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Appendix A

# A PRACTICAL APPROACH FOR MINIMUM TIME CONTROL OF THE SPACECRAFT CONTROL LABORATORY EXPERIMENT (SCOLE)

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# A PRACTICAL APPROACH FOR MINIMUM TIME CONTROL OF THE SPACECRAFT CONTROL LABARATORY EXPERIMENT (SCOLE)

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## 1. INTRODUCTION

The Space COntrol Labaratory Experiment (SCOLE) is a challenge for control engineering applications. This is a result of the system dynamics, the available measurement information, the actuator capabilities and finally the specified performance requirements set.

Results on the use of Model Reference Adaptive Control have already been reported [1]. In view of the necessity for rapid response, this work deals with an optimal control formulation, with a minimum time requirement and constrained input.

In Section 2 a mathematical statement of the problem is presented. The time optimal control formulation is presented in Section 3, and the reasons that make such an approach not promising are discussed. As a result, a pseudo time-optimal control algorithm is presented in Section 4. In Section 5, the proposed approach is tested to see if it satisfies the design specifications, and finally in Section 6, discussion and suggestions for futher research are provided.

## 2. BRIEF STATEMENT OF THE PROBLEM

The antenna-beam system to be controlled is modelled as:

$$\dot{x} = A \cdot x + B \cdot u \qquad A \in \Re^{12\times 12}, B \in \Re^{12\times 3}$$

Only the 12th order approximation of the distributed parameter system is considered here. The states are organized in the form of modes, that is $x_{2i-1}$:position, $x_{2i}$:velocity, $i = 1, 2, ..., 6$. The modes are dynamically decoupled, as can be seen from the structure of matrix $A$:

$$A = \begin{bmatrix} J_1 & 0 & 0 & \dots \\ 0 & J_2 & 0 & \dots \\ \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & J_6 \end{bmatrix} \qquad J_i = \begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta_i\omega_j \end{bmatrix}$$

However, the coupling is introduced by matrix $B$ where all three controls affect all the modes:

$$B = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_6 \end{bmatrix} \qquad B_i = \begin{bmatrix} 0 & 0 & 0 \\ B_{i1} & B_{i2} & B_{i3} \end{bmatrix}$$

The output to be controlled is

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = C_p \cdot x \qquad C_p \in \Re^{2\times 12}$$

where $y_1$, $y_2$ are the $X$ and $Y$ coordinates of the tip of the antenna, and

$$C_p = \begin{pmatrix} C_{p11} & 0 & C_{p13} & 0 & \dots & C_{p16} & 0 \\ C_{p21} & 0 & C_{p23} & 0 & \dots & C_{p26} & 0 \end{pmatrix}$$

The actual measured output is

$$y_m = C_m \cdot x \qquad C_m \in \Re^{3\times 12}.$$

where

$$C_m = \begin{pmatrix} 0 & C_{m11} & 0 & C_{m13} & 0 & \dots & C_{m16} \\ 0 & C_{m21} & 0 & C_{m23} & 0 & \dots & C_{m26} \\ 0 & C_{m31} & 0 & C_{m33} & 0 & \dots & C_{m36} \end{pmatrix}$$

Obviously, the output matrix $C_p$ incorporates at the desired output only the position component states, while $C_m$ maps at the output only the velocity components of the modes.

Given an initial output, such that the displacement $\|y(0)\| = d_0 = 1\ ft$, it is desired to transfer the state vector to $x(t_f) = 0$ in minimum time $t_f$, subject to the constraints

$$|u_i| \leq U \quad i = 1,2,3$$

where $U = 6.25$. The numerical values of matrices $A, B, Cp, C_m$ can be found in [1].

Starting with an initial state at time $t = 0$:

$$x_0^1(0) = [-3.3013 \quad 0 \quad -0.71870 \quad \dots \quad 0]^T$$

corresponding to the initial output

$$y(0) = C_p \cdot x_0^1(0) = \begin{pmatrix} 3.7947 \\ 11.3842 \end{pmatrix} \qquad \|y(0)\| = d_0 = 1\ ft$$

the output of the free response of the system is shown in fig. 1. It is obvious that the settling time is $t_f \geq 1000 \quad sec.$

In order to improve the response of the system, the following design specifications were considered:

- The decision should be a feedback controller.
- If possible, explicit use of observers should be avoided.
- The decision scheme should be implementable in an ON-LINE fashion.

- The design should provide a controller robust with respect to initial state and to parameter uncertainties.

In the sequel, a description of the ways that the problem was attacked is presented, and simulation results are given and analyzed for performance evaluation. Finally suggestions for improvements are given.

## 3. THE MINIMUM TIME PROBLEM

The first approach towards trying to solve the above problem was to formulate it as the simple minimum time control problem:

$$\min_u t_f$$

s.t

$$\dot{x} = A \cdot x + B \cdot u$$

$$x(0) = x_0 \qquad x(t_f) = 0$$

$$|u_i| \leq U \quad i = 1, 2, 3$$

Numerical implementation of this optimal control problem requires its transformation to a Two Point Boundary Value Problem (TPBVP). Such a transformation is obtained by introducing the scalar variable

$$\tau = \frac{t}{t_f}$$

thus transforming the problem to the equivalent

$$\min_u \int_0^1 z \cdot d\tau$$

s.t

$$x' = (A \cdot x + B \cdot u) \cdot z, \qquad (.)' = \frac{d(.)}{d\tau}$$

$$z' = 0$$

$$x(0) = x_0 \qquad x(1) = 0$$

$$|u_i| \leq U \quad i = 1, 2, 3$$

Solution of the above problem using multiple shooting methods, was not possible due to numerical difficulties. The major problem is the almost marginal stablitiy of the system that makes necessary the use of very small sampling intervals, and thus a very large number of intervals and parameters.

Another approach to solving the above problem was that of [2], which in fact transforms the infinite dimensional problem to a finite dimensional one. A series of Linear Programs have to be solved. Similarly, the large number of variables along with the resulting constraints that are extremely restricting, made the solution impossible.

## 4. THE PSEUDO-MIMIMUM TIME PROBLEM

All the above problems that led to numerical instabilities convinced us that finding a minimum time control with the optimal control formulation presented above, is a very difficult task. Its application would be even harder due to the fact that it is open-loop control and therefore non-robust. Additionally, minimum time control trajectories obtained for reduced order subsystems showed that the controller saturation levels were changing signs every 3ms making the implementation impossible. Therefore another approach was adopted; namely, the LQ Regulator with fixed function of final state [3]. Such an approach is defined as follows:

SYSTEM

$$x_{k+1} = A \cdot x_k + B \cdot u_k$$

PERFORMANCE INDEX

$$J = \frac{1}{2} x_N^T \cdot S \cdot x_N + \frac{1}{2} \sum_{k=0}^{N-1} (x_k^T \cdot Q \cdot x_k + u_k^T \cdot R \cdot u_k)$$

$$S_N \geq 0, \quad Q \geq 0, \quad R > 0$$

FINAL STATE CONSTRAINT

$$C_p \cdot x_N = 0$$

OPTIMAL CONTROL LAW

$$K_k = (B^T \cdot S_{k+1} \cdot B + R)^{-1} \cdot B^T \cdot S_{k+1} \cdot A \qquad S_N : given$$

$$S_k = A^T \cdot S_{k+1} \cdot (A - B \cdot K_k) + Q$$

$$V_k = (A - B \cdot K_k)^T \cdot V_{k+1} \qquad V_N = C_p^T$$

$$P_k = P_{k+1} - V_{k+1}^T \cdot B \cdot (B^T \cdot S_{k+1} \cdot B + R)^{-1} \cdot B^T \cdot V_{k+1} \qquad P_N = 0$$

$$K_k^u = (B^T \cdot S_{k+1} \cdot B + R)^{-1} \cdot B^T$$

$$G_k = -(K_k - K_k^u \cdot V_{k+1} \cdot P_k^{-1} \cdot V_k^T)$$

$$u_k = G_k \cdot x_k$$

Obviously this is a state feedback controller. The measurements matrix $C_m$ contains linear combinations of the velocities of the modes. Therefore state $x$ can be derived only by using an observer. An effort to avoid observer utilization is reported in the next section.

The approach, described above, allows much flexibility in the selection of the parameters of the matrices $S, Q, R$. Since the output constraint considers only the linear combination of the position components of the states, the state can be reduced by penalizing the final state, i.e by having a large S matrix. In the present set of experiments, $S$ was $50000 * I$. The selection of the $R$ matrix directly affects the maximum value of the control trajectories. However since the bounds in all controls are the same, $R$ was chosen diagonal with the same elements for all controls. In the present set of simulations $R$ was $I$. Finally, the selection of $Q$ matrix, provides some valid flexibility since it is obvious from the control structure that the control values are directly proportional to the states. An iterative adjustment was made using

$$q_{ii} = \frac{\max_{t\in[0,t_f]}(x_i^2)}{\sum_{i=1}^{n} \max_{t\in[0,t_f]}(x_i^2)}$$

Finally, the number of steps $N$ that define the final time $t_f$, have to be found experimentally based on the selected strategy.

As an example, consider again the initial state

$$x_0^1 = [-3.3013 \quad 0 \quad -0.71870 \quad \dots \quad 0]^T$$

corresponding to $\|y(0)\| = 1 \quad ft$. Using the proposed controller, the system's responce can be shown by the trajectories of the two components of the output $y_k = C_p \cdot x_k$ that are shown in fig.2. The corresponding optimal control trajectories $u_{ik}^* \quad i = 1, 2$ are shown in fig. 3 (because $u_3$ is of the order of $10^{-1}$, it was not shown). Inspection shows that the output is lead to zero while the input satisfies the constraints. The settling final time was found to be $t_f = 130 \quad sec$ which is considerably less than the settling time of $t_f \geq 1000 \quad sec$ of the free system of fig.1.

