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SIMULATION OF SPACECRAFT ATTITUDE DYNAMICS USING TREETOPS AND MODEL-SPECIFIC COMPUTER CODES

John E. Cochran, Jr.,* T. S. No† and
Norman G. Fitz-Coy†

Auburn University, Alabama 36849

ABSTRACT

The simulation of spacecraft attitude dynamics and control using the generic, multi-body code called TREETOPS and other codes written especially to simulate particular systems is discussed. Differences in the methods used to derive equations of motion--Kane's method for TREETOPS and the Lagrangian and Newton-Euler methods, respectively, for the other two codes--are considered. Simulation results from the TREETOPS code are compared with those from the other two codes for two example systems. One system is a chain of rigid bodies; the other consists of two rigid bodies attached to a flexible base body. Since the computer codes were developed independently, consistent results serve as a verification of the correctness of all the programs. Differences in the results are discussed. Results for the two-rigid-body, one-flexible-body system are useful also as information on multi-body, flexible, pointing payload dynamics.

INTRODUCTION

Since the launch of Explorer I and the realization, based on its anomalous attitude time history,[1] that a spacecraft generally could not be considered a rigid body, the field of spacecraft attitude dynamics and control has developed to the point that many methods of analysis[2,3,4,5,6] and numerous attitude dynamics and control simulation codes[7,8,9,] are now available. The volume of literature in the area of spacecraft attitude dynamics is great enough that we will not attempt to review even the part more directly concerned with multibody spacecraft. The purpose of this paper is merely to consider some methods for developing equations of motion for multi-body spacecraft and to compare results obtained from a rather general digital simulation code called TREETOPS[8] with those from simulations which are model-specific.

First, we will consider the use of the Newton-Euler method, the Lagrangian method with quasi-coordinates[10,11] and Kane's method[4] for deriving equations of motion for both rigid and flexible multi-body spacecraft models. Second, we will discuss, briefly, the computer codes used to obtain comparative results. Third, we will present some examples of results obtained from the computer codes.

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*Professor, Department of Aerospace Engineering.

†Graduate Student, Department of Aerospace Engineering.

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METHODS FOR DERIVING EQUATIONS OF MOTION

To illustrate the use of the three methods for deriving equations of motion, we adopt the simple two-body model shown in Fig. 1. Our motivation for doing this is that a chain configuration will be considered in the examples. Body B_1, of mass m_1, is rigid and body B_2, of mass m_2, is either rigid, or flexible, at our convenience. The bodies have centers of mass of C_1 and C_2, respectively, and move with respect to an inertial frame N in which a dextral, orthogonal coordinate system, OXYZ, with its associated unit vectors $\hat{\underline{n}}_j$, j=1,2,3, is fixed. Body B_j has a centroidal inertia dyadic $\underline{\underline{I}}_j$. For body B_1, $\underline{\underline{I}}_1$ is constant. If we decide that B_2 is rigid, $\underline{\underline{I}}_2$ is also constant. But, generally, $\underline{\underline{I}}_2$ varies with time when B_2 is flexible.

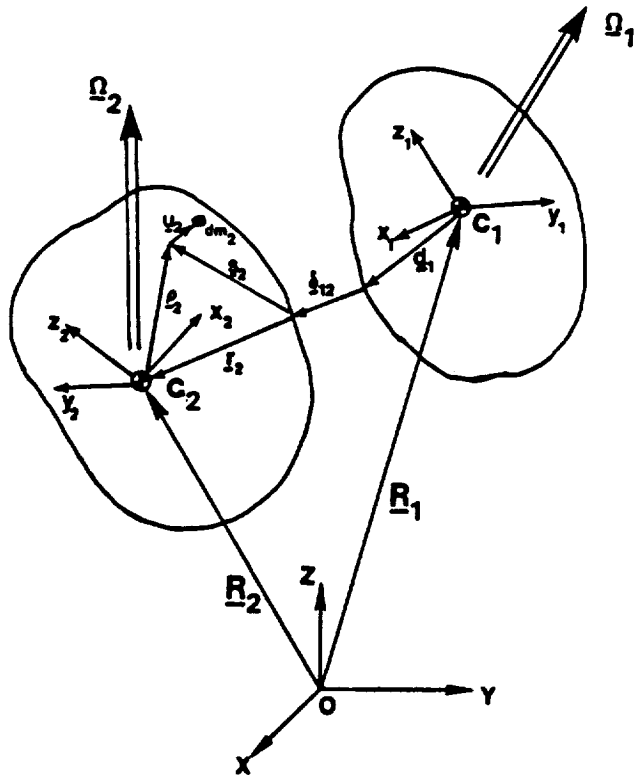

Fig. 1 Two-body system for example.

We let $\underline{R}_j$ denote the position vector from 0 to C_j, $\underline{\rho}_j$ the vector from C_j to an arbitrary element of mass dm_j in body B_j when that body is undeformed and $\underline{u}_j$ denote the displacement of dm_j from the position it occupies when body B_j is deformed. In addition, we let $\underline{\Omega}_j$ denote the angular velocity of a coordinate system, $C_j x_j y_j z_j$, in body B_j. For j=1, the $C_1 x_1 y_1 z_1$ coordinate system is <u>fixed</u> in B_1. For j=2, we may let the $C_2 x_2 y_2 z_2$ system be such that $\int \tilde{\underline{u}}_2 \tilde{\underline{u}}_2 dm_2$ (where $\tilde{\underline{u}}_2$ is the skew-symmetric matrix

counterpart of $\underline{u}_2 x$) is diagonal, or some other condition can be used to define the orientation of $C_2 x_2 y_2 z_2$ in B_2.[2]

For convenience, we let $\underline{u}_2 = \sum_{k=1}^{n} \underline{\phi}_k q_k$, where the $\underline{\phi}_k$ are modal vectors and are functions of the undeformed coordinates of dm_2.

Newton-Euler Equations

The Newton-Euler method is to write equations for the translation and rotation of each body subject to external, or active,[4] forces and moments and forces and moments of constraint.

