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SHAPES OF STAR-GAS WAVES IN SPIRAL GALAXIES

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Density-wave profile shapes are influenced by several effects. By solving viscous fluid equations, the nonlinear effects of the gas and its gravitational interaction with the stars can be analyzed. The stars are treated through a linear theory developed by Lin and coworkers. Short wavelength gravitational forces are important in determining the gas density profile shape With the inclusion of disk finite thickness effects, the gas gravitational field remains important, but is significantly reduced at short wavelengths. Softening of the gas equation of state results in an enhanced response and a smoothing of the gas density profile. A Newtonian stress relation is marginally acceptable for HI gas clouds, but not acceptable for giant molecular clouds.

1. INTRODUCTION

The gas gravitational field can have a significant influence on the dynamics of spiral waves. This paper extends the results of a recent study of star-gas density waves.[1] In that work, clouds form a viscous fluid in which the atomic and molecular gas are are treated as a single component which interacts gravitationally with the stars that form a second component. Steady-state, nonlinear viscous fluid equations were derived and solved for a zero-thickness, isothermal gas disk.

An alternative to the approach described here of directly solving fluid equations for the gas is an N-body simulation for clouds.[2] The steady-state fluid model suppresses time-dependent processes, but hopefully provides some significant detail about mean flow properties. The other approach may provide some time-dependent information, but may have some of the usual noise problems associated with N-body calculations of spirals (see discussion of noise in reference 3).

2. SHOCK SMOOTHING VIA GRAVITY

Perhaps the most surprising result of our recent study was that gas gravity actually smooths gas shock profiles (see Fig. 1). This result is actually not very surprising when one considers the fact that for a fixed peak to average gas density ratio at the solar circle *pressure increases the tendency of gas to shock, while gravity opposes pressure to decrease the tendency to shock.* Intuition may suggest that shocks are avoided when pressure is included, because pressure can prevent streamlines from crossing and thus prevent shocks. In local Galactic flow, away from resonances, streamlines do not cross, even in the absence of pressure. Instead, shocks arise as the flow makes a sonic transition from supersonic to subsonic speeds, relative to the wavefront.[4] Therefore, contrary to intuition, raising the sound speed of the gas somewhat actually makes it easier for gas to make a sonic transition from its upstream supersonic velocity, since there is less change in velocity required for a sonic transition and hence for a shock.

Gas gravity pulls gas into the shock front where there is a high concentration of gas. The gravity counters the effect of pressure forces which decelerate the flow through the strong pressure gradient at the front. As a result, the flow profile becomes smoother and thus the gas pressure gradient less. The effect of gravity can be understood mathematically in terms of the inviscid (nonviscous) flow equation[4]

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$$\frac{1}{r \sin(i)} \frac{\partial u}{\partial \varphi} = u(2\Omega v + f(\varphi))/(u^2 - c^2) \tag{1}$$

where u and v are the gas velocities respectively perpendicular and parallel to the spiral wave in the wave pattern frame, and $f(\varphi)$ is the star-gas gravity perpendicular to the wave front at radius r.

Wherever shocks occur, the denominator in the above expression must vanish, which occurs when the gas makes a sonic transition relative to the density wave front. However, the possible singularity in $\frac{\partial u}{\partial \varphi}$ can be cancelled by a simultaneously vanishing numerator. Shocks can occur at a sonic transition *where rotational, $2\Omega v$, and gravitational, f, forces are unbalanced.*

Fig. 2 shows that gas gravity causes the rotational and gravitational forces to be better balanced at the first sonic point, and hence reduces the strength of the shock (Rotational and gravitational forces always nearly cancel at the second sonic point, since this point corresponds to flow from subsonic to supersonic speeds.) This effect is due to lumpy nature of the gas gravity near the density peak. In other words, short wavelength gravitational forces play a critical role in determining gas profile shapes, since those forces must compete with pressure forces, which are intrinsically short wavelength forces. However, short wavelength gravity is most susceptible to finite thickness corrections.

3. FINITE-THICKNESS EFFECTS

Finite thickness corrections most strongly affect the gravitational forces.[5] The strength of finite-thickness effects depends on the product of the wavenumber and layer thickness. The gas contains more density at high harmonics (short wavelengths) than the stars. Although the gas disk is substantially thinner than the stellar, finite thickness corrections for the high order gas harmonics can be as large as the corrections for the stars at its fundamental mode. A full solution for the nonlinear flow with finite thickness corrections involves solving for vertical motions, since the gas disk thickness is likely to vary significantly in phase. Instead, the finite thickness corrections are approximated by suppressing the phase dependence of the vertical disk structure. A further approximation is that the gas and stars as have a constant density within their respective layers.[5] We define thickness $H_j = 0.5\sigma_j(r)/\rho_j(r, z = 0)$, for species j. For the gas, the thickness H_g is chosen as 110 pc, a value about midway between that for atomic (150 pc) and molecular gas (65 pc), which contribute about equally to the local surface density.[6] For the stars, a thickness H_* of 700 pc is adopted.[7]

For each harmonic n, the gravitational forces in the star and gas dynamical equations are modified by the correction factor $\int_{-\infty}^{\infty} \rho_{0i}\phi_{nj}(z)/(\sigma_i\phi_{nj}(H = 0))$ where i and j can represent stars or gas. This correction factor works well in the linear flow case.[8]

The gas gravity is significantly softened as a result of this correction. However, our current best model of the solar circle with about 14% gas ($70M_\odot/pc^2$ total, $10M_\odot/pc^2$ gas)[9] in Fig. 1 still shows important effects of the gas gravity. If thickness H_g for the gas layer had been over 250 pc, the gas density profile would have been noticeably steeper.