## 5. DESIGN SPECIFICATIONS TESTING

The proposed approach might be considered an ON-LINE procedure, because the set of feedback gains $G_k$ can be computed fast. This is so, because the computation of $G_k$'s is not iterative, but it is straightforward. It is also a feedback policy because the control input is given by $u_k = G_k \cdot x_k$. However this is state feedback, and not output feedback as required. Additionally, a problem with this approach is that although the output is going to be lead to zero for every initial state, the input $u_k$ is not guaranteed to be bounded by $U$. If the only considered measurements are $y_{mk} = C_m \cdot x_k$, (i.e linear combinations of velocities), then either the explicit or the implicit use of observers is unavoidable.

Assuming that the desired ouput $y_k = C_p \cdot x_k$ is available, an attempt to obtain an approximate estimate of the state, was the following: Matrix $C_p$ is not square, and therefore an invertible submatrix $C_p' \in \Re^{2\times 2}$ of $C_p$ was sought, so that

$$x_k' = (C_p')^{-1} \cdot y_k$$

and

$$\hat{x}_k = [x_k'(1) \quad 0 \quad x_k'(2) \quad 0 \quad \ldots \quad 0]^T$$

could be used as an approximation to $x_k$. By considering the first two modes of the system as dominant, matrix $C_p'$ was constructed as

$$C_p' = [C_{pi1} \quad C_{pi3}].$$

Still, this approach did not seem to work as can be seen by the following example:

Assume that at time $t = 0$ the system is at state

$$x_0^2 = [0 \quad 0 \quad 0 \quad 0 \quad 0.3018 \quad 0 \quad 56.0972 \quad 0 \quad \ldots \quad 0]^T.$$

corresponding to an ouput, $y_0 = C_p \cdot x_0^2 = [0.7589 \quad 2.2768]^T$ with $\|y_0\| = 2.4 \quad in$. Obviously, the magnitude of this initial deviation is smaller than the 12 $in$ requirement, but this was done because of memory constraints of the computing system used. According to the above scheme, the **estimated** initial state corresponding to $y_0$ was

$$\hat{x}_0^1 = (C_p')^{-1} \cdot y_0 = [-0.6603 \quad 0 \quad -0.14370 \quad \ldots \quad 0]^T.$$

This was the state that the controller actually "observed" at $t = 0$. Using $\hat{x}_0^1$ as an initial estimate of the state, a set of gain matrices $G_k$ $k = 1, 2, \ldots N$ was obtained by the algorithm of section 4. $N$ was chosen in a such a way that the input

$$u_k = G_k \cdot x_k$$

of the **fictitious** system:

$$x_{k+1} = A \cdot x_k + B \cdot u_k \qquad x_0 = \hat{x}_0^1$$

are bounded by $U$. Since boundedness depends on the initial state, we denote the set of gains obtained for this fictitious system by $G_k(\hat{x}_0^1)$ $k = 1, 2, ...N$, in order to show that these gains were selected based on $\hat{x}_0^1$.

The response of the output of the fictitious system is shown in fig. 4, and the inputs $u_i \quad i = 1, 2, 3$ of the fictitious system which are always less than $U = 6.25$ are shown in figures 5,6 and 7 respectively. The final time for the fictitious system was found to be $t_f = 13 \quad sec$.

However, the initial state was not $\hat{x}_0^1$ but actually $x_0^2$ which was essentially unknown to the controller. The control gains $G_k(\hat{x}_0^1)$ were calculated based on the estimate $\hat{x}_0^1$ of the initial state, and unfortunately, the control inputs $u_k = G_k \cdot [(C_p')^{-1} \cdot y_k]$ that were actually fed to the system, exceeded by far the bound $U = 6.25$, while the actual output $y_k$ was not lead to zero. Even an increase of the final time $t_f$ from 13 $sec$ to $t_f = 100 \quad sec$ at the fictitious system (which meant the calculation of a whole new set of gains $G_k(\hat{x}_0^1)$ $k = 1, 2, ....N'$) did not give better results, as can be seen in figures 8 and 9 for $y(1)$,$y(2)$,$u(1)$ and $u(2)$ respectively.

Finally, the robustness properties to parameters variation were tested by increasing the resonant frequencies of the system by 10%. Using as initial state the previously used $x_0^2$, the set of gains obtained for the nominal system for $t_f = 13 \; sec$ and finally assuming state feedback, the "perturbed" system's output was lead to zero as shown in fig. 10, however the control (fig. 11) exceeded $U$. The responces for this perturbed system do not seem to be much different than these of the nominal (fig. 4,5,6). An increase of the final time to $t_f = 15 \quad sec$ seems to resolve the problem since the output is lead to zero (fig. 12) and the control is now bounded (fig. 13). These can be compared to the corresponding reponses of the nominal system for $t_f = 15 \quad sec$ cited at (fig.14 and 15).

## 6. DISCUSSION

It is obvious, that the proposed scheme requires knowledge of the full system state for efficient implementation. In this case the system performs acceptably since the output is lead to zero, while the state, because of the penalty imposed by matrix $S$ is lead close enough to zero. Additionally, the properties of the control scheme with respect to system parameters variation are satisfactory if a larger final time is chosen for the nominal system. The on-line calculation of $G(t)$ will not be a big issue if current parallel computation schemes are utilized [4].

In the case that the only available measurement is $y_m = C_m \cdot x$, which is a linear combination of the velocities of the system, then no obvious way to guarantee performance exist other than use of an observer.

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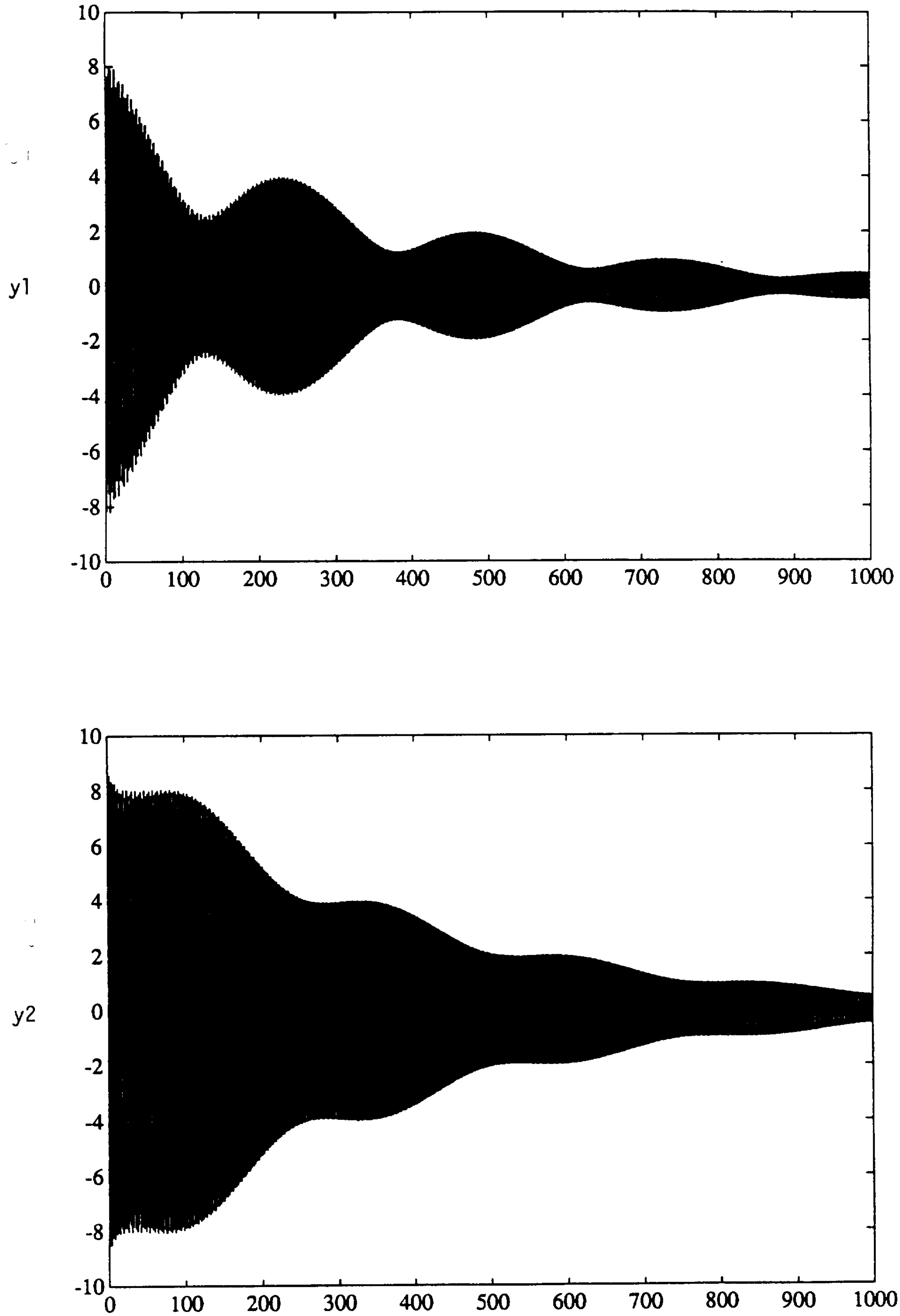



