# EXTENDED KALMAN FILTER FOR ATTITUDE ESTIMATION
## OF THE
## EARTH RADIATION BUDGET SATELLITE

by

Julie Deutschmann[*] and Itzhack Y. Bar-Itzhack[**]
NASA - Goddard Space Flight Center
Greenbelt, MD 20771

## Abstract

This paper describes the design and testing of an Extended Kalman Filter (EKF) for ground attitude determination, misalignment estimation and sensor calibration of the Earth Radiation Budget Satellite (ERBS). Attitude is represented by the quaternion of rotation and the attitude estimation error is defined as an additive error. Quaternion normalization is used for increasing the convergence rate and for minimizing the need for filter tuning. The paper presents the development of the filter dynamic model, the gyro error model and the measurement models of the Sun sensors, the IR horizon scanner and the magnetometers which are used to generate vector measurements. The filter is applied to real data transmitted by ERBS sensors. Results are presented and analyzed and the EKF advantages as well as sensitivities are discussed. On the whole the filter meets the expected synergism, accuracy and robustness.

## I. INTRODUCTION

An important part of spacecraft ground support is attitude determination, sensor alignment, and sensor calibration. In the past, at Goddard Space Flight Center (GSFC) in the Flight Dynamics Division (FDD) each task was performed separately, usually using a relatively small state . The use of more sophisticated algorithms has been suggested in the literature, but they have not yet been tested with real spacecraft data for ground processing in Flight Dynamics.

The purpose of this study was to design and test an Extended Kalman Filter (EKF). The filter was designed for the Earth Radiation Budget Satellite (ERBS). ERBS is equipped with the following sensors which are used for attitude determination: 2 redundant Inertial Reference Units (IRUs) each containing 3 single-axis gyroscopes, 2 digital fine Sun sensors (FSSs), 2 infra-red (IR) horizon scanners, and 1 three-axis magnetometer. The state estimated by the filter consists of the attitude parameters (quaternion), sensor misalignments for the Sun sensor, magnetometer and gyros, biases for the Sun sensor, horizon scanner, magnetometer, and gyros, and scale factor corrections for the Sun sensor, magnetometer, and gyros. The filter was tested using real spacecraft data transmitted to Earth by ERBS.

Kalman filters have not been used for ground attitude processing in the FDD at GSFC. The current ground support software implements single frame and batch estimators and, as mentioned before, much of the calibration effort is performed separately from the

attitude determination. The EKF designed for ERBS allows for all of the calibration to be performed along with the attitude determination.

The use of the extended Kalman filter (EKF) for spacecraft attitude determination has been dealt with quite extensively in the past. Kau et al. [1] as well as Farrell [2], for example, used an ad-hoc solution to the problem of estimating the Euler angles directly from vector measurements. A more general approach to this problem was presented in [3]. The problem of estimating the direction cosine matrix directly from vector measurements was discussed in [4]. The filter which was required there was a linear one with some adaptation. A general analytic exposition of the use of the EKF for spacecraft attitude determination was given by Lefferts, Markley and Shuster [5]. Reference [6] dealt with the problem of estimating the attitude quaternion from vector measurements. Basically, the estimated quantity was the difference between the best known value of the quaternion and its true value. This difference was defined as a four component additive quantity. Because of this definition the estimate of the quaternion is not necessarily normal unless it converges to the correct quaternion. It was found that normalization of the estimated quaternion during the filtering process speeds up convergence and eliminates the need for filter tuning. In other references, e.g. [5], [7] and for on board attitude determination software which is used in LANDSAT 4 and is planned to be used in the GRO and EP spacecraft a multiplicative quaternion difference is used. Since it is assumed that this difference quaternion is small and as for small rotations the scalar part of the quaternion is close to 1, those algorithms are estimating only three attitude - error components. Obviously, estimation of an additive quaternion error of four parameters plus the induced normality constraint is equivalent to estimating three parameters. Because of our good experience with the additive quaternion error approach [6] we chose to implement this approach in the present EKF algorithm.

In the next section we introduce the algorithms developed for the ERBS EKF.

## II. THE EXTENDED KALMAN FILTER ALGORITHM

The EKF algorithm is based on the following assumed models: System model:

$$\dot{\underline{X}} = \underline{f}(\underline{X}(t),t) + \underline{w}(t) \qquad (2.1)$$

Measurement Model:

$$\underline{z}_k = \underline{h}_k(\underline{X}(t_k)) + \underline{v}_k \qquad (2.2)$$

where:

$\underline{X}(t)$ = state vector.
$\underline{w}(t)$ = zero mean white process.
$\underline{v}_k$ = zero mean white sequence.

The EKF algorithm is as follows [8]. The measurement update of the state estimate and of the estimation error covariance are performed as follows:

[*] Aerospace Engineer, Attitude Analysis Section, Flight Dynamic Analysis Branch.

[**] Professor, on leave from the Aerospace Department of Technion -Israel Institute of Technology. Member, Technion Space Research Institute, Haifa 32000, Israel.

## State Estimation Update:

$$\hat{X}_k(+) = \hat{X}_k(-) + K_k[z_k - \underline{h}_k(\hat{X}_k(-))] \qquad (2.3)$$

where the Gain Matrix is evaluated as follows:

$$K_k = P_k(-)H_k^T(\hat{X}_k(-))[H_k(\hat{X}_k(-))P_k(-)H_k^T(\hat{X}_k(-)) + R_k]^{-1}$$

$$\dots (2.4)$$

## Error Covariance Update:

$$P_k(+) = [I - K_kH_k]P_k(-)[I - K_kH_k]^T + K_kR_kK_k^T \quad (2.5)$$

The propagation of the state estimate and the error covariance are accomplished using:

## State Estimation Propagation:

$$\dot{\hat{X}}(t) = \underline{f}(\hat{X}(t),t) \qquad (2.6)$$

## Error Covariance Propagation:

$$\dot{P}(t) = F(\hat{X}(t),t)P(t) + P(t)F^T(\hat{X}(t),t) + Q(t) \qquad (2.7)$$

where

$$F(\underline{X}(t),t) = \frac{\partial \underline{f}(\underline{X}(t),t)}{\partial \underline{X}(t)} \bigg|_{\underline{X}(t)=\hat{\underline{X}}(t)} \qquad (2.8a)$$

$$H(\underline{X}(-)) = \frac{\partial \underline{h}(\underline{X}(t))}{\partial \underline{X}(t)} \bigg|_{\underline{X}(t)=\hat{\underline{X}}(t)} \qquad (2.8b)$$

$R_k$ = covariance matrix of white sequence.
$Q$ = spectral density matrix of $\underline{w}(t)$.

The EKF rather than the linear KF algorithm must be used because the measurement vectors obtained from the sensors are non-linear functions of the state vector. The state vector was selected to be:

$$\underline{X} = \begin{bmatrix} \underline{q} \\ \underline{S_g} \\ \underline{\theta_g} \\ \underline{b_g} \\ \underline{\theta_s} \\ \underline{C_s} \\ \underline{b_s} \\ \underline{d_h} \\ \underline{S_m} \\ \underline{\theta_m} \\ \underline{b_m} \end{bmatrix} \begin{array}{l} \text{4 quaternion components} \\ \text{3 gyro scale factor errors} \\ \text{6 gyro misalignment angles} \\ \text{3 gyro biases} \\ \text{3 FSS misalignment angles} \\ \text{2 FSS scale factor errors} \\ \text{2 FSS biases} \\ \text{2 IR horizon scanner biases} \\ \text{3 magnetometer scale factors} \\ \text{6 magnetometer misalignments} \\ \text{3 magnetometer biases} \end{array} \quad (2.9)$$

Following the tradition of the NASA Goddard's Flight Dynamics Division we used vector measurements to update the EKF. (It should be noted that this is not a must but rather a choice). The **effective measurements** which are used to update the filter are defined as follows

$$\underline{y} = M_{AT}\underline{W}_{T',meas} - A(\hat{q})\underline{y}_I \qquad (2.10)$$

where:

$\underline{y}$ = effective measurements.
$M_{AT}$ = transformation matrix from the nominal (non-misaligned) sensor to body coordinates.
$W_{T',meas}$ = unit vector as measured by the sensor in the sensor misaligned coordinates.
$A(\hat{q})$ = transformation matrix from the inertial to the body coordinates as a function of the estimated quaternion.
$\underline{y}_I$ = the measured unit vector as known in the inertial coordinates.

While the traditional EKF algorithm updates the state estimate according to (2.3), we use $\underline{y}$ (as computed in (2.10)) to update the state estimate as follows

$$\hat{X}_k(+) = \hat{X}_k(-) + K_k\underline{y}_k \qquad (2.11)$$

To reconcile this apparent deviation from the ordinary EKF algorithm, define $d\underline{z}_k$ as follows

$$d\underline{z}_k = \underline{z}_k - \underline{h}_k(\hat{X}_k(-)) \qquad (2.12)$$

then (2.3), the state update equation in the ordinary EKF algorithm, reads

$$\hat{X}_k(+) = \hat{X}_k(-) + K_kd\underline{z}_k \qquad (2.13)$$

Next define $\underline{x}(t_k)$ as

$$\underline{X}(t_k) = \hat{X}_k(-) + \underline{x}(t_k) \qquad (2.14)$$

expand (2.2) in a Taylor series expansion about $\hat{X}_k(-)$ and omit terms of second and higher order of $\underline{x}(t_k)$. This yields

$$\underline{z}_k = \underline{h}_k(\hat{X}_k(-)) + H_k\underline{x}(t_k) + \underline{v}_k \qquad (2.15)$$

where $H_k$ is as defined in (2.8b). When $\underline{z}_k$ from (2.15) is substituted into (2.12) we obtain

$$d\underline{z}_k = H_k\underline{x}(t_k) + \underline{v}_k \qquad (2.16)$$

that is, $d\underline{z}_k$ **is linearly related to** $\underline{x}(t_k)$. An inspection of (2.13) reveals that the EKF estimates $\underline{x}(t_k)$, which according to (2.16) is linearly related to the **effective** measurement $d\underline{z}_k$, and then adds the estimate, $\underline{x}(t_k)$, to $\hat{X}_k(-)$, the best estimate of $\underline{X}(t_k)$. As will be seen in the ensuing, also our use of $\underline{y}$, as defined in (2.10), in the state update equation, (2.11), amounts to estimating $\underline{x}(t_k)$, which is linearly related to $\underline{y}$, and adding the estimate to $\hat{X}_k(-)$. In fact, to show the latter we only have to show that $\underline{x}(t_k)$ is linearly related to $\underline{y}$. This will indeed be shown in Section IV.

