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Summary

Algebraic expressions in terms of elementary mathematical functions are derived for power absorption and dissipation by eddy currents and magnetic hysteresis in ferrite spheres. Skin depth is determined by using a variable inner radius in descriptive integral equations.

Numerical results are presented for sphere diameters less than one wavelength. A generalized power absorption parameter for both eddy currents and hysteresis is expressed in terms of the independent parameters involving wave frequency, sphere radius, resistivity, and complex permeability.

In general, the hysteresis phenomenon has a greater sensitivity to these independent parameters than do eddy currents over the ranges of independent parameters studied herein.

Introduction

Damping of electromagnetic waves by iron oxides differs distinctly from that by iron (refs. 1 and 2). At low wave frequencies large persistent eddy currents can be induced in iron because of its high electrical conductivity, a process that converts field energy into heat as ohmic losses. At high wave frequencies, however, the oscillating currents cannot effectively penetrate the surface of iron.

Ferromagnetic and ferrimagnetic oxides (ref. 3, pp. 24 and 25), on the other hand, with their low conductivity and large skin depth (ref. 4) can be effective in absorbing field energy by eddy currents even at high wave frequencies. Their low weight densities, approximately 60 percent that of iron (ref. 2), make them attractive for flight applications.

Associated with a harmonic wave motion is a relatively large hysteresis effect due to frictional resistance in the crystalline structure (Weiss domains) and to radiation from electron spin reorientation (ref. 5, p. 16), again resulting in dissipation of the applied field energy. These damping mechanisms cause a phase shift between the applied magnetic field B_A and the resulting magnetization M, which enters the analysis by use of a complex susceptibility, $\chi = \chi' - j\chi''$ (ref. 5, p. 100). Dissipation of field energy in the study reported herein was treated with a complex isotropic permeability $\mu = \mu' - j\mu'' = 1 + \chi' - j\chi''$ in the ferrite spheres.

Large ratios of wavelength λ to sphere radius a are of main interest. Scattering is then of the Rayleigh type, which is proportional to sphere diameter divided by wavelength to the fourth power (ref. 6, p. 457). Therefore diffraction can be neglected.

Analysis

Power absorbed per sphere due to internal eddy currents P_{ec} plus that due to hysteresis P_{hy} is

$$P_{ec} + P_{hy} = \frac{1}{2}\int_V \tau \mathbf{i} \bullet \bar{\mathbf{i}}\, dV + \frac{1}{2}\int_V \mathbf{H} \bullet \frac{\partial \bar{\mathbf{B}}}{\partial t}\, dV \qquad (1)$$

The linear nature of Maxwell's equations permits P_{ec} and P_{hy} to be treated as separate individual events. Symbols are defined in appendix D.

The objective now is to evaluate these two integrals in terms of elementary algebraic functions and the basic parameters of the applied wave along with the material properties of the ferrites.

Wave-Particle Interaction

Consider an applied transverse electromagnetic wave with a propagation vector κ and mutually orthogonal electric and magnetic field vectors $\mathbf{E}_A$ and $\mathbf{B}_A$ (fig. 1) having a harmonic time dependence $e^{j\omega t}$. The magnetic vector potential relation $\nabla \times \mathbf{A} = \mathbf{B}$ used with the applied wave motion of figure 1 gives

$$\mathbf{A}_A = \hat{\varphi}\ \mathbf{B}_A r \sin\theta \qquad (2)$$

The large λ/a values of interest herein permit neglect of Maxwell's displacement current inside the spheres (ref. 7, p. 368). Then with use of an internal magnetic potential $\mathbf{A}_i$ and the laws of Faraday, Ohm, and Ampere,

$$\nabla^2 \mathbf{A}_i = j\omega\mu\tau^{-1}\mathbf{A}_i$$

With the assumption that $\mathbf{A}_i$ is independent of azimuthal angle φ (as on p. 374 of ref. 7), this equation reduces to

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial \mathbf{A}_i}{\partial r}\right) + \frac{(1-\cos^2\theta)^{1/2}}{r^2}\frac{\partial^2\left[(1-\cos^2\theta)^{1/2}\mathbf{A}_i\right]}{\partial \cos^2\theta} = j\omega\mu\tau^{-1}\mathbf{A}_i \qquad (3)$$

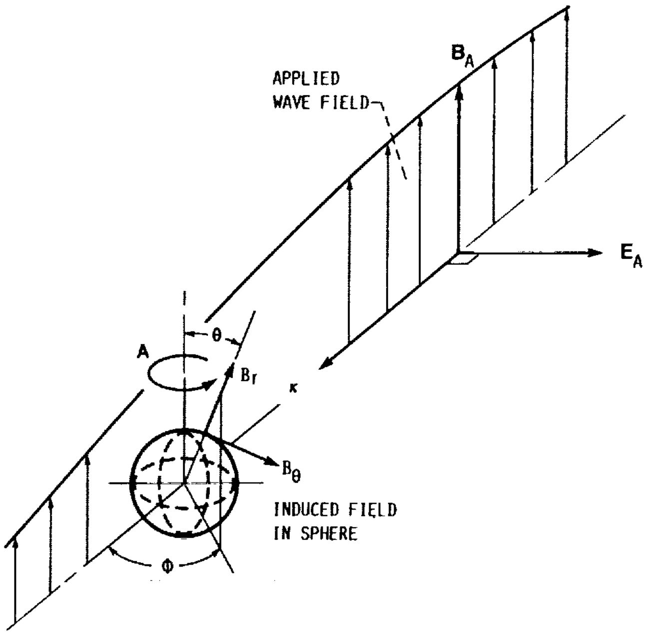


Figure 1.—Transverse wave interacting with a sphere. Wavelength λ much greater than sphere radius a.

Solving equation (3) by separation of variables gives

$$\mathbf{A}_i = \hat{\varphi}\,\frac{1}{2}\,B_A C r^{-1/2} I_{3/2}(r\sqrt{jp})\,\sin\theta \tag{4}$$

where

$$p = \frac{\omega\mu}{\tau} \tag{5}$$

and

$$\mu = \mu' - j\mu'' \tag{6}$$

By convention, a negative sign is used before j in the definition of complex permeability μ. The imaginary part μ'' must then be a positive number for a loss of wave energy.

The contribution of the applied wave plus the vanishing influence of the sphere on the external vector potential with radial distance is

$$\mathbf{A}_o = \hat{\varphi}\,\frac{1}{2}\,B_A\,(r + Dr^{-2})\,\sin\theta \tag{7}$$

Here conductivity is assumed to be negligible, and permeability outside the sphere μ_o is considered to be real.

The constants C and D of equations (4) and (7) are next to be determined. Solutions must be matched at the surface $r = a$ so that (ref. 7, p. 376).

$$\mathbf{A}_o(a) = \mathbf{A}_i(a) \tag{8}$$

To satisfy conservation of magnetic flux across this boundary, $\mathbf{B} \bullet \hat{n}$ must equal zero and thus by Stokes' theorem

$$\mu\,\frac{\partial(r\mathbf{A}_o)}{\partial r} = \mu_o\,\frac{\partial(r\mathbf{A}_i)}{\partial r} \qquad \text{at } r = a \tag{9}$$

This equation implies that there is no surface current and that the current enters the sphere as a continuous distribution, which is the true case for microscopic detail.

Substituting equations (4) and (7) into both equations (8) and (9) gives, in terms of Bessel functions,

$$a^3 + D = a^{3/2}C\left[I_{-1/2}\,(a\sqrt{jp}) - \frac{1}{r\sqrt{jp}}\,I_{1/2}\,(a\sqrt{jp})\right] \tag{10}$$

and

$$(2a^3 - D)\,\mu = -\,\mu_o a^{3/2}C\left[I_{-1/2}(a\sqrt{jp}) - \left(a\sqrt{jp} + \frac{1}{a\sqrt{jp}}\right) I_{1/2}\,(a\sqrt{jp})\right] \tag{11}$$

Equations (10) and (11) can be combined to solve for C and D as

$$C = \frac{3\mu a^{5/2}\sqrt{jp}}{(\mu-\mu_o)a\sqrt{jp}\;I_{-1/2}(a\sqrt{jp}) + \left[\mu_o(1+jpa^2)-\mu\right]\,I_{1/2}(a\sqrt{jp})} \tag{12}$$

and

$$D = a^3\,\frac{a\sqrt{jp}\;(2\mu+\mu_o)\;I_{-1/2}(a\sqrt{jp}) - \left[\mu_o(1+jpa^2)+2\mu\right]\,I_{1/2}(a\sqrt{jp})}{a\sqrt{jp}\;(\mu-\mu_o)\;I_{-1/2}(a\sqrt{jp}) + \left[\mu_o(1+jpa^2)-\mu\right]\,I_{1/2}(a\sqrt{jp})} \tag{13}$$

Thus the equations to determine the potentials $\mathbf{A}_i$ and $\mathbf{A}_o$ keep the same form as in reference 7, to which they reduce as μ'' goes to zero.

Eddy Currents

By using Ohm's law in Faraday's equation

$$\nabla \times \mathbf{E} = \tau \nabla \times \mathbf{i} = -\frac{\partial \mathbf{B}}{\partial t} = -j\omega \nabla \times \mathbf{A}_i$$

a solution of which, for current $\mathbf{i}$ inside a sphere, is

$$\mathbf{i} = i\hat{\varphi} = \frac{-j\omega \mathbf{A}_i}{\tau} \tag{14}$$

The power absorbed per unit volume of the sphere by eddy currents is

$$\frac{dP_{ec}}{dV} = \frac{1}{2}\,\tau \mathbf{i} \bullet \bar{\mathbf{i}} \tag{15}$$

where

$$dV = r^2 \sin\theta \; d\theta \; dr \; d\varphi \tag{16}$$

Substituting equations (4) and (14) into equation (15) and integrating the result over θ from 0 to π gives the power absorbed per sphere as

$$P_{ec} = \frac{\pi\omega^2 B_A^2 C\bar{C}}{3\tau} \int_d^a I_{3/2}(r\sqrt{jp}) \; I_{3/2}(r\sqrt{-j\bar{p}}) \; r \; dr \tag{17}$$

if the radial distance to the lower limit of integration d is set equal to zero. Varying d allows the amount of power absorption versus wave penetration distance into the sphere to be determined for a study of "skin depth."