Fig. 1 Output y1 and y2 of the FREE response of the system

yfree

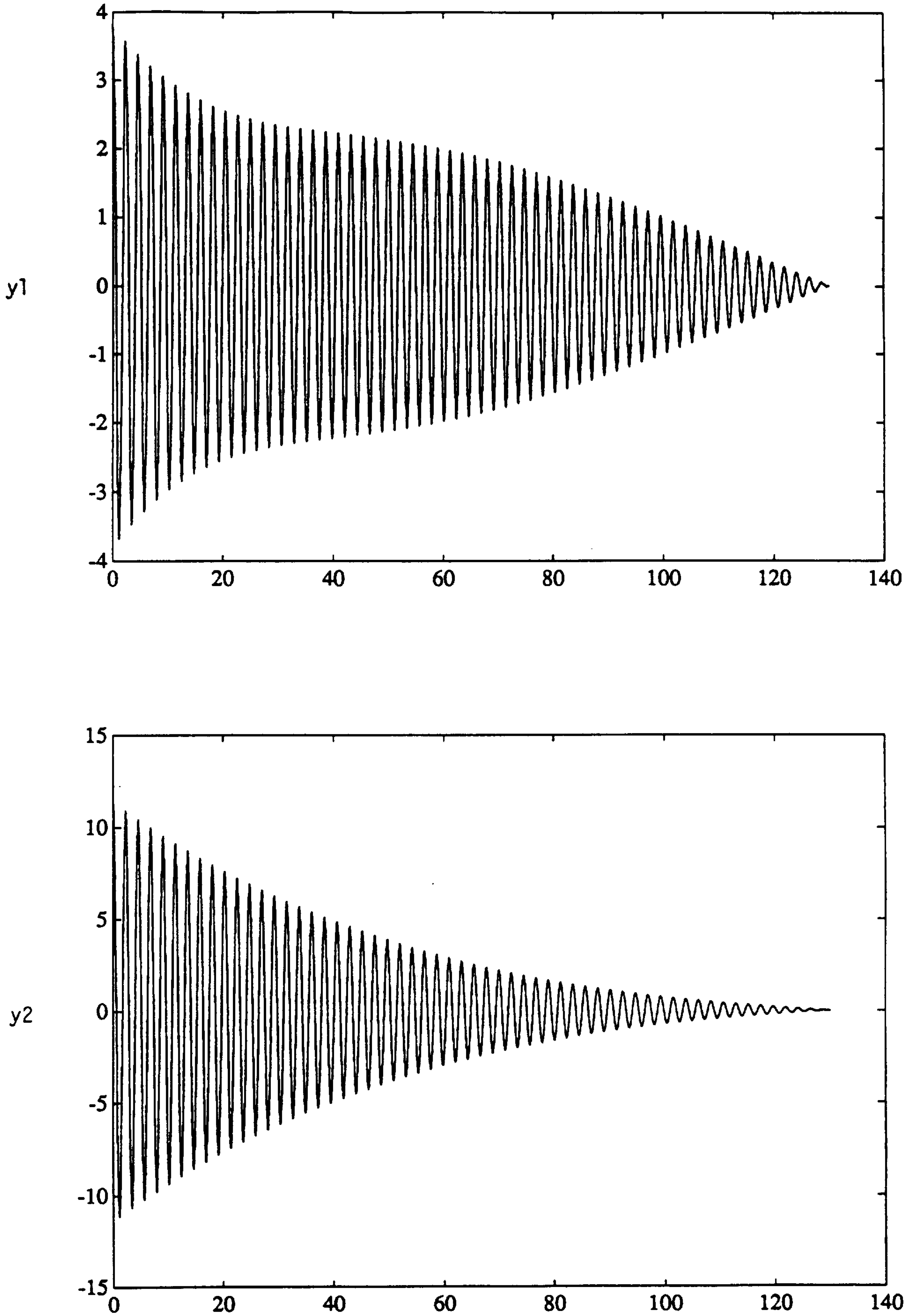



Fig. 2. Output y1 and y2 of the system under feedback control

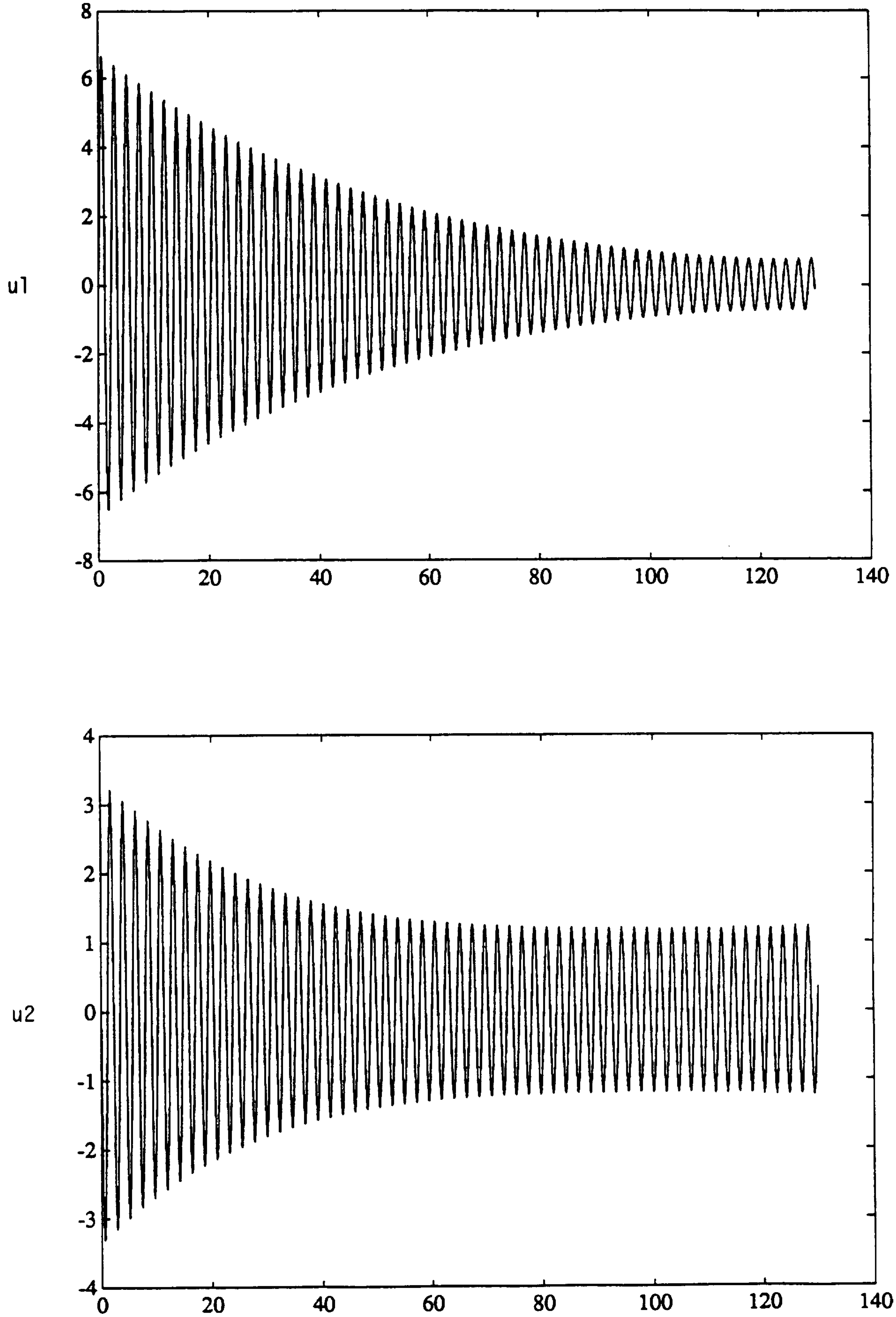