For body j, if $\underline{F}_{je}$ and $\underline{F}_{jc}$ are, respectively, the external force on body j and the constraint force,

$$m_j \ddot{\underline{R}}_j = \underline{F}_{je} + \underline{F}_{jc} \quad , \quad j = 1,2, \tag{1}$$

Also, for body 1, if $\underline{M}_{1e}$ and $\underline{M}_{1c}$ are the external moment and constraint moment, respectively, we have,

$$\underline{\underline{I}}_1 \cdot \dot{\underline{\Omega}}_1 + \underline{\Omega}_1 x \, \underline{\underline{I}}_1 \cdot \underline{\Omega}_1 = \underline{M}_{1e} + \underline{M}_{1c} \tag{2}$$

The equations of motion for body B_2 are somewhat different, of course, if it is flexible. First, to obtain an independent equation for each q_ℓ, we may take the fundamental equation for the acceleration of dm_2,

$$(\ddot{\underline{R}}_2 + \ddot{\underline{\rho}}_2 + \ddot{\underline{u}}_2) dm_2 = d\underline{f}_2 \quad , \tag{3}$$

where $d\underline{f}_2$ is the force on dm_2, expand $\ddot{\underline{\rho}}_2$ and $\ddot{\underline{u}}_2$, dot multiply by $\underline{\phi}_\ell$ and integrate over the mass of the body to get

$$\begin{aligned} \int \underline{\phi}_\ell dm_2 \cdot \ddot{\underline{R}}_2 = {} & \underline{\Omega}_2 \cdot \int \underline{\phi}_\ell x [(\underline{\rho}_2 + \underline{u}_2) x \, \underline{\Omega}_2] dm_2 \\ & - \int [\underline{\phi}_\ell x (\underline{\rho}_2 + \underline{u}_2)] dm_2 \cdot \dot{\underline{\Omega}}_2 \\ & - 2 \int \underline{\phi}_\ell x \, \mathring{\underline{u}}_2 \, dm_2 \cdot \underline{\Omega}_2 \\ & - \int \underline{\phi}_\ell \cdot \overset{\infty}{\underline{u}}_2 \, dm_2 = \int \underline{\phi}_\ell \cdot d\underline{f}_2 \end{aligned} \tag{4}$$

where $\overset{\circ}{\underline{u}}_2$ and $\overset{\infty}{\underline{u}}_2$ are the time derivatives of $\underline{u}_2$ in the coordinate system $C_2x_2y_2z_2$. Here,

$$\underline{Q}_\ell = \int \underline{\phi}_\ell \cdot d\underline{f}_2 \tag{5}$$

contains contributions due to the external forces on B_2. Also, if $\underline{\phi}_\ell$ is not compatible with the constraints, then $\underline{Q}_\ell$ will contain terms due to the constraint forces.

An equation for the rotational motion of B_2 is also required. To find one, we may cross $\underline{\rho}_2 + \underline{u}_2$ into Eq. (4) and integrate over the mass of B_2 to get

$$\begin{aligned} &- \int (\underline{\rho}_2+\underline{u}_2) \times[(\underline{\rho}_2 \times \underline{u}_2) \times \dot{\underline{\Omega}}_2]dm_2 - \underline{\Omega}_2 \times \int (\underline{\rho}_2+\underline{u}_2)\times[(\underline{\rho}_2+\underline{u}_2)\times \underline{\Omega}_2]dm_2 \\ &- 2 \int (\underline{\rho}_2+\underline{u}_2) \times(\overset{\circ}{\underline{u}}_2 \times \underline{\Omega}_2)dm_2 \\ &- \int (\underline{\rho}_2+\underline{u}_2) \times \overset{\infty}{\underline{u}}_2 \, dm_2 \\ &= \int (\underline{\rho}_2+\underline{u}_2) \times d\underline{f}_2 = \underline{M}_{2e} + \underline{M}_{2c} \end{aligned} \tag{6}$$

If B_2 is rigid, we can reduce Eq. (6) to

$$\underline{\underline{I}}_2 \cdot \dot{\underline{\Omega}}_2 + \underline{\Omega}_2 \times \underline{\underline{I}}_2 \cdot \underline{\Omega}_2 = \underline{M}_{2e} + \underline{M}_{2c} \tag{7}$$

We will consider that B_1 and B_2 are coupled together with a hinge which allows rotation of B_2 with respect to B_1 with three degrees of freedom. In such a case, we can consider $\underline{M}_{1c} = -\underline{M}_{2c}$ to be a function of state variables such as the components of $\underline{\Omega}_{2/1} = \underline{\Omega}_2 - \underline{\Omega}_1$ and the angles used to describe the relative rotational motion.

The constraint force $\underline{F}_{1c} = -\underline{F}_{2c}$ can be found by subtracting the first of Eqs. (1) from the second and simplifying, i.e.,

$$\underline{F}_{1c} = \frac{1}{2}(\underline{F}_{1e} - \underline{F}_{2e} - m_2 \ddot{\underline{R}}_2 + m_1 \ddot{\underline{R}}_1) \tag{8}$$

From Eq. (8) and Eqs. (1) we can obtain verifications of the well known fact that the center of mass of the system, C, moves according to

$$\ddot{\underline{R}} = \frac{1}{M}(\underline{F}_{1e} + \underline{F}_{2e}) \tag{9}$$

where $\underline{R} = (m_1\underline{R}_1 + m_2\underline{R}_2)/M$ and $M = m_1 + m_2$.

Equations (2), (4), (6) and (9) define the motion except for that of C_2. One way to get an equation for the motion of C_2 is to write (see Fig. 2)

$$m_2\underline{c}_2 + m_1\underline{c}_1 = \underline{0} \tag{10}$$

and

$$\underline{c}_1 - \underline{c}_2 = -\underline{d}_1 - \underline{r}_2 \tag{11}$$

