NASA Technical Memorandum 101581

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November 1989

(NASA-TM-101581) CHARACTERISTIC IMPEDANCE OF MICROSTRIP LINES (NASA) 59 p CSCL 17B

N90-13694

Unclas
G3/32 0252605

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by

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Abstract

The dyadic Green's function for a current embedded in a grounded dielectric slab is used to analyze microstrip lines at millimeter wave frequencies. The dyadic Green's function accounts accurately for fringing fields and dielectric cover over the microstrip line. Using Rumsey's reaction concept, an expression for the characteristic impedance is obtained. The numerical results are compared with results reported by others.

Introduction

In recent years there has been a great deal of interest in analyzing microstrip lines at millimeter wave frequencies due to use of these lines in monolithic phased array systems.

Microstrip lines were initially analyzed using a quasi-static approach and later by a waveguide model to study dispersion characteristics [4]. However, these models do not take into account losses due to radiation and surface wave excitation. These losses can be significant at high frequencies and some attempts [1-3] have been made to account for these losses. In reference 1, the current distribution on the microstrip discontinuity is determined by solving the electric field integral equation. From a knowledge of the current distribution, characteristics such as impedance and guide wavelength can then be determined. The microstrip line embedded in a multilayer dielectric has also been analyzed using a generalized spectral domain Green's function [3].

In this paper, the dyadic Green's function for a current source embedded in a grounded dielectric slab is used to determine the field due to the microstrip line current. Using Rumsey's reaction concept [5], the characteristic impedance of the microstrip line embedded in a dielectric slab is then determined. The numerical results obtained using the present method are compared with earlier published data.

Symbols

d	Total thickness of dielectric.
$\mathbf{E}$	Electric field vector.
E^i	Incident electric field.
f_0	Frequency.
$f_x(y)$	Transverse distribution function.
I_n	Complex amplitude of n^{th} pulse.
I_0	Total input current.
$\mathbf{J}(x,y)$	Surface electric current density vector.
j	$\sqrt{-1}$.
$J_x(x,y)$	x-component of $\mathbf{J}(x,y)$.
$J_x(k_x,k_y)$	Bidimensional Fourier transform of $J_x(x,y)$.
J_t, J_{xt}	Surface current distribution on microstrip.
J_u, J_{xu}	Surface current distribution on microstrip induced by x-polarized plane wave.
k	Wave propagation constant in dielectric, $k_0\sqrt{\varepsilon_r}$.
k_0	Wave propagation constant in free space.
K_e	Dominant mode propagation constant.
k_x	Fourier transform variable with respect to x.
k_y	Fourier transform variable with respect to y.
N	Number of pulses in y-direction.
W	Width of microstrip line.
$\mathbf{x},\mathbf{y},\mathbf{z}$	Unit vector along x-,y-,z-axis respectively.
y_n	$-W/2 + (2n-1)\Delta y/2$.
Z_0	Characteristic impedance of microstrip line.
z'	Microstrip location from ground plane.
α	Imaginary part of K_e.
β	Real part of K_e.
Δy	W/N.
ε_r	Relative permittivity of dielectric substrate.
λ_0	Wavelength in free space.
η_0	Characteristic impedance of free space.

Theory

The geometry of a microstrip line embedded in a grounded dielectric slab is shown in figure 1 along with the notation to be used. In order to determine the characteristic impedance, consider a microstrip line being excited by a current source backed by a perfect electric conductor at the x=0 plane, as shown in the current source looks into a semi-infinite transmission line, the characteristic impedance of the line would be the same as the input impedance seen by the source. Assuming the strip width, W, is much smaller than the operating wavelength, the y-component of the strip current can be neglected. The surface current, J(x,y), on the z=z′ plane may then be represented by

$$J(x,y) = \underline{x} f(y) \exp\left(-jK_e|x|\right) \qquad -\infty \le x \le \infty \qquad (1)$$

where $K_e=\beta+j\alpha$ is the dominant mode propagation constant along the strip. The range of x is extended to $-\infty$ due to the image of the strip behind the conducting plane at x=0 and the magnitude of x is used to indicate propagation away from the source at x=0. The transverse distribution, f(y), in equation 1 may be assumed to be of the form

$$f(y) = \sum_{n=1}^{N} I_n \, P_n(y) \qquad (2)$$

where

$$P_n(y) = 1/\Delta y \quad \text{for} \quad \left(y_n - \frac{\Delta y}{2}\right) \le y \le \left(y_n + \frac{\Delta y}{2}\right)$$
$$= 0 \quad \text{otherwise.}$$

From the continuity condition, the distribution f(y) must satisfy

$$\int_{-W/2}^{W/2} f(y)\, dy = I_0 \tag{3}$$

The pulse amplitudes, I_n, in equation 2 are determined, as explained later, by using the integral equation method in conjunction with the method of moments.

If **E**(x,y,z) is the electric field due to the strip current, **J**(x,y), then the input impedance seen by the source is obtained from

$$Z_{in} = Z_0 = -\frac{1}{I_0^2} \int_{-W/2}^{W/2} \int_0^{\infty} \mathbf{J}(x,y) \bullet \mathbf{E}(x,y,z')\, dx\, dy \tag{4}$$

This approach thus differs from the solution to the multilayer transmission line problem [3], where the characteristic impedance is obtained after dividing the average voltage by the total current on the strip. Furthermore, by taking into account the actual current distribution in the y-direction, it is expected that the present approach would give more accurate results. Using the dyadic Green's function for a current embedded in a grounded dielectric slab [6], the x-component of the electric field **E**(x,y,z) is obtained as

$$\mathbf{x} \bullet \mathbf{E}(x,y,z') = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} Q(k_x,k_y)\ J_x(k_x,k_y) \cdot \exp(jk_x x + jk_y y)\ dk_x\, dk_y \tag{5}$$

where,

$$Q(k_x,k_y) = (k^2-k_x^2)\mathcal{G}_1(k_x,k_y) + jk_x^2\mathcal{G}_2(k_x,k_y) \qquad (6)$$

$$\mathcal{G}_1(k_x,k_y) = \left\{\frac{\sin(k_I z')}{(k_I z')}\right\} \cdot \left\{\frac{k_I\cos(k_I(d-z')) + jk_{II}\sin(k_I(d-z'))}{k_I\cos(k_I d) + jk_{II}\sin(k_I d)}\right\} \qquad (7a)$$

$$\mathcal{G}_2(k_x,k_y) = \left\{\frac{-k_I^2\sin(k_I z')}{\varepsilon_r k_{II}\cos(k_I d) + jk_I\sin(k_I d)}\right\} \cdot \left\{\frac{\sin(k_I z')}{(k_I z')}\right\}\left\{\frac{(\varepsilon_r-1)}{k_I\cos(k_I d) + jk_{II}\sin(k_I d)}\right\} \qquad (7b)$$

$$k_I = \begin{cases} \sqrt{k^2-k_x^2-k_y^2} & (k_x^2+k_y^2) \le k^2 \\ -j\sqrt{k_x^2+k_y^2-k^2} & (k_x^2+k_y^2) > k^2 \end{cases}$$

$$k_{II} = \begin{cases} \sqrt{k_0^2-k_x^2-k_y^2} & (k_x^2+k_y^2) \le k_0^2 \\ -j\sqrt{k_x^2+k_y^2-k_0^2} & (k_x^2+k_y^2) > k_0^2 \end{cases}$$

In equation 5, η_0 is the free space characteristic impedance, $\mathcal{G}_1$ and $\mathcal{G}_2$ are derived in reference 6, and $J_x(k_x,k_y)$ is given by

$$J_x(k_x,k_y) = \int_{W/2}^{W/2}\int_{-\infty}^{\infty} f(y)\,\exp(-jK_e|x|-jk_x x-jk_y y)\,dx\,dy \qquad (8)$$

From equations 2 and 8, and assuming α to be a finite negative value, the integral in equation 8 may be evaluated in closed form as

$$J_x(k_x,k_y) = \sum_{n=1}^{N} jI_n \left[\frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)}\right] \cdot \left[\frac{1}{k_x - K_e} - \frac{1}{k_x + K_e}\right] \exp(-jk_y y_n) \qquad (9)$$

Substituting equation 5 into equation 4 and after some mathematical manipulations, the characteristic impedance is obtained as

$$Z_o = \frac{j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^{N} \frac{jI_n}{I_0^2} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left\{ \left[\frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)}\right] \cdot \left[\frac{1}{k_x - K_e} - \frac{1}{k_x + K_e}\right] Q(k_x,k_y) \cdot \mathcal{J}_x(-k_x,-k_y)\ \exp(-jk_y y_n) \right\} dk_x\ dk_y \qquad (10)$$

where

$$\mathcal{J}_x(-k_x,-k_y) = \int_{-W/2}^{W/2}\int_{0}^{\infty} f(y)\ \exp(-jK_e x + jk_x x + jk_y y)\ dx\ dy$$

$$= \sum_{m=1}^{N} jI_m \left[\frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)}\right]\left[\frac{\exp(jk_y y_m)}{k_x - K_e}\right] \qquad (11)$$

Substituting equation 11 into equation 10 gives

$$Z_0 = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{I_n I_m}{I_0^2}$$

$$\cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[\frac{\sin(k_y \Delta y/2)}{(k_y \Delta y/2)}\right]^2 \left[\frac{1}{(k_x - K_e)^2} - \frac{1}{k_x^2 - K_e^2}\right] \right.$$

$$\left. \cdot\, Q(k_x, k_y)\ \exp(jk_y(y_m - y_n)) \right\} dk_x\, dk_y \qquad (12)$$

Making a change of variables such that:

$$\zeta = \frac{k_x}{k_0} \quad ; \quad \xi = \frac{k_y}{k_0} \quad ; \quad \nu = \frac{K_e}{k_0} \quad ,$$

equation 12 is rewritten as

$$Z_0 = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{I_n I_m}{I_0^2}$$

$$\cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[\frac{\sin(\xi\ k_0 \Delta y/2)}{(\xi\ k_0 \Delta y/2)}\right]^2 \left[\frac{1}{(\zeta - \nu)^2} - \frac{1}{\zeta^2 - \nu^2}\right] \right.$$

$$\left. \cdot\, \mathcal{Q}(\zeta, \xi)\ \exp(j\xi k_0(y_m - y_n)) \right\} d\zeta\ d\xi \qquad (13)$$

where, $\mathcal{Q}(\zeta,\xi) \equiv Q(k_0\zeta, k_0\xi)$.

Note that the integrand in equation 13 has poles at $\zeta=\nu$ of order two and simple poles at $\zeta=\pm\nu$. The integration with respect to ζ can be evaluated using the residue theorem. In order to evaluate the integration with respect to ζ, consider the following contour integration,

$$\int_C \left\{ \left(\frac{1}{\gamma-\nu}\right)^2 - \left(\frac{1}{\gamma^2-\nu^2}\right) \right\} Q(\gamma,\xi)\, d\gamma \qquad (14)$$

where ζ is the real part of γ, and C is the closed contour to be selected. The function $Q(\gamma,\xi)$ has branch cuts at

$$\gamma = \pm \sqrt{1 - \xi^2}$$

and poles corresponding to the roots of the equations:

$$k_I \cos(k_I d) + jk_{II} \sin(k_I d) = 0$$

$$\varepsilon_r k_{II} \cos(k_I d) + jk_I \sin(k_I d) = 0$$

These poles correspond to the surface wave modes that would be launched by the semi-infinite microstrip line. These surface wave poles and the branch cuts may be plotted in the complex γ-plane as shown in figure 3. The contour C should be selected such that the propagation of the surface waves in the x>0 direction is assured. For this condition, the surface wave poles on the positive real axis must be included in the contour C. The required contour C is shown in figure 3. The integration in equation 14 can be written as

$$\int_C = \int_{C_1} + \int_{C_2} + \int_{C_3} = -2\pi j \{ R_1 + R_2 + R_3 \} \qquad (15)$$

where,

$R_1 = \sum$ Residues of Q at surface wave poles
R_2 = Residue of $Q/(\gamma-\nu)^2$ at $\gamma=\nu$
R_3 = Residue of $Q/(\gamma^2-\nu^2)$ at $\gamma=\nu$.

In equation 15, the integration along C_2 is zero because of the radiation condition, the integration along the branch cut, C_3, represents a radiation field, and the integration along C_1 is the required integration to be evaluated with respect to ζ. Since the radiation field due to the microstrip line would be very small, the integration along C_3 will be neglected. For the present case, the contribution due to surface wave modes will also be neglected. The integration with respect to ζ becomes

$$\int_{-\infty}^{\infty} \left\{ \left(\frac{1}{\zeta-\nu}\right)^2 - \left(\frac{1}{\zeta^2-\nu^2}\right) \right\} Q(\zeta,\xi)\, d\zeta$$

$$= -2\pi j \left\{ \left.\frac{\partial Q(\zeta,\xi)}{\partial \zeta}\right|_{\zeta=\nu} - \left.\frac{Q(\zeta,\xi)}{2\zeta}\right|_{\zeta=\nu} \right\} \qquad (16)$$

Substituting equation 16 into equation 13, the expression for Z_0 reduces to

$$Z_0 = \frac{-\eta_0 k_0 z'}{2\pi k^2} \sum_{n=1}^{N} \sum_{m=1}^{N} \frac{I_n I_m}{I_0^2} \int_0^{\infty} \left\{ \left[\frac{\sin\,(k_0 \xi \Delta y/2)}{(k_0 \xi \Delta y/2)}\right]^2 \right.$$

$$\left. \cdot \left(\frac{2\partial Q(\zeta,\xi)}{\partial \zeta} - \frac{Q(\zeta,\xi)}{\zeta}\right)\right|_{\zeta=\nu} \cos\left(\xi(m-n)k_0 \Delta y\right) \Bigg\}\, d\xi \qquad (17)$$

From the characteristic equation (27), it can be seen that the term

$$\sum_{n=1}^{N} \sum_{m=1}^{N} \frac{I_n I_m}{I_0^2} \int_0^{\infty} \left\{ \left.\frac{Q(\zeta,\xi)}{\zeta}\right|_{\zeta=\nu} \right.$$

$$\left. \cdot \left[\frac{\sin\,(k_0 \xi \Delta y/2)}{(k_0 \xi \Delta y/2)}\right]^2 \cos\left(\xi(m-n)k_0 \Delta y\right) \right\} d\xi$$

in equation 17 is always zero.

For a uniform current distribution in the y-direction $(I_n = I_0)$, the expression in equation 17 becomes

$$Z_0 = \frac{-\eta_0 k_0 z'}{\pi k^2} \int_0^\infty \left\{ \left. \frac{\partial Q(\zeta,\xi)}{\partial \zeta} \right|_{\zeta=\nu} \left[\frac{\sin\left(k_0 \xi \Delta y/2\right)}{\left(k_0 \xi \Delta y/2\right)} \right]^2 \right\} d\xi \qquad (18)$$

For a nonuniform current distribution of

$$f(y) = \frac{2I_0/(\pi W)}{\sqrt{1 - (2y/W)^2}} \qquad (19)$$

the characteristic impedance of the transmission line becomes

$$Z_0 = \frac{-\eta_0 k_0 z'}{\pi k^2} \int_0^\infty \left\{ \left. \frac{\partial Q(\zeta,\xi)}{\partial \zeta} \right|_{\zeta=\nu} J_0^2\left(\xi k_0 W/2\right) \right\} d\xi \qquad (20)$$

where $J_0(\cdot)$ denotes the Bessel function of the first kind of zero order.

Evaluation of K_e and I_n

In order to determine K_e, consider the infinite microstrip line as shown in figure 1. The surface current distribution on the strip may be assumed to be of the form

$$\mathbf{J}_t(x,y) = \mathbf{x}\ J_{xt}(x,y) = \mathbf{x}\ f(y)\ \exp(-jK_e x) \qquad -\infty \le x \le \infty \qquad (21)$$

where f(y) is as given in equation 2.

