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ANALYSIS OF THE FLIGHT DYNAMICS OF THE SOLAR MAXIMUM MISSION (SMM) OFF-SUN SCIENTIFIC POINTING*

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ABSTRACT

This paper presents the algorithms created and implemented by the Goddard Space Flight Center's (GSFC's) Solar Maximum Mission (SMM) attitude operations team to support large-angle spacecraft pointing at scientific objectives. The mission objective of the post-repair SMM satellite was to study solar phenomena. However, because the scientific instruments, such as the Coronagraph/Polarimeter (CP) and the Hard X-ray Burst Spectrometer (HXRBS), were able to view objects other than the Sun, attitude operations support for attitude pointing at large angles from the nominal solar-pointing attitudes has been required. Subsequently, attitude support for SMM has been provided for scientific objectives such as Comet Halley, Supernova 1987A, Cygnus X-1, and the Crab Nebula. In addition, the analysis has been extended to include the reverse problem, computing the right ascension and declination of a body given the off-Sun angles. This analysis has led to the computation of the orbits of seven new solar comets seen in the field-of-view (FOV) of the CP. The activities necessary to meet these large-angle attitude-pointing sequences, such as slew sequence planning, viewing-period prediction, and tracking-bias computation are described. Analysis is presented for the computation of manuevers and pointing parameters relative to the SMM-unique, Sun-centered reference frame. Finally, science data and independent attitude solutions are used to evaluate the large-angle pointing performance.

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1. INTRODUCTION

This paper presents

- Algorithms used in support of large-angle attitude pointing maneuvers by the Solar Maximum Mission (SMM) spacecraft to nonsolar scientific objectives
- A brief history of the mission and its primary scientific objectives
- Background of the problem of pointing at celestial objects, especially as pertains to the SMM

The algorithms used to compute the necessary parameters for the observations are derived and results are presented that show the algorithms worked correctly, with accuracy significantly better than that specified by the SMM project. This work was performed by the Flight Dynamics Division (FDD) attitude determination and control analysts in the Flight Dynamics Facility (FDF) at Goddard Space Flight Center (GSFC).

1.1 MISSION HISTORY

The SMM spacecraft was launched in February 1980 from the Eastern Test Range into an approximately circular low-Earth orbit, with an inclination of nearly 28 degrees (deg) (Reference 1). The spacecraft functioned normally until November 1980 when the standard reaction wheel (SRW) package that controls the spacecraft attitude failed. To preserve the mission, the spacecraft was put into a spin, approximately 1 deg per second (sec), about the minor principal axis. While the spacecraft was in this spin mode, very little scientific work was accomplished. In April 1984, the spacecraft was repaired in-orbit by astronauts aboard the Space Transportation System (STS) orbiter. The entire modular attitude control system (MACS) was replaced, and the spacecraft returned to normal operations.

Some time before the repair mission, scientists at the GSFC Laboratory for Astronomy and Solar Physics proposed that the SMM be used to observe Comet Halley for approximately 1 month before and after perihelion of the comet's orbit (February 9, 1986). During this time, the comet, as seen from the Earth, would be between 7 and 45 deg from the Sun, and very little useful ground-based observation was anticipated.

1.2 SMM SCIENTIFIC OBJECTIVES AND PAYLOAD

The original scientific objective of the SMM was to study solar phenomena, especially the solar maximum of 1981. The mission was planned to be three-axis stabilized to keep the scientific payload pointed at the Sun. However, because of the SRW failure, most of the scientific instruments missed the solar maximum. Since the repair of the spacecraft, a tremendous amount of solar radiation data at a variety of wavelengths has been amassed. In addition, beginning with the Comet Halley observations, several interesting nonsolar targets have been and are being studied.

A Coronagraph/Polarimeter (CP), designed to study the solar corona in the visible spectrum, was used to observe Comet Halley and the Moon. In addition, seven solar grazing comets have been discovered during routine operation of the CP. A Hard X-Ray Burst Spectrometer (HXRBS) was pointed at Cygnus X-1 (a suspected black hole) and the Crab Nebula. The Gamma Ray Spectrometer (GRS) was used to study gamma ray output from Supernova 1987A and the Ultra-Violet Spectrometer (UVSP) observed a transit of the Sun by Mercury. All of these observations required attitude excursions from the primary solar objective and provided valuable data for the scientific community on a wide range of celestial objectives.

1.3 MANEUVER REQUIREMENTS

The goal of this analysis is to provide algorithms for attitude control of the SMM so that the scientific payload points at the desired celestial body and tracks it, even if it is moving relatively fast, to keep the body in the instrument field of view (FOV). In addition, the periods during which the celestial body is visible to the spacecraft (i.e., not occulted by the Earth) must be determined.

