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A GYROSCOPE CALIBRATION ANALYSIS FOR THE GAMMA RAY OBSERVATORY (GRO)*

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ABSTRACT

The attitude analysts of the Flight Dynamics Division (FDD) are responsible for calibrating, among other sensors, inertial reference units (IRU), a crucial activity for accurate attitude determination. The IRU calibration utility (IRUCAL) for the Gamma Ray Observatory (GRO) spacecraft, based on an algorithm developed by P. Davenport, includes user-specified weighting matrices for the measurements, for the a priori misalignments, and for the a priori biases. By assigning "large" values to the appropriate a priori weighting matrix elements, one can choose to adjust only the biases, only the misalignments, or some combination of the two. Different weight matrices produce vastly different biases and misalignments for the same measurement.

Current documentation and software do not adequately address the calculation and use of the optimal weight matrices involved in calibrating the IRU. This study investigates several facets of the GRO IRU calibration as it relates to the bias and misalignment weighting matrices. The physical meaning and use of the bias and misalignment weight matrices in IRU calibration are examined. The relation of the weighting and the final biases, misalignments, and their corrections are pursued.

Ultimately, methods for determining reliable, realistic weighting matrices to be used in the GRO IRUCAL utility are determined. Possible correlations among observation uncertainties are also explored. For the undetermined case where the maneuvers are insufficient to identify all calibration parameters, the weighting matrices allow as much information as possible to be extracted from the measurements. Finally, applicable simulated flight data are used, incorporating the appropriate calibration maneuvers, to test the weighting matrices in the IRUCAL utility, and examine correlation effects.

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1. INTRODUCTION

The gyro calibration process contains many subtleties. Data can be used and interpreted several ways; identical data can be processed through the same software yet could achieve vastly different calibration results. A paper in the May 1988 Flight Mechanics and Estimation Theory Symposium (Reference 1) investigates the gyroscope calibration for the Hubble Space Telescope, using the same algorithm as the Gamma Ray Observatory (GRO) software. Last year's paper, by Davenport and Welter, examines the selection of the loss function weight matrix when more accurate attitude sensor information is available in pitch and yaw than roll; the study, however, ignores the use of a priori information, assigning a zero weight to that term in the loss equation. The GRO mission will not encounter such situations during normal operations but could benefit from incorporating a priori information into the gyro calibration effort. This paper examines the careful use of a priori gyro information and covariance in calibration for the GRO mission and also considers any implications for future missions.

The GRO mission will employ an inertial reference unit (IRU) consisting of three two-degree-of-freedom gyros built by Teledyne Systems Company. This National Aeronautics and Space Administration (NASA) standard IRU, DRIRU-II, has flown successfully on several missions, including the Solar Maximum Mission (SMM). Each gyro in the IRU contains two orthogonal sensing axes and are oriented to provide redundant sensing about each of these axes. References 2 and 3 contain a more detailed description of the IRU.

The general method that the Flight Dynamics Facility (FDF) will use to calibrate the IRU from the ground comes from the algorithm used by SMM. A period of fixed-inertial attitude will be followed by an attitude maneuver. A period of constant attitude will then follow the maneuver. An attitude solution will be determined using fixed-head star tracker (FHST) data for both periods of fixed attitude. Gyro data are collected and compared to the attitude solution generated by the FHSTs. Biases, misalignments, and scale factors can then be determined. The basic mathematics for gyro calibration is presented first as background for the reader. Further documentation is referenced for a more thorough discussion.

The search for a priori information begins with past missions. To anticipate the kinds of biases and misalignments GRO's IRU might experience during launch, past missions that used and calibrated the DRIRU-II in flight were examined. Unfortunately, Landsat did not fully calibrate its DRIRU-II because of the nature of the mission. SMM, however, used a calibration scheme similar to GRO's; the information from SMM's early mission could, therefore, be applied to the GRO effort. Unfortunately, some information is not available from the SMM calibration effort, so workarounds were developed where possible. A complete plan is, therefore, offered in Section 7 of this study so that all information will be available from GRO's early mission. These data can be analyzed for future launches to help establish appropriate a priori information to be incorporated into IRU calibration efforts for future missions.

During the GRO mission, flight dynamics analysts will be using the IRUCAL utility (part of the GRO Attitude Ground Support System (AGSS)) to process gyro data and calibrate the IRU. In this utility, the user is given the option of including a priori information. The user is also allowed to weigh this information as he/she chooses. IRUCAL is sensitive to these weights, as identical information weighted differently results in dissimilar calibration results. Section 6 of this study explores this sensitivity by performing simulations. The dynamic simulator will create gyro data that will be processed through the AGSS. These processed data will then be put through simulations involving the IRUCAL, including different a priori estimates and weights. Different weighting schemes will be incorporated, noting the sensitivity of the results to weighting changes.