Using equation 8, the Fourier transform of $\mathbf{J}_t(x,y)$ is obtained as

$$\mathbf{J}_t(k_x,k_y) = \mathbf{x}\ J_{xt}(k_x,k_y)$$

$$= \mathbf{x} \sum_{n=1}^{N} I_n 2\pi \left[\frac{\sin\,(k_y \Delta y/2)}{(k_y \Delta y/2)}\right]$$

$$\cdot\ \delta\left(k_x + K_e\right)\ \exp\left(-jk_y y_n\right) \qquad (22)$$

where $\delta(\cdot)$ is a delta function and is the result of assuming a traveling wave current in the x-direction on the strip as denoted by the exponential factor in equation 21.

The x-component of the electric field radiated by J_{xt} is obtained by replacing $J_x(k_x,k_y)$ with $J_{xt}(k_x,k_y)$ in equation 5. Then by equating the x-component of the electric field to zero on the surface of the strip, since the total field must be zero and there is no incident field, an electric field integral equation is obtained

$$0 = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^{N} I_n 2\pi$$

$$\cdot \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[\frac{\sin (k_y \Delta y/2)}{(k_y \Delta y/2)}\right] Q(k_x,k_y)\ \delta(k_x+K_e) \right.$$

$$\left. \cdot \exp(-jk_y y_n)\ \exp(jk_x x+jk_y y) \right\} dk_x\ dk_y \qquad (23)$$

By setting x=0 in equation 23 and selecting $P_m(y)$ as a testing function and multiplying both sides of equation 23 by $P_m(y)$ yields

$$0 = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^{N} I_n 2\pi$$

$$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[\frac{\sin (k_y \Delta y/2)}{(k_y \Delta y/2)}\right] Q(k_x,k_y)\ \delta(k_x+K_e) \right.$$

$$\left. \cdot P_m(y)\ \exp(-jk_y y_n)\ \exp(jk_y y) \right\} dk_x\ dk_y \qquad (24)$$

Integrating both sides of equation 24 with respect to y and rearranging gives

$$0 = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^{N} I_n 2\pi \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left\{ \left[\frac{\sin (k_y \Delta y/2)}{(k_y \Delta y/2)}\right] \right.$$

$$\cdot Q(k_x,k_y)\ \delta(k_x+K_e)\ \exp(-jk_y y_n)$$

$$\left. \cdot \left[\int_{y_m-\Delta y/2}^{y_m+\Delta y/2} P_m(y)\ \exp(jk_y y)\ dy \right] \right\} dk_x\ dk_y \qquad (25)$$

Using the property,

$$\int_{-\infty}^{\infty} f(x)\ \delta(x+x')\ dx = f(-x')$$

of a delta function, equation 25 can be written in the form

$$0 = \frac{-j\eta_0 k_0 z'}{2\pi\ k^2} \sum_{n=1}^{N} I_n \int_{-\infty}^{\infty} \left\{ Q(k_x,k_y)\Big|_{k_x=-K_e} \cdot \left[\frac{\sin\ (k_y \Delta y/2)}{(k_y \Delta y/2)}\right]^2 \exp(-jk_y(y_n-y_m)) \right\} dk_y \qquad (26)$$

Since $Q(k_x,k_y)=Q(k_x,-k_y)$, equation 26 may be rewritten as

$$0 = \frac{-j\eta_0 z'}{\pi\ \varepsilon_r} \sum_{n=1}^{N} I_n \int_{0}^{\infty} \left\{ \mathcal{Q}(\zeta,\xi)\Big|_{\zeta=-\nu} \cdot \left[\frac{\sin\ (\xi k_0 \Delta y/2)}{(\xi k_0 \Delta y/2)}\right]^2 \cos(\xi k_0 \Delta y(m-n)) \right\} d\xi \qquad (27)$$

where $\xi=k_y/k_0$, $\zeta=k_x/k_0$, $\nu=K_e/k_0$, and $\mathcal{Q}(\zeta,\xi)\equiv Q(k_0\zeta,k_0\xi)$.

By selecting $m=1,2,3 \cdots N$, equation 27 may be written in matrix form as

$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \cdots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} \cdots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} \cdots & Z_{3N} \\ \vdots & \vdots & \vdots \ \ddots & \vdots \\ Z_{N1} & Z_{N2} & Z_{N3} \cdots & Z_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix} \qquad (28)$$

where the components of the square matrix are given by

$$\mathcal{Z}_{mn} = \int_0^\infty \left\{ Q(\zeta,\xi)\Big|_{\zeta=-\nu} \left[\frac{\sin\ (\xi k_0 \Delta y/2)}{(\xi k_0 \Delta y/2)}\right]^2 \cdot \cos\left(\xi k_0 \Delta y(m-n)\right)\right\} d\xi \qquad (29)$$

For a nontrivial solution of equation 28, the determinant of the square matrix in equation 28 must be set to zero, therefore,

$$\begin{vmatrix} \mathcal{Z}_{11} & \mathcal{Z}_{12} & \mathcal{Z}_{13} & \cdots & \mathcal{Z}_{1N} \\ \mathcal{Z}_{21} & \mathcal{Z}_{22} & \mathcal{Z}_{23} & \cdots & \mathcal{Z}_{2N} \\ \mathcal{Z}_{31} & \mathcal{Z}_{32} & \mathcal{Z}_{33} & \cdots & \mathcal{Z}_{3N} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \mathcal{Z}_{N1} & \mathcal{Z}_{N2} & \mathcal{Z}_{N3} & \cdots & \mathcal{Z}_{NN} \end{vmatrix} = 0 \qquad (30)$$

Equation 30 is solved for K_e in a numerical procedure by selecting N and then starting with an estimate of $K_e \approx k$ and iterating K_e until equation 30 is satisfied to the desired accuracy.

In order to determine the current distribution in the transverse direction, it is assumed that the infinite microstrip line shown in figure 1 is excited by an x-polarized plane wave. The surface current induced on the strip may be assumed to be of the form

$$\mathbf{J}(x,y) = \mathbf{x}\ J_{xu}(x,y) = \sum_{n=1}^{N} I_n\ P_n(y)\ \mathbf{x} \qquad (31)$$

If $E_{xu}(x,y,z)$ is the x-component of the scattered electric field due to $J_{xu}(x,y)$, and $E_x^i(x,y,z)$ is the incident electric field, the boundary condition of zero tangential electric field on the conducting strip gives

$$-E_x^i(x,y,z') = E_{xu}(x,y,z') \tag{32}$$

$E_{xu}(x,y,z')$ can be obtained from equations 5,6 and 8. Equation 32 then becomes

$$-E_x^i(x,y,z') = \frac{-j\eta_0 k_0 z'}{(2\pi)^2 k^2} \sum_{n=1}^{N} I_n \int_{-\infty}^{\infty} \left\{ Q(k_x,k_y)\Big|_{k_x=0} \cdot \left[\frac{\sin\ (k_y \Delta y/2)}{(k_y \Delta y/2)}\right] \exp(-jk_y y_n) \right\} dk_y \tag{33}$$

Equation 33 is an integral equation with I_n as the unknown complex values to be determined. Equation 33 can be solved by selecting $P_m(y)$ as a testing function and applying the method of moments to yield

$$\sum_{=1}^{N} I_n\ Z_{mn} = -\ \Delta y\ E_0^i(z') \qquad \text{for } m=1,2,3,\cdots,N \tag{34}$$

where $E_0^i(z')$ is the x-polarized plane-wave electric field incident on the strip from the z-direction (normal incidence) and Z_{mn} is given by

$$Z_{mn} = \frac{-j\eta_0 k_0^2 z' \Delta y}{\pi\ \varepsilon_r} \int_0^{\infty} \left\{ Q(\zeta,\xi)\Big|_{\zeta=0} \cdot \left[\frac{\sin(\xi k_0 \Delta y/2)}{(\xi k_0 \Delta y/2)}\right] \cos(\xi(m-n)k_0 \Delta y) \right\} d\xi \tag{35}$$

Equation 34 can be written in matrix notation as

$$
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ \vdots \\ I_N \end{bmatrix}
\begin{bmatrix} Z_{11} & Z_{12} & Z_{13} \cdots & Z_{1N} \\ Z_{21} & Z_{22} & Z_{23} \cdots & Z_{2N} \\ Z_{31} & Z_{32} & Z_{33} \cdots & Z_{3N} \\ \vdots & \vdots & \vdots \ddots & \vdots \\ Z_{N1} & Z_{N2} & Z_{N3} \cdots & Z_{NN} \end{bmatrix}
=
\begin{bmatrix} -\Delta y \; E_0^i \\ -\Delta y \; E_0^i \\ -\Delta y \; E_0^i \\ \vdots \\ -\Delta y \; E_0^i \end{bmatrix}
\tag{36}
$$

and the complex values of I_n are obtained by matrix inversion.

Numerical Results

In order to obtain numerical results, the semi-infinite integrals appearing in equations 17, 18, 20, 29 and 35 are evaluated using Gauss Quadrature integration techniques and the upper limit on the integration is truncated to a large finite value. From numerical results, it was found that the upper limit on the integration depended upon frequency and could vary from $t \approx 200$ (for high frequencies) to $t \approx 1000$ (for frequencies below 5 GHz). Because of the complexity of the expression for $Q(k_x, k_y)$, the derivative, $\partial Q/\partial k_x$ or $\partial Q/\partial \zeta$, is evaluated numerically.

In order to obtain meaningful numerical results, it is desirable to apply convergence tests to I_n, K_e and Z_0. The matrix equation is solved for I_n for various values of N and plotted in figure 4. It is observed that a stable solution for I_n is obtained when $N \geq 25$. Figure 5 shows the variation of K_e and Z_0 as a function of N. It is noted that $N \geq 25$ also gives converged values for these quantities.

To verify the present formulation, the characteristic impedance and the guide wavelength of a microstrip line with $W/z' = 1.0$, $d = z' = -.04$ cm, and $\varepsilon_r = 10$, is computed using equations 13 and 22 for N=25. These results are presented in figure 6 as a function of frequency along with results reported earlier [5]. Using the present method, Z_0 and K_e are also computed for uniform current distribution and nonuniform (see eq.14) current distributions and presented in figure 6. The results obtained by the present method with uniform distribution are in excellent agreement with the results reported earlier [4]. However, Z_0 and K_e calculated by taking into account the actual current distribution (as obtained from equation 36) is believed to give more accurate results than the results obtained with uniform current distribution. It is also observed from curves C & D of figure 6, that the assumed nonuniform distribution is close to the actual calculated current distribution and may be used instead of the pulse distribution in order to save computation time. To compare the present method with the approach in reference 3, the characteristic impedance of a microstrip line with d=z′ and d=4z′, ε_r=2.53, f_0=3GHz is computed as a function of strip width, W/z′, and presented in figures 7 and 8. It is noted that there is a small discrepancy between the two results in figure 8. This may be attributed to the different type of current distribution assumed in the two cases. With an assumption of uniform current distribution in the y-direction, the results obtained by the present method are also presented in figure 7. There is good agreement between the results obtained by the two methods. Figure 8 shows the characteristic impedance of a microstrip line covered with a dielectric slab.

The results obtained by the present method are also compared in figure 9 with data taken from reference 1. For frequencies greater than 42 GHz, the dielectric thickness becomes greater than a quarter-wavelength and the current distribution in the y-direction may not be of the assumed form. Furthermore, for thicknesses greater than a quarter-wavelength, the microstrip line would act as a radiator rather than a transmission line.

It is interesting to study the effect of distribution in the y-direction on the guide wavelength and the characteristic impedance of a microstrip line. It is expected that for a narrow strip, the distribution may be closer to the assumed nonuniform function (equation 19); whereas, for a wider strip, the distribution is expected to be closer to uniform, except near the strip edges. To verify this expectation, the guide wavelength and the characteristic impedance of a microstrip line with $d=z'=-.05$ wavelengths, $\varepsilon_r=10$, and the strip width in the range of 0.025 to 0.330 wavelengths were computed and presented in figure 10. The guide wavelength computed with the assumed nonuniform distribution and pulse distribution are almost the same for a narrow strip ($W/\lambda_0<0.05$). For wider strips ($W/\lambda_0>0.27$) the guide wavelength computed using the uniform distribution and the pulse distribution approach the same values. From figure 10, it is observed that the characteristic impedance does not significantly depend upon the transverse distribution of the current.

For microwave integrated circuit design, alumina substrate with dielectric constants in the range of 10 to 12.8 are often used. In order to design a microstrip line on these substrates, an accurate knowledge of the guide wavelength and the characteristic impedance is essential. In figures 11 and 12, the guide wavelength and the characteristic impedance are presented as a function of frequency for various design parameters. Calculations for other parameters may be obtained with the aid of the computer code given in the appendix.

Conclusion

Using the dyadic Green's function and Rumsey's reaction concept, an expression for the characteristic impedance of a microstrip line embedded in a grounded dielectric slab has been derived. The characteristic impedance determined by the present method is found to be in excellent agreement with earlier published results. Numerical results for a microstrip line on alumina substrate is presented at millimeter wave frequencies. A computer code listing is included for calculating the characteristic impedance and propagation constant for microstrip lines.

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1. P. B. Katehi and N. G. Alexopoulos: "Frequency-Dependent Characteristics of Microstrip Discontinuities in Millimeter Wave Integrated Circuits", IEEE Trans. on Microwave Theory and Techniques, Vol. MTT-33, No. 10, pp. 1029-1035, October 1985.
2. R. W. Jackson and D. M. Pozar: "Full Wave Analysis of Microstrip Open End and Gap Discontinuities", IEEE Trans. on Microwave Theory and Techniques, Vol MTT-33, No. 10, pp. 1036-1042, October 1985.
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4. K. C. Gupta, R. Garg and I. J. Bahl: *Microstrip Lines and Slotlines*, Artech House, Dedham, MA, 1979.
5. R. F. Harrington: *Time Harmonic Electromagnetic Fields*, McGraw-Hill Book Company, NY, 1961.
6. M. D. Deshpande and M. C. Bailey: "Integral Equation Formulation of Microstrip Antennas", IEEE Trans. on Antennas and Propagation, Vol. APS-30, No. 4, July 1982.

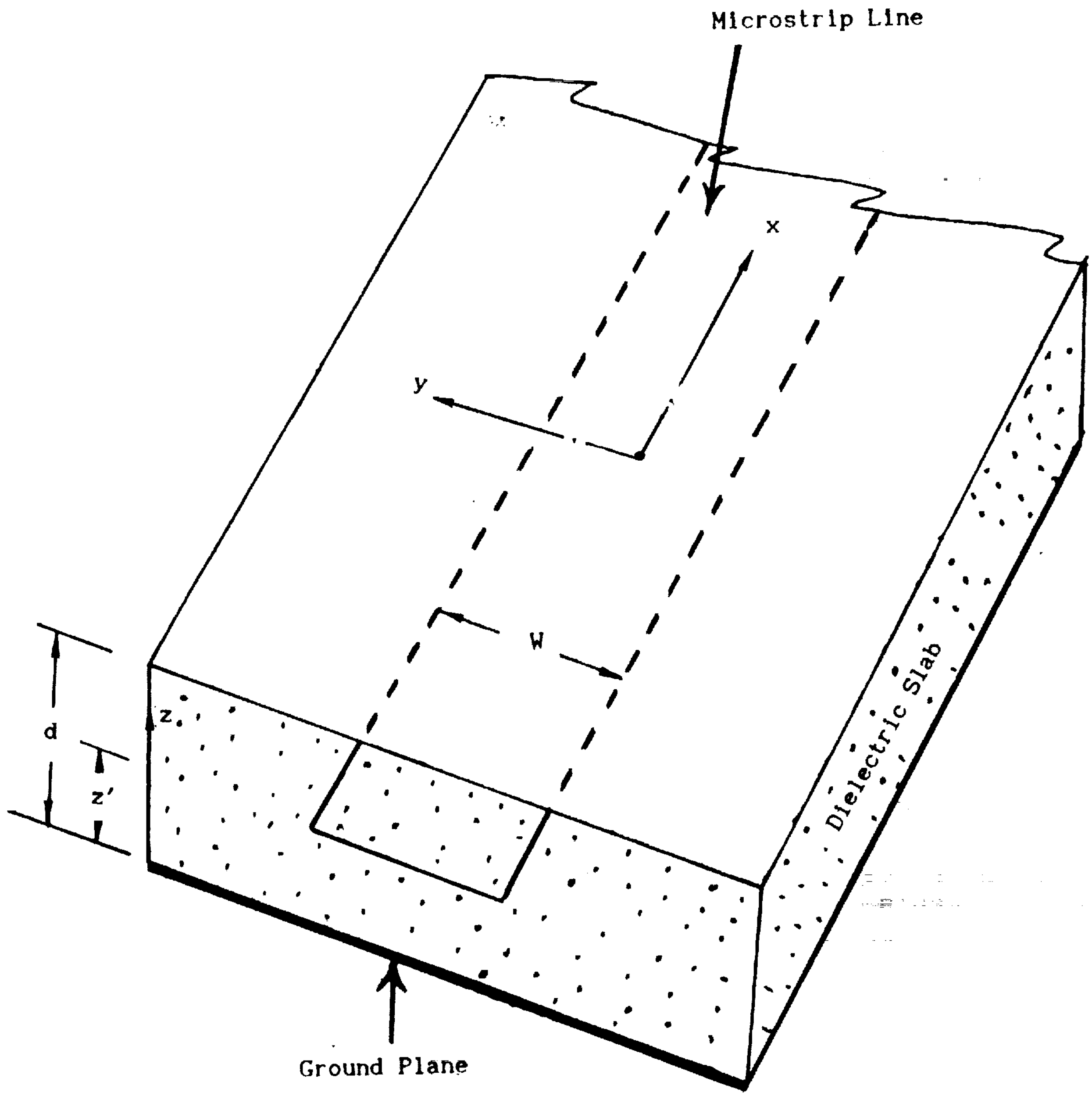

Figure 1. Geometry of microstrip line.