Requirements for attitude support of the large-angle attitude pointing maneuvers were not defined until several years after the spacecraft had been launched. For this reason, mathematical algorithms were developed for each nonsolar objective, as needed. The algorithms were later combined to form one software system that could efficiently plan the large-angle off-solar pointing maneuvers for any celestial body. The software was designed to plan the maneuver sequence, compute the viewing periods available, and determine the tracking parameters. Once computed, these data are uplinked to the spacecraft.

The constraints imposed by the SMM experimenters were few and were concerned with getting the best observations possible. To accomplish normal pointing, an accuracy of a tenth of a degree was desired. Another consideration was the determination accuracy in the viewing periods. Because of uncertainties in the spacecraft ephemeris, pointing accuracy, and spacecraft gyro drift, accuracy in computing the start and stop times of the viewing periods was estimated to be 10 sec (3σ).

Consideration for spacecraft safety placed other constraints on the maneuvers. The most important constraint was that the spacecraft had to remain in a power-positive mode to ensure survival of the spacecraft following any observation period contingency. This effectively meant that the body-fixed Solar Panel Array would always need to be absorbing at least as much energy as the spacecraft was using. During nominal operation, the solar panels are approximately perpendicular to the spacecraft-to-Sun vector. As a result of the analysis by the power engineers, the maximum off-Sun pointing angle was set at 65 deg.

2. PROBLEM ANALYSIS

2.1 THE SMM REFERENCE FRAMES

The SMM spacecraft attitude is represented in a solar reference frame, referred to as the SUN frame. The X-axis of the SUN frame is defined by the apparent spacecraft-to-Sun unit vector, $\underline{V}_{ss}$. The Y-axis is the unitized cross-product of the Sun's North Pole vector,

$\underline{S}_{np}$, and $\underline{V}_{ss}$. The Z-axis completes the right-handed coordinate system. The equations for the unit vectors along the Sun frame axes are (Reference 2)

$$\underline{X} = \underline{V}_{ss} \tag{1}$$

$$\underline{Y} = a\,(\underline{S}_{np} \times \underline{X}) \tag{2}$$

$$\underline{Z} = a\,(\underline{S}_{np} - (\underline{S}_{np} \cdot \underline{X})\,\underline{X}) \tag{3}$$

where

$$a = [1 - (\underline{S}_{np} \cdot \underline{X})^2]^{-1/2} \tag{4}$$

Figure 1 illustrates the SUN coordinate system.

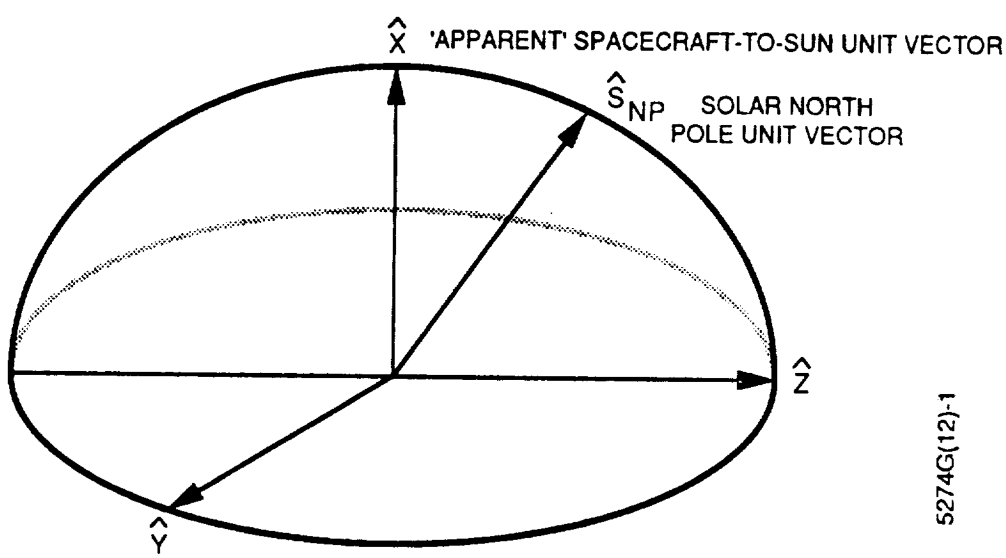

Figure 1. Solar Coordinate System

The transform, A, from geocentric inertial (GCI) coordinates to the SUN frame, is computed from the actual spacecraft and Sun positions with consideration for the Earth and spacecraft velocities and the speed of light to correct for aberration. Thus, for attitude determination, reference vectors must be transformed using A to get the reference vectors in the SUN frame. Due to the spacecraft's orbit about the Earth and the Earth's orbit about the Sun, the SUN frame is constantly changing. Thus, each time the attitude of the spacecraft is computed by the spacecraft's onboard computer (OBC) or the ground system, the SUN frame has to be recomputed.

