NASA Contractor Report 185123
M/NA/TURB-1

Application of Large Eddy Interaction Model to a Mixing Layer

S.N.B. Murthy
Purdue University
West Lafayette, Indiana

November 1989

Prepared for
Lewis Research Center
Under Grant NAG3-592

National Aeronautics and
Space Administration

TABLE OF CONTENTS

1. Introduction

Research efforts on the development of the so-called large eddy interaction model (LEIM) have been devoted to the following:

(a) setting up of dynamical equations for velocity and scalar quantities and correlations;

(b) examining methods of solution of the dynamical equations with minimal modeling at the spectral level; and

(c) examining data sets on mixing layers that are suitable for utilizing the LEIM as a metric for determining transport and production of turbulence quantities.

The two-dimensional mixing layers under consideration fall under two categories: (i) the mixing fluids differ in velocity and density, the latter due to composition; and (ii) the mixing fluids differ in velocity and temperature. It is assumed that the fluids are incompressible and non-reactive. Thus, the dynamical equations for velocity and scalar fluctuations and correlations are common for density and temperature.

The principal modeling required in the use of dynamical equations is for eddy-eddy interactions. Alternative schemes of modeling are then of interest. Regarding the need for establishing a relation between velocity and temperature fluctuations, similarity may be invoked. In view of the nature of the LEIM, such similarity is most effective if invoked at the spectral level between velocity and scalar quantities.

An attempt has been made at setting up a computational program for a developing mixing layer. There is considerable sensitivity to initial conditions in the prediction of turbulence quantities, although there is some success in predicting mean flow development.

In respect of data sets, the two main problems are (i) the lack of adequate definition of initial conditions and (ii) the lack of data on mean flow development.

These preclude adequate comparison between predictions and experimental data.

1.1. Background

The large eddy interaction hypothesis (LEIH) is based on the work of J. Lumley (Refs. 1 and 2) on the rational representation of turbulence and its development. The LEIH rests on two postulates: (i) selected eddies are adequate for representation of a number of features of turbulence; and (ii) turbulence development in a flow is the result of the action of mean strain in the presence of eddy-eddy interactions that constitute turbulent transport processes and of viscous transport.

The application of a model based on LEIH to curved wall boundary layers is discussed in Refs. 3-6. Other application to channel flows is discussed in Refs. 7-9. Recent developments on J. Lumley's work are contained in Refs. 10-14.

A general critique on experimental work on mixing layers can be found in Ref. 15.

1.2. Outline of Report

Chapter 2 of the Report deals with the rational representation of turbulence and its application to mixing layers. The method of obtaining predictions based on the LEIH model is presented in Chapter 3. An approach to the problem of predicting mixing layer development utilizing the LEIH is outlined in Chapter 4.

2. THE LEIH AND MODEL DEVELOPMENT

The LEIH is an outcome of J. Lumley's proposal for rational representation of turbulence (Refs. 1 and 2), which has also been adapted for use by A.A. Townsend (Ref. 16) with his own formalism. A.A. Townsend had earlier (Ref. 17) proposed that turbulent shear flows have a "double structure" with a noticeable, well organized structure called large eddies and a less organized, smaller scale turbulence. The large eddies are said to contain only a small part of the total kinetic energy, but determine the turbulent-non-turbulent interface and hence the intermittency. There is expected to be a large difference between the scale of large eddies and the scales of other eddies. The effect of the smaller eddies may be visualized as the presence of eddy viscosity. A.A. Townsend was thus able to establish a scheme for determining the structure of large eddies.

An objective definition for such large eddies has been provided by J. Lumley. The importance of such a definition is two-fold: (i) it leads to a model for inhomogeneous flows and (ii) it also provides a rational basis for the recovery of the characteristic, generally orderly structure of large eddies in various flows based on detailed space-time based measurements. In other terminology, it represents a means of examining time-wise unsteady, space-wise steady development of the structure of a given turbulent flow.

2.1. Notation

In order to make the notation compact and easily recognized, the following is utilized.

Throughout Cartesian summation notation is employed, where repeated indices imply that the terms containing them must be summed over all possible coordinate indices. An overdot $\dot{}$ indicates partial derivative with respect to time and a dot inside the parenthesis $(\cdot)$ indicates that the quantity is a function of whatever are the relevant coordinates.

The isotropic tensors are defined by

$\delta_{ij} = 1$ and 0, if $i = j$ and otherwise

and

$\epsilon_{ijk} = 1,\ -1$ and 0, when ijk form the sequence 123123, 321321 and otherwise.

Mean velocity, pressure and temperature are denoted by U_i, P and T, while turbulence velocity, pressure and temperature are denoted by u_i, p and θ.

All other symbols and operations are explained locally.

2.2. Application of Proper Orthogonal Decomposition Theorem

The turbulence problem is posed as one of identifying a structure in a random vector field. The proper orthogonal decomposition theorem of M. Loeve (Ref. 18) is invoked for application.

The vector field u_i is expanded in terms of a candidate structural quantity, ϕ_i^n , n = 1,2,...etc., and coefficient, α_n, as follows :

$$u_i(\bullet) = \Sigma\, \alpha_n\, \phi_i^n(\bullet) \tag{2.1}$$

where, denoting the complex conjugate by $(\ )^*$,

$$\alpha_n = \int u_i(\bullet)\, \phi_i^{n*}(\bullet)\, d(\bullet) \tag{2.2}$$

and the random coefficients are uncorrelated:

$$\overline{\alpha_n \alpha_m^*} = \begin{cases} 0, & n \neq m \\ \lambda^{(n)}, & n = m \end{cases} \tag{2.3}$$

It is claimed as a hypothesis that ϕ_i is occurring in a recognizable form in a given ensemble of random vector fields, u_i. One considers the quantity, namely

$$\alpha = \frac{\int \phi_i^*(\bullet)\, u_i(\bullet)\, d(\bullet)}{\left[\int \phi_j(\bullet)\, \phi_j^*(\bullet)\, d(\bullet)\right]^{1/2}} \tag{2.4}$$

and its statistics. The best ϕ_i is then taken to be that which gives the largest magnitude of α in some average sense, the sign being irrelevant. In other words, ϕ_i is selected such that $|\overline{\alpha}|^2$ is maximized. According to the calculus of variations, the expansion of (2.1) is then optimal since, on account of $|\bar{\alpha}|^2$ being maximized, each term leaves as little as possible of $u_i(\cdot)$ in the succeeding terms. Noting that the large eddies are physically well "separated" from the rest, the first term of the expansion can be claimed to be the representation of large eddies.

Defining R_{ij} as

$$R_{ij}(\bullet, \bullet') = \overline{u_i(\bullet)\ u_j(\bullet')} \tag{2.5}$$

one can write

$$R_{ij}(\bullet, \bullet') = \Sigma\ \lambda^{(n)} \phi_i^{(n)}(\bullet)\ \phi_j^{(n)}(\bullet') \tag{2.6}$$

It may be noted that in (2.3) $\lambda^{(n)}$ are non-negative and their sum is finite. Furthermore, $\phi_i^{(n)}$ are ortho-normal, i.e.,

$$\int \phi_i^{(p)}(\bullet)\ \phi_i^{(q)*}(\bullet)\ d(\bullet) = \delta_{pq} \tag{2.7}$$

This is to say that the structures of various orders have nothing in common with one another.

Now, one considers the possibility of u_i or R_{ij} being homogeneous in certain coordinate directions and inhomogeneous in the remaining. For example, the vector field may be homogeneous in x_3 and inhomogeneous in x_1 and x_2, as in a two-dimensional boundary layer or a two dimensional mixing layer. Then one can invoke the Harmonic Orthogonal Decomposition Theorem (M. Loeve, Ref. 18) and note that expansion in harmonic functions is possible in any homogeneous directions, while the proper orthogonal decomposition applies in the remaining.

It is clear that there is a difficulty in a totally homogeneous flowfield. Considering a wave number-frequency space, the more broadband in wave number and frequency is the structure, the less representative will the definition be. On the other hand, if the flowfield is totally inhomogeneous, one has to appeal to $\phi_i^{(1)}$. In the case that there is at least one direction in which the field is homogeneous, the flow can be represented in terms of a denumerable set of motions. The large eddy then is that organized motion of largest scale in the inhomogeneous directions and possessing the greatest energy. A strong peak is expected at a wave number-frequency combination away from the origin and that corresponds to the large eddy.

2.3. Some Comments

2.3.1. The eddies in the turbulent flowfield are represented in terms of flow quantities treated as random vector fields and hydrodynamic/thermodynamic quantities, such as pressure, temperature and concentration, as random fields. Various correlations between velocities and between velocities and scalars can then be constructed.

Each field is represented in terms of a structure and associated random coefficients representing amplitudes. Upon decomposition, a flowfield with some inhomogeneities can be represented in terms of a denumerable set of eddies, each of which is rational being optimal in the expansion. Each contains the maximum energy it can and each is of the largest scale possible. There is no similarity among them. The largest of them is the large eddy.

2.3.2. As stated earlier, the Orthogonal Decomposition Theorem reduces to the Harmonic Decomposition Theorem when the flowfield is homogeneous, and also, when it is stationary or periodic. This implies that in any direction in which the flowfield is homogeneous, stationary or periodic, the flowfield in that direction can be expressed by Fourier analysis. In the remaining directions, one has to use the Proper Orthogonal Decomposition Theorem.

J. Lumley points out (Ref. 2) that eigen functions are not confined to one region of space and time and therefore they cannot correspond to physical eddies as they are usually visualized.