The final portion of this study presents conclusions and recommendations. Unfortunately, little is documented concerning past use of the DRIRU-II and in-flight calibration as far as actual early mission data. Some data could, however, be deduced and incorporated into the GRO IRU calibration effort. The study furnishes a detailed list and schedule of early calibration activities. During GRO's launch it will prove crucial to keep track of various information not only to test out new a priori weighting schemes but to provide critical information for missions still in the planning stages.

2. GRO GYRO CALIBRATION ALGORITHM

This discussion of the GRO gyro calibration algorithm closely follows Reference 1, and an alternate derivation can be found in Reference 4. The calibration algorithm for GRO assumes that a three-vector **R** is output from the gyros, and the measured angular velocity is given by

$$\mathbf{\Omega}_M = G_0 \mathbf{R} - \mathbf{D}_0 \qquad (1)$$

where G_0 is the 3-by-3 scale factor/alignment matrix and $\mathbf{D}_0$ is the drift-rate bias. The algorithm determines a correction matrix M to G_0 and a correction to the bias, **d**. Ideally, the true angular rate Ω is found using the corrected alignment matrix G and bias **D** via the following equations:

$$G = M G_0 \qquad (2a)$$

$$\mathbf{D} = M \mathbf{D}_0 + \mathbf{d} \qquad (2b)$$

$$\mathbf{\Omega} = G \mathbf{R} - \mathbf{D} = M \mathbf{\Omega}_M - \mathbf{d} \qquad (2c)$$

Let $\mathbf{m} = M - I$, for I the 3-by-3 identity matrix and define the difference between the measured and true angular rate ω by

$$\omega = \mathbf{\Omega}_M - \mathbf{\Omega} = -\mathbf{m}\,\mathbf{\Omega}_M + \mathbf{d} \qquad (3)$$

The elements of **m** and **d** are the parameters solved for in the calibration algorithm. These parameters can be related to attitude solutions as determined through data from sensors such as star trackers if gyro output data are available between attitude solutions.

Again following Reference 1, let attitude solutions at time t be denoted by Q(t) in quaternion form. The quaternion time derivative is given by

$$Q'(t) = Q(t)\, q(\Omega/2) \qquad (4)$$

where $q(\Omega/2)$ is a quaternion with vector component $\Omega/2$ and scalar component zero. Let Q be the quaternion representing the true rotation for a maneuver and Q_M be the quaternion representing the rotation as determined by the gyro output. The attitude error quaternion δQ expressing the rotation from the gyro-determined postmaneuver attitude to the true postmaneuver attitude is given by

$$\delta Q = Q_M\,(Q_M^{-1}\, Q)\, Q_M^{-1} = Q\, Q_M^{-1} \qquad (5)$$

Applying the chain rule of differentiation to Equation (5) above gives

$$\delta Q' = Q\, q\,(\Omega/2)\, Q_m^{-1} + Q\, q^{-1}\,(\Omega_M/2)\, Q_m^{-1} \qquad (6)$$

Noting that

$$q^{-1}\,(\Omega_M/2) = q\,(-\Omega_M/2)$$

and using Equation (3) results in

$$\delta Q' = Q\, q\,(-\omega/2)\, Q_m^{-1} \qquad (7)$$

Integrating Equation (7) over the maneuver gives

$$\delta Q - 1 = \int Q\, q\,(-\omega/2)\, Q_m^{-1}\, dt \qquad (8)$$

where 1, the identity quaternion, is the constant of integration. Let Q_{R1} and Q_{R2} be the reference quaternions at the beginning and end of the maneuver (for GRO, these come from the Fine Attitude Determination System (FADS)) so that

$$Q = Q_{R1}^{-1}\, Q_{R2}$$

Similarly, define the gyro propagated quaternions Q_{G1} and Q_{G2} so that

$$Q_M = Q_{G1}^{-1}\, Q_{G2}$$

Equation (8) then becomes

$$(Q_{R1}^{-1}\, Q_{R2})\,(Q_{G2}^{-2}\, Q_{G1}) - 1 = \int Q\, q\,(-\omega/2)\, Q_M^{-1}\, dt \qquad (9)$$

The first order, Q can be replaced by Q_M in Equation (9). When this substitution is made, the integrand becomes the quaternion representation for the rotation of the vector $-\omega/2$ from the spacecraft coordinates at time t to the premaneuver spacecraft coordinates. Dropping the scalar portion of Equation (9) and substituting for Q gives

$$\mathbf{Z}_i = \frac{1}{2} \int T_i \, \omega \, dt \tag{10}$$

where $\mathbf{Z}_i$ is the vector component of δQ, T_i is the (time-dependent) matrix transforming vectors to premaneuver spacecraft coordinates, and i is a subscript designating maneuver number. By dropping the scalar part of Equation (9), an approximation is made equating 1 to the cosine of the error rotation angle. Because of these approximations, the calibration algorithm is by nature an iterative process.

