

GÖRTLER INSTABILITY ON AN AIRFOIL

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INTRODUCTION

Görtler vortices arise in boundary layers along concave surfaces due to centrifugal forces. These counter-rotating streamwise vortices are one of the three known flow instabilities which lead to boundary-layer transition. Advanced supercritical Laminar Flow Control wings have concave regions on the lower surface near the leading and trailing edges. Görtler vortices coupled with T-S waves and crossflow vortices may play an important role in triggering early transition.

In earlier studies the linear development of Görtler vortices was reduced to an eigenvalue problem assuming the flow to be parallel or quasi-parallel (refs. 1-4). The shapes of the perturbation velocity components were assumed invariant in the streamwise direction while their amplitudes were assumed to grow at a common rate. The major differences in the approach, details of the formulations, as well as the computational results are discussed extensively by Herbert (ref. 5). In each of these investigations, a unique neutral curve was obtained. The major limitation of this method is that it cannot be used to determine the development of Görtler vortices in the presence of variable curvature, suction and pressure gradients. In such a general case it is necessary to solve the governing partial differential equations as an initial value problem as developed by Hall (ref. 6).

BACKGROUND

Hall obtains multiple neutral curves that depend strongly on the initial condition and their location (Fig. 1). However his conclusions are misleading because his initial conditions are mathematically correct but physically meaningless as shown in figures 2a - 2c below. If a physically meaningful vortex perturbation is introduced as the initial condition, then these multiple curves will coalesce into one curve. It will be shown subsequently that the resulting growth rates agree well with results obtained from the solution of the eigenvalue problem for the case of constant curvature.

