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COBE NONSPINNING ATTITUDE PROPAGATION*

D. Chu
Computer Sciences Corporation (CSC)

ABSTRACT

The Cosmic Background Explorer (COBE) spacecraft will exhibit complex attitude motion consisting of a spin rate of approximately -0.8 revolution per minute (rpm) about the x-axis and simultaneous precession of the spin axis at a rate of one revolution per orbit (rpo) about the nearly perpendicular spacecraft-to-Sun vector. The effect of the combined spinning and precession is to make accurate attitude propagation difficult and the 1-degree (3σ) solution accuracy goal problematic.

To improve this situation, an intermediate reference frame is introduced, and the angular velocity divided into two parts. The "nonspinning" part is that which would be observed if there were no rotation about the x-axis. The "spinning" part is simply the x-axis component of the angular velocity. The two are propagated independently and combined whenever the complete attitude is needed. This approach is better than the usual "one-step" method because each of the two angular velocities look nearly constant in their respective reference frames. Since the angular velocities are almost constant, the approximations made in discrete time propagation are more nearly true.

To demonstrate the advantages of this "nonspinning" method, attitude is propagated as outlined above and is then compared with the results of the one-step method. Over the 100-minute COBE orbit, the one-step error grows to several degrees while the nonspinning error remains negligible.

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COBE ATTITUDE AND THE NEED FOR MORE ACCURATE PROPAGATION

The attitude of the Cosmic Background Explorer (COBE) is three-axis stabilized with the minus x-axis maintained 94 degrees from the Sun line (Figure 1). The spacecraft pitches about that line at a rate of 1 revolution per 100-minute orbit period. It also spins about the minus x-axis once every 75 seconds. COBE attitude can be expressed as an Euler 3-2-1 (pitch-roll-yaw) rotation sequence with respect to a rotating Earth-Sun coordinate frame with the z-axis pointing toward the Sun and the y-axis pointing along the cross product of the Sun and Earth vectors. Nominal pitch and roll are then 0 and –4 degrees, respectively.

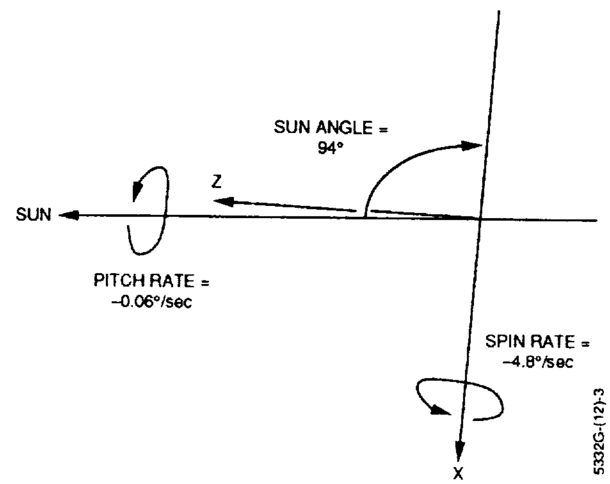

Figure 1. COBE Attitude Profile

Propagating the attitude from samples of the angular velocity assumes that the angular velocity remains constant in the body frame over the sample interval. The COBE gyro sampling intervals are 0.5, 1, 2, or 4 seconds, depending on the telemetry format and data rate. For COBE, which spins as much as 19.2 degrees per gyro sample, the pitch component of the angular velocity can change direction significantly. Under these circumstances, the usual method of propagation, which handles the total incremental rotation at once, introduces errors that accumulate over time and become unacceptably large.

This paper describes a variation on the usual one-step propagation that reduces this error by introducing an intermediate nonspinning coordinate frame. The advantage of the nonspinning frame is that the angular velocities used for its propagation and for the subsequent transformation to the body frame vary much less between gyro samples. The associated equations for gyro calibration follow along with numerical estimates of the improvement in COBE propagation accuracy.