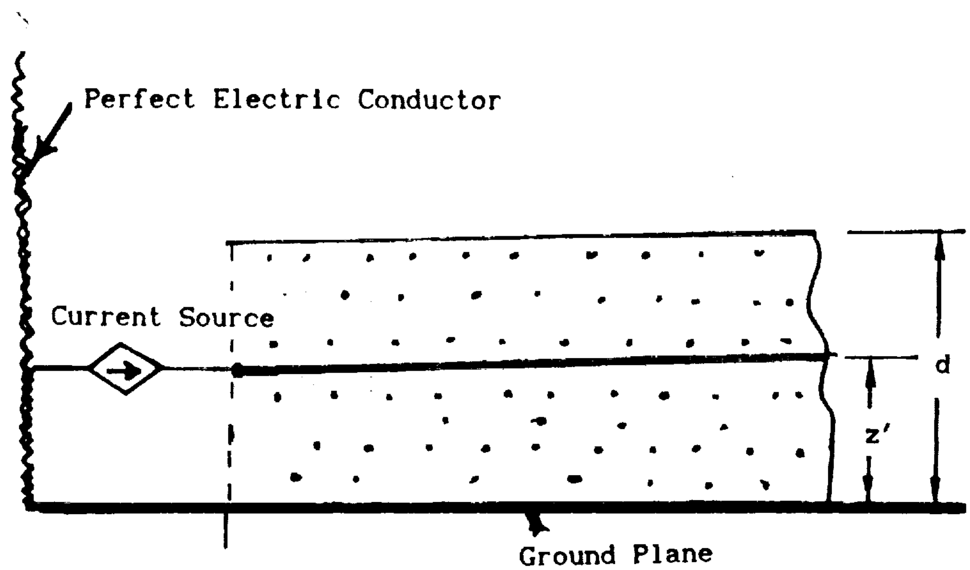

Figure 2. Current source exciting microstrip line.

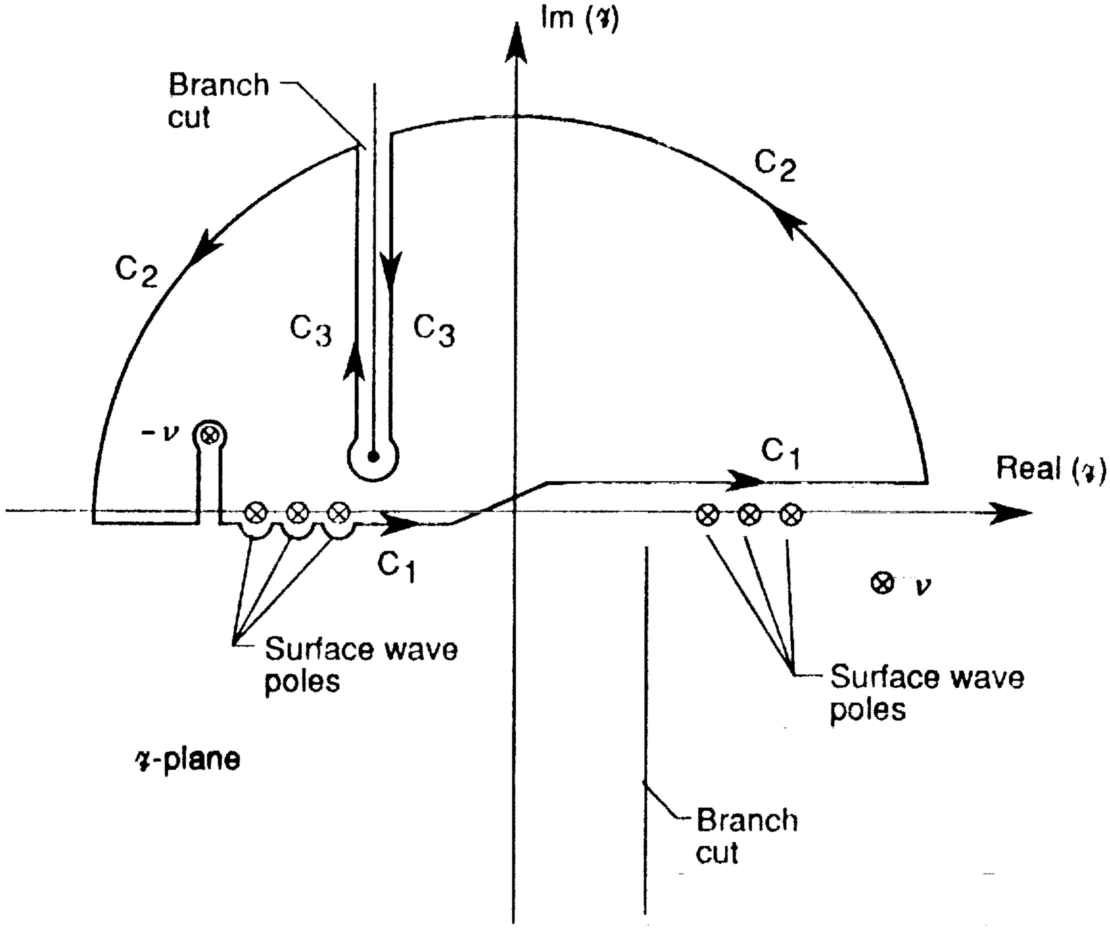

Figure 3. Complex γ-plane with surface wave poles and branch cuts.

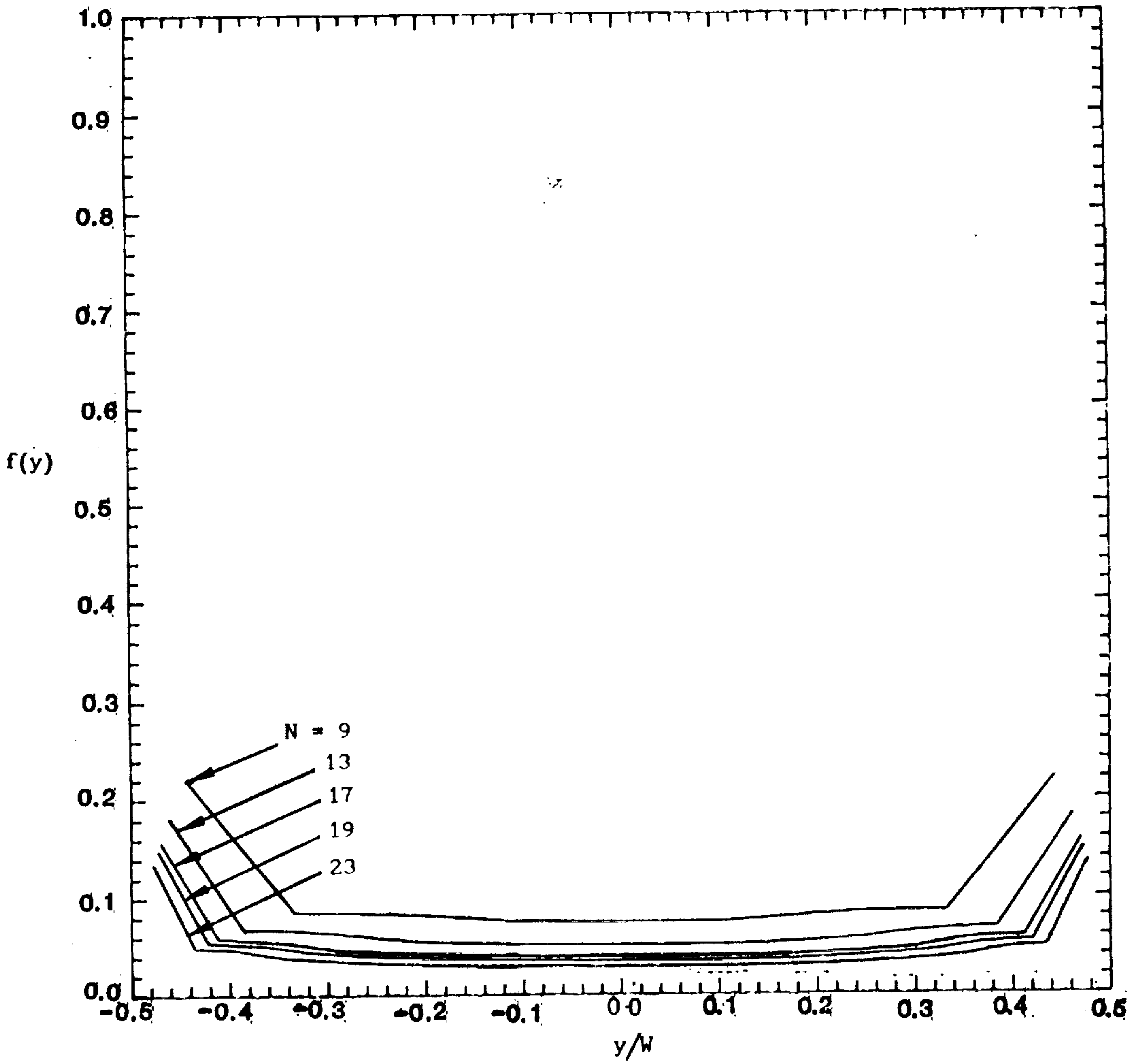

Figure 4. Current distribution in the transverse direction versus number of pulses. (ε_r=10, W=z', z'=d=0.08 cm, f=11.936 GHz).

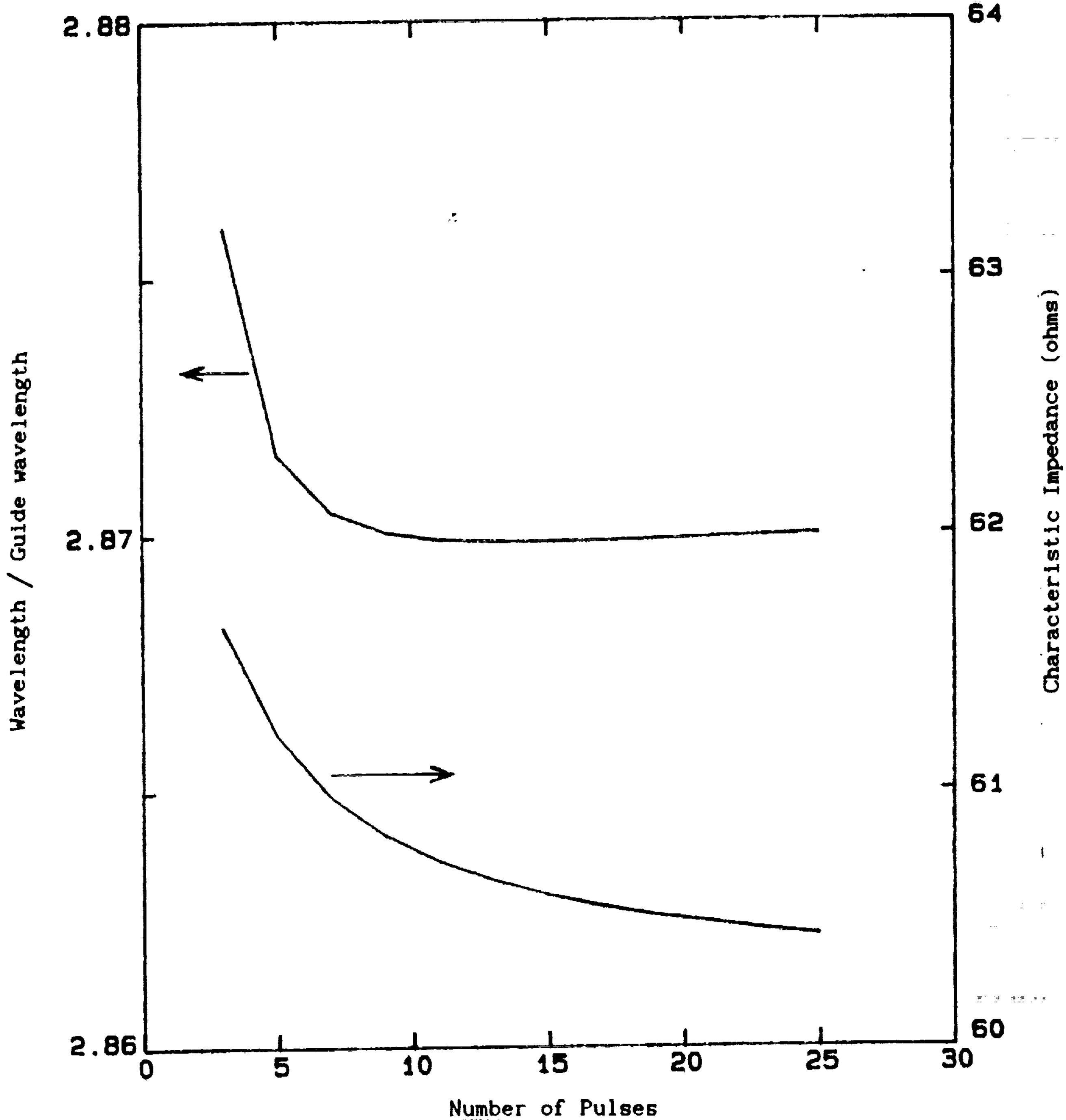

Figure 5. Variation of characteristic impedance and normalized guide wavelength of microstrip line as a function of number of transverse pulses. ($\epsilon_r = 10$, W=z', z'=d=0.04 cm, f=60 GHz).