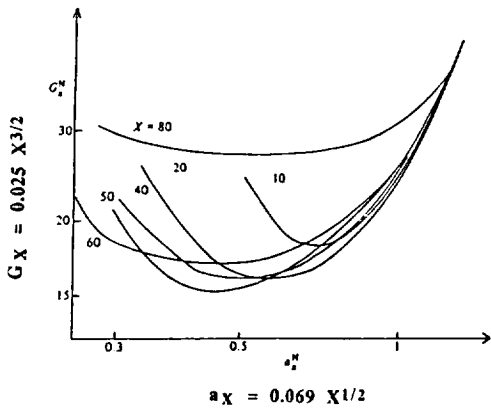


Figure 1

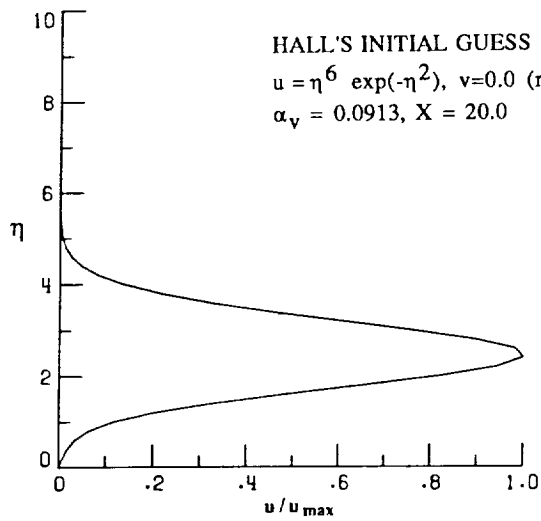


Figure 2a

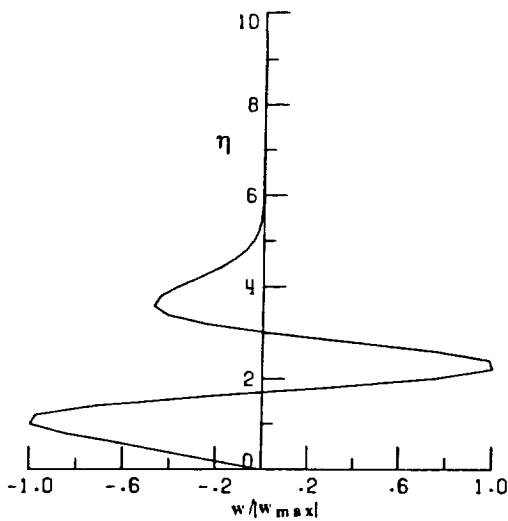


Figure 2b

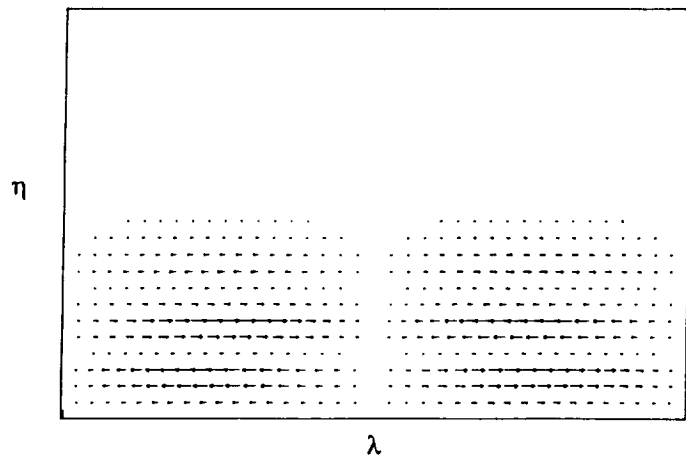


Figure 2c

GOVERNING EQUATIONS

The perturbation form and their linearized governing equations with appropriate boundary and initial conditions are given in figure 3. A second order accurate, implicit, iterative finite-difference scheme is used to solve the perturbation equations for the Blasius boundary layer. The governing equations are the same as those developed by Hall (ref. 6) but physically meaningful initial conditions have been used in the computations.

● **DISTURBANCE FORM**

$$\begin{aligned} U(X,Y,Z) &= U(X,Y)\cos(\alpha_V Z) \\ V(X,Y,Z) &= V(X,Y)\cos(\alpha_V Z) \\ W(X,Y,Z) &= W(X,Y)\sin(\alpha_V Z) \\ P(X,Y,Z) &= P(X,Y)\cos(\alpha_V Z) \end{aligned}$$

● **GOVERNING EQUATIONS**

$$G_V = 2I/R \sqrt{U_\infty I/\nu}$$

$$\begin{aligned} U_X + V_Y + \alpha_V W &= 0 \\ uU_X + u_X U + vU_Y + Vu_Y - U_{YY} + \alpha_V^2 U &= 0 \\ uV_X + v_X U + vV_Y + Vv_Y + P_Y + G_V Uu - V_{YY} + \alpha_V^2 V &= 0 \\ uW_X + vW_Y - \alpha_V P - W_{YY} + \alpha_V^2 W &= 0 \end{aligned}$$

● **BOUNDARY CONDITIONS**

$$\begin{aligned} U(X,0) &= V(X,0) = W(X,0) = 0 \\ U(X,\infty) &= V(X,\infty) = W(X,\infty) = 0 \end{aligned}$$

● **INITIAL CONDITION**

$$\begin{aligned} U &= U(Y) \\ V &= V(Y) \\ W &= W(Y) \end{aligned} \quad \text{AT } (X = \bar{X})$$

Figure 3

NORMALIZED PERTURBATION VELOCITIES

Figure 4 below shows the normalized u - and v -perturbation velocities obtained from the present method and by solving the eigenvalue problem (ref. 3). Computed results based on Hall's initial guess have also been included to show the effect of physically incorrect input on the solution. The u -, v -, and w -perturbation velocities are assumed to grow at a common rate in the eigenvalue problem. If this approximation is true for the physical problem, then the v -perturbation velocity has to grow very rapidly to match the correct shape and amplitude if it is assumed to be zero initially as in reference 6. This may explain the behavior of the v -perturbation velocity in the following figures when it is assumed zero initially.