## III. THE DYNAMICS MODEL

The states which vary in time are the attitude parameters and bias states which are modeled as Markov rather than as bias states. (The reason for this modeling will be discussed later). The scale factors and misalignments are assumed to be constant in time.

The attitude matrix is given in terms of the quaternion, q, as follows

$$A = \begin{bmatrix} q_1^2-q_2^2-q_3^2+q_4^2 & 2(q_1q_2+q_3q_4) & 2(q_1q_3-q_2q_4) \\ 2(q_1q_2-q_3q_4) & -q_1^2+q_2^2-q_3^2+q_4^2 & 2(q_2q_3+q_1q_4) \\ 2(q_1q_3+q_2q_4) & 2(q_2q_3-q_1q_4) & -q_1^2-q_2^2+q_3^2+q_4^2 \end{bmatrix} \qquad (3.1)$$

The quaternion changes in time according to [8, pp. 511, 512]

$$\dot{q} = \Omega q \qquad (3.2)$$

where:

$$\Omega = \frac{1}{2}\begin{bmatrix} 0 & w_z & -w_y & w_x \\ -w_z & 0 & w_x & w_y \\ w_y & -w_x & 0 & w_z \\ -w_x & -w_y & -w_z & 0 \end{bmatrix} \qquad (3.3)$$

and where $w_{x,y,z}$ are the components of the spacecraft angular velocity vector resolved in the spacecraft coordinates. The true quaternion of the spacecraft propagates in time according to (3.2). We cannot compute q precisely since we do not know precisely the initial quaternion nor do we know $\underline{w}$ precisely as it is a measured vector and the measurement contains errors. The measured angular velocity can be written as

$$\bar{w} = \underline{w} + d\underline{w} \qquad (3.4)$$

where

$\bar{w}$ = gyro reading.
$\underline{w}$ = true angular velocity.
$d\underline{w}$ = vector of gyro errors.

Since the true quaternion propagates according to (3.2) we propagate the estimated quaternion in a similar manner; that is, we propagate it according to

$$\dot{\hat{q}} = \Omega\hat{q} \qquad (3.5)$$

where $\bar{\Omega}$ has the form of (3.3) but its elements are the elements of the measured angular rate $\bar{w}$. Now a matrix $d\Omega$ can be defined as follows

$$\Omega = \bar{\Omega} - d\Omega \qquad (3.6)$$

Substitution of (3.6) into (3.2) results in

$$\dot{q} = \bar{\Omega}q - d\Omega q \qquad (3.7)$$

When (3.5) is subtracted from (3.7) we obtain

$$\dot{q} - \dot{\hat{q}} = \bar{\Omega}(q - \hat{q}) - d\Omega q \qquad (3.8)$$

As discussed in the introduction, we define an additive quaternion error as follows

$$dq = q - \hat{q} \qquad (3.9)$$

Then (3.8) can be written as

$$d\dot{q} = \bar{\Omega}dq - d\Omega q \qquad (3.10)$$

A matrix, B, can be defined as follows

$$B = \frac{1}{2}\begin{bmatrix} -q_4 & q_3 & q_2 \\ -q_3 & -q_4 & q_1 \\ q_2 & -q_1 & -q_4 \\ q_1 & q_2 & q_3 \end{bmatrix} \qquad (3.11)$$

and used in (3.10). However, since q itself is not known, we use its estimate, $\hat{q}$ to compute (3.11). When this is done, we can write (3.10) as follows

$$\boxed{d\dot{q} = \bar{\Omega}dq + \hat{B}d\underline{w}} \qquad (3.12)$$

where $\hat{B}$ is computed as in (3.11) using $\hat{q}$ rather than $q$. Equation (3.12) is the dynamics equation of the additive quaternion error.

Equation (3.12) cannot be used as a dynamics model in an EKF since the vector of gyro errors, $d\underline{w}$, is not a white noise vector. It could be modeled though as a linear system excited by a white noise. Consequently this linear model can be augmented with the dynamics model of (3.12). The augmented model is linear and is driven by a white noise vector hence the model can legitimately used by the EKF [8]. To accomplish that we use the following standard gyro error model.

$$\begin{bmatrix} dw_x \\ dw_y \\ dw_z \end{bmatrix} = \begin{bmatrix} S_{gx} & 0 & 0 \\ 0 & S_{gy} & 0 \\ 0 & 0 & S_{gz} \end{bmatrix}\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} + \begin{bmatrix} 0 & \theta_{gxy} & \theta_{gxz} \\ \theta_{gyx} & 0 & \theta_{gyz} \\ \theta_{gzx} & \theta_{gzy} & 0 \end{bmatrix}\begin{bmatrix} w_x \\ w_y \\ w_z \end{bmatrix} + \begin{bmatrix} b_{gx} \\ b_{gy} \\ b_{gz} \end{bmatrix}$$

$$+ \begin{bmatrix} n_{g1x} \\ n_{g1y} \\ n_{g1z} \end{bmatrix} \qquad (3.13)$$

where

$$\underline{S}_g^T = [S_{gx}, S_{gy}, S_{gz}] \qquad (3.14a)$$

$$\underline{\theta}_g^T = [\theta_{gxy}, \theta_{gxz}, \theta_{gyx}, \theta_{gyz}, \theta_{gzx}, \theta_{gzy}] \qquad (3.14b)$$

$$\underline{b}_g^T = [b_{gx}, b_{gy}, b_{gz}] \qquad (3.14c)$$

$$\underline{n}_{g1}^T = [n_{g1x}, n_{g1y}, n_{g1z}] \qquad (3.14d)$$

and T denotes the transpose, $\underline{S}_g$, $\underline{\theta}_g$ and $\underline{b}_g$ are as explained in (2.9) and $\underline{n}_{g1}$ is a white noise vector. We can write (3.13) as follows

$$\begin{bmatrix} dw_x \\ dw_y \\ dw_z \end{bmatrix} = \begin{bmatrix} w_x & 0 & 0 & w_y & w_z & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & w_y & 0 & 0 & 0 & w_x & w_z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & w_z & 0 & 0 & 0 & 0 & w_x & w_y & 0 & 0 & 1 \end{bmatrix}\underline{x}^* + \begin{bmatrix} n_{g1x} \\ n_{g1y} \\ n_{g1z} \end{bmatrix}$$

$$\dots (3.15)$$

where

$$\underline{x}^{*T} = [\underline{S}_g^T, \underline{\theta}_g^T, \underline{b}_g^T] \qquad (3.16)$$

Define the following matrices

$$U = \begin{bmatrix} w_x & 0 & 0 \\ 0 & w_y & 0 \\ 0 & 0 & w_z \end{bmatrix} \qquad W = \begin{bmatrix} w_y & w_z & 0 & 0 & 0 & 0 \\ 0 & 0 & w_x & w_z & 0 & 0 \\ 0 & 0 & 0 & 0 & w_x & w_y \end{bmatrix}$$

$$\dots (3.17a) \qquad\qquad \dots (3.17b)$$

then (3.15) can be written as

$$d\underline{w} = [U|W|I]\underline{x}^* + \underline{n}_{g1} \qquad (3.18)$$

The vectors $\underline{S}_g$ and $\underline{\theta}_g$ contained in $\underline{x}^*$ are constants, therefore

$$\dot{\underline{S}}_g = 0 \quad (3.19a) \qquad \dot{\underline{\theta}}_g = 0 \quad (3.19b)$$

The gyro bias vector, $\underline{b}_g$, may actually be time-varying so they are more adequately modeled as Markov states as follows [8].

$$\frac{d}{dt}\begin{vmatrix} \bar{b}_{gx} \\ b_{gy} \\ b_{gz} \end{vmatrix} = \begin{vmatrix} -1/t_g & 0 & 0 \\ 0 & -1/t_g & 0 \\ 0 & 0 & -1/t_g \end{vmatrix}\begin{vmatrix} \bar{b}_{gx} \\ b_{gy} \\ b_{gz} \end{vmatrix} + \begin{vmatrix} \bar{n}_{g2x} \\ n_{g2y} \\ n_{g2z} \end{vmatrix} \quad (3.20)$$

where $t_g$ is the time constant of the Markov states and

$$\underline{n}_{g2}{}^T = [n_{g2x}, \ n_{g2y}, \ n_{g2z}] \quad (3.21)$$

is the white noise vector which drives the Markov states. Define the matrix $T_g$ as follows

$$T_g = \begin{vmatrix} -1/t_g & 0 & 0 \\ 0 & -1/t_g & 0 \\ 0 & 0 & -1/t_g \end{vmatrix} \quad (3.22)$$

then (3.20) can be written as

$$\dot{\underline{b}}_g = T_g \underline{b}_g + \underline{n}_{g2} \quad (3.23)$$