The SMM spacecraft attitude is represented as the transformation from the SUN frame to the SMM body frame. As shown in Figure 2, the fine-pointing Sun sensor (FPSS) and the scientific instruments are at one end of the spacecraft. During normal operations, the attitude control system is designed to point this end of the spacecraft at small areas of the Sun. The X-axis (roll) of the spacecraft body frame is defined to coincide with the direction of the FPSS boresight. Two orthogonal axes perpendicular to the FPSS boresight and corresponding to the FPSS outputs are defined as the Y- (pitch) and Z- (yaw) axes of the body frame. This alignment is advantageous because the spacecraft pitch and yaw rotations are read directly from the output of the FPSS when the Sun is in the FOV of the FPSS, which is 4-square deg. Thus, during normal operations, the spacecraft is observing the Sun, and the attitude accuracy in pitch and yaw is 5 arc-sec (3σ). The roll is determined by the fixed-head star trackers (FHSTs) to an accuracy of approximately 30 arc-sec (3σ). During periods of solar occultation or spacecraft pitch or yaw offpoint, the Sun is not in the FPSS FOV, and the attitude of all three axes is determined by the FHSTs to an accuracy of approximately 30 arc-sec (3σ). Most of the scientific instrument boresights are parallel to the boresight of the FPSS; thus, the output from the FPSS can be used directly to determine where the instruments are pointing to within 5 arc-sec when the Sun is being observed.

The SMM attitude matrix, B, is the matrix that transforms a vector from the SUN frame to the SMM body frame. If the roll angle is ϕ, the pitch angle θ, and the yaw angle β, the matrix B is (Reference 2)

$$B = T_3(\beta)\, T_2(\theta)\, T_1(\phi) \tag{5}$$

where T_i are the individual matrices for each Euler rotation on the intermediate axes. Thus,

$$B = \left[\begin{array}{c|c|c} \cos\beta\cos\theta & \cos\beta\sin\theta\sin\phi + \sin\beta\cos\phi & -\cos\beta\sin\theta\cos\phi + \sin\beta\sin\phi \\ -\sin\beta\cos\theta & -\sin\beta\sin\theta\sin\phi + \cos\beta\cos\phi & \sin\beta\sin\theta\cos\phi + \cos\beta\sin\phi \\ \sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{array}\right] \tag{6}$$

2.2 CALCULATION OF POINTING MANEUVERS

Pointing at a celestial object involves rotating the SMM to align its X-axis with the spacecraft-to-target vector. In general, pointing from an initial target to any other target on the celestial sphere can be accomplished by consecutive rotations about any two orthogonal axes (Reference 3). Thus, any pair of rotations of roll, pitch, or yaw will suffice. For convenience, the first maneuver is chosen to be a roll. After this, the four choices for the second maneuver are positive or negative pitch or yaw. The magnitude of any of these second maneuvers is simply the angular distance from the Sun to the target objective. For this algorithm, the roll corresponding to each choice of second rotations is computed, and the smallest roll is chosen.