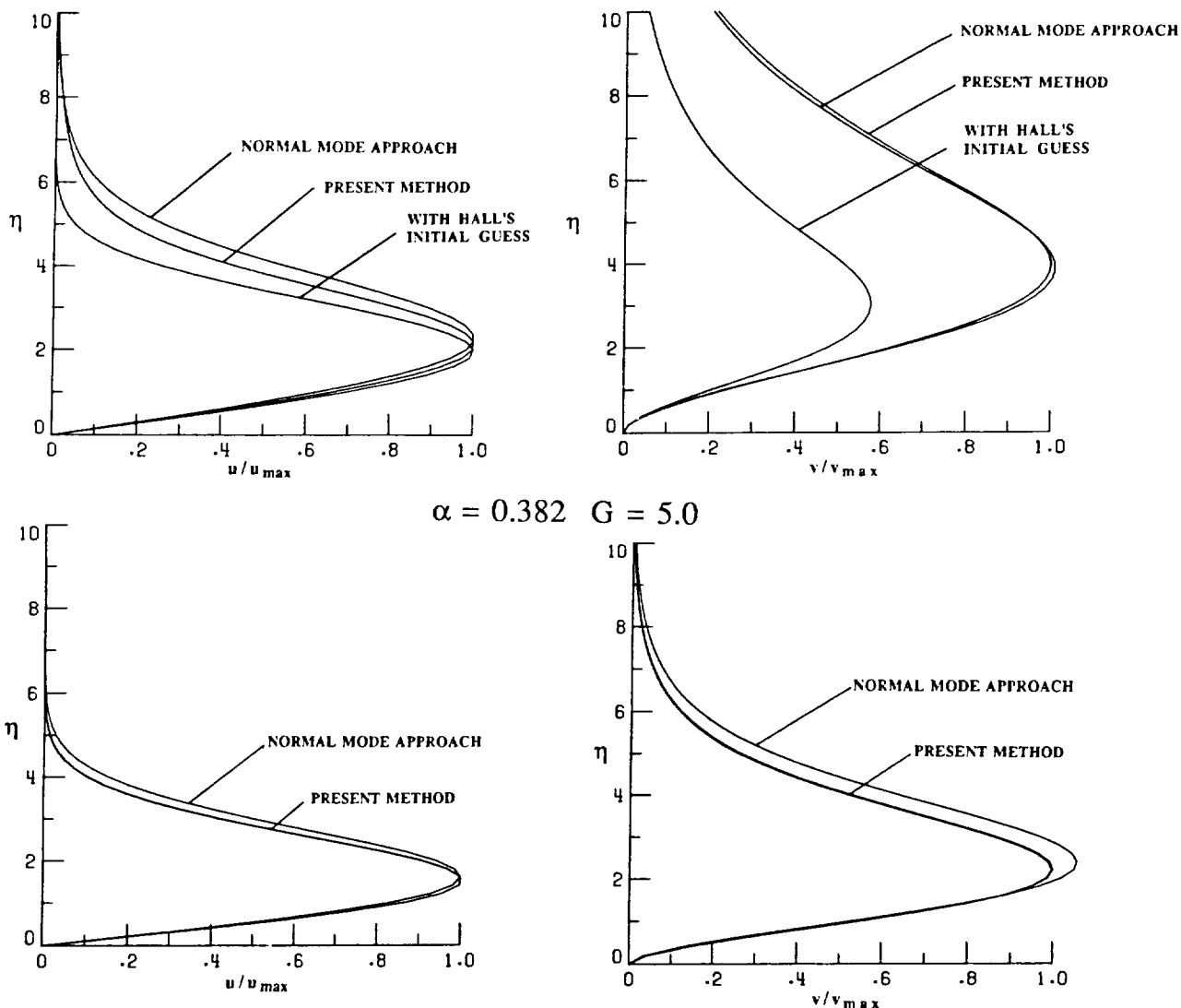


Figure 4

VARIATION OF AMPLIFICATION RATE WITH GÖRTLER NUMBER

This conclusion is further reinforced by figure 5 showing the variation of the amplification rates with Görtler number. A number of computational experiments showed that whenever the growth rates β_u and β_v matched (as assumed in the normal mode approach) the computed results from the initial value problem merged with results obtained from the normal mode approach, indicating that the assumptions made in the normal mode approach are reasonable for this problem (also, see ref. 2).