The other bias states in the fine Sun sensor, IR horizon scanner, and magnetometer which are listed in (2.9) and will be mentioned in the development of the sensor error models, are also modeled as Markov states as follows. Define the following matrices

$$T_s = \begin{vmatrix} -1/t_s & 0 \\ 0 & -1/t_s \end{vmatrix} \qquad T_h = \begin{vmatrix} -1/t_h & 0 \\ 0 & -1/t_h \end{vmatrix}$$

$$\dots \ (3.24a) \qquad \qquad \dots \ (3.24b)$$

$$T_m = \begin{vmatrix} -1/t_m & 0 & 0 \\ 0 & -1/t_m & 0 \\ 0 & 0 & -1/t_m \end{vmatrix} \quad (3.24c)$$

then

$$\dot{\underline{b}}_s = T_s \underline{b}_s + \underline{n}_s \quad (3.25a)$$

$$\dot{\underline{b}}_h = T_h \underline{b}_h + \underline{n}_h \quad (3.25b)$$

$$\dot{\underline{b}}_m = T_m \underline{b}_m + \underline{n}_m \quad (3.25c)$$

where

$$\underline{b}_s^T = [b_A, \ b_B] \qquad \underline{b}_h^T = [d_r, \ d_p] \qquad \underline{b}_m^T = [b_{mx}, \ b_{my}, \ b_{mz}]$$

$$\dots \ (3.26)$$

These vectors denote "biases" as defined in (2.9). The scale factor and misalignment states of the sensors which also are a part of the state vector listed in (2.9), are assumed constant. That is

$$\dot{\underline{\theta}}_s = 0 \qquad \dot{\underline{C}}_s = 0 \qquad \dot{\underline{S}}_m = 0 \qquad \dot{\underline{\theta}}_m = 0 \quad (3.27)$$

where

$$\underline{\theta}_s^T = [\theta_{sx}, \ \theta_{sy}, \ \theta_{sz}] \qquad \underline{C}_s^T = [C_A, \ C_B]$$

$$\underline{S}_m^T = [S_{mx}, \ S_{my}, \ S_{mz}]$$

$$\underline{\theta}_m^T = [\theta_{mxy}, \ \theta_{mxz}, \ \theta_{myx}, \ \theta_{myz}, \ \theta_{mzx}, \ \theta_{mzy}]$$
$$\dots \ (3.28)$$

The seven sensor states (of the Sun sensor, IR horizon scanner and magnetometer) which are listed in (3.26) and in (3.28), are augmented with the quaternion error and gyro states to form the attitude augmented state vector, $\underline{X}$. This vector is that shown in (2.9) when $\underline{q}$ is replaced by $d\underline{q}$. The differential equation which governs the propagation of $\underline{X}$ is obtained by combining the linear differential equations of the components of the attitude augmented state vector. Accordingly the augmentation of (3.11), (3.18), (3.19), (3.23), (3.25) and (3.27) yields



$$\dots \ (3.29a)$$

which is of the form

$$\dot{\underline{X}} = F(\hat{\underline{X}})\underline{X} + \underline{n} \quad (3.29b)$$

The spectral density of the elements of the white noise driving Markov states in $\underline{X}$ is related to the individual states they drive according to the well known relation [8] $Q_i = 2/T_i S_{i,o}^2$ where $Q_i$ is the spectral density of the white noise driving state $i$, $T_i$ is the time constant of this Markov state and $S_{i,o}$ is the initial standard deviation of the state. The matrix $F(\hat{\underline{X}})$ is the one defined in (2.8a).

The estimation problem dealt with in this paper is characterized by a linear dynamics equation. The system dynamics is determined by (3.5), (3.19), (3.23), (3.25) and (3.27). It is easy to see that when these equations are augmented into one equation we obtain an equation of the form

$$\dot{\underline{X}} = f(t)\underline{X} + \underline{n} \quad (3.30)$$

where $\underline{X}$ is given by (2.9) and $f(t)$ is the following

matrix

$$
f(t) = \begin{bmatrix}
\underline{0} & & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & T_g & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & T_s & & & & & \\
& & & & & & T_h & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & \\
& & & & & & & & & & T_m
\end{bmatrix} \qquad (3.31)
$$

The white noise vector $\underline{n}$ is of no consequence when dealing with the role of (3.30) in the estimation process since according to (2.6) the propagation of $\underline{\hat{x}}$ requires only the evaluation of f(t).

## IV. THE MEASUREMENT MODEL

As mentioned in Section II the **effective measurements** which are used to update the filter are defined as follows

$$
\boxed{\underline{y} = M_{AT}\underline{W}_{T',meas} - A(\underline{\hat{q}})\underline{V}_I} \qquad (4.1)
$$

where:

$\underline{y}$ = effective measurements.
$M_{AT}$ = transformation matrix from the nominal (non-misaligned) sensor to body coordinates.
$W_{T',meas}$ = unit vector as measured by the sensor in the sensor misaligned coordinates.
$A(\underline{\hat{q}})$ = transformation matrix from the inertial to the body coordinates as a function of the estimated quaternion.
$\underline{V}_I$ = the measured unit vector as known in the inertial coordinates.

In the ideal (nominal) situation the sensor is well aligned and, in addition, introduces no measurement errors. Also, $\underline{\hat{q}}$, the estimate of $\underline{q}$ is perfect and is, thus, equal to $\underline{q}$ itself. Therefore, using (4.1), we obtain

$$
\underline{y} = M_{AT}\underline{W}_{T',meas} - A(\underline{\hat{q}})\underline{V}_I = M_{AT'}\underline{W}_{T'} - A(\underline{q})\underline{V}_I = 0 \quad (4.2)
$$

Any deviation from the nominal will be reflected in $\underline{y}$. If the deviations are small, then $\underline{y}$ will be related linearly to them. It is our purpose in this section to derive the linear relations between the effective measurement $\underline{y}$ and those deviations which are actually the error states in $\underline{x}$ (whose time behavior was given in (3.25 and 3.27)).

Let us denote the two terms on the right-hand side of (4.1) as follows

$$
\underline{W}_A = M_{AT}\underline{W}_{T',meas} \quad (4.3a) \qquad \underline{V}_A = A(\underline{\hat{q}})\underline{V}_I \quad (4.3b)
$$

Consider first $\underline{W}_A$. The ideal sensor measures in its misaligned coordinates the vector $\underline{W}_{T'}$. Since the sensor is not ideal, it adds to the measured vector the error term $d\underline{W}_{T'}$, hence

$$
\underline{W}_{T',meas} = \underline{W}_{T'} + d\underline{W}_{T'} \qquad (4.4)
$$

Substitution of (4.4) into (4.3a) yields

$$
\underline{W}_A = M_{AT}(\underline{W}_{T'} + d\underline{W}_{T'}) \qquad (4.5)
$$

Now

$$
M_{T'A} = M_{T'T}M_{TA} \qquad (4.6)
$$

For small misalignment angles

$$
M_{T'T} = I + \theta \qquad (4.7)
$$

where

$$
\theta = \begin{bmatrix}
0 & \theta_z & -\theta_y \\
-\theta_z & 0 & \theta_x \\
\theta_y & -\theta_x & 0
\end{bmatrix} \qquad (4.8)
$$

From (4.6)

$$
M_{AT} = M_{AT'}M_{T'T} \qquad (4.9)
$$

Substitution of (4.7) into (4.9) yields

$$
M_{AT} = M_{AT'}(I + \theta) \qquad (4.10)
$$

When (4.10) is substituted into (4.5) and the term containing products of errors is dropped, the following is obtained

$$
\underline{W}_A = M_{AT'}\underline{W}_{T'} + M_{AT'}\theta\underline{W}_{T'} + M_{AT'}d\underline{W}_{T'} \qquad (4.11)
$$

Next we address $\underline{V}_A$ defined in (4.3b). Using the definition of $d\underline{q}$ in (3.9) we can write

$$
A(\underline{\hat{q}}) = A(\underline{q} - d\underline{q}) \qquad (4.12)
$$

Using Taylor series expansion $A(\underline{\hat{q}})$ can be approximated to within first order terms as follows

$$
A(\underline{\hat{q}}) = A(\underline{q}) - \sum_{i=1}^{4} \frac{\partial A(\underline{q})}{\partial q_i}\bigg|_{\underline{q}} dq_i \qquad (4.13)
$$

substitution of (4.13) into (4.3b) yields

$$
\underline{V}_A = A(\underline{q})\underline{V}_I - \sum_{i=1}^{4} \frac{\partial A(\underline{q})}{\partial q_i}\bigg|_{\underline{q}} \underline{V}_I \, dq_i \qquad (4.14)
$$

Note that the derivatives have to be evaluated at $\underline{q}$ which is unknown. Therefore, as usual, we use $\underline{\hat{q}}$ instead. This is based on the assumption that $d\underline{q}$ is small enough such that $\underline{\hat{q}}$ is close enough to $\underline{q}$. Define

$$
G_i(\underline{\hat{q}}) = \frac{\partial A(\underline{q})}{\partial q_i}\bigg|_{\underline{\hat{q}}} \qquad (4.15)
$$

then using (3.1) we obtain

$$
G_1 = \begin{bmatrix}
-\hat{q}_1 & \hat{q}_2 & \hat{q}_3 \\
\hat{q}_2 & -\hat{q}_1 & \hat{q}_4 \\
\hat{q}_3 & -\hat{q}_4 & -\hat{q}_1
\end{bmatrix} \qquad
G_2 = \begin{bmatrix}
-\hat{q}_2 & \hat{q}_1 & -\hat{q}_4 \\
\hat{q}_1 & \hat{q}_2 & \hat{q}_3 \\
\hat{q}_4 & \hat{q}_3 & -\hat{q}_2
\end{bmatrix}
$$

$$
\dots \quad (4.16a) \qquad\qquad \dots \quad (4.16b)
$$

$$G_3 = \begin{vmatrix} -\hat{q}_3 & \hat{q}_4 & \hat{q}_1 \\ -\hat{q}_4 & -\hat{q}_3 & \hat{q}_2 \\ -\hat{q}_1 & \hat{q}_2 & \hat{q}_3 \end{vmatrix} \qquad G_4 = \begin{vmatrix} -\hat{q}_4 & \hat{q}_3 & -\hat{q}_2 \\ -\hat{q}_3 & \hat{q}_4 & \hat{q}_1 \\ \hat{q}_2 & -\hat{q}_1 & \hat{q}_4 \end{vmatrix}$$

$$\cdots \quad (4.16c) \qquad\qquad \cdots \quad (4.16e)$$

Further define

$$\underline{h}_i = G_i \underline{V}_I \qquad (4.17)$$

Then (4.14) can be written as

$$\underline{V}_A = A(\underline{q})\underline{V}_I - [\; \underline{h}_1 \mid \underline{h}_2 \mid \underline{h}_3 \mid \underline{h}_4 \;] \begin{vmatrix} dq_1 \\ dq_2 \\ dq_3 \\ dq_4 \end{vmatrix} \qquad (4.18)$$