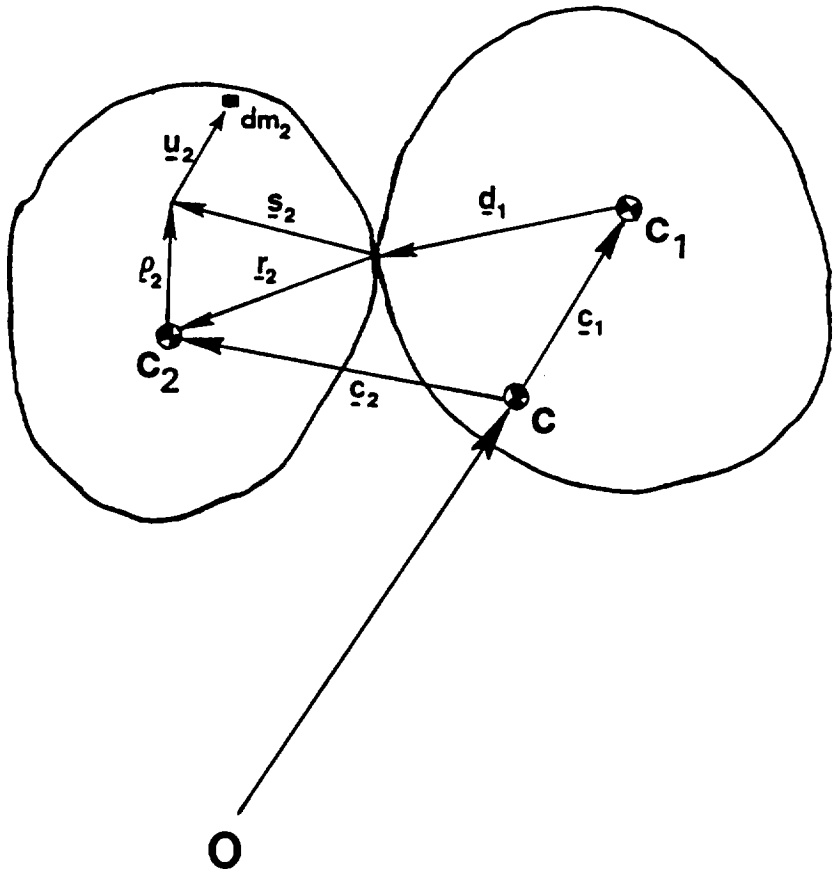

Fig. 2 Position vectors.

Then, since

$$\underline{r}_2 = \int (\underline{s}_2 + \underline{u}_2)dm_2 \tag{12}$$

and from Eq. (10),

$$\underline{c}_1 = -(m_2/m_1)\underline{c}_2, \tag{13}$$

we find that

$$M/m_1\ \underline{c}_2 = \underline{d}_1 + \frac{1}{m_2}\int(\underline{s}_2 + \underline{u}_2)dm_2 \tag{14}$$

Our dependent variables are $\underline{R}$, $\dot{\underline{R}}$, $\underline{\Omega}_1$, $\underline{\Omega}_2$, the q_k, k=1,2,...,n, and suitable orientation variables for B_1 and the relative angular orientation of B_2 with respect to B_1.

Lagrange's Method

An ad hoc procedure based directly on Newton's equations of motion is not as attractive to many analysts as one which includes a "recipe" for obtaining the desired result. For complex dynamical systems subject to holonomic constraints a modification of Lagrange's method often leads more easily, or at least more directly, to equations of motion which are first order in the derivatives of "quasi-coordinates." The quasi-coordinates are introduced by Whittaker by homogeneous differential forms in generalized coordinates. For our example, we can take as generalized coordinates θ_{1j} and θ_{2j}, j=1,2,3, Euler angles which define the attitude of B_1 with respect to the OXYZ system and the attitude of B_2 with respect to B_1, respectively, the q_j, j=1,2,...,n, associated with the vibrational modes and the coordinates of the center of mass of B_1 and B_2. Then, for convenience, we define

$$\underline{q}^* = (X_1\ Y_1\ Z_1\ X_2\ Y_2\ Z_2\ \theta_{11}\ \theta_{12}\ \theta_{13}\ \theta_{21}\ \theta_{22}\ \theta_{23}\ q_1\ q_2\ \cdots\ q_n)^T \tag{15}$$

An N=12 + n vector of quasi-coordinates, $\underline{\pi}$, can be defined by

$$d\underline{\pi} = A\ d\underline{q}^* \tag{16}$$

where A is a non-singular NxN matrix of functions of the q_k^*, k=1,2,...,N, and possibly the time.

In particular, the π_k can be chosen so that

$$\underline{\Omega}_1 = \begin{bmatrix} d\pi_7/dt \\ d\pi_8/dt \\ d\pi_9/dt \end{bmatrix} \tag{17}$$

and

$$\underline{\Omega}_2 = \begin{bmatrix} d\pi_{10}/dt \\ d\pi_{11}/dt \\ d\pi_{12}/dt \end{bmatrix} \tag{18}$$

The other π_k may be identical to the other original generalized coordinates, or we may take

$$\begin{aligned} d\pi_1/dt &= \dot{X}_1 \\ d\pi_2/dt &= \dot{Y}_1 \\ d\pi_3/dt &= \dot{Z}_1 \end{aligned} \tag{19}$$

and use Eqs. (10)-(14) to write the components of the vector $\underline{r} = x_{12}\,\hat{i}_1 + y_{12}\,\hat{j}_1 + z_{12}\,\hat{k}_1$ from C_1 to C_2 in terms of the Euler angles θ_{2j} and the q_k. This equation would be a vector (3x1) holomonic constraint.

Lagrange's equations using the q_k^* are, in matrix form,

$$\frac{d}{dt}\left[\frac{\partial T}{\partial \dot{\underline{q}}^*}\right] - \frac{\partial T}{\partial \underline{q}^*} = \underline{Q}^T \,, \tag{20}$$

where T is the kinetic energy of the system and $\underline{Q}$ is an N vector of generalized forces.