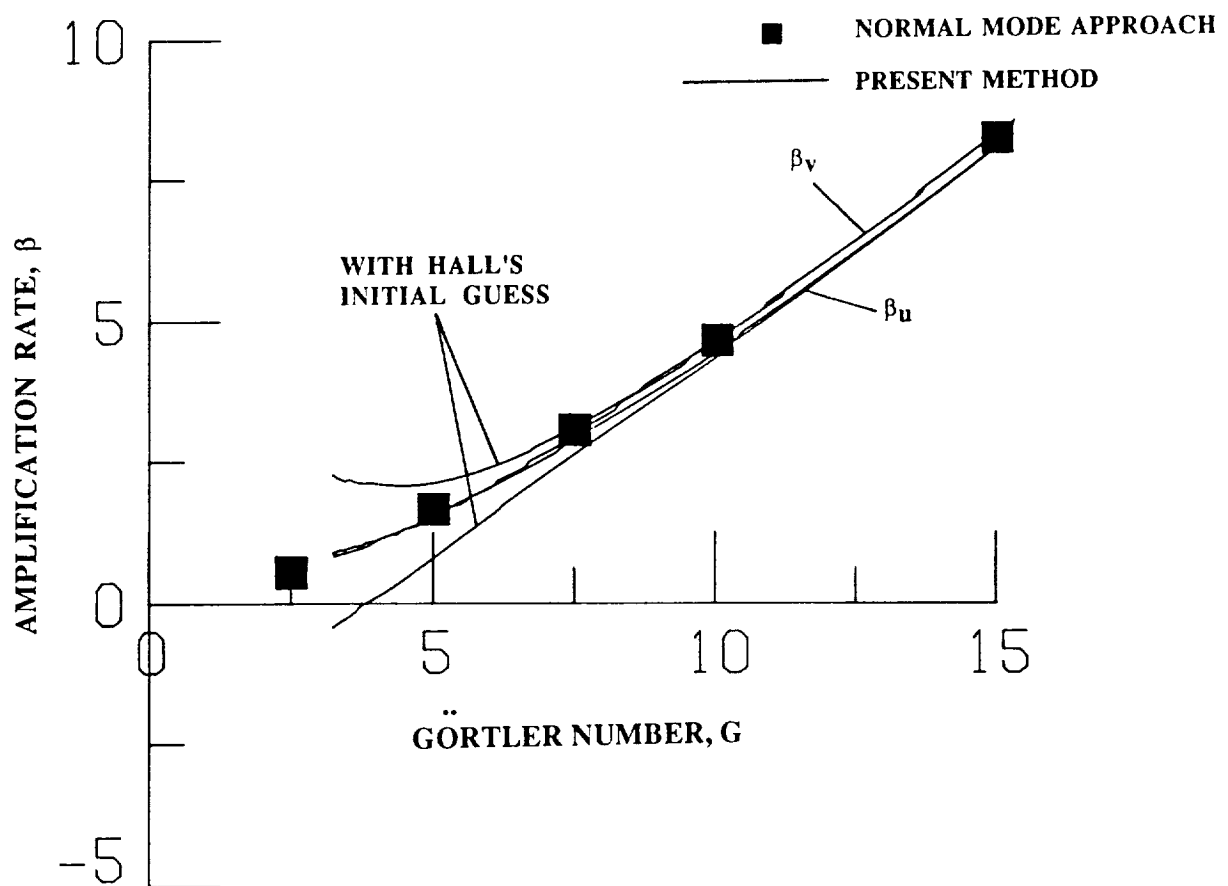


Figure 5

EFFECT OF VARIABLE CURVATURE DISTRIBUTION ON GÖRTLER VORTICES

We now look at the growth/damping of Görtler vortices in the presence of a variable curvature distribution (Fig. 6). A Blasius boundary layer is assumed for the mean flow. The normal mode approach is not applicable to this problem. Computations were carried out for a number of curvature distributions, but only one case is considered here. Typical normalized perturbation functions and the perturbation velocity field along the span over one wavelength are shown in the following pages (Figs. 7a - 7f) for different streamwise locations. Note that a negative value of the Görtler number G_v denotes convex curvature. The Görtler vortices appear to lift off at the beginning of the convex region and a secondary, weaker vortex pair begins to emerge near the surface. The original vortex changes sign in this region and we observe counter-rotating vortices in the spanwise as well as normal direction. Further studies on more realistic problems are in progress.