Fig. 3. Feedback inputs u1 and u2 of the system

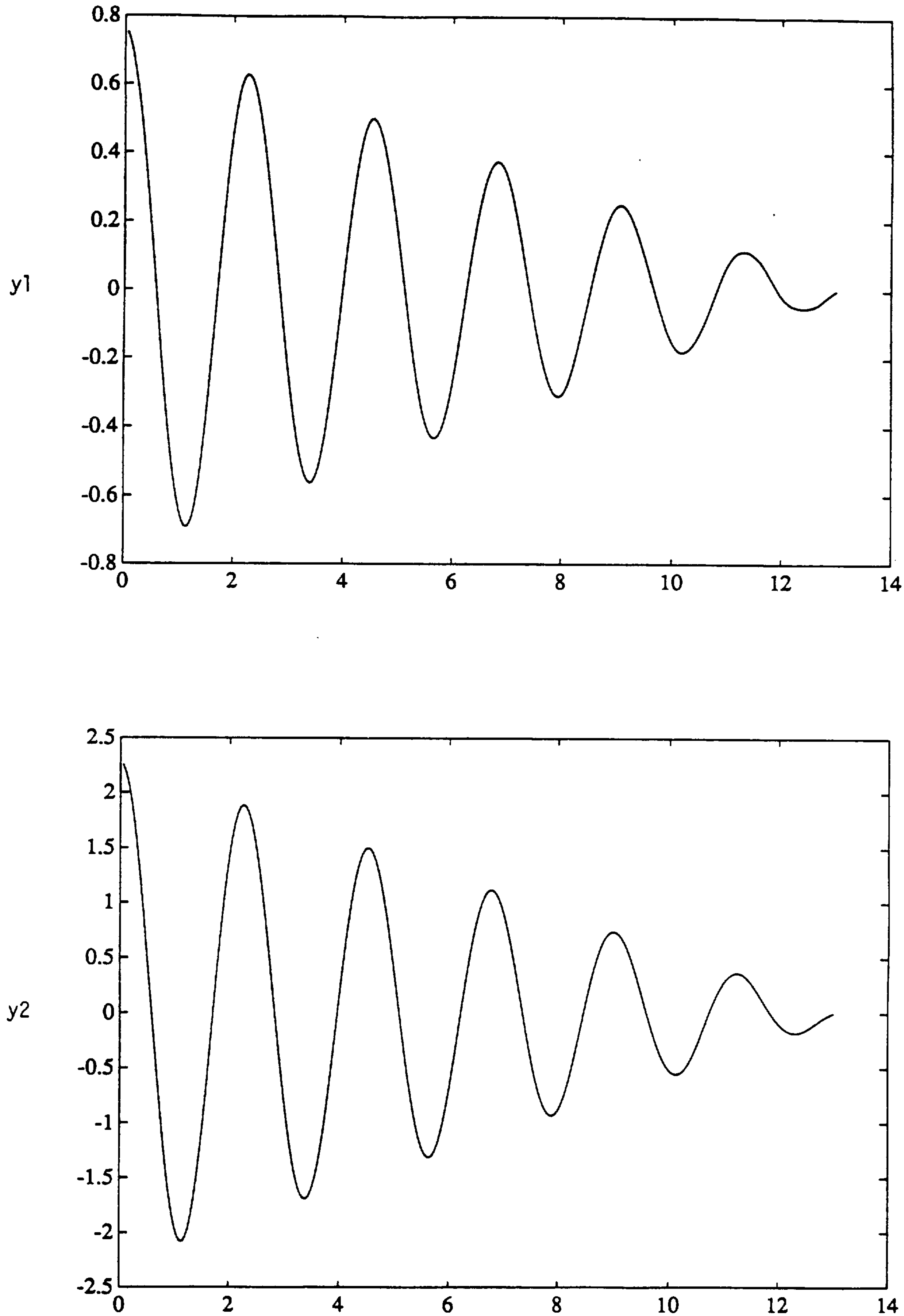



Fig. 4. Ouput y1 and y2 of EXPECTED response

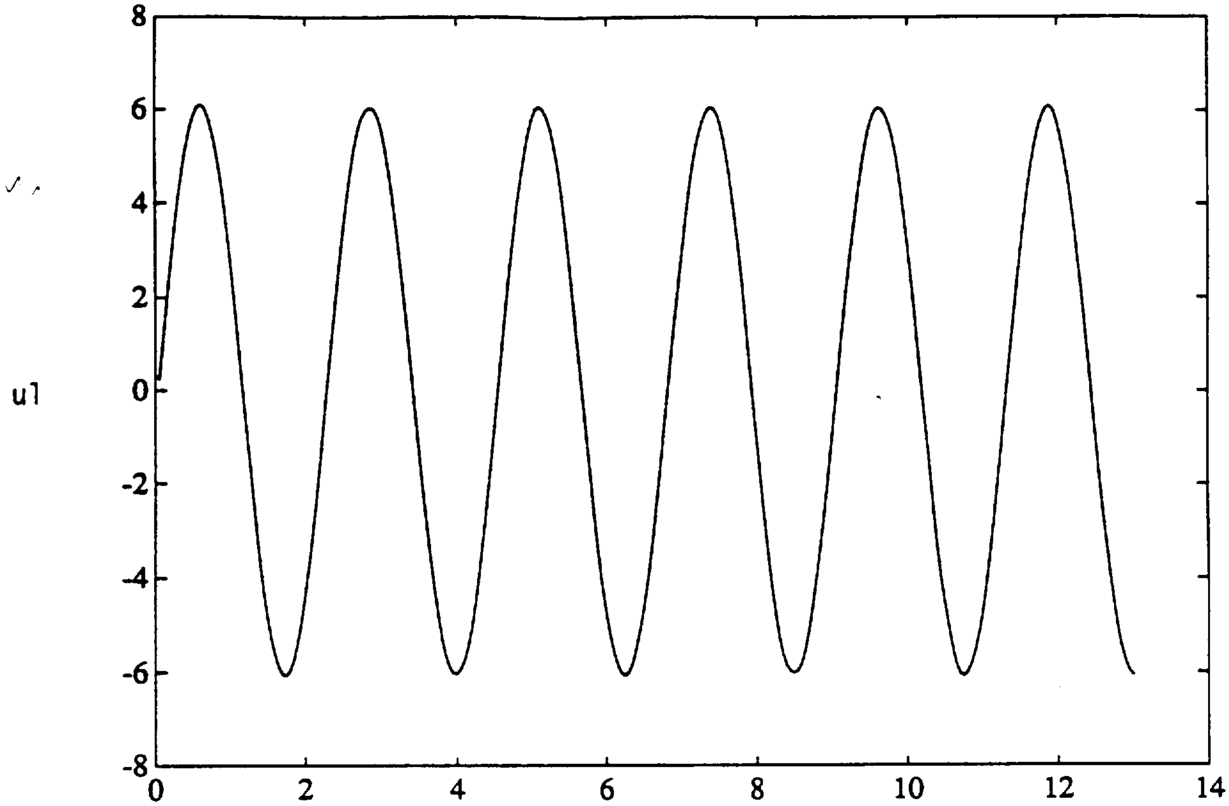



Fig. 5. EXPECTED feedback input u1

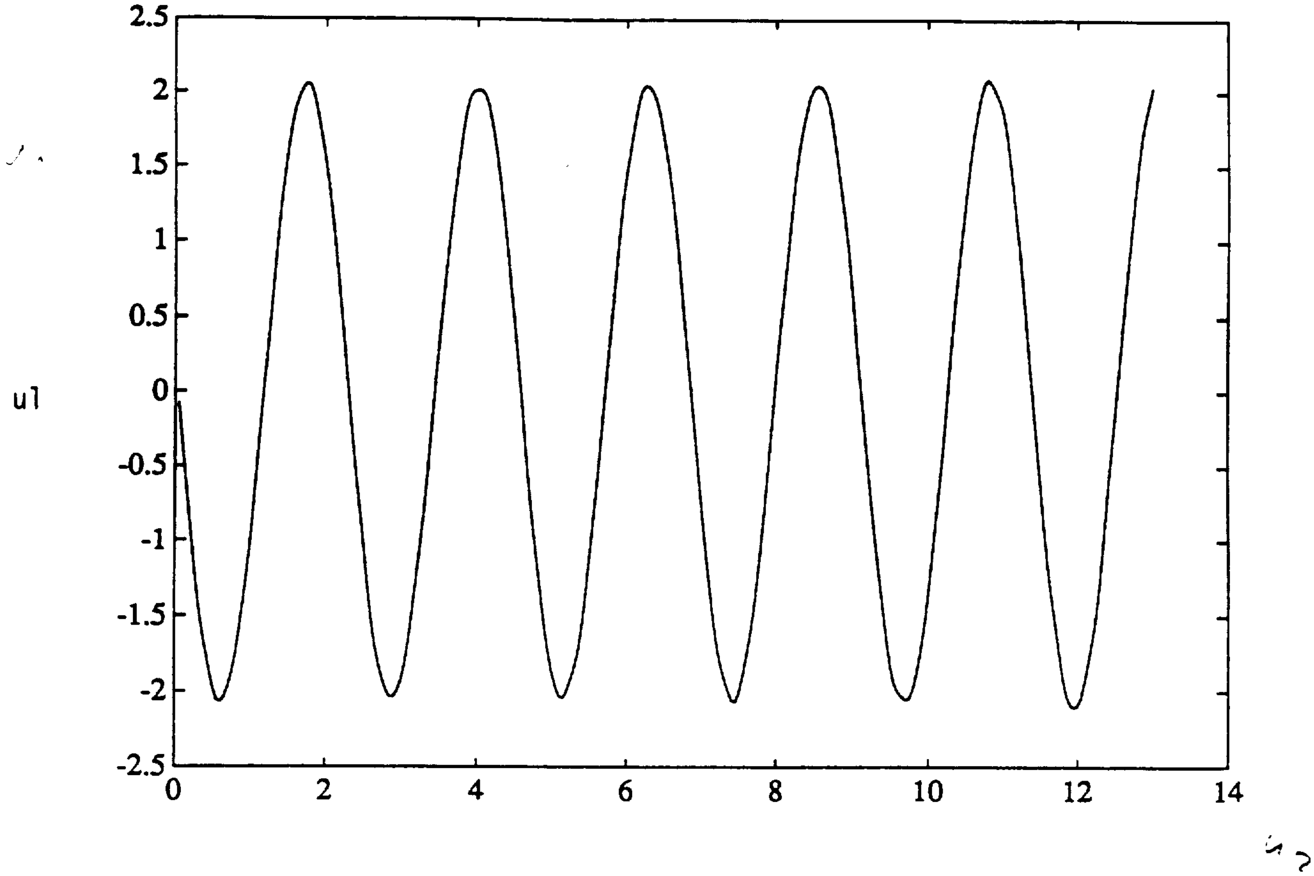