If we let

$$\underline{\Omega} = d\underline{\pi}/dt \tag{21}$$

and

$$B = A^{-1} \,, \tag{22}$$

then Eqs. (20) may be transformed into

$$\frac{d}{dt}\left[\frac{\partial \bar{T}}{\partial \underline{\Omega}}\right] + \left[\frac{\partial \bar{T}}{\partial \underline{\Omega}}\right] C^T - \frac{\partial \bar{T}}{\partial \underline{q}^*} D^T = \underline{N}^T \,, \tag{23}$$

where $\bar{T}$ is expressed using $\underline{\Omega}$ and $\underline{q}^*$, C and D are NxN matrices, and $\underline{N}$ is a generalized force matrix. For a single rigid body of mass m_1 centroidal inertia dyadic $\underline{\underline{I}}_1$ and center of mass velocity $\underline{V}_1$,

$$T = 1/2\ \underline{\Omega}_1 \cdot \underline{\underline{I}} \cdot \underline{\Omega}_1 + 1/2\ m_1 \underline{V}_1 \cdot \underline{V}_1 . \tag{24}$$

Or, in matrix form, body-fixed basis, we have

$$T = 1/2\ \underline{\Omega}_1^T\ \underline{\underline{I}}_1\ \underline{\Omega}_1 + 1/2\ m_1 \underline{V}_1^T\ \underline{V}_1 \tag{25}$$

Thus, if we let $\underline{\Omega}^T = (\underline{\Omega}_1^T\ \underline{V}_1^T)$,

$$T = \frac{1}{2}\ \underline{\Omega}^T \begin{bmatrix} J & 0 \\ 0 & m_1 I \end{bmatrix} \underline{\Omega} \tag{26}$$

where I is the 3x3 identity matrix and J is the inertia matrix, then

$$\frac{\partial \bar{T}}{\partial \underline{\Omega}} = \underline{\Omega}^T \begin{bmatrix} J & 0 \\ 0 & m_1 I \end{bmatrix} \tag{27}$$

Note that $\partial \bar{T}/\partial \underline{\Omega}$ does not contain the q_k^*, explicitly, and

$$\frac{d}{dt} \left(\frac{\partial \bar{T}}{\partial \underline{\Omega}}\right)^T = \begin{bmatrix} J & 0 \\ 0 & m_1 I \end{bmatrix} \dot{\underline{\Omega}} \tag{28}$$

In this case, we have

$$C = \begin{bmatrix} \tilde{\underline{\Omega}}_1 & \\ & 0 \end{bmatrix} \tag{29}$$

and

$$\underline{N} = \begin{bmatrix} \underline{F} \\ \underline{M} \end{bmatrix} \tag{30}$$

For the two-body example, we get a matrix equation of the form

$$M \begin{bmatrix} \dot{\underline{V}}_1 \\ \dot{\underline{V}}_2 \\ \dot{\underline{\Omega}}_1 \\ \dot{\underline{\Omega}}_2 \\ \ddot{\underline{q}} \end{bmatrix} = \underline{N} \ . \tag{31}$$

Kane's Method

Kane's method for deriving equations of motion is based on the use of "partial velocities and partial angular velocities" (see Ref. 4, pp. 87-90 and Chapter Four) to extract from Newton's equations of motion a sufficient set of equations of motion in terms of chosen variables, the so-called "generalized speeds" and and finding partial coordinates. Kane's procedure for a system of N particles with n degrees of freedom consists of (1) choosing generalized speeds, generalized velocities and partial angular velocities; (2) writing $\underline{F}_i$, the resultant force on each particle, m_i, in the system; (3) writing the acceleration, $\underline{a}_i$, i=1,2,...n; (3) dotting each of the partial velocities ($\underline{V}_r$) in turn, into $\underline{F}_i - m_i\underline{a}_i = \underline{0}$ and summing over the particles. The basic equation used is

$$F_r + F_r^* = 0 \quad (r=1,\ldots,n) \ , \tag{34}$$

where

$$F_r = \sum_{i=1}^{N} \underline{V}_r \cdot \underline{R}_i \quad (r=1,2,\ldots,n) \tag{35}$$

are the generalized active forces and

$$F_r^* = \sum_{i=1}^{N} \underline{V}_r \cdot (-m_i \underline{a}_i) \quad (r=1,2,\ldots,n) \tag{36}$$

are the generalized inertia forces.

For our two-body example, we may use the equations,

$$\underline{\Omega}_\ell = \Omega_{\ell 1}\hat{\underline{i}}_\ell + \Omega_{\ell 2}\hat{\underline{j}}_\ell + \Omega_{\ell 3}\hat{\underline{k}}_\ell \ , \quad \ell=1,2, \tag{37}$$

$$\underline{V} = \dot{X}\,\hat{\underline{n}}_1 + \dot{Y}\,\underline{n}_2 + \dot{Z}\,\hat{\underline{n}}_3 \tag{38}$$

and

$$\dot{\underline{u}}_2 = \sum_{k=1}^{n} \underline{\phi}_k \dot{q}_k + \underline{\Omega}_2 \times \sum_{k=1}^{n} \underline{\phi}_k q_k \ , \tag{39}$$

where

$$\underline{\phi}_k = \phi_{k1} \hat{\underline{i}}_2 + \phi_{k2} \hat{\underline{j}}_2 + \phi_{k3} \hat{\underline{k}}_2 \ , \tag{40}$$

to identify the partial velocities $\hat{\underline{n}}_r$, r=1,2,3, of C; partial angular velocities $\hat{\underline{i}}_\ell, \hat{\underline{j}}_\ell, \hat{\underline{k}}_\ell$, ℓ=1,2, of Body ℓ; and the partial velocities $\phi_{k1}\hat{\underline{i}}_2$, $\phi_{k2}\hat{\underline{j}}_2$ and $\phi_{k3}\hat{\underline{k}}_2$, k=1,2,...,n, of the elements of B_2 due to deformation.

By writing the acceleration of an element in each boedy, as we did in the Newton-Euler method, we can obtain the $\underline{V}_r$ to substitute into Eq. (36). The equations are basically the same in form as those found using the Newton-Euler method. However, the procedure is well defined rather than ad hoc.

COMPUTER CODES

Four digital computer programs for simulating multi-body dynamics have been used to obtain the results which follow. There is a model-specific program written to simulate the system shown in Fig. 3. The three-body satellite (actually a sounding rocket payload[12]) consists of a rigid body to which are attached two booms carrying sphere. Equations of motion[12] were obtained directly from Newton's laws and programmed in a special code.

A second program[10] called MBODY was developed to model a chain of rigid bodies. The equations for this more general model were derived using Lagrange's equation with quasi-coordinates.

TREETOPS, the third program to simulate example systems, is based on equations of motion obtained by applying Kane's method. The latest version, which apparently is still in the development stage, contains rather general models of flexible bodies interconnected in a tree topology and of active control elements.

TREETOPS is intended to be useful control system analysis tool.