$$\alpha_v = 0.0913$$

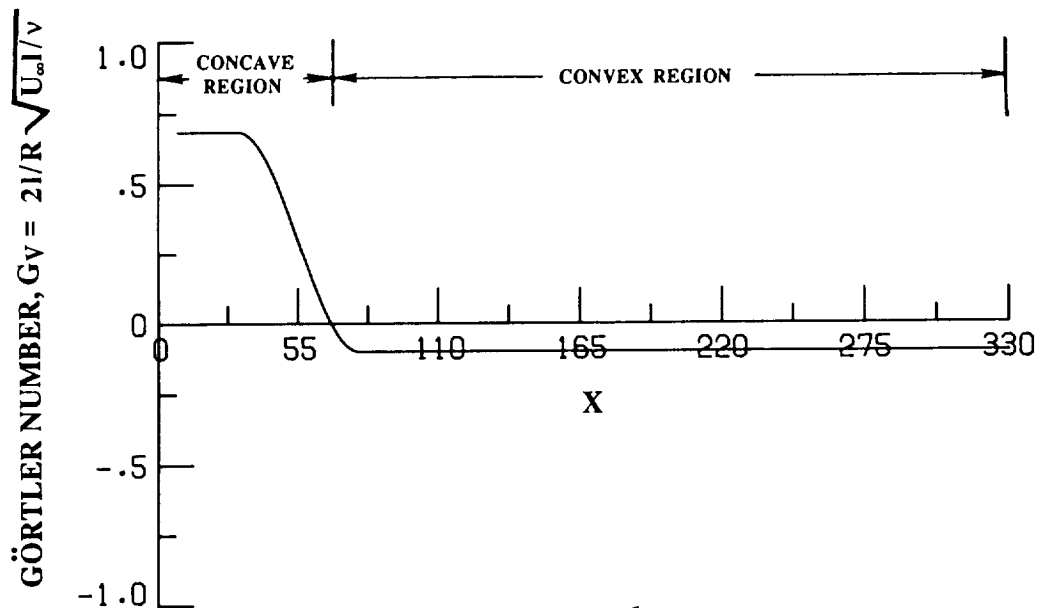


Figure 6

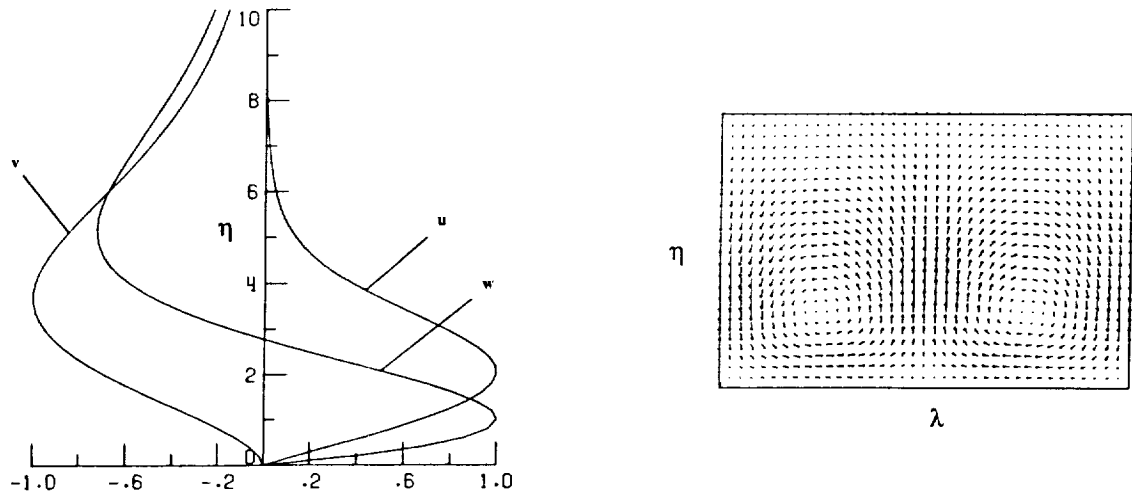


Fig. 7a $G_v = 0.685$, $X = 20.2$, $\alpha_v = 0.0913$

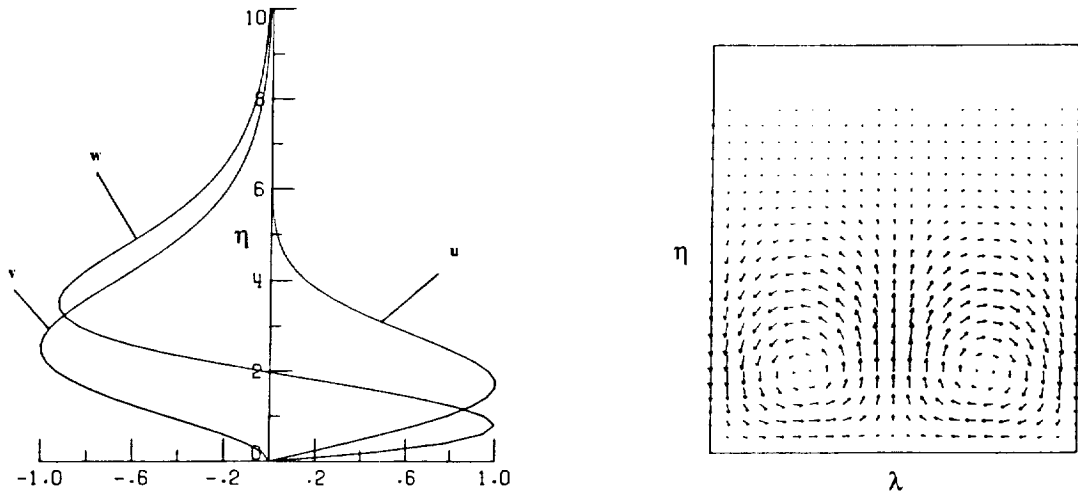


Fig. 7b $G_v = 0.165$, $X = 60.2$, $\alpha_v = 0.0913$