NOTATION AND THE USUAL ONE-STEP ATTITUDE PROPAGATION

Attitude is represented here by the orthogonal inertial-to-body coordinate transformation matrix, $A_{B/I}$, and the kinematic equation for its propagation (Reference 1, p. 512) is

$$\dot{A}_{B/I} = -\underset{\sim}{\omega}_B \, A_{B/I} \tag{1}$$

The dot $(\cdot)$ above $A_{B/I}$ indicates differentiation with respect to time. Since the angular velocity, ω_B, is not constant, Equation (1) is solved numerically. Still, a formal solution may be written as

$$A_{B/I}(t) = \Phi_{B/I}(t, t_o)\, A_{B/I}(t_o) \tag{2}$$

$\Phi_{B/I}$ is the attitude propagation matrix satisfying the differential equation:

$$\dot{\Phi}_{B/I} = -\underset{\sim}{\omega}_B \, \Phi_{B/I} \tag{3}$$

and the initial condition:

$$\Phi_{B/I}(t_o, t_o) = I \tag{4}$$

where I is the identity matrix.

The subscripts I and B refer here to the inertial and body coordinates in which a vector or matrix is expressed. The vector ω_B, for example, is the angular velocity in body coordinates, and $\underset{\sim}{\omega}_B$ is the antisymmetric matrix derived from it.

$$\underset{\sim}{\omega}_B = \begin{bmatrix} 0 & -\omega_{B3} & \omega_{B2} \\ \omega_{B3} & 0 & -\omega_{B1} \\ -\omega_{B2} & \omega_{B1} & 0 \end{bmatrix} \tag{5}$$

The slash (/) indicates a transformation from the frame on the right to that on the left. Thus, $A_{B/I}$ is the inertial-to-body coordinate transformation matrix.

NONSPINNING INTERMEDIATE FRAME AND TWO-STEP PROPAGATION

In body coordinates, the spin component of the COBE angular velocity, ω_B'', is constant, while the pitch component, ω_B', varies with time:

$$\omega_B(t) = \omega_B'(t) + \omega_B'' \tag{6}$$

If the spin axis is denoted by $\hat{s}_B$, these two portions of the angular velocity can be computed as follows:

$$\omega_B' = (I - \hat{s}_B (\hat{s}_B)^T)\, \omega_B \tag{7}$$

$$\omega''_B = \hat{s}_B (\hat{s}_B)^T \omega_B \tag{8}$$

An intermediate, nonspinning coordinate system denoted by the subscript N can be introduced that is defined by the propagation equation

$$\dot{A}_{N/I} \equiv -\underset{\sim}{\omega}'_N A_{N/I} \tag{9}$$

where

$$\omega'_N \equiv (A_{B/N})^T \omega'_B \tag{10}$$

Since the magnitude of ω'_B is less than that of ω_B, ω'_B does not change as much as ω_B does between gyro samples. Thus, all other things being the same, the error in propagating $A_{N/I}$ should be less than the error in propagating $A_{B/I}$. If the propagation from the nonspinning frame to the body frame can be done perfectly, as is plausible since the spin rate is constant, the total propagation error for this two-step method should also be less than that for the usual one-step method.

To complete the propagation of $A_{B/I}$, it remains to compute $A_{B/N}$, which transforms from the nonspinning to the body frame:

$$A_{B/N} = A_{B/I} (A_{N/I})^T \tag{11}$$

By the product rule for differentiation, the $\dot{A}_{B/I}$ is

$$\dot{A}_{B/N} = \dot{A}_{B/I} (A_{N/I})^T + A_{B/I} (\dot{A}_{N/I})^T \tag{12}$$

Substituting for $\dot{A}_{B/I}$ and $\dot{A}_{N/I}$ from Equations (1), (6), and (9) and combining attitude transformations yields

$$\dot{A}_{B/N} = -(\underset{\sim}{\omega}'_B + \underset{\sim}{\omega}''_B) A_{B/N} + A_{B/N} \underset{\sim}{\omega}'_N \tag{13}$$

Noting that $\underset{\sim}{\omega}'_N$ is the similarity transformation of $\underset{\sim}{\omega}'_B$,

$$\underset{\sim}{\omega}'_N = (A_{B/N})^T \underset{\sim}{\omega}'_B A_{B/N} \tag{14}$$

gives the differential equation for the propagation of $A_{B/N}$:

$$\dot{A}_{B/N} = -\underset{\sim}{\omega}''_B A_{B/N} \tag{15}$$

With equations for both $A_{B/N}$ and $A_{N/I}$, the complete attitude, $A_{B/I}$, can be propagated.