A fourth program, called FMBODY, has been developed along the same lines as MBODY to handle flexible as well as rigid bodies. This code has not been fully checked out, but some results from it are included in the next section.

EXAMPLES

Simulation results for several example spacecraft models have been generated using MBODY, TREETOPS, the model-specific code and FMBODY. Results for three spacecraft models are presented here.

The first model, depicted in Fig. 3, consists of a rigid body and two rigid booms. Physical data for the model, which is intended to represent the SPEAR-1 sounding rocket payload,[12] are given in Table 1.

Table 1. Physical Characteristics of Model 1.

1. Main Body

   Mass: 300 kg

   Moments of Inertia:

$$I_{xx} = 100 \text{ kg-m}^2$$
$$I_{yy} = 400 \text{ kg-m}^2$$
$$I_{zz} = 400 \text{ kg-m}^2$$

   Distance from Boom Attachment Point to Center of Mass of Main Body: 6 m

2. Booms

   Length: 2 m

   Moments of Inertia (Rods Neglected):

$$I_{xx} = I_{yy} = I_{zz} : \quad 10 \text{ kg-m}^2$$

Table 2 gives the initial conditions for two cases in which the booms rotate from positions parallel to the main body's axis of symmetry toward orientations in which booms are perpendicular to the symmetry axis. In both cases, the system is initially spinning about its symmetry axis and the external torque is zero throughout the motion. In Case I, the deployment is symmetric, since the booms initially have equal and opposite angular velocities with respect to the main body. In Case II, the booms start with different magnitude relative angular rates.

Figure 4 shows the spin rate (Ω_{11}) time history for Case I. Although it is not representative of an actual deployment, the booms rotate through approximately 190 deg in 5 s. The results for the SPECIAL PROGRAM and MBODY are in exact agreement. The Ω_{11} from TREETOPS begins to disagree with the other results at around 2.5 s, but the values at t = 5 s, all look the same.

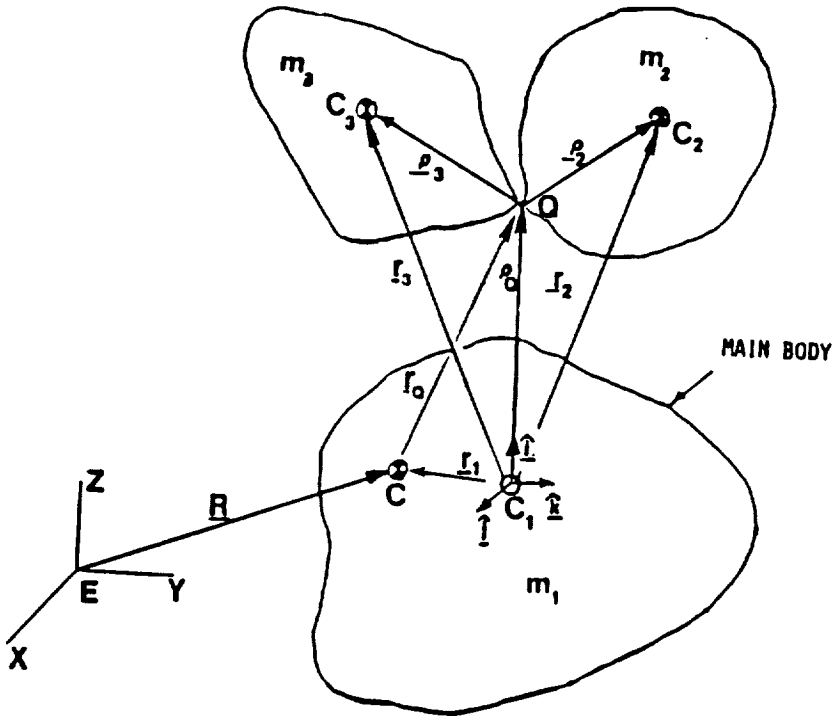

Fig. 3 SPEAR-1 rigid body model.

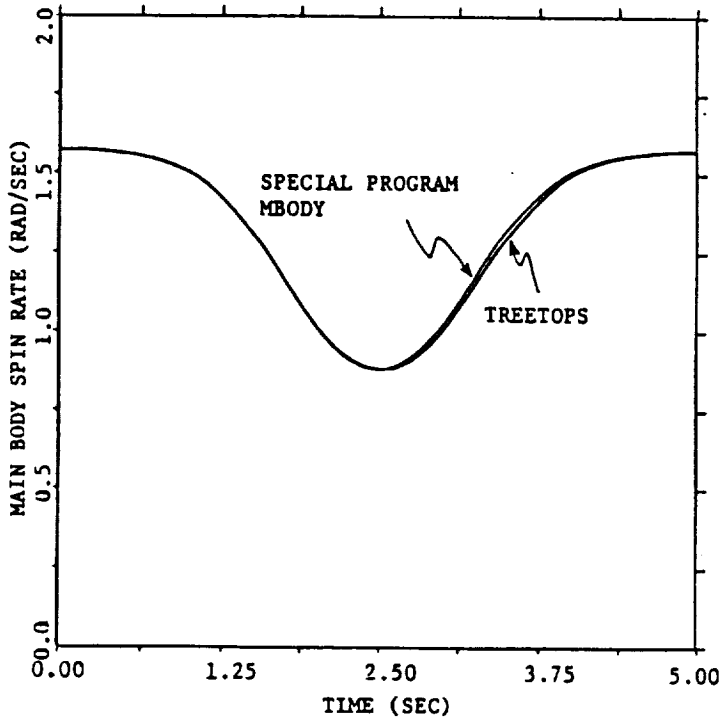

Fig. 4. Spin rate time history (Case I).

The same characteristic is seen in the deployment rate time histories shown in Fig. 5. The small differences in the TREETOPS results are reflected in

Table 2. Initial Conditions for Example 1 Results

CASE I: Main Body Spinning/Symmetric Deployment of Booms

$\underline{\Omega}_1(0) = (90 \quad 0 \quad 0)^T$ deg/sec

$\Omega_{2/1}(0) = (0 \quad 0 \quad 10)^T$ deg/sec

$\Omega_{3/1}(0) = (0 \quad 0 \quad -10)^T$ deg/sec

CASE II: Main Body Spinning Asymmetric Deployment of Booms

$\underline{\Omega}_1(0) = (90 \quad 0 \quad 0)^T$ deg/sec

$\Omega_{2/1}(0) = (0 \quad 0 \quad 10)^T$ deg/sec

$\Omega_{3/1}(0) = (0 \quad 0 \quad -5)^T$ deg/sec

the plot of H, the magnitude of the angular momentum of the system about its center of mass, versus time shown in Fig. 6. The reasons for the small variations in H have not been determined, but it is conjectured that they are due to lack of numerical precision or the way in which constraints are enforced.

