factors affecting this length scale and since it is even difficult in many practical applications to give a unique and accurate description of the length scale, the use of a transport equation for the length scale in the two-equation model of Elghobashi and Abou-Arab (1983) is expected to give better results with fine particles ($d_S < \eta$). However, it should be noticed that the former model and any other similar models contain some empirical constants specific for various flow conditions, and they require the solution for an additional transport equation. It can be expected that the present model combined with an appropriate length scale equation e.g. dissipation rate equation will simulate better most of the important features of multiphase turbulent flows, particularly the fluid particle interaction. In that case the number of closure transport equations will increase to three (if two velocity scale, k_P and k_T, and one length scale transport equations) or four transport equations (if two velocity scales and two length scale, l_P and l_T, are adopted).

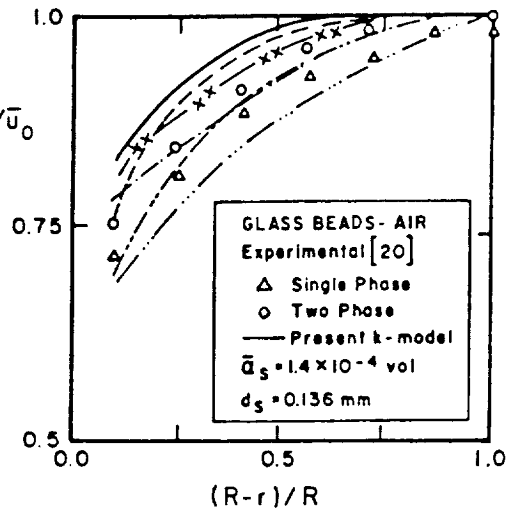


Figure 3. Computed and measured [20] mean velocity distribution of air in air-solid pipe flow (—· single-phase, --- 2 Eqs. k-ε model [12], -·- 1 Eq. k-model [27], -xx- 1 Eq. k-model [12], -····- 1 Eq. ν_t-model [26]).

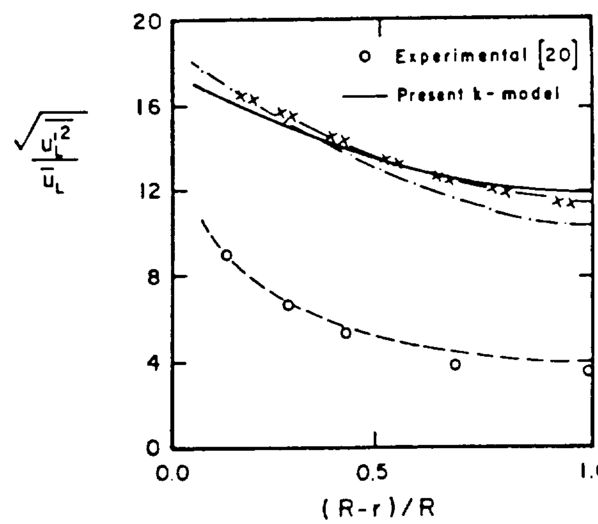


Figure 4. Predicted radial distribution of the turbulence intensity of air in air-solid pipe flow using different turbulence models (keynote as in Fig. 3).

7. SAMPLE OF RESULTS

Figures 3 to 6 compare the present predictions with LDA-measurements for single and two-phase turbulent pipe flow (see Maeda et al (1980)) and turbulent round water jet laden with uniform-size solid particles (see Parthasarathy and Faeth (1987)). Both flows are oriented vertically downward. These flows are axisymmetrical. The concentration profiles are given as input data based on experimental results.

In these flow situations the average eddy size ℓ_e was varying from about one d_S to few hundreds

d_S. The corresponding representative eddy size in the transfer range was only a fraction of particle diameter d_S in the pipe flow case. In the jet flow case the mean size of large eddies and the Kolmogrov length scale were also varied in a wide range. These scales are field variables, and they depend upon the flow configuration, location in the flow domain and particle dimensions.

Two-phase flow solutions were obtained by solving the flow governing equations in their modeled form which are described in the previous section and given in Appendix B. The numerical procedure used for these predictions is based on a developed version of the Genmix-Code of Spalding (1977). However, since the main objective of this paper is to give a complete description of a developed turbulence closure for multiphase flows and due to the space limitation the details of this numerical approach will not be given here. The CPU Time for the two considered flow cases was about 4 minutes on a VAX 760 Mini-Computer and 2 minutes on IBM 3084.