GYRO CALIBRATION

In addition to reducing the propagation error due to the finite gyro sampling interval, it is usually necessary to calibrate the gyros to reduce systematic errors in the sensed angular velocity. Because calibration involves the same type of computations as propagation, it is also done more accurately with a two-step method.

With one-step propagation, gyro calibration errors, Δa, can be found using the solution to the following error propagation equation (Reference 2, p. 4-13):

$$\dot{\theta}_B = -\underset{\sim}{\omega}_B \, \theta_B + \Delta\omega_B \tag{16}$$

where θ_B is the attitude error expressed in axis and angle form, and the angular velocity error, $\Delta\omega_B$, is related to the gyro calibration errors by the matrix $G(\omega_B)$.

$$\Delta\omega_B = G(\omega_B)\,\Delta a \tag{17}$$

The solution to the error equation has the form

$$\theta_B\,(t) = \Gamma_{B/I}\,(t,\ t_o)\,\Delta a + \Phi_{B/I}\,(t,\ t_o)\,\theta_B\,(t_o) \tag{18}$$

where the variational matrix, $\Gamma_{B/I}$, transforms the gyro calibration errors into contributions to the attitude error

$$\Gamma_{B/I}\,(t, t_o) = \int_{t_o}^{t} \Phi_{B/I}\,(t, \tau)\; G\,(\omega_B)\; d\tau \tag{19}$$

The matrix $\Gamma_{B/I}$ then serves as the partial derivative of the propagated attitude error with respect to the gyro parameter errors:

$$\frac{\partial\theta_B}{\partial\Delta a} = \Gamma_{B/I}\,(t,\ t_o) \tag{20}$$

If one knows θ_B at times t and t_o, Δa can be found from Equation (18).

A corresponding variational matrix is needed for calibration with the nonspinning propagation method. Because the two methods are different, there is no reason to expect the variational matrices to be the same. Both steps of the nonspinning propagation, however, follow the same kind of propagation equation as does the one-step method, and the corresponding error equations can be applied to each step separately:

$$\dot{\theta}'_N = \Delta\omega'_N - \underset{\sim}{\omega}'_N \, \theta'_N \tag{21}$$

$$\dot{\theta}''_B = \Delta\omega''_B - \underset{\sim}{\omega}''_B \, \theta''_B \tag{22}$$

Here, θ'_N is the error in the nonspinning propagation expressed in the nonspinning frame. θ''_B is the corresponding error in the spinning propagation expressed in body coordinates. The angular velocity errors, $\Delta\omega'_N$ and $\Delta\omega''_B$, are defined as follows:

$$\Delta\omega'_N \equiv A_{N/B}\,[I - \hat{s}_B\,(\hat{s}_B)^T]\;G\,(\omega_B)\,\Delta a \tag{23}$$

$$\Delta\omega_B'' \equiv \hat{s}_B (\hat{s}_B)^T G(\omega_B) \Delta a \tag{24}$$

θ_N' and θ_B'' can be solved for as in the one-step propagation to give solutions of the form

$$\theta_N' = \Gamma_{N/I} \Delta a + \Phi_{N/I} \theta_N'(t_o) \tag{25}$$

$$\theta_B'' = \Gamma_{B/N} \Delta a + \Phi_{B/N} \theta_B''(t_o) \tag{26}$$

where the variational matrices $\Gamma_{N/I}$ and $\Gamma_{B/N}$ are computed as follows:

$$\Gamma_{N/I} = \int_{t_o}^{t} \Phi_{N/I}(t,\tau) \Delta\omega_N' \, d\tau \tag{27}$$

$$\Gamma_{B/N} = \int_{t_o}^{t} \Phi_{B/N}(t,\tau) \Delta\omega_B'' \, d\tau \tag{28}$$

The total propagation error, θ_B, then equals

$$\theta_B = A_{B/N} \theta_N' + \theta_B'' \tag{29}$$

This gives the partial derivative of the attitude error with respect to the gyro calibration errors as

$$\Gamma_{B/I} = \Gamma_{B/N} + \Phi_{B/N} \Gamma_{N/I} \tag{30}$$

which can be used in Equation (18) to solve for Δa.