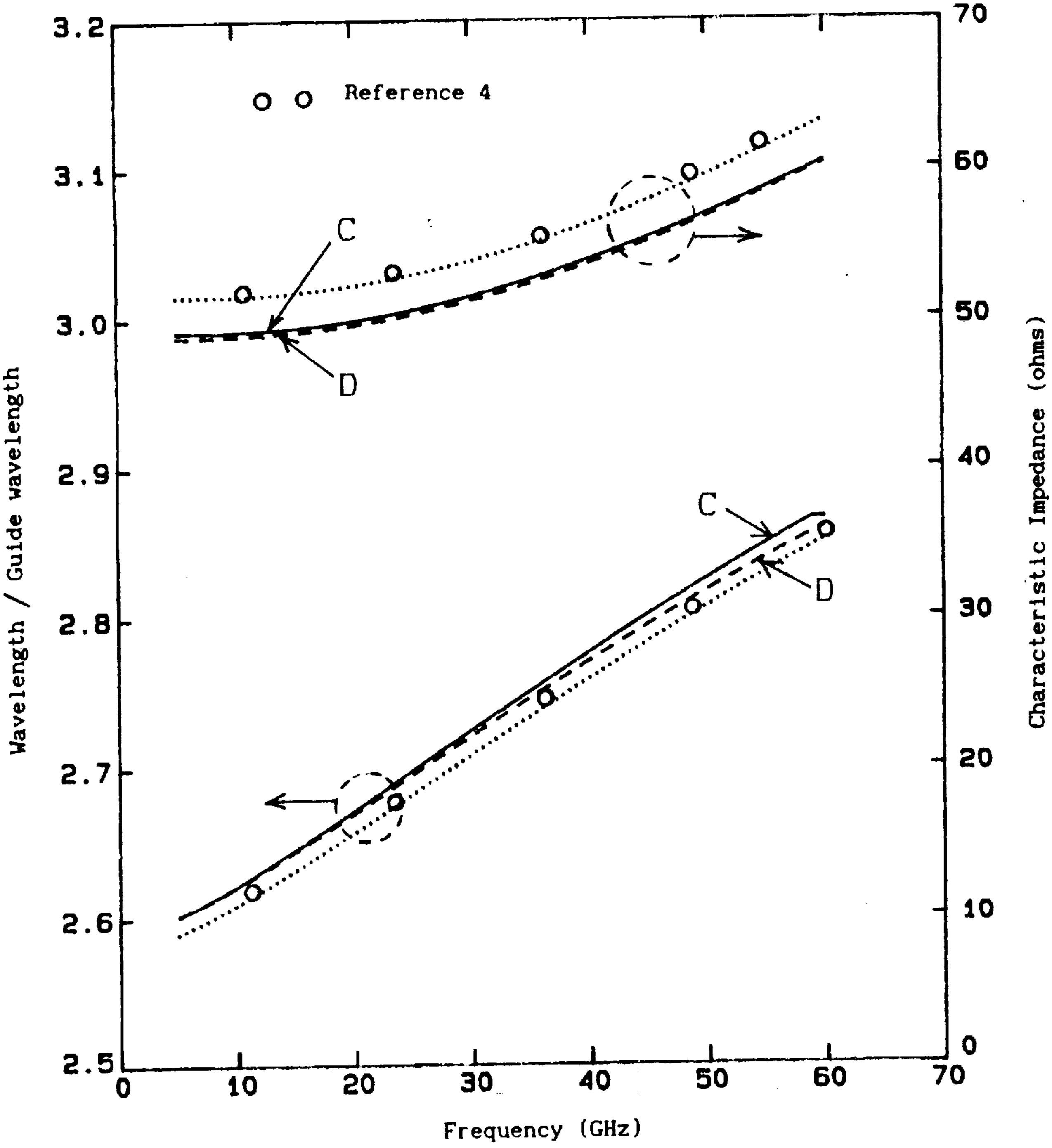

Figure 6. Characteristic impedance and normalized guide wavelength of microstrip line as a function of frequency. (······ uniform, - - - - nonuniform, ——— N=25 pulses).

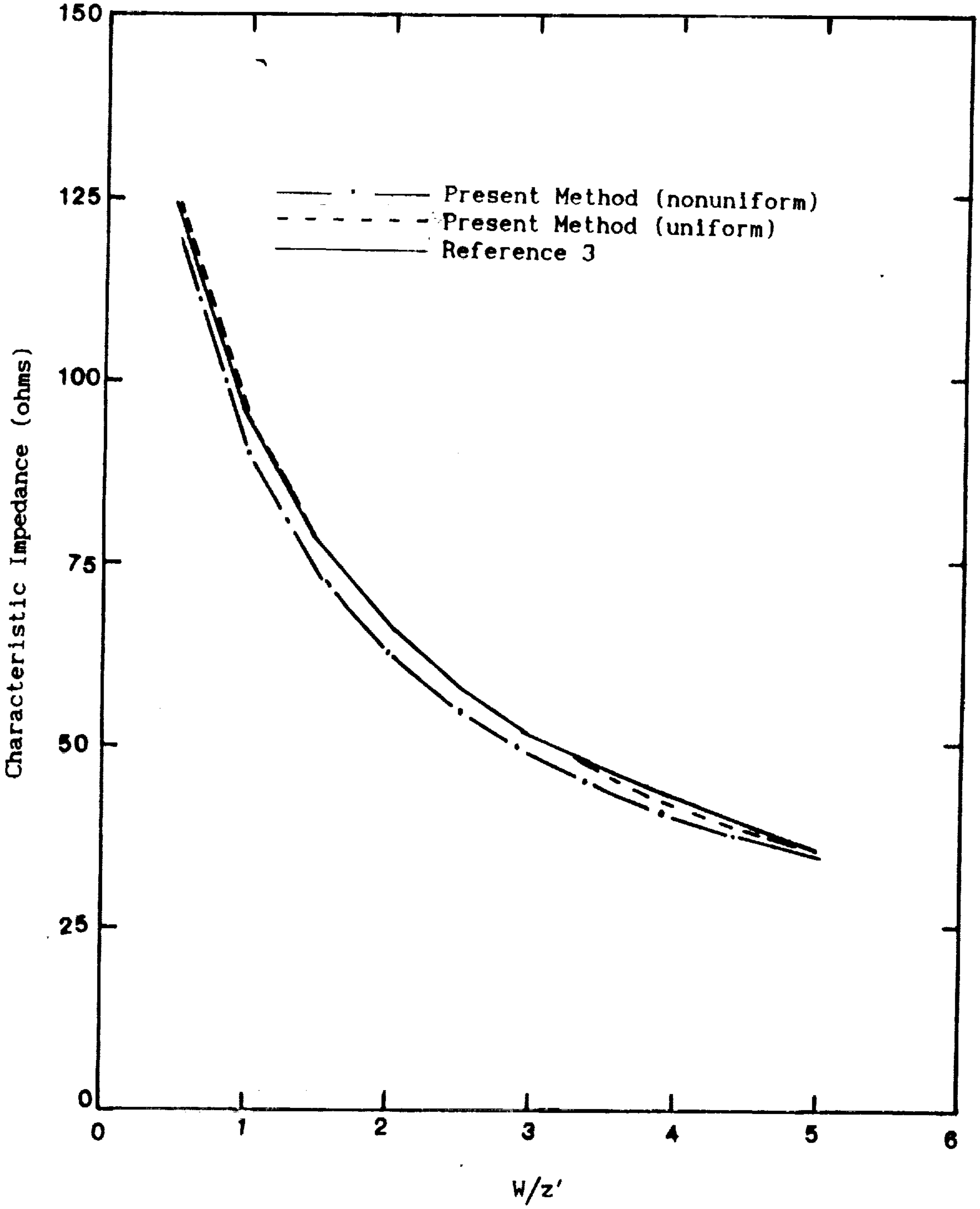

Figure 7. Characteristic iImpedance of covered microstrip line. (d = 0.64 cm, z'= 0.16 cm, ε_r = 2.53, f = 3 GHz).

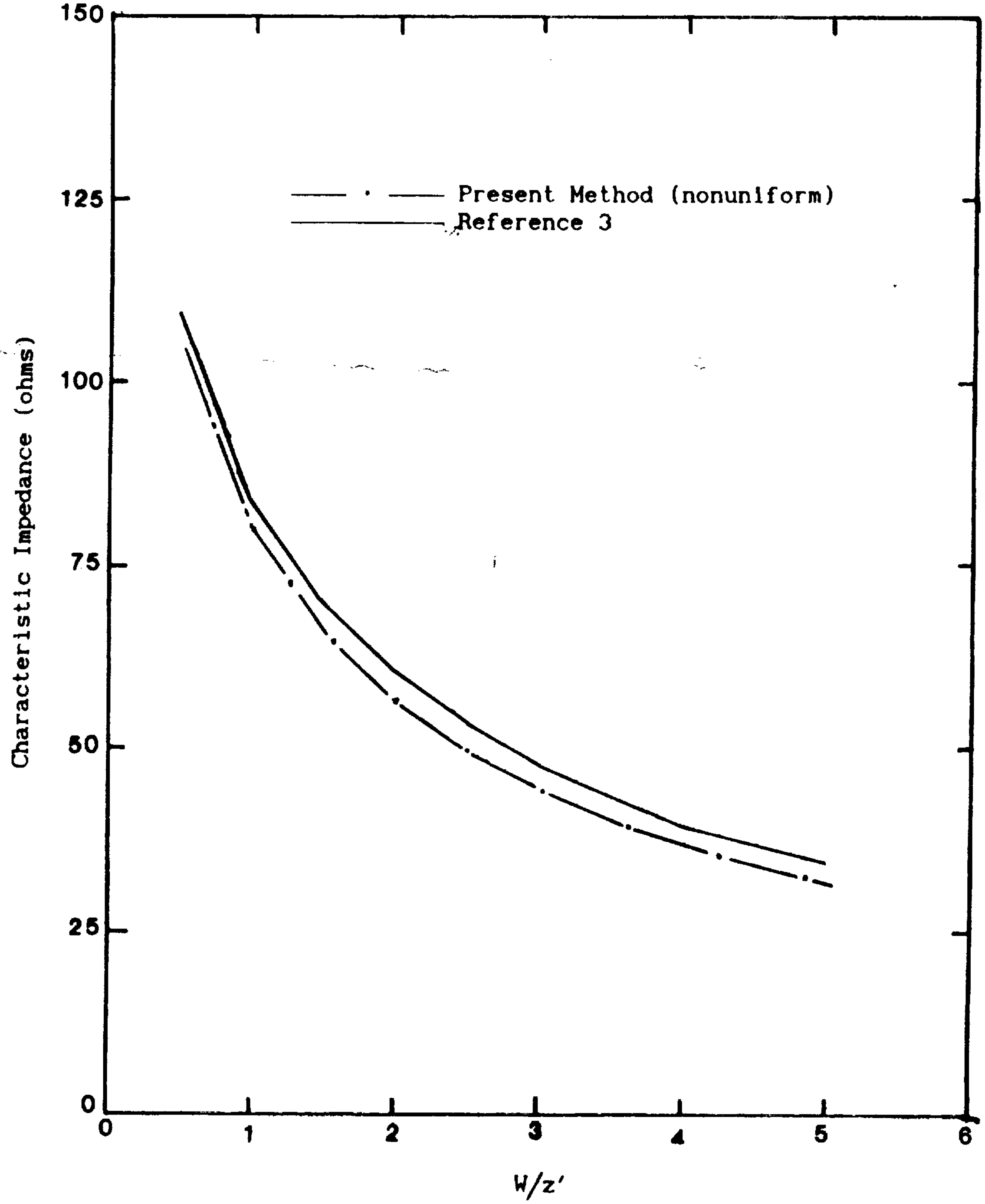

Figure 8. Characteristic impedance of microstrip line. ($z' = d = 0.16$ cm, $\varepsilon_r = 2.53$, $f = 3$ GHz).

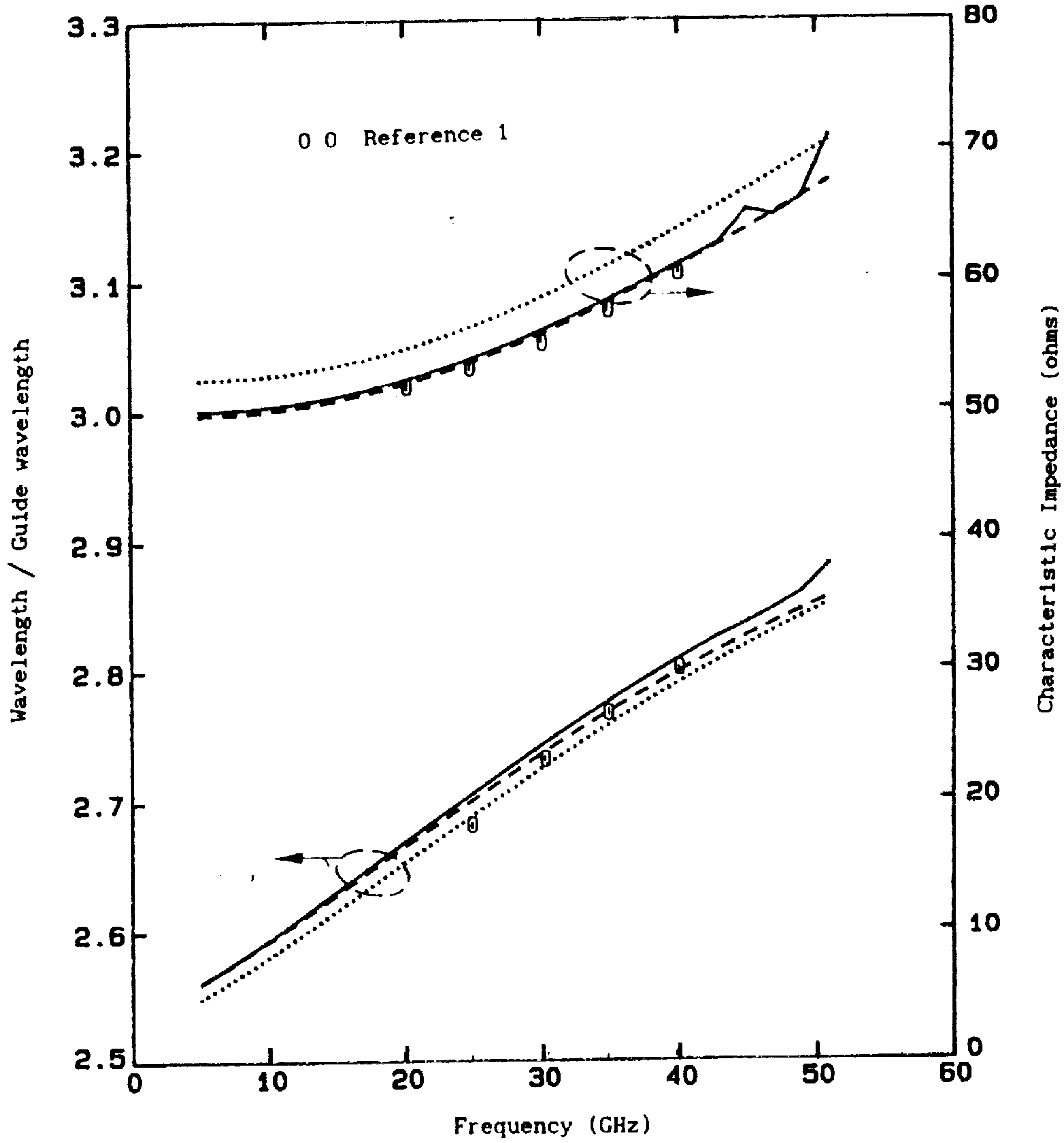

Figure 9. Characteristic impedance and normalized guide wavelength of microstrip line as a function of frequency. (ε_r = 9.6, W = z', z' = d = 0.6 cm) (······ uniform, - - - - nonuniform, ——— N=25 pulses).