Case II is an asymmetric deployment of the booms. The results for spin rate (Ω_{11}) are similar (see Figs. 7 and 8) to those for Case I and again there is some difference in the results from MBODY and the Special Program and TREETOPS. The difference is more evident in the results for H given in Fig. 9.

Example 2

The model for the second example is a uniform flexible beam to which two rigid bodies are coupled. Figure 10 shows the geometry of the system and Table 3 gives the values of system constants used to obtain the numerical results. This model is intended to represent a simple multi-body pointing spacecraft. Bodies B_2 and B_3 are those which are to be pointed. The base body, B_1, is flexible and, for the purposes of this example is uncontrolled. Only two mode shapes were used in this example. The motion of the system is described by the inertial position of the center of mass of B_1, the attitude of B_1 (θ_{1j}, j=1,2,3), the attitudes of B_2 and B_3 with respect to B_1 (θ_{2j} and θ_{3j}, j=1,2,3) and the generalized coordinates q_k, k=1,2,3,4.

The system is initially quiescent. At t=0, torques are applied to B_2 and B_3 about axes parallel to the y-axis and passing through the points of attachment of B_2 and B_3, respectively.

The time histories of the angles θ_{j3}, j=1,2,3, are shown in Fig. 11. Figure 12 shows the time history of the two non-zero generalized coordinates q_{11} and q_{12}, for deformation in the x-direction. As expected, the base body rotated clockwise around the y-axis. It also translated in the z-direction.

Results from TREETOPS for this example have not been obtained as of this writing since a new version of TREETOPS was installed recently on a VAX 785 at Auburn University and a few problems have not been resolved. Additional results will be available soon.

Table 3. Physical Characteristics of Model for Example 2.

Body 1

Mass: 500 kg

Moments of Inertia:

I_{xx} = 4333.33 kg-m^2

I_{yy} = 4333.33 kg-m^2

I_{zz} = 333.33 kg-m^2

Stiffness Characteristics:

EI = 100 N-m^2 ,

Uniform

Dimensions:

a = b = 2 m, c = 10 m
d = 1 m, h = 2 m
d_1 = d_2 = 1 m

Bodies 2 and 3

Mass: 100 kg

Moments of Inertia:

I_{xx} = 12.5 kg-m^2

I_{yy} = 39.6 kg-m^2

I_{zz} = 39.6 kg-m^2

Dimensions:

d = 1 m, h = 2 m

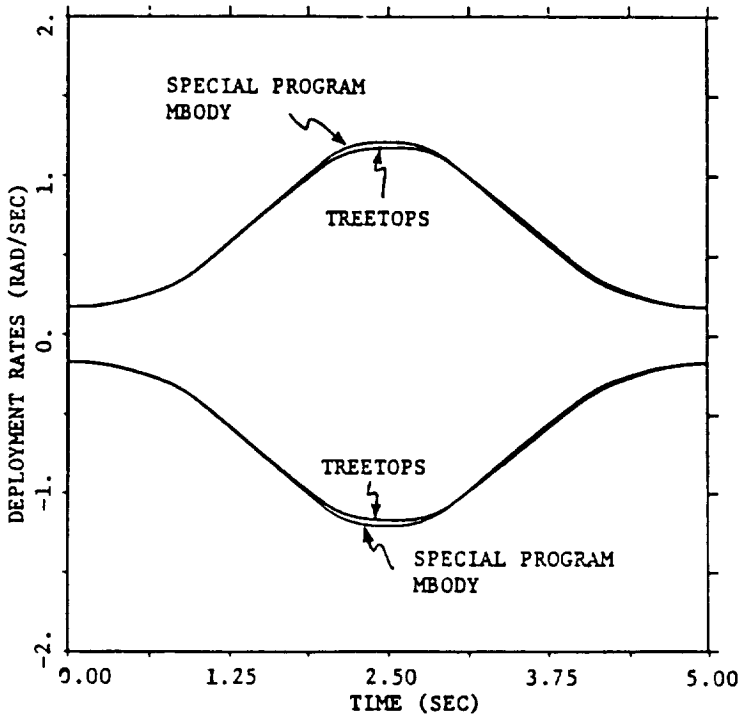

Fig. 5 Deployment rate time history (Case I).

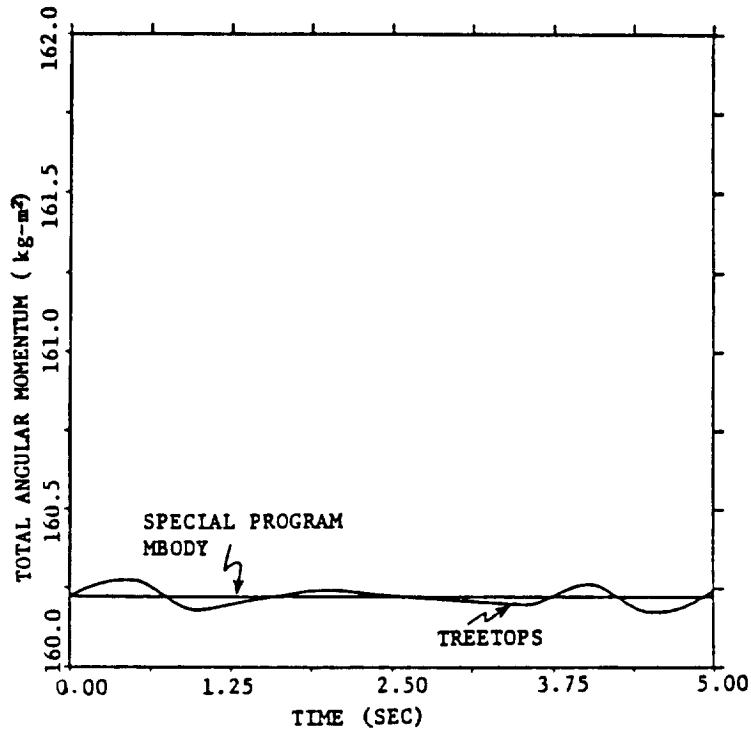

Fig. 6 Magnitude of the total angular momentum (Case I).

