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MULTIPLE PATHS TO SUBHARMONIC LAMINAR BREAKDOWN IN A BOUNDARY LAYER

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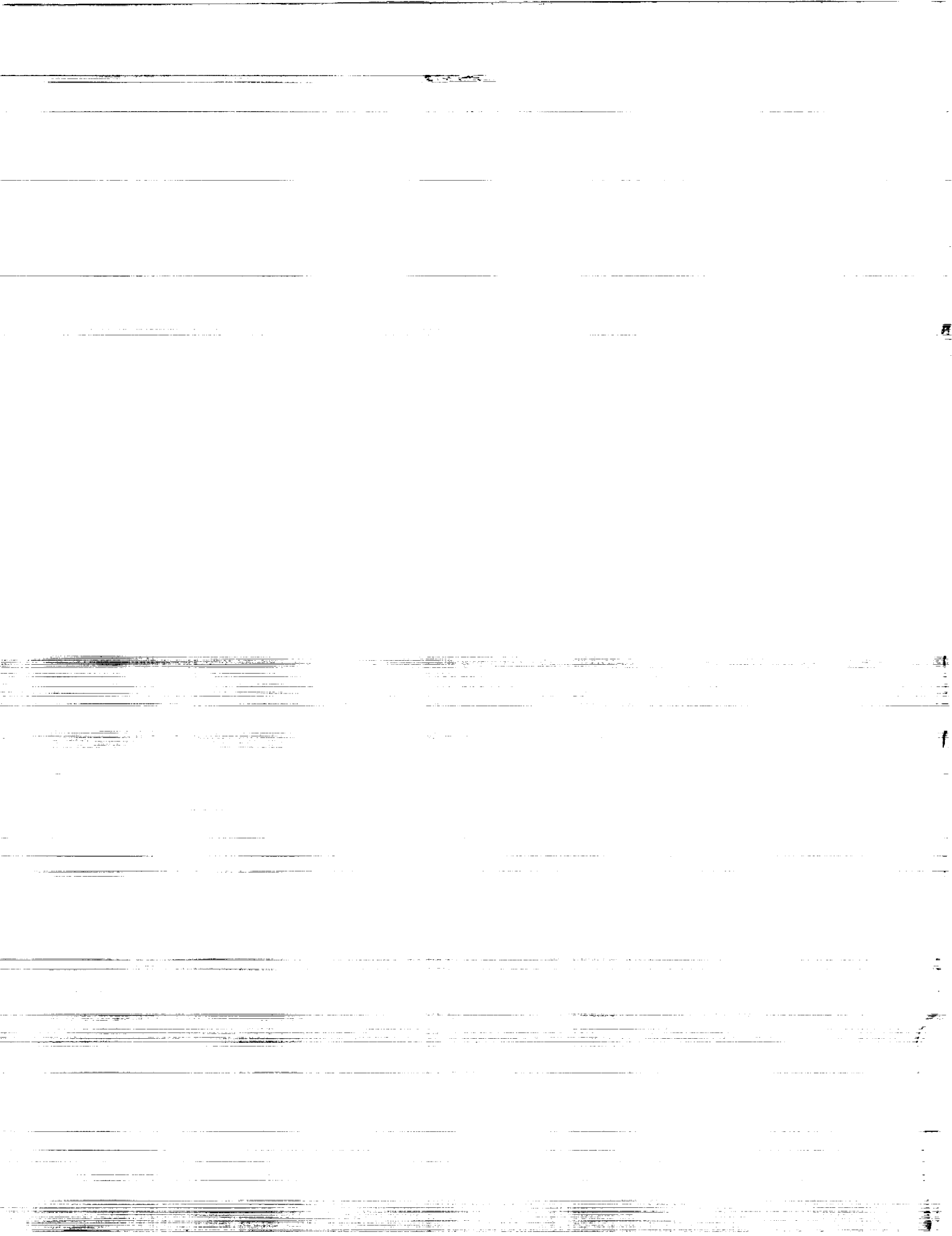G3/34 0251708