However, he also points out (Refs. 1 and 2) that, following a suggestion of A. Townsend, one can utilize an extension of the shot-noise-effect expansion of S.O. Rice (Ref. 19) and obtain an unambiguous representation of eddies. Now, any homogeneous function may be written in the form, namely

$$u_i(x) = \int f_i(x-x')\ g(x')\ dx' \tag{2.8}$$

where f_i is a deterministic function and g is a stochastic function that is white, i.e. uncorrelated in non-overlapping intervals. Thus, the flow is decomposed into characteristic eddies, g_i, occurring at uncorrelated intervals. One can then use the extended Campbell Theorem and write

$$R_{ij}(\xi) = \int f_i(x)\ f_j(x+\xi)\ dx \tag{2.9}$$

Taking the Fourier transform, one can express the spectrum as

$$\Phi_{ij}(k) = \hat{f_i}\ \hat{f_j} \tag{2.10}$$

where (^) indicates the transform.

2.3.3. L. Sirovich (Ref. 20) points out that another method, namely the method of snapshots or strobes may be utilized in place of the shot-noise method. We shall return to the alternative method later.

3. DYNAMICS OF LARGE EDDIES

In any turbulent shear flow it is hypothesized that the large eddies interact with themselves and with all the other eddies in the presence of mean strain in the fluid flow. A dynamical equation can be constructed for any parameter associated with the large eddies, wherein a balance is invoked among production, advection, diffusion and dissipation processes. Turbulence development in the flow, including mean flow development, is assumed to be governed by the dynamical equation along with other relevant flow describing equations.

As noted by J. Lumley (Ref. 1), for the completely inhomogeneous case,

$$\overline{a_m^* u_i(\bullet)} = \lambda^{(m)}\, \phi_i^{(m)}(\bullet) \tag{3.1}$$

which operation is a consequence of Eqs. 2.1 - 2.3. Substituting Eq. 3.1 into the Navier-Stokes equations, one obtains a dynamical equation as follows.

$$\dot{\phi}_i^{(n)} + U_{i,j}\,\phi_j^{(n)} + \phi_{i,j}^{(n)}\,U_j + \sum_{p,q} \overline{(a_n^*\,a_p\,a_q)}\Big/\lambda^{(n)} + \sum_{p,q} (a_n^*\,a_p\,a_q)\Big/\lambda^{(n)} \bullet \phi_{i,j}^{(p)}\,\phi_j^{(q)} = -\frac{1}{\rho}\,\pi_i^{(n)} + \nu\,\phi_{i,jj}^{(n)} \tag{3.2}$$

$$\phi_{i,i}^{(n)} = 0 \tag{3.3}$$

$$\overline{a_m^* p}\Big/\lambda^{(m)} = \pi^{(m)} \tag{3.4}$$

Here U_i, ρ and π represent the mean velocity, density and pressure, and n and m represent the order. The first two terms on the left side of Eq. 3.2 represent the advective derivative. The third term on the left is the result of interaction with the mean strain. The next term denotes interaction with all other eddies. The two terms on the right hand side of Eq. 3.2 are related to the pressure gradient and the viscous action.

Now, the central problem of Eq. 3.2 is again the so-called closure problem because of the term denoting eddy-eddy interactions, which requires some form of approximation. Several alternatives are as follows, although others may be feasible:

(i) a direct interaction approximation of the type given by R.H. Kraichnan (Ref. 21);

(ii) a random phase approximation of the type given by S.F. Edwards (Ref. 22);

(iii) a type of approximation given by W. Heisenberg (Ref. 23) which, although proven unsatisfactory in the case of homogeneous turbulence, has shown some promise in inhomogeneous flows; thus, A.S. Monin and A.M. Yaglom (Ref. 24) indicate that the higher order eigenfunctions may be considered as acting like a viscosity on the lower order ones; the higher and the lower order terms are assumed to be sufficiently different from each other based on their spectral separation. This has been pursued extensively in recent times by N. Aubry (Refs. 10-12); and

(iv) direct modeling of the term on the basis of its relation to turbulence transport (Refs. 1,2,3 and 7).

It may be recalled again that L. Sirovich follows a slightly different procedure.

3.1. Partially Inhomogeneous Case

In shear flows where there exist approximately homogeneous directions, for example x_1 and x_3 directions while x_2 is inhomogeneous, and where the turbulence is stationary, one can write for the fluctuating field, as stated earlier, the following:

$$u_i = \sum_n \exp\left[i(k_i x_i + \omega t)\right] \bullet \phi_i^{(n)} \, a_n \, dk_1 \, dk_3 \, d\omega \tag{3.5}$$

$$= \int f_i \, g_i \, dx' \tag{2.8}$$

and

$$f_i = \int \psi_i^{(1)} \, dk_1 \ldots \tag{3.6}$$

It will be observed that Eq. 3.6 denotes the deterministic function. The dynamical equation for $\psi^{(1)}$ then becomes the following:

$$\dot{\psi}_i^{(1)} + \psi_{i,j}^{(1)} U_j + U_{i,j} \psi_j^{(1)} = -p_{,i}^{(1)} + (\psi_{i,l}^{(1)} u_k u_l T)_{,k} \tag{3.7}$$

where T_s is a time scale, proportional to the ratio of turbulent energy to dissipation rate. It should be pointed out that viscous dissipation is not included in Eqn. 3.7. Equation 3.7 has the same features as Eq. 3.2.

3.2. Eddy-Eddy Interactions

It is clear from Eq. 3.2 that in incompressible flow, whether or not scalar quantities are present such as temperature and species concentration, the equations are closed insofar as pressure and viscous forces are concerned. Of those pressure can be established by including Poisson's equation. The only term that is unclosed is that pertaining to eddy-eddy interactions.

First, it is useful to note that Eq. 3.2 is a coupled, infinite set of eigen value equations. In principle, they should yield the eigenvalues $\overline{a_n^* a_p a_q}/\lambda^{(n)}$ when solved simultaneously. The solutions yield moments of higher order in terms of those of lower order.

A brief discussion follows on the modeling of the interaction term.

3.2.1. Lumley's original suggestion

Based on W. Heisenberg's suggestion (Ref. 23), J. Lumley introduced the hypothesis that $a_p \phi_i^{(p)}$ extracts, through some form of viscous action, energy from $a_n \phi_i^{(n)}$, $n<p$, as a function of time at appropriate scales. One can then write:

$$\overline{a_n^* \, a_p \, a_q} = 0, \qquad p \neq q$$

and

$$- \overline{a_n^* \, a_p \, a_q} \; \phi_i^{(p)} \; \phi_j^{(q)} = \left[\Theta_{np} \, \delta_{ij} + \lambda^{(n)} \, \nu_T^{(p)} \, (\phi_{i,j}^{(n)} + \phi_{j,i}^{(n)}) \right] \delta_{pq}, \qquad p,q > n \tag{3.8}$$

Unfortunately, there is no mechanism for the extraction of energy in the formalism itself. Therefore, at best one can consider the first mode and write the following.

$$\phi_i^{(1)} + U_{i,j} \, \phi_j^{(1)} + \phi_{i,j}^{(1)} \, U_j + \frac{\overline{a_1^* a_1^2}}{\lambda_1} \, \phi_{i,j}^{(1)} \, \phi_j^{(1)} = - \frac{1}{\rho} \, \pi_{ij}^{(1)} + \left(\nu + \sum_{1}^{\infty} \nu_T^{(p)} \right) \phi_{i,jj}^{(1)} \tag{3.9}$$

and

$$\phi_{i,i}^{(1)} = 0$$

One introduces in addition the Poisson equation.

Three important remarks of J. Lumley regarding the foregoing are as follows.

(i) The interaction of close wave numbers does not seem to be fully accounted for. It is of course possible to proceed assuming that there is no correlation between even close wave numbers.

(ii) The amplitude parameter, namely $(a_1^* a_1^?)/\lambda_1$ cannot be determined. The argument is that amplitude may change without a change in structure while the disturbance remains neutral at the same ν_T. However, since amplitude change does cause structural change, one has to adopt some other argument, such as the necessity of a maximum value of ν_T, to determine the amplitude factor.

(iii) One could write

$$a_1^* a_1^2 / \lambda^{(1)} = \beta \sqrt{\lambda^{(1)}} \tag{3.10}$$

where β is in the nature of a structural constant, a skewness factor.

Equation 3.10, along with a value for ν_T, provides a basis for examining shear flows.

3.2.2. An eddy viscosity model

The nonlinear eddy interaction term, representing turbulent transport and dissipation, may in general be expressed as a function of the local value $\phi_i^{(1)}$ for the large eddy and of its first few derivatives (Ref. 25):

$$\sum_{p,q=1}^{\infty} \phi_i^{(p)} \phi_j^{(q)} = fn\left(\phi_i^{(1)}, \phi_j^{(1)}, \phi_{k,i}^{(1)}, \phi_{k,ii}^{(1)}\right) \tag{3.11}$$

Various types of approximations to transport can be obtained as special cases of Eq. 3.11.

In particular the commonly-employed gradient diffusion hypothesis (Ref. 26) yields the following.

$$\sum_{p,q=1}^{\infty} \phi_i^{(p)} \phi_j^{(q)} = \epsilon_{ik} \phi_{k,j}^{(1)} \tag{3.12}$$

where ϵ_{ij} is the anistropic eddy viscosity.

Since the left hand side of Eq. 3.12 is symmetric with respect to i and j, the eddy viscosity can in the most general case be a fourth order tensor. However even the fourth order viscosity, ϵ_{ijkl}, is known to be not satisfactory in all respects, such as the violation of the condition of invariance with respect to rotation of the coordinate system (Ref. 27). Usually a second order eddy viscosity, ϵ_{ij}, is assumed to be sufficient to account for the anisotropic nature of an inhomogeneous turbulent flow.

It may be pointed out that the use of eddy viscosity implies an infinite velocity of transport for the transfer of energy from large eddy interactions to subsequent smaller eddy interactions.

The application of this approximation to curved wall boundary layers is discussed in Refs. 3 and 4.

3.2.3. Finite velocity of transport

An alternative to the (generally employed but questioned) gradient diffusion model is to assume that some form of "bulk convection" forms the basis for turbulent transport (Refs. 17, 28 and 29).