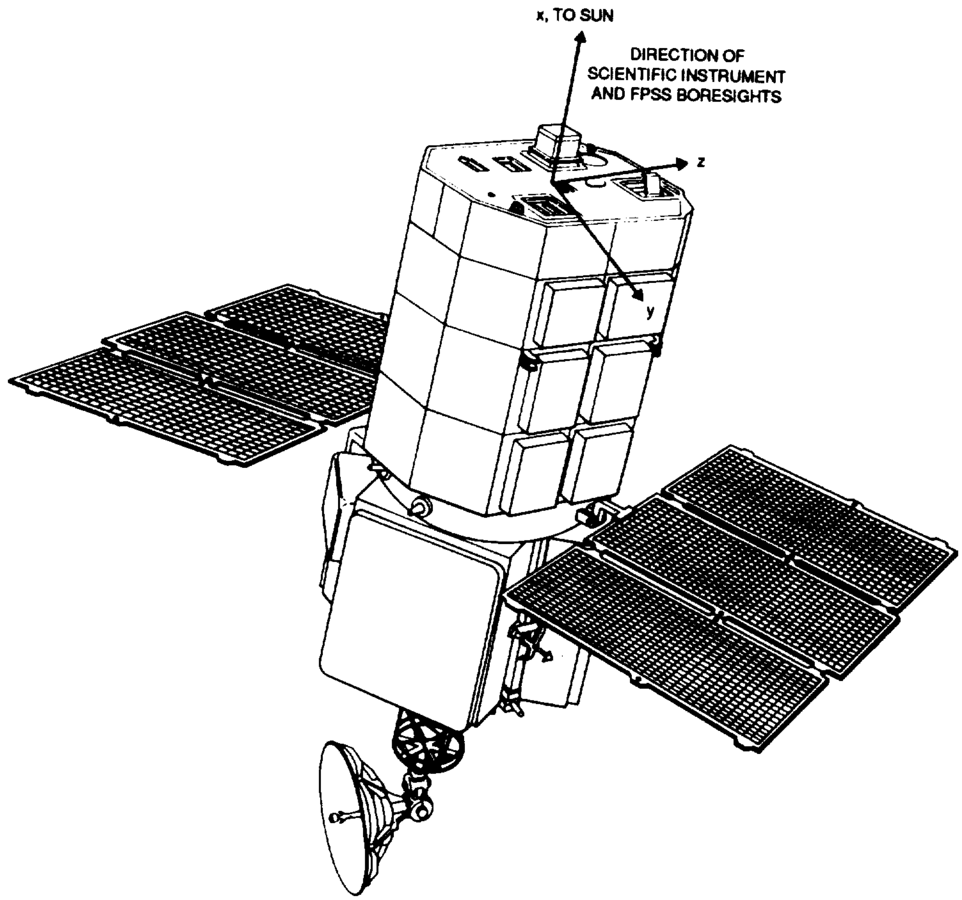

Figure 2. Scientific and FPSS Boresight Directions

To compute the maneuvers, the roll-yaw combination is computed first. The attitude matrix, B, for this case is $T_3(\beta)\ T_1(\phi)$ or

$$B = \left[\begin{array}{c|c|c} \cos\beta & \sin\beta\cos\phi & \sin\beta\sin\phi \\ -\sin\beta & \cos\beta\cos\phi & \cos\beta\sin\phi \\ 0 & -\sin\phi & \cos\phi \end{array}\right] \tag{7}$$

This transforms vectors from the SUN frame to the body frame after the maneuvers have been completed.

Since the scientific instrument boresights are parallel to the X-axis, the representation of the celestial target vector in the body frame after the maneuver is desired to be

$$\underline{V}_{scb} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

The GCI coordinate representation of the celestial target vector, unchanged by the maneuver is

$$\underline{V}_{gci} = \begin{bmatrix} V_{gci1} \\ V_{gci2} \\ V_{gci3} \end{bmatrix} \tag{9}$$

To represent the celestial target vector in the SUN frame, the matrix A is calculated for the time of the maneuvers. The representation of the celestial target vector, $\underline{V}_s$, in the Sun frame is

$$\underline{V}_s = A\,\underline{V}_{gci} \tag{10}$$

while $\underline{V}_{scb}$ is related to $\underline{V}_s$ as follows:

$$\underline{V}_s = B^T\,\underline{V}_{scb} \tag{11}$$

or

$$\begin{bmatrix} V_{s1} \\ V_{s2} \\ V_{s3} \end{bmatrix} = \left[\begin{array}{c|c|c} \cos\beta & -\sin\beta & 0 \\ \sin\beta\cos\phi & \cos\beta\cos\phi & -\sin\phi \\ \sin\beta\sin\phi & \cos\beta\sin\phi & \cos\phi \end{array}\right] \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \tag{12}$$

Thus, $\underline{V}_s$ can be computed from Equation (10) and substituted into the left-hand side of Equation (12). The unknowns in Equation (12) are then the maneuver angles β and ϕ. Further reduction of this equation leads to

$$V_{s1} = \cos\beta \tag{13}$$

$$V_{s2} = \sin\beta\cos\phi \tag{14}$$

$$V_{s3} = \sin\beta\sin\phi \tag{15}$$

Dividing Equation (15) by Equation (14)

$$V_{s3}/V_{s2} = \tan \phi \tag{16}$$

Thus, the equations for the roll-yaw maneuvers are

$$\text{Yaw} = \beta = \cos^{-1} (V_{s1}) \tag{17}$$

$$\text{Roll} = \phi = \tan^{-1} (V_{s3}/V_{s2}) \tag{18}$$

The sign of the roll is resolved by using the function DATAN2 to compute the inverse tangent. The roll-pitch sequences are calculated from the above roll-yaw results by adding and subtracting 90 deg from the roll and selecting the sign of the pitch accordingly. As stated earlier, the maneuver sequence requiring the smallest roll rotation is the favored sequence. For targets that are further than 65 deg from the Sun, the maneuver to the attitude closest to the celestial body is computed. In this case the angular distance from the target to the planned closest attitude is also computed and delivered to the experimenters.

2.3 COMET POSITION DETERMINATION

To date, seven Sun-grazing comets have been discovered in the CP FOV during normal observations of the solar corona. The CP uses an occulting disk on its sensor FOV to block out the main part of the Sun, enabling it to observe the solar corona. These comets could not have been observed without this special instrumentation of the CP. Several observations of each comet in CP instrument coordinates have been reported. In support of these observations, it is desired to transform the CP comet coordinate positions to GCI coordinates to determine the orbits of the comets.

The determination of the comet positions in GCI coordinates is the inverse of the maneuver planning. It is assumed that the CP boresight is parallel to the X-axis of the spacecraft. The comet position in CP coordinates is transformed to an equivalent spacecraft roll-pitch maneuver sequence that would point the spacecraft directly at the comet. The spacecraft, however, is never actually maneuvered; the maneuver is only computed so that the maneuver planning procedure can be reversed, and the comet position computed. The position vector in the Sun frame is then determined from the roll-pitch angles. The comet position vector in GCI coordinates is computed by multiplying the comet position vector in the Sun frame by the transpose of A at the time of the observation.