Fig. 6. EXPECTED feedback input u2

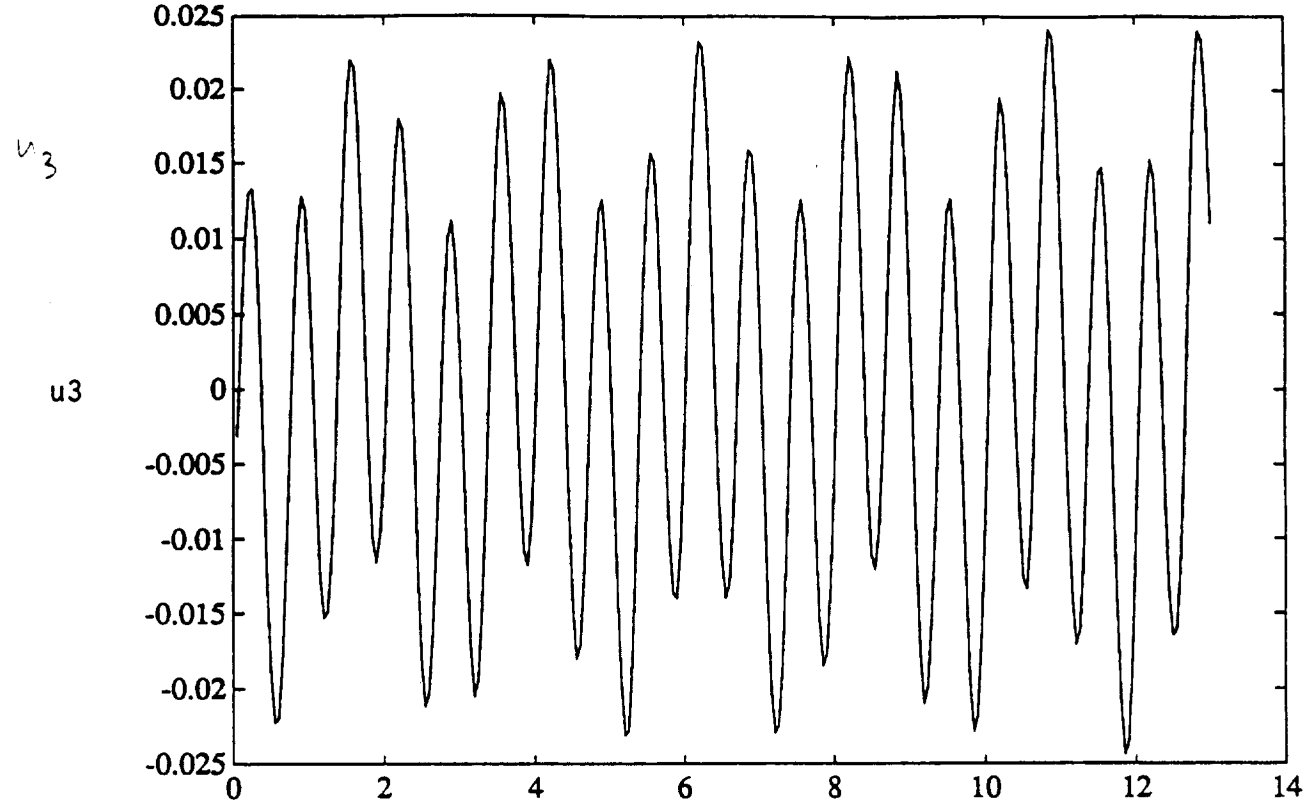



Fig. 7. EXPECTED feedback input u3

$u_3$

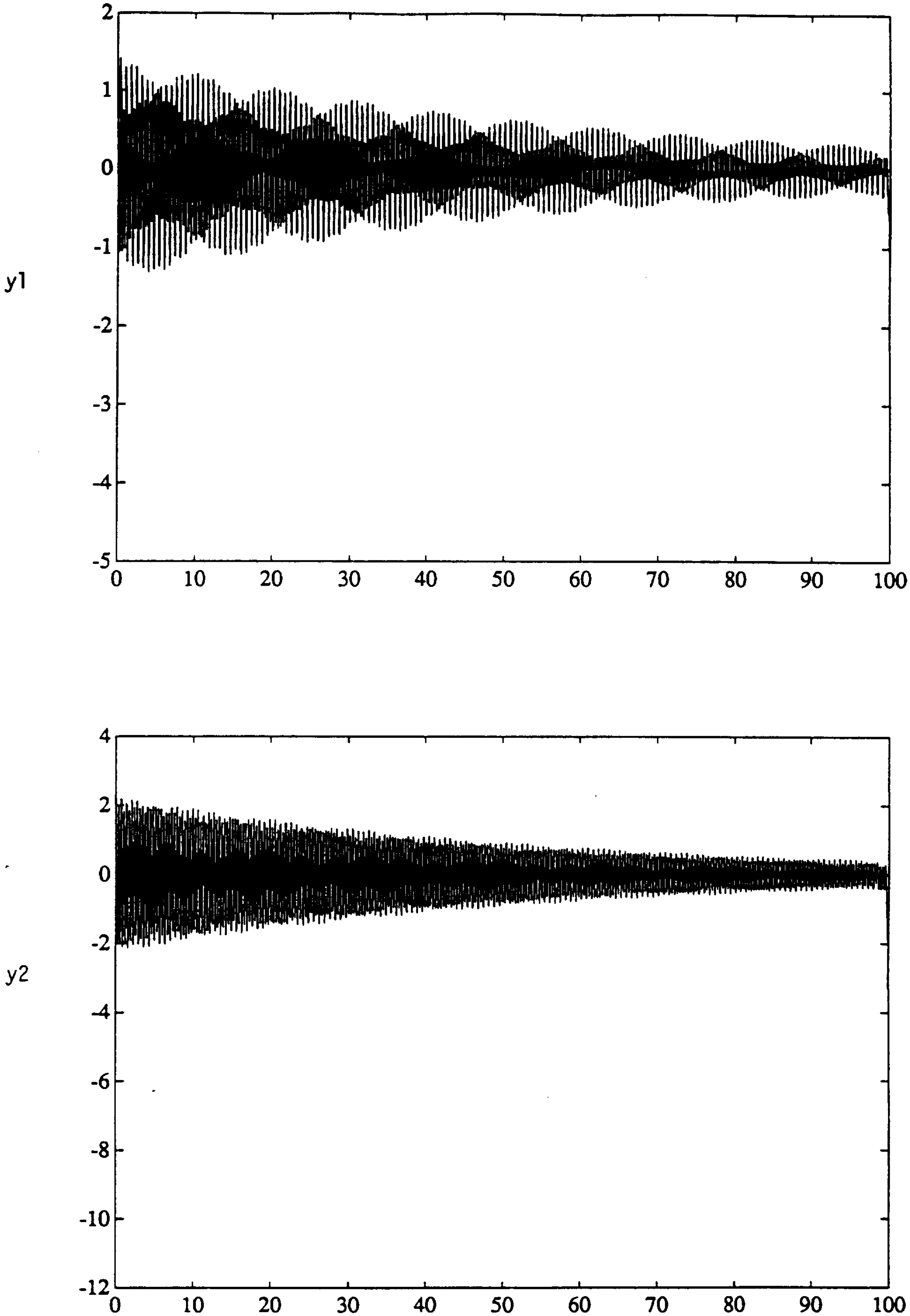
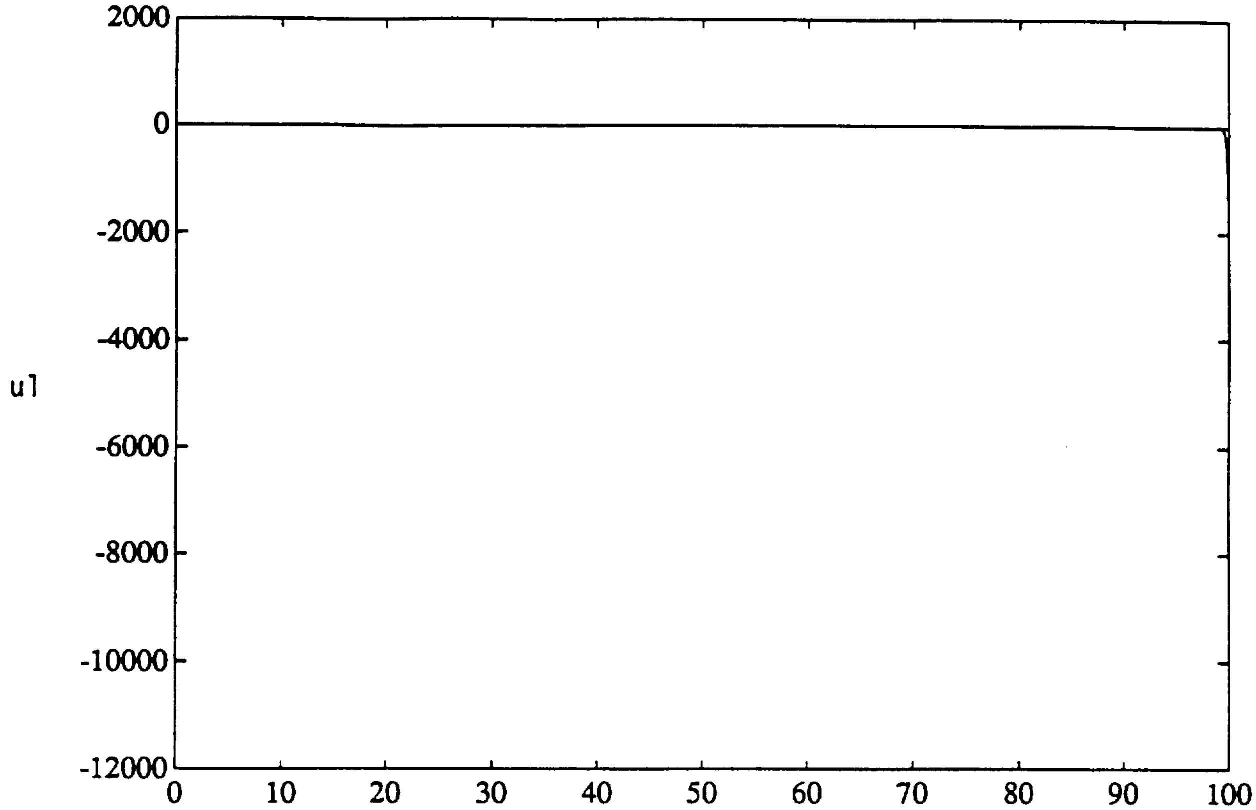



