verified by model tests of both advanced and baseline designs for the UH-1, AH-64, and UH-60 helicopters (refs. 19-22). A more sophisticated hover analysis which includes wake effects may be used in the future if the trends predicted by such an analysis are verified for a wide range of configurations, i.e., different taper ratios, taper initiation points, twist distributions, etc.

ROTOR BLADE DYNAMIC DESIGN

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Design Considerations

The rotor dynamic design considerations are essentially limitations on the vibratory response of the blades which in turn limit the dynamic excitation of the fuselage by forces and moments transmitted to the hub. The following quantities associated with the blade response are subject to design constraints: blade frequencies, vertical and inplane hub shear, rolling and pitching moments, and aeroelastic stability margin.

Frequencies.- The blade natural frequencies are required to be separated from multiples of the rotor speed. A typical constraint is written as

$$\omega_{Li} \le \omega_i \le \omega_{ui} \tag{10}$$

where ω_i is a blade frequency, and ω_{Li}, ω_{ui} are lower and upper bounds of the ith frequency. Generally, ω_{Li} and ω_{ui} are $n\Omega \pm \delta$ where n is an integer, Ω is the rotor speed, and δ is a tolerance usually about 10 percent of $n\Omega$ (e.g., ref. 6).

Vertical hub shear.- The transmitted vertical hub shear S is to be made as small as possible. This requirement may be handled either as part of the objective function wherein it is minimized (ref. 6), or as a constraint where the vertical hub shear is required to be less than some specified value (ref. 23). In the first approach, letting N denote the number of blades in the rotor

$$|S_k| \rightarrow \min \qquad k = N,\ 2N,\ \ldots \tag{11}$$

In the second approach

$$|S_k| \leq \epsilon \qquad k = N,\ 2N,\ \ldots \tag{12}$$

where ϵ is a positive value.

Only blade shear responses at multiples of $N\Omega$ contribute to the transmitted vertical hub shear. The vertical blade shear at all other frequencies cancel out in the summation process. In other words

$$S_k\Big|_{tot} = \begin{cases} NS_k & k = N,\ 2N,\ \ldots \\ 0 & \text{All other } k \end{cases} \tag{13}$$

At the same time, for a finite hinge offset, the blade vertical shear at other frequencies contributes to the transmitted hub moments.

Hub moments.- Two types of moments are generated at the hub due to blade motion. The first is due to distributed blade bending moments and the second is due to couples involving the blade shear forces at the hinge offset of the blade. Each type of moment has both a rolling and pitching component at the hub.

Inplane hub shear.- In the approaches described herein, the inplane hub shear is handled in the same way as the vertical hub shear. Specifically, in the first approach,

$$|H_k| \rightarrow \min \qquad k = N,\ 2N, \ldots\ . \tag{14}$$

in the second approach

$$|H_k| \leq \epsilon \qquad k = N,\ 2N, \ldots \tag{15}$$

For an N-bladed rotor, the total transmitted shear at the hub is non-zero only at frequencies which are multiples of $N\Omega$. However, in this case, the transmitted hub shear is made up of contributions from the blade responses at the following multiples of the rotor speed: $N \pm 1,\ 2N \pm 1,\ \ldots$ For example, in a four-bladed rotor,

$$\left.\begin{aligned} X_4 &= (2F_5 - 2F_3) \cos 4\Omega t \\ Y_4 &= (2F_5 - 2F_3) \sin 4\Omega t \end{aligned}\right\} \tag{16}$$

where X_4 and Y_4 are orthogonal components of in-plane forces. F_3 and F_5 are amplitudes of tangential forces at the blade root at frequencies 3Ω and 5Ω, respectively. Thus X_4 and Y_4 play the roles of H_k in equations (14) and (15).

Rotor aeroelastic and aeromechanical stability.- The constraint for positive system aeromechanical stability relies on knowledge of fixed system characteristics and rotor frequency placement. Specifically, the rotor's lower modes, especially lead-lag, should not have fixed-system values which coalesce with the fuselage roll or pitch degrees of freedom, either on the ground or in flight.

Additionally, aeroelastic stability constraints for the isolated rotor in hover as developed by Friedmann (ref. 6) require that

$$\eta_k \le \eta_{Lk} < 0 \tag{17}$$

where η_k is the real part of the kth complex eigenvalue and η_{Lk} is its limiting value.

Analysis Considerations

For the purpose of dynamic response analyses, the rotor blade is modeled as a beam undergoing coupled flap-lag-torsion motion in response to harmonically varying airloads. The beam is assumed to rotate at constant rotor speed which gives rise to centrifugal loading and stiffness effects. It is anticipated that either a finite-element analysis (e.g., ref. 24) or CAMRAD (ref. 14) will be used for the dynamic calculations. These calculations include mode shapes and (complex) eigenvalues, steady-state response (displacements), blade loads, and transmitted hub loads and moments.

The governing matrix equation for vibration response of a finite-element modeled structure is

$$M\ddot{X} + C\dot{X} + KX = F \tag{18}$$

where M is the mass matrix

C is the damping matrix

K is the stiffness matrix

X is the vector of displacements and rotations

F is the applied force vector

The stiffness matrix K for a rotor blade has the form

$$K = K_E + K_C + K_D$$

where K_E is the linear elastic stiffness matrix

K_C is a centrifugal stiffness matrix

K_D is the differential stiffness matrix and contains stresses associated with centrifugal forces

Detailed discussions and explicit forms for K_C and K_D are available in reference 24.

Equation (18) may be solved by modal superposition. The modal analysis produces the natural frequencies and damping needed in the phase 1 constraints (eqs. 10, 12, 15). Additional analyses are used to calculate the blade loads and transmitted hub loads based on modal expansions of the blade response and are outlined in reference 23.

Derivatives of the dynamic response quantities which appear in the constraints are needed. Expressions for most of these derivatives are given in reference 23. For example, analytical derivatives of the frequencies are given by

$$\frac{\partial \omega^2}{\partial v} = \phi^T\left(\frac{\partial K}{\partial v} - \omega^2 \frac{\partial M}{\partial v}\right)\phi \tag{19}$$

The alternative to finite-element analysis is the modified Galerkin approach in CAMRAD. The advantage of the latter approach is that it resides in the same code that will be used for the aerodynamic analysis. The disadvantage is that the method does not ordinarily generate the matrices M, C, and K which are needed for the analytical derivatives (e.g., eq. (19)). Thus, the modified Galerkin approach may require the use of finite difference derivatives. This was done in reference 7 without any ill effects. Nevertheless, studies are underway to find ways to generate equivalent M, C, and K matrices based on the modified Galerkin method and use these in the calculations of analytical derivatives.

ROTOR BLADE STRUCTURAL DESIGN

Mark W. Nixon

In this section the structural design of rotor blades is discussed. The various topics associated with the structural design include constraints, load cases, and analyses.

Design Constraints

The constraints associated with traditional structural design can be categorized as aerodynamic, autorotation, buckling, frequency, and material strength. As discussed in reference 10, some of these constraints are based on maintaining characteristics required by other disciplines involved in the integrated optimization. Constraints associated with aerodynamics, autorotation and frequency are not addressed in this section, since they are addressed in other sections of the paper.

Of the remaining structural constraints, the most important is the material strength constraint. All stresses in the blade structure must be less than the design allowable stress of the material for all load cases. To account for stress interactions, a failure criterion such as Tsai-Hill (ref. 25) is calculated based on the material limit allowable stresses. The governing equation is