The integration is performed in appendix A and the constant $C\bar{C}$ is determined in appendix B. This results in

$$P_{ec} = \frac{\omega^2 B_A^2 C\bar{C}}{3\tau\sqrt{|p|}\; \mathcal{R}e\, p} \Bigg[(S_a - S_d)\; \mathcal{I}m\sqrt{jp} + (s_a - s_d)\; \mathcal{R}e\sqrt{jp} - \left(\frac{C_a - c_a}{a} - \frac{C_d - c_d}{d} \right) \frac{\mathcal{R}e\, p}{|p|} \Bigg] \tag{18}$$

where r equals a and d in

$$S_r = \sinh\,(2r\; \mathcal{R}e\sqrt{jp}) \tag{19}$$

$$s_r = \sin\,(2r\; \mathcal{I}m\sqrt{jp}) \tag{20}$$

$$C_r = \cosh\,(2r\; \mathcal{R}e\sqrt{jp}) \tag{21}$$

and

$$c_r = \cos(2r\; \mathcal{I}m\sqrt{jp}) \tag{22}$$

in the upper and lower limits of integration, respectively, and

$$\begin{aligned} C\bar{C} = 9\pi \left|\mu^2\right| \left|p\right|^{3/2} a^6 \Bigg/ \Bigg(& U_g^2 \left|p\right| a^2 (C_a + c_a) \\ & + \left[U_g^2 + a^2\mu_o^2 \left(a^2 \left|p\right|^2 - 2\; \mathcal{I}m\; p \right) \right] (C_a - c_a) - 2a \\ & \times \left\{ U_g^2 + \left[(S_a + s_a)\; \mathcal{R}e\sqrt{\frac{p}{2}} - (S_a - s_a)\; \mathcal{I}m\sqrt{\frac{p}{2}} \right] \right\} \\ & + 2a^3\mu_o \left[(S_a - s_a)\; \mathcal{R}e \left(U\bar{p}\sqrt{\frac{p}{2}} \right) \right. \\ & \left. + (S_a + s_a)\; \mathcal{I}m \left(U\bar{p}\sqrt{\frac{p}{2}} \right) \right] \Bigg) \end{aligned} \tag{23}$$

where $U = \mu - \mu_o$ and $U_g^2 = \mu\bar{\mu} - 2\mu_o\; \mathcal{R}e\; \mu + \mu_o^2$. These equations reduce to those of reference 7 as distance d and permeability μ'' go to zero.

Hysteresis Loss

Energy of the applied electromagnetic wave is also dissipated in the continual attempt of the applied oscillating wave motion to alter the Weiss crystalline structure and the electron spin orientations inside the ferrite spheres. This is illustrated by the lag in $\mathbf{B}$ to the changes of $\mathbf{H}$ in a hysteresis loop.

The energy change per unit time P_{hy} per unit volume in a hysteresis loop is

$$\mathbf{H} \bullet \frac{\partial \bar{\mathbf{B}}}{\partial t} = \frac{1}{2}\, j\omega \mathbf{H} \bullet \bar{\mathbf{B}}$$

for a harmonic time dependence. For an isotropic permeability $\bar{\mathbf{B}} = \bar{\mu}\bar{\mathbf{H}} = (\mu' + j\mu'')\,\bar{\mathbf{H}}$. Thus the wave power loss per unit volume due to hysteresis is

$$\begin{aligned} \frac{dP_{hy}}{dV} &= \frac{1}{2}\,\omega\mu''\,(\mathbf{H} \bullet \bar{\mathbf{H}}) \\ &= \frac{1}{2}\,\omega\mu'' H^2 \\ &= \frac{\mu''\omega}{2|\mu|^2} \left(|B_r|^2 + |B_\theta|^2 \right) \end{aligned} \tag{24}$$

which is consistent with that on page 100 of reference 5. By using equation (4)

$$\mathbf{B} = \nabla \times \mathbf{A}_i = \frac{CB_A}{r^{3/2}} \begin{bmatrix} \cos\theta \, I_{3/2}(r\sqrt{jp}) \\ \frac{1}{2} \sin\theta \left[I_{3/2}(r\sqrt{jp}) - r\sqrt{jp} \right] I_{1/2}(r\sqrt{jp}) \\ 0 \end{bmatrix} \tag{25}$$

Substituting equations (16) and (25) into equation (24) and integrating over $d\varphi$ gives

$$P_{hy} = \frac{\pi\omega\mu''}{|\mu|^2} B_A^2 C\bar{C} \left\{ \int_d^a \int_0^\pi \frac{\cos^2\theta + \frac{1}{4}\sin^2\theta}{r} I_{3/2}(r\sqrt{jp}) \right.$$

$$\times I_{3/2}(r\sqrt{-j\bar{p}}) \sin\theta \, d\theta \, dr$$

$$+ \frac{1}{4} |p| \int_d^a \int_0^\pi r I_{1/2}(r\sqrt{jp}) I_{1/2}(r\sqrt{-j\bar{p}}) \sin^3\theta \, d\theta \, dr$$

$$- \frac{1}{4} \int_d^a \int_0^\pi \left[\sqrt{jp}\, I_{1/2}(r\sqrt{jp}) I_{3/2}(r\sqrt{-j\bar{p}}) + \sqrt{-j\bar{p}} \right.$$

$$\left. \left. \times I_{1/2}(r\sqrt{-j\bar{p}}) I_{3/2}(r\sqrt{jp}) \right] \sin^3\theta \, dr \right\} \tag{26}$$

Using

$$\int_0^\pi (\cos^2\theta + \tfrac{1}{4}\sin^2\theta) \sin\theta \, d\theta = 1$$

and

$$\frac{1}{4} \int_0^\pi \sin^3\theta \, d\theta = \frac{1}{3}$$

in equation (26) gives

$$P_{hy} = \frac{\pi\omega\mu''}{|\mu|^2} B_A^2 C\bar{C}$$

$$\times \left\{ \int_d^a \frac{1}{r\sqrt{jp}} I_{3/2}(r\sqrt{jp}) I_{3/2}(r\sqrt{-j\bar{p}}) \sqrt{jp} \, dr \right.$$

$$+ \frac{1}{3} \int_d^a r\sqrt{-j\bar{p}}\, I_{1/2}(r\sqrt{jp}) I_{1/2}(r\sqrt{-j\bar{p}}) \sqrt{jp} \, dr$$

$$- \frac{1}{3} \int_d^a \left[\sqrt{jp}\, I_{1/2}(r\sqrt{jp}) I_{3/2}(r\sqrt{-j\bar{p}}) + \sqrt{-j\bar{p}} \right.$$

$$\left. \left. \times I_{1/2}(r\sqrt{-j\bar{p}}) I_{3/2}(r\sqrt{jp}) \right] dr \right\} \tag{27}$$

The integrations over r are shown in appendix C. The resulting expression (C18) can be written as

$$P_{hy} = \frac{\omega\mu''}{|\mu|^2} B_A^2 C\bar{C} \left| \left(\frac{\alpha^2 + \beta^2}{6} - 1 \right) \frac{C_r + c_r}{\sqrt{|p|}r} \right.$$

$$+ \frac{1}{3} \left[\frac{\alpha S_r + \beta s_r}{|p| r^2} - \frac{C_r - c_r}{(\sqrt{|p|}r)^3} + \frac{S_r}{\alpha} - \frac{s_r}{\beta} \right]$$

$$+ \left(\frac{4}{3} + \frac{\beta^2}{6} - \frac{\alpha^2}{2} \right) \alpha \, \mathrm{Shi}(2r\, \mathcal{R}e\sqrt{jp})$$

$$\left. - \left(\frac{4}{3} + \frac{\alpha^2}{6} - \frac{\beta^2}{2} \right) \beta \, \mathrm{Si}(2r\, \mathcal{I}m\sqrt{jp}) \right|_{r=d}^{a} \tag{28}$$

where use was made of definitions (19) to (22) and

$$\alpha = \frac{2\, \mathcal{R}e\sqrt{jp}}{|p|} \tag{29}$$

$$\beta = \frac{2\, \mathcal{I}m\sqrt{jp}}{|p|} \tag{30}$$

The hyperbolic sine and sine integrals can be expressed as

$$\mathrm{Shi}(X) = X + \frac{X^3}{3\times 3!} + \frac{X^5}{5\times 5!} + \frac{X^7}{7\times 7!} \tag{31}$$

and

$$\mathrm{Si}(X) = X - \frac{X^3}{3\times 3!} + \frac{X^5}{5\times 5!} - \frac{X^7}{7\times 7!} \tag{32}$$

(ref. 8, eqs. 672.11 and 431.11).

Generalized Parameters

The dimensionless power parameter $\tau P / (\omega^2 a^5 B_A^2)$ for both eddy currents and hysteresis can be written in terms of two independent dimensionless parameters: a normalized permeability

$$\mu_n = \frac{\mu' - j\mu''}{\mu_o}$$

and a combination of the remaining physical variables $\mathcal{A}_o = a^2 p_o$, where $p_o = \omega\mu_o/\tau$.