Least-squares techniques can be applied to Equation (10). Rewrite Equation (10) as

$$\mathbf{Z} = H \, \mathbf{x} \tag{11}$$

where **Z** is composed of the (assumed) n $\mathbf{Z}_i$ vectors for n maneuvers, and **x** is defined as shown below:

$$\mathbf{Z} = \{\mathbf{Z}_1^T, \mathbf{Z}_2^T, \ldots, \mathbf{Z}_n^T\}^T \tag{12a}$$

$$\mathbf{x} = \{\mathbf{m}_{11}, \mathbf{m}_{12}, \mathbf{m}_{13}, \mathbf{m}_{21}, \mathbf{m}_{22}, \mathbf{m}_{23}, \mathbf{m}_{31}, \mathbf{m}_{32}, \mathbf{m}_{33}, \mathbf{d}_1, \mathbf{d}_2, \mathbf{d}_3\} \tag{12b}$$

H is a 3n-by-12 matrix of the form

$$H = \begin{bmatrix} U_1 & -Y_1 \\ \vdots & \vdots \\ U_n & -Y_n \end{bmatrix} \tag{13}$$

where each U_1 is a 3-by-9 matrix with components given by

$$(U_{j,k+3(i-1)})_1 = \frac{1}{2} \int (T_{jk})_1 \, (\Omega_M)_1 \, dt \tag{14}$$

and each Y_1 is a 3-by-3 matrix given by

$$(Y_{jk})_1 = \frac{1}{2} \int (T_{jk})_1 \, dt \tag{15}$$

An observed value for Z derived from combination of Q's from the GRO FADS and the gyro's and is assumed to be of the form

$$\mathbf{Z}_{obs} = H \, \mathbf{x}_t + \mathbf{v} \tag{16}$$

where $\mathbf{x}_t$ contains the true bias and alignment parameters and $\mathbf{v}$ is the measurement error in $\mathbf{Z}_{obs}$. The loss function J for the least-squares solution is chosen to be

$$J = \frac{1}{2} \{\mathbf{E}^T W \mathbf{E} + (\mathbf{x} - \mathbf{x}_a)^T S_a (\mathbf{x} - \mathbf{x}_a)\} \tag{17}$$

where

$$\mathbf{E} = \mathbf{Z}_{obs} - H\,\mathbf{x} \tag{18}$$

W and S_a are symmetric nonnegative definite weighting matrices, and $\mathbf{x}_a$ is an a priori estimate of $\mathbf{x}$. $\mathbf{x}^*$, the least-squares solution for $\mathbf{x}$, is given by setting the derivative of the loss function with respect to $\mathbf{x}$ equal to zero,

$$0 = H^T W [\mathbf{Z}_{obs} - H\,\mathbf{x}^*] - S_a [\mathbf{x}^* - \mathbf{x}_a] \tag{19}$$

or

$$\mathbf{x}^* = \{H^T W H + S_a\}^{-1} [H^T W \mathbf{Z}_{obs} + S_a\,\mathbf{x}_a] \tag{20}$$

Define

$$\delta\mathbf{x} = \mathbf{x}_t - \mathbf{x}^*$$

and substitute Equation (16) into Equation (19) to give

$$\delta\mathbf{x} = \{H^T W H + S_a\}^{-1} [H^T W\,\mathbf{v} - S_a (\mathbf{x}_t - \mathbf{x}_a)] \tag{21}$$

Let

$$N = \{H^T W H + S_a\}^{-1}$$

If the correlation between $\mathbf{v}$ and $(\mathbf{x}_t - \mathbf{x}_a)$ is assumed to be zero, the covariance for $\delta\mathbf{x}$ can be written as

$$< \delta\mathbf{x}\, \delta\mathbf{x}^T > = N \{H^T W < \mathbf{v}\,\mathbf{v}^T > W H + S_a < (\mathbf{x}_t - \mathbf{x}_a)(\mathbf{x}_t - \mathbf{x}_a)^T > S_a\} N \tag{22}$$

If the optimal weightings,

$$W = < \mathbf{v}\,\mathbf{v}^T >^{-1} \tag{23a}$$

and

$$S_a = < (\mathbf{x}_t - \mathbf{x}_a)(\mathbf{x}_t - \mathbf{x}_a)^T >^{-1} \tag{23b}$$

are used, Equation (22) reduces to

$$< \delta x \, \delta x^T > = N \tag{24}$$

3. CROSS-CORRELATION OF ERRORS

Equation (23a) above identifies the optimal weighting of the maneuvers for gyro calibration as the W matrix given by

$$W = < v \, v^T >^{-1} \tag{25}$$

where v = measurement error in $\mathbf{Z_{obs}}$

The GRO FADS computes the error for an attitude in a form appropriate for computing the error, v, for a maneuver. A maneuver, or calibration interval, is a time interval with gyro-rate information and accurate attitude solutions (available at the end points of the interval). The set of angles (measured in radians) of the small rotation carrying the true attitude matrix into the measured attitude matrix in the current spacecraft body frame defines the covariance of the error angle vector. The FADS computes this covariance for each attitude solution. Combining the attitude errors at each end of the maneuver comprises the total measurement error for the $\mathbf{Z_{obs}}$ corresponding to a single maneuver.