Finally, define

$$H_q = [\; \underline{h}_1 \mid \underline{h}_2 \mid \underline{h}_3 \mid \underline{h}_4 \;] \qquad (4.19)$$

then (4.18) can be written as

$$\underline{V}_A = A(\underline{q})\underline{V}_I - H_q d\underline{q} \qquad (4.20)$$

Now recall the definition of $\underline{W}_A$ and $\underline{V}_I$ as shown in (4.3). From these definitions it is obvious that we may substitute (4.11) and (4.20) into (4.1). When this is done and in view of (4.2), we obtain

$$\underline{y} = H_q d\underline{q} + M_{AT'}\theta\underline{W}_{T'} + M_{AT'}d\underline{W}_{T'} \qquad (4.21)$$

Note from (4.8) that

$$\theta = -[\underline{\varrho}x] \qquad (4.22)$$

therefore (4.21) can be written as

$$\underline{y} = H_q d\underline{q} - M_{AT'}[\underline{\varrho}x]\underline{W}_{T'} + M_{AT'}d\underline{W}_{T'} \qquad (4.23)$$

The matrix $M_{AT'}$ is not known to us; however, we do know $M_{AT}$. It is easy to see that using the latter rather than the former does not affect the accuracy to any meaningful degree. For identical reasoning we use $\underline{W}_{T',meas}$ rather than $\underline{W}_{T'}$. When these changes are made and the order of the cross product is changed in (4.23), we obtain

$$\boxed{\underline{y} = H_q d\underline{q} + M_{AT}[\underline{W}_{T',meas}x]\underline{\varrho} + M_{AT}d\underline{W}_{T'}} \qquad (4.24)$$

While (4.1) indicates how to generate the effective measurement $\underline{y}$ which updates the estimate, (4.24) indicates the linear relationship between $\underline{y}$, the attitude errors, the misalignment errors of the sensor whose measurements are being used and $d\underline{W}_{T'}$, the total error generated by the sensor. The derivation of (4.24) is the first stage in finding the measurement matrix, H, (defined in (2.8b)) for each of the sensors used onboard ERBS. In order to conclude the development which will yield those H matrices, we have to express $d\underline{W}_{T'}$ in terms of the error states of each sensor which constitute a part of $\underline{X}$ shown in (2.9). This is done next.

### Fine Sun Sensor (FSS) Measurement Model

The Sun sensor measures the tangents of the two angles of the vector from the spacecraft to the sun.

These two angles are A and B. Using the measured quantities $(\tan A)_m$ and $(\tan B)_m$, the unit vector measured by the sensor is computed as follows

$$\underline{W}_{T',meas} = [1 + (\tan A)_m^2 + (\tan B)_m^2]^{-1/2} \begin{vmatrix} (\tan A)_m \\ (\tan B)_m \\ 1 \end{vmatrix} \qquad (4.25)$$

Let $u_m = (\tan A)_m$ and $v_m = (\tan B)_m$ then (4.25) becomes

$$\underline{W}_{T',meas} = (1 + u_m^2 + v_m^2)^{-1/2} \begin{vmatrix} u_m \\ v_m \\ 1 \end{vmatrix} \qquad (4.26)$$

Perturbation of (4.26) yields the following vector of errors for the measured sun vector.

$$d\underline{W}_{T'} = (1 + u_m^2 + v_m^2)^{-1/2} \begin{vmatrix} du \\ dv \\ 0 \end{vmatrix}$$
$$- (1 + u_m^2 + v_m^2)^{-3/2} \begin{vmatrix} u_m^2 du + u_m v_m dv \\ u_m v_m du + v_m^2 dv \\ u_m du + v_m dv \end{vmatrix} \qquad (4.27)$$

Let

$$Q = (1 + u_m^2 + v_m^2)^{-1/2}$$
$$W_{11} = Q - Q^3 u_m^2 \qquad W_{12} = -Q^3 u_m v_m$$
$$W_{21} = -Q^3 u_m v_m \qquad W_{22} = Q - Q^3 v_m^2 \qquad (4.28)$$
$$W_{31} = -Q^3 u_m \qquad W_{32} = -Q^3 v_m$$

Then (4.27) can be written as

$$d\underline{W}_{T'} = \begin{vmatrix} W_{11} & W_{12} \\ W_{21} & W_{22} \\ W_{31} & W_{32} \end{vmatrix} \begin{vmatrix} du \\ dv \end{vmatrix} = W_s \begin{vmatrix} du \\ dv \end{vmatrix} \qquad (4.29)$$

The measured quantities $(\tan A)_m$ and $(\tan B)_m$ can be written as

$$(\tan A)_m = \tan A + C_A \tan A + b_A + n_A \qquad (4.30a)$$
$$(\tan B)_m = \tan B + C_B \tan B + b_B + n_B \qquad (4.30b)$$

where

$C_A, C_B$ = scale factor errors
$b_A, b_B$ = biases modeled as Markov states in (3.25a)
$n_A, n_B$ = white noise

From (4.30) and the definition of $u_m$ and $v_m$ we realize that

$$du = C_A \tan A + b_A + n_A \qquad (4.31a)$$
$$dv = C_B \tan B + b_B + n_B \qquad (4.31b)$$

When (4.31) is substituted into (4.29) the following is obtained

$$dW_{T'} = W_s \begin{vmatrix} C_A \tan A \\ C_B \tan B \end{vmatrix} + W_s \begin{vmatrix} b_A \\ b_B \end{vmatrix} + W_s \begin{vmatrix} n_A \\ n_B \end{vmatrix} \quad (4.32)$$

which can be written as

$$dW_{T'} = \begin{vmatrix} W_s \begin{vmatrix} \tan A & 0 \\ 0 & \tan B \end{vmatrix} & W_s \end{vmatrix} \begin{vmatrix} C_A \\ C_B \\ b_A \\ b_B \end{vmatrix}$$

$$+ W_s \begin{vmatrix} n_A \\ n_B \end{vmatrix} \quad (4.33)$$

This is then substituted into (4.24) resulting in

$$Y = \begin{vmatrix} H_q & \begin{matrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{matrix} & M_{AT}[W_{T',meas}x] \end{vmatrix}$$

$$M_{AT}W_s \begin{vmatrix} \tan A & 0 \\ 0 & \tan B \end{vmatrix} \quad M_{AT}W_s \begin{vmatrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{vmatrix} X$$

$$+ M_{AT}W_s \begin{vmatrix} n_A \\ n_B \end{vmatrix} \quad (4.34)$$

Equation (4.34) gives the measurement matrix, H, for the FSS which is used in computing the gain matrix and updating the covariance matrix. Since tanA and tanB are not available to us, we use, respectively, $(\tan A)_m$ and $(\tan B)_m$ instead. Since the measured and the true quantities are close, this change practically introduces no error.

## IR Horizon Scanner Measurement Model

The horizon scanner measures the roll and pitch of the spacecraft with respect to the geodetic coordinate system (GDS), i.e. it measures the direction of the nadir vector. The horizon scanner misalignment errors are assumed to be small with respect to roll and pitch errors, to be additive to roll and pitch and indistinguishable from them. The unit vector in the direction of the nadir in the GDS is given as

$$Z_{GDS}^T = [0, 0, 1] \quad (4.35)$$

In body coordinates this vector is given as

$$Z_{body} = \begin{vmatrix} -\cos(r)\sin(p) \\ \sin(r) \\ \cos(r)\cos(p) \end{vmatrix} \quad (4.36)$$

where r is the roll angle and p is the pitch angle. As mentioned, this is the measured vector; that is

$$W_{T',meas} = \begin{vmatrix} -\cos(r)\sin(p) \\ \sin(r) \\ \cos(r)\cos(p) \end{vmatrix}_{meas} \quad (4.37)$$

which is equal to the true vector plus error. The error vector is obtained by perturbing (4.36). The perturbation yields

$$dW_{T'} = \begin{vmatrix} \sin(r)\sin(p)dr - \cos(r)\cos(p)dp \\ \cos(r)dr \\ -\sin(r)\cos(p)dr - \cos(r)\sin(p)dp \end{vmatrix} \quad (4.38)$$

Let

$$W_h = \begin{vmatrix} \sin(r)\sin(p) & -\cos(r)\cos(p) \\ \cos(r) & 0 \\ -\sin(r)\cos(p) & -\cos(r)\sin(p) \end{vmatrix} \quad (4.39)$$

then (4.38) can be written as

$$dW_{T'} = W_h \begin{vmatrix} dr^* \\ dp^* \end{vmatrix} \quad (4.40)$$

We characterize the horizon scanner errors as bias (modeled as Markov process in (3.25b)) plus white noise; that is,

$$dr^* = d_r + n_{hr} \quad (4.41a)$$

$$dp^* = d_p + n_{hp} \quad (4.41b)$$

where $d_r$ and $d_p$ are the roll and pitch biases and $n_{hr}$ and $n_{hp}$ are the roll and pitch white measurement noise components. When (4.41) are substituted into (4.40), the following is obtained

$$dW_{T',meas} = W_h b_h + W_h n_h \quad (4.42)$$

where $b_h$ is as defined in (3.26) and $n_h^T = [n_{hr}, n_{hp}]$. Since $Z_{body}$ is already in body coordinates, $M_{AT}$ given in (4.1) and in (4.24) is the identity matrix. Since the horizon scanner was assumed not to have misalignment error the term containing misalignment angles in (4.24) is not needed. The model for the horizon scanner is given in (4.43) below. Again $y$ is computed using (4.1).

$$Y = \begin{vmatrix} H_q & \begin{matrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{matrix} & W_h & \begin{matrix} 0 & \dots & 0 \\ 0 & \dots & 0 \\ 0 & \dots & 0 \end{matrix} \end{vmatrix} X + W_h n_h \quad (4.43)$$

Equation (4.43) yields the H matrix to be used with IR horizon scanner measurements. Similarly to the evaluation of the Sun sensor H matrix, we use the measured roll and pitch to evaluate $W_h$ in (4.43).