For eddy currents with the lower limit of integration d of equation (A10) set equal to zero

$$\frac{\tau P_{ec}}{\omega^2 a^5 B_A^2} = \frac{3\pi|\mu_n|^2 \mathcal{A}}{\mathcal{D}\ \mathcal{R}e\ \mathcal{A}} \left[S_a\ \mathcal{I}m\sqrt{j\mathcal{A}} + s_a\ \mathcal{R}e\sqrt{j\mathcal{A}} - (C_a - c_a)\frac{\mathcal{R}e\ \mathcal{A}}{|\mathcal{A}|} \right] \tag{33}$$

where $\mathcal{A} = \mathcal{A}_o\mu_n$. The arguments of the functions S_a, s_a, C_a, and c_a defined in equations (19) to (22) can be expressed in terms of the real and imaginary parts of $2\sqrt{j\mathcal{A}}$. The constant $\mathcal{D}$ is defined as

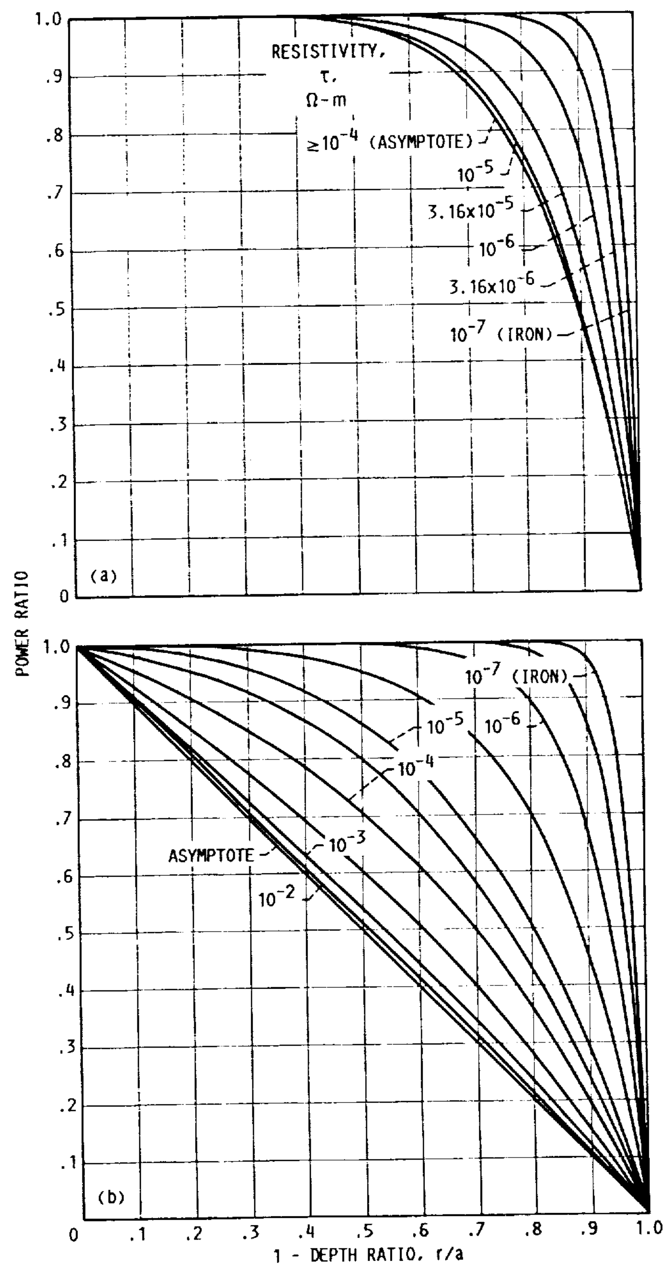
$$\begin{aligned}\mathcal{D} &= U_{gn}^2\ |\mathcal{A}|\ (C_a + c_a) \\ &+ \left[U_{gn}^2 + 2\ \mathcal{I}m(\mathcal{A}\bar{\mu}_n) + |\mathcal{A}|^2 - 2\ \mathcal{I}m\ \mathcal{A}\right] \\ &\times (C_a - c_a) - \sqrt{2} \\ &\times \Big\{U_{gn}^2 \left[(S_a + s_a)\ \mathcal{R}e\sqrt{\mathcal{A}} - (S_a - s_a)\ \mathcal{I}m\sqrt{\mathcal{A}}\right] \\ &- (S_a - s_a)\ \mathcal{R}e(U_n\ \bar{\mathcal{A}}\sqrt{\mathcal{A}}) \\ &- (S_a + s_a)\ \mathcal{I}m(U_n\ \bar{\mathcal{A}}\sqrt{\mathcal{A}})\Big\}\end{aligned} \tag{34}$$

where $U_n = \mu_n - 1$ and $U_{gn}^2 = |\mu_n|^2 + (1 - 2\ \mathcal{R}e\ \mu_n)$. The corresponding hysteresis equation is

$$\begin{aligned}\frac{\tau P_{hy}}{\omega^2 a^5 B_A^2} &= \frac{9\pi\mu''|\mu_n|\sqrt{\mathcal{A}}}{\mathcal{D}} \Bigg\{\left(\frac{\alpha^2+\beta^2}{6} - 1\right)\frac{C_a + c_a}{\sqrt{|\mathcal{A}|}} \\ &+ \frac{1}{3}\left[\frac{\alpha S_a - \beta s_a}{|\mathcal{A}|} - \frac{C_a - c_a}{|\mathcal{A}|^{3/2}} - \frac{S_a}{\alpha} - \frac{s_a}{\beta}\right] \\ &+ \left(\frac{4}{3} + \frac{\beta^2}{6} - \frac{\alpha^2}{2}\right)\alpha\ \mathrm{Shi}(2\ \mathcal{R}e\sqrt{\mathcal{A}}) \\ &- \left(\frac{4}{3} + \frac{\alpha^2}{6} - \frac{\beta^2}{2}\right)\beta\ \mathrm{Si}(2\ \mathcal{I}m\sqrt{\mathcal{A}}\Bigg\}\end{aligned} \tag{35}$$

Results and Discussion

For physical clarity it is advantageous to first relate results directly to the basic independent variables τ, ω, μ, and r as well as to a in figures 2 to 4. The ranges of generalized parameters $\mathcal{A}_o$ and a/λ are indicated in the legends of these figures. The plotting variables in figures 5 and 6 are dimensionless power absorption parameters that are functions



(a) Power absorbed by eddy currents.
(b) Power absorbed by hysteresis. Curves are shown at increments of powers of the square root of 10.

Figure 2.—Effect of varying resistivity τ with wave frequency $f = 10^7$ Hz; radius of sphere $a = 10^{-4}$ m, and normalized permeability $\mu/\mu_o = 100 - j10$. Parameters $a/\lambda = 3.3 \times 10^{-6}$ and $\mathcal{A}_o = a^2\omega\mu_o/\tau = 8\pi^2 \times 10^{-8}/\tau$.

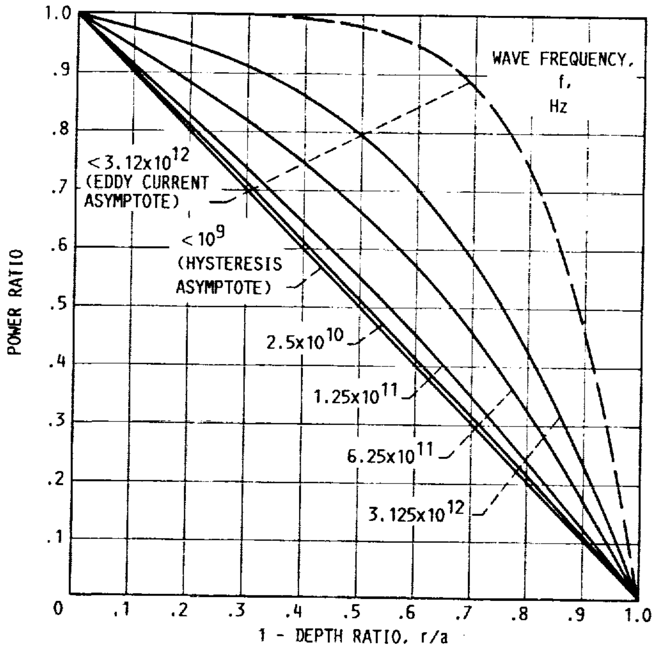


Figure 3.—Effect of varying wave frequency f with radius of sphere $a = 10^{-4}$ m; resistivity $\tau = 10$ Ω-m; and normalized permeability $\mu/\mu_o = 100-j10$. The curves reach asymptotes at $f < 10^9$ for hysteresis and $f < 3\times10^{12}$ for eddy currents. Parameters $a/\lambda = 3.3 \times 10^{-13} f$ and $\mathcal{A}_o = a^2\omega\mu_o/\tau = 8\pi^2 \times 10^{-16} f$.

