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**STABILITY THEORY APPLICATIONS TO
LAMINAR-FLOW CONTROL**

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APPLICATION OF STABILITY THEORY TO LAMINAR FLOW CONTROL

In order to design LFC* configurations, one needs reliable methods for boundary-layer transition prediction. Among the available methods, there are correlations based upon R_θ , shape factors, Görtler number and crossflow Reynolds number. These correlations derived from experimental information have limited scope. The most advanced transition prediction method is based upon linear stability theory in the form of the e^N method which has proven to be successful in predicting transition in two- and three-dimensional boundary layers and, in particular, studying the sensitivity of boundary-layer transition to various control parameters such as pressure gradient, suction, and wall temperature.

*Laminar-flow Control (LFC).

- LFC - DELAY OF BOUNDARY-LAYER TRANSITION USING MEANS SUCH AS PRESSURE GRADIENT, SUCTION, WALL TEMPERATURE, ETC.

- NEED FOR TRANSITION PREDICTION METHODS

- AVAILABLE METHODS
 - CORRELATIONS BASED UPON R_θ , SHAPE FACTORS, GÖRTLER NUMBER, CROSSFLOW REYNOLDS NUMBER, ETC.

 - PREDICTION METHODS BASED UPON BOUNDARY-LAYER STABILITY THEORY

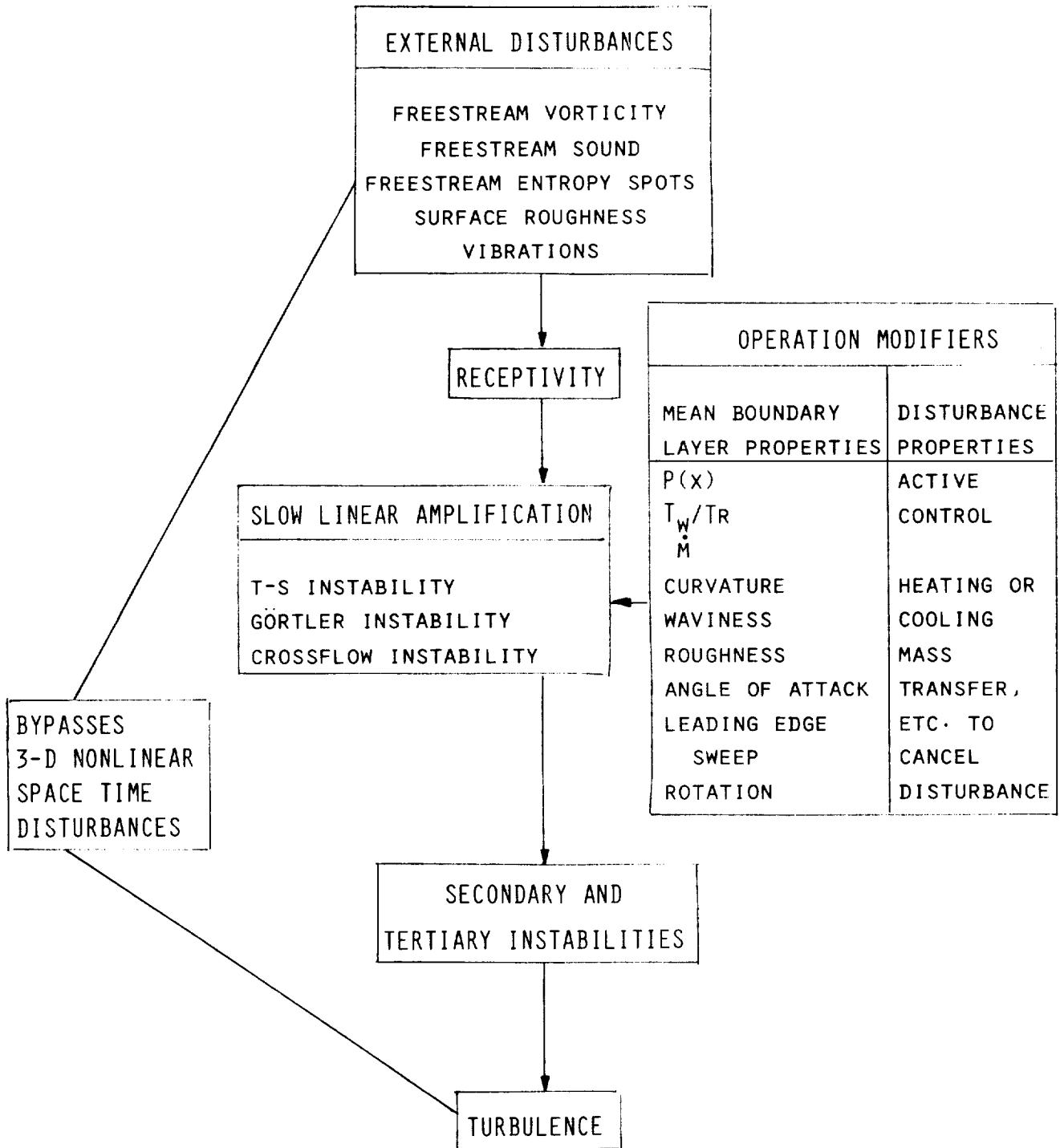
EVOLUTIONARY PATHS IN LAMINAR/TURBULENT TRANSITION

There are various stages involved in the transition process. External disturbances in the form of freestream vorticity, sound, entropy spots, surface roughness and surface vibrations get internalized in the boundary through a process known as 'receptivity' -- a phrase first coined by Morkovin (Ref. 1). These internalized small disturbances begin to grow past a critical Reynolds number. At first the disturbances grow exponentially (according to linear theory) in the form of Tollmien-Schlichting (T-S), Görtler or crossflow waves until nonlinearity sets in and then secondary and perhaps tertiary instabilities in the flow cause transition. We are beginning to understand more and more about receptivity and nonlinear stages now. We know, for example, that flow nonhomogeneities play an important role in receptivity process (Refs. 2-5). In recent years, considerable progress has been made in understanding nonlinear stages of transition process (Refs. 6-10). More advances will certainly be made both in the field of receptivity and nonlinear breakdown mechanism. But transition essentially depends upon the disturbance environment and it is the lack of detailed quantitative characterization of the disturbance environment that we will always have to rely upon empirical information for transition prediction in practical situations.