## Magnetometer Measurement Model

The three magnetometers mounted orthogonally to one another measure the Earth's magnetic field components along each of their axes. This arrangement of sensors is identical to the three gyro arrangement which measure the spacecraft's angular rate. The magnetometer error sources are also identical to the gyro error sources which are: scale factor errors, misalignments, bias (modeled as Markov process) and white measurement noise. Therefore the magnetometer errors can be represented by the same model as for the gyros. Therefore, in analogy to (3.13), we write the following expression for the errors introduced by the magnetometers

$$\begin{bmatrix} dB_x \\ dB_y \\ dB_z \end{bmatrix} = \begin{bmatrix} S_{mx} & 0 & 0 \\ 0 & S_{my} & 0 \\ 0 & 0 & S_{mz} \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} + \begin{bmatrix} 0 & \theta_{mxy} & \theta_{mxz} \\ \theta_{myx} & 0 & \theta_{myz} \\ \theta_{mzx} & \theta_{mzy} & 0 \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} + \begin{bmatrix} b_{mx} \\ b_{my} \\ b_{mz} \end{bmatrix}$$
$$+ \begin{bmatrix} n_{mx} \\ n_{my} \\ n_{mz} \end{bmatrix} \quad (4.44)$$

where $B_x$, $B_y$ and $B_z$ are the magnetometer measurements and

$$\underline{S}_m^T = [S_{mx}, S_{my}, S_{mz}] \quad (4.45a)$$

$$\underline{\theta}_m^T = [\theta_{mxy}, \theta_{mxz}, \theta_{myx}, \theta_{myz}, \theta_{mzx}, \theta_{mzy}] \quad (4.45b)$$

$$\underline{b}_m^T = [b_{mx}, b_{my}, b_{mz}] \quad (4.45c)$$

$$\underline{n}_m^T = [n_{mx}, n_{my}, n_{mz}] \quad (4.45d)$$

$\underline{S}_m$, $\underline{\theta}_m$ and $\underline{b}_m$ are the vectors of magnetometer scale factors, misalignments and biases, and $\underline{n}_m$ is the white measurement noise vector. The magnetometer output vector $\underline{B}_{meas}$ can be written as

$$\underline{B}_{T,meas} = \underline{B}_T + d\underline{B} \quad (4.46)$$

where $\underline{B}_T$ is the true magnetic field vector in the assumed magnetometer coordinates (note the difference between these sensors and the FSS and IR horizon scanner). We can write (4.44) as follows

$$\begin{bmatrix} dB_x \\ dB_y \\ dB_z \end{bmatrix} = \begin{bmatrix} B_x & 0 & 0 & B_y & B_z & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & B_y & 0 & 0 & 0 & B_x & B_z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & B_z & 0 & 0 & 0 & 0 & B_x & B_y & 0 & 0 & 1 \end{bmatrix} \underline{x}^+ + \begin{bmatrix} n_{mx} \\ n_{my} \\ n_{mz} \end{bmatrix}$$

$$\dots \quad (4.47)$$

where

$$\underline{x}^{+T} = [\underline{S}_m^T, \underline{\theta}_m^T, \underline{b}_m^T] \quad (4.48)$$

Define the matrix B' as follows

$$B' = \begin{bmatrix} B_x & 0 & 0 & B_y & B_z & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & B_y & 0 & 0 & 0 & B_x & B_z & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & B_z & 0 & 0 & 0 & 0 & B_x & B_y & 0 & 0 & 1 \end{bmatrix} \quad (4.49)$$

We may write then (4.47) as

$$d\underline{B} = B'\underline{x}^+ + \underline{n}_m \quad (4.50)$$

Note that $d\underline{B}$ is not identical to $d\underline{W}_T$, the way the latter is defined in (4.4) since $d\underline{B}$ contains the magnetometer misalignment errors whereas $d\underline{W}_T$ does not. For this reason $d\underline{B}$ cannot be substituted for $d\underline{W}_T$ in (4.24). We have, then, to use the basic definition of $\underline{y}$ as applied to the magnetometer readings in order to formulate the linear relationship between $\underline{y}$ and the magnetometer errors which constitute $\underline{x}^+$. For magnetometer measurements we formulate our effective measurement, $\underline{y}$, as follows

$$\boxed{\underline{y} = M_{AT}\underline{B}_{T,meas} - A(\hat{\underline{q}})\underline{V}_I} \quad (4.51)$$

Substitute (4.46) into (4.51) and in view of (4.3b) also substitute (4.20) into (4.51). This results in

$$\underline{y} = M_{AT}\underline{B}_T + M_{AT}d\underline{B} - A(\underline{q})\underline{V}_I + H_q d\underline{q} \quad (4.59)$$

Note that the first and third terms on the right-hand side of (4.59) cancel one another. Then when (4.50) is substituted into (4.59) we obtain the desired result

$$\underline{y} = H_q d\underline{q} + M_{AT}B'\underline{x}^+ + M_{AT}\underline{n}_m \quad (4.60)$$

or more explicitly

$$\underline{y} = \begin{bmatrix} H_q & \begin{matrix} 0 & \dots\dots\dots & 0 \\ 0 & \dots\dots\dots & 0 \\ 0 & \dots\dots\dots & 0 \end{matrix} & M_{AT}B' \end{bmatrix} + M_{AT}\underline{n}_m \quad (4.61)$$

Equation (4.61) yields the H matrix to be used with the magnetometer measurement updates. As for the previous sensors, we use the measured magnetometer data to evaluate B'. Finally, note again that the effective magnetometer measurements which have to be processed by the EKF are computed using (4.51) and not (4.1).

Measurement Error
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The main component in the magnetometer noise vector, $\underline{n}_m$, is the quantization error. Its nature and characterization is explained as follows. The output of the magnetometers is received in the telemetry stream as

$$\underline{N}^T = [N_x, N_y, N_z] \quad (4.62)$$

with $\underline{N}$ in counts. The ith component of $\underline{N}$ ($i=x,y,z$) is obtained from the actual measured components as [9]

$$N_i = INT(B_{i,meas}/K_i) \quad (4.63)$$

where INT means "the integer part of". Obviously, a certain part of the measured value is lost due to the INT operation: that is

$$INT(B_{i,meas}/K_i) + n_i = B_{i,meas}/K_i \quad (4.64)$$

The nature of the INT operation is such that $n_i$ can vary between 0 to 1. Moreover, the distribution of the chopped off value is uniform over the range 0 to 1. It is then easy to show that

$$E(n_i) = 0.5 \quad (4.65a) \qquad Var(n_i) = 1/12 \quad (4.65b)$$

Substituting (4.63) into (4.64) yields

$$N_i = B_{i,meas}/K_i - n_i \quad (4.66)$$

It is easy to see why in order to calculate the magnetometer readings on the ground the following computation is performed

$$B_{i,comp} = K_i[N_i + 0.5] \quad (4.67)$$

Substituting (4.66) into (4.67) yields
$$B_{i,comp} = K_i[B_{i,meas}/K_i - n_i + 0.5] \quad (4.68)$$

Define the measurement noise of magnetometer i as

$$n_{m,i} = K_i(0.5 - n_i) \quad (4.69)$$

then, in view of (4.65),

$$E\{n_{m,i}\} = 0 \quad (4.70a) \qquad Var\{n_{m,i}\} = K_i^2/12 \quad (4.70b)$$

where E denotes the expected value and Var denotes the variance. From (4.48) and (4.69)

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$$B_{i,comp} = B_{i,meas} + n_{m,i} \qquad (4.71)$$

and in vector form

$$\underline{B}_{comp} = \underline{B}_{meas} + \underline{n}_m \qquad (4.72)$$

From (4.46), (4.47), (4.61) and (4.72) it is obvious that the noise vector to be use in (4.61) is the vector $\underline{n}_m$ as defined in (4.69) whose expected value and variance are defined in (4.70)

## V. QUATERNION NORMALIZATION

The quaternion which represents attitude is a normal one. It was found [6] that forcing normalization on the estimated quaternion is advantageous since it speeds up convergence and eliminates the need for filter tuning. It was found in the present work too that normalization has these benefits. As shown in [6], normalization of the quaternion is equivalent to removing a portion of the estimate. This part that is removed must be accounted for in the next stage of the filtration. The handling of the normalization in this work is not identical to the one in [6] since here the covariance and state are propagated by solving their respective differential equations and not by using the state transition matrix as is the case in [6]. There the part of the estimate which is removed by normalization is propagated using the state transition matrix and is considered at the next measurement update of the state estimate. Here though the normalization is done in between measurements. After the state estimate update by the horizon scanner measurement, the quaternion is normalized as follows

$$\hat{q}_{IR}^{*}(+) = \hat{q}_{IR}(+)/|\hat{q}_{IR}(+)| \qquad (5.1)$$

where the subscript IR denotes the fact that the quaternion estimate being dealt with is at the time point where the IR horizon scanner measurement are considered, (+) denotes the a-posteriori estimate and the superscript * denotes the resultant normal quaternion. It can be shown [6] that the normalization removes the following part from $\hat{q}_{IR}(+)$

$$d\underline{q}_N = \hat{q}_{IR}(+)\hat{q}_{IR}^{T}(+)d\hat{q}_{IR} \qquad (5.2)$$

where $d\hat{q}_{IR} = K_{IR}\underline{Y}_{IR}$ is the estimate of $d\underline{q}$ which is computed using the kalman gain and the effective measurement of the scanner. Now when the FSS measurements are processed next, the estimate of the quaternion is updated as follows

$$d\hat{q}_{FSS}(+) = d\hat{q}_N + K_{FSS}[\underline{Y}_{FSS} - H_{FSS}d\hat{q}_N] \qquad (5.3a)$$

$$\hat{q}_{FSS}(+) = \hat{q}_{IR}^{*}(+) + d\hat{q}_{FSS}(+) \qquad (5.3b)$$

where $\hat{q}_{FSS}(+)$ is the quaternion estimate after its update by the FSS measurements. If no normalization is performed, $\hat{q}_{IR}^{*}(+) = \hat{q}_{IR}(+)$, $d\hat{q} = 0$ and (5.3) change accordingly. In any event, the quaternion, $\hat{q}_{FSS}(+)$, is used as the a priori estimate of $\underline{q}$ for the magnetometer update, if available, or else is propagated to the next time point.