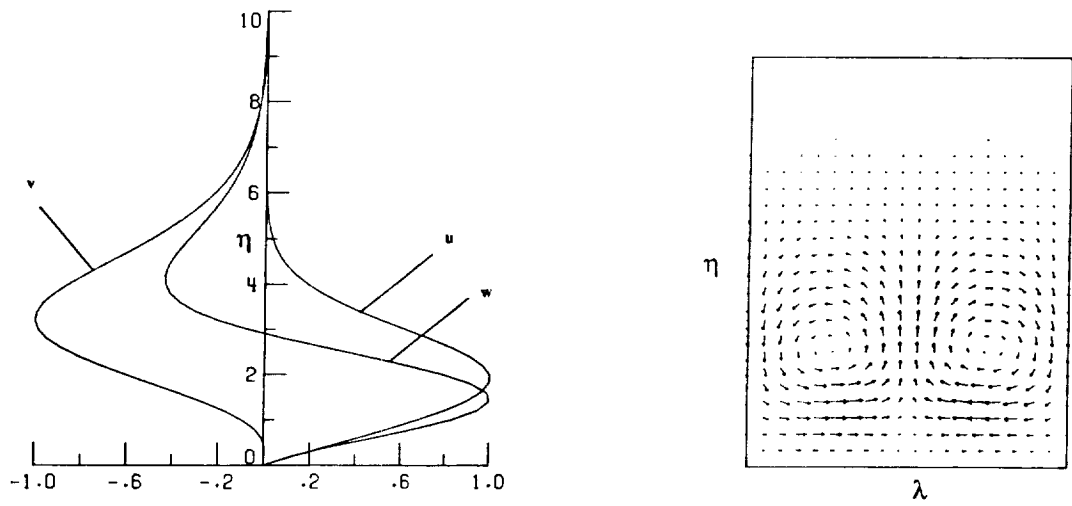


Fig. 7c $G_v = -0.10$, $X = 80.2$, $\alpha_v = 0.0913$

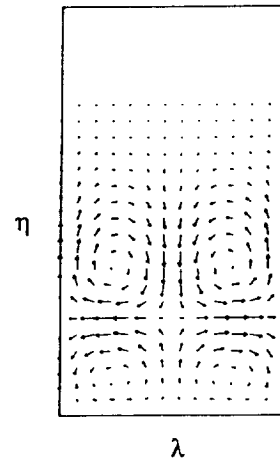
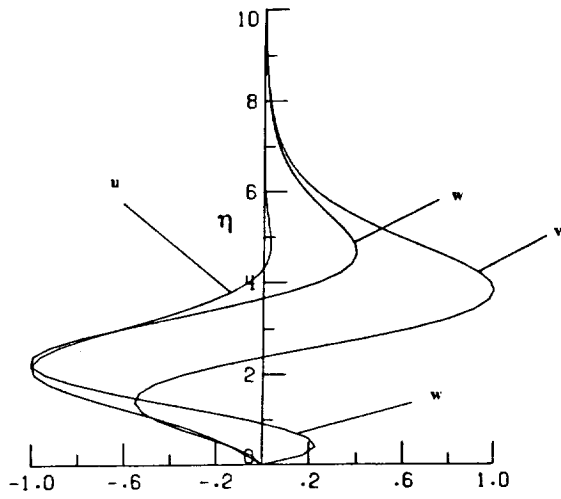


Fig. 7d $G_v = -0.10$, $X = 160.2$, $\alpha_v = 0.0913$