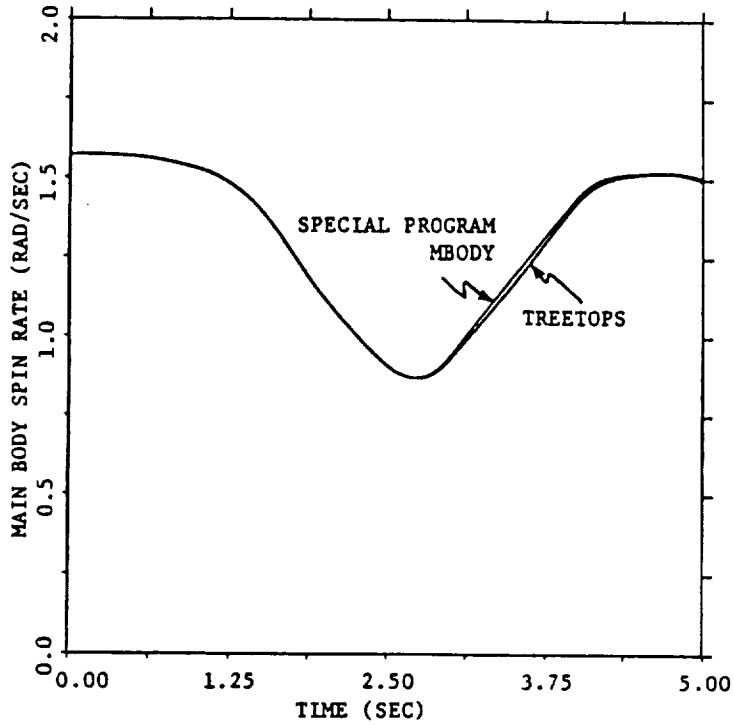

Fig. 7 Spin rate time history (Case II).

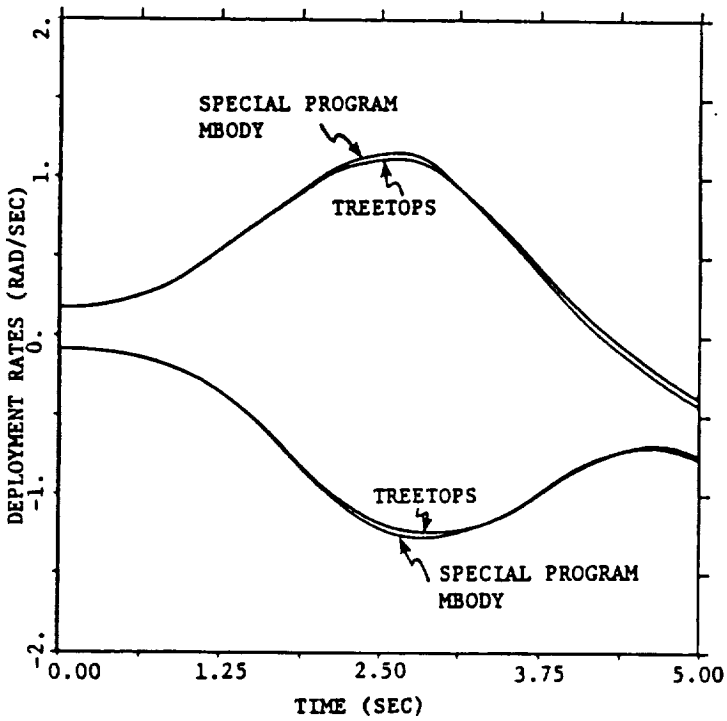

Fig. 8 Deployment rate time history (Case II).

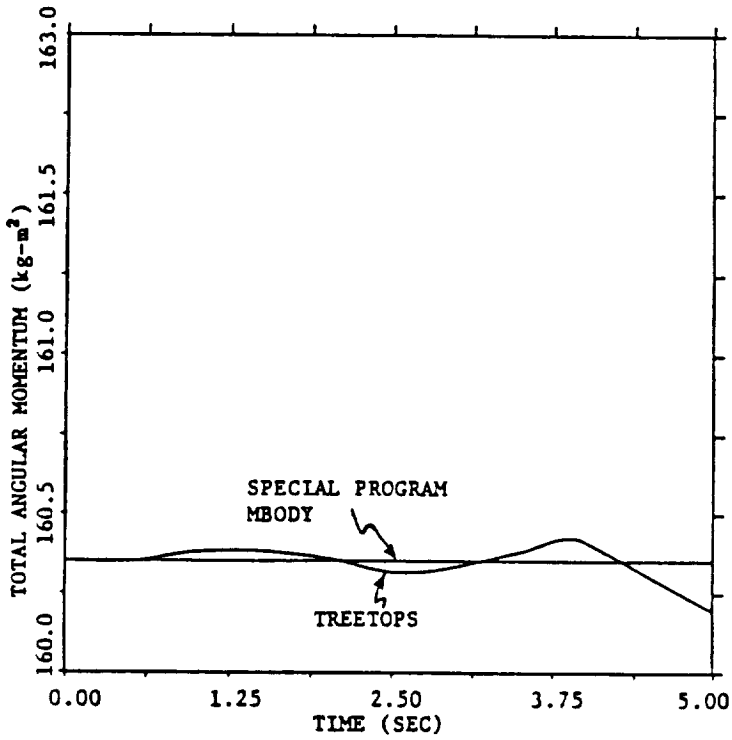

Fig. 9 Magnitude of the total angular momentum (Case II).