Case I: Gas-Solid Vertical Pipe Flow:

Figure 3 shows a comparison between the experimental data and the present predictions using five different models of turbulence namely 1. One-equation k-model of Roco and Mahadevan (1986). 2. One-equation ν_t-model of Roco and Balakrishnan (1985). 3. The k-equation as given in the two-equation k-ε model of Elghobashi and Abou-Arab (1983). 4. The two-equation model of Elghobashi and Abou-Arab (1983). and 5. The present one-equation k-model. The figure displays the mean axial velocity distribution in the fully developed zone of the pipe flow for single and two-phase cases. The differences between the predictions of all one-equation turbulence models and experiments is mainly caused by the general algebraic expression adopted for the turbulence length scale which was not optimized or adjusted. In the present computation the concentration profiles are assumed based on previous experimental data. The inlet concentration distribution is taken to be similar to that given by the best curve fit after the experimental data of Soo (1967). Figure 4 compares the calculated turbulence intensity defined as $\sqrt{\overline{u_L'^2}(=\frac{2}{3}k_L)} \; / \; \bar{u}_L$ with its measured values. The near wall treatment is based on a modified form for the law of wall (see Abou-Ellail and Abou-Arab (1984). Lee and Chung (1987)) and the particle slip condition at the pipe wall.

The present model as compared with the one equation models of Refs. [12] and [27] predicts slightly higher values for the mean flow quantities. However, it must be mentioned that the last k-models contain empirical constants in the dissipation term of the k equation. Concerning the fluctuating flow quantities the present model gives slightly better results for the turbulence intensity in the near wall region than other one-equation models. The expression for the turbulence length scale was not optimized. It is also expected that the current model will give better predictions for coarse ($d_S > \eta$) and heavy suspension flows for which no set of comprehensive 2D data is available for comparison.

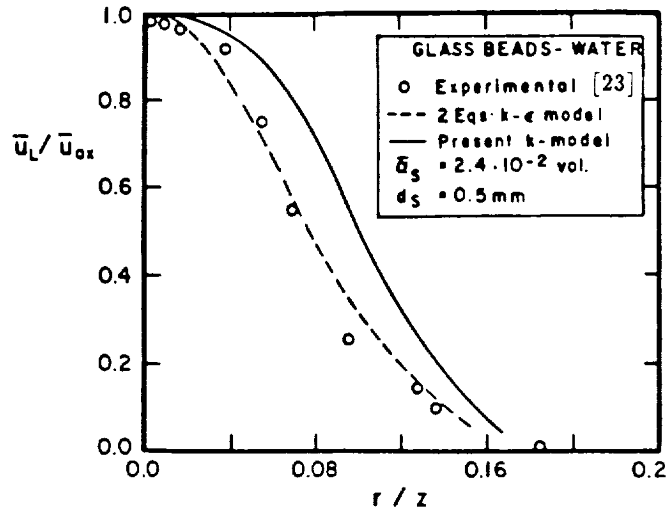


Figure 5. Computed and measured [23] mean velocity distribution in water-solid jet flow.

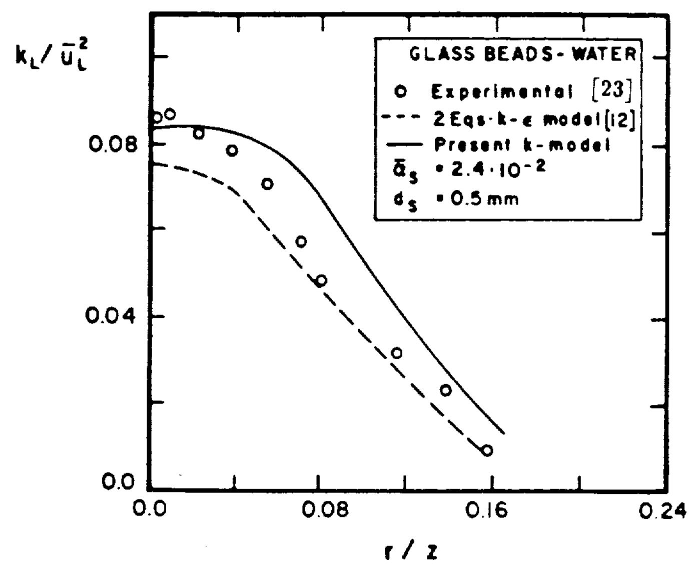


Figure 6. Predicted radial distribution of the turbulence intensity of water in water-solid jet flow.