Fig. 8. ACTUAL response of output y1 and y2



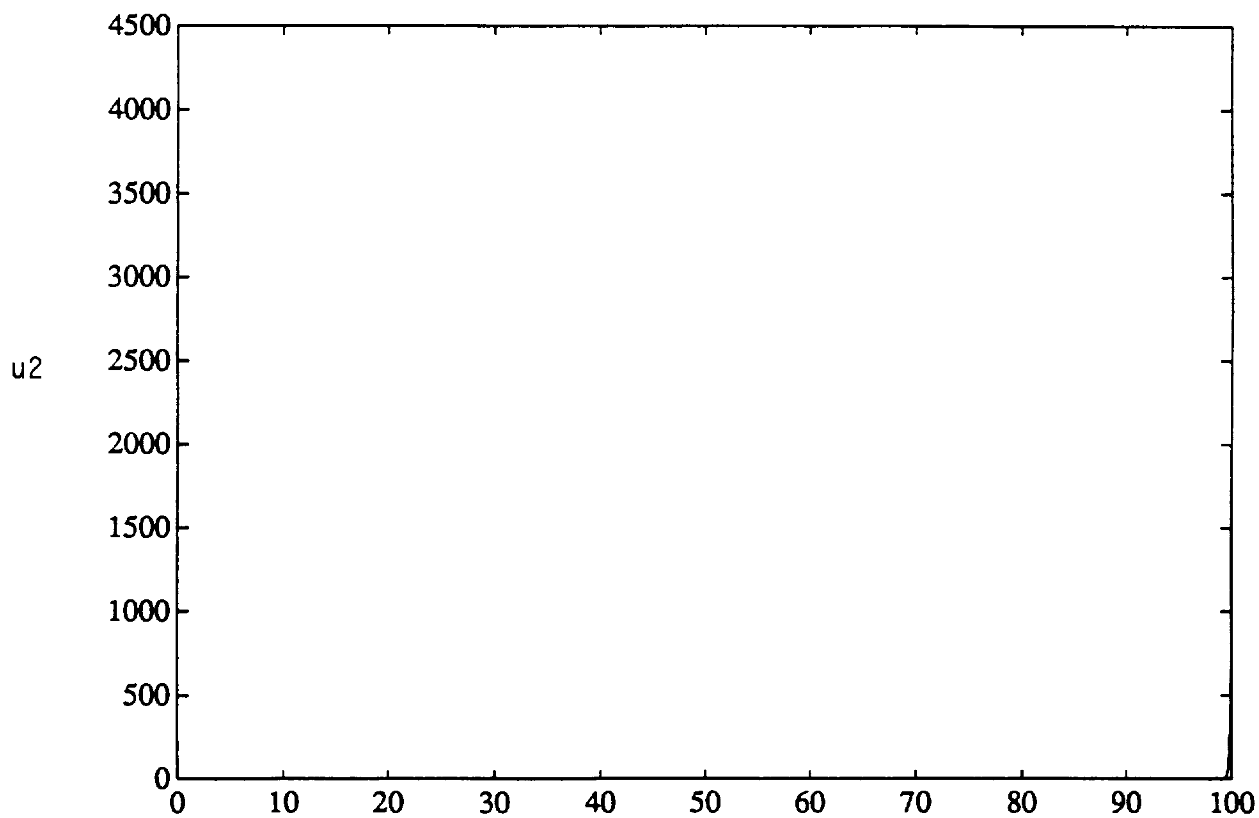




Fig. 9. ACTUAL feedback inputs u1 and u2

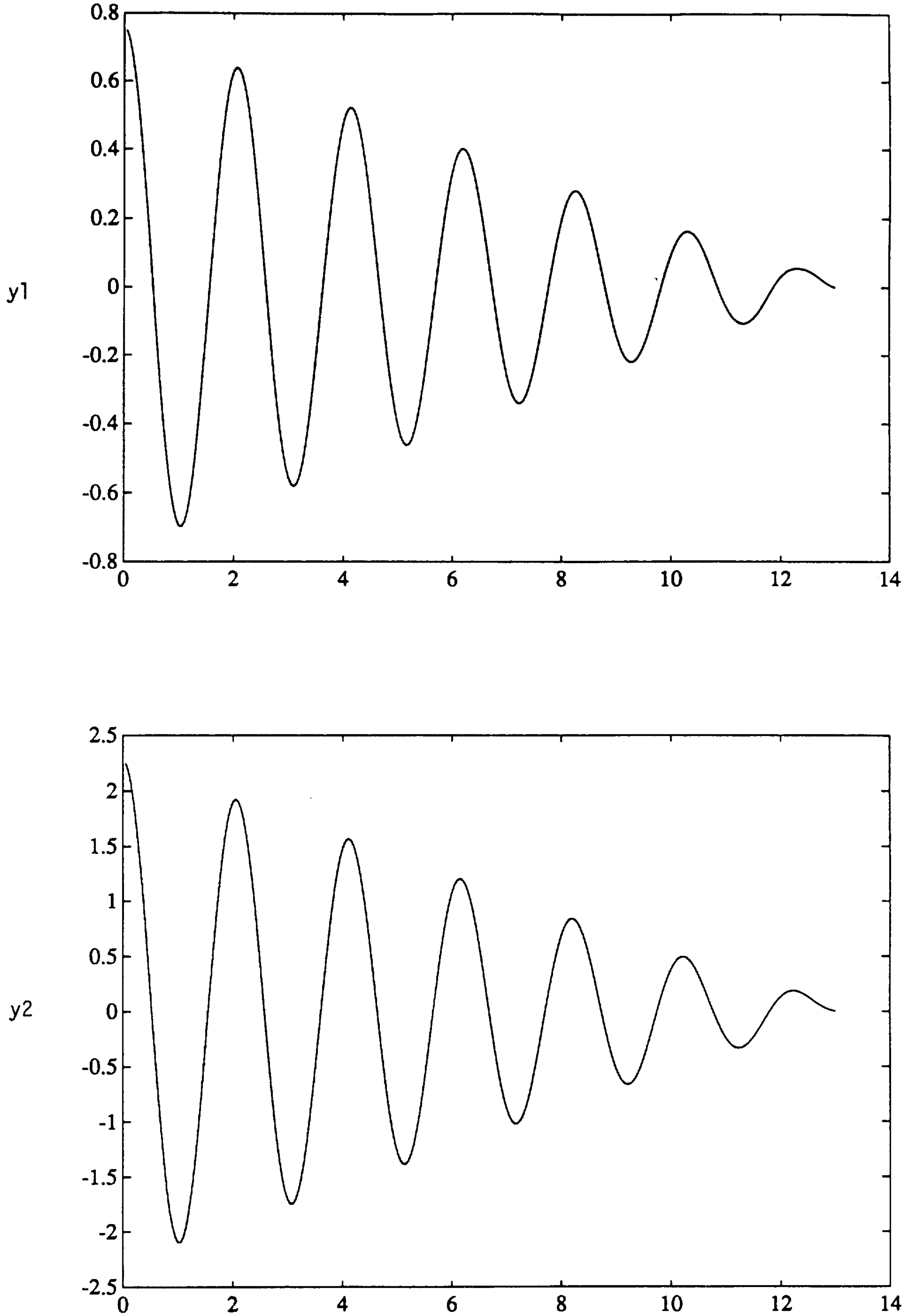


Fig. 10. Output response y1 and y2 of system with $t_f$=13 sec

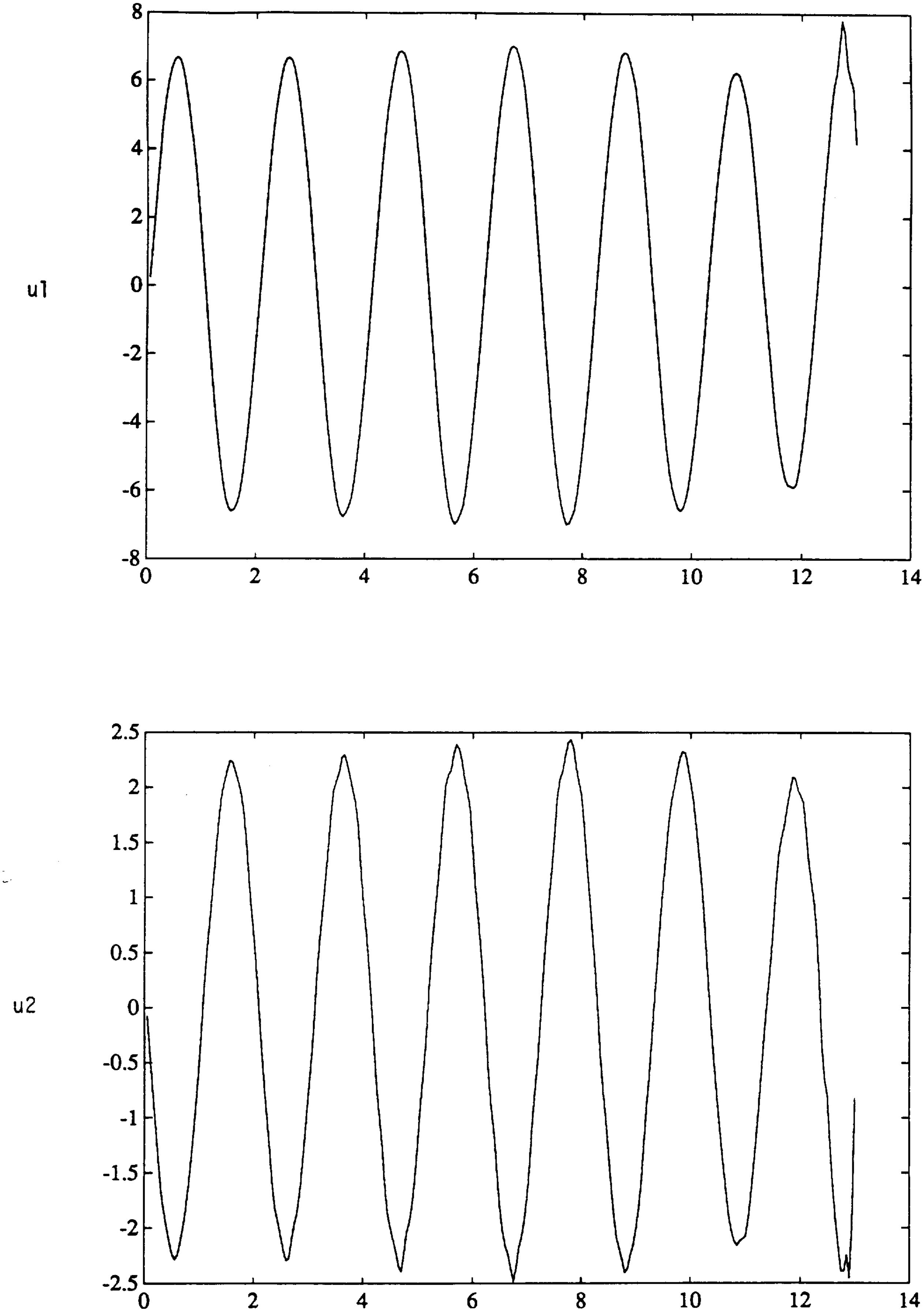