## VI. COMPENSATION

When propagating the state estimate and the covariance, we use the measured angular velocity. We know, however, that the propagated values are not accurate since the gyro outputs contain errors. As we estimate those errors, we can do better if we correct the gyro outputs for estimated errors. This operation is known as calibration.

We also want to compensate the measurements obtained from the FSS, the IR horizon scanner, and the magnetometers which are all orientation measuring devices whose outputs are used to update the filter. The reason we want to compensate these sensors' outputs is different in nature than the reason for compensating the gyro outputs. Rewrite (4.1) and (2.11)

$$\underline{Y} = M_{AT}\underline{W}_{T',meas} - A(\hat{q})\underline{V}_I \qquad (6.1)$$

$$\hat{\underline{X}}_k(+) = \hat{\underline{X}}_k(-) + K_k\underline{Y}_k \qquad (6.2)$$

The term $K_k\underline{Y}_k$ is nothing but the estimate of $\underline{x}$ defined in (2.14) as

$$\underline{X}(t_k) = \hat{\underline{X}}(-) + \underline{x}(t_k) \qquad (6.3)$$

That is, in (2.11) we estimate the difference between the true value of $\underline{X}$ and its latest estimate, and add the estimate of the difference to the latest estimate of $\underline{X}$ to form its updated estimate. Now let us consider an error term in one of the sensor measurements, say a bias. This bias is a part of $\underline{W}_{T',meas}$ and thus, as indicated in (6.1), bears its signature on $\underline{Y}$. Consequently, if certain observability conditions are met, it is estimated and added to the state estimate as indicated in (6.3). If no compensation takes place, the next time the measurements of this sensor will be processed the bias will again be estimated and added to the previous estimate of this bias, thus creating a too large and hence wrong estimate. The correct way, then, to handle this case is to eliminate the estimate of the bias from $\underline{W}_{T',meas}$. This way only residual bias which has not been estimated yet will be present in $\underline{y}$ as shown in (6.1). Only the estimate of this residual will, then, be added to the existing estimate of the bias, which is a part of $\hat{\underline{X}}$, yielding a correction to the previous estimate. This logic holds for the other error states too. The way we carried out the compensation is outlined in the ensuing.

### Gyro Compensation

From (3.4)

$$\underline{w} = \bar{\underline{w}} - d\underline{w} \qquad (6.4a)$$

therefore

$$\hat{\underline{w}} = \bar{\underline{w}} - d\hat{\underline{w}} \qquad (6.4b)$$

Rewrite (3.18)

$$d\underline{w} = [U|W|I]\begin{bmatrix} \underline{S}_g \\ \underline{\theta}_g \\ \underline{b}_g \end{bmatrix} + \underline{n}_{g1} \qquad (6.5)$$

Since U and W are functions of $\underline{w}$, and since the noise vector is of zero mean, a good estimate of $d\underline{w}$ is obtained from (6.5) as

$$d\hat{\underline{w}} = [U(\hat{\underline{w}})|W(\hat{\underline{w}})|I]\begin{bmatrix} \hat{\underline{S}}_g \\ \hat{\underline{\theta}}_g \\ \hat{\underline{b}}_g \end{bmatrix} \qquad (6.6)$$

This estimate can then be used in (6.4b) to yield an estimate of $\underline{w}$ which is then used in the propagation algorithm instead of the raw gyro outputs.

### FSS Compensation

Consider

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$$\underline{y} = M_{AT}\underline{W}_{T',meas} - A(\hat{\underline{q}})\underline{V}_I \qquad (6.7)$$

and recall (4.3)

$$\underline{W}_A = M_{AT}\underline{W}_{T',meas} \quad (6.8a) \qquad \underline{V}_A = A(\hat{\underline{q}})\underline{V}_I \quad (6.8b)$$

As explained earlier, $\underline{y}$ is a linear function of $\underline{x}$ which is the difference between $\hat{\underline{X}}$ and $\underline{X}$. We want $\underline{x}$ to go to zero when $\hat{\underline{X}}$ approaches $\underline{X}$. Indeed, when $\hat{\underline{X}}$ approaches $\underline{X}$, $\hat{\underline{q}}$ approaches $\underline{q}$ and $\underline{V}_A$ computed in (6.8b) really yields the components of the unit vector in the Sun's direction when resolved in body coordinates. However,

even when $\hat{\underline{X}}$ is equal to $\underline{X}$, $\underline{W}_{T',meas}$ will not be equal to $\underline{W}_T$ and therefore $\underline{W}_A$ computed in (6.8a) will not be equal to $\underline{V}_A$ and thus $\underline{y}$ will not be zero. (This fact shouldn't be confused with the fact that $\underline{y}$ goes to zero when the errors themselves go to zero). Only when $\underline{W}_{T',meas}$ goes to $\underline{W}_T$ as $\hat{\underline{X}}$ goes to $\underline{X}$, will $\underline{y}$ go to zero as it should. To achieve this use (4.4) to note that

$$\underline{W}_T = M_{TT'}(\underline{W}_{T',meas} - d\underline{W}_{T'}) \qquad (6.9)$$

As shown in (4.7), for small FSS misalignment angles

$$M_{T'T} = I + \Theta \qquad (6.10)$$

Transposing (6.10) we obtain

$$M_{TT'} = I - \Theta \qquad (6.11)$$

which when substituted into (6.9) yields

$$\underline{W}_T = (I - \Theta)(\underline{W}_{T',meas} - d\underline{W}_{T'}) \qquad (6.12)$$

Therefore a reasonable estimate of $\underline{W}_T$ is

$$\hat{\underline{W}}_T = (I - \hat{\Theta})(\underline{W}_{T',meas} - d\hat{\underline{W}}_{T'}) \qquad (6.13)$$

Replacing $\underline{W}_{T',meas}$ in (6.7) by $\hat{\underline{W}}_T$ yields the desired result

$$\underline{y} = M_{AT}(I - \hat{\Theta})(\underline{W}_{T',meas} - d\hat{\underline{W}}_{T'}) - A(\hat{\underline{q}})\underline{V}_I \quad (6.14)$$

where, in view of (4.32), $d\hat{\underline{W}}_{T'}$ is computed as follows

$$d\hat{\underline{W}}_{T'} = W_s\begin{vmatrix} \hat{C}_A(tanA)_m \\ \hat{C}_B(tanB)_m \end{vmatrix} + W_s\begin{vmatrix} \hat{b}_A \\ \hat{b}_B \end{vmatrix} \qquad (6.15)$$

Note that initially when our estimate of $\underline{x}$ is zero, (6.14) is reduced to (6.1).

## IR Horizon Scanner Compensation

The arguments made in outlining the FSS compensation are also valid for the horizon scanner only that here $d\underline{W}_{T'}$ is different and there are no sensor misalignments (they are considered to be a part of the bias errors). Following (4.42), we compute for the horizon scanner

$$d\hat{\underline{W}}_{T'} = \hat{Q}_h\hat{\underline{b}}_h \qquad (6.16a)$$

and

$$\hat{\Theta} = 0 \qquad (6.16b)$$

and substitute them into (6.14). The result is then the compensated effective measurement of the IR horizon sensor.

## Magnetometer Compensation

As shown in (4.51) which is rewritten below, $\underline{y}$ is computed differently for the magnetometer measurements; namely

$$\underline{y} = M_{AT}\underline{B}_{T,meas} - A(\hat{\underline{q}})\underline{V}_I \qquad (6.17)$$

where

and

$$\underline{B}_{T,meas} = \underline{B}_T + d\underline{B} \qquad (6.18a)$$

$$d\underline{B} = B'\underline{x}^+ + \underline{n}_m \qquad (6.18b)$$

$$\underline{x}^{+T} = [\underline{S}_m^T, \underline{\theta}_m^T, \underline{b}_m^T] \qquad (6.18c)$$

Following the rationale behind the FSS and IR horizon scanner and in view of (6.18), we compensate the magnetometer readings as follows. Compute

$$\hat{\underline{x}}^{+T} = [\hat{\underline{S}}_m^T, \hat{\underline{\theta}}_m^T, \hat{\underline{b}}_m^T] \qquad (6.19a)$$

$$d\hat{\underline{B}} = B'\hat{\underline{x}}^+ \qquad (6.19b)$$

where B' is computed according to (4.49) using the uncompensated outputs of the magnetometers. Next compute the compensated magnetometer measurements

$$\hat{\underline{B}}_T = \underline{B}_{T,meas} - d\hat{\underline{B}} \qquad (6.19c)$$

which are used to compute the effective measurement as follows

$$\hat{\underline{y}} = M_{AT}\hat{\underline{B}}_T - A(\hat{\underline{q}})\underline{V}_I \qquad (6.19d)$$

## VII. THE COMPLETE ERBS EKF ALGORITHM

The models developed in the previous section were implemented into a program written in Fortran. The data used in the program is actual spacecraft data transmitted to Earth by ERBS.

## MEASUREMENT UPDATES

The program is set up to compute an update initially and then propagate. The updates are performed for each sensor individually, the horizon scanner update is performed first, followed by the sun sensor, and the magnetometer. If any sensor data are not available the program bypasses that sensor and goes on to the next. In between each sensor update, the updated state and covariance are set to the a-priori values going into the next sensor update. If no sensor data are available, the a-posteriori state and covariance are set equal to the a-priori and are propagated to the next time point.

Below is a summary of the algorithm and how it is applied to each update and to the propagation.

## IR Update

Compute H:

$$H = \begin{vmatrix} H_q \end{vmatrix}\begin{vmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{vmatrix} \quad W_h \quad \begin{vmatrix} 0 & \cdots & 0 \\ 0 & \cdots & 0 \\ 0 & \cdots & 0 \end{vmatrix} \quad (7.1)$$

Compute $\underline{W}_{T',meas}$:

$$\underline{W}_{T',meas} = \begin{vmatrix} -\cos(r)\sin(p) \\ \sin(r) \\ \cos(r)\cos(p) \end{vmatrix}_{meas} \qquad (7.2)$$

Compensate $\underline{W}_{T',meas}$:

$$\hat{\underline{W}}_T = \underline{W}_{T',meas} - \hat{\underline{q}}_h \hat{\underline{b}}_h \qquad (7.3)$$

Compute residual and uncertainty of residual:

$$\underline{y} = \hat{\underline{W}}_T - A(\hat{q})\underline{V}_{I,ir} \qquad (7.4)$$

$$(u_{ir})_j = \{[HP(-)H^T + R_{ir}]\}_j^{1/2} \qquad (7.5)$$

where the subscript ir denotes quantities pertinent to the horizon scanner, $\underline{V}_{I,ir}$ is the spacecraft-to-earth unit vector obtained from the ephemeris and j denotes the j<u>th</u> element on the main diagonal of the residual covariance matrix.