Introducing anisotropic, transport velocity scales, v_{ij}, to represent transfer of energy from a high turbulence intensity region to one of lower turbulence intensity, one can write as follows:

$$\sum_{p,q=1}^{\infty} \phi_i^{(p)} \phi_j^{(q)} \propto V_{ij} \phi_j^{(1)} \tag{3.13}$$

The transport velocity scales must again be symmetric.

The application of this approximation to curved wall boundary layers is discussed in Refs. 3 and 5.

It may be pointed out that the foregoing method of relating the nonlinear term to local turbulence intensity through the use of a finite transport scale converts the dynamical equation into a hyperbolic one. Refs. 3 and 5 then discuss a characteristic field velocity associated with the characteristics. Considering other assumptions introduced in Ref. 29. one can write the characteristic velocity in the following form:

$$V^* = \frac{1}{2}\left(V_c^2 - 2a_1\,\overline{u_iu_j}\right)^{1/2} \tag{3.14}$$

where V_c and a_1 are empirical quantities introduced when modeling diffusion of turbulence kinetic energy and turbulence energy itself, respectively. The velocity V^* then represents the characteristic speed associated with local changes in shear stress.

It is interesting in this connection that one can also deduce a characteristic velocity for turbulent transport of the temperature field in a mixing layer based on Ref. 30. The characteristic velocity becomes

$$V^* = \overline{v^2} \tag{3.15}$$

where $\overline{v^2}$ is the constant of proportionality introduced in modeling triple correlations. It is also equal to the turbulent normal stress. Then, assuming isotropy in transport velocity scales, one can write

$$V^* = \frac{1}{2}\,v_{22} \tag{3.16}$$

The role of the transport velocity scales, v_{ij}, in the transport of eddy-eddy interactions is analogous to that of $\overline{v^2}$ in the transport of temperature fluctuations, but the two velocity scales are different.

One can also examine the transport of velocity-temperature correlations.

The precise analogy between characteristic velocities for momentum and scalar quantities such as temperature is yet to be explored. We will return to this subject later.

3.2.4. Aubry's adaptation

The main thesis of N. Aubry (Ref. 10) is that energy transfer from mode to mode can be modeled by a representation adapted from Heisenberg's spectral model (Ref. 23). Eventually, this model is utilized for establishing the dynamical processes associated with the characteristic structures of turbulent shear flows; we return to this topic in Section 3.3.

Heisenberg's model in isotropic and homogeneous turbulence is based on the following assumption: in a given energy spectrum of turbulence, the energy transfer to wave numbers larger than a cut-off value k may be set equal to an effective transport coefficient $\nu_T(k, T)$, where T(k) is a nonlinear spectral transfer term; the transport coefficient is a measure of the action on eddies of wave number smaller than k. There is a notional similarity with augmentation of thermal agitation.

Aubry then introduces the assumption that one can average the influence of the small scales on the large ones; to quote, "The smaller scales extract energy from the larger ones by a global viscous action on each unresolved mode."

To proceed, one may adopt the same notation as N. Aubry: contributions less and greater than the cut-off between the large and the small scales are denoted by subscripts $<$ and $>$ respectively; an average value is indicated by $< >$; and additional subscripts to the average denote ranges over which averages are obtained.

Assuming that the small scale stress tensor is again linearly proportional to the strain rate tensor of the large scales, one can write the following.

$$\tau_{ij>} = -2\,\nu_T\,S_{ij<} \tag{3.17}$$

where

$$S_{ij<} = -\frac{1}{2}\left(u_{i<,j} + u_{j<,i}\right) \tag{3.18}$$

In terms of mean and fluctuating components of velocity, U and u_i, for instance, one can set up a dynamical equation as follows, exactly as was done in Eq. 3.2:

$$\dot{u}_{i<} + u_{i<,j}\,U_j + U_{i<,j} \bullet u_{j<} + u_{i<,j}\,u_j - <u_{i<,j}\,u_{j<}>$$

$$= -\frac{1}{\rho}\,p + \frac{1}{3}\,\delta_{ij}\left(<u_{k>}\,u_{k>}> - \ll u_{k>}\,u_{k>}\gg\right)_{,j} + (\nu + \nu_T)\,u_{i<,jj} \tag{3.19}$$

which applies to the small wave number part.

There arise two terms in Eq. 3.19 that require some form of modeling. One of them is combined with the pressure term. In order to calculate this, it is assumed that the fluctuations of the kinetic energy of small scales are proportional to the rate of loss of energy by the large scales to the small scales.

The other term requiring modeling in Eq. 3.19 is associated with ν_T. Again, it is assumed that ν_T can be made proportional to the kinetic energy distribution of the higher modes over the shear flow under consideration and a characteristic length scale; the latter is set proportional to the ratio of kinetic energy and dissipation energy.

It is clear that in both cases, the assumptions are the usual ones and therefore appear as artifices that are justified by their success. A further controversy pointed out by J. Lumely relates to the neglect of any influence of the mean strain in Eq. 3.17. Finally, it is useful to note that in sub-grid scale modeling, P. Moin and J. Kim (Ref. 31) introduce the following expression for the small scale stress tensor.

$$\tau_{ij\,>} = -\nu_T\left(S_{ij} - <S_{ij}>_<\right) - \nu_T' < S_{ij}>_< \tag{3.20}$$

which can be compared with Eq. 3.17.

3.2.5. Developments in the use of (S, ν_T).

It will be recalled that in Section 3.2.1, a suggestion was made that the amplitude parameter can be related to a factor, β, Eq. 3.10, which is in the nature of a skewness factor. Neither β nor ν_T are strictly "physical" quantities and we introduce the notation S and ν_T to represent them as in the nature of the associated quantities.

S.K. Hong (Refs. 7, 8 and 9) has examined the possibility of simulating channel flows with a model based on the LEIH utilizing S and ν_T as the transport and dissipation factors.

The formalism may be summarized as follows.

The mean velocity is decomposed into an ensemble average component and a fluctuating component. The latter is written in terms of orthogonal functions, $\phi_i^{(n)}$, $n = 1, 2, 3, \ldots$. Thus

$$u_i(\underline{x},t) = \sum_{n=1}^{\infty} \alpha_n\, \phi_i^{(n)}(\underline{x},t) \tag{3.21}$$

$$\int \phi_i^{(p)}\, \phi_i^{(q)}\, d\underline{x}\, dt = 0, \qquad p \neq q, \qquad i = 1, 2, 3. \tag{3.22}$$

where α_n are random coefficients with units of velocity and are uncorrelated with one another. Hence, one can write the following.

$$\overline{\alpha_n} = 0$$

and

$$\overline{\alpha_m \alpha_n} = \lambda^{(n)} \delta_{mn}$$

where $\lambda^{(n)}$ are all positive. Now, the $\phi_i^{(n)}$ are assumed to be orthonormal functions, implying

$$\int \phi_i^{(p)} \phi_i^{(q)} \, d\underline{x} \, dt = \delta_{pq}$$

and, further, that none of the $\phi_i^{(n)}$ is identically zero. It can then be shown that α_n are Fourier coefficients given by

$$\alpha_n = \int u_i(\underline{x},t) \, \phi_i^{(n)}(\underline{x},t) \, d\underline{x} \, dt \tag{3.23}$$

where i = 1.

If one considers the integral of the turbulent kinetic energy of the u_1–component across the whole flow, one can write

$$\frac{1}{2} \overline{u^2} \, d\vec{k} \, dt = \frac{1}{2} \sum_{n=1}^{\infty} \lambda^{(n)} \tag{3.24}$$

The n–th mode distributes the energy in space and time according to its functional form.

In the case of turbulent flow, a dynamical equation corresponding to the Navier-Stokes equation becomes the following:

$$\dot{u}_i + U_j \, u_{i,j} + U_{i,j} \, u_j + \left(u_i u_j - \overline{u_i u_j}\right)_{,j} = -\frac{1}{\rho} p_{,i} + \nu \, u_{i,jj} \tag{3.25}$$

Introducing the orthogonal decomposition of Eq. 3.22, after some algebraic manipulation, one obtains a dynamical equation for $\tilde{\phi}_i^{(n)}$ as follows; this is similar to Eq. 3.8:

$$\dot{\tilde{\phi}}_i^{(n)} + U_j\, \tilde{\phi}_{i,j} + U_{i,j}\, \tilde{\phi}_{i,j}^{(n)} + \Big\{ \qquad \Big\}_{,j} = \tilde{\Pi}_{,i}^{(n)} + \nu\, \phi_i^{(n)}{}_{,jj} \tag{3.26}$$

where

$$\tilde{\phi}_i^{(n)} = \sqrt{\lambda^{(n)}}\; \phi_i^{(n)} \tag{3.27}$$

$$\Big\{ \qquad \Big\}_{,j} = \left\{ \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\overline{\alpha_n\, \alpha_p\, \alpha_q}}{[\lambda^{(n)} \lambda^{(p)} \lambda^{(q)}]^{1/2}}\; \tilde{\phi}_i^{(p)}\, \tilde{\phi}_j^{(q)} \right\} \tag{3.28}$$

and

$$\tilde{\pi}^{(n)} = -\,\rho^{-1}\, \overline{\alpha_n p} \Big/ \sqrt{\lambda^{(n)}} \tag{3.29}$$

It may be observed that the pressure fluctuation term, $\tilde{\pi}^{(n)}$, has no restrictions other than determinacy; in fact, one can utilize the Poisson equation simultaneously with Eq. 3.26.