Fig. 11. Inputs u1 and u2 of system with $t_f$ = 13 sec

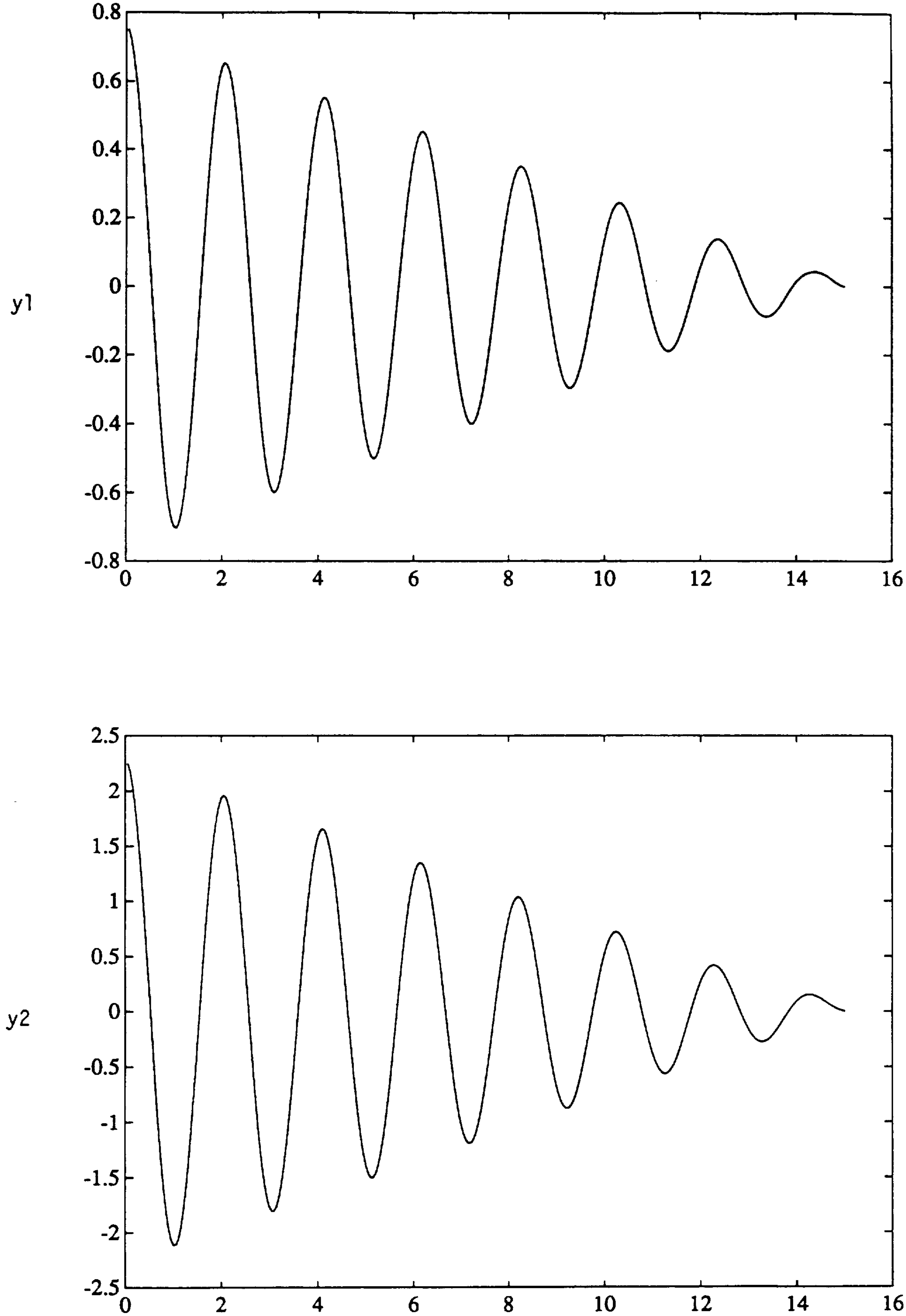



Fig. 12. Output response y1 and y2 of system with $t_f$= 15 sec

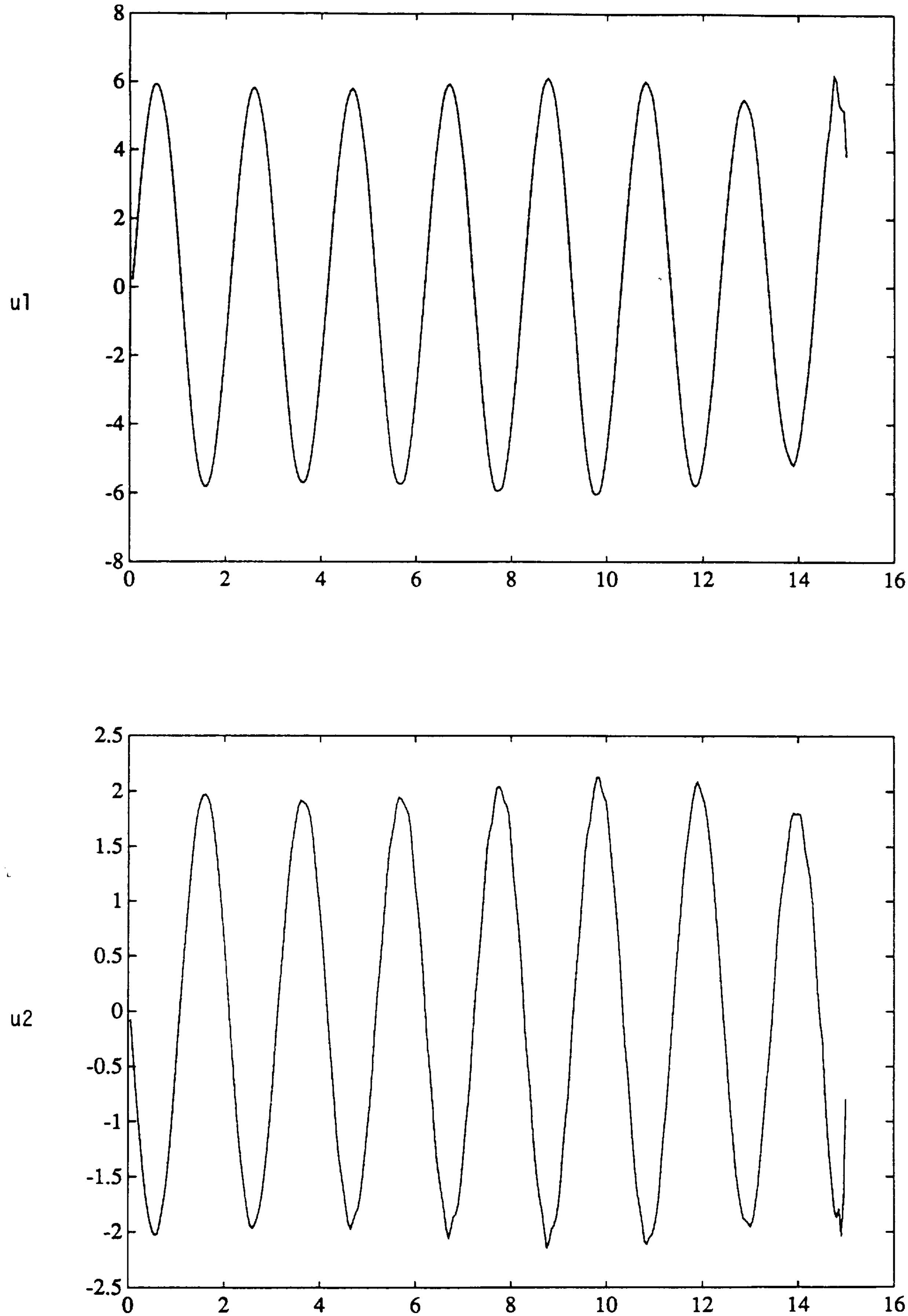
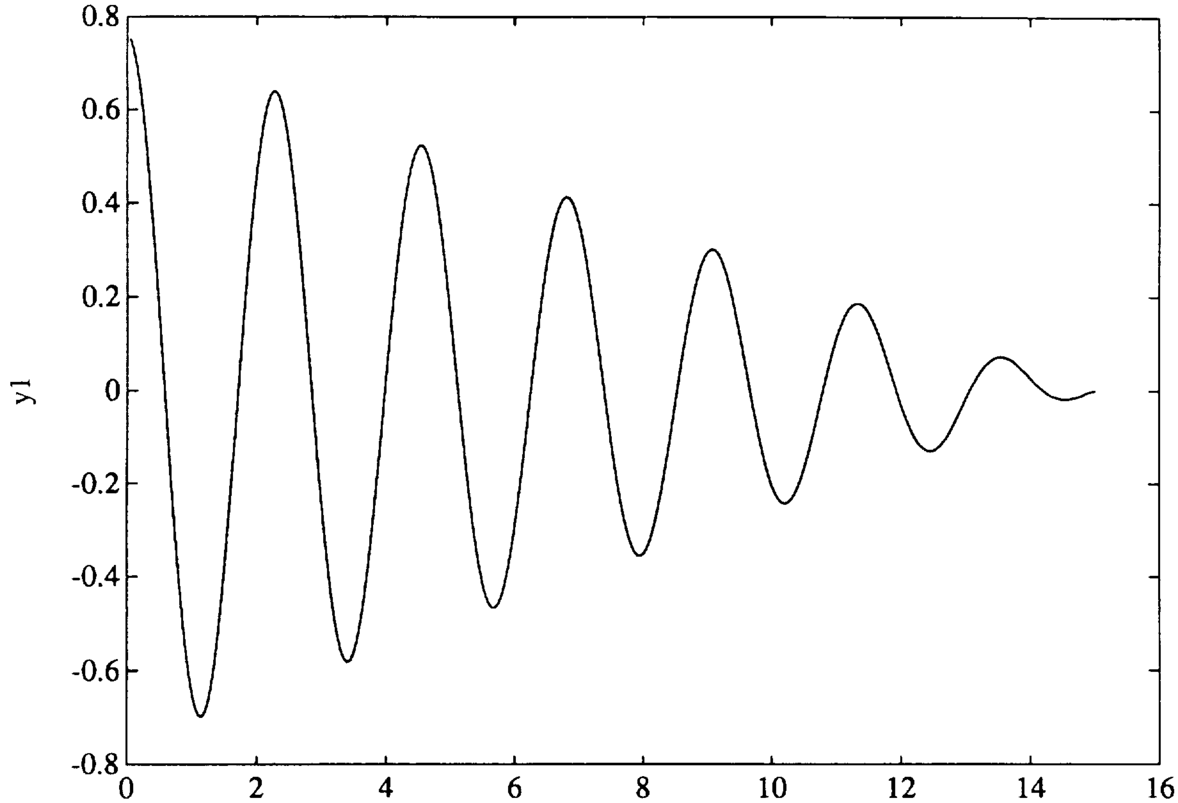