As long as the calibration interval is large enough so that the attitude solutions incorporate different star vectors, the attitude errors at the ends of the maneuver will be uncorrelated. The weight matrix is then given by

$$W = \{P_i + T_{if} \, P_f \, T_{if}^T\}^{-1} \tag{26}$$

where

P_i = covariance of the initial error angle vector

P_f = covariance of the final error angle vector

T_{if} = transformation from spacecraft body frame at end of maneuver to body frame at start of maneuver

A difficulty arises for the more general case where $\mathbf{Z_{obs}}$ contains multiple intervals. When an attitude solution is used at both the end of one interval and the start of another, the covariance of v contains significant cross-correlation terms. GRO's IRU calibration algorithm neglects these cross-correlation terms.

In general, the effect of cross-correlations remains small as long as the algorithm is used efficiently. When given a time interval containing accurate attitude solutions (at the end points and inside the interval), those who calibrate the GRO IRU should choose their calibration interval with care. Only one calibration interval of maximum length should be formed; the original time interval should not be broken into two or more calibration intervals, thereby avoiding using one attitude solution for two calibration intervals.

4. INFORMATION FROM PAST MISSIONS

The IRU calibration algorithm for GRO is capable of including the weighting of the a priori estimates of the components of **x**. From Equation (23b), the optimal weighting matrix associated with this a priori information is

$$S_a = < (x_t - x_a)(x_t - x_a)^T >^{-1} \quad (27)$$

Since the a priori values of **x** will always be zero, S_a reduces to the inverse of the covariance of x_t. The following two scenarios for estimating S_a are expected to occur during GRO's mission, the first of which is the focus of this section:

- GRO's gyros will first be calibrated on the ground. Therefore, when GRO is first deployed, the effects of the launch shock on the IRU will be the major contributor to the uncertainty in x_t.
- Following the first calibration of the gyros, the a priori information becomes simply the covariance of the $\delta\mathbf{x}$ from the previous calibration (propagated to a time appropriate for the current calibration effort).

Due to the lack of rigorous analysis on launch shock effects, only a rough estimate of the statistics of x_t after launch is considered. For this reason, several simplifications are incorporated. The first assumption is that the change in the alignment and bias for each gyro channel from the ground calibration values to the first in-flight calibration is assumed to be a normal random variable with zero mean.

The value of x_t depends on the error in the prelaunch calibration, the change due to launch shock, and all other effects occurring before the first calibration in orbit. Along with the space environment, launch shock is assumed to be the dominant effect. The best source of launch shock information should be past missions that also flew a DRIRU-II. Two missions have flown these IRUs: SMM and Landsat.

Unfortunately, because of the nature of its attitude during mission (constant 1 revolution per orbit (RPO)), Landsat did not calibrate the misalignments, as GRO's algorithm must, but depended solely on bias adjustments to meet accuracy requirements. Therefore, this study relies on SMM as the prime source of information on gyro performance during launch.

Following the development in Reference 5, gyro drift is neglected and an inertial frame is defined as the body frame at the start of the maneuver. The equation

$$\Omega_{inertial} = R\, M_o\, \Omega_M \quad (28)$$

where

R = matrix representing a rotation from the current body frame to the current inertial frame

M_o = alignment correction matrix with perfect reference attitudes

expresses the transformation of the gyro-measured angular velocity into the inertial frame. During a commanded roll slew, the control system will try to rotate the spacecraft about the roll axis (as sensed by the gyros) at a constant rate. The following relations hold

$$\Omega_M = \left(\frac{d\phi}{dt}, 0, 0\right)^T \tag{29}$$

$$R = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \tag{30}$$

where ϕ = roll slew angle at time t from start of maneuver

Integrating Equation (28) yields

$$\theta(M_1^o) = \int_0^t \Omega_{inertial}\, dt \tag{31}$$

$$= T(\phi)\, M_1^o \tag{32}$$

or

$$\theta(M_1^o) = \begin{bmatrix} \phi\, M_{11}^o \\ \sin(\phi)\, M_{21}^o - (1 - \cos\phi)\, M_{31}^o \\ (1 - \cos\phi)\, M_{21}^o + \sin(\phi)\, M_{31}^o \end{bmatrix} \tag{33}$$

where M_1^o is the first column of M^o and

$$T(\phi) = \int_0^\phi R\, d\phi = \begin{bmatrix} \phi & 0 & 0 \\ 0 & \sin\phi & -(1 - \cos\phi) \\ 0 & (1 - \cos\phi) & \sin\phi \end{bmatrix} \tag{34}$$