For the first mode, the dynamical equations for $\tilde{\phi}_i^{(n)}$ and $\tilde{\pi}^{(n)}$ become the following.

$$\dot{\tilde{\phi}}_i^{(1)} + U_j\, \tilde{\phi}_{i,j}^{(1)} + U_{i,j}\, \tilde{\phi}_i^{(1)} + \Big\{ \qquad \Big\}_{,j} = \tilde{\pi}_{,i}^{(1)} + \nu\, \tilde{\phi}_{i,jj}^{(1)} \tag{3.30}$$

and

$$\tilde{\pi}^{(1)}_{,jj} = 2U_{j,k}\,\tilde{\Phi}^{(1)}_{k,j} + \Big[\qquad\Big]_{,kj} \tag{3.31}$$

Again,

$$\Big\{\qquad\Big\} = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \frac{\overline{\alpha_1\,\alpha_p\,\alpha_q}}{\left[\lambda^{(1)}\lambda^{(p)}\lambda^{(q)}\right]^{1/2}}\,\tilde{\Phi}_j^{(p)}\,\tilde{\Phi}_k^{(q)} \tag{3.32}$$

and

$$\Big[\qquad\Big] = \sum_{p=1}^{\infty}\sum_{q=1}^{\infty} \frac{\overline{\alpha_1\,\alpha_p\,\alpha_q}}{\lambda^{(1)}\lambda^{(p)}\lambda^{(q)}}\,\tilde{\Phi}_j^{(q)}\,\phi_k^{(p)} \tag{3.33}$$

The central problem, then, is to model the term in Eq. 3.32 and Eq. 3.33. Either term represents modal interactions giving rise to transport and dissipation. It is proposed to write

$$\Big\{\qquad\Big\} = \frac{\overline{\alpha_1^3}}{\lambda^{(1)3/2}}\,\tilde{\Phi}_i^{(1)}\,\tilde{\Phi}_j^{(1)} - \nu_T\,\tilde{\Phi}_{i,j}^{(1)} \tag{3.34}$$

$$= S\,\tilde{\Phi}_i^{(1)}\,\tilde{\Phi}_j^{(1)} - \nu_T\,\tilde{\Phi}_{i,j}^{(1)} \tag{3.35}$$

wherein S and ν_T are in the nature of skewness and eddy viscosity. Considering S as a structural factor involved with transport, ν_T is associated with damping. In general S is close to one numerically except for the influence of inhomogeneities in flow. It will be observed that the skewness is associated with the nonlinear term in the dynamical equation. The term itself, therefore, must not be neglected. Once the term is retained, it becomes essential to have an additional dissipation in the form of ν_T. The method, accordingly, requires two parameters to be selected. There is no direct, rational procedure for defining or selecting the parameters. However, an indirect justification can be obtained and is discussed later.

Now, considering a shear flow, such as a channel flow in which the inhomogeneity may be considered to be confined to the y-direction, in standard Cartesian coordinates, one can define spectral functions as follows.

$$\hat{\phi}_i\ (k_i,y,k_3,t) = \frac{1}{(2\pi)^2} \iint \left\{ \sqrt{\lambda}\ \phi_i^{(1)}\ (x,y,z,t)\ \exp\left[-\hat{i}(k_1 x + k_3 z) \right] \right\} dx\, dz \qquad (3.36)$$

and

$$\hat{\pi}\ (k_1,y,k_3,t) = \frac{1}{(2\pi)^2} \iint \sqrt{\lambda}\ \pi^{(1)} \exp\left[-\hat{i}(k_1 x + k_3 z) \right] \Big\} dx\, dz \qquad (3.37)$$

A similar approach can be adopted for a two-dimensional mixing layer, which is discussed further in Section 5.

4. RECOVERY OF CHARACTERISTIC STRUCTURES

The existence of some form of structure in turbulence has been postulated for several decades based on various types of observations. However, studies of their roles in different flows and of their physical structure has become increasingly fashionable over the past fifteen years or so, as pointed out by H. Liepmann and expounded in Refs. 32. There are obviously controversies in regard to the importance of their role, their existence in fully-developed turbulence and in various types of shear flows and their strength. Nevertheless in actual fluid machinery, where turbulence may not attain a fully developed state, and in the case of mixing layers in particular, there is reason to believe that (Kelvin-Helmholtz) instability-generated large structures may display coherence and a specific role based on their structure and the gradual break-down of their structure. Regarding fluid flow machines, it is however, necessary to point-out that various (inherent) disturbances may in fact disorganize the structure. The mixing layer has, in any case, been postulated by A. Roshko (Ref. 33) to be quite distinctive in this respect among inhomogeneous shear flows.

J. Lumley and F. R. Payne (Ref. 34) obtained the large eddy structure in the wake of a cylinder within the premise of the theory put forward by J. Lumley, Ref. 1. It is the first accomplishment of the demonstration of (the large) eddy structure starting from classical, statistical, structure theory of turbulence and utilizing the most rationally-based representation of the structure. It is important to point out that no phase information was generated in that early work and therefore nothing was said about the coherence or the phase-relations governing the structures in the mean fluid flow. In any case, the possibilities for determining a structure became clear and A. Townsend has a critique, in proper perspective, of this approach in Ref. 16.

A method of extending the earlier analysis to the determination of coherent structures has been outlined in Ref. 3. A modification to the shot-noise approach of J. Lumely utilizing a snap-shot approach has been outlined by L. Sirovich in Ref. 20. A modification to the recommendation in Ref. 3 has been worked out by N. Aubry, Refs. 10 and 11. One should also note the important investigations reported by P. Moin in Ref. 35. Finally, M. N. Glauser and W. K. George (Ref. 36) have attempted a recovery of structures in an axisymmetric jet, utilizing experimental results along with orthogonal decomposition analysis. It is considered important to review this body of investigations if only briefly.

4.1. Lumley's suggestion for recovery of phase information

In Section 2.3.2, the spectrum of ϕ_{ij} has been given by Eq. 2.10. That equation permits the determination of f to within a phase angle given by the following.

$$\hat{f}_\alpha = \phi_{\alpha\alpha}^{1/2} \exp\left[i\theta(k)\right] \tag{4.1}$$

In other words the phase or the overlap and spacing information cannot be obtained from second order statistics. It is then shown how the phase information can be obtained from third order statistics,

$$u(t)\ u(t+\tau_1)\ u(t+\tau_2)$$

The procedure is elaborated in Ref. 3.

In order to obtain overlap and spacing, one again has to utilize higher order correlations of $g_i(t)$. Once the phase of f has been determined, one can Fourier transform $u_i(t)$ and the individual values of the transform can lead to obtaining the transform of g(t).

One point to note is that an important piece of information is required, from experiments or empiricism, about g, the stochastic function.

4.2. P. Moin's Application to Channel Flow

The objective (Ref. 35) was to extract coherent structure details for a channel flow in a two-dimensional plane (x, y), y being the inhomogeneous direction normal to the wall. The structures were assumed to be sprinkled randomly in the spanwise direction. No attempt was made to obtain phase information. The numerical results were compared with those from large eddy simulation model.

It is not clear how the mean flow is prescribed or obtained in the foregoing calculation.

4.3. Sirovich Method

In Section 2.3.3. we mentioned briefly the recommendation of L. Sirovich for the use of the so-called snapshot method (Ref. 20) as an alternative to the use of a shot-noise method utilized by J. Lumely.

The methodology depends upon the availability of sufficiently extensive information from numerical or physical experiments in which highly resolved flows are followed in time. A coherent structure is associated with the eigenfunctions of the correlation operator. The main problem then is establishing a method for deriving the eigenfunctions. Several aspects of the method become clear in the next section.

It may be stated that no numerical results are presented by L. Sirovich.

4.4. N. Aubry's Approach

We continue here the discussion presented in Section 3.2.4.

D. Ruelle and F. Takens (Ref. 37) proposed a model of turbulence as a deterministic, chaotic regime of flow reached after a small number of bifurcations, rather than one involving indefinitely many modes. The proposal was based on a possible connection between dynamical systems theory and turbulence. It is generally accepted that chaotic, dissipative, dynamical systems eventually display features of a strange attractor that is of low dimension. L. Sirovich suggests an optimal description for the attractor and also some form of an upper bound to the dimension of the attractor. Now, N. Aubry (working quite independently) has investigated the question of "whether the dynamics of a complex turbulent flow can be described ..... by a finite, possibly small, number of modes". The complex turbulent flow chosen for illustration is the wall region of a turbulent boundary layer in a channel flow, where large scale, horseshoe shaped, structures have been shown to play an important role. These organized, coherent structures appear in a fine-grained turbulent background.

A.K.M.F. Hussain (Ref. 38) has stated that "the motion of coherent structures .... is likely to be low-dimensional". The problem is to extract deterministic structures in a description of turbulence based on the rational representation of J. Lumley.

One question, raised by L. Sirovich, pertains to the fact that coherent structures evolve in time. The original suggestion of J. Lumely (Ref. 2) was to construct a coherent structure as an appropriate superposition of eigenfunctions, albeit utilizing higher-order statistics for calculating the coefficients. This approach is unsuitable for determining evolving structures. L. Sirovich therefore utilizes Galerkin projection, in addition, as stated earlier, to abandoning the shot-noise method. N. Aubry also has utilized Galerkin projection along with proper orthogonal composition. The Galerkin projection is an effective means of reducing truncation errors and also, leads to a set of ordinary differential equations for the coefficients.

A significant point here is the claim to establishing "a unique relationship between the correlation tensor and the unsteady flow that produces it," which had been thought impossible by B. J. Cantwell (Ref. 39).

The method is implemented by N. Aubry to determine the characteristic eddies in three dimensions together with their temporal dynamics. The flow is considered steady, but periodic in two directions, while being inhomogeneous normal to the wall.