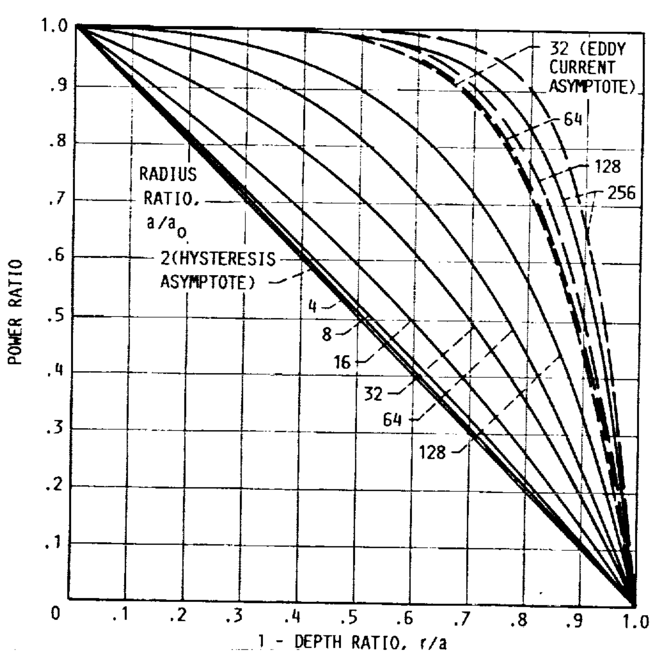


Figure 4.— Effect of varying radius of sphere a from 2×10^{-4} to 256×10^{-4} m with normalizing radius $a_o = 10^{-4}$ m; wave frequency $f = 10^9$ Hz; resistivity $\tau = 10$ Ω-m; and normalized permeability $\mu/\mu_o = 100-j10$. Parameter a/λ ranges from 6.7×10^{-4} to 8.6×10^{-2} and parameter $\mathcal{A}_o$ from 3.2×10^{-5} to 0.5.

of real and imaginary parts of permeability for fixed values of $\mathcal{A}_o$.

Skin Depth

For good absorption of wave energy the electromagnetic waves must penetrate the spheres to a sizable depth. The fraction of total power absorbed by a sphere as a function of the lower radial integration limit $r = d$ in equations (18) and (28) is used as an indication of skin depth in figures 2 to 4.

Figure 2 shows the effects of resistivity τ ranging from a typical value for iron of 10^{-7} Ω-m to values that lie on the asymptotic curve. This curve was reached as τ increased beyond 10^{-4} Ω-m for eddy currents and beyond 1 for hysteresis. Held constant were frequency at 10^7 Hz, permeability ratio at $100-j10$, and radius at 10^{-4} m. In general the wave penetration depth was considerably greater for hysteresis than for eddy currents.

Decreasing frequency $f = \omega/2\pi$ gave the same trends in figure 3 as increasing τ, as expected from the basic parameter $a^2\omega\mu_o/\tau$. Here τ held constant at 10 Ω-m. The ratio a/λ varied from approximately 1 to less than 10^{-3} in the curves shown for hysteresis. For eddy currents, however, $a/\lambda < 1$ yielded curves that lie on their asymptote. Curves for $a > \lambda$ looked reasonable but violated the neglect-of-displacement-current assumption and therefore are not shown.

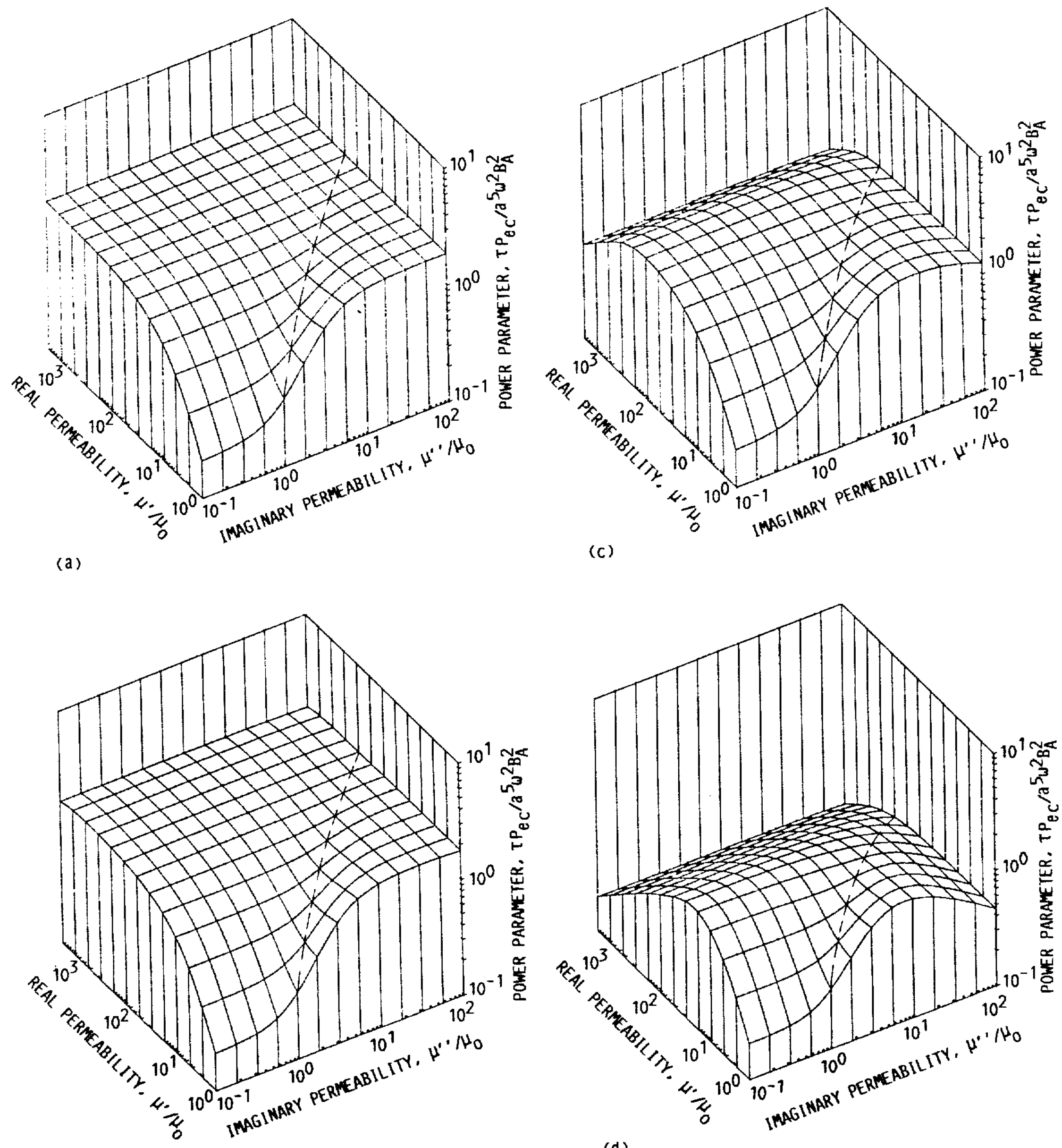
The curves of various size spheres in figure 4 are for radii less than 0.1 wavelength and therefore are well within the validity of the neglect-of-displacement-current assumption. Radius was varied by multiples of 2 from 10^{-4} to 256×10^{-4} m. Again τ and ω were held constant at 10 and $2\pi\times10^9$, respectively. The eddy current curves, shown dashed, are asymptotic at $a<3.2\times10^{-3}$; the hysteresis curves are asymptotic at $a<2\times10^{-4}$.

Power Absorptions P_{ec} and P_{hy}

The same parameter, $\tau/a^5\omega^2B_A^2$, factors out of both the hysteresis equation (28) and the eddy current equation (18). Typical results, shown in figures 5 and 6 for these cases, are, however, quite different.

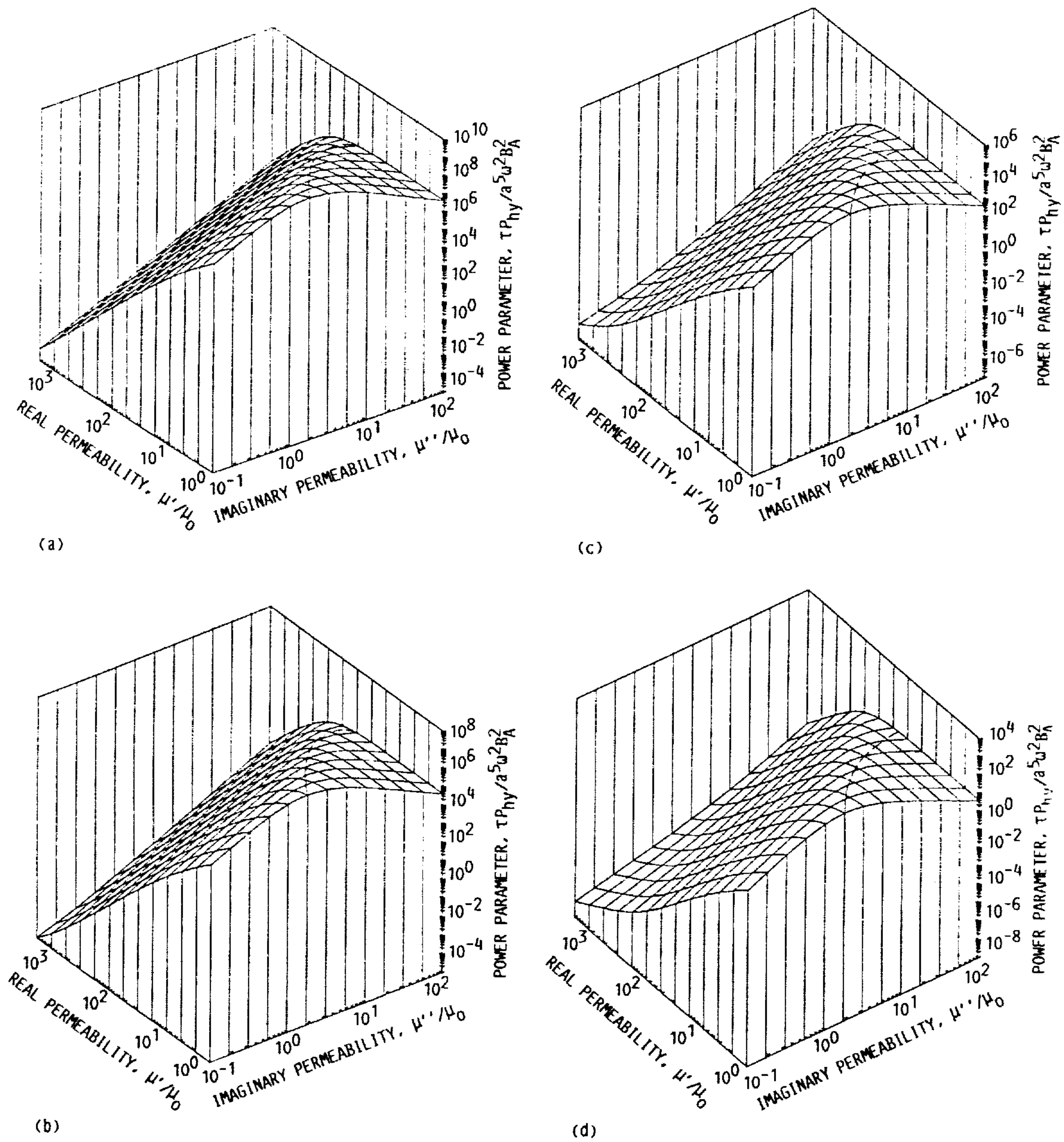
The eddy current power parameter was insensitive to $\mathcal{A}_o$ over the wide range of $10^{-7} < \mathcal{A}_o < 10^{-2}$ as shown in figures 5(a) and (b), but gradually decreased with increasing $\mathcal{A}_o$ beyond 10^{-2}, as shown in figures 5(c) and (d). These three-dimensional plots are symmetric about a diagonal curve (shown dashed) where $\mu' = \mu''$. This curve also represents a local minimum of the eddy current power parameter at low μ' and μ''.