Transition may also take place through nonlinear mechanisms by passing the usual linear mechanism. An example is the swept attachment line boundary layer which exhibits subcritical transition (Ref. 11). However, if the initial disturbance level is kept low the linear process (exponential growth) is, in general, involved and its extent (in terms of distance along the body and total amplification) is quite large in comparison with the nonlinear process and this essentially leads to the success of the e^N method.

It is at the linear state that control, whether 'passive' (through boundary layer modification) or 'active' (through disturbance cancellation) is possible. Though some CFD studies indicate possibility of control at nonlinear stages too (Ref. 12). An LFC designer, however, ought to be conservative and keep the amplitudes low.

EVOLUTIONARY PATHS IN LAMINAR/TURBULENT TRANSITION



THE e^N METHOD FOR TRANSITION PREDICTION

The e^N method was first used by A.M.O. Smith in 1952* (Ref. 13) for Görtler instability on concave surfaces, though the work remained classified and was not published until 1955 (Ref. 14). By that time, both Smith and van Ingen (Ref. 15, 16) independently had shown that, for two-dimensional flows, the e^N method could correlate low disturbance experimental data with N approximately 9 and the method came to be known as the e^9 method.

The necessary steps involved in application of the e^N method are: (1) computation of mean boundary layer profiles accurately, (2) computation of linear amplification rate by an "appropriate stability model," and (3) integration of the growth rate from onset of instability x_0 to transition initiation location x_T . The value of the integral is equal to the exponent in e^N and is commonly known as the "N factor."

- CALCULATE MEAN BOUNDARY-LAYER PROFILES
- CALCULATE LINEAR AMPLIFICATION RATE BY USING "APPROPRIATE STABILITY MODEL"
- TRANSITION OCCURS WHEN DISTURBANCES IN THE BOUNDARY LAYER ARE FIRST AMPLIFIED BY A FACTOR e^N , WHERE

$$N = \ln(A/A_0) = \int_{x_0}^{x_T} (\text{LINEAR AMPLIFICATION RATE}) dx$$

(SMITH, 1952)

*Smith, A.M.O.: Design of the DESA-2 Airfoil. Douglas Aircraft Co., ES17117, AD143008, 1952. (Reference 13 mentions 1952 reference.)

REVIEW OF LINEAR STABILITY THEORY

The first question one asks is: "What is the 'appropriate stability model' for computation of the linear growth rates?" The simplest of the model is the Orr-Sommerfeld equation, which is a fourth-order ordinary differential equation in the disturbance stream function ϕ derived from the Navier-Stokes equation written in the cartesian coordinates x, y, z , where y is the normal boundary layer coordinate and x and z are in the plane parallel to the surface. In deriving this equation it is assumed that mean flow profiles such as U in the direction of X and W in the direction of Z are functions of y only. This is the well-known 'parallel flow' assumption. The disturbance is assumed to have a waveform with wave numbers α, β in x and z directions respectively and ω is the disturbance frequency.

We have an eigenvalue problem, given by the dispersion relation, meaning that nontrivial solution of the Orr-Sommerfeld equation exists only for certain combinations of α, β , and ω . In general, α, β, ω can all be complex. However, we can talk in terms of temporal or spatial theories where either $e^{-\alpha_1 x - \beta_1 z} e^{\omega_1 t}$ or $e^{\omega_1 t}$ is set to unity. The Orr-Sommerfeld equation is a model equation for T-S or crossflow disturbances in incompressible flows.

• ORR-SOMMERFELD EQUATION (INCOMPRESSIBLE FLOW)

$$\left(\frac{d^2}{dy^2} - (\alpha^2 + \beta^2) \right)^2 \phi = iR[(\alpha U + \beta W - \omega) \left\{ \frac{d^2}{dy^2} - (\alpha^2 + \beta^2) \right\} \phi - (\alpha U'' + \beta W'')]$$

B. C. $\phi(0) = \phi'(0) = 0; \quad \phi = 0, \quad \phi' = 0 \quad \text{WHEN } y = \infty$

- DERIVED FROM NAVIER-STOKES EQUATION USING PARALLEL FLOW ASSUMPTION AND BY ASSUMING

$$u(x, y, z, t) = U(y) + \phi_y(y) e^{i(\alpha x + \beta z - \omega t)} e^{-\alpha_1 x - \beta_1 z} e^{\omega_1 t}$$

- EIGENVALUE PROBLEM: $\omega = \omega(\alpha, \beta)$
- TEMPORAL THEORY
- SPATIAL THEORY

LINEAR STABILITY THEORY (CONCLUDED)

When effects of curvature (body or streamline) are important, as in the Görtler problem, then the governing equations become sixth-order.

The governing equations for compressible stability with or without curvature are, in general, eighth-order. There, for hypersonic flows, one needs to worry about real gas effects. Some recent calculations (Ref. 17) at Mach 10 show their significance.

Boundary layer flows, in general, are nonparallel. For comparison with stability experiments on quantities such as disturbance eigenfunctions and growth rates, it is advisable to use nonparallel stability theory (Ref. 18). Since the e^N method is essentially a correlation with experimental data, it is not necessary to use nonparallel theory for transition prediction purposes. Use of nonparallel theory, say for two-dimensional boundary-layer flows analyzed by Smith (Ref. 15), will simply shift the value of N from 9 to some other value (say 12) and the method would have been known as the e^{12} method.