State and Covariance Update

Compute K:

$$K_k = P_k(-)H_k^T[H_kP_k(-)H_k^T + R_k]^{-1} \qquad (7.6)$$

Update $\hat{\underline{X}}$:

$$\hat{\underline{X}}_k(+) = \hat{\underline{X}}_k(-) + K_k\underline{y}_k \qquad (7.7)$$

Update P:

$$P_k(+) = (I-K_kH_k)P_k(-)(I-K_kH_k)^T + K_kR_kK_k^T \qquad (7.8)$$

## Sun Sensor Update

Compute H:

$$H = \begin{vmatrix} H_q & \begin{matrix} 0 & .. & 0 \\ 0 & .. & 0 \\ 0 & .. & 0 \end{matrix} & M_{AT}[\underline{W}_{T',meas}{}^x] & M_{AT}W_s\begin{vmatrix} tanA & 0 \\ 0 & tanB \end{vmatrix} \end{vmatrix}$$

$$\begin{vmatrix} M_{AT}W_s & \begin{matrix} 0 & ...... & 0 \\ 0 & ...... & 0 \\ 0 & ...... & 0 \end{matrix} \end{vmatrix} \qquad (7.9)$$

Compute $\underline{W}_{T',meas}$:

$$\underline{W}_{T',meas} = (1 + (tanA)_m^2 + (tanB)_m^2)^{-1/2} \begin{vmatrix} (tanA)_m \\ (tanB)_m \\ 1 \end{vmatrix} \qquad , \qquad \quad ... \quad (7.10)$$

Compensate $\underline{W}_{T',meas}$:

$$\hat{\underline{W}}_T = (I - \hat{\theta})( \underline{W}_{T',meas} - W_s\begin{vmatrix} \hat{C}_A(tanA)_m \\ \hat{C}_B(tanB)_m \end{vmatrix} - W_s\begin{vmatrix} \hat{b}_A \\ \hat{b}_B \end{vmatrix} ) \qquad ... \quad (7.11)$$

Compute residual and uncertainty of residual:

$$\underline{y} = \hat{\underline{W}}_T - A(\hat{q})\underline{V}_{I,fss} \qquad (7.13)$$

$$(u_{fss})_j = \{[HP(-)H^T + R_{fss}]\}_j^{1/2} \qquad (7.14)$$

where the subscript fss denotes quantities pertinent to the fine Sun sensor, $\underline{V}_{I,fss}$ is the spacecraft-to-Sun unit vector obtained from a Solar-Lunar-Planetary file and j denotes the j<u>th</u> element on the main diagonal of the residual covariance matrix. The state and covariance are then updated as in (7.6) through (7.8).

## Magnetometer Update

Compute H:

$$H = \begin{vmatrix} H_q & \begin{matrix} 0 & ............. & 0 \\ 0 & ............. & 0 \\ 0 & ............. & 0 \end{matrix} & M_{AT}B' \end{vmatrix} \qquad (7.15)$$

Compute $\underline{B}_{T',meas}$:

$$\underline{B}_{T',meas} = \begin{vmatrix} B_x \\ B_y \\ B_z \end{vmatrix} \qquad (7.16)$$

Compensate $\underline{B}_{T',meas}$:

$$\hat{\underline{B}}_T = \underline{B}_{T',meas} - \hat{B}'\begin{vmatrix} \hat{S}_m \\ \hat{\theta}_m \\ \hat{b}_m \end{vmatrix} \qquad (7.17)$$

Compute residual and uncertainty of residual:

$$\underline{y} = \hat{\underline{B}}_T - A(\hat{q})\underline{V}_{I,mag} \qquad (7.18)$$

$$(u_{mag})_j = \{[HP(-)H^T + R_{mag}]\}_j^{1/2} \qquad (7.19)$$

where the subscript mag denotes quantities pertinent to the magnetometers, $\underline{V}_{I,mag}$ is a unit vector in the direction of the magnetic field obtained from a 1980 International Geomagnetic Reference Field model available in a Fortran subroutine and j denotes the j<u>th</u> element on the main diagonal of the residual covariance matrix. The state and covariance are then updated as in (7.6) through (7.8).

## STATE AND COVARIANCE PROPAGATION

After all the sensor updates are performed, the state and covariance are propagated using a fourth order Runge-Kutta routine. The state and covariance are propagated ahead using the gyro data at the time of the update and the gyro data one second (nominally) ahead. Before propagating, though, the gyro data is compensated as follows.

$$\hat{\underline{w}} = \underline{w} - [U|W|I]\begin{vmatrix} \hat{S}_g \\ \hat{\theta}_g \\ \hat{b}_g \end{vmatrix} \qquad (7.20)$$

State Propagation

$$\hat{\underline{X}}(t) = \underline{f}(\hat{\underline{X}}(t),t) \qquad (7.21)$$

Covariance Propagation

$$\dot{P}(t) = F(\hat{\underline{X}}(t),t)P(t) + P(t)F^T(\hat{\underline{X}}(t),t) + Q(t) \qquad (7.22)$$

## VIII. RESULTS

### Reference Solution

The reference we used for comparison was the attitude solution obtained from the batch estimator used on the ground for operational attitude determination for ERBS. Table 8.1 shows the attitude solutions in the GDS and uncertainties for three different conditions (cases). Case 1 used sensor standard deviations of: FSS = 0.002 deg, IR = 0.2 deg,

MAG = 0.5 (unit vector). An orbit's worth of data was used to compute the solution. Case 2 used the same amount of data as the first but the sensor standard deviations were: FSS = 0.1 deg, IR = 0.5 deg, MAG = 0.5 (unit vector). Case 3 had the same sensor standard deviations as the second but only 30 seconds of data were used. Obviously, the reference solution is not unique although the real solution is.

Table 8.1  Final Attitude Solutions
and Uncertainties (deg.)

|  | Case 1 | | Case 2 | | Case 3 | |
|---|---|---|---|---|---|---|
|  | Attitude | Unc. | Attitude | Unc. | Attitude | Unc. |
| Yaw | -0.294 | 0.0135 | -0.262 | 0.004 | -0.731 | 0.012 |
| Roll | 0.400 | 0.0118 | 0.421 | 0.005 | 0.316 | 0.070 |
| Pitch | 0.650 | 0.173 | 0.420 | 0.004 | 0.384 | 0.007 |

## Filter Solution

Since ERBS is not inertially fixed, it is not very enlightening to see the variation in the quaternion. Therefore, in order to compare the filter solution to the batch solution, the estimated quaternion was converted to roll, pitch, and yaw in the GDS. We used the filter first to estimate attitude only. Figure 1 shows the yaw solution in the GDS and is a typical example of the behavior when estimating attitude only.
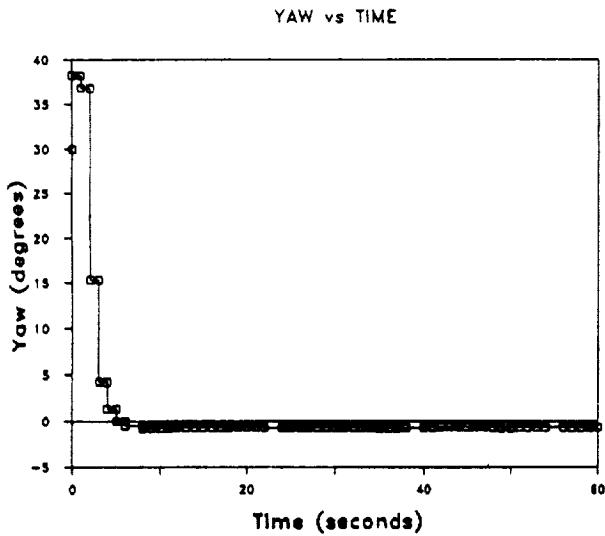
YAW vs TIME