The CP uses a two-dimensional polar coordinate system. The first coordinate is the counterclockwise rotation about the X-axis (roll) from the positive Z-axis (yaw). Thus, this is the negative roll necessary to align the positive Z-axis (yaw) with the Sun-to-comet vector. The second coordinate is the angular separation between the center of the Sun and the comet, along the positive Z-axis (yaw). This corresponds to a negative pitch angle. The result is the roll-pitch angle pair that would be required to maneuver the spacecraft and align the boresight with the spacecraft-to-comet vector.

If the matrix, B', for a roll (ϕ)-pitch (θ) maneuver from a null attitude in the Sun frame to the comet is

$$B' = \begin{bmatrix} \cos\theta & \sin\theta\sin\phi & -\sin\theta\cos\phi \\ 0 & \cos\phi & \sin\phi \\ \sin\theta & -\cos\theta\sin\phi & \cos\theta\cos\phi \end{bmatrix} \quad (19)$$

then the position vector, $\underline{V}_c$, of the comet in the Sun frame is simply

$$\underline{V}_c = B'^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (20)$$

or

$$\underline{V}_c = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ \sin\theta\sin\phi & \cos\phi & -\cos\theta\sin\phi \\ -\sin\theta\cos\phi & \sin\phi & \cos\theta\cos\phi \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \quad (21)$$

This assumes that the spacecraft attitude is zero roll, pitch, and yaw so that the SUN frame is aligned to the body frame. If the spacecraft is at a non-nominal attitude, the matrix B must be recomputed, taking into account the spacecraft attitude at the time of the observation. The comet position in GCI coordinates, $\underline{V}_{cgci}$, is then

$$\underline{V}_{cgci} = A^T \underline{V}_c \quad (22)$$

The vector, $\underline{V}_{cgci}$, is then reduced to right ascension and declination (mean of 1950) and delivered to the International Astronomical Union (IAU), where it is used for orbit computations.

2.4 CELESTIAL BODY VIEWING PERIOD PREDICTION

Because of the geometry of the SMM orbit, every point in the celestial sphere of the spacecraft is occulted by the Earth at some time in one revolution of the ascending node of the spacecraft's orbit. Thus, a list of periods when the celestial target can be viewed by the spacecraft is necessary for all times that observations of nonsolar targets are being planned. The computation of these periods is done by comparing the relevant angles in the geometry of the spacecraft, Earth, and the target objective.

The unit vector, $\underline{W}_{sce}$, from the center of the Earth to the spacecraft, is constantly changing as the spacecraft orbits the Earth and is known from the spacecraft ephemeris. The vector from the Earth to the target body, $\underline{W}_{eb}$, is normally computed from the right ascension and declination of the target body in the Earth's celestial sphere. To compute the viewing periods, the spacecraft-to-target vector, $\underline{W}_{sb}$ is needed. Since the distance to the viewing target in most cases has not been relevant, the Earth-to-target vector is represented as a unit vector. Thus, vector addition of the Earth-to-target vector and the Earth-to-spacecraft vector does not give the spacecraft-to-target vector. Instead, it is assumed that the parallax of the target body's position due to the spacecraft's orbit around the Earth is negligible, and, therefore,

$$\underline{W}_{eb} \approx \underline{W}_{sb} \tag{23}$$

For objects out of the solar system and the outer planets, this approximation will yield results that are well within the constraints. For the inner planets and comets, this approximation leads to an error in the vector of between 1 arc-sec and 20 arc-sec. This will translate to an error of less than 1 sec in the occultation times, which can be considered negligible compared to the 10-sec constraint on the accuracy. If observations of the Moon are planned, the position error caused by the above assumption can reach 1 deg. This will translate to approximately a 15-sec error in the occultation times, which is greater than the maximum error allowed. To resolve this, the Earth-to-Moon vector obtained from lunar ephemerides is used to calculate $\underline{W}_{sb}$ directly by vector addition.

Once $\underline{W}_{sb}$ and $\underline{W}_{sce}$ are known, the angle, Φ, between the vectors is computed by taking the inverse cosine of the dot product of the two vectors as

$$\Phi = \cos^{-1} (\underline{W}_{sb} \cdot \underline{W}_{sce}) \tag{24}$$

This geometry is illustrated in Figure 3.