- SIXTH-ORDER SYSTEM (INCOMPRESSIBLE FLOW)
 - EFFECT OF CURVATURE (BODY AND STREAMLINE)
 - EFFECT OF ROTATION

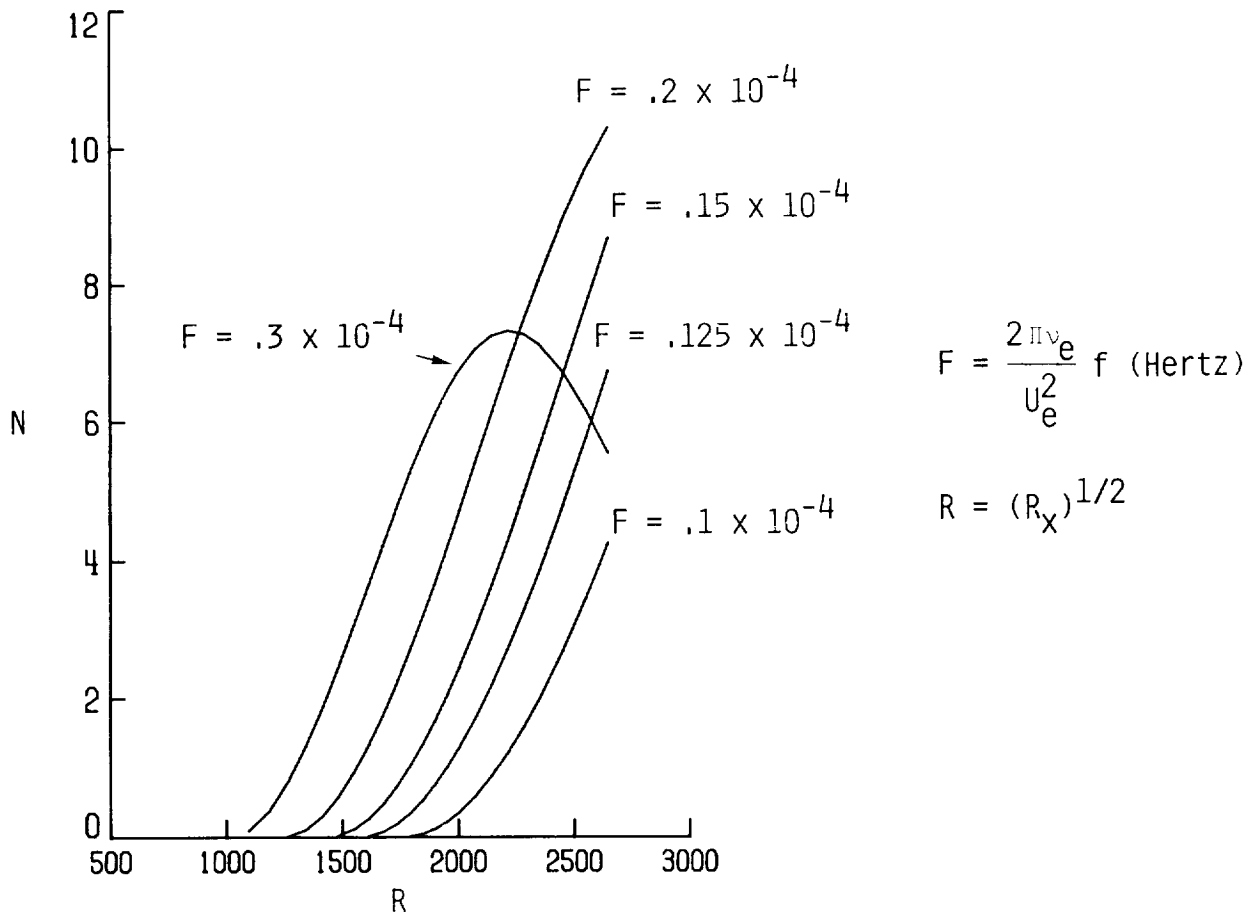
- COMPRESSIBLE STABILITY
 - EIGHTH-ORDER SYSTEM OF EQUATION
 - PHYSICAL AND TRANSPORT PROPERTIES (PERFECT OR REAL GAS)

- NONPARALLEL STABILITY

COMPUTATION OF N FACTORS

An example of a typical e^N calculation is provided in this figure. In reality disturbances develop in the form of 'wave-packets,' but then questions regarding initial conditions and the origins of these packets arise. So for the e^N purposes, it is common to consider monochromatic waves. Calculations for a fixed frequency are performed and repeated for others. When a frequency first reaches an N factor of, say 10, transition is said to initiate. In this figure, for example, transition takes place at R of about 2500 where a frequency $F = .2 \times 10^{-4}$ first reaches $N = 10$. To compare experimental transition data, one could generate an N versus an F curve at the transition location, and the peak of such a curve then gives a relevant N factor.

10^0 SHARP CONE, $M_\infty = 1.5$

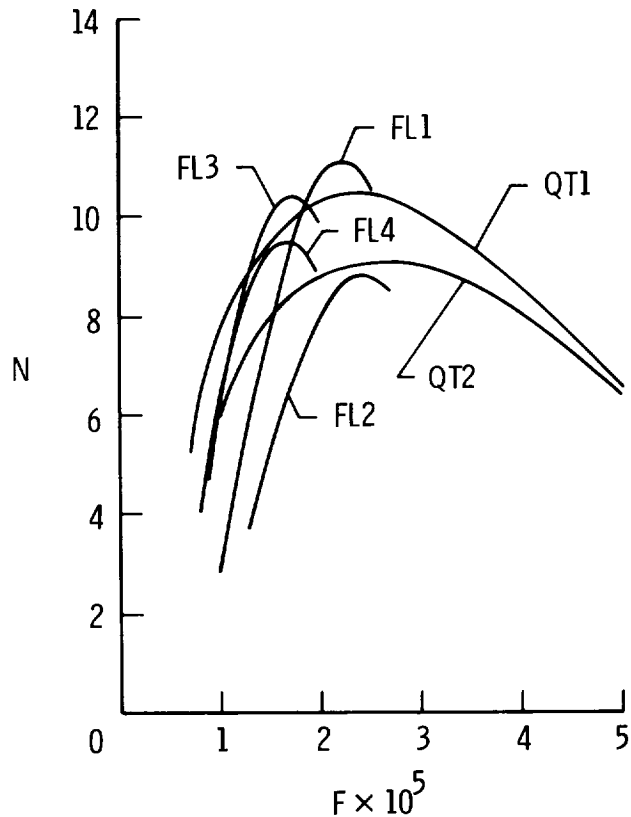


TRANSITION N FACTORS FOR QUIET TUNNEL AND F-15 CONE DATA

The N versus F curves have been generated for the experimental transition data listed in the table. These data are for 10° sharp cones from F-15 flight and the Mach 3.5 Langley quiet tunnel. Calculations are made using adiabatic wall conditions to closely match the experimental conditions. Eigenvalues are computed using the full eight-order system. Note that the peak of all the curves for the first six test cases listed in the table lie between about 9 and 11. So the e^N method (with N from 9-11) is successful in correlating with experimental transition data at Mach 1.2 to 3.5. For the last case (QT3) listed in the table, the N factor was calculated to be 6 at the last computational station indicating no transition. This is consistent with the experiment where flow was still laminar at the last measurement station and the cone was not long enough to have transition. External disturbances in these experiments are believed to be low - a necessary condition for the success of the e^N method. Correlation with experimental data from conventional supersonic wind tunnels would yield low values of N (2-4) since they have high level of freestream disturbances.