An issue in the method is the manner of prescribing the mean flow. A mean velocity profile is obtained from a physical experiment for a fully developed channel flow. However, this cannot be introduced into the formalism wherein one obtains, starting from the Navier-Stokes equations, the following for a fully-developed channel flow:

$$<u_1u_2>_{,2} = -\rho^{-1}p_{,1} + \nu U_{,22} \qquad (4.2)$$

and

$$<u_2^2>_{,2} = -\rho^{-1}p_{,2} \tag{4.3}$$

Expressing the wall mean pressure gradient in terms of friction velocity and integrating Eq. 4.2., one obtains

$$U = \frac{1}{\nu}\int_o^{x_2} <u_1u_2> dx_2 + \frac{u_\tau^2}{\nu}\left(x_2 - \frac{x_2^2}{2H}\right) \tag{4.4}$$

where H is the half-height of the channel and u_τ, the friction velocity. N. Aubry points out that the U-profile introduced needs to be consistent with Eq. 4.3. In other words the velocity profile must show the interaction between the Reynolds stress and itself. Physically, the shear stress term is related to the intensity of the structures, which is inversely proportional to Reynolds stress. However, this raises a problem in selecting the U-profile objectively.

It may be pointed out that both P. Moin and N. Aubry obtain the structure based on the large eddy represented by the first model. The question of choosing and using a certain number of modes is addressed in the next section.

4.5. Multiple-mode Analysis

J. Lumely's rational representation of turbulence can be summed up as follows: the large eddy is that structure which has the largest mean square projection on the velocity field. It can be defined through orthogonal decomposition of the field of interest. Its structure can be established utilizing a dynamical equation. In defining it, one can introduce a truncation at any low mode number; for example, the first mode can be considered as the large eddy. On the other hand, one may want to include several modes. Each mode must contain the largest amount of energy possible. However, there is no rational basis for determining the energy transfer from mode to higher mode. In principle, one can consider a set of dynamical equations, a set of eigenvalue equations, which is solved to obtain eigenvalues of $\overline{a_n^* a_p a_q}/\lambda^{(n)}$. In practice, it is simpler to obtain the required information from physical experiment. This procedure of utilizing data from physical experiments has been utilized by W. K. George (Refs. 40-42) for determining coherent structures in an axisymmetric jet mixing layer. No attempt was made to obtain phase information, and therefore, only the local structure has been obtained.

In the axisymmetric jet studied there is stationarity with respect to time and also periodicity and symmetry in the azimuthal direction; inhomogeneity exists in one direction. In the inhomogeneous direction one uses orthogonal decomposition after fitting the homogeneous directions with harmonic eigenfunctions.

A scalar two-dimensional version of orthogonal decomposition has been derived:

$$\int \phi(x,x',\omega)\ \phi^{n}(x',\omega)\ dx' = \lambda^{(n)}(\omega)\ \phi^{(n)}(x,\omega) \tag{4.5}$$

The characteristic eddy is determined from the dominant eigenfunctions as

$$f(x,t) = \int_{-\infty}^{\infty} e^{-i\omega t} \left(\frac{\lambda^{(1)}(\omega)}{2\pi}\right)^{1/2} \left| \phi^{(1)}(x,\omega) \right| d\omega \tag{4.6}$$

Experimental data are utilized for the cross spectrum.

Several modes have been superposed to examine the resulting structure. That combination which appears most acceptable compared to experimental data is retained. It is clear that no rational procedure is available for determining the higher order modes or selecting a combination of them.

We have referred to this fact earlier in Section 3.2.4. Referring to Eq. 3.8, assuming from experimental observations that $\phi_i^{(1)}$ is weak, a solution can be constructed in the form:

$$\phi_i^{(1)} \phi_{o,i}^{(1)} + \epsilon \phi_{1,i}^{(1)} + \epsilon^2 \phi_{2,i}^{(1)} + \ . \ . \ . \qquad (4.6)$$

where

$\epsilon = a_1^* a_1^2 / \lambda^{(1)}$

In Eq. 4.6, the first term determines the (neutral) stability of the mean motion to small disturbances. The parameter ν_T can be selected on this basis (Ref. 16). However it turns out that at least the second term must be retailed when the flow is such that the mean velocity profile is stable with respect to small disturbances.

Now, a similar optimization can be carried out, in principle, to determine a succession of modes; however, as stated earlier, this may not be effective.

The evolution of modes within the framework of Navier-Stokes equations can be determined noting that Fourier decomposition of the equations leads to a triad structure in Fourier space (Ref. 43). The triad interaction represents the nonlinear process of energy exchange between the modes. The interaction depends upon the triad geometry, the orientations of the amplitude vectors with respect to the triad plane and the phase relations among the modes. A basic study on the generation of various modes and determination of their energy content is reported in Ref. 44.

It may be added here that W.K. George and N. Aubry also construct the streamline pattern within a coherent structure of the flows considered by them.

Finally, one may note that several scales of interest can be constructed utilizing the values of $\phi_i^{(n)}$.

The scale obtainable most directly is the average Taylor scale (Ref. 1),

$$l^{(T)}_{\text{average}} = \left[\frac{3}{5} \int \phi^{(n)}_{i,j} \, \phi^{(n)}_{i,j} \right] d(\bullet) \tag{4.7}$$

Based on spatial velocity autocorrelation, one can define the Taylor microscale in the form

$$\lambda^2 = \overline{u_i^2} \Big/ \overline{(u_{i,i})^2} \tag{4.8}$$

The integral length scale can be written as

$$L_e = \left(\overline{u_i^2}\right)^{3/2} \Big/ \epsilon \tag{4.9}$$

where ϵ is the dissipation rate of energy. A time scale can then be defined in the form

$$L_e / u_i = \tau_{to}$$

5. A MODEL FOR A MIXING LAYER

We consider a mixing layer, as shown in Fig. 1, involving two-dimensional mixing of two streams of Newtonian, compressible, turbulent fluids differing in mean velocity and temperature. The general interest is in predicting (a) rate of spreading of the mixing layer, (b) intermittency at the two boundaries, (c) coherent structures with their streamline pattern and (d) critical interaction between coherent structures. Our objective here is limited to examining a formalism of LEIH that is applicable to a mixing layer.

5.1. Describing Equations

Two sets of describing equations for the mean flow quantities are provided in Appendix I: (1) mass-averaged compressible flow equations (Ref. 45), and (2) equations with no change in mean density and neglect of density fluctuations as small (Ref. 16). The latter set represents a physical case of mixing of two flows with a small difference in temperature or concentration, and therefore of interest in addition to being a useful test case for the application of the proposed model.

It may be pointed out that it has repeatedly been stated, for example in Ref. 47, that density non-uniformity influences only the mean flow quantities and turbulence is unaffected by density fluctuations in thin mixing layers. These assumptions are equivalent to a local equilibrium approximation and can be applied, with great computational convenience, whereby they are shown to be adequate.

5.2. Representation of turbulence

We consider two main types of representations: (1) as in earlier work, large eddies in one group and all other eddies in another group, and (2) large eddies, energy containing eddies and all other eddies. The latter has some resemblance to the model of Ref. 46.

It is also possible to visualize yet another representation with active, inactive and all other eddies. This is not of particular interest in the case of mixing layers.

The two representations of interest in mixing layers are discussed in Appendix II.

5.3. Temperature Field

In the given formulation, it becomes necessary to model the eddy-eddy interactions separately with respect to velocity and temperature. Including turbulent viscosity and heat diffusivity, this entails assigning three skewness-like factors for velocity, temperature and correlations of the two. The Poisson equation for pressure fluctuation is unaltered in incompressible flow.

Regarding the skewness factors, one possibility is to invoke a type of structural similarity between velocity and temperature that has been discussed extensively in

Refs. 48-49. In summary, the analogy suggests that

$$\overline{q^2}/\overline{\theta^2} \bigg/ \left[\frac{\partial T}{\partial y} \bigg/ \frac{\partial U}{\partial y}\right]$$

is a constant across different parts of the mixing layers provided the averaging is done over appropriate time scales.

The possibility of using such an analogy thus rests on investigating eddy-eddy interactions over a number of time scales starting with the time scale associated with the generation of large scale structures, through the time scale based on a characteristic length scale and a velocity of motion of large scale structures, ultimately to dissipation time scale. An approach of this type is discussed in Ref. 50. A method of incorporating such an approach into the large eddy interaction model was under investigation at the close of the grant period.

References

1. J.L. Lumley: The Structure of Inhomogeneous Turbulent Flows in *Atmospheric Turbulence and Radio Wave Propagation*, A.M. Yaglom and V.I. Takarski, eds., Moscow Nauka, 1967.
2. J.L. Lumley: Coherent Structures in Turbulence in *Transition and Turbulence*, R.E. Meyer ed., Academic Press, New York, 1981.
3. S.K. Hong: Large Eddy Interactions in Curved Wall Boundary Layers - Model and Applications, Ph.D. Thesis, Purdue University, Aug. 1983.
4. S.K. Hong and S.N.B. Murthy: Structure of Turbulence in Curved Wall Boundary Layers, AIAA Paper 83-0457, Jan. 1983.
5. S.K. Hong and S.N.B. Murthy: Effective Velocity of Transport in Curved Wall Boundary Layers, *AIAA J.*, Vol. 24, March 1986.
6. S.K. Hong and S.N.B. Murthy: Pressure-Strain Correlations in Curved Wall Boundary Layers, *AIAA J.*, Vol. 24, June 1986.
7. S.K. Hong: Large Eddy Interactions in a Turbulent Channel Flow, NASA Tech. Memo. 86757, September 1985.
8. S.K. Hong and F.R. Payne: Dynamics of Large Eddies in Turbulent Channel Flows, Technical Report No. 246, Prepared under NASA Ames Research Center Grant No. NCC-2-386, June 1987.
9. S.K. Hong and F.R. Payne: Computation of Turbulent Channel Flow Using Large Eddy Interaction Model, 5th International Conference on Numerical Methods in Laminar and Turbulent Flow, Montreal, Canada, July 1987.
10. S. Herzog: The Large Scale Structure of the Near-Wall Region of Turbulent Pipe Flow, Ph.D. Thesis, Cornell University, 1986.
11. N. Aubry: A Dynamical System/Coherent Structure Approach to the Fully Developed Turbulent Wall Layer, Ph.D. Thesis, Cornell University, May 1987.
12. N. Aubry, P. Holmes, J. Lumley and E. Stone: Models for Coherent Structures in the Wall Layer, in *Advances in Turbulence*, ed. G. Comte-Bellow and J. Mathieu, Springer, New York, 1987.
13. S.L. Chuang: Second Stage Eddy Viscosity Calculation, *Applied Nonlinear Analysis*, Academic Press, 1979.
14. N. Aubry, et al: The Dynamics of Coherent Structures in the Wall Region of a Turbulent Boundary Layer, *J. Fluid Mech.*, Vol. 192, 1988.
15. S.M. Matsutani: An Experimental Investigation of Mixing and Chemical Reaction in a Plane Mixing Layer, Stanford University Report No. HTGL-T-246, March 1985.
16. A.A. Townsend: *The Structure of Turbulent Shear Flow*, First Edition, Cambridge University Press, 1956.
17. A.A. Townsend: *The Structure of Turbulent Shear Flow*, Second Edition, Cambridge University Press, 1976.
18. M. Loeve: *Probability Theory*, Second Edition, D. Van Nostrand Co., 1960.
19. S.O. Rice: Mathematical Analysis of Random Noise, *Bell Systems Tech. J.*, Vol. 23, 1944.