Let M be the calculated correction matrix containing errors and write

$$M_1 - M_1^o = dM \tag{35}$$

The effect of errors in M_1 on gyro-measured attitude changes is given by

$$\epsilon = \left. \frac{d\theta(M_1^o)}{dM_1} \right|_{M_1 = M_1^o} dM_1 = T(\phi)\, dM_1 \tag{36}$$

Solving Equation (34) for M_{11}, M_{21}, and M_{31}, yields

$$dM_{11} = \frac{\epsilon_1}{\phi} \tag{37a}$$

$$dM_{21} = \left(\frac{\epsilon_2}{2}\right)\left[\cot\left(\frac{\phi}{2}\right) + 1\right] \tag{37b}$$

$$dM_{31} = \left(\frac{\epsilon_3}{2}\right)\left[\cot\left(\frac{\phi}{2}\right) - 1\right] \tag{37c}$$

These equations can be used to give the roll gyro calibration uncertainty in M_{11}, M_{21}, and M_{31} due to the reference attitude uncertainties. For a pitch calibration slew through the angle ϕ_2, the corresponding expressions are

$$dM_{12} = \left(\frac{1}{2}\right)\left[\epsilon_1 \cot\left(\frac{\phi_2}{2}\right) - \epsilon_2\right] \tag{38a}$$

$$dM_{22} = \frac{\epsilon_2}{\phi_2} \tag{38b}$$

$$dM_{32} = \left(\frac{1}{2}\right)\left[\epsilon_2 \cot\left(\frac{\phi_2}{2}\right) + \epsilon_1\right] \tag{38c}$$

Similarly for a yaw calibration maneuver of angle ϕ_2

$$dM_{13} = \left(\frac{1}{2}\right)\left[\epsilon_1 \cot\left(\frac{\phi_3}{2}\right) + \epsilon_2\right] \tag{39a}$$

$$dM_{23} = \left(\frac{1}{2}\right)\left[\epsilon_2 \cot\left(\frac{\phi_3}{2}\right) - \epsilon_1\right] \tag{39b}$$

$$dM_{33} = \frac{\epsilon_3}{\phi_3} \tag{39c}$$

The following results for the first two calibrations (in February and July 1980) of the SMM gyros are taken from Reference 6 and converted to their **M** and **d** forms. Each gyro has two channels. There exist, therefore, eight possible gyro configurations; A1B1C1, A2B1C1, etc. The configurations A1B1C1 and A2B2C2 together contain information from all six gyro channels. Reference 3 quotes an accuracy of 0.005 deg (1σ) for the SMM FHST attitude solutions.

Equations (37) through (39) with all components of ϵ equal to 1.23E–4 radians (0.005 deg spherical uncertainty at the start and end of the calibration interval) were used to compute the uncertainty in the calibration coefficients. Also, a roll slew of 90 deg, a pitch slew of 25 deg, and a yaw slew of 25 deg were assumed (Reference 6). The original calibration plan for SMM calibration plan called for a 55-minute (min) inertial period for the bias calibration (Reference 3), which results in a 2E–6 deg/sec accuracy. Table 1 shows the calibration results and their approximate uncertainties.

Table 1. SMM Calibration Results

CALIBRATED VALUES	A1B1C1 (Feb)	A1B1C1 (Jul)	A2B2C2 (Feb)	A2B2C2 (Jul)	ACCURACY
VALUES x 10^{-4}					
M_{11}	22.0	21.0	1.9	4.0	0.8
M_{21}	10.9	9.8	7.1	6.0	0.6
M_{31}	2.9	4.0	3.3	5.4	0.6
M_{12}	-9.85	-9.11	-8.88	-7.85	0.6
M_{22}	-1.08	2.83	-7.67	-3.39	3.0
M_{32}	-47.4	-47.1	-39.9	-40.4	3.0
M_{13}	-9.67	-6.17	-7.58	-4.59	3.0
M_{23}	26.0	26.2	34.2	34.6	3.0
M_{33}	-10.1	-15.7	-3.34	-8.51	3.0
VALUES x 10^{-6} DEG/SEC					
d_1	-40.828	-39.292	-34.904	-34.493	2.0
d_2	107.80	107.94	-89.301	-87.852	2.0
d_3	-5.8081	-6.0539	-6.7913	-6.7340	2.0

For this study, the measurement uncertainty is sufficiently small to neglect its effect on any conclusions drawn. Several useful observations can be made from the SMM-tabulated calibration results.