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MULTIPLE PATHS TO SUBHARMONIC LAMINAR BREAKDOWN IN A BOUNDARY LAYER

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ABSTRACT

Numerical simulations demonstrate that laminar breakdown in a boundary layer induced by the secondary instability of two-dimensional Tollmien-Schlichting waves to three-dimensional subharmonic disturbances need not take the conventional lambda vortex/high-shear layer path.
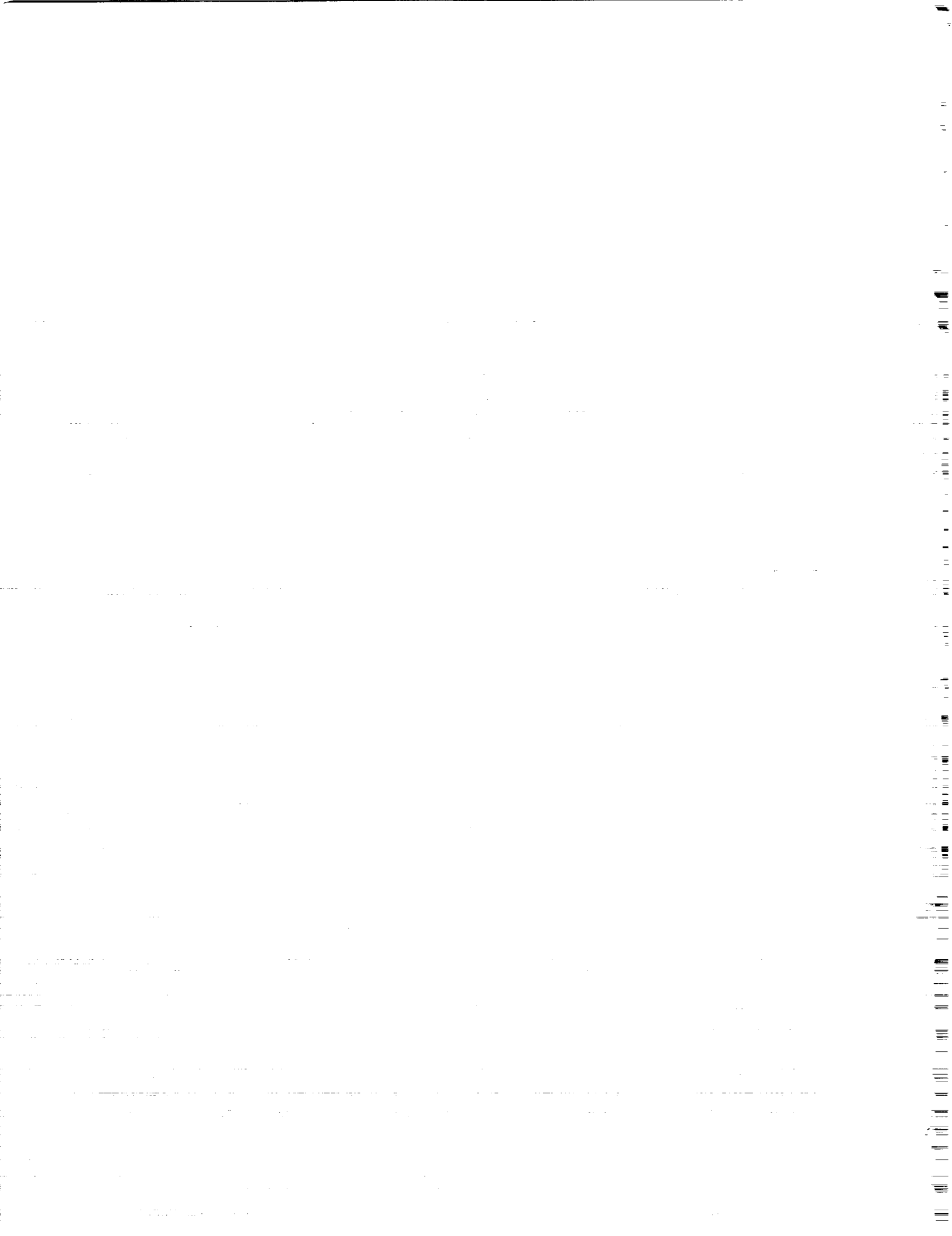
The classical boundary-layer transition experiments of the early 1960's [1], [2], [3] led to the perception that the laminar-turbulent transition process in a Blasius boundary layer may be viewed as a sequence of instabilities. The first two instabilities in the sequence, called the primary and the secondary instabilities, are linear at onset and are now very well understood. The primary instability in an incompressible flat-plate boundary layer is known as a Tollmien-Schlichting (TS) wave. The TS wave, as it equilibrates nonlinearly, becomes susceptible to what is known as the secondary instability. This is truly three-dimensional in character and is thoroughly reviewed by Herbert [4]. The known secondary instabilities are essentially of three kinds – the fundamental secondary instability, the subharmonic secondary instability, and the detuned secondary instability. In the first case, both the primary and the secondary wave have the same streamwise wavelength, and a streamwise vortex system on the scale of the primary wave (a "peak-valley splitting") ensues in which the so-called lambda vortices are aligned in the streamwise direction. The nonlinear evolution of the fundamental secondary instability leads to detached high-shear layers. This stage suffers a tertiary instability manifested by a kink in the high-shear layer and an associated peak in the streamwise perturbation oscillogram, usually called a "spike". Also associated with this stage are hairpin vortex elements. Thereafter, multiple spikes appear through what is possibly a rapid sequence of instabilities, and turbulent spot formation becomes imminent. This particular scenario, which is now known to have dominated all of the classical experiments, is called the K-type breakdown.

The breakdown arising from a subharmonic secondary instability was observed experimentally much later [5]-[6]. In the subharmonic case, the streamwise wavelength of the secondary wave is double that of the primary wave, and the resulting pattern of lambda vortices is staggered. The detuned secondary instability involves so-called combination modes, i.e., a pair of secondary waves whose wavenumbers combine to yield the wavenumber of the primary wave. To date, high-shear layers and their associated spikes have not been observed experimentally for either the subharmonic or the detuned breakdowns. Instead, these instabilities appear to lead to breakdown by a rapid broadening of the spectrum, not just to the shorter wavelengths that are associated with the formation of "spikes", but also to even longer streamwise wavelengths [7].