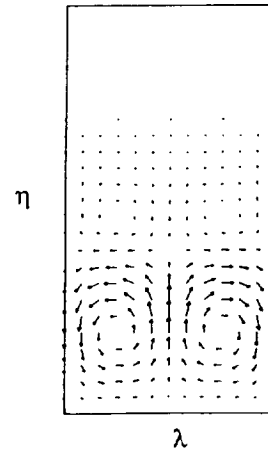
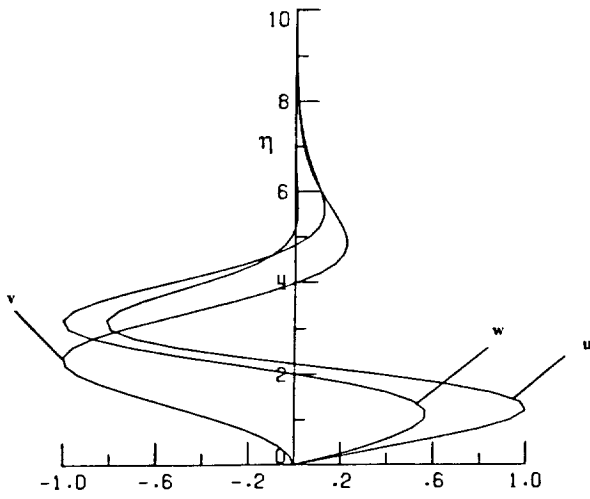


Fig. 7e $G_v = -0.10$, $X = 200.2$, $\alpha_v = 0.0913$

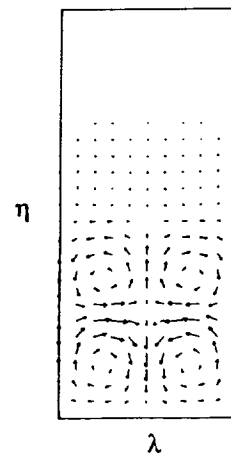
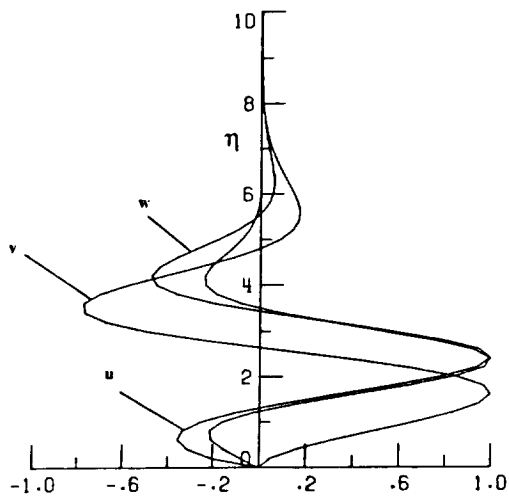