Case II: Turbulent Round Water Jet Laden with Uniform-size Solid Particles

Comparison between experimental data and numerical predictions of different turbulence closures are given in Figures 5 and 6.

The two-phase flow measurements on velocity, concentration and turbulence correlations used for comparisons are taken from Parthasarathy & Faeth (1987). Different axial locations within the round jet, between eight and fourty jet diameters from the injection nozzle are

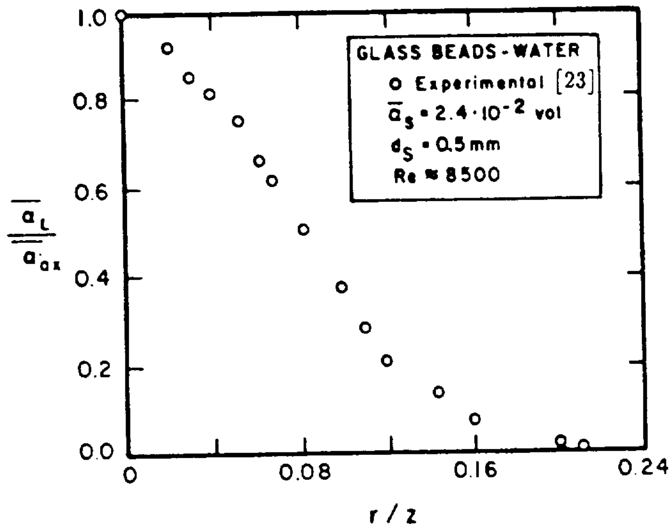


Figure 7. Radial distribution of solid phase concentration in water-solid jet flow.

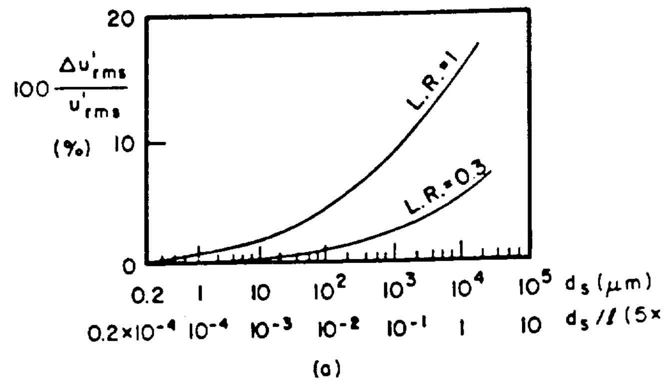
considered. The radial distribution of solid concentration given in Figure 7 are at eight diameters from the nozzle.

The comparison shows that the two-equation two-phase k-ε model of Elghobashi and Abou-Arab (1983) describes both flows better than the one-equation mass/time averaged turbulence model. However, it is important to mention here that the presently developed closure uses only one transport equation and without adjusting any empirical coefficient. At the same time, the present closure is in its early stages and more refinements and validation tests are required, especially for coarse particles two-phase flows for which one would expect that the present k-formulation will provide improved predictions. The difference between the predictions of the mean and fluctuating flow velocity components as obtained by the present k-formulation and that of Ref. [12] depends on the particle size relative to its surrounding eddies. Figure 8 illustrates this difference at two different loading ratios for the pipe flow at a radial distance (R - r)/R equal 0.1.

8. CONCLUDING REMARKS

The turbulence closure presented in this paper for dilute suspension flow is based on the fluid turbulence kinetic energy equation. The main features of this model are:

i) Two velocity scales are adopted in computation for large and medium size eddies, corresponding to the turbulence production and transfer range, respectively. They are expressed into the governing equations by a specific local mass/time averaging. On this basis the spatial and temporal transfer rates of the thermodynamic quantities and the particle-eddy interaction are better estimated.