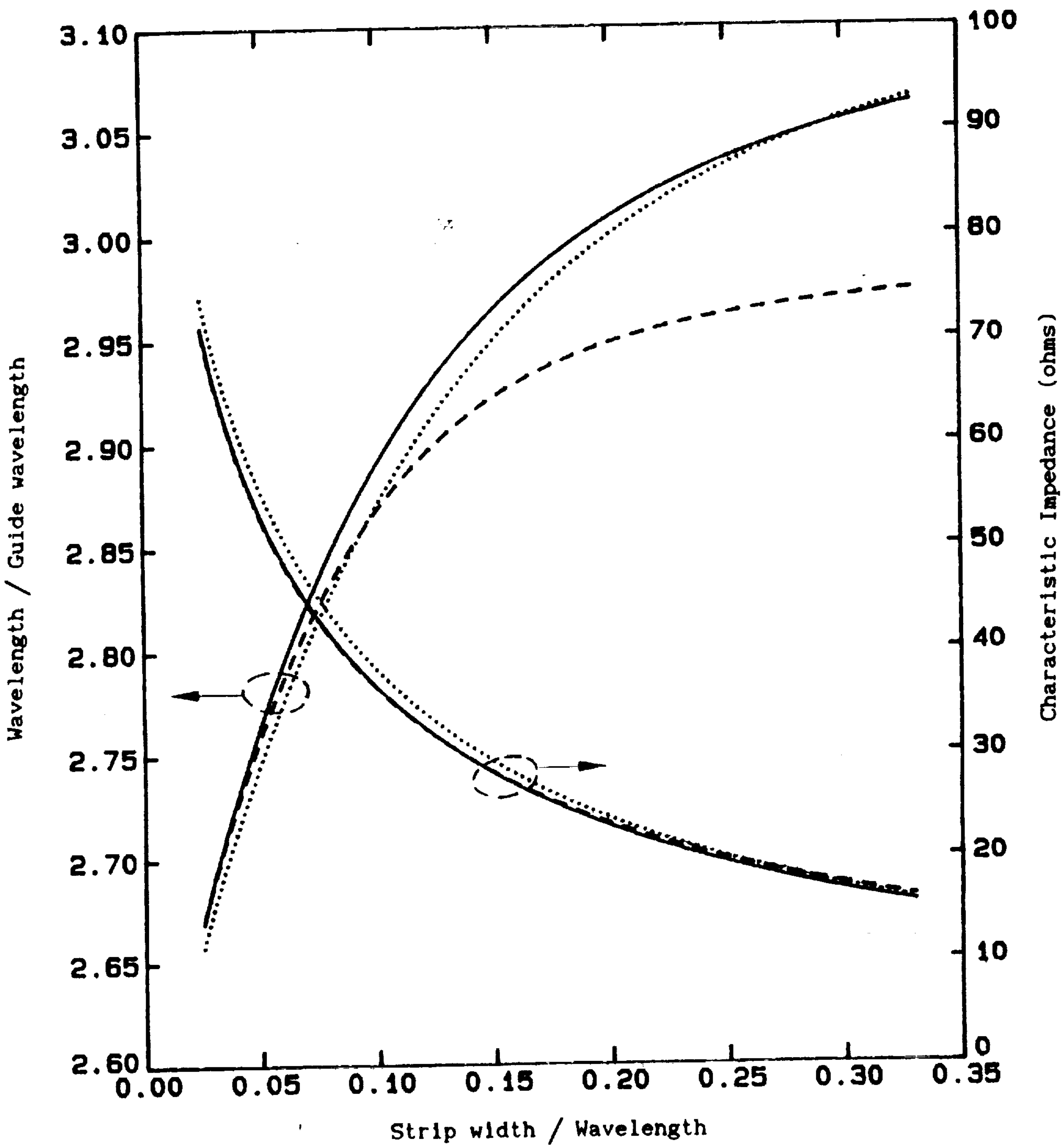

Figure 10. Characteristic impedance and normalized guide wavelength of microstrip line as a function of strip width.
(ε_r = 10, z' = d = 0.05 λ_o)
(······ uniform, - - -·- nonuniform, ——— N=25 pulses).

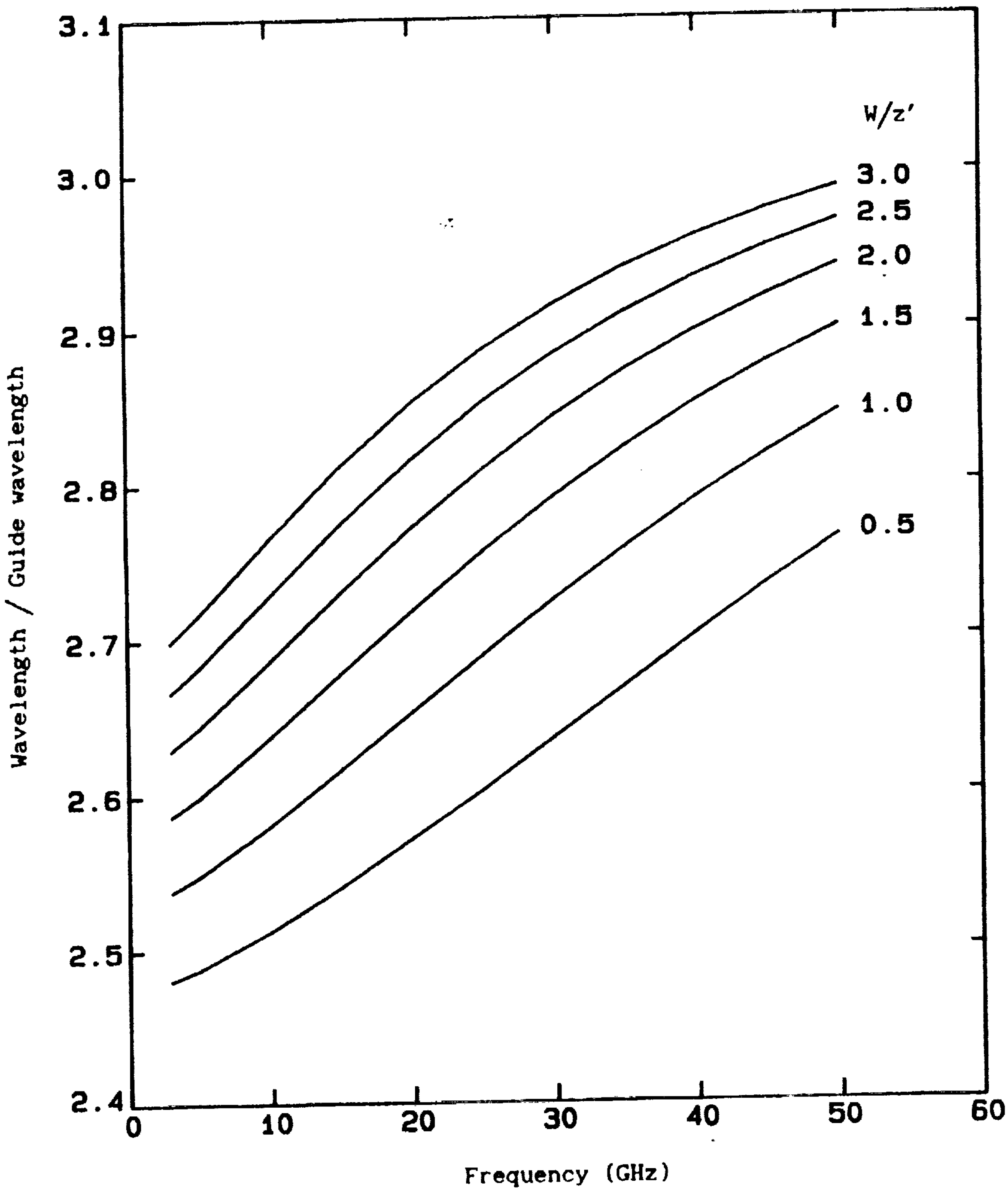

Figure 11a. Normalized guide wavelength of microstrip line as a function of frequency versus strip-width/strip-height. ($\varepsilon_r = 9.6$, $z' = d = 0.06$ cm).

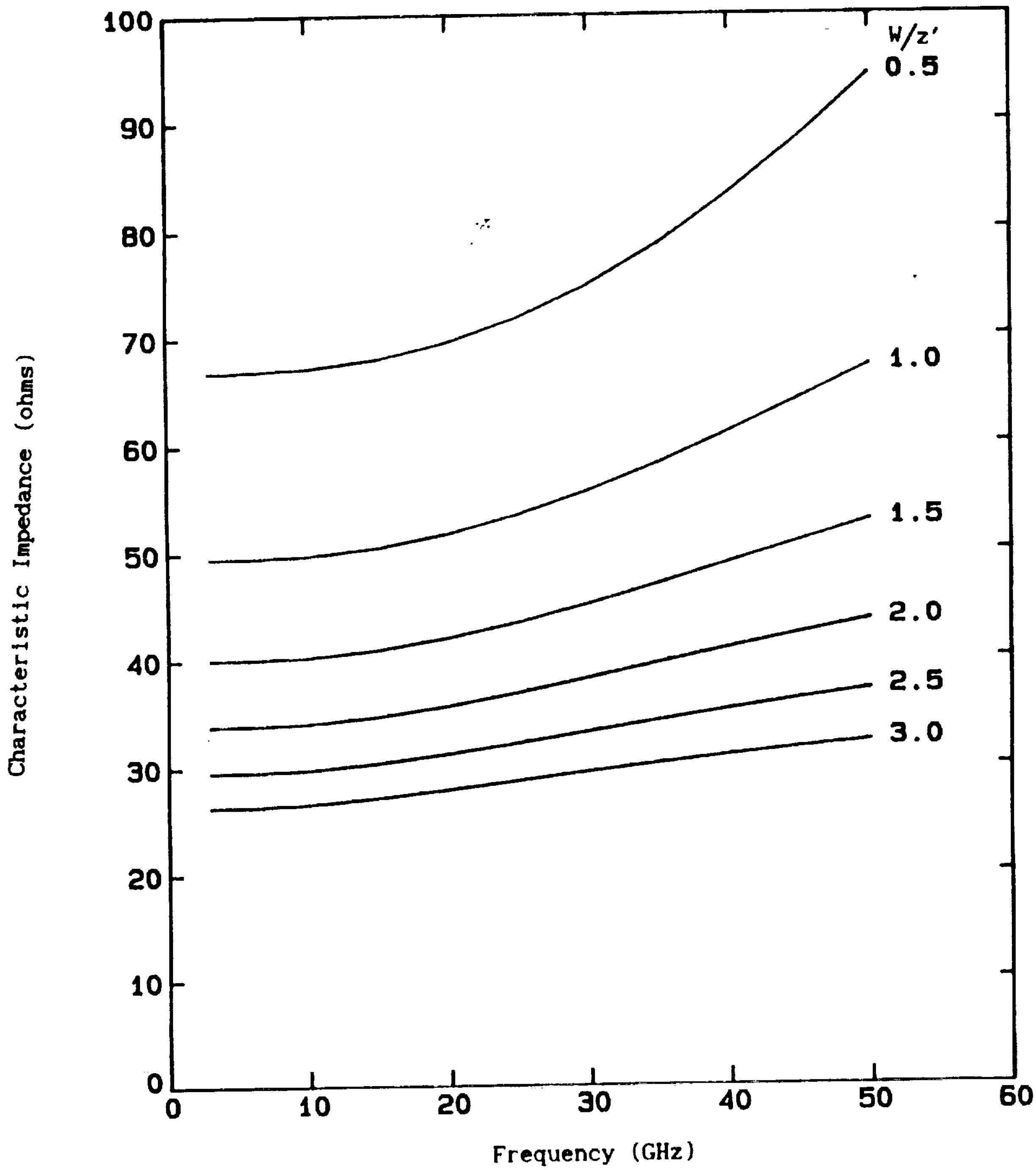

Figure 11b. Characteristic impedance of microstrip line as a function of frequency versus strip-width/strip-height. (ϵ_r = 9.6, z' = d = 0.06 cm).

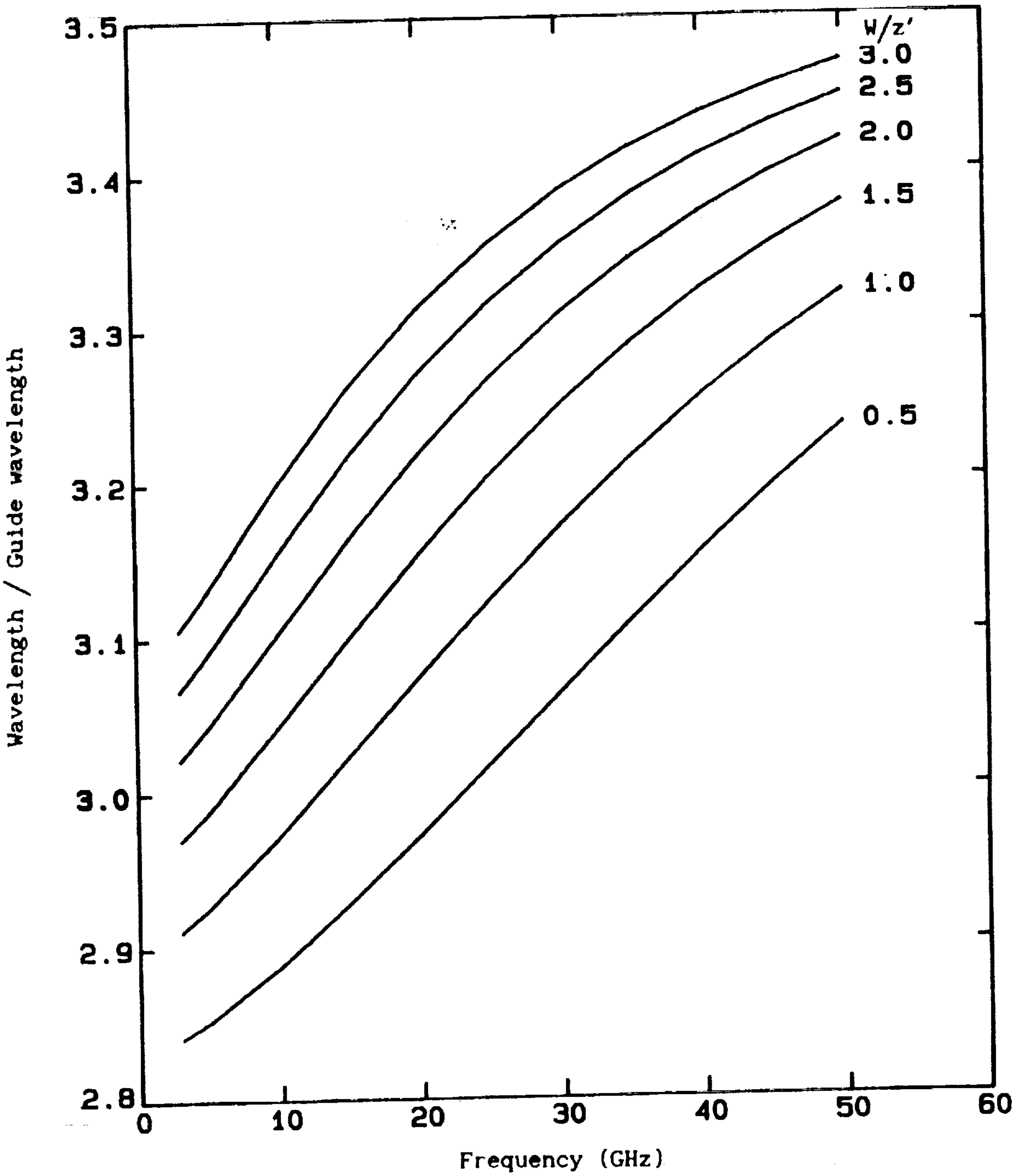

Figure 12a. Normalized guide wavelength of microstrip line as a function of frequency versus strip-width/strip-height. (ε_r = 12.8, z' = d = 0.06 cm).