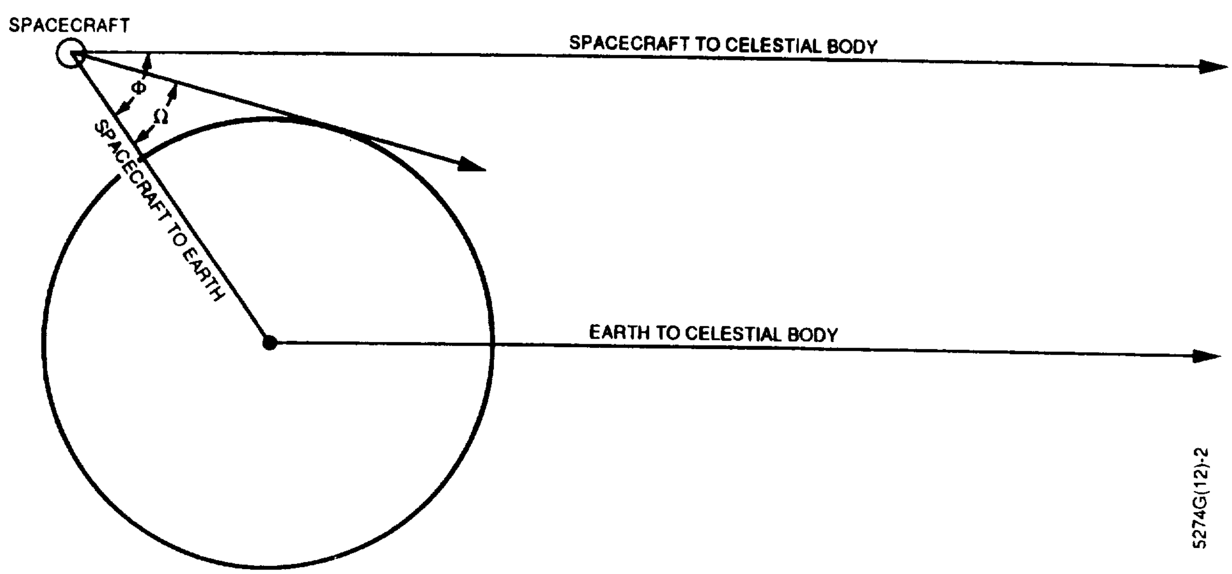

Figure 3. Occultation Geometry

To compute whether the celestial target is blocked by the Earth, Φ is compared to the subtended Earth half angle, Ω. The computation of Ω assumes a perfectly circular

Earth, with a radius equal to the equatorial radius. This translates to a maximum of 8 sec for celestial objects with declinations near positive or negative 90 deg. However, nearly all of the celestial targets observed by the SMM have had declinations close to the Sun's. Thus, the typical error induced by the Earth's oblateness is typically near 3 or 4 sec.

The half angle subtended by the Earth is given by

$$\Omega = \sin^{-1} (R/D) \tag{25}$$

where R is the equatorial radius of the Earth and D is the magnitude of the Earth-to-spacecraft vector obtained from the spacecraft ephemeris file. The relationship of the computed angles is also shown in Figure 3. The only variable on the right-hand side of Equation (25) is D. This varies with the orbital eccentricity and more slowly with the rate of decay of the spacecraft's altitude above the Earth. Thus, for SMM, Ω changes very slowly and is approximately 69 deg. The celestial object is considered to be occulted by the Earth from the spacecraft if Ω is larger than Φ. Conversely, if Φ is larger than Ω, the celestial object is considered in view of the spacecraft. This makes sense intuitively, since at times the spacecraft is between the Earth and the celestial object, Φ will be obtuse, while if the Earth is between the spacecraft and the celestial object, Φ will be acute.

Earth occultation computations are performed every 4 minutes (min) for the period of interest. When a change occurs, a binary search is used to narrow the change time to the nearest second, which will typically have a maximum error (3σ) of less than 10 sec (Reference 4).

2.5 TRACKING PARAMETER COMPUTATION

Targets not in the solar system, such as distant stars, do not move significantly relative to the SMM due to the spacecraft's orbit about the Earth or the Earth's motion about the Sun. Thus, during observations of very distant objects, provisions need not be made to track motion of the celestial target due to parallax and proper motion. However, bodies in heliocentric (planets or comets) or geocentric (the Moon) orbits will move through an angle δ, which can be represented as a rotation about a unit vector, $\underline{e}$, in the spacecraft inertial reference unit (IRU) coordinate frame as illustrated in Figure 4. This motion is due to the spacecraft's motion about the Earth, the Earth's motion about the Sun, and the target objects motion in its orbit about the Sun or Earth. During an observation period at a constant attitude, this motion will cause the celestial target to move through, and eventually out of, the scientific sensor FOV. To compensate, the spacecraft is rotated about the axis $\underline{e}$ by the angle δ from the beginning to the end of the observation period. This motion is called tracking.

Tracking by the spacecraft is accomplished by computing the angle, δ, and axis of rotation, $\underline{e}$, of the target body in the IRU sensor frame of reference and combining δ and $\underline{e}$ with the observation time Δt to form a command gyro rate bias vector (historically called BGDT) which is uplinked to the spacecraft. These BGDTs cause the gyros to show motion opposite to the way the target is moving. This apparent (in that the spacecraft is not actually moving) motion causes the SRWs to counteract the apparent motion and move the spacecraft in the direction of the celestial objective. Thus, tracking is accomplished. To compute the BGDTs, the transformation matrix, H, between the SMM body-frame attitude at the start of the observation period and the attitude at the end of the observation period is computed and used to determine average rates about the three IRU axes.