The eddy current power parameter varied over less than 3 decades, but the hysteresis power parameter varied from 12 to 7 decades, depending on $\mathcal{A}_o$; as μ' and μ'' were varied over 3 decades in figures 5 and 6.



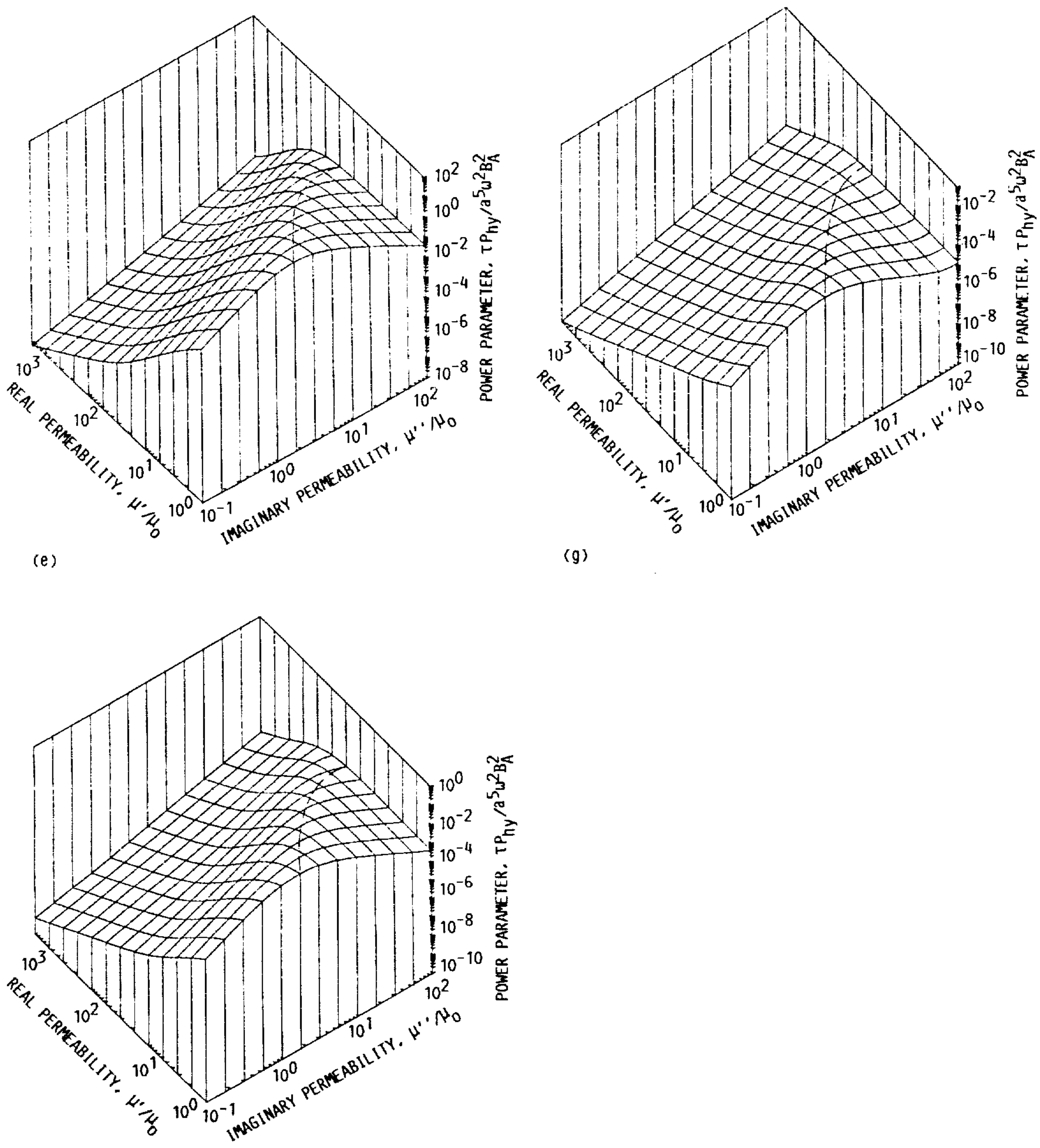
(a) $\mathcal{A}_o = 10^{-7}$.
(b) $\mathcal{A}_o = 10^{-2}$.
(c) $\mathcal{A}_o = 10^{-1}$.
(d) $\mathcal{A}_o = 1$.

Figure 5.—Power absorption by eddy currents for various values of $\mathcal{A}_o = a^2\omega\mu_o/\tau$. Dashed line denotes a diagonal and local minimum.



(a) $\mathcal{A}_o = 10^{-7}$.
(b) $\mathcal{A}_o = 10^{-6}$.
(c) $\mathcal{A}_o = 10^{-5}$.
(d) $\mathcal{A}_o = 10^{-4}$.

Figure 6.—Power absorption by hysteresis for various values of $\mathcal{A}_o = a^2\omega\mu_o/\tau$. Dashed line locates peak values of power parameter.



(e) $\mathfrak{A}_o = 10^{-3}$.
(f) $\mathfrak{A}_o = 10^{-2}$.
(g) $\mathfrak{A}_o = 10^{-1}$.

Figure 6.—Concluded.

Real and imaginary permeability played similar roles in the eddy current formulation, since they entered P_{ec} only through μ. Starting from the dashed diagonal line of figure 5 the parameter $\tau P_{ec}/a^5\omega^2 B_A^2$ increased to the same peak values with increasing μ' or μ''.

Power absorption by hysteresis, however, was directly proportional to a multiplier μ'' in addition to its dependence on μ. This offset the symmetry of dependence on μ' and μ'' present through the parameter p in equations (18) and (28) and made absorption much more sensitive to hysteresis than to eddy currents. As a result an upper elevation (shown dashed) of the three-dimensional plots of $\tau P_{hy}/a^5\omega^2 B_A^2$ formed close to the diagonal $\mu''/\mu_o = \mu'/\mu_o$ in the range $10^{-7} \leq \mathcal{A}_o \leq 10^{-1}$.

As a practical example consider a resistance τ of 10^4 Ω-m, a B_A of 1 gauss (10^{-4} tesla), and a frequency ω of $2\pi \times 10^{10}$ rad/sec. Then the peak power *per sphere* absorbed by hysteresis varies from about 0.2 to 14 W as the sphere radius a varies from 0.1 mm to 1 cm in figures 6(a) to (e). On the other hand the maximum power per sphere absorbed by eddy currents varies from about 1.4×10^{-10} to 1.4 W for the same τ, B_A, and range of a in figures 5(a) and (b).

For this same example let μ'' approach zero in figure 5 while holding μ' equal to μ_o. The power *per unit volume* by eddy currents then reduces to 0.2 W/cm^3 for the same τ, B_A, and ω values. This power absorption is in close agreement with results obtained from the sphere equation on page 135 of reference 5.

Concluding Remarks

Expressions were obtained, in terms of simple algebraic functions, for power absorption of electromagnetic waves in ferrite spheres by eddy currents and hysteresis. Skin depth was especially large for hysteresis and was significant for eddy currents, indicating good absorption for the small spheres of special interest herein.

Generalized parameters were found that reduced the five independent variables wave frequency $\omega/2\pi$, sphere radius a, resistivity τ, and real μ' and imaginary μ'' permeability divided by free-space permeability μ_o to three independent generalized parameters $\mathcal{A}_o = a^2\omega\mu_o/\tau$, μ'/μ_o, and μ''/μ_o and two power loss parameters $\tau P_{ec}/a^5\omega^2 B_A^2$ and $\tau P_{hy}/a^5\omega^2 B_A^2$ for small spheres. Working curves were presented for obtaining power losses from input to the independent parameters.