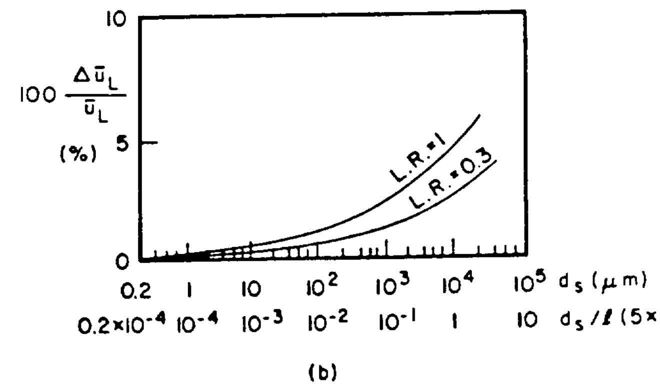


Figure 8. Differences between the present k-formulation and that of Ref. [12] for different mass loading ratios L.R.: (a) fluctuating and (b) mean axial velocity.

ii) Spectral analysis of the interaction mechanism between particles and most energetic eddies provide analytical correlations for closure. The particle response and the modulation of turbulent eddying motion is given as a function of the particle-fluid density and size ratios.

iii) To keep the number of transport equations of the turbulence closure and the number of empirical constants as minimum as possible, the length scales l_P and l_T are described using algebraic expressions. Relations between these scales (Eqs. 40 and 47) are suggested.

iv) The model does not introduce additional empirical constants to the closure of the velocity scale equation.

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APPENDIX - A

Mass/Time Averaged conservation Equations

Mass averaging the conservation equations of mass and momentum over a flow component (K) one obtains a new system of equations for mean velocity, concentration and kinetic energy of turbulence.

The point instantaneous conservation equation can be written for any flow component (K) or for the entire mixture in the folowing general form

$$\frac{\partial}{\partial t}(\rho\psi) + \nabla\cdot(\rho\psi\underline{u}) + \nabla\cdot\underline{J} - S = 0 \qquad \text{(A1)}$$

where: ρ = density
$\underline{u}$ = velocity vector
ψ = transported quantity
$\underline{J}$ = flux vector for ψ
S = source term

Let assume $\psi = u_{Ki}$. By splitting each flow property into mean and turbulent fluctuating component $(\bar{u}_{Ki} + u^t_{Ki})$ and mass/time or double time averaging the equation (A1), one obtains the following momentum equations in the i-th direction for an incompressible phase (K) without mass exchange with other flow components (see Roco and Shook (1985)):

$$\underbrace{\rho_K \frac{\partial}{\partial t}(\bar{\alpha}_K\bar{u}_{Ki} + \overline{\alpha^t_K u^t_{Ki}})}_{\text{Time Rate}} + \underbrace{\rho_K \frac{\partial}{\partial x_j}(\bar{\alpha}_K\bar{u}_{Ki}\bar{u}_{Kj})}_{\text{Mean Flow Convection}}$$

$$= \underbrace{\rho_K\bar{\alpha}_K\bar{b}_{Ki}}_{\text{Body Force}} - \underbrace{\frac{\partial}{\partial x_i}(\bar{\alpha}_K\bar{p}_K + \overline{\alpha^t_K p^t_K}) + \bar{p}_K\frac{\partial\bar{\alpha}_K}{\partial x_i} + \overline{p^t_K\frac{\partial\alpha^t_K}{\partial x_i}}}_{\text{Pressure Effect}}$$

$$+ \underbrace{\frac{\partial}{\partial x_j}[\bar{\alpha}_K\bar{\tau}_{K\Sigma_{ji}}}_{\text{Frictional Effect}} \underbrace{-\rho_K\bar{\alpha}_K\cdot\overline{u^t_{Ki}u^t_{Kj}}}_{\text{Inertial Effect}}$$

$$\underbrace{-\rho_K\overline{\alpha^t_K(u_{Ki}u_{Kj})^t} + \overline{\alpha^t_K\tau^t_{K\Sigma_{ji}}}\,]}_{\text{Collisional Effect}} + \underbrace{(\overline{I_{Ki}})_{-K}}_{\text{Interactions with (-K)}} \qquad \text{(A2)}$$

where

K = phase (or generally a flow component)
$\bar{f}_K$ = mass/time average of f over K
i, j = 1,2,3 (Cartesian coordiantes)
b_{Ki} = body force in the i-th direction
$(\overline{I_{Ki}})_{-K}$ = projection in the i-th direction of the interaction vector $(\overline{\underline{I}_K})_{-K}$

The interaction term as it stands for solid/liquid drag is given by

$$(I_{Si})_L = (I_{Li})_S = 0.75\alpha_S\rho_L\frac{C_{DS}}{d_S}\frac{|u_{Ly}-u_{Sy}|(u_{Ly}-u_{Sy})}{(1-\alpha_S)^{1.7}} \qquad \text{(A3)}$$

This drag term takes a simple form for Reynolds numbers less than unity

$$(I_{Si})_L = -(I_{Li})_S = \alpha_S(18\mu_L/d_S^2)(u_{Li}-u_{Si}) \qquad \text{(A4)}$$

The transverse effects caused by the presence of other solid particles, Saffman force and Ho and Leal inertial force are neglected. Their importance is small in dilute suspension turbulent flows with fine particles.