Numerical simulations of boundary-layer transition have produced the characteristic high-shear layers and spikes of the K-type breakdown [8],[9], [10], [11] and the early secondary instability stage of subharmonic transition [12]. In this paper we report numerical simulations which address the details of the laminar breakdown induced by subharmonic secondary instability.

Numerical simulations of boundary-layer transition have been reviewed recently by Zang and Hussaini [9]. In the present simulations the parallel flow approximation is employed and the evolution is tracked in time rather than in space. The focus is on the vicinity of some point x_0 and the approximation is that the displacement thickness is constant in x (although not necessarily in t), the base flow is strictly in the streamwise direction and it is given by $\mathbf{u}_b(y,t) = (u_B(y\sqrt{x_0/X}),0,0)$, where $u_B(\eta)$ is the Blasius velocity profile which follows from the similar boundary-layer equations, and $X = x_0 + c_f t$; c_f is the speed of a moving computational frame. Lengths are scaled by the displacement thickness, δ_0^*, at x_0, and

1

velocities by the free-stream velocity, u_∞. In these units, $X/x_0 = 1 + \frac{d^2}{Re} c_f t$, where $d = 1.72$. Although the boundary layer is assumed to be parallel, it is permitted to thicken in time [12], with the speed of the computational frame, c_f, equal to the phase speed of the 2D wave. The numerical simulations reported here were performed with the boundary-layer version of the algorithm described in [13], with the nonlinear terms treated in skew-symmetric form [14].

If α and β denote the fundamental wavenumbers in the streamwise and spanwise directions, respectively, then the fundamental wavelengths in these directions are given by $L_x = 2\pi/\alpha$ and $L_z = 2\pi/\beta$. The imposed periodicity lengths are $s_x L_x$ and $s_z L_z$, where s_x and s_z are integers which specify the number of subharmonics that are permitted in each direction. (In the cases presented in this paper $s_z = 1$ and $s_x = 1, 2$, or 9.) The rational numbers $k_x = \hat{k}_x/s_x$ and $k_z = \hat{k}_z/s_z$ label the Fourier wavenumbers in the numerical representations with respect to the fundamental wavenumbers α and β.

Case	Re	wave	α	β	ω	ϵ	θ	s_x
KKVF	1100	TS	0.250	0.000	.08624 + .00333i	0.018	0°	1
	1100	TS	0.250	0.209	.09396 - .01263i	0.008		
KKVS	1100	TS	0.250	0.000	.08624 + .00333i	0.018	0°	2
	1100	SQ	0.125	0.209	.03385 - .01946i	0.008		
KLSH	730	TS	0.254	0.000	.09292 + .00160i	0.00622	36°	2
	730	SQ	0.127	0.241	.03925 - .02254i	1.78×10^{-5}		
KLSL	730	TS	0.254	0.000	.09292 + .00160i	0.00622	36°	2
	730	SQ	0.127	0.241	.03925 - .02254i	4.5×10^{-6}		
KLD	730	TS	0.254	0.000	.09292 + .00160i	0.00622	36°	9
	730	SQ	0.113	0.241	.03630 - .02082i	1.78×10^{-5}		

Table 1: Nonlinear Simulations

The initial conditions for the simulations are of the form

$$\mathbf{u}(\mathbf{x}, 0) = \Re e\{\mathbf{u}_b(y, 0) + \epsilon_{2D} \, e^{i\theta}\mathbf{u}_{2D}(y)e^{i\alpha x}$$

$$+\frac{1}{2}\epsilon_{3D} \, \mathbf{u}_{3D}^{+}(y) \, e^{i[(\alpha/s_x)x+\beta z]} + \frac{1}{2}\epsilon_{3D} \, \mathbf{u}_{3D}^{-}(y) \, e^{i[(\alpha/s_x)x-\beta z]}\}, \tag{1}$$

where $\mathbf{u}_{2D}(y)$ and $\mathbf{u}_{3D}^{\pm}(y)$ are the least stable linear modes of the relevant equation (Orr-Sommerfeld or Squire) for the prescribed wavenumbers α and β. These eigenfunctions are normalized so that their maximum streamwise amplitudes are 1, and, when used in the form given in Eq. (1), these maxima occur at $x = 0$ and $z = 0$. Thus, ϵ_{2D} and ϵ_{3D} measure the maxima of the streamwise fluctuations of the 2-D and oblique disturbances relative to the freestream velocity. The angle θ measures the phase shift between the two-dimensional and the oblique waves.

The parameters of five simulations are given in Table 1. They were chosen with particular experiments in mind. The table indicates whether the initial wave component is a TS wave or a Squire (SQ) wave. The temporal frequency, ω, is included for reference. The direct numerical simulations were performed with sufficient resolution to meet the criterion recommended by Krist and Zang [15]; this typically required grids on the order of $192 \times 96 \times 256$ by the laminar breakdown stage. (However, spanwise symmetry was exploited to halve the number of grid points in that direction.)