The eddy current power parameter was insensitive to $\mathcal{A}_o$ over wide ranges, $10^{-7} < \mathcal{A}_o < 10^{-2}$, and gradually decreased with increasing $\mathcal{A}_o$ beyond 10^{-2}. Real and imaginary permeability played similar roles in eddy current formulation, since they entered P_{ec} only through complex $\mu = \mu' - j\mu''$. Power absorption by hysteresis, however, was directly proportional to μ'' in addition to its dependence on complex μ. This offset the symmetry of dependence on μ' and μ'' and made absorption much more sensitive to hysteresis than to eddy currents.

From results obtained by using the equations derived, it appears that ferrite spheres provide a good means of damping extraneous electromagnetic waves and isolating electronic circuitry, especially in flight applications where weight is a consideration.

The components of the induced magnetic field, B_r and B_θ, diminished rapidly with radial distance as r^{-3} and r^{-2}, respectively. Except for closely packed spheres, there should then be little interaction between small spheres suspended in an insulator. The intentional degradation of an electromagnetic wave by its penetration into such a medium should then be closely proportional to the distance of wave travel and the number density of spheres.

Lewis Research Center
National Aeronautics and Space Administration
Cleveland, Ohio, June 13, 1989

Appendix A
Evaluation of Integral in Equation (17) for Power Loss by Eddy Currents

The equation of the modified Bessel function $I_m(kr)$ can be written as

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dI_m(kr)}{dr}\right] = \left(k^2 + \frac{m^2}{r^2}\right) I_m(kr) \qquad \text{(A1)}$$

and its conjugate as

$$\frac{1}{r}\frac{d}{dr}\left[r\frac{dI_m(\bar{k}r)}{dr}\right] = \left(\bar{k}^2 + \frac{m^2}{r^2}\right) I_m(\bar{k}r) \qquad \text{(A2)}$$

where

$$\left.\begin{aligned} k &= \sqrt{jp} = (1+j)\sqrt{\frac{p}{2}} \\ \bar{k} &= \sqrt{-j\bar{p}} = (1-j)\sqrt{\frac{\bar{p}}{2}} \end{aligned}\right\} \qquad \text{(A3)}$$

Multiplying equation (A1) by $I_m(\bar{k}r)$ and equation (A2) by $I_m(kr)$ and forming the integral of the second of these equations subtracted from the first gives

$$\int_d^a \left\{ I_m(\bar{k}r)\frac{d}{dr}\left[r\frac{dI_m(kr)}{dr}\right] - I_m(kr)\frac{d}{dr}\left[r\frac{dI_m(\bar{k}r)}{dr}\right]\right\} dr = j(p+\bar{p})\int_d^a I_m(kr)\, I_m(\bar{k}r) r\, dr$$

Using integration by parts on the left side yields

$$\left| I_m(\bar{k}r) r\frac{dI_m(kr)}{dr} - I_m(kr) r\frac{dI_m(\bar{k}r)}{dr}\right|_d^a - j(p+\bar{p})\int_d^a I_m(kr) I_m(\bar{k}r) r\, dr = \int_d^a \left[r\frac{dI_m(kr)}{dr}\frac{dI_m(\bar{k}r)}{dr} - r\frac{dI_m(\bar{k}r)}{dr}\frac{dI_m(kr)}{dr}\right] r\, dr = 0$$

Finally

$$j(p+\bar{p})\int_d^a I_m(kr) I_m(\bar{k}r) r\, dr = \left| r\left[I_m(\bar{k}r) k I_m'(kr) - I_m(kr)\bar{k} I_m'(\bar{k}r)\right]\right|_d^a \qquad \text{(A4)}$$

where the prime marks denote differentiation with respect to the arguments of the Bessel functions.

Substituting equation (A4) into equation (17) with $m = 3/2$ gives

$$P_{ec} = \frac{\pi\omega^2 B_A^2 C\bar{C}}{3j\tau(p+\bar{p})} \left| r\left[k I_{3/2}(\bar{k}r) I_{3/2}'(kr) - \bar{k} I_{3/2}(kr) I_{3/2}'(\bar{k}r)\right]\right|_{r=d}^a \qquad \text{(A5)}$$

Using $m = 3/2$ in

$$I_m'(z) = I_{m-1}(z) - \frac{m}{z} I_m(z) \qquad \text{(A6)}$$

of reference 9 (p. 79, eq. (3)) to eliminate the derivatives in equation (A5) yields

$$P_{ec} = \frac{\pi\omega^2 B_A^2 C\bar{C}}{3j\tau(p+\bar{p})} \left| r\left[k I_{3/2}(\bar{k}r) I_{1/2}(kr) - \bar{k} I_{3/2}(kr) I_{1/2}(\bar{k}r)\right]\right|_d^a$$

Then using $m = 1/2$ in equation (A6) and reference 9 (p. 79, eq. (1)) gives

$$P_{ec} = \frac{\pi\omega^2 B_A^2 C\bar{C}}{3j\tau(p+\bar{p})}$$

$$\times \left| r\left[kI_{-1/2}(\bar{k}r)I_{1/2}(kr) - \bar{k}I_{-1/2}(kr)I_{1/2}(\bar{k}r)\right] - \frac{j(p+\bar{p})}{|p|} I_{1/2}(\bar{k}r)I_{1/2}(kr) \right|_d^a$$

which can be written as

$$P_{ec} = \frac{2\pi\omega^2 B_A^2 C\bar{C}}{3\tau(p+\bar{p})} \left| \mathcal{I}m\left[rkI_{-1/2}(\bar{k}r)I_{1/2}(kr)\right] - \frac{\mathcal{R}e\, p}{|p|} I_{1/2}(\bar{k}r)I_{1/2}(kr) \right|_d^a \quad \text{(A7)}$$

Utilizing the relations

$$I_{-1/2}(\bar{z})\, I_{1/2}(z) = \frac{2}{\pi}\frac{\cosh\bar{z}\ \sinh z}{|z|} = \frac{e^{2\mathcal{R}e z} - e^{-2\mathcal{R}e z}}{2\pi|z|} + \frac{e^{2j\mathcal{I}m z} - e^{-2j\mathcal{I}m z}}{2\pi|z|} = \frac{1}{\pi|z|}\left[\sinh(2\ \mathcal{R}e\ z) + j\sin(2\ \mathcal{I}m\ z)\right] \quad \text{(A8)}$$

and

$$I_{1/2}(z)I_{1/2}(\bar{z}) = \frac{2}{\pi}\frac{\sinh z\ \sinh\bar{z}}{|z|} = \frac{e^{2\mathcal{R}e z} + e^{-2\mathcal{R}e z}}{2\pi|z|} - \frac{e^{2j\mathcal{I}m z} + e^{-2j\mathcal{I}m z}}{2\pi|z|} = \frac{1}{\pi|z|}\left[\cosh(2\ \mathcal{R}e\ z) - \cos(2\ \mathcal{I}m\ z)\right] \quad \text{(A9)}$$

in equation (A7) with $z = kr = \sqrt{jp}\ r$ and $\bar{z} = \bar{k}r$ gives

$$P_{ec} = \frac{2\omega^2 B_A^2 C\bar{C}}{3\tau(p+\bar{p})} \left| \mathcal{I}m\left\{\sqrt{\frac{jp}{|p|}}\left[\sinh(2r\ \mathcal{R}e\sqrt{jp}) + j\sin(2r\ \mathcal{I}m\sqrt{jp})\right]\right\} - \frac{\mathcal{R}e\, p}{r\,|p|^{3/2}} \times \left[\cosh(2r\ \mathcal{R}e\sqrt{jp}) - \cos(2r\ \mathcal{I}m\sqrt{jp})\right] \right|_d^a$$

$$= \frac{\omega^2 B_A^2 C\bar{C}}{3\tau\ \mathcal{R}e\, p} \left| \frac{1}{\sqrt{|p|}}\ \mathcal{I}m\left[(\mathcal{R}e\sqrt{jp} + j\ \mathcal{I}m\sqrt{jp}) \times \left(\sinh(2r\ \mathcal{R}e\sqrt{jp}) + j\sin(2r\ \mathcal{I}m\sqrt{jp})\right)\right] - \frac{\mathcal{R}e\, p}{r|p|^{3/2}}\left[\cosh(2r\ \mathcal{R}e\sqrt{jp}) - \cos(2r\ \mathcal{I}m\sqrt{jp})\right] \right|_d^a$$

Finally

$$P_{ec} = \frac{\omega^2 B_A^2 C\bar{C}}{3\tau\sqrt{|p|}\ \mathcal{R}e\, p} \left| (\mathcal{I}m\sqrt{jp})\ \sinh(2r\ \mathcal{R}e\sqrt{jp}) + (\mathcal{R}e\sqrt{jp})\ \sin(2r\ \mathcal{I}m\sqrt{jp}) - \frac{\mathcal{R}e\, p}{r\,|p|} \times \left[\cosh(2r\ \mathcal{R}e\sqrt{jp}) - \cos(2r\ \mathcal{I}m\sqrt{jp})\right] \right|_d^a \quad \text{(A10)}$$

For $d = 0$ and $\mu'' = 0$ (therefore $p = \mathcal{R}e\, p$), equation (A10) reduces to that of reference 7 (p. 378):

$$P_{ec} = \frac{\omega^2 B_A^2 C\bar{C}}{3\tau p^{3/2} a}\left[\frac{u}{2}(S + s) - (C - c)\right]$$

where $u = a\sqrt{2p}$, $S = \sinh\mu$, $s = \sin\mu$, $C = \cosh\mu$, $c = \cos\mu$, and, by using equation (A3), $\mathcal{I}m\sqrt{jp} = \mathcal{R}e\sqrt{jp} = \sqrt{p/2}$.