NUMERICAL SIMULATION

Although the large COBE gyro sampling interval can be expected to degrade one-step propagation accuracy, it is useful to know how much of an effect it actually has. The nonspinning method must justify its additional computation with significantly better accuracy.

To compute the propagation error for each method, the pitch rate, $\dot{\phi}$, roll rate, $\dot{\theta}$, and spin rate, $\dot{\psi}$, measured with respect to the inertial frame are assumed to be constant.

$$\dot{\phi} = -360°/6000 \text{ sec} = -0.06°/\text{sec} \tag{31}$$

$$\dot{\theta} = 0 \tag{32}$$

$$\dot{\psi} = -0.8 \text{ rpm} = -4.8°/\text{sec} \tag{33}$$

The exact attitude is then found by first performing the pitch rotation, ϕ, followed by the roll and spin rotations, θ and ψ.

$$\phi = (t - t_o)\,\dot{\phi} + \phi_o \tag{34}$$

$$\theta = -4° \tag{35}$$

$$\psi = (t - t_o)\,\dot{\psi} + \psi_o \tag{36}$$

The one-step attitude can be propagated from the following formula for the angular velocity (Reference 1, p. 765):

$$\omega_B\,(t)^T = [\dot{\psi} - \sin(\theta)\,\dot{\phi},\ \cos(\theta)\ \sin(\psi)\,\dot{\phi},\ \cos(\theta)\ \cos(\psi)\,\dot{\phi}] \tag{37}$$

The two angular velocities for the nonspinning propagation are as follows:

$$\omega_N'\,(t)^T = [0,\ \cos(\omega_{B1}t)\,\omega_{B2} - \sin(\omega_{B1}t)\,\omega_{B3},\ \sin(\omega_{B1}t)\,\omega_{B2} + \cos(\omega_{B1}t)\,\omega_{B3}] \tag{38}$$

$$\omega_B''{}^T = [\dot{\psi} - \sin(\theta)\,\dot{\phi},\ 0,\ 0] \tag{39}$$

The one-step and nonspinning attitudes are then compared to the exact attitude, and the angular differences are computed. A plot of the one-step propagation error for 0.5-second sampling intervals is shown in Figure 2 for a timespan of one orbit. While the one-step error grows to a maximum value of 2.4 degrees, the nonspinning error remains less than 0.003 degree.

The results show that, even for the smallest gyro sampling interval, which is 0.5 second, the one-step propagation errors are quite large. This is counterintuitive. Since a constant spin can be propagated without error, it would be expected that adding a much smaller constant pitch rate would have a negligible effect. The flaw in that argument is that, although the spin is constant in the body frame, the pitch angular velocity is not. It is constant in the inertial frame. The assumption about the size of the error is also slightly misplaced. Small pitch rates do produce slowly growing propagation errors, but because the orbit is correspondingly longer, they have more time to grow.

Even more surprising than the size of the error is its oscillation. Rather than grow without limit, the propagation error peaks at the middle of the orbit. The reason is that while the actual pitch angular velocity moves continuously in the body frame, the sampled pitch angular velocity is fixed in the direction it has at the start of the interval. Thus, the sampled value lags the true value and introduces a roll component of angular velocity (Figure 3). Over an orbit period, the roll direction changes by 360 degrees, and the propagation error that builds up in the first half of the orbit decreases over the second half.