The first two simulations were performed under the conditions of the K-type experiment of Kovasznay, Komoda, and Vasudeva [2]. The KKVF and KKVS simulations addressed the fundamental and subharmonic laminar breakdowns, respectively, with the former simulation corresponding to the actual experiment. Figure 1 demonstrates that at these high initial amplitudes both types of breakdown exhibit strong detached high-shear layers. The figure shows the normal shear, $\partial u/\partial y$, in the spanwise symmetry plane after $3\frac{7}{8}$ periods of the 2-D wave for the KKVF simulation, and after $4\frac{7}{8}$ periods for the KKVS simulation. In both cases a strong detached high-shear layer is present and by the time of the illustrations each has already undergone two roll-ups. (The figure corresponds to the two-spike stage.) Three-

dimensional flow-field visualizations of both cases reveal that the transition is associated with the formation of a large-scale lambda vortex/detached high-shear layer for which the breakdown occurs first in the symmetry plane and then spreads outwards in the spanwise direction. (The vortex lines for the KKVS case shown the left frame of Figure 2, for example, indicate the presence of a strong large-scale lambda vortex.)

The subharmonic instability evolves more slowly in this case, as is typical of secondary instabilities at large amplitudes of the primary wave. Nevertheless, the KKVS simulation does furnish unmistakeable evidence that strong detached high-shear layers are not exclusively the province of K-type breakdowns. Continuation of these two simulations to later times shows that the strong similarity of the breakdowns remains, except, of course, that in the former case the patterns are aligned in the streamwise direction, whereas in the latter case they are staggered with twice the streamwise wavelength. We shall refer to this type of breakdown, which in this case is common to both the fundamental and subharmonic instabilities, as a primary-scale breakdown in deference to the scale of the vortical patterns (relative to the scale of the primary wave) which ultimately roll-up.

The next two simulations were under the conditions of the Kachanov and Levchenko [6] experiments, which focused on subharmonic instabilities. The initial amplitudes here, particularly for the three-dimensional components, were much smaller than those for the other experiment. Indeed, the amplitudes were so low that laminar breakdown did not occur until after the primary wave had passed Branch II of the neutral curve. (In the KKVF and KKVS cases the breakdowns occurred well before Branch II was reached.) The KLSH and KLSL cases differed only in the amplitude of the initial 3-D perturbation.

Figure 3 illustrates the time evolution of the normalized kinetic energy [9] in the 2-D and 3-D components for these two simulations. On this scale Branch II is reached at $t = 675$, as evidenced by the subsequent decay of the primary wave. However, the primary wave eventually resumes growth because of feedback from the subharmonic waves [16].

Of more immediate interest for this paper is the character of the ultimate laminar breakdown in these two cases. This is documented in Figure 4. The KLSL case does not break down in the same manner as the two simulations discussed previously. The symmetry plane contains only a very weak detached shear layer, and three-dimensional visualizations, such as the vortex lines for the KLSL case shown in the right frame of Figure 2, reveal that much smaller scale vortices form away from the symmetry plane. These are associated with some moderately strong detached shear layers. Eventually, strong shear does develop in the symmetry plane, but it originates near the wall, as shown in Figure 5. Thus, this subharmonic breakdown does not follow the classical paradigm of a breakdown which commences in the symmetry, or "peak" plane and spreads outward in the spanwise direction. This breakdown is more diffuse, occurring at many spanwise locations, in some places in detached shear layers but in other places, such as the symmetry plane, in the near-wall region. We shall refer to it as a small-scale breakdown, since the vortices which eventually break down are on a much smaller scale than the primary wave.

The KLSH simulation exhibits a stronger detached high-shear layer in the symmetry plane than the KLSL case. The shear layer of the latter case does eventually exhibit the familiar roll-up, although smaller scale features form off the symmetry plane as well. This is

an intermediate case between the distinct primary-scale breakdown of the KKVS simulation and the small-scale breakdown of the KLSL run.

The principal factor determining the type of laminar breakdown which occurs appears to be the amplitude of the primary wave at the stage of incipient three-dimensionality, i.e., when the three-dimensional component reaches a level comparable to that of the primary. When the primary amplitude is large at incipient three-dimensionality, a primary-scale breakdown occurs; when it is small, a small-scale breakdown ensues; and when it is at an intermediate level, the breakdown has a mixture of both features. This speculation is supported by two additional numerical simulations. When the KLSL simulation is repeated, but with the boundary-layer thickness held fixed at its initial value, then the breakdown is manifestly a primary-scale one. Here the primary wave is always unstable and reaches a sufficient amplitude for a primary-scale breakdown to occur. Moreover, if an even larger amplitude is chosen for the KLSH case, for example, with $\epsilon_4 = 1.78 \times 10^{-4}$, then a clean primary-scale breakdown develops. Thus the small-scale breakdown of the KLSL simulation is associated with its small initial amplitudes and the eventual stability of the primary TS wave past Branch II.

To date there have been no experimental observations of detached high-shear layers for subharmonic breakdowns. Such shear layers would be as easy to identify in the KKVS case as they were in the actual experiment [2] corresponding to the KKVF simulation. However, no experiments focusing on subharmonic behavior appear to have been done at such large amplitudes.