Appendix B
Determination of Constant $C\bar{C}$ Used in Equations (18) and (28)

The scalar product of constant C in equation (12) times its complex conjugate can be expressed as

$$C\bar{C} = \frac{9\mu\bar{\mu}\ \sqrt{p\bar{p}}\ a^5}{A\bar{A} + B\bar{B} + A\bar{B} + \bar{A}B} \tag{B1}$$

where the constants in the denominator are defined by

$$A\bar{A} = \left[\mu\bar{\mu} - 2\mu_o\ \Re e\ \mu + \mu_o^2\right] \times |p|a^2 I_{-1/2}(\sqrt{jp}\ a) I_{-1/2}(\sqrt{-j\bar{p}}\ a) \tag{B2}$$

$$B\bar{B} = \left[\mu - \mu_o\ (1 + jpa^2)\right]\left[\bar{\mu} - \mu_o(1 - j\bar{p}a^2)\right] \times I_{1/2}\ (\sqrt{jp}\ a) I_{1/2}\ (\sqrt{-j\bar{p}}\ a) \tag{B3}$$

$$A\bar{B} = (\mu - \mu_o)a\ \sqrt{jp}\ \left[\mu_o(1 - j\bar{p}a^2) - \bar{\mu}\right] \times I_{-1/2}(\sqrt{jp}\ a)\ I_{1/2}(\sqrt{-j\bar{p}}\ a)$$

and

$$\bar{A}B = (\bar{\mu} - \mu_o\ )\ a\ \sqrt{-j\bar{p}}\ \left[\mu_o(1 + jpa^2) - \mu\right] \times I_{-1/2}(\sqrt{-j\bar{p}}a)\ I_{1/2}(\sqrt{jp}\ a) \tag{B5}$$

The product of Bessel functions in equation (B2) can be written by using reference 9 as

$$I_{-1/2}(\sqrt{jp}\,a)\ I_{-1/2}(\sqrt{-j\bar{p}}a) = \frac{2}{\pi\ \sqrt{|p|}\ a}\cosh(\sqrt{jp}\ a) \times \cosh(\sqrt{-j\bar{p}}\ a)$$

$$= \frac{1}{2\pi\sqrt{|p|}\,a}\left(e^{\sqrt{jp}\,a} + e^{-\sqrt{jp}\,a}\right) \times \left(e^{\sqrt{-j\bar{p}}\,a} + e^{-\sqrt{-j\bar{p}}\,a}\right)$$

and therefore

$$I_{-1/2}(\sqrt{jp}\ a)\ I_{-1/2}(\sqrt{-j\bar{p}}\ a) = \frac{1}{\pi\sqrt{|p|}\ a} \times \left[\cosh(2\ \Re e\sqrt{jp}\ a) + \cos(2\ \Im m\sqrt{jp}\ a)\right] \tag{B6}$$

Since

$$I_{-1/2}(\sqrt{jp}\ a)\ I_{1/2}(\sqrt{-j\bar{p}}\ a) = \overline{I_{-1/2}(\sqrt{-j\bar{p}}\ a)\ I_{1/2}(\sqrt{jp}\ a)}$$

it follows from equation (A8) that

$$I_{-1/2}(\sqrt{jp}\ a)\ I_{1/2}(\sqrt{-j\bar{p}}\ a) = \frac{1}{\pi a\sqrt{|p|}} \times \left[\sinh(2a\ \Re e\ \sqrt{jp}) - j\ \sin(2a\ \Im m\ \sqrt{jp})\right] \tag{B7}$$

Using equations (A9) and (B7) in equations (B3) and (B4), respectively, along with equations (B6) and (A8) in equations (B2) and (B5) reduces the Bessel function expressions in equations (B2) to (B5) to simple hyperbolic and trigonometric functions defined in equations (19) to (23). This results in the expressions

$$A\bar{A} = a\sqrt{|p|}\ U_g^2\ \frac{C_a - c_a}{\pi} \tag{B8}$$

$$BB = \left[U_g^2 + \mu_o^2\ (a^4\ |p|^2 - 2a^2\ \Im m\ p)\right]\frac{C_a - c_a}{\pi a\sqrt{|p|}} \tag{B9}$$

and

$$\tilde{A}B + A\bar{B} = \frac{2}{\pi\sqrt{|p|}}\left\{-U_g^2\left[(S_a + s_a)\ \mathcal{R}e\sqrt{\frac{p}{2}} - (S_a - s_a)\ \mathcal{I}m\sqrt{\frac{p}{2}}\right]\right.$$

$$\left. + a^2\mu_o\left[(S_a - s_a)\ \mathcal{R}e\left(U\bar{p}\sqrt{\frac{p}{2}}\right) + (S_a + s_a)\ \mathcal{I}m\left(U\bar{p}\sqrt{\frac{p}{2}}\right)\right]\right\} \qquad \text{(B10)}$$

Substituting equations (B8) to (B10) into equation (B1) gives equation (23).

Appendix C
Evaluation of Integrals in Equation (27) for Power Loss by Hysteresis

Equation (27) can be written as

$$\frac{|\mu|^2 P_{hy}}{\pi\omega\mu'' B_A^2 C\bar{C}} = \mathcal{I}_1 + \frac{1}{3}(\mathcal{I}_2 - \mathcal{I}_3 - \mathcal{I}_4) \qquad \text{(C1)}$$

Here

$$\mathcal{I}_1 = \int \frac{1}{z} I_{3/2}(z)\, I_{3/2}(\bar{z})\, dz$$

and by using reference 10 (p. 443)

$$\mathcal{I}_1 = \frac{2}{\pi}\int \frac{1}{z|z|}\left(\cosh z - \frac{1}{z}\sinh\right)\left(\cosh \bar{z} - \frac{1}{\bar{z}}\sinh \bar{z}\right) dz$$

In like manner

$$\mathcal{I}_2 = \frac{2}{\pi}\int \bar{z} I_{1/2}(z) I_{1/2}(\bar{z})\, dz = \frac{2}{\pi}\int \frac{\bar{z}}{|z|}\sinh z \sinh \bar{z}\, dz$$

$$\mathcal{I}_3 = \int I_{1/2}(z)\, I_{3/2}(\bar{z})\, dz$$

$$= \frac{2}{\pi}\int \frac{1}{|z|}\sinh z\left(\cosh \bar{z} - \frac{1}{\bar{z}}\sinh \bar{z}\right) dz$$

and

$$\mathcal{I}_4 = \bar{\mathcal{I}}_3 = \frac{2}{\pi}\int \frac{1}{|z|}\sinh \bar{z}\left(\cosh z - \frac{1}{z}\sinh z\right) d\bar{z}$$

where

$$z = r\sqrt{jp} \quad \text{and} \quad \bar{z} = r\sqrt{-j\bar{p}} \qquad \text{(C2)}$$

Next, replace the hyperbolic functions by their exponential forms, rearrange, and reduce as in the following example for $\mathcal{I}_1$.

$$\mathcal{I}_1 = \frac{1}{2\pi}\int\left[e^z + e^{-z} - \frac{1}{z}(e^z - e^{-z})\right] \times \left[e^{\bar{z}} + e^{-\bar{z}} - \frac{1}{\bar{z}}(e^{\bar{z}} - e^{-\bar{z}})\right]\frac{dz}{z|z|}$$

$$= \frac{1}{2\pi}\int\left[\left(1 - \frac{1}{z}\right)e^z + \left(1 + \frac{1}{z}\right)e^{-z}\right] \times \left[\left(1 - \frac{1}{\bar{z}}\right)e^{\bar{z}} + \left(1 + \frac{1}{\bar{z}}\right)e^{-\bar{z}}\right]\frac{dz}{z|z|}$$

$$= \frac{1}{2\pi}\int\Bigg[\left(1 + \frac{1}{z\bar{z}} - \frac{1}{z} - \frac{1}{\bar{z}}\right)e^{2\mathcal{R}_e z} + \left(1 + \frac{1}{z\bar{z}} + \frac{1}{z} + \frac{1}{\bar{z}}\right)e^{-2\mathcal{R}_e z} + \left(1 - \frac{1}{z\bar{z}} - \frac{1}{z} + \frac{1}{\bar{z}}\right)e^{2j\mathcal{I}_m z} + \left(1 - \frac{1}{z\bar{z}} + \frac{1}{z} - \frac{1}{\bar{z}}\right)e^{-2j\mathcal{I}_m z}\Bigg]\frac{dz}{z|z|}$$

$$= \frac{1}{2\pi}\int\Bigg[\frac{z\bar{z}+1}{z\bar{z}}\left(e^{2\mathcal{R}_e z} + e^{-2\mathcal{R}_e z}\right) + \frac{z\bar{z}-1}{z\bar{z}}\left(e^{2j\mathcal{I}_m z} + e^{-2j\mathcal{I}_m z}\right) - \frac{z+\bar{z}}{z\bar{z}}\left(e^{2\mathcal{R}_e z} - e^{-2\mathcal{R}_e z}\right) + \frac{z-\bar{z}}{z\bar{z}}\left(e^{2j\mathcal{I}_m z} - e^{-2j\mathcal{I}_m z}\right)\Bigg]\frac{dz}{z|z|}$$

$$\mathcal{I}_1 = \frac{1}{\pi}\int\Bigg[\frac{z\bar{z}+1}{z\bar{z}}\cosh(2\,\mathcal{R}_e\, z) + \frac{z\bar{z}-1}{z\bar{z}}\cos(2\,\mathcal{I}_m\, z) - \frac{2\,\mathcal{R}_e\, z}{z\bar{z}}\sinh(2\,\mathcal{R}_e\, z) - \left[\frac{2\,\mathcal{I}_m\, z}{z\bar{z}}\sin(2\,\mathcal{I}_m\, z)\right]\frac{dz}{z|z|} \qquad \text{(C3)}$$