10° SHARP CONE

CASE I.D.	M_∞	U_e / v_e $\times 10^6 / m$	Re_{tr} $\times 10^6$
FL1	1.20	9.16	6.99
FL2	1.35	9.28	5.59
FL3	1.60	11.55	7.86
FL4	1.92	14.19	7.26
QT1	3.5	29.46	8.08
QT2	3.5	19.94	6.74
QT3	3.5	9.50	-

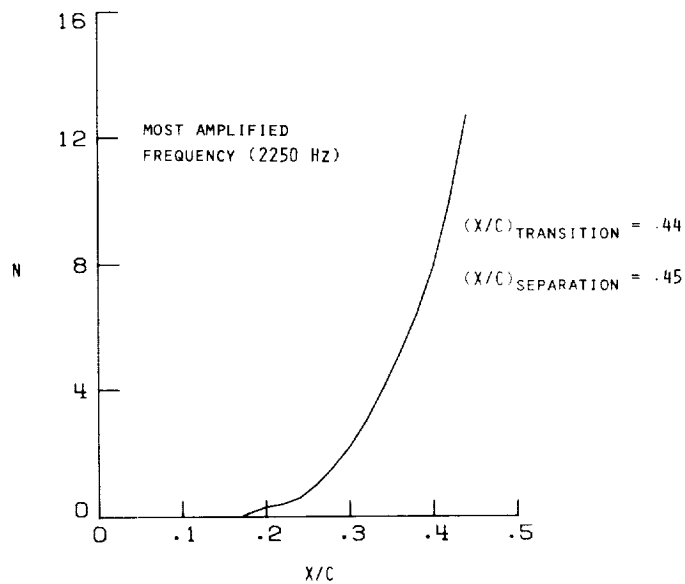


TRANSITION IN A BOUNDARY LAYER APPROACHING SEPARATION

When flow approaches separation, the validity of both the boundary-layer theory and the Orr-Sommerfeld equation becomes questionable. However, amplification rates become large due to the development of inflectional velocity profiles and any error committed by the e^N method in predicting transition becomes small in terms of surface distance. The figure presents results for the most amplified frequency in the boundary layer over the Beechcraft T-34C NLF glove. In the experiment (Ref. 19), transition took place at $X/C = .44$ and separation occurred at $X/C = .45$. At $X/C = .44$, the N factor is 12.8. N increases very rapidly beyond $X/C = .4$ due to inflectional streamwise velocity profiles (note that this is an unswept airfoil). If an N of 10 had been used to predict onset of transition in this experiment, a value of $(X/C)_{\text{transition}} = .42$ will result as compared to $.44$ observed in the experiment.

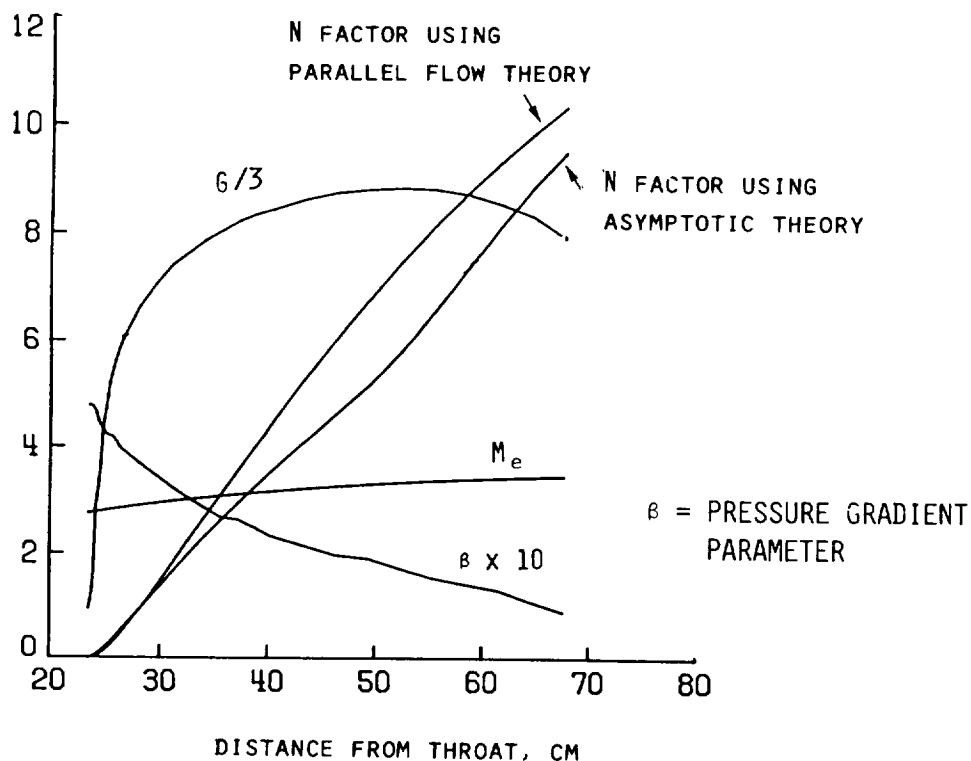
BEECHCRAFT T-34C NLF GLOVE

$M = 0.27$, $R = 12.6 \times 10^6$, $C_L = 0.35$



AMPLIFICATION OF GÖRTLER VORTICES ALONG A MACH 3.5 NOZZLE WALL

The question of Görtler or centrifugal instability has been a subject of controversy for many years. The attempts have been made to obtain a critical value of the Görtler number. It was pointed out by Hall (Ref. 20) that parabolic PDE's need to be solved for this problem in which case neutral curves are not unique since they depend upon initial conditions. If one thinks of transition taking place at an N of 9-11, then the parameters that are involved which give such a growth are not significantly affected by the region of controversy (low wavenumber region) and computations may be made using parallel flow theory. We tested this for various quiet tunnel test runs where transition on the nozzle walls takes place due to the amplification of Görtler vortices. The transition location could be correlated in those cases with an N factor of 9-11 (see Ref. 21). The success of parallel flow theory implies that perhaps asymptotic theory will also be successful. In this figure, N factor results are presented for Mach 3.5 nozzle wall using parallel (Ref. 21) and asymptotic (Ref. 22) theories. Both theories give results that are quite close for design purposes. The asymptotic theory requires an order of magnitude less computer time since eigenvalue computations are not involved.