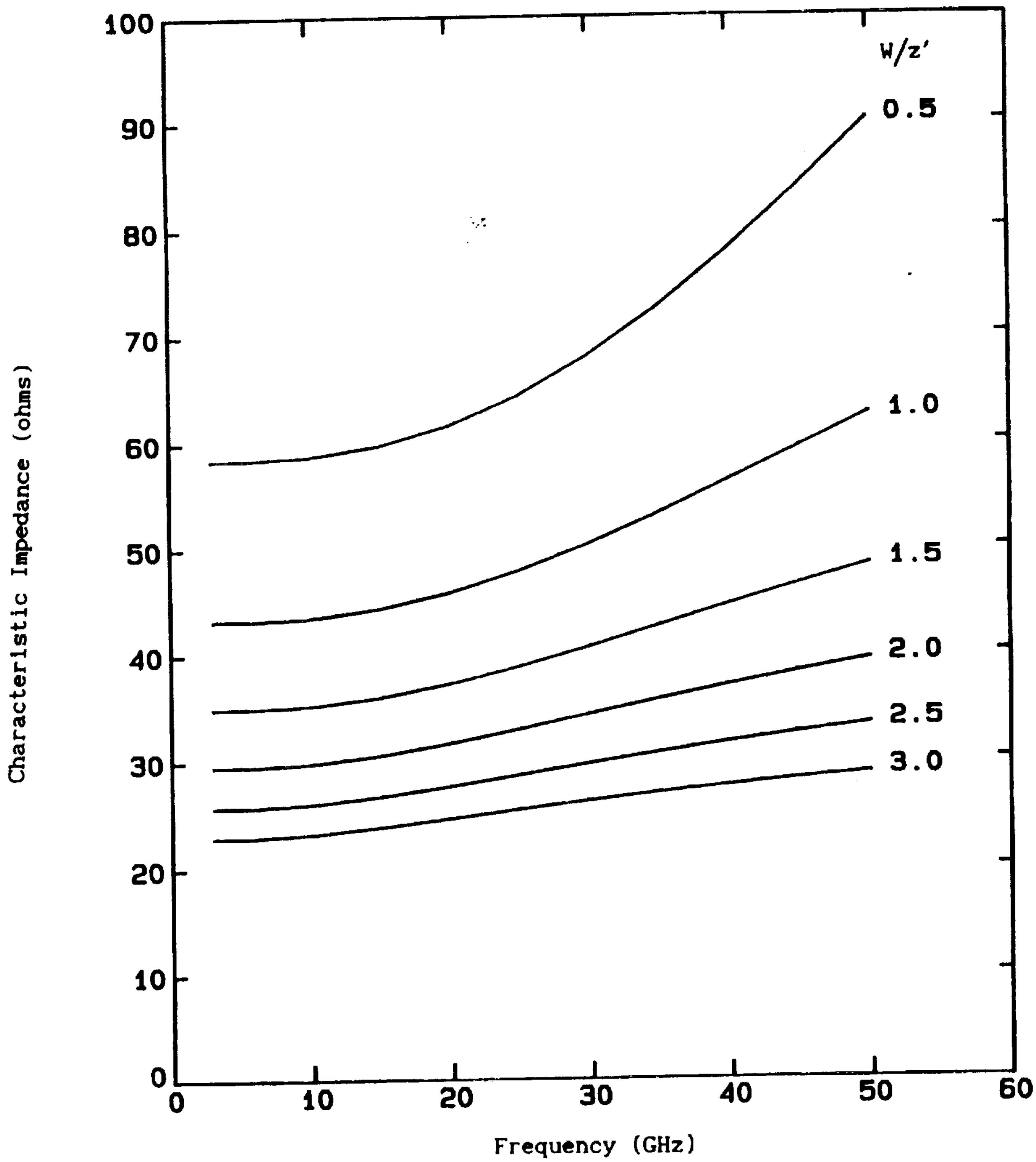

Figure 12b. Characteristic impedance of microstrip line as a function of frequency versus strip-width/strip-height. (ϵ_r = 12.8, z' = d = 0.06 cm).

Appendix
Instruction for using computer code

The main program MSTRIP.FOR calculates the characteristic impedance, Z_0, and the dominant mode propagation constant, K_e, of a microstrip line embedded in a grounded dielectric slab.

The main program must be linked to the following subroutines:

ZTEMP.FOR
SRMN.FOR
MATINV.FOR
MATROW.FOR
RSTRIP1.FOR
RSTRIP.FOR
MZO.FOR
DFKX.FOR

<u>INPUT DATA</u> :

NU	= Integer selecting the type of current distribution. 1 for pulse distribution. 2 for uniform distribution. 3 for nonuniform distribution.
MM	= Number of pulses assumed in the pulse distribution. (maximum number = 242).
ED	= Dielectric slab thickness in free space wavelengths.
EZP	= Height of strip above ground plane in free space wavelengths.
E	= Dielectric constant of slab.
E1	= Loss tangent of dielectric slab material.
WSTRIP	= Minimum microstrip width in free space wavelengths.
WMAX	= Maximum microstrip width in free space wavelengths.

<u>OUTPUT DATA</u> :

For pulse distribution, the transverse current distribution is output to file FOR010.DAT.

The characteristic impedance and dominant mode propagation constant as a function of microstrip width are output to file FOR013.DAT.

BRIEF OUTLINE OF METHOD :

The current distribution is assumed in the form;

$$J_x(x,y) = f(y)\exp(-JK_e x)$$

$$J_y(x,y) = 0.$$

For the above current distribution, the program calculates the dominant mode propagation constant and the characteristic impedance for one of three cases:

$$f(y) = I_0/W \qquad \text{(uniform)}$$

$$f(y) = \frac{2I_0}{\pi W\sqrt{1-(2y/W)^2}} \qquad \text{(nonuniform)}$$

$$f(y) = I_n P_n(y) \quad n=1,2,\cdots,MM \qquad \text{(pulses).}$$

For the distributions, the program calculates the K_e first by calling subroutine RSTRIP1. The subroutine RSTRIP1 solves equation 27 for K_e. RSTRIP1 uses standard technique to find zeros of a polynomial.

For the pulse distribution, the main program solves the matrix equation given in equation 36. This is done by computing mutual impedances by calling subroutine SRMN. The mutual impedance matrix is inverted using MATINV. I_n are found using MATROW. Equation 28 is then solved for K_e by using subroutine RSTRIP1.