20. L. Sirovich: Turbulence and the Dynamics of Coherent Structures: Parts I, II and III, *Quarterly of App. Maths.*, Vol. XLV, October 1987.

21. R.H. Kraichnan: Direct-Interaction Approximation for Shear and Thermally Driven Turbulence, *Phys. Fluids*, Vol. 7, 1964.

22. S.F. Edwards: The Theoretical Dynamics of Homogeneous Turbulence, *J. Fluid. Mech.*, Vol. 18, 1964.

23. W. Heisenberg: On the Theory of Statistical and Isotropic Turbulence, *Proc. Roy. Soc.*, London, Series A195, 1948.

24. A.S. Monin and A.M. Yaglom: *Statistical Fluid Mechanics: Mechanics of Turbulence*, MIT Press, Cambridge, Mass., 1975.

25. S. Corrsin: Limitations of Gradient Transport Models in Random Walks and in Turbulence, in *Advances in Geophysics*, Academic Press, 1974.

26. T. Cebeci and A.M.O. Smith: *Analysis of Turbulent Boundary Layers*, Academic Press, New York, 1974.

27. J. O. Hinze: *Turbulence*, McGraw Hill, New York, 1976.

28. J. F. Keffer: The Uniform Distortion of a Turbulent Wake, *J. Fluid Mech.*, Vol. 22, 1955.

29. P. Bradshaw, D.H. Ferris and N.P. Atwell: Calculation of Boundary Layer Development Using the Turbulent Energy Equation, *J. Fluid Mech.*, Vol. 28, 1967.

30. A.F. Kurbatskii and N.N. Yanenko: On the Modeling of Effects of Negative Production of Temperature-Fluctuation Intensity in the Turbulent Mixing Layer, *Journal of Fluid Mechanics*, Vol. 130, Sept. 1983.

31. P. Moin and J. Kim: Numerical Investigation of Turbulent Channel Flow, *J. Fluid Mech.*, 118, 1982.

32. B.J. Cantwell: Organized Motion in Turbulent Flow, *Ann. Rev. Fluid Mech.*, Vol. 13, 1981.

33. A. Roshko: A New Look at Turbulent Mixing Layer, *AIAA J.*, Vol. 14, 1976.

34. F.R. Payne and J.L. Lumley: Large Eddy Structure of the Turbulent Wake Behind a Circular Cylinder, *Physics of Fluids*, Vol. 10, 1967.

35. P. Moin: Probing Turbulence via Large Eddy Simulation, *AIAA*, 84-0174, Jan. 1984.

36. H.P. Bakewell and J. L. Lumley: Viscous Sublayer and Adjacent Wall Region in Turbulent Pipe Flow, *Phys. Fluids*, Vol. 10, 1967.

37. D. Ruelle and F. Takens: On the Nature of Turbulence, *Comm. Math. Phys.*, Vol. 82, 1971.

38. A.K.M.F. Hussain: Coherent Structures and Turbulence, *J. Fluid Mech.*, Vol. 173, 1986.

39. B.J. Cantwell: Organized Motion in Turbulent Flow, *Ann. Rev. Fluid Mech.*, Vol. 13, 1981.

40. S.J. Leib, M.N. Glauser and W.K. George: An Application of Lumley's Orthogonal Decomposition to the Axisymmetric Turbulent Jet Mixing Layer, Proc. 9th Rolla Symposium, 1986.

41. M.N. Glauser, S.J. Leib and W.K. George: Coherent Structures in Axisymmetric Jet Mixing Layer, Proc. 5th Symposium on Turbulent Shear Flow, Cornell University, 1985.

42. M.N. Glauser and W.K. George: Orthogonal Decomposition of the Axisymmetric Jet Mixing Layer Including Azimuthal Dependence, Proc. 5th Symposium on Turbulent Shear Flow, Cornell University, 1985.

43. J. Lee: Development of Mixing and Isotrophy in Inviscid Homogeneous Turbulence, *J. Fluid Mech.*, Vol. 120, 1982.

44. J.G. Brasseur and S. Corrsin: Spectral Evolution of the Navier-Stokes Equations for Low Order Couplings of Fourier Modes, in

45. P. Libby and F.A. Williams (Eds.): *Turbulent Reactive Flows*, Springer, 1979.

46. K. Hanjalic, B.E. Launder and R. Schiestel: Multiple-Time Scale Concepts in Turbulent Transport Modeling, Second Symposium on Turbulent Shear Flows, 1979.

47. Tsan-Hsing Shih, J. Lumley and J. Janicka: Second-Order Modelling of a Variable Density Mixing Layer, *J. Fluid Mech.*, Vol. 180.

48. L. Fulacher: Spectral Analogy Between Temperature and Velocity Fluctuations in Several Turbulent Flows, *Int. J. Heat Mass Transfer*, Vol. 27, 1984.

49. F. Anselmat and R. A. Antonia: Joint Statistics Between Temperature and Its Dissipation, *Phys. Fluids*, Vol. 28, 1985.

50. W.D. McComb: Reformulation of the Statistical Equations for Turbulent Shear Flow, *Physical Review A*, Vol. 26, Aug. 1982.

APPENDIX I

BASIC EQUATIONS

1. General Describing Equations

Standard terminology as in Reference 45 is employed throughout.

Considering a multi-component gas mixture involving N species and M elements, the equation for conservation of individual species may be written as follows.

$$\frac{\partial}{\partial t}(\rho\, y_i) + \frac{\partial}{\partial x_k}(\rho\, u_k\, y_i) = \frac{\partial}{\partial x_k}\left(\rho D\, \frac{\partial y_i}{\partial x_k}\right) + \dot{w}_i \qquad i = 1, 2, \ldots, N \tag{I.1}$$

It will be observed that a single diffusion coefficient is utilized. The last term in the foregoing equation represents the chemical source term.

The equation for overall conservation of mass becom s:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_k}(\rho\, u_k) = 0 \tag{I.2}$$

The conservation of momentum is given by the following:

$$\frac{\partial}{\partial t}(\rho\, u_i) + \frac{\partial}{\partial x_k}(\rho\, u_k\, u_i) = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_k}\tau_{ik} \tag{I.3}$$

here p is the hydrostatic pressure, a thermodynamic variable. The viscous tensor is given by

$$\tau_{ij} = \mu\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right) - \frac{2}{3}\,\mu\,\frac{\partial u_k}{\partial x_k}\,\delta_{ij} \tag{I.4}$$

The conservation of energy can be expressed as follows:

$$\frac{\partial}{\partial t}(\rho h) + \frac{\partial}{\partial x_k}(\rho u_k h) = \frac{\partial p}{\partial t} + \frac{\partial}{\partial x_k}\left\{\frac{\mu}{\sigma}\frac{\partial h}{\partial x_k} + \mu\left(\frac{1}{Sc} - \frac{1}{\sigma}\right)\sum_{i=1}^{N} h_i \frac{\partial Y_i}{\partial x_k}\right\} \tag{I.5}$$

The mixture enthalpy, h, can be expressed by the relation

$$h = \sum_{i=1}^{N} y_i h_i \tag{I.6}$$

where

$$h_i = c_p T + \Delta_i \tag{I.7}$$

which accounts for the variations over the range of interest in temperature and heat content changes due to chemical transformation. The Prandtl and the Schmidt numbers are denoted by σ and Sc, respectively.

In most flow of gases involving chemical reaction, one can expect a density change, the density variations arising due to inhomogeneities in temperature or composition. It is then common practice to utilize Favre averaging, wherein all quantities, except pressure, are mass-averaged. The equations based on Favre-averaging corresponding to equations I.1 - I.3 and 1.5 become the following in the absence of variation with time.

$$\frac{\partial}{\partial x_k}(\tilde{\rho}\,\tilde{u}_k\,\tilde{Y}_i) = \frac{\partial}{\partial x_k}\left(\bar{\rho} D\,\overline{\frac{\partial Y_i}{\partial x_k}} - \overline{\rho u_k'' Y_i''}\right) + \overline{\dot{w}_i} \tag{I.8}$$

$$\frac{\partial}{\partial x_k}(\tilde{\rho}\,\tilde{u}_k) = 0 \tag{I.9}$$

$$\frac{\partial}{\partial x_k}(\tilde{\rho}\,\tilde{u}_k\,\tilde{u}_i) = -\frac{\partial \bar{p}}{\partial x_i} + \frac{\partial}{\partial x_k}(\tilde{\tau}_{ik} - \overline{\rho u_k'' u_i''}) \tag{I.10}$$

$$\frac{\partial}{\partial x_k}(\tilde{\rho}\,\tilde{u}_k\,\tilde{h}\,) = \frac{\partial}{\partial x_k}\left(\mu\,\frac{\overline{\partial h}}{\partial x_k} - \overline{\rho u_k'' h''}\right) \tag{I.11}$$

It mav be noted that the equation of state

$$p = \rho\, R_o\, T \sum_{i=1}^{N} \frac{Y_i}{W_i} \tag{I.12}$$

becomes

$$\overline{p} = R_o \sum_{i=1}^{N} (\bar{\rho}\;\tilde{T}\;Y_i + \overline{\rho T'' Y_i''}\,)\;\frac{1}{w_i} \tag{I.13}$$

When the molecular weights of different species do not vary greatly, one can write

$$\overline{p} = \tilde{\rho}\, R\, \tilde{T} \tag{I.14}$$

where R is the mass-based gas constant applicable to the system under consideration.