At lower amplitudes experimental reports [6], [7] suggest breakdown by spectral broadening to even longer scales and by various sum and difference interactions. The final simulation, KLD, allowed for both effects by leaving room in the computational domain for 9 streamwise subharmonics of the primary wave and by setting the streamwise wavenumber of the initial three-dimensional component to be $\frac{4}{9}$ the the streamwise wavenumber of the primary wave. Figure 6 shows the evolution of various harmonics. This simulation is indicative of a de-tuned, or combination, secondary instability which involves both the $(4/9, 1)$ and the $(5/9, 1)$ modes. Note that although only the former is included in the initial conditions, the latter mode is quickly generated and both modes eventually evolve together. Notice also the rapid development of such other sum and difference harmonics as $\frac{3}{9}, \frac{6}{9}, \frac{13}{9}$, and $\frac{14}{9}$. Flow field visualizations reveal that this particular simulation leads to a primary-scale breakdown. This type of simulation is much more expensive than those which allow for only one or two streamwise harmonics, so a more complete parameter study has not yet been performed. We suspect, however, that at sufficiently low amplitudes of the initial three-dimensional component a small-scale breakdown is also possible for the detuned instability.

The simulations reported here aimed to illustrate that there are several paths to laminar breakdown in a boundary layer. The basic two types are primary-scale and small-scale, although features of both types can be present in a given case.