Equation (A2) contains terms due to the unsteady flow, mean flow convection, diffusion, pressure, body force, as well as frictional, inertial and collisional effects. The mean form of the turbulence kinetic energy governing equation is obtained by subtracting from the steady state instantaneous momentum equation for a component K = L the corresponding mean flow equations, and then multiplying the resulting difference equation by $(u'_{Ki} + u''_{Ki})$. By averaging one obtain the kinetic energy equation for a flow component K = L. This equation reads

$$\underbrace{\rho_K\bar{\alpha}_K\bar{u}_{Kj}\left(\frac{\partial\overline{u'^2_{Ki}}/2}{\partial x_j} + \frac{\partial\overline{u''^2_{Ki}}/2}{\partial x_j}\right)}_{\text{Group \#1 (Convection)}} + \underbrace{\frac{\partial}{\partial x_j}[\rho_K\bar{\alpha}_K(\overline{u'_{Kj}u'_{Ki}u'_{Ki}}}_{\text{Group \#2 (Velocity Diffusion)}}$$

$$+ \overline{u''_{Kj}u'_{Ki}u'_{Ki}} + \overline{u'_{Kj}u'_{Ki}u''_{Ki}} + \overline{u''_{Kj}u''_{Ki}u'_{Ki}} + \overline{u'_{Kj}u'_{Ki}u''_{Ki}}$$

$$+ \overline{u'_{Kj}u''_{Ki}u''_{Ki}} + \overline{u''_{Kj}u'_{Ki}u''_{Ki}} + \overline{u''_{Kj}u''_{Ki}u''_{Ki}})]$$

$$+ \underbrace{[\frac{\partial}{\partial x_j}(\rho_K\overline{\alpha'_K u'_{Ki}u'_{Ki}u'_{Kj}} + \overline{\alpha''_K u''_{Ki}u''_{Ki}u''_{Kj}})}_{\text{Group \#3 (Higher Order Correlations)}}$$

+ other 4th order and minor terms)]=

$$- \bar{\alpha}_K \frac{\partial}{\partial x_i} (\overline{u'_{Ki} P'_K} + \overline{u''_{Ki} P'_K} + \overline{u'_{Ki} P''_K} + \overline{u''_{Ki} P''_K})$$

Group #4 (Pressure Diffusion)

$$+ \bar{\alpha}_K (\overline{P'_K \frac{\partial u'_{Ki}}{\partial x_i}} + \overline{P''_K \frac{\partial u''_{Ki}}{\partial x_i}} + \overline{P'_K \frac{\partial u''_{Ki}}{\partial x_i}} + \overline{P''_K \frac{\partial u'_{Ki}}{\partial x_i}}$$

Group #5 (Extra Production and Transfer)

$$- (\overline{u''_{Ki} \alpha'_K \frac{\partial p'_K}{\partial x_i}} + \overline{u''_{Ki} \alpha''_K \frac{\partial p'_K}{\partial x_i}} + \overline{u''_{Ki} \alpha''_K \frac{\partial p''_K}{\partial x_i}}$$

$$+ \overline{\alpha''_K u'_{Ki} \frac{\partial p''_K}{\partial x_i}} + \overline{\alpha'_K u'_{Ki} \frac{\partial p''_K}{\partial x_i}})$$

$$- (\rho_K \bar{\alpha}_K \overline{u'_{Ki} u'_{Kj}} \frac{\partial \bar{u}_{Ki}}{\partial x_j} + \rho_K \bar{\alpha}_K \overline{u''_{Ki} u''_{Kj}} \frac{\partial \bar{u}_{Ki}}{\partial x_j})$$

Group #6 (Production)

$$- \left[\bar{u}_{Ki} \overline{\alpha'_K u'_{Kj}} \frac{\partial \bar{u}_{Ki}}{\partial x_j} + \bar{u}_{Kj} \overline{\alpha'_K u'_{Ki}} \frac{\partial \bar{u}_{Ki}}{\partial x_j} \right.$$