TRANSITION IN 3-D BOUNDARY LAYERS

Stability computations are very sensitive to the details of the mean boundary-layer profiles which, therefore, have to be computed accurately. Anything that affects boundary-layer profile shape also indirectly affects their stability. So, inviscid solution has to be accurately prescribed and should be free of any unwanted wiggles. Boundary-layer computation is a rather trivial matter for two-dimensional flows but this is certainly not the case for three-dimensional configurations. To date, almost all swept-wing computations have been done using conical similarity using computer codes similar to the one due to Kaups and Cebeci (Ref. 23). While comparing the stability calculations with experimental data, one has to know if the conical similarity assumption, which requires straight isobars, is valid. If spanwise pressure gradient is present, the computed crossflow and thus, the crossflow instability will be in error by an unknown magnitude.

- MEAN FLOW
 - INVISCID
COMPUTED, EXPERIMENTAL

 - BOUNDARY LAYER
FULLY 3-D, CONICAL ASSUMPTION

- STABILITY EQUATIONS
 - EFFECT OF CURVATURE (BODY, STREAMLINE)

- UNSTEADY VERSUS STEADY DISTURBANCES NEAR THE LEADING EDGE

TRANSITION IN 3-D BOUNDARY LAYERS (CONTINUED)

In the stability of three-dimensional boundary layers, the question that immediately arises is how waves propagate in these boundary layers or the bottomline question 'how to compute N?'

One can start with spatial stability theory. There are five parameters: real parts of α , β , ω and imaginary parts of α , β . Two conditions are provided by the dispersion relation itself. Since we consider monochromatic waves, the real part of ω is also fixed. So, two more conditions need to be specified. Nayfeh (Ref. 24) and Cebeci and Stewartson (Ref. 25) independently derived a condition that the group velocity ratio ought to be real. This fixes direction of growth. It seems reasonable to follow disturbances that grow the most, so the second condition is that the growth rate should be a maximum. This fixes the wave angle. However, this angle may vary as the boundary layer develops. By providing these conditions, all the arbitrariness in the problem has been eliminated and the N factor calculation may proceed.

● WAVE-PROPAGATION (OR HOW TO COMPUTE N?)

- SPATIAL STABILITY

5 PARAMETERS: REAL (α, β, ω); $\text{Im}(\alpha, \beta)$

TWO CONDITIONS PROVIDED BY DISPERSION RELATION

FOR FIXED REAL (ω), TWO CONDITIONS NEEDED

NAYFEH (1979), CEBECI AND STEWARTSON (1979):

(1) GROUP VELOCITY RATIO ($\omega_\beta/\omega_\alpha$) IS REAL

(2) MAXIMIZE GROWTH RATE $\sigma = -\alpha_i - \beta_i(\omega_\beta/\omega_\alpha)$

TRANSITION IN 3-D BOUNDARY LAYERS (CONCLUDED)

Alternatively one may use temporal stability. Now there are four parameters: real (α, β, ω) and imaginary part of ω or ω_i . Again, two conditions are provided by the dispersion relation. For fixed real ω , one can maximize ω_i to follow waves that amplify the most. When this maximum is computed, it turns out (and it can also be shown mathematically) that group velocity ratio is automatically real. One also needs group velocity transformations to obtain spatial growth rates for computation of N factors. This scheme is commonly referred to as the envelope method and is built in computer codes SALLY (Ref. 26) and COSAL (Ref. 27). N factor results from this approach have been found to be quite close to the ones obtained using the spatial approach outlined in the previous figure.

A third approach is the one that is commonly used by Boeing and is called the N_{CF}/N_{T-S} approach. In this approach, different methods of integration are used for crossflow and T-S waves. The crossflow waves are assumed always to be stationary and are subjected to the condition that the curl of the wavenumber vector vanishes - a condition that is strictly only true for conservative wave systems. A boundary layer is not considered to be such a system. The direction of growth is the same as the external streamline direction. The T-S waves, on the other hand, always orient themselves at some fixed angle with respect to the external streamline. The direction of growth is again taken as the external streamline. This approach then results in two sets of N factors, N_{CF} for crossflow disturbances and N_{T-S} for T-S waves as described above. The N factors are then correlated with experimental transition data on swept-back wings.

- TEMPORAL STABILITY

4 PARAMETERS: REAL (α, β, ω); $\text{Im}(\omega)$

FOR FIXED REAL (ω):

(1) MAXIMIZE ω_i

→ GROUP VELOCITY RATIO AUTOMATICALLY REAL

(2) SPATIAL GROWTH RATE: $\sigma = \frac{\omega_i}{|\mathbf{v}_g|}$; $\mathbf{v}_g = \left(\frac{\partial \omega}{\partial \alpha} \mathbf{r}, \frac{\partial \omega}{\partial \beta} \mathbf{r} \right)$