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Fundamental

Subharmonic

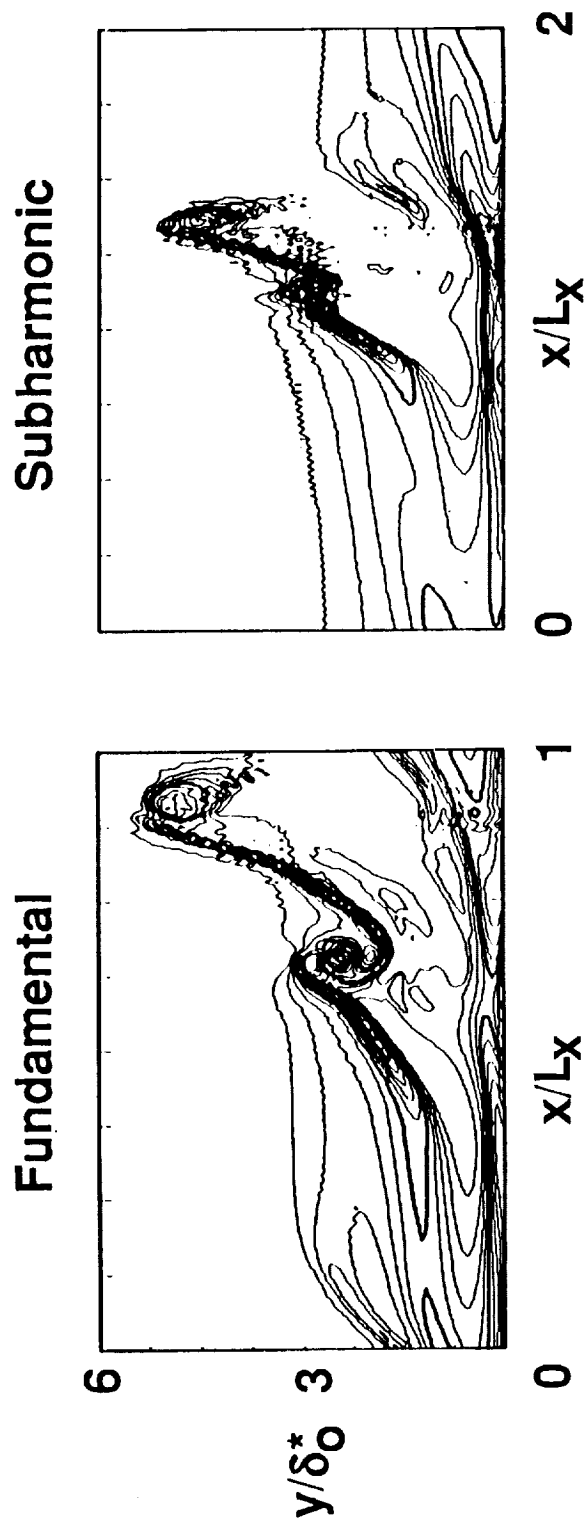y/δ_o^*

x/L_x

x/L_x

Figure 1. Vertical shear in the symmetry plane for the KKVF (left) simulation at $t = 3\frac{7}{8}$ and the KKVS (right) simulation at $t = 4\frac{7}{8}$ periods. The contour interval is 0.10 .
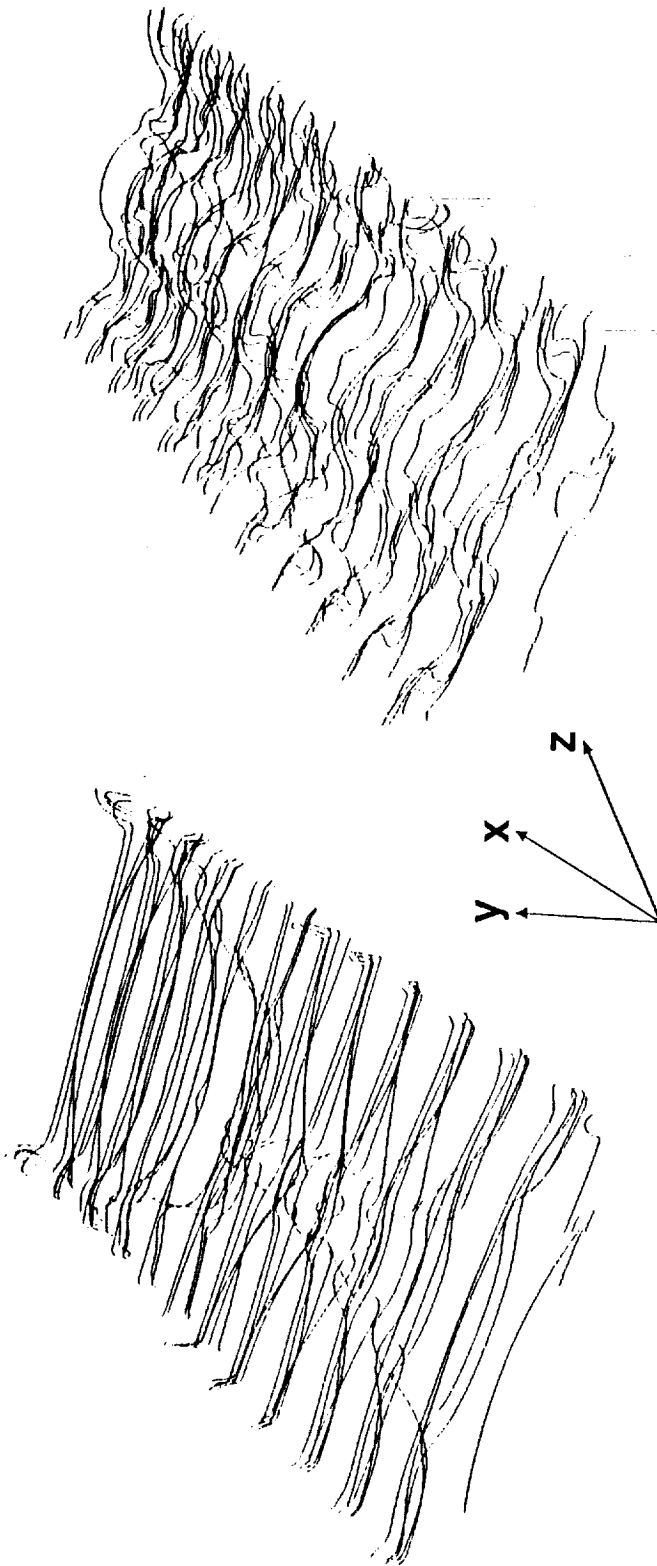
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Small Scale