It has now become well recognized that several difficulties arise in the course of Favre-averaging. Such difficulties pertain in general to the various nonlinear terms that arise in the describing equations. Unfortunately, there is no clear method of resolving the difficulties on a reasonably universal basis even with an appeal to experimental results. Yet another problem in Favre-averaging is that the molecular transport terms need to be modeled specifically for the Favre-averaged form of the equations, and such models cannot be the same as in the original equations.

2. Simplified Equations with No Variations in Density

We consider in the following a particular case of a flowfield with temperature (or concentration fluctuations) in which both the flow Mach number and the density variations are negligibly small.

Equations for mean and fluctuating quantities may then be written as follows.

Mean flow

$$\frac{\partial U_i}{\partial t} + U_k \frac{\partial U_i}{\partial x_k} = - \frac{\partial p}{\partial x_i} - \frac{\partial \overline{u_i u_k}}{\partial x_k} + \nu \frac{\partial^2 U_i}{\partial x_k^2} \tag{I.15}$$

Mean temperature

$$\frac{\partial T}{\partial t} + U_k \frac{\partial T}{\partial x_k} = - \frac{\partial \overline{u_i \theta}}{\partial x_k} + K \frac{\partial^2 T}{\partial x_k^2} \tag{I.16}$$

Intensity of velocity fluctuations

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\,\overline{q^2}\right) + U_k \frac{\partial}{\partial x_k}\left(\frac{1}{2}\,\overline{q^2}\right) + \frac{\partial}{\partial x_k}\left(\overline{p' u_k} + \frac{1}{2}\,\overline{q^2 u_k}\right) = - \overline{u_i u_k} \frac{\partial U_i}{\partial x_k} + \nu \overline{u_i \frac{\partial^2 u_i}{\partial x_k^2}} \tag{I.17}$$

Variance of temperature fluctuations

$$\frac{\partial}{\partial t}\left(\frac{1}{2}\,\overline{\theta^2}\right) + U_k \frac{\partial}{\partial x_k}\left(\frac{1}{2}\,\overline{\theta^2}\right) + \frac{\partial}{\partial x_k}\left(\frac{1}{2}\,\overline{\theta^2 u_k}\right) = - \overline{\theta u_k} \frac{\partial T}{\partial x_k} + K \frac{\partial^2}{\partial x_k^2}\left(\frac{1}{2}\overline{\theta^2}\right) - K \overline{\left(\frac{\partial \theta}{\partial x_k}\right)^2} \tag{I.18}$$

Mean heat flux

$$\frac{\partial}{\partial t}(\overline{\theta u_i}) + U_k \frac{\partial}{\partial x_k}(\overline{\theta u_i}) + \frac{\partial}{\partial x_k}(\overline{\theta u_i u_k}) = -\overline{u_i u_k}\frac{\partial T}{\partial x_k} - \overline{\theta u_k}\frac{\partial U_i}{\partial x_k} - \overline{\theta \frac{\partial p}{\partial x_i}}$$

$$+ \frac{\partial}{\partial x_k}\left(\overline{K u_i \frac{\partial \theta}{\partial x_k}} + \overline{\nu \theta \frac{\partial U_i}{\partial x_k}}\right) - (\nu + K)\overline{\frac{\partial u_i}{\partial x_k}\frac{\partial \theta}{\partial x_k}} \qquad \text{(I.19)}$$

It may be noted that p′ represents the pressure fluctuation. Further, it may be of interest to observe that the second term on the right hand side of Equation (I.17) represents the rate of destruction of temperature fluctuation by molecular diffusion.

Fluctuating flow

$$\frac{\partial u_i}{\partial t} + U_k \frac{\partial u_i}{\partial x_k} = -\frac{1}{\rho}\frac{\partial p'}{\partial x_i} - \frac{\partial U_i}{\partial x_k} u_k - \frac{\partial}{\partial x_j}(u_i u_k - \overline{u_i u_k}) + \nu \frac{\partial^2 u_i}{\partial x_k^2} \qquad \text{(I.20)}$$

Fluctuating pressure governed by Poisson equation

$$\frac{\partial^2 p}{\partial x_j^2} = 2\frac{\partial U_i}{\partial x_k}\frac{\partial u_i}{\partial x_k} + \frac{\partial^2}{\partial x_k \partial x_j}(u_i u_j - \overline{u_i u_j}) \qquad \text{(I.21)}$$

Fluctuating temperature

$$\frac{\partial \theta}{\partial t} + U_k \frac{\partial \theta}{\partial x_k} = -\frac{\partial}{\partial x_j}(u_i \theta - \overline{u_i \theta}) + K\frac{\partial^2 T}{\partial x_k^2} \qquad \text{(I.22)}$$

APPENDIX II

APPLICATION TO MIXING LAYER

We consider the mixing layer studied originally by J. Laufer (NACA Report 1174, 1954) and H. Liepmann. The layer is formed by a single stream flowing over a stationary fluid. In the problem considered here, it is assumed that the stationary fluid is at a higher temperature then the flowing fluid, while both fluids are incompressible and at constant densities.

The plan of attack has been as follows: (a) to obtain the mean velocity field and the shear stress distribution in the mixing layer; (b) to invoke spectral analogy between velocity and temperature fields and thereby establish the mean temperature field and temperature variance distribution; and finally, to predict the velocity-temperature correlation distribution which could be checked against experimental results. It may be said at the outlet that no calculations could be completed within the project period.

II.1. Formulation of the Problem.

Referring to Appendix I, Eqs. I.15, I.20 and I.21 can be solved to obtain mean flow, turbulence intensity of velocity of fluctuations and Reynolds stress. The modelling required pertains to eddy-eddy interactions. Eq. I.17 can be utilized to check the modelling.

In Eq. I.18, the analogy between velocity and temperature intensities can be invoked. One then solves Eqs. I.18, I.22 and I.16 to obtain $\overline{u_R \theta}$ and mean temperature as functions of x_j. Returning to Eq. I.22 it is possible to establish the skewness factor and turbulent conductivity parameter that are compatible with the analogy between velocity and temperature.

Finally, it is possible to extract the pdf of velocity and temperature fluctuations and compare them with measured data.

In analyzing the flowfield one may start with assuming that the flowfield is homogeneous in the spanwise direction and then proceed to a formulation in which the flowfield is fully inhomogeneous. Such a procedure is especially useful in extracting physical structures.

It may be pointed out that the entire formulation is based upon concentrating attention on the largest eddy represented by the first mode and a few wave numbers in the overall spectrum. Therefore, the analogy between temperature and velocity intensities may in fact be partially valid. This may cause errors in the individual contributions of production, dissipation and transport to the stresses and the velocity-temperature correlations. A multi-modal analysis, and on the other hand, requires some experimental input.

II.2. Orthogonal Decomposition and Spectral Analysis

The orthogonal decomposition and spectral analysis can be illustrated utilizing the case of velocity. As stated elsewhere, the instantaneous velocity can be written in terms of a time-mean value and its fluctuation:

$$\overline{U_i} = U_i + u_i \qquad \text{(II.1)}$$

where the velocity fluctuation can be represented in a series in terms of orthogonal functions:

$$u_i(\underline{x},t) = \sum_{n=1}^{\infty} \alpha_n(t)\ \phi_i^{(n)}(\underline{x}) \qquad n = 1, 2, 3, \ldots \qquad \text{(II.2)}$$

The random coefficients, $\alpha_n(t)$, are uncorrelated with each other. Hence,

$$\overline{\alpha_m \alpha_n} = \lambda_n^2(t) \bullet \delta_{mn}, \qquad \delta_{mn} = \text{kronecker delta,} \tag{II.3}$$

the overbar representing an ensemble average. Assuming that the determinate $\phi_i^{(n)}$ are orthonormal functions

$$\int \phi_i^{(p)} \phi_i^{(q)} \, dx = \delta_{pq} \tag{II.4}$$

It then follows that the two-point velocity correlation, R_{ij}, can be expressed in the form, namely:

$$R_{ij} \; (\underline{x}, \underline{x}'; t) = \sum_{n=1}^{\infty} \; \lambda_n^2 \; \phi_i^{(n)}(\underline{x}) \bullet \phi_j^{(n)}(\underline{x}') \tag{II.5}$$

Now, one can set up dynamical equations for the n^{th} mode by an appeal to Navier-Stokes equations. After some manipulation, one obtains the following dynamical equation.

$$\frac{\partial \tilde{\phi}_i^{(n)}}{\partial t} + U_j \frac{\partial \tilde{\phi}_i^{(n)}}{\partial x_j} + \frac{\partial U_i}{\partial x_j} \left\{ \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\alpha_n \alpha_p \alpha_q}{\left[\lambda^{(n)} \lambda^{(p)} \lambda^{(q)} \right]^{1/2}} \tilde{\phi}_i^{(p)} \; \tilde{\phi}_j^{(q)} \right\} = \frac{\partial \tilde{p}}{\partial x_i} + \nu \frac{\partial^2 \tilde{\phi}_i^{(n)}}{\partial x_j^2} \tag{II.6}$$

where

$$\tilde{\phi}_i^{(n)} = \sqrt{\lambda^{(n)}} \; \phi_i^{(n)} \quad \text{and} \quad \tilde{\pi}^{(n)} = -\frac{1}{\rho} \; \frac{\overline{\alpha_n p}}{\sqrt{\lambda^{(n)}}}$$

Also, the Poisson equation for pressure fluctuation becomes the the following.

$$\frac{\partial^2 \tilde{\pi}^{(n)}}{\partial x_j^2} = \frac{2\,\partial U_j}{\partial x_k} \frac{\partial \tilde{\phi}_k^{(n)}}{\partial x_j} + \frac{\partial^2}{\partial x_k \partial x_j} \left\{ \sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\alpha_i \alpha_p \alpha_q}{\left[\lambda^{(i)} \lambda^{(p)} \lambda^{(q)}\right]^{1/2}} \tilde{\phi}_j^{(q)} \tilde{\phi}_k^{(p)} \right\} \tag{II.7}$$

The continuity conditions yields

$$\frac{\partial \tilde{\phi}_j^{(n)}}{\partial x_j} = 0 \tag{II.8}$$

Eqs. II.6-II.8 can be specialized to the first mode, n = 1, considered on the large eddy.