- BOEING'S N_{CF}/N_{TS} APPROACH

N_{CF}

- (1) IRROTATIONALITY OF WAVE NUMBER VECTOR
- (2) GROWTH IN THE DIRECTION OF EXTERNAL STREAMLINE

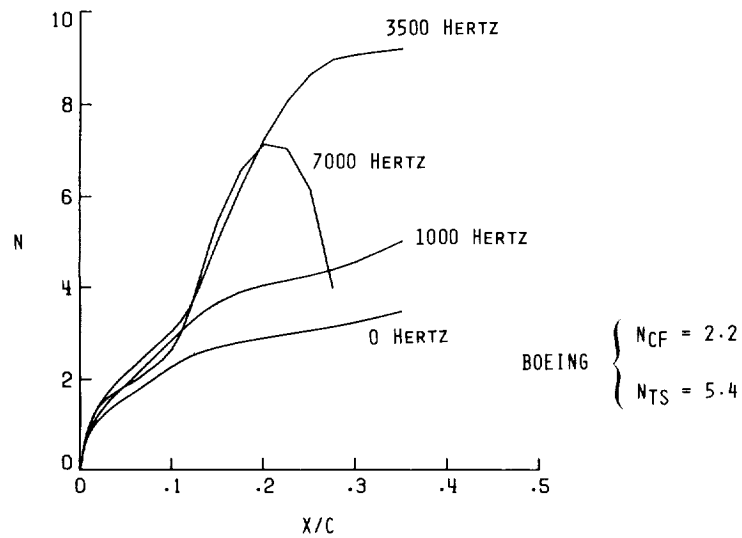
N_{TS}

- (1) FIXED WAVE-ANGLE
- (2) GROWTH IN THE DIRECTION OF EXTERNAL STREAMLINE

N FACTORS FOR F-111 TEST CASE NO. 19

If one uses the first or the second approach and computes N factors for a range of frequencies without a priori labelling the waves as crossflow or T-S, then most often it turns out that N for the most amplified wave is around 9-11. An example, using the envelope method, is provided in this figure for F-111 Test Case No. 19 where the computed N factor is about 9. The corresponding Boeing calculation yields $N_{CF} = 2.2$ and $N_{T-S} = 5.4$. However, in cases where transition is closer to the leading edge and the c_p distribution is such that large growth takes place very near the leading edge, then the envelope method will give very high N's if curvature terms are not included in the analysis. The reason is that the correct stability equations do contain curvature terms but it is for simplicity that they are ignored. However, very near the leading edge both the body and streamline curvature have a dominant role and they ought to be in the governing equations. To make a convincing case for the importance of streamline and body curvature, we present two cases in the next two figures.

$$M = 0.83, \quad \Lambda = 16.1^\circ, \quad C_L = 0.379, \quad RE_c = 23.3 \times 10^6$$

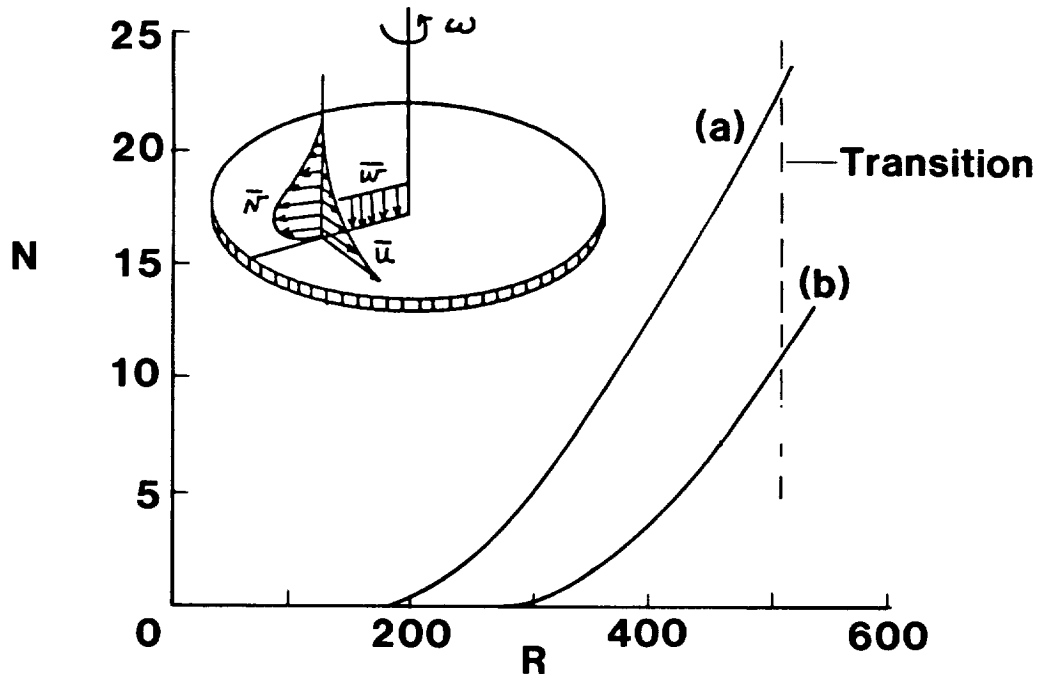


N FACTORS FOR ROTATING DISK FLOW

The first of these cases is the classical problem of a disk rotating in an otherwise quiescent ambient. The mean flow that develops on the disk has an exact solution to the Navier-Stokes equations and is also subjected to the crossflow instability and for that reason has long been used as a model problem for the swept leading-edge flow. Cebeci and Stewartson (Ref. 25) using the Orr-Sommerfeld equation as the stability model found that N comes out to be about 20. Their result suggested that perhaps the e^N method, which worked so well for two-dimensional flows, will not work for three-dimensional boundary layers. However, it was shown by Malik, Wilkinson, and Orszag (Ref. 28) that when the full sixth-order stability model is used, including the streamline curvature effects and Coriolis force (an effect present due to rotation), then N drops to about 11 which is in line with the 2-D values. The importance of the full sixth-order system was also demonstrated by the wavepacket computations of Mack (Ref. 29) for the Wilkinson-Malik disk experiment (Ref. 30). There, Mack noted that he could simulate all the fine details of the experiment only when he used the sixth-order system of Malik, Wilkinson, and Orszag.