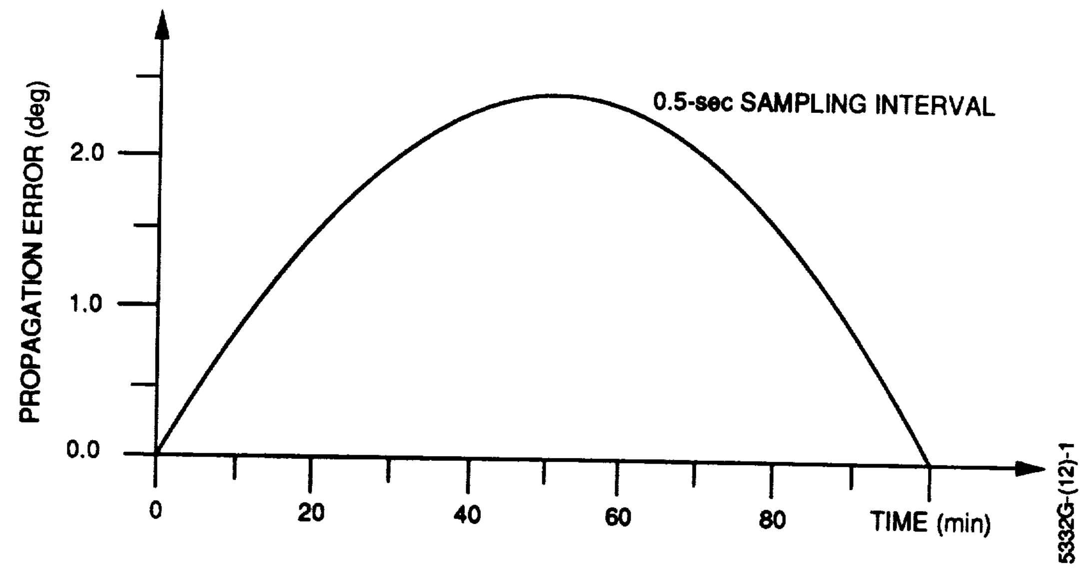

Figure 2. One-Step Propagation Error Time Dependence

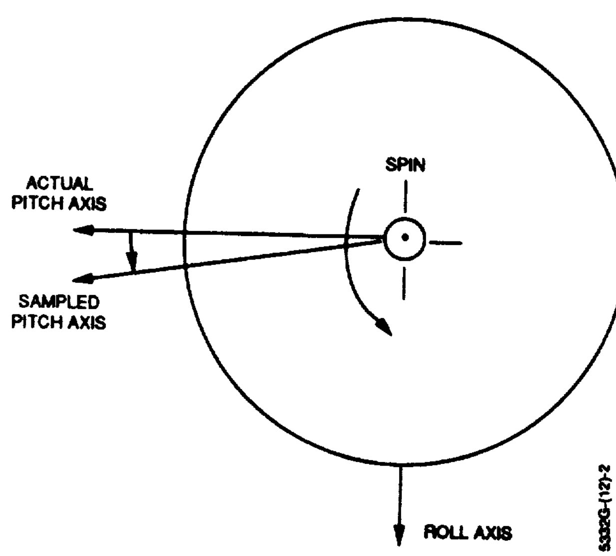

Figure 3. Sampling Introduces a Roll Angular Velocity

SUMMARY

The nonspinning propagation method described here is a means of trading computation for accuracy when the body angular velocity changes direction between gyro samples. This method is currently implemented for batch attitude determination in the COBE flight dynamics support system (Reference 3, p. 3.1.2.42-1), where it is needed to meet the attitude determination accuracy goal of 1 degree (3σ). Further investigation should still be done on understanding the effects of nonconstant roll, interpolating the angular velocity to the midpoint of the sampling interval, and using higher order numerical integration methods.

Whether nonspinning propagation is worth the extra work for other missions depends on the magnitude and form of the angular velocity, the gyro sampling rate, and the accuracy requirements. The unexpectedly large errors that would have been observed for COBE, however, argue for consideration of this effect whenever a three-axis stabilized spacecraft undergoes a compound rotational motion, such as spinning and pitching.

ACKNOWLEDGMENTS

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