The requirements for ground calibration indicate that the absolute alignment error be less than some relatively large tolerance. The GRO specification stipulates the absolute alignment errors be less than 300 arc-seconds (arc-sec) (Reference 2). GRO's prelaunch gyro alignment is, therefore, required to be within this tolerance of the nominal alignment.

A more stringent requirement exists on the measured ground alignment. For GRO, the orientation of the gyro axes to the system mounting interface is required to be known to within ±10 arc-sec. The SMM calibration results assume the nominal alignment and no bias initially. Therefore, the results contain the corrections due to the large absolute alignment errors. This correction can contain directional biases; the x-axis misalignment is likely to be similar for both configurations.

The calibration results show a 0.91 sample correction between the A1B1C1 and A2B2C2 misalignment terms (M_{ij}). The degree to which this effect is due to launch shock cannot be determined without the measured values for the prelaunch misalignments, and unfortunately, these results are no longer available. The sample mean and standard deviation are −1.8 and 21 (x 10^{-4}), respectively, for the A1B1C1 configuration and -2.3 and 19 (x 10^{-4}), respectively, for the A2B2C2 configuration. During the first calibration for the GRO DRIRU-II, the SMM results indicate that for the correction made to the nominal alignment matrix a zero mean can reasonably be assumed, and an uncertainty on the order of 20 (x 10^{-4}) can be expected.

Although the M_{ii} terms contain scale factor and misalignment effects, this estimation treats the M_{ij} identically. For the current analysis, no further effort seems worthwhile; however, it should be noted that assuming that the M_{ij} terms are normal variables allows one to assign a confidence level of 90 percent to the assumption that the standard deviation of the M_{ij} for A1B1C1 is less than 34 (x 10^{-4}).

When the initial estimate of the GRO alignment is taken from the more accurate measured prelaunch alignments, a substantially lower variance in the alignment results can be expected. Environmental testing performed on the DRIRU-II shows that the gyro uncertainties due to any environmental effects are small compared to the absolute alignment errors. The alignment shifts due to environmental tests of Reference 6 reflect a standard deviation of 17 arc-sec, with the absolute alignment errors producing a standard deviation of 111 arc-sec. A 17 arc-sec change in alignment (first order) corresponds to roughly a 0.8 (x 10^{-4}) change in M_{ij} (i not equal to j for alignment effects). The GRO requirement (Reference 5) is for alignment stability to within 20 arc-sec.

Similarly, scale factor changes across the environmental testing were on the order of 120 parts per million (ppm) compared to absolute alignment errors on the order of 1300 ppm. The diagonal elements of M are roughly 1.2 (x 10^{-4}) for a 120 ppm change in scale factor. Uncertainties comparable to this are on the order of the likely measurement errors for GRO calibration, 0.008 deg 1σ attitude solution uncertainty.

For the gyro biases, the environmental tests showed results similar to the alignment data. For the different test temperatures, the environmental change in the biases gave standard deviations ranging from 4 to 8 (x 10^{-6} deg/sec). The standard deviation for the bias itself was on the order of 42 (x 10^{-6} deg/sec). Again, the tested environmental stability was on the order of the attitude solution uncertainties, with the total bias correction an order of magnitude larger.

The following conclusions are drawn considering the a priori weighting matrix for the first in-flight calibration of GRO. For nominal initial calibration alignments and biases, large uncertainties (with respect to the expected measurement errors) should be assigned to the a priori estimates. The calibration algorithm will not be sensitive to the exact uncertainties used in this case; values commensurate with the SMM values are appropriate. If the ground-measured alignments and biases are assumed to be the initial values, uncertainties that agree with the environmental stability requirements are probably appropriate. The a priori alignments and biases will then be weighted roughly equally to the in-flight measurements. Without access to the ground measured alignments and biases for SMM, assuming such small uncertainty due to launch shock, seems presumptuous. For GRO, however, maneuvers sufficient to determine the alignment/scale factor matrices and biases are planned. It is suggested that both approaches be implemented as well as a third approach using the ground-measured alignments and biases and applying large a priori uncertainties. The results for all three approaches should then be analyzed for consistency.

5. PROPAGATION OF COVARIANCE MATRIX

As noted previously, a method is needed to propagate the covariance matrix of the calibration solution **x**. After the first planned set of maneuvers for GRO, it is likely that only partial information will be available for subsequent calibration updates. In this situation, the GRO algorithm operates optimally if the a priori uncertainties are known.

Immediately after the first in-flight calibration, the covariance of the solved-for biases and misalignments can be computed. The uncertainty in the solved-for values then increases with time. The random walk acts as the standard model to describe the time variation of the estimated state vector (for gyro calibration, the 12 vector of the misalignment terms M_{ij} and the bias vector corrections).