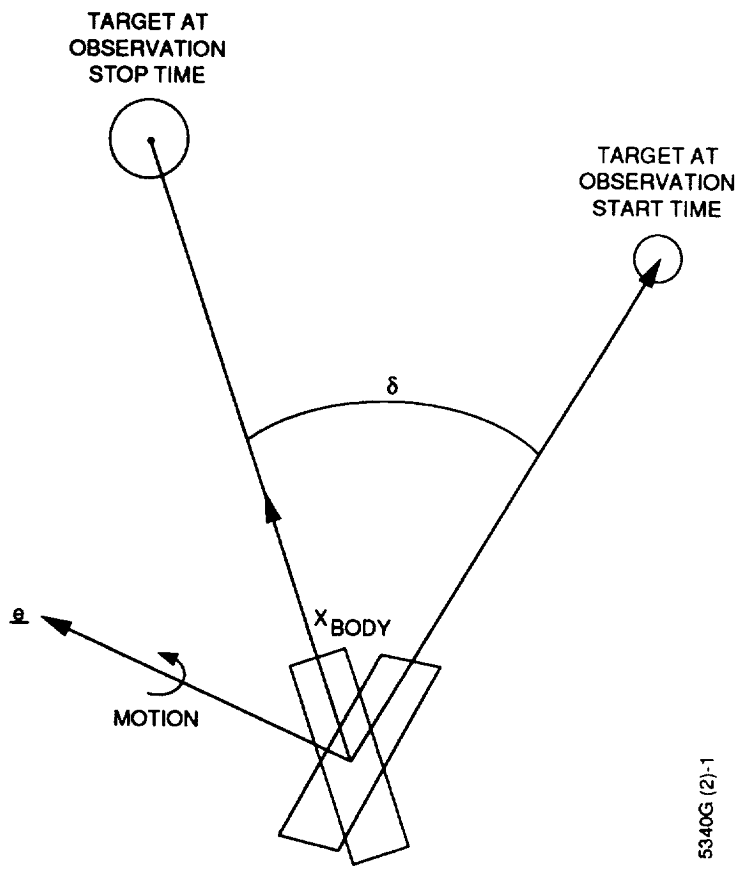

Figure 4. Tracking Geometry

To compute H, the maneuvers to point the spacecraft at the target are computed by the methods described earlier at the observation start and stop times. The maneuvers are assumed to be relative to an attitude of roll, pitch, and yaw equal to zero. Thus, using Equation (6), the SUN-frame-to-body-frame matrices, B_1 and B_2, at the start and stop times are computed. However, the coordinate frame that B is referenced to at all times on the SMM mission is the SUN frame, which, as stated above, is not inertial. The attitude motion for target tracking must be referenced to an inertial frame. For this, the GCI-to-SUN frame matrix, A, is used. In addition, because the BGDTs are for the IRUs, they must be represented in the IRU sensor frame. Defining G_1 and G_2 to be the GCI-to-IRU transformation matrices at the start and stop times, respectively, and M to be the IRU-to-body transformation matrix, then

$$G_1 = M^T B_1 A_1 \tag{26}$$

$$G_2 = M^T B_2 A_2 \tag{27}$$

and

$$G_2 = H G_1 \tag{28}$$

Solving for H yields

$$\begin{aligned} H &= G_2 G_1^T = M^T B_2 A_2 (M^T B_1 A_1)^T \\ &= M^T B_2 A_2 A_1^T B_1^T M \end{aligned} \tag{29}$$

If the Sun frame were an inertial frame, A_1 would be equal to A_2, and Equation (29) would be simplified. Now that H has been found, δ and $\underline{e}$ can be computed from H, and the BGDTs can be computed.

To compute δ and $\underline{e}$, H is written as an Euler rotation with unit vector $\underline{e}$ in the IRU frame and rotation angle δ. Thus, using the Euler convention for a general rotation, H is (Reference 4)

$$H = \cos(\delta)\, I + (1 - \cos(\delta))\, \underline{e}\underline{e}^T - \sin(\delta)\, [[\underline{e}]] \tag{30}$$

where $[[\underline{e}]]$ is the skew symmetric matrix representation of a vector. The general form for a three-component vector is

$$[[\underline{x}]] = \begin{bmatrix} 0 & -x_3 & x_2 \\ x_3 & 0 & -x_1 \\ -x_2 & x_1 & 0 \end{bmatrix} \tag{31}$$