The eddy-eddy interactions are then modelled as follows.

$$\sum_{p=1}^{\infty} \sum_{q=1}^{\infty} \frac{\alpha_1 \alpha_p \alpha_q}{\lambda_1} \phi_i^{(p)} \phi_j^{(q)} = \frac{\overline{\alpha_1^3}}{\lambda_1} \phi_i^{(1)} \phi_j^{(1)} - \nu_T \left\{ \frac{\partial(\lambda_i \phi_i^{(1)})}{\partial x_j} + \frac{\partial(\lambda_i \phi_j^{(1)})}{\partial x_i} \right\} \tag{II.9}$$

where

$$\frac{\alpha_1^3}{\lambda_1} = S \tag{II.10}$$

the skewness factor of the random coefficient; and ν_T is a form of damping coefficient referred to as eddy viscosity for convenience.

II.1.1. An Alternative Formulation

At this stage, we have also considered an alternative formulation. It consists in including two types of eddies, one that may be referred to as large and the other as small. it is assumed that the large and the small eddies are sufficiently separated.

The velocity fluctuation is then written in the following form.

$$u_i(\underline{x},t) = \alpha\phi_i\ (\underline{x},t) + \alpha'\phi_i'(\underline{x},t) \tag{II.11}$$

The two deterministic functions are orthogonal in the (x,t) space of each other. The random coefficients do not have any correlations whatsoever between them, that is

$$\overline{\alpha\alpha'} = 0$$

The Reynolds stress tensor then becomes

$$\overline{u_iu_j} = \overline{\alpha^2}\ \phi_i\ \phi_j + \overline{\alpha'^2}\ \phi_i'\ \phi_j' \simeq (\sqrt{\overline{\alpha^2}}\ \phi_i)\ \ (\overline{\alpha'^2}\ \phi_j) \tag{II.12}$$

The approximation is an important one to proceed further.

The eddy-eddy interactions again involve a skewness factor and an eddy viscosity. The second-order eddy viscosity tensor ϵ_{ij} is written in the following form. Defining

$$S_1 = \frac{\overline{\alpha^2\alpha'}}{\overline{\alpha^2}\ \sqrt{\overline{\alpha'^2}}}\ ; \qquad S_2 = \frac{\overline{\alpha\alpha'^2}}{\overline{\alpha'^2}\ \sqrt{\overline{\alpha^2}}}$$

we write

$$S_1(\phi_i'\ \phi_j - \phi_i\ \phi_j') - S_2(\phi_i'\ \phi_j') = -\ \epsilon_{ik}\left(\frac{\partial\phi_k}{\partial x_j} + \frac{\partial\phi_j}{\partial x_k}\right) - \epsilon_{jk}\left(\frac{\partial\phi_k}{\partial x_i} + \frac{\partial\phi_i}{\partial x_k}\right) \tag{II.13}$$

There are some complex arguments in setting up Eq. (II.13). In any case, this approach has not been developed further.

II.1.2. Temperature Fluctuations

In a manner similar to the foregoing, it is also possible to decompose the temperature fluctuation, θ and the correlations $\overline{u_i\theta}$. Thus

$$\theta\ (x,t) = \sum_{n=1}^{\infty} \beta_n(t)\ \gamma^{(n)} \qquad n = 1, 2, 3, \ldots \tag{II.14}$$

and

$$\overline{u_i\theta} = \sum_{n=1}^{\infty} \sigma_n\ \phi_i^{(n)}(x)\ \gamma^{(n)} \tag{II.15}$$

A dynamical equation can also be constructed for θ corresponding to Eq. II.6. However, as stated earlier, it is proposed to invoke a similarity between temperature and velocity intensities.

II.1.2. Spectral Decomposition

Assuming that the flowfield is homogeneous in the z-direction, spanwise with respect to the developing mixing layer, one can construct the spectrum as follows. It may be observed that a more general formulation in which the flowfield is inhomogeneous in all three directions is easily set up formally. However, it is worthwhile starting with the simpler formulation.

$$\hat{\phi}_i^{(1)}(k_3; x,y,z,t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{\phi}_i^{(1)}\ (x,y,z,t)\ \exp(-\hat{i}k_3 z)\ dz \tag{II.16}$$

where k_3 is the (scalar) wave number corresponding to the z-direction. For any z, then, turbulence contains various length scales whose sizes are proportional to $1/k_e$. The spectrum, $\phi_i^{(1)}$ (k_3), is the integrated value of the first mode, $\phi_i^{(1)}$ (z), corresponding only to $1/k_3$, over the entire homogeneous field, z. Inversely, one can write

$$\hat{\phi}_i^{(1)}(x,y,z,t) = \int_{-\infty}^{\infty} \hat{\phi}_i^{(1)}(k_3;\, x,y,z,t)\, \exp(ik_3 z)\, dz \qquad \text{(II.17)}$$

The spectrum is a complex variable which may be divided into real and imaginary parts as follows.

$$\hat{\phi}_i = (\hat{\phi}_i)_R + i(\hat{\phi}_i)_I$$

Proceeding similarly with respect to $\hat{\pi}$ and $\hat{\theta}$, one can write

$$\begin{aligned}
\hat{\phi}_1 &= P_1 + \hat{i}P_2 \\
\hat{\phi}_2 &= P_3 + \hat{i}P_4 \\
\hat{\phi}_3 &= P_5 + \hat{i}P_6 \\
\hat{\pi} &= P_7 + \hat{i}P_8 \\
\theta &= P_9 + \hat{i}P_{10}
\end{aligned} \qquad \text{(II.18)}$$

It is important to recognize here that, while the flowfield is assumed to be two-dimensional, with homogeneity being imposed in the third direction, turbulence is treated correctly as being three-dimensional.

Various quantities such as $\overline{u_i u_j}$, $\overline{u_i^2}$, $\overline{q^2}$, $\overline{uv}/\overline{q^2}$, orientation of principal axes and anisotropy can be expressed in the mixed $(x,y,\, k_3;\, t)$ space. Similarly, the various turbulence quantities related to temperature, $\overline{\theta^2}$, $\overline{u_i\theta}$, can also be expressed in spectral space.

II.2. Analogy Between Temperature and Velocity

The analogy is expressed in the following form

$$\overline{q^2}\Big/\overline{\theta^2}\Big/\left[\frac{\partial T}{\partial y}\Big/\frac{\partial U}{\partial y}\right] = \beta \qquad \text{(II.19)}$$

where β is in general of 0(1), but is taken as equal to unity in the first instance.

As stated earlier, $\overline{q^2}$ is expressed in spectral form in the first instance, while simultaneously establishing the mean flow distribution. The spectrum of $\overline{\theta^2}$ can be related to the spectrum of $\overline{q^2}$ but only with a knowledge of $\partial T/\partial y$. thus, it is necessary to solve Eqs. I.18, I.22 and I.16 simultaneously to determine mean temperature and $\overline{u_k\theta}$. It may be pointed out that in accomplishing this in practice, it is useful to equate $\partial T/\partial y$ to $\partial U/\partial y$ and thus use the spectrum of $\overline{q^2}$ as the spectrum of $\overline{\theta^2}$. One can then proceed to modify the analogy through successive iterations.

National Aeronautics and Space Administration

Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-185123		

4. Title and Subtitle
Application of Large Eddy Interaction Model to a Mixing Layer

5. Report Date
November 1989

6. Performing Organization Code

7. Author(s)
S.N.B. Murthy

8. Performing Organization Report No.
M/NA/TURB-1

10. Work Unit No.
505-62-21

9. Performing Organization Name and Address
Purdue University
School of Mechanical Engineering
West Lafayette, Indiana 47907

11. Contract or Grant No.
NAG3-592

13. Type of Report and Period Covered
Contractor Report
Final

12. Sponsoring Agency Name and Address
National Aeronautics and Space Administration
Lewis Research Center
Cleveland, Ohio 44135-3191

14. Sponsoring Agency Code

15. Supplementary Notes
Project Manager, Thomas J. VanOverbeke, Internal Fluid Mechanics Division, NASA Lewis Research Center.

16. Abstract
The large eddy interaction model (LEIM) is a statistical model of turbulence based on the interaction of selected eddies with the mean flow and all of the eddies in a turbulent shear flow. It can be utilized as the starting point for obtaining physical structures in the flow. The possible application of the LEIM to a mixing layer formed between two parallel, incompressible flows with a small temperature difference is developed by invoking a detailed similarity between the spectra of velocity and temperature.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
CFD; Eddy; Interaction; Model; Mixing; Layer	Unclassified – Unlimited Subject Category 02

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No of pages	22. Price*
Unclassified	Unclassified	50	A03

NASA FORM 1626 OCT 86

*For sale by the National Technical Information Service, Springfield, Virginia 22161