Primary Scale

Figure 2. Vortex lines for the KKVS simulation at $t = 4\frac{3}{4}$ periods (left), and for the KLSL simulation at $t = 23\frac{1}{2}$ periods (right).
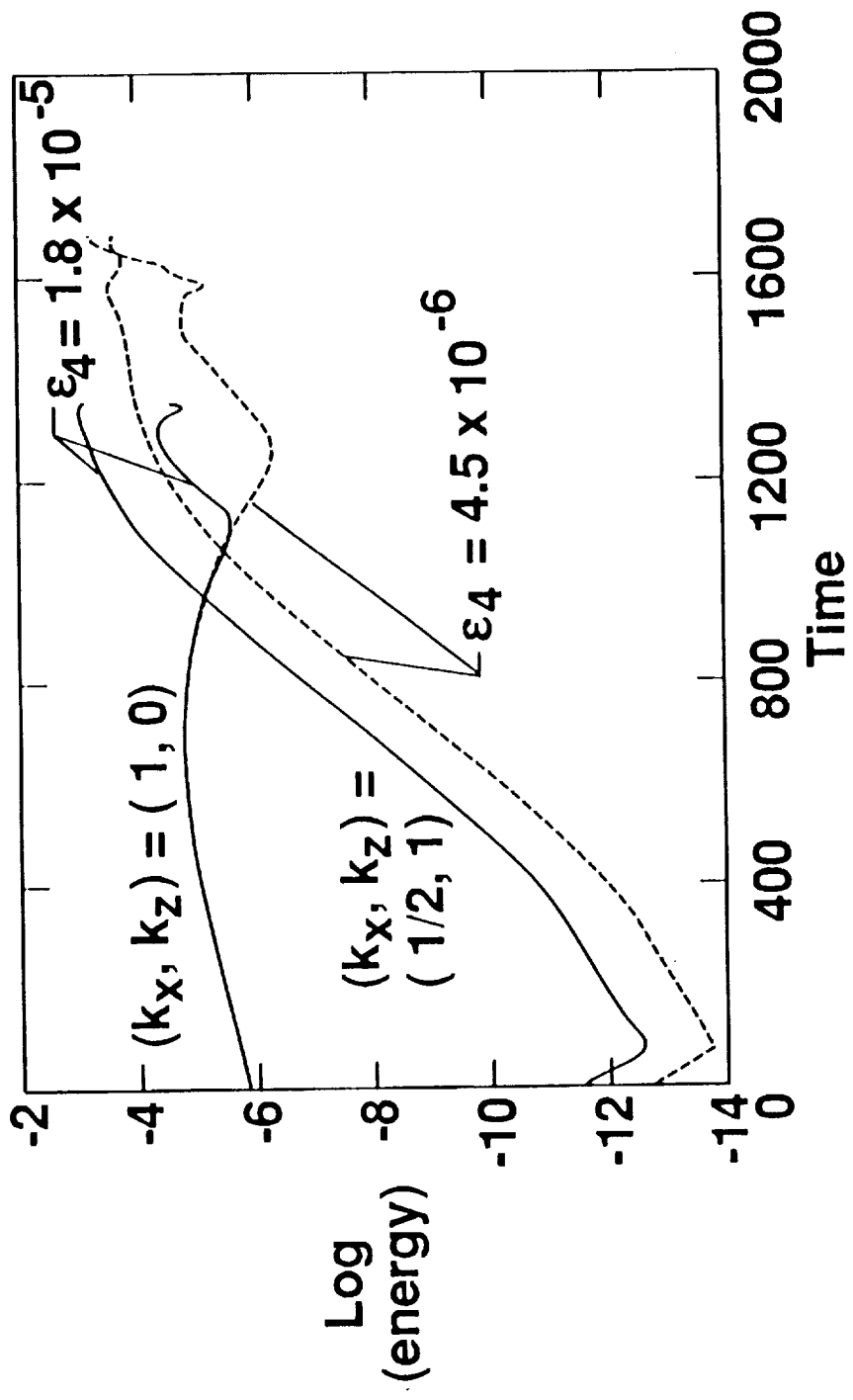
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Figure 3. Evolution of the kinetic energy in the $(1, 0)$ and $(1/2, 1)$ harmonics for the KLSH and KLSL simulations.
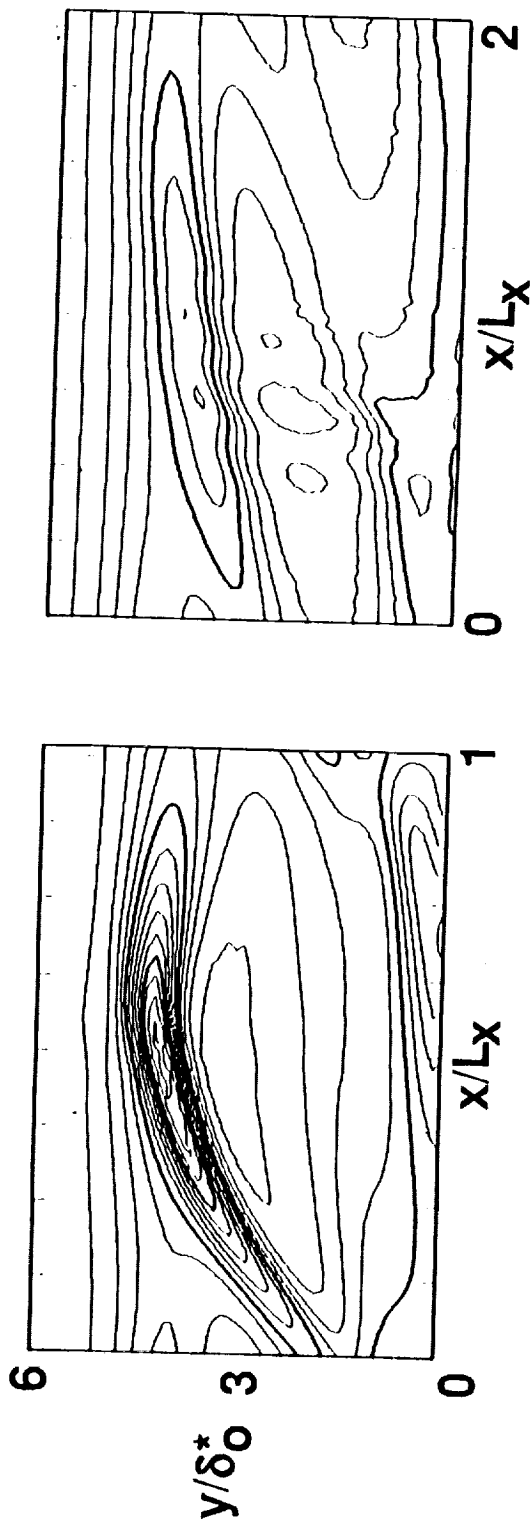
Figure 4. Vertical shear in the symmetry plane for the KLSH (left) simulation at $t = 19$ and