The characteristic impedance for either distribution is computed by calling subroutine MZO.

```
      PROGRAM MSTRIP
C***********************************************************************
C**                                                                 ***
C**    This program calculates characteristic impedance of a        ***
C**    microstrip line embedded in a dielectric slab.               ***
C**                                                                 ***
C**    This program gives characteristic impedance of a micro-      ***
C**    strip line for three types of transverse current distri-     ***
C**    butions.  (1) Uniform, (2) Nonuniform with singularities     ***
C**    at the strip edges, (3) General distribution with pulse      ***
C**    approximations.                                              ***
C**                                                                 ***
C**    This program gives characteristic impedance of a micro-      ***
C**    strip line as a function of strip width.  It also gives      ***
C**    the transverse current distribution.                         ***
C**                                                                 ***
C***********************************************************************
      DIMENSION BTM(10),CZPAT(242),CZMN(242,242),CI(242)
     1,CZT1(242,242),CZT2(242,242),CZT3(242,242),CVT(242)
     2,CVT1(242),CI1(242),CI2(242)
      COMPLEX CJ,CJE1,CZPAT,CZMN,ZPATR,CF1,CZT1,CZT2,CZT3,CVT,CVT1
      COMMON CJ,CJE1,PI,TWOPI,PI02,DELZ,DELD,DELDZ,DELW,XX,B,E,E1
C***********************************************************************
C**                                                                 ***
C** ****************     INPUT DATA     *******************************
C**                                                                 ***
C**   NU = INTEGER  AND SELECTS TYPE OF CURRENT DISTRIBUTION IN     ***
C**        TRANSVERSE DIRECTION.                                    ***
C**                                                                 ***
C**   NU = 1 FOR PULSE DISTRIBUTION                                 ***
C**                                                                 ***
C**      = 2 FOR UNIFROM DISTRIBUTION                               ***
C**                                                                 ***
C**      = 3 FOR NON-UNIFORM DISTRIBUTION                           ***
C**                                                                 ***
C**      > or = 4 EXIT                                              ***
C**                                                                 ***
C**  MM = NUMBER OF PULSES FOR NU = 1                               ***
C**     = ANY NUMBER WHEN NU IS NOT EQUAL TO ONE.                   ***
C**       MAXIMUM VALUE OF MM MUST BE LESS THAN 242.                ***
C**                                                                 ***
C**  ED = DIELECTRIC SLAB THICKNESS IN WAVELENGTH                   ***
C**                                                                 ***
C**  EZP = HIGHT OF MICROSTRIP FROM GROUND IN WAVELENGTH            ***
C**        (EZP MUST BE LESS THAN ED)                               ***
C**                                                                 ***
C**  E  = DIELECTRIC CONSTANT OF SLAB.                              ***
C**                                                                 ***
C**  WSTRIP = WIDTH OF MICROSTRIP IN WAVELENGTH                     ***
C**                                                                 ***
C**  WMAX   = MAXIMUM WIDTH OF STRIP IN WAVELENGTH                  ***
C**                                                                 ***
C***********************************************************************
```

```
      WRITE(5,*) 'GIVE THE VALUE OF NU'
      WRITE(5,*)'FOR PULSE DISTRIBUTION NU=1, FOR UNIFORM DISTRIBUTION
     1NU=2, FOR NONUNIFORM DISTRIBUTION NU=3, NU GREATER THAN 3 EXIT.'
      READ(5,*)NU
      WRITE(5,*)'GIVE THE VALUE OF MM'
      WRITE(5,*)'MM = 1 TO 242'
      READ(5,*)MM
      WRITE(5,*)'GIVE THE VALUE OF DILECTRIC SLAB THICKNESS
     1 IN WAVELENGTH'
      READ(5,*)ED
      WRITE(5,*)'GIVE THE POSTION OF STRIP FROM GROUND IN
     1 WAVELENGTH'
      READ(5,*)EZP
      WRITE(5,*)'GIVE DIELECTRIC CONSTANT AND LOSS TANGENT
     1 OF SLAB MATERIAL'
      READ(5,*)E,E1
      WRITE(5,*)'GIVE STRIP WIDTH,  MAXIMUM STRIP WIDTH IN
     1 WAVELENGTH'
      READ(5,*)WSTRIP,WMAX
      IF(NU.GT.3)GO TO 105
      DELTX=0.001
      CJ=(0.,1.)
      PI=2.*ASIN(1.0)
      PI02=PI*0.5
      TWOPI=2.*PI
      CJE1=CJ*(E-CJ*E*E1-1.)
      SQRE=SQRT(E)
      WRITE(13,*)'CHARACTERISTIC IMPEDANCE, PROPAGATION CONSTANT'
      WRITE(13,*)'          OF MICROSTRIP LINE                 '
      WRITE(13,*)'SLAB THICKNESS ED = ',ED
      WRITE(13,*)'POSITION OF STRIP EZP =',EZP
      IF(NU.EQ.1)WRITE(13,*)'DISTRIBUTION PULSE WITH MM =',MM
      IF(NU.EQ.2)WRITE(13,*)'UNIFORM DISTRIBUTION'
      IF(NU.EQ.3)WRITE(13,*)'NONUNIFORM DISTRIBUTION'
      WRITE(13,*)'**STRIPWIDTH**','**PROPAGATION CONST.**', '**IMP.**'
 103  DELD=TWOPI*ED
      DELZ=TWOPI*EZP
      DELW=TWOPI*WSTRIP
      DELDZ=DELD-DELZ
      DELWY=DELW/MM
      B=0
C*****************************************************************
C**                                                          ****
C**      If dielectric constant of the slab = 1              ****
C**      then avoid surface wave  calculation.               ****
C**                                                          ****
C*****************************************************************
      IF(E.EQ.1.)GO TO 501
      NE=1
      NNMAX=1
      CALL ZTEMP(BTM,NE,NNMAX,E,DELD)
      B=BTM(1)
```

```
C*****************************************************************
C**                                                           ***
C**      If NU = 1 calculate mutual impedance matrix using    ***
C**      subroutine SRMN.  When NU not equal to 1 mutual      ***
C**      matrix is not required.                              ***
C**                                                           ***
C*****************************************************************
      IF(NU.NE.1)GO TO 502
 501  CALL SRMN(MM,CZPAT)
      DO 10 I=1,MM
      DO 10 J=1,MM
      IF(J.LT.I)GO TO 10
      CZMN(I,J)=CZPAT(J-I+1)
  10  CONTINUE
      DO 20 I=1,MM
      DO 20 J=1,MM
      IF(J.LE.I)GO TO 20
      CZMN(J,I)=CZMN(I,J)
  20  CONTINUE
      DO 30 I=1,MM
      DO 30 J=1,MM
      CZT1(I,J)=CZMN(I,J)
  30  CONTINUE
      EDK=SQRE*DELZ
      DKK=SQRE*DELD
      CF1=2.*CJ*SIN(EDK)*CEXP(CJ*DELZ)/(SQRE*COS(DKK)+CJ*SIN(DKK))
      DO 50 I=1,MM
      CVT(I)=-CF1
  50  CONTINUE
      CALL MATINV(CZT1,MM,CZT2)
      CALL MATROW(CZT2,MM,CVT,CVT1)
      S1=0.
      DO 301 I=1,MM
      FACT=CABS(CVT1(I))
      S1=S1+FACT
 301  CONTINUE
      DO 302 I=1,MM
      CVT1(I)=CVT1(I)/S1
 302  CONTINUE
      DO 303 I=1,MM
      CI(I)=CABS(CVT1(I))
 303  CONTINUE
C*********************************************************************
C**                                                               ***
C**   CI(I) is the current distribution in the transverse direction***
C**                                                               ***
C*********************************************************************
      WRITE(10,*) 'TRANSVERSE CURRENT DISTRIBUTION'
      WRITE(10,*) '******* Y/W ******','*******CURRENT AMP.****'
      DO 304 I=1,MM
      DWY=(2*I-1)
      DWY=DWY*DELWY*0.5/DELW
      WRITE(10,*)DWY,CI(I)
 304  CONTINUE
```

```
C********************************************************************
C**                                                              ***
C**   The transverse current distribution calculation for NU =1  ***
C**   is completed here.  The CI(I) as a function of DWY is      ***
C**   outputted at FOR010.DAT file.  The CI(I) is not computed   ***
C**   when NU is not equal to one.                               ***
C**                                                              ***
C********************************************************************
 502  NNMAX=1
C********************************************************************
C**                                                             ****
C**   For the current distribution CI(I) the SUBROUTINE RSTRIP1 ****
C**   calculates dominant mode propagation constant for the micro- *
C**   strip line.                                               ****
C**   RSTRIP1 solves equation (17) for Ke.                      ****
C**   RSTRIP1 must be linked to RTSRIP SUBROUTINUE              ****
C**                                                             ****
C********************************************************************
      CALL RSTRIP1(NU,NNMAX,MM,DELWY,DELD,DELDZ,DELZ,DELW,
     1E,E1,BTM,CI)
      XX=BTM(1)
      CALL MZO(NU,DELTX,ZPATR,CI,MM)
      WRITE(13,*)WSTRIP,XX,ZPATR
      WSTRIP=WSTRIP+0.005
      IF(WSTRIP.GT.WMAX)GO TO 104
      GO TO 103
 105  WRITE(5,*) 'CURRENT DISTRIBUTION NOT SELECTED PROPERLY'
 104  STOP
      END
```

```
      SUBROUTINE ZTEMP(BTM,NE,MAX,E,DK)
      DIMENSION BTM(1)
      SQRE=SQRT(E)
      DELTA=0.1
      B1=1.0
      B2=1.0+DELTA
      IF(B2.GT.SQRE)B2=SQRE
      BT=B2
      N=0
10    D1=SQRT(E-B1*B1)
      DK1=DK*D1
      S1=SIN(DK1)
      C1=COS(DK1)
      F1=E*SQRT(B1*B1-1.0)*C1-D1*S1
      D2=SQRT(ABS(E-B2*B2))
      DK2=DK*D2
      S2=SIN(DK2)
      C2=COS(DK2)
      F2=E*SQRT(B2*B2-1.0)*C2-D2*S2
      FT=F2
      IF(F1.LT.0.0.AND.F2.LT.0.0) GO TO 15
      IF(F1.GT.0.0.AND.F2.GT.0.0) GO TO 15
12    AF1=ABS(F1)
      AF2=ABS(F2)
      BP=B1+(B2-B1)*AF1/(AF1+AF2)
      IF(ABS(B2-B1).LE.1.E-04) GO TO 14
      DP=SQRT(E-BP*BP)
      DKP=DK*DP
      SP=SIN(DKP)
      CP=COS(DKP)
      FP=E*SQRT(BP*BP-1.0)*CP-DP*SP
      IF(ABS(FP).LE.1.E-04) GO TO 14
      IF(F1.LT.0.0.AND.FP.LT.0.0) GO TO 13
      IF(F1.GT.0.0.AND.FP.GT.0.0) GO TO 13
      B2=BP
      F2=FP
      GO TO 12
13    B1=BP
      F1=FP
      GO TO 12
14    N=N+1
      IF(N.GT.MAX) GO TO 17
      BTM(N)=BP
15    B1=BT
      IF(ABS(B1-SQRE).LE.1.E-05) GO TO 16
      B2=B1+DELTA
      IF(B2.GT.SQRE)B2=SQRE
      BT=B2
      GO TO 10
16    NE=N
17    RETURN
      END
```

```
      SUBROUTINE SRMN(MM,ZPATR)
      DIMENSION U1(3),U2(10),R1(3),R2(10),U(13),R(13)
      DIMENSION BTM(1),SUMRR(242),ZPATR(242)
      EQUIVALENCE (U1(1),U(1)),(U2(1),U(4)),(R1(1),R(1)),(R2(1),R(4))
      COMMON CJ,CJE1,PI,TWOPI,PI02,DELZ,DELD,DELDZ,DELW,XX,B,E,E1
      COMPLEX CJ,CJE1,CF1,SUMRR,ZPATR,CCFA
      DATA U1/0.11270166537925,.5,0.88729833462074/,U2/.01304673574141
     A,.06746831665550,.16029521585048,.28330230293537,.42556283050918
     B,.57443716949081,.71669769706462,.83970478414951,.93253168334449
     C,.98695326425858/,R1/.27777777777777,.44444444444444,.2777777777
     D7777/,R2/.03333567215434,.07472567457529,.10954318125799,.134633
     E35965499,.14776211235737,.14776211235737,.13463335965499,.109543
     F18125799,.07472567457529,.03333567215434/
      DELWY=DELW/MM
      SQRE=SQRT(E)
      DWKO2=0.5*DELWY
      CONST=377.7*DELZ*DELWY/PI
      DO 201 I=1,MM
      SUMRR(I)=(0.,0.)
      ZPATR(I)=(0.,0.)
201   CONTINUE
      NF=1
      NQ=5
      XX1=0.0001
      XX2=1.-0.0001
  100 DELT=(XX2-XX1)/FLOAT(NQ)
      DO 80 K=1,NQ
      XI=K-1
      FF=XX1+XI*DELT
      DO 80 JL=4,13
      UU=U(JL)*DELT+FF
      BSQ=UU*UU
      BET=UU
      BETA=UU
      DWKB2=BETA*DWKO2
      AKX1=BSQ
      IF(E.EQ.1.)GO TO 501
      IF(AKX1-1.)10,10,12
10    B1=SQRT(1.-AKX1)
      BE=SQRT(E-AKX1)
      AZ=BE*DELZ
      AD=DELD*BE
      ADZ=DELDZ*BE
      SINZ=SIN(AZ)
      COSD=COS(AD)
      SIND=SIN(AD)
      COSDZ=COS(ADZ)
      SINDZ=SIN(ADZ)
      SINZ1=1.0
      IF(ABS(AZ).LT.1.E-05)GO TO 21
      SINZ1=SINZ/AZ
21    CF1=SINZ1*(BE*COSDZ+CJ*B1*SINDZ)/(BE*COSD+CJ*B1*SIND)
      GO TO 20
12    IF(AKX1-E)14,14,15
```

```
14    B1=SQRT(AKX1-1.)
      BE=SQRT(E-AKX1)
      AZ=BE*DELZ
      AD=BE*DELD
      ADZ=BE*DELDZ
      SINZ=SIN(AZ)
      COSD=COS(AD)
      SIND=SIN(AD)
      COSDZ=COS(ADZ)
      SINDZ=SIN(ADZ)
      SINZ1=1.
      IF(ABS(AZ).LT.1.E-05)GO TO 22
      SINZ1=SINZ/AZ
22    CF1=SINZ1*(BE*COSDZ+B1*SINDZ)/(BE*COSD+B1*SIND)
      GO TO 20
15    B1=SQRT(AKX1-1.)
      BE=SQRT(AKX1-E)
      AZ=DELZ*BE
      AD=DELD*BE
      ADZ=DELDZ*BE
      EP1=0.0
      IF(ABS(AZ).GT.20.)GO TO 23
      EP1=EXP(-2.*AZ)
23    EPD=0.
      IF(ABS(AD).GT.20.)GO TO 24
      EPD=EXP(-2.*AD)
24    EPDZ=0.
      IF(ABS(ADZ).GT.20.)GO TO 25
      EPDZ=EXP(-2.*ADZ)
25    CF1=(1.-EP1)*(BE*(1.+EPDZ)+B1*(1.-EPDZ))
      CF1=CF1/(BE*(1.+EPD)+B1*(1.-EPD))
      CF1=CF1/(2.*AZ)
      GO TO 20
501   IF(AKX1-1.)502,502,503
502   BE=SQRT(1.-AKX1)
      AZ=BE*DELZ
      SINZ=SIN(AZ)
      IF(ABS(AZ).LT.1.E-05)GO TO 504
      SINZ1=SINZ/AZ
504   CF1=SINZ1*(COS(AZ)-CJ*SINZ)
      GO TO 20
503   BE=SQRT(AKX1-1.)
      AZ=BE*DELZ
      EZ=0.
      IF(ABS(AZ).GT.20.)GO TO 505
      EZ=EXP(-2.*AZ)
505   CF1=(1.-EZ)/(2.*AZ)
20    DWKA2=DWKB2
      F1=1.0
      IF(ABS(DWKA2).LT.1.E-05)GO TO 301
      F1=SIN(DWKA2)/DWKA2
301   CCFA=-CJ*CONST*CF1
      CCFA=CCFA*F1*F1
      DO 202 I=1,MM
```

```
      SUMRR(I)=SUMRR(I)+CCFA*R(JL)*COS(2.*DWKA2*(1-I))
202   CONTINUE
80    CONTINUE
      DO 203 I=1,MM
      SUMRR(I)=SUMRR(I)*DELT
203   CONTINUE
      DO 204 I=1,MM
      ZPATR(I)=ZPATR(I)+SUMRR(I)
204   CONTINUE
      DO 205 I=1,MM
      SUMRR(I)=(0.,0.)
205   CONTINUE
      CPOLE=0.0001
      NF=NF+1
      IF(NF.EQ.2)XX1=1.+CPOLE
      IF(NF.EQ.2)XX2=SQRE-CPOLE
      IF(NF.EQ.2)GO TO 100
      IF(NF.EQ.3)XX1=SQRE+CPOLE
      IF(NF.EQ.3)XX2=XX1+1.
      IF(NF.EQ.3)GO TO 100
      IF(XX2.LT.5.0.AND.XX2.GT.0.)DINC=1.0
      IF(XX2.LT.25.0.AND.XX2.GT.5.0)DINC=5.
      IF(XX2.LT.100.0.AND.XX2.GT.25.0)DINC=25.0
      IF(XX2.GT.100.0)DINC=100.0
      XX1=XX2
      XX2=XX1+DINC
      IF(XX2.LT.2000.0)GO TO 100
      RETURN
      END
```

```
      SUBROUTINE MATINV(SUM1,N,SUM2)
      DIMENSION IST(242),JST(242)
      COMPLEX SUM1(242,242),SUM2(242,242)
      DO 1 I=1,N
      IST(I)=0
    1 JST(I)=0
      DO 30 IND=1,N
      H=0.0
      DO 10 I=1,N
      DO 3 KCH=1,N
      IF(IST(KCH).EQ.I)GO TO 10
    3 CONTINUE
      DO 9 J=1,N
      DO 4 KCH=1,N
      IF(JST(KCH).EQ.J) GO TO 9
    4 CONTINUE
      IF(H.GE.CABS(SUM1(I,J))) GO TO 9
      H=CABS(SUM1(I,J))
      IM=I
      JM=J
    9 CONTINUE
   10 CONTINUE
      IST(IND)=IM
      JST(IND)=JM
      DO 20 I=1,N
      DO 20 J=1,N
      IF(I-IM)5,6,5
    6 SUM2(I,J)=-SUM1(I,J)/SUM1(IM,JM)
      GO TO 20
    5 IF(J-JM)7,8,7
    8 SUM2(I,J)=SUM1(I,J)/SUM1(IM,JM)
      GO TO 20
    7 SUM2(I,J)=SUM1(I,J)-SUM1(I,JM)*SUM1(IM,J)/SUM1(IM,JM)
   20 CONTINUE
      SUM2(IM,JM)=1./SUM1(IM,JM)
      DO 25 I=1,N
      DO 25 J=1,N
   25 SUM1(I,J)=SUM2(I,J)
   30 CONTINUE
      DO 35 I=1,N
      IF(IST(I).EQ.JST(I))GO TO 35
      DO 45 J=1,N
   45 SUM2(JST(I),J)=SUM1(IST(I),J)
   35 CONTINUE
      DO 26 I=1,N
      DO 26 J=1,N
   26 SUM1(I,J)=SUM2(I,J)
      DO 50 I=1,N
      IF(IST(I).EQ.JST(I))GO TO 50
      DO 55 J=1,N
   55 SUM2(J,IST(I))=SUM1(J,JST(I))