(a) Orr-Sommerfeld equation

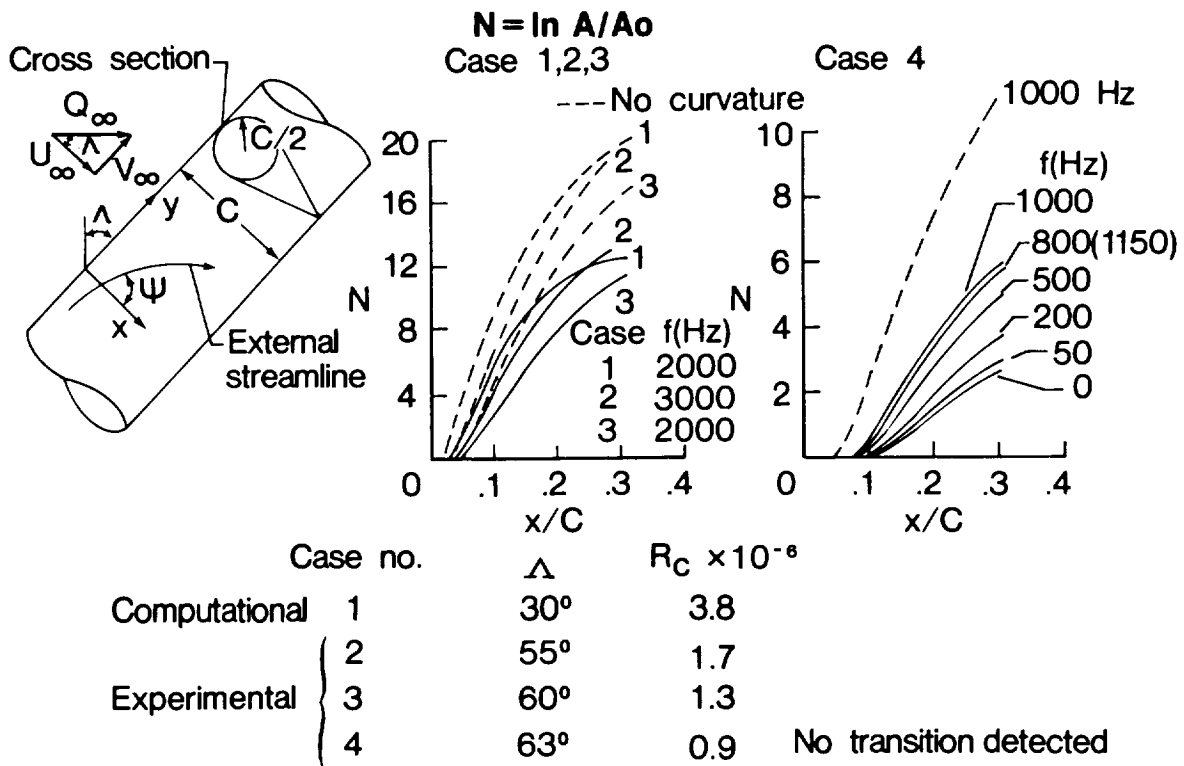
(b) Sixth-order equation



N FACTORS FOR SWEEP CYLINDER

Another case for the inclusion of curvature terms in the stability model may be made by considering the experiment of Poll (Ref. 31) on a swept cylinder which simulates the leading edge of a swept wing. For simplicity, let us concentrate on cases 3 and 4 in the figure. For case 3, when computations of the N factor are carried to the transition location without curvature terms, N factor is about 17. If the curvature terms (both body and streamline) are included, then N drops to around 11. The most amplified waves are not stationary, though the theory does predict the correct wave-length of the stationary disturbances measured from oil-flow photographs.

In case 4, flow was still laminar at the last measured station. Without curvature, an N above 10 is computed. With curvature, an N of 6 is computed indicating no transition. The most amplified wave in this computation was about 1000 Hertz. Poll, with a hot-wire, observed disturbances with a frequency of about 1050 Hertz. The unsteady disturbances have also been observed in the recent experiments of Bippes and Nitschke-Kowsky (Ref. 32.).



CALIBRATION OF e^N METHOD FOR TRANSITION PREDICTION/LFC DESIGN

The list of cases where e^N works is quite long. This includes the work of A.M.O. Smith and others. The conclusions from these applications are that when the mean flow is correct and the linear stability equations include dominant physical effects, N is of $O(9-11)$ for a low disturbance environment.

LOW-SPEED

- AXIS. (INCL. HEATING IN WATER, PRESSURE GRADIENT STABILIZATION)
- CONCAVE (GÖRTLER)
- ROTATING DISK
- 2-D WINGS (FLIGHT)
- 3-D (SWEPT WING, FLIGHT & W.T.)
- SWEPT L.E. REGION (CONVEX CURV. SURFACE AND IN-PLANE STREAMLINE CURV.)

HIGH-SPEED

- AXIS. (FLIGHT & W.T.)
- GÖRTLER
- SWEPT LEADING EDGE

CONCLUSIONS FROM THESE APPLICATIONS:

- WHEN LINEAR THEORY HAS CORRECT PHYSICS, THEN $N \sim O(9-11)$ FOR BACKGROUND DISTURBANCES OF $O(.05\%)$

POSSIBLE STREAM/WALL DISTURBANCES CRITICAL TO BOUNDARY-LAYER TRANSITION

However, the list of things that can affect transition is also very long. For that reason, the e^N method is not a general method for transition prediction. However, it is applicable to LFC studies since there a designer will strive hard to minimize all kinds of disturbances in order to obtain long runs of laminar flow.

- ROUGHNESS
 - DISCRETE
 - DISCONTINUOUS
 - TWO-DIMENSIONAL
 - THREE-DIMENSIONAL
 - STEPS
 - GAPS
 - PARTICLE IMPACT/EROSION
 - CORROSION
 - LEAKAGE
- ACOUSTIC ENVIRONMENT
 - ATTACHED FLOW
 - SEPARATED FLOW
 - PROPULSION SYSTEM
 - VORTEX SHEDDING
- PARTICLES
 - ICE CLOUDS
 - RAIN
 - ALGAE
 - SUSPENSIONS
 - FAUNA (INSECTS, FISH, ETC.)
- WALL WAVINESS
 - TWO-DIMENSIONAL
 - THREE-DIMENSIONAL
 - SINGLE WAVE
 - MULTIPLE WAVE
 - DISTORTION UNDER LOAD
- SURFACE AND DUCT VIBRATION
- STREAM FLUCTUATIONS AND VORTICITY
 - PROPELLER WAKES
 - OCEAN SURFACE
 - BODY WAKES (FISH/AIRCRAFT)
 - HIGH SHEAR AREAS (WEATHER FRONTS/JET STREAM EDGES/OCEAN CURRENTS)
- LFC SYSTEM-GENERATED DISTURBANCES
 - VORTEX SHEDDING (BLOCKED SLOTS, HOLES, PORES)
 - ACOUSTIC OR CHUGGING
 - PORE DISTURBANCES
 - NON-UNIFORMITIES

WAVE INTERACTION IN BOUNDARY LAYERS

The possibility of wave-interactions is a matter of great concern to an LFC designer. While there are many possible regions of interactions, only the cases where crossflow or Görtler is finite-amplitude and T-S is infinitesimally small will be discussed here. Reed (Ref. 33) developed a theory to compute such interactions on X-21 wing and showed that in the presence of finite-amplitude crossflow vortices, T-S waves are excited. The N factor for these T-S waves jumps from about 0.5 to 8.5 due to what is commonly known as 'double exponential growth' (Ref. 34). However, it was pointed out by Malik (Ref. 35) that the excited waves have unphysically long wavelengths at finite Reynolds numbers. Later, Reed* did not find the explosive growth of T-S waves (observed in Ref. 33) in other swept-wing boundary layers.