Figure 1:  YAW vs. TIME
No Calibration States

The filter was then run with the full state starting at an a-priori attitude of zero degrees yaw, roll, and pitch. Figures 2 and 3 show the behavior of the yaw and pitch. Roll is similar to pitch. Figure 4 shows the estimation of the Z component of the gyro bias and Figure 5 shows an example of the residual behavior.
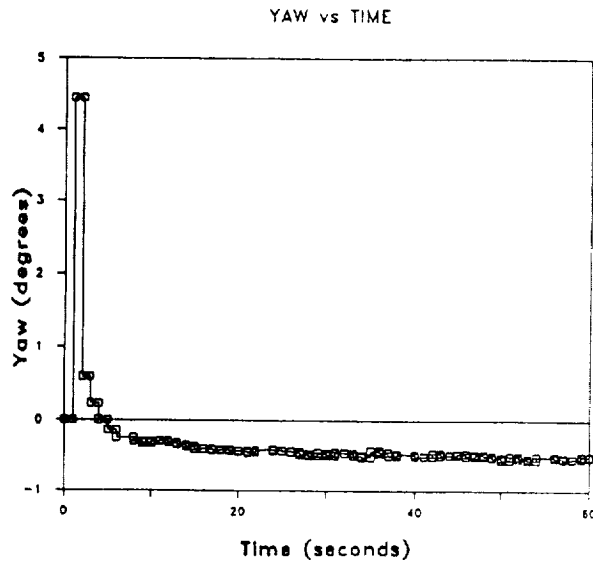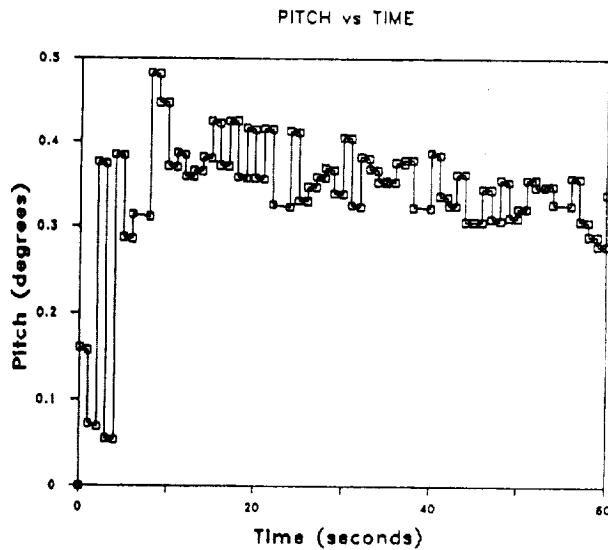
YAW vs TIME
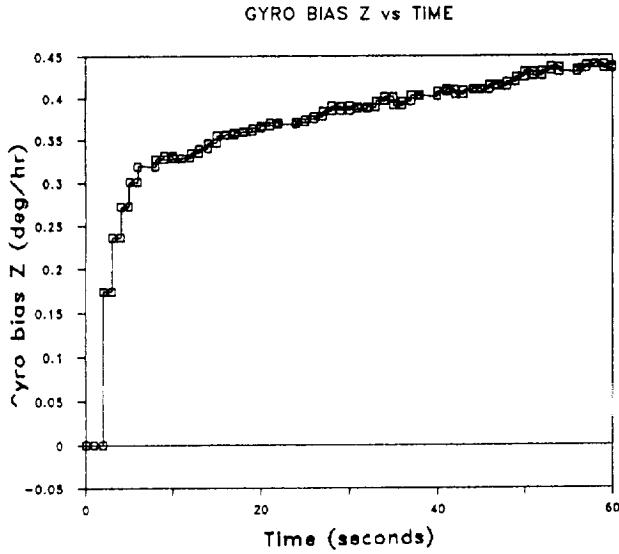


Figure 2:  YAW vs. TIME
Full State

PITCH vs TIME



Figure 3:  PITCH vs. TIME
Full State

GYRO BIAS Z vs TIME



Figure 4: GYRO BIAS Z vs. TIME
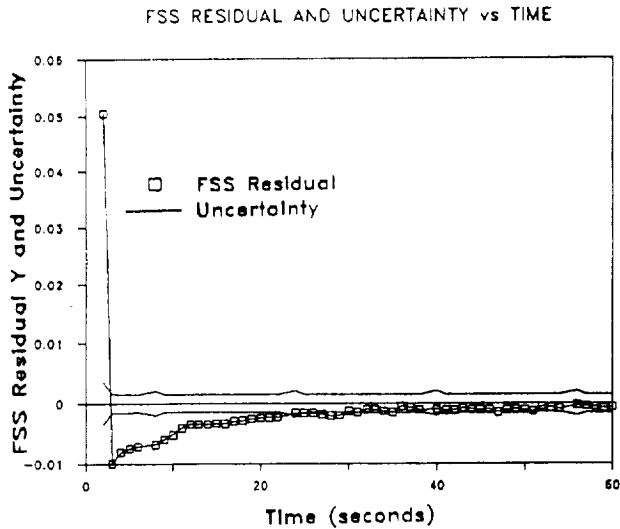Full State

FSS RESIDUAL AND UNCERTAINTY vs TIME



Figure 5: FSS RESIDUALS Y vs. TIME
Full State

Table 8.2 gives the final attitude solutions and the various calibration states (after 60 sec of data) for 2 different sets of initial conditions (which actually differ only in the FSS and IR bias uncertainty).

Table 8.2: Attitude and Calibration States
for Varying Initial Conditions

| | Case 1 | | Case 2 | |
|---|---|---|---|---|
| | | | | |
| Yaw | -0.512 | | -0.676 | |
| Roll | 0.354 | | 0.406 | |
| Pitch | 0.339 | | 0.253 | |
| | Value | Unc. | Value | Unc. |
| GYRO | | | | |
| SF | 0.194E-4 | 0.01 | 0.360E-5 | 0.01 |
| | -0.598E-4 | 0.00998 | 3.898E-5 | 0.00998 |
| | -0.786E-4 | 0.01 | -4.384E-5 | 0.01 |
| $\theta$ (deg) | 0.62E-4 | 0.057 | 3.2E-4 | 0.057 |
| | 0.34E-4 | 0.057 | 0.50E-4 | 0.057 |
| | 0.028E-4 | 0.057 | 0.013E-4 | 0.057 |
| | 0.15E-4 | 0.057 | 0.046E-4 | 0.057 |
| | -0.070E-4 | 0.057 | 0.052E-4 | 0.057 |
| | -14.6E-4 | 0.057 | -10.2E-4 | 0.057 |
| Bias | -0.0211 | 1.995 | -0.0984 | 1.995 |
| (Deg/hr) | 0.0094 | 1.995 | -0.0096 | 1.995 |
| | 0.4381 | 1.995 | 0.3037 | 1.995 |
| FSS | | | | |
| $\theta$ | 0.0648 | 0.050 | 0.0017 | 0.056 |
| (Deg) | 0.0466 | 0.054 | 0.0114 | 0.056 |
| | 0.0896 | 0.057 | 0.0140 | 0.057 |
| SF | -0.0235 | 0.00607 | -0.00557 | 0.009 |
| | -0.0177 | 0.00830 | 0.00044 | 0.00954 |
| Bias | 0.1732 | 0.086 | 0.077 | 0.151 |
| (Deg) | 0.0812 | 0.058 | -0.0468 | 0.0709 |
| IR | | | | |
| Bias | 0.2115 | 0.032 | 0.0340 | 0.0472 |
| (Deg) | -0.2377 | 0.033 | -0.0840 | 0.046 |
| MAG | | | | |
| Bias | -0.499 | 0.699 | -0.597 | 0.698 |
| (mg) | -1.479 | 0.816 | -1.237 | 0.816 |
| | 0.289 | 0.911 | 0.223 | 0.911 |
| | (other magnetometer states are negligible) | | | |

Initial Uncertainties:

Gyro
----
SF = 3*0.01
$\theta$ = 6*0.057 deg
bias = 2.0*3 deg/hr

FSS
---
    Θ = 3*0.057 deg,
    SF = 2*0.01
  Bias = 2*0.1 in case 1,
         2*0.316 in case 2

IR
--
  Bias = 2*0.1 in case 1,
         2*0.316 in case 2

MAG
---
    Bias = 3*1 mGauss

## IX. CONCLUSIONS

The ERBS EKF shows good, quick convergence properties when estimating only attitude. The filter is robust in that it can overcome initial attitude errors of up to 30 degrees (it may even go higher but 30 degrees was the limit of our testing). When the remaining calibration states are added, and the sensor measurements are compensated for their calibration states the filter is not very robust. Starting the filter with a large initial attitude error would be outside of the linear region and the filter is not expected to give good behavior in those conditions.

The attitude solutions estimated by the filter show some oscillation. Since the results presented are from real spacecraft data the filter would be expected to follow more closely the oscillations in the data whereas a batch solution would have these oscillations averaged out.

We found, when estimating the entire state, that the results were dependent on the initial uncertainties due to a lack of observability. The batch solution which was used as our basis of comparison also was dependent on initial conditions. It could not be used as a true reference.

The ability of the filter to quickly converge to an attitude solution from a large initial error demonstrates the feasibility of using an EKF for ground attitude processing in FDD, particularly in a real-time situation. Since all the states cannot be estimated simultaneously due to a lack of observability, more investigation into the a-priori uncertainties is necessary in order to achieve a desired accuracy in the final calibration states.

## X. FUTURE WORK

In this work, the batch solution served only as a basis for comparison. It cannot be treated as a true reference. Simulated data will be used in the future, which will provide a true reference. From there further studies of the different calibration states and the ability of the filter to estimate them can be determined. At the time of this writing, the runs using simulated data were still being debugged.

A further enhancement of the filter state will need to be made to include more than one sensor of a single type. Currently the filter only estimates calibration parameters for the sensors with coverage; no switching is done when the coverage changes over to another sensor of the same type.

As mentioned previously, the ability to overcome large initial attitude errors makes the filter attractive for real-time operations. A real-time EKF will be developed which estimates attitude and possibly gyro calibration states only.

## References

1. Kau, S., Kumar, K.S.P. and Granley, G.B., "Attitude Determination Via Nonlinear Filtering," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-5, Nov. 1969, pp. 906 - 911.

2. Farrell, J.L., "Attitude Determination by Kalman Filtering," Automatica, Vol. 6, 1970, pp. 419 - 430.

3. Bar-Itzhack, I.Y. and Idan, M., "Recursive Attitude Determination from Vector Observations: Euler Angle Estimation," Journal of Guidance, Control, and Dynamics, Vol. 10, March - April 1987, pp. 152-157.

4. Bar - Itzhack, I.Y. and Reiner, J., "Recursive Attitude Determination from Vector Observations: DCM Identification," Journal of Guidance, Control, and Dynamics, Vol. 7, Jan. - Feb. 1984, pp. 51 - 56.

5. Lefferts, E.J., Markley, F.L., and Shuster, M.D., "Kalman Filtering for Spacecraft Attitude Estimation," Journal of Guidance, Control, and Dynamics, Vol. 5, Sept. - Oct. 1982, pp. 417 - 429.

6. Bar - Itzhack, I.Y. and Oshman, Y., "Recursive Attitude Determination from Vector Observation: Quaternion Estimation," IEEE Transactions on Aerospace and Electronic Systems, Vol. AES-21, Jan. 1985, pp. 128 - 136.

7. Murrell, J.W., "Precision Attitude Determination for Multimission Spacecraft," Paper No. 78-1248, AIAA Guidance and Control Conference, Palo-Alto, CA., August 7-9, 1978.

8. Gelb, A., (ed.), Applied Optimal Estimation, M.I.T. Press, Cambridge MA, 1988.

9. Wertz J. R., Spacecraft Attitude Determination and Control, D. Reidel Publishing Company, Dordrecht Holland, 1978.

# FLIGHT MECHANICS/ESTIMATION THEORY SYMPOSIUM

## SESSION 4