   50 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE MATROW(SUM2,N,SUM,SUM5)
      COMPLEX SUM2(242,242),SUM(242),SUM5(242)
      DO 110 I=1,N
      SUM5(I)=(0.,0.)
110   CONTINUE
      DO 120 I=1,N
      DO 120 J=1,N
      SUM5(I)=SUM2(I,J)*SUM(J)+SUM5(I)
120   CONTINUE
      RETURN
      END
```

```
      SUBROUTINE RSTRIP1(NU,NNMAX,MM,DELWY,DK,DZP,ZPK,DWSTRIP,
     1E,E1,BTM,CI)
      DIMENSION BTM(1),CI(242)
      COMPLEX ZPATRR
      SQRE=SQRT(E)
      N=0
      DELTA=0.1
      B1=SQRE
      B2=B1-DELTA
10    CALL RSTRIP(NU,MM,DELWY,DK,DZP,ZPK,DWSTRIP,E,E1,B1,ZPATRR,CI)
      F1=REAL(ZPATRR)
      CALL RSTRIP(NU,MM,DELWY,DK,DZP,ZPK,DWSTRIP,E,E1,B2,ZPATRR,CI)
      F2=REAL(ZPATRR)
      FT=F2
      IF(F1.LT.0.0.AND.F2.LT.0.0) GO TO 15
      IF(F1.GT.0.0.AND.F2.GT.0.0) GO TO 15
12    AF1=ABS(F1)
      AF2=ABS(F2)
      BP=B1-(B1-B2)*AF1/(AF1+AF2)
      IF(ABS(B2-B1).LE.1.E-04) GO TO 14
      CALL RSTRIP(NU,MM,DELWY,DK,DZP,ZPK,DWSTRIP,E,E1,BP,ZPATRR,CI)
      FP=REAL(ZPATRR)
      IF(ABS(FP).LE.1.E-04) GO TO 14
      IF(F1.LT.0.0.AND.FP.LT.0.0) GO TO 13
      IF(F1.GT.0.0.AND.FP.GT.0.0) GO TO 13
      B2=BP
      F2=FP
      GO TO 12
13    B1=BP
      F1=FP
      GO TO 12
14    IF(ABS(FP).LT.1.E-04)GO TO 19
      GO TO 15
19    N=N+1
      BTM(N)=BP
      IF(N.GE.1)GO TO 17
15    B1=B2
      B2=B1-DELTA
      GO TO 10
16    NE=N
17    RETURN
      END
```

```
      SUBROUTINE RSTRIP(NU,MM,DELWY,DK,DZP,ZPK,DWSTRIP,E,E1,
     1                  XX,ZPATR,CI)
      DIMENSION U1(3),U2(10),R1(3),R2(10),U(13),R(13),BTM(1),CI(242)
      EQUIVALENCE (U1(1),U(1)),(U2(1),U(4)),(R1(1),R(1)),(R2(1),R(4))
      COMPLEX CJ,CJE1,CF1,CF2,SUMRR,ZPATR,CCFA
      DATA U1/0.11270166537925,.5,0.88729833462074/,U2/.01304673574141
     A,.06746831665550,.16029521585048,.28330230293537,.42556283050918
     B,.57443716949081,.71669769706462,.83970478414951,.93253168334449
     C,.98695326425858/,R1/.27777777777777,.44444444444444,.2777777777
     D7777/,R2/.03333567215434,.07472567457529,.10954318125799,.134633
     E35965499,.14776211235737,.14776211235737,.13463335965499,.109543
     F18125799,.07472567457529,.03333567215434/
      CJ=(0.,1.)
      PI=2.*ASIN(1.0)
      TWOPI=2.*PI
      PIO2=PI/2.
      SQRE=SQRT(E)
      DWKO2=0.5*DWSTRIP
      DWKY2=0.5*DELWY
      CJE1=CJ*(E-CJ*E1*E-1.)
      SUMRR=(0.,0.)
      ZPATR=(0.,0.)
      CONTINUE
      NF=1
      NQ=5
      XX1=0.0001
      XX2=1.-0.0001
  100 DELT=(XX2-XX1)/FLOAT(NQ)
      DO 80 K=1,NQ
      XI=K-1
      FF=XX1+XI*DELT
      DO 80 JL=4,13
      UU=U(JL)*DELT+FF
      BSQ=UU*UU
      BET=UU
      BETA=UU
      DWKB2=BETA*DWKO2
      DYKB2=BETA*DWKY2
      AKX1=XX*XX+BSQ
      IF(AKX1-1.)10,10,12
10    B1=SQRT(1.-AKX1)
      BE=SQRT(E-AKX1)
      AZ=BE*ZPK
      AD=DK*BE
      ADZ=DZP*BE
      SINZ=SIN(AZ)
      COSD=COS(AD)
      SIND=SIN(AD)
      COSDZ=COS(ADZ)
      SINDZ=SIN(ADZ)
      SINZ1=1.0
      IF(ABS(AZ).LT.1.E-05)GO TO 21
      SINZ1=SINZ/AZ
21    CF1=SINZ1*(BE*COSDZ+CJ*B1*SINDZ)/(BE*COSD+CJ*B1*SIND)
```

```
      CF2=SINZ1*SINZ*BE*BE/(BE*COSD+CJ*B1*SIND)
      CF2=CF2/(E*B1*COSD+CJ*BE*SIND)
      GO TO 20
 12   IF(AKX1-E)14,14,15
 14   B1=SQRT(AKX1-1.)
      BE=SQRT(E-AKX1)
      AZ=BE*ZPK
      AD=BE*DK
      ADZ=BE*DZP
      SINZ=SIN(AZ)
      COSD=COS(AD)
      SIND=SIN(AD)
      COSDZ=COS(ADZ)
      SINDZ=SIN(ADZ)
      SINZ1=1.
      IF(ABS(AZ).LT.1.E-05)GO TO 22
      SINZ1=SINZ/AZ
 22   CF1=SINZ1*(BE*COSDZ+B1*SINDZ)/(BE*COSD+B1*SIND)
      CF2=SINZ1*SINZ*BE*BE/(BE*COSD+B1*COSD)
      CF2=CJ*CF2/(B1*E*COSD-BE*SIND)
      GO TO 20
 15   B1=SQRT(AKX1-1.)
      BE=SQRT(AKX1-E)
      AZ=ZPK*BE
      AD=DK*BE
      ADZ=DZP*BE
      EP1=0.0
      IF(ABS(AZ).GT.20.)GO TO 23
      EP1=EXP(-2.*AZ)
 23   EPD=0.
      IF(ABS(AD).GT.20.)GO TO 24
      EPD=EXP(-2.*AD)
 24   EPDZ=0.
      IF(ABS(ADZ).GT.20.)GO TO 25
      EPDZ=EXP(-2.*ADZ)
 25   CF1=(1.-EP1)*(BE*(1.+EPDZ)+B1*(1.-EPDZ))
      CF1=CF1/(BE*(1.+EPD)+B1*(1.-EPD))
      CF1=CF1/(2.*AZ)
      CF2=-CJ*BE*BE*(1.-EP1)*(1.-EP1)*EPDZ/AZ
      CF2=CF2/(BE*(1.+EPD)+B1*(1.-EPD))
      CF2=CF2/(E*B1*(1.+EPD)+BE*(1.-EPD))
 20   DWKA2=DWKB2
      IF(NU.NE.1)GO TO 501
      F11=SIN(DYKB2)
      F1=1.
      IF(ABS(DWKA2).LT.1.E-05)GO TO 301
      F1=SIN(DWKA2)/DWKA2
 301  F2=1.
      IF(ABS(DYKB2).LT.1.E-05)GO TO 302
      F2=F11/DYKB2
 302  GO TO 508
 501  F3=0.5*DWKA2
      IF(ABS(F3).LT.1.E-05)GO TO 506
      F3=SIN(F3)/F3
```

```
506  F4=1.
     IF(ABS(DWKA2).LT.1.E-05)GO TO 507
     F4=SIN(DWKA2)/DWKA2
507  F1=1.
     F2=1.
     IF(ABS(DWKA2).LT.1.E-05)GO TO 503
     IF(NU.EQ.2)F2=F4
     IF(NU.EQ.3)CALL BJOR(DWKA2,F2,IERR)
     IF(NU.EQ.4)F2=2.*F4-F3*F3
503  F2=F2*F2
508  CCFA=((E-XX*XX)*CF1-CJE1*XX*XX*CF2)
     CCFA=CCFA*F1*F2
     IF(NU.NE.1)GO TO 504
     S1=0.
     DO 303 I=1,MM
     XARG=MM/2
     XARG1=I
     XARG2=0.5
     XARG=XARG-(XARG1-XARG2)
     FACT=CI(I)*COS(2.*DYKB2*XARG)
     S1=S1+FACT
303  CONTINUE
     GO TO 505
504  S1=1.
505  SUMRR=SUMRR+CCFA*R(JL)*S1
80   CONTINUE
     SUMRR=SUMRR*DELT
     ZPATR=ZPATR+SUMRR
     SUMRR=(0.,0.)
     CPOLE=0.0001
     NF=NF+1
     IF(NF.EQ.2)XX1=1.+CPOLE
     IF(NF.EQ.2)XX2=SQRE-CPOLE
     IF(NF.EQ.2)GO TO 100
     IF(NF.EQ.3)XX1=SQRE+CPOLE
     IF(NF.EQ.3)XX2=XX1+1.
     IF(NF.EQ.3)GO TO 100
     IF(XX2.LT.5.0.AND.XX2.GT.0.)DINC=1.0
     IF(XX2.LT.25.0.AND.XX2.GT.5.0)DINC=5.
     IF(XX2.LT.100.0.AND.XX2.GT.25.0)DINC=25.0
     IF(XX2.GT.100.0)DINC=100.0
     XX1=XX2
     XX2=XX1+DINC
     IF(XX2.LT.1000.0)GO TO 100
     RETURN
     END
```

```
      SUBROUTINE MZO(NU,DELTX,ZPATR,CI,MM)
C***********************************************************************
C**   This program computes the characteristic impedance of micro-  **
C**   line.  This program calls subroutine DFKX to calculate        **
C**   derivatives of the integrand appearing in equations (12),     **
C**   (13), and (14).                                               **
C**                                                                 **
C**                                                                 **
C**   INPUT PARAMETERS:                                             **
C**                                                                 **
C**   NU = 1 FOR PULSE DISTRIBUTION                                 **
C**      = 2 FOR UNIFORM DISTRIBUTION                               **
C**      = 3 FOR NONUNIFORM DISTRIBUTION                            **
C**                                                                 **
C**   DELTX = A CONSTANT IN THE RANGE 0.01-0.001                    **
C**                                                                 **
C**   CI = CURRENT DISTRIBUTION AMP.                                **
C**         IT IS ZERO IF NU=2 OR NU=3                              **
C**                                                                 **
C**    MM = NUMBER OF PULES                                         **
C**                                                                 **
C**   OUTPUT PARAMETERS:                                            **
C**                                                                 **
C**   ZPATR = CHARACTERISTIC IMPEDANCE OF MICROSTRIP LINE           **
C**                                                                 **
C***********************************************************************
      DIMENSION U1(3),U2(10),R1(3),R2(10),U(13),R(13)
      DIMENSION BTM(1),CI(242)
      EQUIVALENCE (U1(1),U(1)),(U2(1),U(4)),(R1(1),R(1)),(R2(1),R(4))
      COMMON CJ,CJE1,PI,TWOPI,PIO2,DELZ,DELD,DELDZ,DELW,XX,B,E,E1
      COMPLEX CJ,CJE1,CF1,CF2,SUMRR,ZPATR,CCFA,DF
      DATA U1/0.11270166537925,.5,0.88729833462074/,U2/.01304673574141
     A,.06746831665550,.16029521585048,.28330230293537,.42556283050918
     B,.57443716949081,.71669769706462,.83970478414951,.93253168334449
     C,.98695326425858/,R1/.27777777777777,.44444444444444,.2777777777
     D7777/,R2/.03333567215434,.07472567457529,.10954318125799,.134633
     E35965499,.14776211235737,.14776211235737,.13463335965499,.109543
     F18125799,.07472567457529,.03333567215434/
      CPOLE=0.0001
      SQRE=SQRT(E)
      DELWY=DELW/MM
      DWKO2=0.5*DELW
      DYKO2=0.5*DELWY
      CONST=-377.7*DELZ/(PI*E)
      SUMRR=(0.,0.)
      ZPATR=(0.,0.)
      NF=1
      NQ=5
      XX1=0.0001
      XX2=1.-0.0001
  100 DELT=(XX2-XX1)/FLOAT(NQ)
      DO 80 K=1,NQ
      XI=K-1
      FF=XX1+XI*DELT
```

```
      DO 80 JL=4,13
      UU=U(JL)*DELT+FF
      BSQ=UU*UU
      BET=UU
      BETA=UU
      DWKB2=BETA*DWKO2
      DYKB2=BETA*DYKO2
      CALL DFKX(XX,DELTX,DELD,DELDZ,DELZ,E,CJ,CJE1,BETA,DF)
      AKX1=XX*XX+BSQ
      IF(AKX1-1.)10,10,12
10    B1=SQRT(1.-AKX1)
      BE=SQRT(E-AKX1)
      AD=DELD*BE
      AZ=DELZ*BE
      ADZ=DELDZ*BE
      SINZ=SIN(AZ)
      SIND=SIN(AD)
      COSD=COS(AD)
      SINDZ=SIN(ADZ)
      COSDZ=COS(ADZ)
      SINZ1=1.
      IF(ABS(AZ).LT.1.E-05)GO TO 201
      SINZ1=SINZ/AZ
201   CF1=SINZ1*(BE*COSDZ+CJ*B1*SINDZ)
      CF1=CF1/(BE*COSD+CJ*B1*SIND)
      CF2=BE*BE*SINZ1*SINZ/(BE*COSD+CJ*B1*SIND)
      CF2=CF2/(E*B1*COSD+CJ*BE*SIND)
      GO TO 20
12    IF(AKX1-E)14,14,18
14    B1=SQRT(AKX1-1.)
      BE=SQRT(E-AKX1)
      AD=DELD*BE
      AZ=DELZ*BE
      ADZ=DELDZ*BE
      SINZ=SIN(AZ)
      SIND=SIN(AD)
      COSD=COS(AD)
      SINDZ=SIN(ADZ)
      COSDZ=COS(ADZ)
      SINZ1=1.
      IF(ABS(AZ).LT.1.E-05)GO TO 202
      SINZ1=SINZ/AZ
202   CF1=SINZ1*(BE*COSDZ+B1*SINDZ)/(BE*COSD+B1*SIND)
      CF2=CJ*BE*BE*SINZ1*SINZ/(BE*COSD+B1*SIND)
      CF2=CF2/(E*B1*COSD-BE*SIND)
      GO TO 20
18    B1=SQRT(AKX1-1.)
      BE=SQRT(AKX1-E)
      AD=DELD*BE
      AZ=DELZ*BE
      ADZ=DELDZ*BE
      AZ2=2.*AZ
      AD2=2.*AD
      ADZ2=2.*ADZ
```

```
      EZ=0.
      IF(ABS(AZ2).GT.15.)GO TO 203
      EZ=EXP(-AZ2)
203   EDD=0.
      IF(ABS(AD2).GT.15.)GO TO 204
      EDD=EXP(-AD2)
204   EDZ=0.
      IF(ABS(ADZ2).GT.15.)GO TO 205
      EDZ=EXP(-ADZ2)
205   CF1=(1.-EZ)*(BE*(1.+EDZ)+B1*(1.-EDZ))
      CF1=CF1/(BE*(1.+EDD)+B1*(1.-EDD))
      CF1=CF1/AZ2
      CF2=BE*BE*(1.-EZ)*(1.-EZ)*EDZ/AZ
      CF2=-CJ*CF2/(BE*(1.+EDD)+B1*(1.-EDD))
      CF2=CF2/(E*B1*(1.+EDD)+BE*(1.-EDD))
20    IF(NU.NE.1)GO TO 501
      S1=0.
      DO 302 I=1,MM
      DO 302 J=1,MM
      XARG=(I-J)
      FACT=CI(I)*CI(J)*COS(2.*DYKB2*XARG)
      S1=S1+FACT
302   CONTINUE
      F2=1.
      IF(ABS(DYKB2).LT.1.E-05)GO TO 303
      F2=SIN(DYKB2)/DYKB2
303   GO TO 502
501   S1=1.
      F3=0.5*DWKB2
      IF(ABS(F3).LT.1.E-05)GO TO 507
      F3=SIN(F3)/F3
507   F2=1.
      IF(ABS(DWKB2).LT.1.E-05)GO TO 508
      F2=SIN(DWKB2)/DWKB2
508   IF(NU.EQ.2)F2=F2
      IF(NU.EQ.3)CALL BJOR(DWKB2,F2,IERR)
      IF(NU.EQ.4)F2=2.*F2-F3*F3
502   CCFA=((E-XX*XX)*CF1-CJE1*XX*XX*CF2-XX*DF)/XX
      CCFA=CONST*DF*F2*F2*S1
      SUMRR=SUMRR+CCFA*R(JL)
80    CONTINUE
      SUMRR=SUMRR*DELT
      ZPATR=ZPATR+SUMRR
      SUMRR=(0.,0.)
      NF=NF+1
      IF(NF.EQ.2)XX1=1.+CPOLE
      IF(NF.EQ.2)XX2=B-CPOLE
      IF(NF.EQ.2)GO TO 100
      IF(NF.EQ.3)XX1=B+CPOLE
      IF(NF.EQ.3)XX2=SQRE-CPOLE
      IF(NF.EQ.3)GO TO 100
      IF(NF.EQ.4)XX1=SQRE+CPOLE
      IF(NF.EQ.4)XX2=XX2+1.0
      IF(NF.EQ.4)GO TO 100
```

```
IF(XX2.LT.5.0.AND.XX2.GT.0.)DINC=1.0
IF(XX2.LT.25.0.AND.XX2.GT.5.0)DINC=5.
IF(XX2.LT.100.0.AND.XX2.GT.25.0)DINC=25.0
IF(XX2.GT.100.0)DINC=100.0
XX1=XX2
XX2=XX1+DINC
IF(XX2.LT.3000.0)GO TO 100
ZPATR=ZPATR/4.
RETURN
END
```

```
      SUBROUTINE DFKX(XX,DELTX,DK,DZP,ZPK,E,CJ,CJE1,BETA,DF)
C*********************************************************************
C**                                                                 **
C**   This program calculates the derivative of integrand          **
C**   appearing in equations (12), (13), and (14).                 **
C**                                                                 **
C*********************************************************************
      DIMENSION DF1(2)
      COMPLEX DF,DF1
      COMPLEX CJ,CJE1,CF1,CF2
      AKX1=XX-DELTX
      I=1
      BSQ=BETA*BETA
  100 AB1=BSQ+AKX1*AKX1
      IF(AB1-1.)10,10,12
   10 B1=SQRT(1.-AB1)
      BE1=SQRT(E-AB1)
      AD=BE1*DK
      AZ=BE1*ZPK
      ADZ=BE1*ADZ
      SINZ=SIN(AZ)
      SIND=SIN(AD)
      COSD=COS(AD)
      SINDZ=SIN(ADZ)
      COSDZ=COS(ADZ)
      SINZ1=1.0
      IF(ABS(AZ).LT.1.E-05)GO TO 21
      SINZ1=SINZ/AZ
   21 CF1=SINZ1*(BE1*COSDZ+CJ*B1*SINDZ)/(BE1*COSD+CJ*B1*SIND)
      CF2=SINZ1*SINZ*BE1*BE1/(BE1*COSD+CJ*B1*SIND)
      CF2=CF2/(E*B1*COSD+CJ*BE1*SIND)
      GO TO 20
   12 IF(AB1-E)14,14,15
   14 B1=SQRT(AB1-1.)
      BE1=SQRT(E-AB1)
      AD=BE1*DK
      AZ=BE1*ZPK
      ADZ=BE1*DZP
      SINZ=SIN(AZ)
      SIND=SIN(AD)
      COSD=COS(AD)
      SINDZ=SIN(ADZ)
      COSDZ=COS(ADZ)
      SINZ1=1.
      IF(ABS(AZ).LT.1.E-05)GO TO 22
      SINZ1=SINZ/AZ
   22 CF1=SINZ1*(BE1*COSDZ+B1*SINDZ)/(BE1*COSD+B1*SIND)
      CF2=SINZ1*SINZ*BE1*BE1/(BE1*COSD+B1*SIND)
      CF2=CJ*CF2/(E*B1*COSD-BE1*SIND)
      GO TO 20
   15 B1=SQRT(AB1-1.)
      BE1=SQRT(AB1-E)
      AZ=BE1*ZPK
      AD=BE1*DK
```

```
      ADZ=BE1*DZP
      EP1=0.0
      IF(ABS(AZ).GT.20.)GO TO 23
      EP1=EXP(-2.*AZ)
23    EPD=0.0
      IF(ABS(AD).GT.20.)GO TO 24
      EPD=EXP(-2.*AD)
24    EPDZ=0.0
      IF(ABS(ADZ).GT.20.)GO TO 25
      EPDZ=EXP(-2.*ADZ)
25    CF1=(1.-EP1)*(BE1*(1.+EPDZ)+B1*(1.-EPDZ))
      CF1=CF1/(BE1*(1.+EPD)+B1*(1.-EPD))
      CF1=CF1/(2.*AZ)
      CF2=-CJ*BE1*BE1*(1.-EP1)*(1.-EP1)*EPDZ/AZ
      CF2=CF2/(BE1*(1.+EPD)+B1*(1.-EPD))
      CF2=CF2/(E*B1*(1.+EPD)+BE1*(1.-EPD))
20    DF1(I)=(E-AKX1*AKX1)*CF1-CJE1*AKX1*AKX1*CF2
      AKX1=XX+DELTX
      I=I+1
      IF(I.LE.2)GO TO 100
      DF=(DF1(2)-DF1(1))/DELTX
      RETURN
      END
```

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Report Documentation Page

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-101581		

4. Title and Subtitle

Characteristic Impedance of Microstrip Lines

5. Report Date

November 1989

6. Performing Organization Code

7. Author(s)

M. C. Bailey and M. D. Deshpande

8. Performing Organization Report No.

10. Work Unit No.

506-44-21-03

9. Performing Organization Name and Address

NASA Langley Research Center
Hampton, VA 23665-5225

11. Contract or Grant No.

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, D. C. 20546-0001

13. Type of Report and Period Covered

Technical Memorandum

14. Sponsoring Agency Code

15. Supplementary Notes

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M. D. Deshpande: Vigyan Research Associates, Inc., Hampton, Virginia.

16. Abstract

The dyadic Green's function for a current embedded in a grounded dielectric slab is used to analyze microstrip lines at millimeter wave frequencies. The dyadic Green's function accounts accurately for fringing fields and dielectric cover over the microstrip line. Using Rumsey's reaction concept an expression for the characteristic impedance is obtained. The numerical results are compared with the results reported earlier.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
microstrip impedance	Unclassified-Unlimited Subject Category 32

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	58	A04

NASA FORM 1626 OCT 86