An earlier theory by Nayfeh (Ref. 36) on Görtler/T-S interaction had shown a similar type of 'double exponential growth' of T-S waves in the presence of finite-amplitude Görtler vortices. According to his theory, T-S waves with spanwise wavelength twice that of the Görtler wavelengths are excited. We have performed a computation to test the Görtler/T-S interaction of the type suggested by Nayfeh's theory. This Navier-Stokes simulation is limited in scope since it uses periodic boundary conditions in the streamwise direction; this implies a parallel boundary layer, which is a common practice for boundary-layer transition simulations on flat plates (Ref. 6 and Ref. 9). However, if the Görtler/T-S interaction is dominated by non-parallel effects, the computation will fail to capture it. Nayfeh (Ref. 36) mentioned that non-parallelism had little effect on the excited T-S wave.

*Reed, H., Arizona State University, private communication, 1986.

- CROSSFLOW/T-S INTERACTION

- REED'S (1984) THEORY OF DOUBLE EXPONENTIAL GROWTH
- CONCERN FOR HYBRID LAMINAR-FLOW CONTROL

- GÖRTLER/T-S INTERACTION

- NAYFEH'S (1981) THEORY OF DOUBLE EXPONENTIAL GROWTH
- CONCERN FOR LFC DESIGN OF CONCAVE SURFACES (QUIET TUNNEL)

NAVIER-STOKES SIMULATION OF GÖRTLER/T-S INTERACTION IN A BOUNDARY LAYER

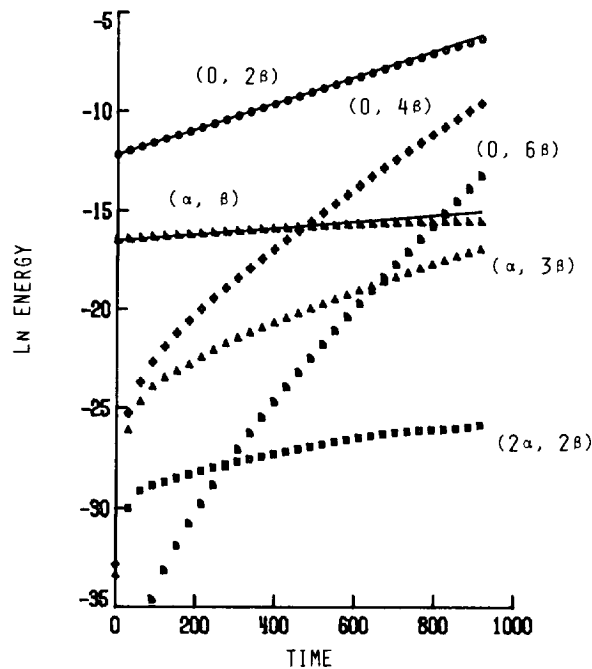
First, computation is made with the Görtler vortex having a 1% initial amplitude which is superposed on the Blasius flow. The Görtler vortex is noted as $(0, 2\beta)$ mode in the figure. Also included in the initial conditions are two oblique T-S waves $(\alpha, \pm\beta)$ with amplitude of .1%. The figure presents energy in various modes as a function of time. For simplicity let us concentrate on the primary Görtler $(0, 2\beta)$ mode and oblique T-S (α, β) mode. The T-S mode does not show any sign of strong instability. Towards the end of the computation, its growth rate actually drops slightly below the linear theory result. A notable feature in the figure is the strong growth of the first harmonic, i.e., $(0, 2\beta)$ mode. This is consistent with the experiment of Aihara and Koyama (Ref. 37).

An error in Nayfeh's paper (Ref. 36) was found by Malik (Ref. 35). When corrected, Nayfeh* finds that the growth rates of the excited T-S waves are small. However, he maintains that strong excitation may take place at some other values of parameters α , β , R and G .

*Nayfeh, A. H., Virginia Polytechnic Institute and State University, private communication, 1987.

$$R = 950, \quad G_\theta = 7.5, \quad \alpha = .103, \quad \beta = .15$$

INITIAL AMPLITUDE OF GÖRTLER $(0, 2\beta)$ MODE = 1%

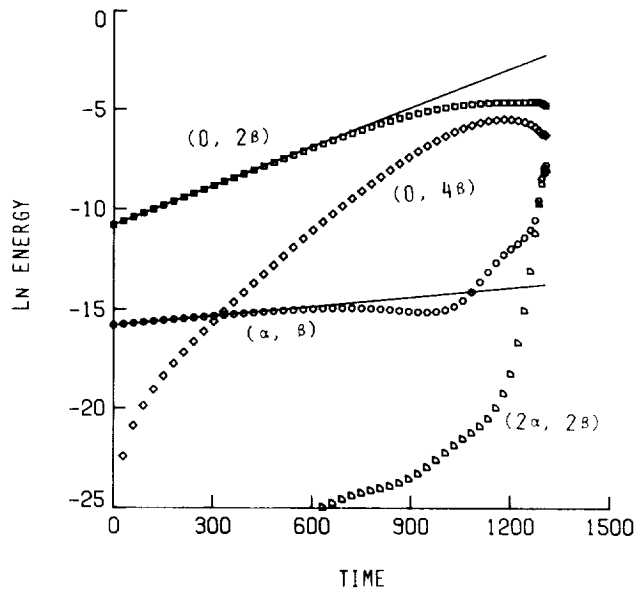


GÖRTLER/T-S INTERACTION WHEN GÖRTLER AMPLITUDE IS LARGE

Another calculation was made with a 2% initial amplitude for the Görtler vortex, and the solution was carried to longer times. The Görtler mode reaches an equilibrium state at which time the (α, β) mode grows fast but then other oblique modes (such as the $(2\alpha, 2\beta)$ mode) also show strong instability. It should be pointed out that at this stage the amplitude of the fundamental has reached in excess of 30%. At these amplitudes interactions are not a concern for the LFC designer. However, we have not yet searched for possible interactions when both Görtler and T-S have about the same finite amplitude.

$$R = 950, \quad G_0 = 7.5, \quad \alpha = .103, \quad \beta = .15,$$

INITIAL AMPLITUDE OF GÖRTLER $(0, 2\beta)$ MODE = 2%



CONCLUSIONS

1. When transition occurs in a low-disturbance environment, the e^N method provides a viable design tool for transition prediction and LFC in both 2-D and 3-D subsonic/supersonic flows. This is true for transition dominated by either T-S, crossflow, or Görtler instability.
2. If Görtler/T-S or crossflow/T-S interaction is present, then the e^N will fail to predict transition. However, there is no evidence of such interaction at low amplitudes of Görtler and crossflow vortices.

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