Group #7 (Extra Production)

$$+ \bar{u}_{Ki} \overline{\alpha''_K u''_{Kj}} \frac{\partial \bar{u}_{Ki}}{\partial x_j} + \bar{u}_{Kj} \overline{\alpha''_K u''_{Ki}} \frac{\partial \bar{u}_{Ki}}{\partial x_j}$$

$$\left. - (\overline{\alpha'_K u'_{Ki}} + \overline{\alpha''_K u''_{Ki}}) \frac{\partial \bar{P}_K}{\partial x_i} + \text{minor terms} \right]$$

$$+ \rho_K \bar{\alpha}_K \nu_K \overline{\frac{\partial u'_{Ki}}{\partial x_j} \frac{\partial u'_{Ki}}{\partial x_j}} + \rho_K \bar{\alpha}_K \nu_K \overline{\frac{\partial u''_{Ki}}{\partial x_j} \frac{\partial u''_{Ki}}{\partial x_j}}$$

Group #8 (Dissipation)

$$+ \rho_K \overline{\alpha'_K \nu_K \frac{\partial u'_{Ki}}{\partial x_j} \frac{\partial u'_{Ki}}{\partial x_j}} + \rho_K \overline{\alpha''_K \nu_K \frac{\partial u''_{Ki}}{\partial x_j} \frac{\partial u''_{Ki}}{\partial x_j}} + \text{mixed correlations}$$

Group #9 (Extra Dissipation & Diffusion)

$$+ \overline{u'_{Ki} (I'_{Ki})}_{-K} + \overline{u''_{Ki} (I''_{Ki})}_{-K} \qquad \text{(A5)}$$

Group #10 (Extra Dissipation)

APPENDIX B

The Modeled Form of the Turbulence Kinetic Energy Equation

The steady-state turbulence kinetic energy equation for the liquid phase (K = L) is:

$$\underset{\text{Convection}}{\rho_L \bar{\alpha}_L \bar{u}_{Lj} \frac{\partial k_L}{\partial x_j}} = \underset{\text{Diffusion}}{\frac{\partial \bar{\alpha}_L}{\partial x_j} \frac{\mu_{Lt}}{\sigma_k} \frac{\partial k_L}{\partial x_j}} + \underset{\text{Production}}{\alpha_L \mu_{Lt} (\frac{\partial \bar{u}_{Li}}{\partial x_j})^2}$$

$$+ \underset{\text{Extra Production}}{\frac{\nu_{Lt}}{\sigma_{\alpha S}} \frac{\partial \bar{\alpha}_L}{\partial x_i} (\frac{\partial \bar{P}_L}{\partial x_i} + \bar{u}_{Lj} \frac{\partial \bar{u}_{Li}}{\partial x_j}) + \frac{\nu_{Lt}}{\sigma_{\alpha S}} \frac{\partial \bar{\alpha}_L}{\partial x_j} (\bar{u}_{Li} \frac{\partial \bar{u}_{Li}}{\partial x_j})}$$

$$- \underset{\text{Dissipation}}{\rho_L \bar{\alpha}_L \epsilon_L}$$

$$+ \underset{\text{Extra Dissipation}}{\frac{18 \mu_L}{d_S^2} (\bar{u}_{Li} - \bar{u}_{Si}) \frac{\nu_{Lt}}{\sigma_{\alpha S}} \frac{\partial \bar{\alpha}_L}{\partial x_i}}$$

$$- \bar{\alpha}_S k_L (1 - \Gamma_{SL} + \Gamma_{RL}) - \frac{18 \mu_L}{d_S^2} (T1 + T2 - T3 - T4) \qquad \text{(B1)}$$

where the expressions for Γ_{SL}, Γ_{RL}, T1, T2, T3 and T4 are given in the text.

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16. ABSTRACT

During the first week of April 1989, a workshop, entitled "Constitutive Relationships and Models in Continuum Theories of Multiphase Flows," was convened at NASA's Marshall Space Flight Center. The purpose of this workshop was to open a dialogue on the topic of constitutive relationships for the partial or per phase stresses, including the concept of solid phase "pressure" and the models used for the exchange of mass, momentum, and energy between the phases in a multiphase flow. This volume is the result of the stated objective of the workshop. The program, abstracts, and text of the presentations made at the workshop are included.

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