For the DRIRU-II, random walk type modeling is used for the short term (roughly 6 hours). However, this type of modeling is not appropriate for the DRIRU-II's long-term behavior. The misalignment and bias corrections appear to be bounded in the long term as opposed to the unbounded behavior of the random walk processes. Reference 7 reports the ground measured 74-month stability value for the serial number 1001 DRIRU-II to be 7.9 arc-sec. The 74-month stability of its scale factor was 58-ppm (dropping the data for the "c" channel due to electronic module changes) and the 66-month stability for the serial number 1004 was 77 ppm. The absolute changes in the gyro biases for these two cases were 0.005 and 0.009 deg/hour, respectively.

Numbers commensurate with these can be used to give conservative estimates of the increase in the state vector corresponding to times of several months or more if the in-flight calibrations results support them. The noise processes leading to the increases in the state vector uncertainty are assumed to be independent. Therefore, only the diagonal terms of the covariance would increase.

The SMM values for misalignment angles can be approximated (in radians to first order) by the off-diagonal values of M. The scale factors (SF_i) resulting from the in-flight calibrations can be computed from the resultant alignment matrix and are displayed in Table 2.

The measurement accuracies are approximately 80 ppm for SF_1 and 300 ppm for SF_2 and SF_3. The error propagation effects are on the order of the measurement noise so that

Table 2. SMM Scale Factors

SCALE FACTOR	A1B1C1 (Feb) (ppm)	A1B1C1 (Jul) (ppm)	A2B2C2 (Feb) (ppm)	A2B2C2 (Jul) (ppm)
SF_1	-2195.6	-2096.1	189.84	399.9
SF_2	-598.40	-727.45	404.31	212.3
SF_3	1706.4	2008.5	683.8	964.5

these effects cannot be observed directly. However, as the derivation of Equations (35) through (42) demonstrates, the attitude solution errors effect on the calculation of M is linear in the attitude error vector ϵ. If, for example, M_{ij} for A1B1C1 in February is subtracted from the February value for A2B2C2, the error due to ϵ cancels, and the result becomes the difference in M_{ij} due to internal effects. If the two configurations are independent, the difference should have a variance given by the sum of the squares of the standard deviation for each configuration for M_{ij}.

The quantity of interest is the change in the difference from February to July. The computations are straightforward, and the sample standard deviation for the six off-diagonal elements of M is 0.38 (x 10^{-4}) or equivalently a misalignment of approximately 7.8 arc-sec. The sample standard deviation for the three scale factors is 74 ppm. These results assume that the M_{ij} have zero means, and the two configurations are independent. A similar process can be applied to the biases, giving a sample standard deviation of 0.78 (x 10^{-6} deg/sec) or 0.003 deg/hour. All the values derived from SMM calibration results are consistent with the reported long-term test results.

The in-flight SMM calibration results support the ground test long-term outcome. Uncertainties based on the ground test results might be applicable to the GRO in-flight calibration effort. Once data are available for GRO, analysis similar to that accomplished in this study can be used to investigate the time propagation of uncertainty for GRO calibration.

6. SUGGESTED CALIBRATION PLAN

This section presents a suggested calibration plan that includes some accepted calibration practices (taken predominantly from the SMM and SM Repair Mission (SMRM), Reference 7) and several suggestions for faster, more accurate gyro calibration. Included also are certain components of calibration, times and initial bias values for example, which should be tracked for future analysis of both the GRO spacecraft and other missions using the same gyro package. Tracking the items could prove to be extremely enlightening for future missions for accurately deducing a priori calibration parameters and their weights. Valuable calibration information should be carefully saved for GRO and all subsequent DRIRU-II missions (particularly STS-launched missions); each case offers one more example of launch and deployment effects.

Basic Procedures

Initially, the gyro biases will be determined while GRO is in a fixed-inertial attitude at the very beginning of the mission. Ideally, at least 1/2 hour of data will be used, comparing the gyro-propagated attitude with a finer FHST solution. This early bias determination will help improve course attitude solutions during orbit night early in the mission.

Once the FHSTs have been calibrated to meet GRO attitude determination requirements, the acquisition of gyro misalignments and scale factors begins. A series of slews are performed. Ideally, six slews of 30 deg each will be performed, for example, beginning in a fixed-inertial attitude, +X axis as the velocity vector and the +Z axis as the orbit normal, then performing a +30-deg roll slew, then back to zero, then a -30-deg roll slew, back to zero and so forth for the other two directions. These are, at this writing, the planned attitude verification slews. The slews should be separated by a period (at least 10 minutes) of fixed-inertial attitude. An attitude solution is determined using the highly accurate FHST data, both before and after the maneuver; it is, therefore, important to plan the maneuvers to ensure star data during the inertial periods between slews. These fine FHST-determined attitude solutions are then compared with the gyro data throughout the maneuver. The IRUCAL utility in the GRO AGSS uses this information to determine the gyro misalignments and scale factors.