In like manner

$$\mathcal{I}_2 = \frac{1}{\pi} \int \left[\cosh(2\ \Re_e\ z) - \cos(2\ \Im_m\ z)\right] \frac{\bar{z}}{z}\, dz \qquad \text{(C4)}$$

and

$$\mathcal{I}_3 = \frac{1}{\pi} \int \frac{\sinh(2\ \Re_e\ z) - j\sin(2\ \Im_m\ z)}{|z|} - \frac{\cosh(2\ \Re_e\ z) - \cos(2\ \Im_m\ z)}{\bar{z}|z|}\, dz \qquad \text{(C5)}$$

Since $\mathcal{I}_4$ is the complex conjugate of $\mathcal{I}_3$, it follows that

$$\mathcal{I}_3 + \mathcal{I}_4 = 2\ \Re_e\ \mathcal{I}_3 \qquad \text{(C6)}$$

Using equations (C2) and (C5) in equation (C6) gives

$$\mathcal{I}_3 + \mathcal{I}_4 = \frac{2}{\pi\sqrt{|p|}} \int\Big\{\Re_e\sqrt{jp}\,\sinh(2\ \Re_e\ z) - \Im_m\sqrt{jp}\,\sin(2\ \Im_m\ z) - \frac{\Re_e\ z\sqrt{jp}}{|z|^2} \times \left[\cosh(2\ \Re_e\ z) - \cos(2\ \Im_m\ z)\right]\Big\} \frac{dr}{r} \qquad \text{(C7)}$$

Substituting equations (C3) to (C7) into equation (C1) and collecting terms of like integrals gives

$$\frac{|\mu|^2 P_{\text{hy}}}{\omega\mu'' B_A^2\, \overline{CC}} = \left(\alpha - \frac{\alpha^3}{6} + \frac{\beta^2\alpha}{6}\right) \times \int \frac{\cosh x}{x^2}\, dx + \left(\beta - \frac{\beta^3}{6} + \frac{\alpha^2\beta}{6}\right) \times \int \frac{\cos y}{y^2}\, dy + \alpha^3 \int \frac{\cosh x}{x^4}\, dx - \beta^3 \int \frac{\cos y}{y^4}\, dy + \frac{1}{3\alpha} \int \cosh x\, dx - \frac{1}{3\beta} \times \int \cos y\, dy - \alpha^3 \int \frac{\sinh x}{x^3}\, dx + \beta^3 \times \int \frac{\sin y}{y^3}\, dy + \frac{\alpha}{3} \int \frac{\sinh x}{x}\, \mathrm{dx} - \frac{\beta}{3} \int \frac{\sin y}{y}\, dy \qquad \text{(C8)}$$

where

$$\left.\begin{aligned} x &= 2\ \Re_e\ z \\ y &= 2\ \Im_m\ z \\ \alpha &= \frac{2\ \Re_e\sqrt{jp}}{\sqrt{|p|}} \\ \beta &= \frac{2\ \Im_m\sqrt{jp}}{\sqrt{|p|}} \end{aligned}\right\} \qquad \text{(C9)}$$

The following expressions enable the integration of equation (C8). They were obtained by integrating by parts along with using expressions from pages 89, 96, 148, and 149 of reference 8.

$$\int \frac{\cosh x}{x^2}\, dx = -\frac{\cosh x}{x} + \text{Shi}\ x \qquad \text{(C10)}$$

$$\int \frac{\cosh x}{x^4}\, dx = -\frac{\cosh x}{3x^3} - \frac{\sinh x}{6x^2} - \frac{\cosh x}{6x} + \frac{1}{6}\,\text{Shi}\ x \qquad \text{(C11)}$$

$$\int \frac{\cosh y}{y^2}\, dy = -\frac{\cos y}{y} - \int \frac{\sin y}{y}\, dy \qquad \text{(C12)}$$

$$\int \frac{\cos y}{y^4}\, dy = -\frac{\cos y}{3y^3} + \frac{\sin y}{6y^2} + \frac{\cos y}{6y} + \frac{1}{6}\,\text{Si}\ y \qquad \text{(C13)}$$

$$\int \frac{\sinh x}{x^3}\, dx = -\frac{\sinh x}{2x^2} - \frac{\cosh x}{2x} + \frac{1}{2}\,\text{Shi}\ x \qquad \text{(C14)}$$

$$\int \frac{\sin y}{y^3}\, dy = -\frac{\sin y}{2y^2} - \frac{\cos y}{2y} + \frac{1}{2}\,\text{Si}\ y \qquad \text{(C15)}$$

where

$$\text{Shi}\,x = \int \frac{\sinh x}{x}\, dx = x + \frac{x^3}{3\times 3!} + \frac{x^5}{5\times 5!} + \frac{x^7}{7\times 7!} + \ldots \qquad \text{(C16)}$$

and

$$\text{Si}\,y = \int \frac{\sin y}{y}\, dy = y - \frac{y^3}{3\times 3!} + \frac{y^5}{5\times 5!} - \frac{y^7}{7\times 7!} + \ldots \qquad \text{(C17)}$$

These expressions then enable the integration of equation (C8) to the following form:

$$\frac{|\mu^2|\, P_{hy}}{\omega\mu'' B_A^2\, C\overline{C}} = \Bigg| \left(\frac{\alpha^3}{6} + \frac{\alpha\beta^2}{6} - \alpha\right)\frac{\cosh x}{x} + \left(\frac{\beta^3}{6} + \frac{\beta\alpha^2}{6} - \beta\right)\frac{\cos y}{y} + \frac{\alpha^3}{3}\frac{\sinh x}{x^2} + \frac{\beta^3}{3}\frac{\sin y}{y^2} - \frac{\alpha 3}{3}\frac{\cosh x}{x^3} + \frac{\beta^3}{3}\frac{\cos y}{y^3}$$

$$+ \frac{1}{3\alpha}\sinh x - \frac{1}{3\beta}\sin y - \left(\frac{4}{3}\alpha + \frac{\beta^2\alpha}{6} - \frac{\alpha^3}{2}\right)\mathrm{Shi}\, x - \left(\frac{4}{3}\beta + \frac{\alpha^2\beta}{6} - \frac{\beta^3}{2}\right)\mathrm{Si}\, y \,\Bigg|_{x_1, y_1}^{x_2, y_2} \qquad \text{(C18)}$$

where x_2 and y_2 are the real and imaginary parts of z (see eq. (C9)) at the outer radius a and x_1 and y_1 are at an arbitrary inner radius of interest.

Appendix D
Symbols*

$\mathcal{A}$	$\mathcal{A}_o\mu_n$
$\mathcal{A}_o$	a^2p_o
A	magnetic vector potential
a	radius of sphere
B	magnetic flux density
B	magnetic field vector
C	constant defined by eq. (12)
C_r	constant defined by eq. (21)
c_r	constant defined by eq. (22)
D	constant defined by eq. (13)
$\mathfrak{D}$	constant defined by eq. (34)
d	radial distance to lower limit of intergation inside sphere
E	electric field vector
f	wave frequency, $\omega/2\pi$
H	magnetic inductance
$\mathcal{I}$	integral in appendix C
$\mathcal{I}m$	imaginary part of a complex quantity
I_m	modified spherical Bessel function where m is an odd integer plus 1/2
i	current
j	$\sqrt{-1}$
k	$\sqrt{jp}$
M	magnetization
n	normal to boundary
P	power
p	parameter $\omega\mu/\tau$
p_o	parameter $\omega\mu_o/\tau$
$\mathcal{R}e$	real part of a complex quantity
r	radial distance from center of sphere
S_r	defined by eq. (19)
s_r	defined by eq. (20)
t	time
U	$\mu-\mu_o$
U_n	μ_n-1
U_g^2	$\lvert\mu\rvert^2 - 2\mu_o\, \mathcal{R}e\, \mu + \mu_o^2$
U_{gn}^2	$\lvert\mu_n\rvert^2 + (1-2\, \mathcal{R}e\, \mu_n)$
V	volume
X	dummy variable in eqs. (31) and (32)
x	$2\, \mathcal{R}e\, z$
y	$2\, \mathcal{I}m\, z$
z	$r\sqrt{jp}$
α	defined by eq. (29)
β	defined by eq. (30)
θ	polar angle
κ	wave propagation vector
λ	wavelength
μ	permeability inside sphere, $\mu' - j\mu''$
μ_n	normalized permeability, μ/μ_o
μ_o	permeability outside sphere (vacuum), $4\pi\times10^{-7}$ henry/meter
τ	resistivity
φ	azimuthal angle
χ	susceptibility
ω	angular velocity

Subscripts:

A	applied wave
a	at a radius equal to a
d	at a radius equal to d
ec	eddy current
hy	hysteresis
i	inside sphere; internal
o	outside sphere
r	radial direction
θ	direction of polar angle

Superscripts:

$'$	real part of complex quantity
$''$	imaginary part of complex quantity
—	complex conjugate
^	unit vector

*All terms are in international units unless stated otherwise.

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16. Abstract

Algebraic expressions in terms of elementary mathematical functions are derived for power absorption and dissipation by eddy currents and magnetic hysteresis in ferrite spheres. Skin depth is determined by using a variable inner radius in descriptive integral equations. Numerical results are presented for sphere diameters less than one wavelength. A generalized power absorption parameter for both eddy currents and hysteresis is expressed in terms of the independent parameters involving wave frequency, sphere radius, resistivity, and complex permeability. In general, the hysteresis phenomenon has a greater sensitivity to these independent parameters than do eddy currents over the ranges of independent parameters studied herein. Working curves are presented for obtaining power losses from input to the independent parameters.

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