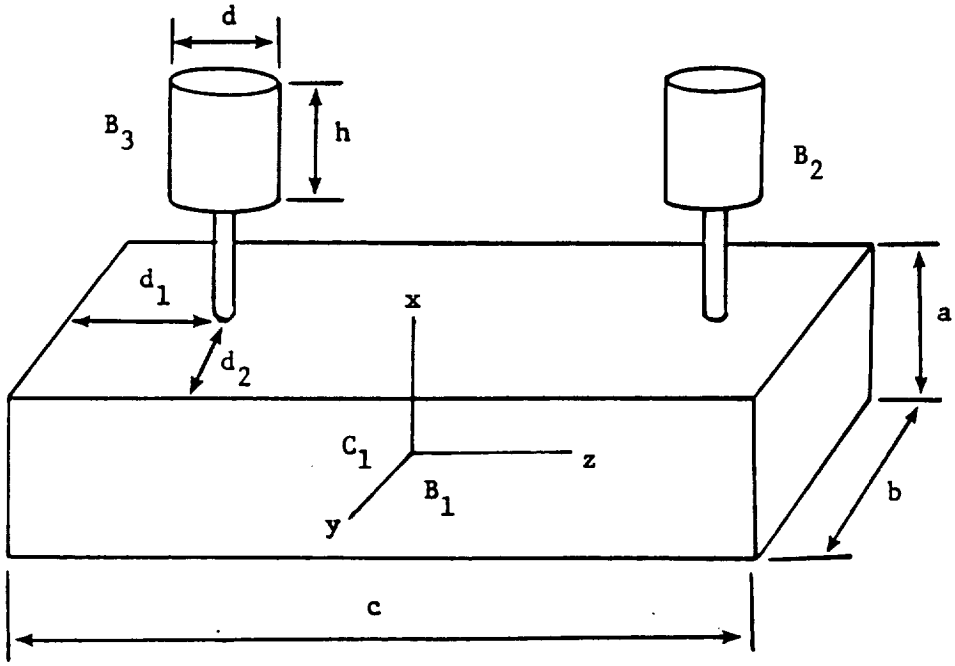

Fig. 10 Model for a multi-body pointing satellite.

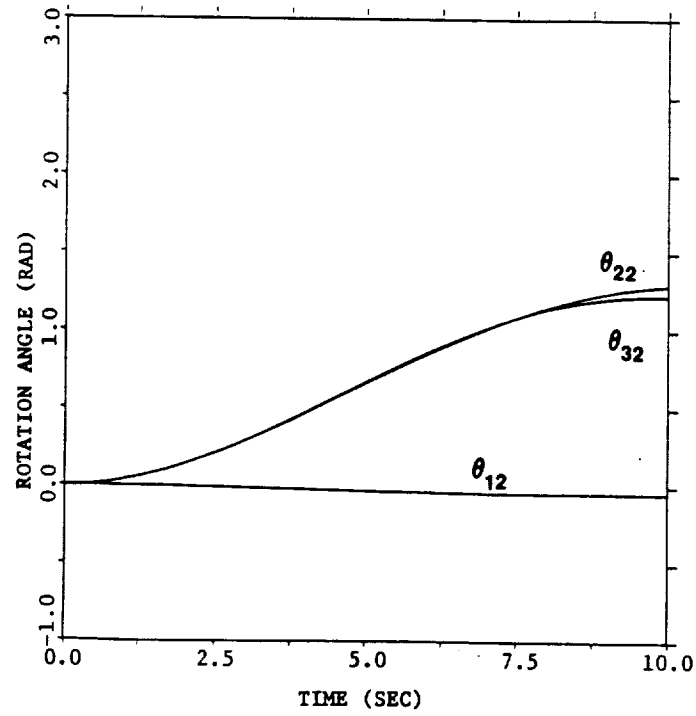

Fig. 11 Attitude angles, bodies 1, 2 and 3.

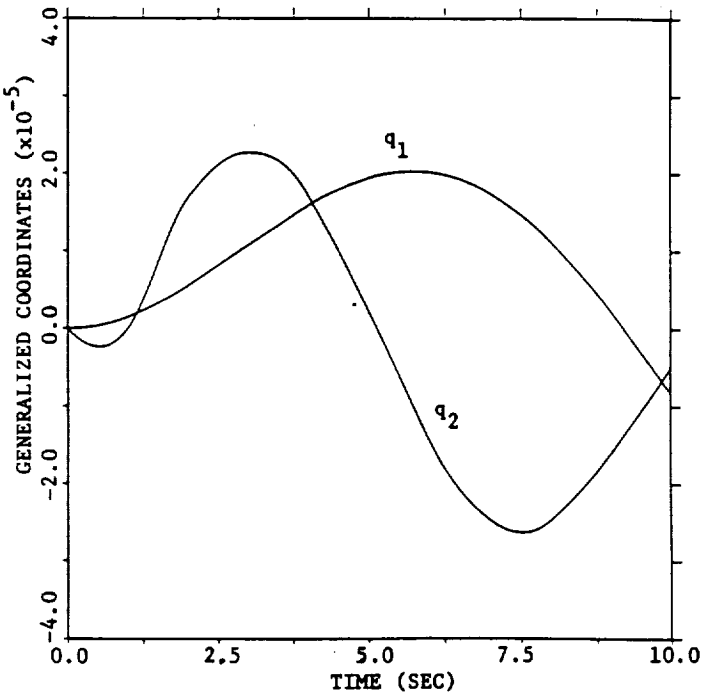

Fig. 12 Generalized coordinates associated with the modes.

CONCLUSIONS

Methods for deriving equations which mathematically model multi-body pointing spacecraft have been discussed. None of the three methods considered appear clearly superior from both the aspects of understanding the system and generating equations.

Results obtained using the model-specific code, based on Newton-Euler equations, and MBODY, based on equations derived using Lagrange's equations and quasi-coordinates, agree to within the numerical precision used. Thus, both of these programs are probably correct. The TREETOPS results differ slightly, but probably not significantly, from those obtained from the model specific code and MBODY. The reason for a non-constant computed angular momentum magnitude may lie in the method for computing the angular momentum. Addition of checking needs to be done to determine the exact course.

For the multi-body pointing spacecraft example, we obtained, and have presented here, results from a program, FMBODY, based on equations derived using Lagrange's equations with quasi-coordinates and flexible body model data. We were not able by the time this paper was submitted to get results from a new, updated TREETOPS program. However, it is expected that we will find that TREETOPS and FMBODY results agree and that TREETOPS requires less CPU time.

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FLIGHT MECHANICS/ESTIMATION THEORY SYMPOSIUM

SESSION 2