As mentioned in Section 3, correlation of errors should be considered during calibration. During a fixed-inertial attitude, it is beneficial to consider observations at the beginning and the end of the span and not to break the interval into two or more spans. Due to the algorithm currently used in the GRO operational software, which ignores the off-diagonal correlation terms, correlation at the shared end points of the smaller intervals discount any benefit from the increased information. To avoid any correlation problems encountered when performing slews, observations at a maneuver's end point should not be used as the beginning point for another maneuver. As long as the slews are separated by at least 10 minutes of fixed-inertial attitude, this should not be a problem.

The gyro calibration process is then complete. Calibration is, however, an iterative process. When the FHST calibration constants are improved, IRUCAL can be rerun using the new FHST information. Calculating the covariance of the gyro calibration solutions, a capability to soon be added to the current operational version of IRUCAL, would be extremely beneficial here. Using the previous results from gyro calibration as the a priori guess for the next iteration ($\mathbf{x}_a$ in Equation (17)) and using the inverse of the propagated covariance to weight the guess (S_a in Equation (17)) would expedite this iterative process.

Calibration Information to Preserve

Some information accumulated during these early calibration phases is not only crucial to GRO postlaunch gyro calibration analysis but also to future missions. As trends begin to emerge from calibration analysis of every mission using the DRIRU-II, accurate assumptions can be made about launch effects and gyro performance. These assumptions in turn can be used to infer a priori calibration coefficient information and weighting. Each launch, with careful documenting, supplies analysts with another case for comparison.

Although each launch and spacecraft is different, with a number of different launches, trends can still be established.

Specific items for consideration are listed below:

- A detailed record of the prelaunch, ground-measured alignments, biases and the times that the measurements were taken
- A detailed record of the first inflight calibration, including the attitude solutions and covariances, time-interval information (average rates, total angles, times), the calibration covariance, and information from all six channels
- A launch acceleration history as it relates to the gyro frame
- Calibration information (as detailed in first calibration above) for all subsequent calibration efforts, to aid in building a time model for the growth in error uncertainty

A series of simulations are planned using the GRO data simulators and operational attitude ground support software to be performed as soon as these tools are available. The GRO Software Simulator (GROSS), which simulates dynamic errors, in conjunction with the GRO telemetry simulator (GROSIM) will generate data for a series of attitude slews separated by inertially fixed intervals. The slews will mimic those planned for early attitude verification where possible (see above). The spacecraft x-axis will point at the Sun and the z-axis parallel to the orbit normal both for simplicity and validity. (During early mission, the GRO Flight Dynamics analysts would like to have the spacecraft x-axis pointing at the Sun to calibrate the Fine Sun Sensor (FSS) and for simplicity.) In this case, inertially fixed intervals (for FHST FADS solutions) will be planned for orbit night to ensure star data (when the FHSTs are viewing away from the Earth and are, therefore, not occulted.)

Data spans will be selected so that correlation effects are small; those with suspected high correlation may also be selected to analyze correlation effects on final calibration results. FADS solutions will be determined during the inertially fixed intervals, while gyroscope data accumulate during the maneuvers. Appropriate a priori calibration estimates will be determined. A series of a priori weights, both correct and with reasonable deviations, will be used with the a priori estimate in IRUCAL. Results from these simulations will be plotted to show sensitivity to a priori weighting changes.

7. CONCLUSIONS AND RECOMMENDATIONS

The current algorithm for GRO gyro calibration does not account for some important yet subtle areas of calibration, while some other useful features are traditionally ignored. The cross correlations of errors does not appear in the IRUCAL process; careful considerations of observation intervals can compensate.

The covariance of the errors in a calibration solution is not currently calculated in the GRO algorithm. This information could, however, prove useful in weighting the a priori state calibration values. With proper, albeit simple, propagation over time, a previous calibration result can be used as an a priori estimate, weighted optimally by the inverse of covariance of the errors. This a priori information, though included in the algorithm (x_a in Equation (17)), has not generally been utilized in past calibration efforts.

Gyro calibration can be a very tricky process. When it is not carefully examined, important information is lost. Every mission using the DRIRU-II can learn about the expected launch effects and performance of their gyro from previous launches. The DRIRU-II gyro package is apparently quite accurate and stable; a priori knowledge could, consequently, be greatly beneficial. Therefore, the careful recording of calibration results of GRO in providing an initial a priori estimate improves the efficiency of the gyro calibration efforts of future missions, and that information for those missions that follow.

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