Fig. 7f $G_v = -0.10$, $X = 260.2$, $\alpha_v = 0.0913$

VARIATION OF ENERGY ALONG THE STREAMWISE DIRECTION

The variation in kinetic energy along the streamwise direction is shown in figure 8. As expected, the energy reaches a maximum at the end of the concave region followed by a rapid damping in the convex zone.

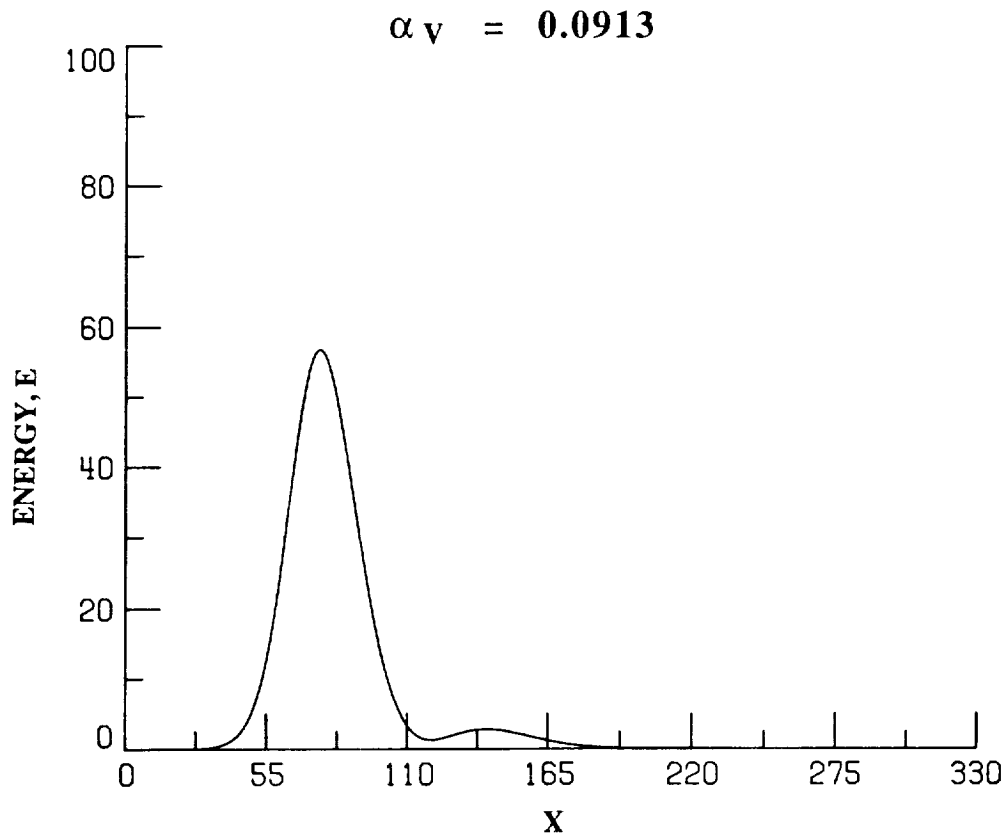


Figure 8

CONCLUSIONS

An effective computational scheme has been developed to study the growth/damping of Görtler vortices along walls of variable curvature.

Computational experiments indicate that when the amplification rates for the u-, v-, and w-perturbations are the same, the finite-difference approach to solve the initial value problem and the normal mode approach give identical results for the Blasius boundary layer on constant curvature concave walls.

The growth of Görtler vortices was rapid in the concave region and was followed by sharp damping in the convex region. However, multiple sets of counter-rotating vortices were formed and remained far downstream in the convex region.

The current computational scheme can be easily extended to more realistic problems including variable pressure gradients and suction effects.

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