Fig. 13. Inputs u1 and u2 of system with $t_f$ = 15 sec



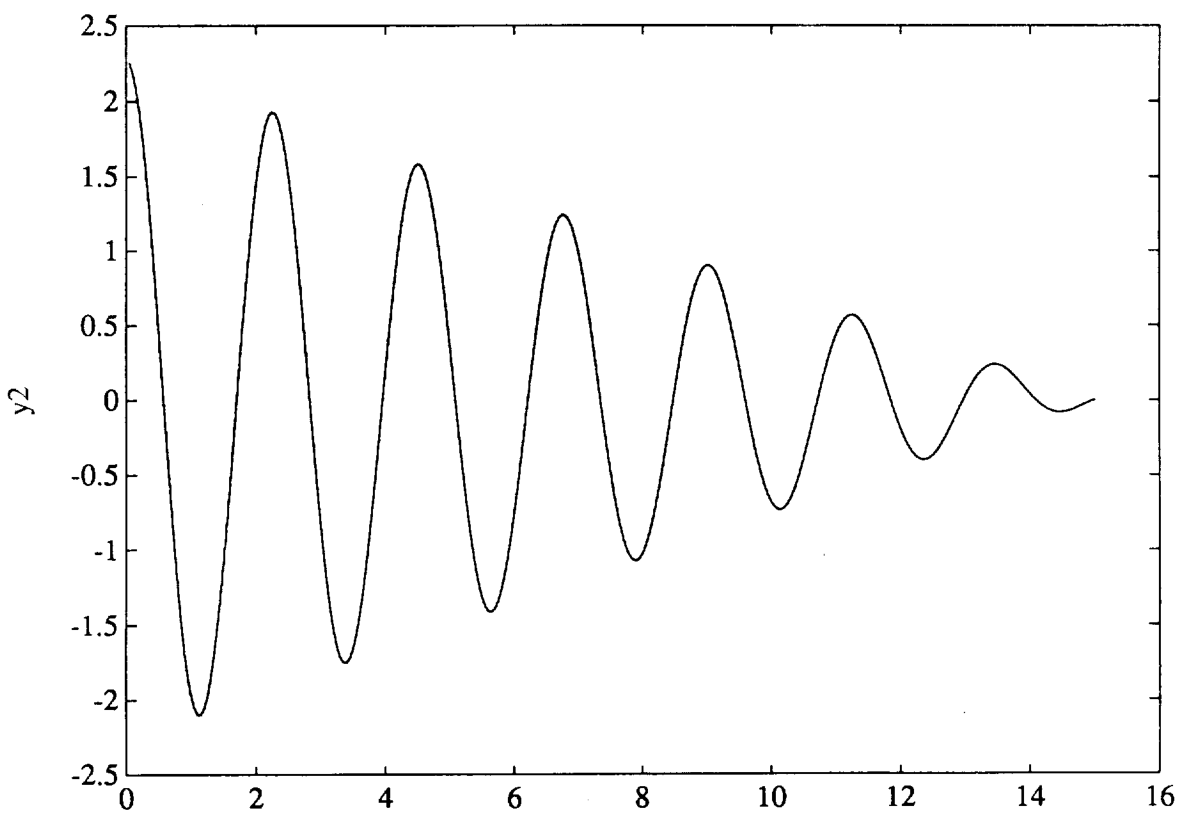




Fig.14 Output response y1 and y2 of nominal system with tf=15 sec

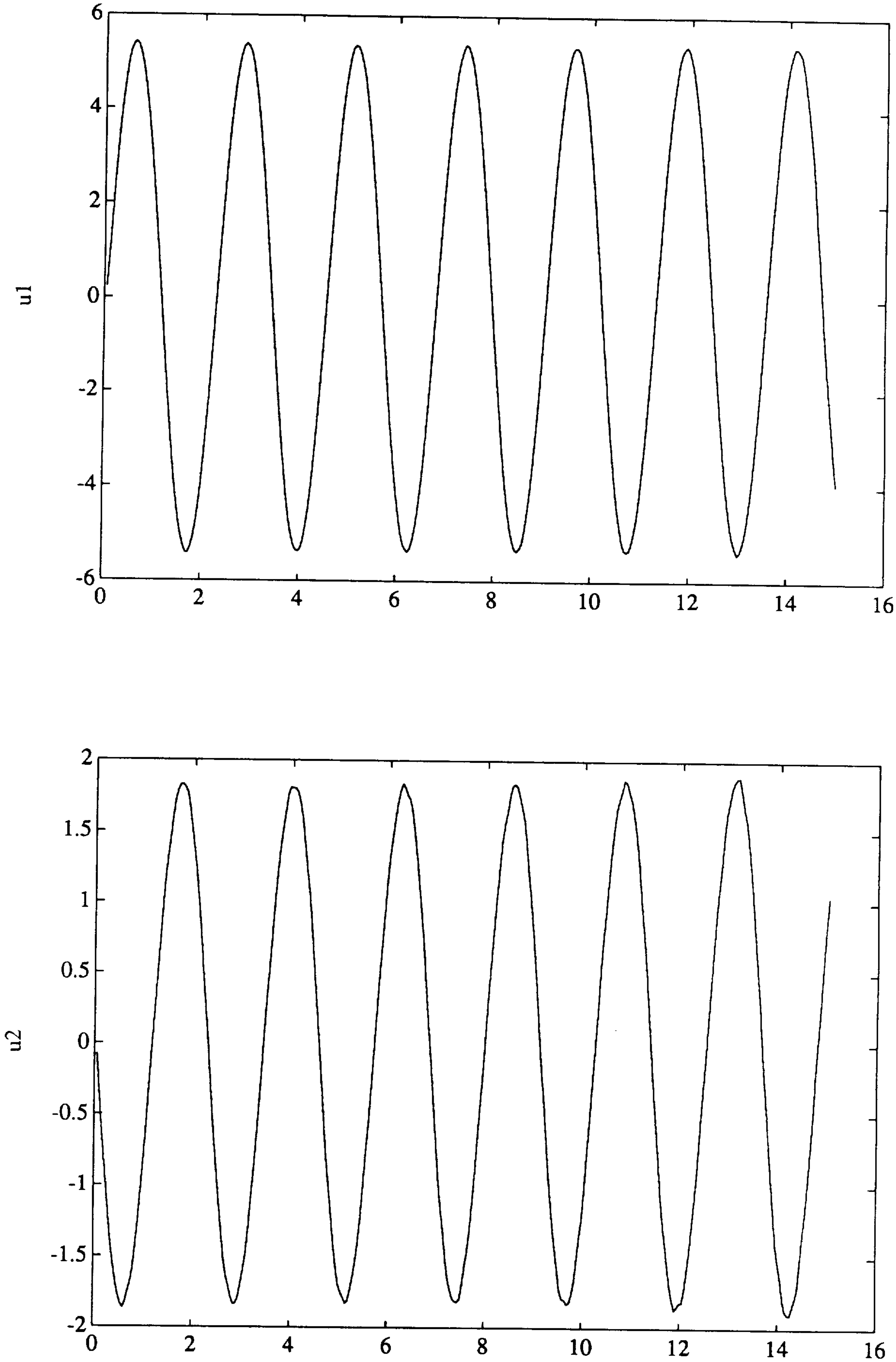



Fig. 15 Inputs u1 and u2 of nominal system with tf=15 sec