Thus, it is desired to manipulate Equation (30) to solve for $\underline{e}$ and δ. Defining the parameter k^2 as

$$k^2 = (H_{23} - H_{32})^2 + (H_{31} - H_{13})^2 + (H_{12} - H_{21})^2 \tag{32}$$

substitution from Equation (30) leads to

$$\begin{aligned} k^2 = {} & [e_2 e_3 (1 - \cos\delta) + e_1 \sin\delta - e_2 e_3 (1 - \cos\delta) + e_1 \sin\delta]^2 \\ & + [e_1 e_3 (1 - \cos\delta) + e_2 \sin\delta - e_1 e_3 (1 - \cos\delta) + e_2 \sin\delta]^2 \\ & + [e_1 e_2 (1 - \cos\delta) + e_3 \sin\delta - e_1 e_2 (1 - \cos\delta) + e_3 \sin\delta]^2 \end{aligned} \tag{33}$$

Some simple algebra leads to

$$
\begin{aligned}
k^2 &= 4e_1^2 \sin^2 \delta + 4e_2^2 \sin^2 \delta + 4e_3^2 \sin^2 \delta \\
&= 4 * \sin^2 \delta * (e_1^2 + e_2^2 + e_3^2)
\end{aligned}
\tag{34}
$$

Since $\underline{e}$ is a unit vector, the last quantity on the right-hand side of Equation (34) is equal to 1. Therefore,

$$
k^2 = 4 * \sin^2 \delta \tag{35}
$$

or taking the square root of both sides and solving for δ

$$
\delta = \sin^{-1} (k/2) \tag{36}
$$

where k is defined by Equation (32).

To solve for $\underline{e}$, the off-diagonal terms in Equation (30) are used. To demonstrate this using e_1,

$$
\begin{aligned}
H_{23} - H_{32} &= e_2\, e_3\, (1 - \cos \delta) + e_1 \sin \delta - e_2\, e_3\, (1 - \cos \delta) + e_1 \sin \delta \\
&= 2 * e_1 * \sin \delta
\end{aligned}
\tag{37}
$$

Solving for e_1

$$
e_1 = (H_{23} - H_{32})/(2 * \sin \delta) \tag{38}
$$

Similarily,

$$
e_2 = (H_{31} - H_{13})/(2 * \sin \delta) \tag{39}
$$

$$
e_3 = (H_{12} - H_{21})/(2 * \sin \delta) \tag{40}
$$

To compute the BGDTs, the angle of rotation in the IRU coordinate frame is divided by the observation time. The rate about each of the IRU axes is then

$$
x' = e_1 * (\delta/\Delta t) \tag{41}
$$

$$
y' = e_2 * (\delta/\Delta t) \tag{42}
$$

$$
z' = e_3 * (\delta/\Delta t) \tag{43}
$$

where Δt is the observation time. These rates are converted to the units of the SMM IRU, radians per 0.512 sec, and uplinked to the spacecraft for use in the control loop.

3. RESULTS AND CONCLUSIONS

3.1 RESULTS

To date, these algorithms have been used to support all the SMM scientific off-Sun maneuvers. Since the computed parameters are designed to support pointing to celestial objects other than the Sun, the performance of the algorithms is ultimately measured by the quality of these observations. Thus far, the science observations have been completed without incident and have yielded excellent results each time. Images obtained from the SMM scientific instruments have always had the target object in the sensor FOV and at the correct times. In addition, for the heliocentric and geocentric objectives, the tracking system has continuously kept the object in view.

As an independent check, attitude determination solutions during the observations have indicated the spacecraft pointing accuracy is within 180 arc-sec. The accuracy of the solutions was approximately 30 arc-sec (3σ). The bulk of the pointing error was due to uncertainty in the IRU bias. During the observations, the spacecraft was inertially stabilized by the IRU package. However, spacecraft attitude drift occurred because of uncertainty in the knowledge of the IRU bias. This bias was calibrated periodically, and the error in the pointing did not exceed 120 arc-sec due to the error in the bias.

Most of the remaining error in the pointing accuracy was due to uncontrollable timing problems with the maneuvers. Since the geometry of the spacecraft, Earth, Sun, and target-body system was constantly changing, the required maneuvers were constantly changing. The algorithm derived above assumes the maneuvers were instantaneous. However, the SMM maneuvered at approximately 5 deg per min; consequently, pointing errors were incurred. In addition, the start time of the maneuvers was calculated at 5-sec intervals. Thus, if the maneuvers did not start at these times, greater error was incurred. The pointing errors due to the timing errors did not exceed 50 arc-sec. Other potential error sources include uncertainties in the position of the target body, the initial spacecraft attitude from which the maneuvers were performed, and the IRU scale factors which controlled the magnitude of a commanded maneuver.

3.2 CONCLUSIONS

Algorithms to support off-Sun maneuvers of the SMM have been presented. These algorithms compute parameters for maneuver planning, viewing period start and stop times, and target tracking rate. All of these items are necessary to maneuver the spacecraft to observe and track the celestial target objective. Because the Sun reference frame is non-inertial, the equations differ from ones normally seen. This work can be adapted to spacecraft that are controlled relative to the inertial frame; however, the primary application is to missions with non-inertial reference frames, such as Earth-pointing (1 revolution per orbit) missions.

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