the KLSL (right) simulation at $t = 23\frac{1}{2}$ periods. The contour interval is 0.05 .
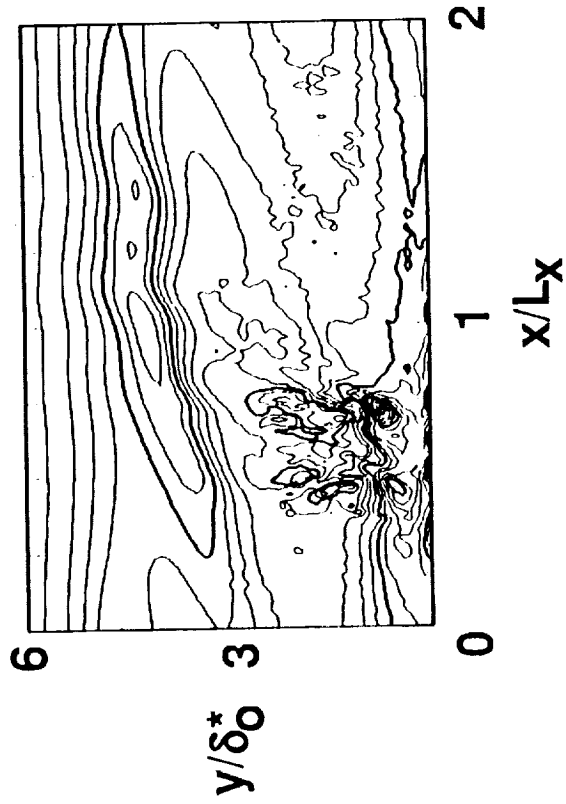
Figure 5. Vertical shear in the symmetry plane for the KLSL simulation at $t = 23\frac{3}{4}$ periods. The contour interval is 0.05.
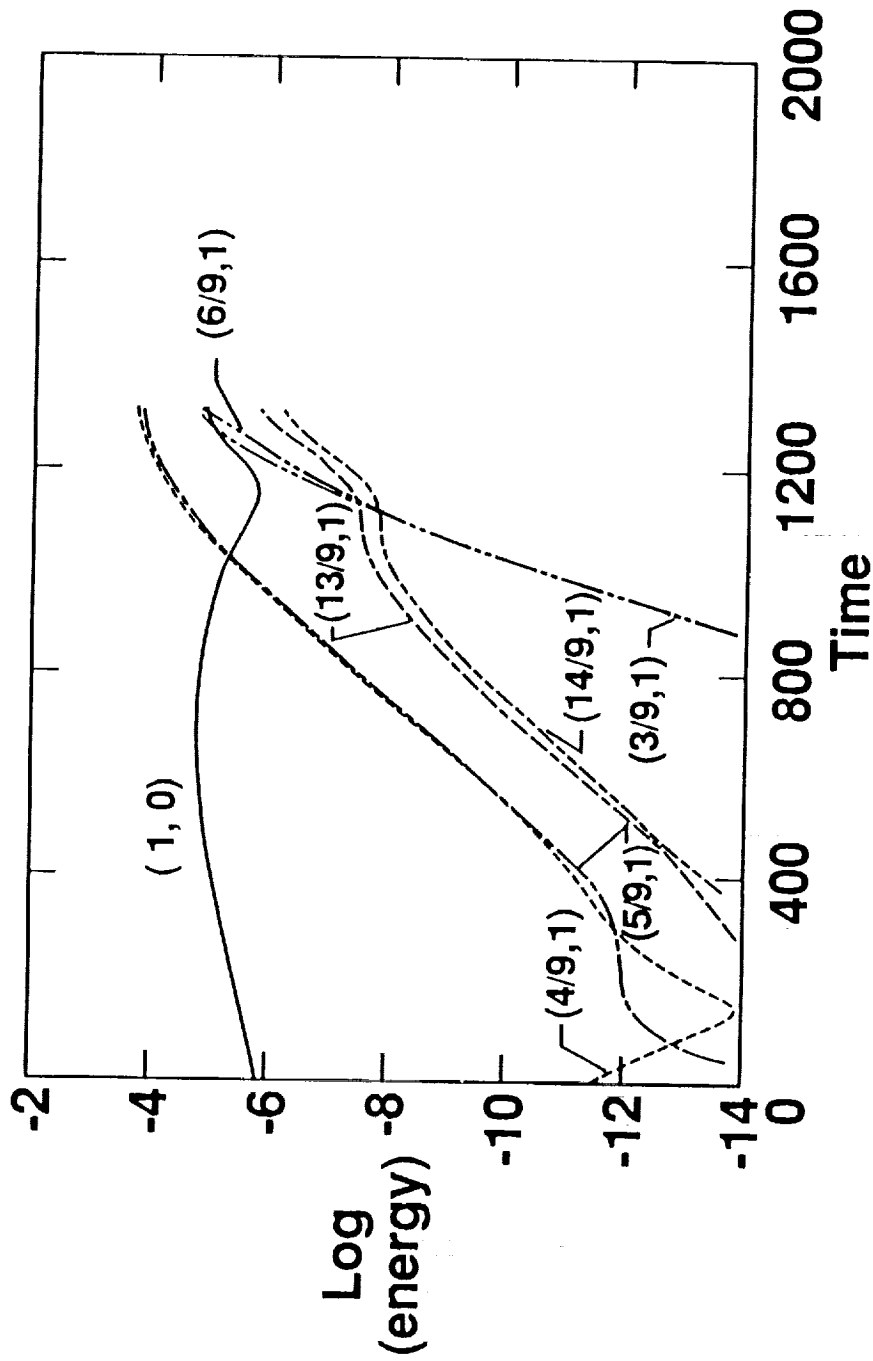
Figure 6. Evolution of the kinetic energy in selected harmonics for the KLD simulation.

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16. Abstract

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