4. SOFTENED EQUATION OF STATE

Several suggestions exist that the equation of state for a fluid of clouds may be softer than isothermal.[10] An extremely softened model was constructed. This model represents energy dissipation through cloud collisions (proportional to the square of the cloud density times the cube of the random velocity) which balances energy injection by supernova explosions (proportional to the local cloud density). (Actually other effects are also present, such as heating through dissipation of differential rotational energy, see equation (16) in reference 12.) This extreme model assumes that the supernova explosions occur so much later than the cloud collisions which form the supernova precursors, that the local energy injection rate depends only on the local cloud density and not the local rate of cloud collisions. This model yields a polytropic pressure relation of $p \propto \sigma^{1/3}$. (At the opposite extreme is a model in which supernova form immediately after cloud collisions. This model yields an approximately isothermal equation of state.)

Numerical results indicate that, like gravity, the softening of the equation of state results in a smoother gas density profile when comparing models with the same peak to average density ratio and the the mean cloud random velocity. The basic reason for the smoothing is that the sound speed of the gas drops upon compression. A lower sound speed means the gas is harder to shock (see section 2). It can be shown that the sonic condition for a shock can be expressed as

$$\sigma_{crit} = (\nu^2/x_{p0})^{1/(\gamma+1)} \qquad (2)$$

where σ_{crit} is the critical gas surface density (normalized to the circular average gas density) needed to produce a shock, $\nu = m(\Omega_p - \Omega)/\kappa$, $x_{p0} = k^2 c_0^2/\kappa^2$, with sound speed c_0 where the normalized density is unity, and $\gamma = \frac{\sigma}{p}\frac{dp}{d\sigma}$. At the solar circle, for $x_{p0} \approx 0.2$, one finds $\sigma_{crit} \approx 1.6$ for the isothermal case and $\sigma_{crit} \approx 2.1$ for $\gamma = 1/3$.

5. VISCOSITY

Viscosity in disks has some new features.[11,12] Gas in a galaxy is typically nonaxisymmetric and highly perturbed. In such a situation, the usual Newtonian stress relation can be applied only when the collision frequency is much greater than any velocity derivatives in the gas flow. From Fig. 3, we see that this condition is marginally satisfied for HI gas clouds.

For giant molecular clouds (GMCs), the collision frequency is more than 5 times smaller than for HI clouds. The Newtonian stress relation cannot be rigorously applied, since the above condition is not well satisfied. Assuming GMCs are long-lived, rotational effects are likely to be important. The *effective mean free path* is likely to be limited by Galactic rotation, since the epicyclic radius of around 300 pc is smaller than the mean free path for cloud collisions.[13] A proper treatment of this problem would require a solution of the Boltzmann or Krook equation, as has been applied in the context of planetary rings.[14,15] In fact, in this regime, the sign of the effective coefficient of shear viscosity can sometimes even be negative.[15]

Simulations with gas cloud mean free paths that are long compared with the epicyclic radius may not yield simple results. For example, the front thickness will not generally be a few collisional mean free paths. Such an effect is seen in some N-body results.[16]

Acknowledgements

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FIGURE CAPTIONS

Figure 1. Normalized gas density as a function of phase (ETA $= 2\,\varphi$) along the solar circle for models with 0.01% and 14.3% gas. In these models, gas disk thickness H_g is 110 pc and stellar disk thickness H_* is 700 pc. The stellar wave amplitude was adjusted so that the ratio of peak to average gas density was 3.6 for the two cases.

Figure 2. In short dashed lines is the total gravitational force per unit mass perpendicular to spiral arms; in long dashed lines is the perturbed rotational force per unit mass. In solid lines is the sum of the perturbed rotational and gravitational forces. The vertical lines mark the location of the two sonic points of the gas flow relative to the spiral wave pattern.

Figure 3. The spatial derivative of the gas velocity perpendicular to spiral arms divided by the local cloud collisional frequency for HI gas clouds is plotted as a function of phase. Mean free path at average surface density is 170 pc; 1-d random velocity is 8 km/sec.
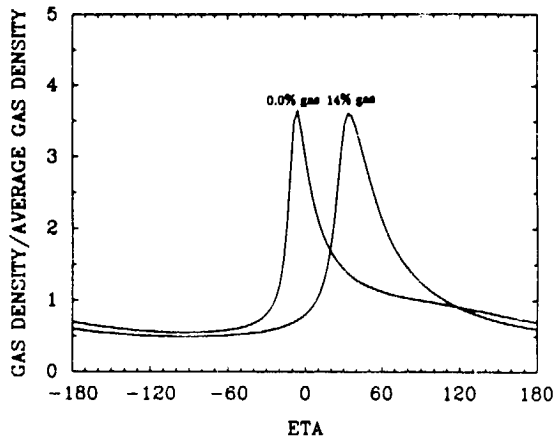
FIGURE 1

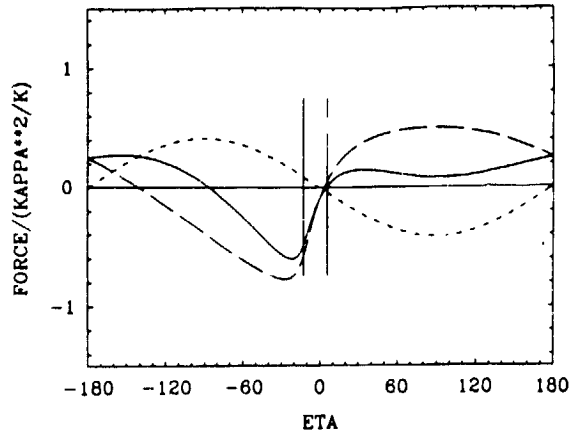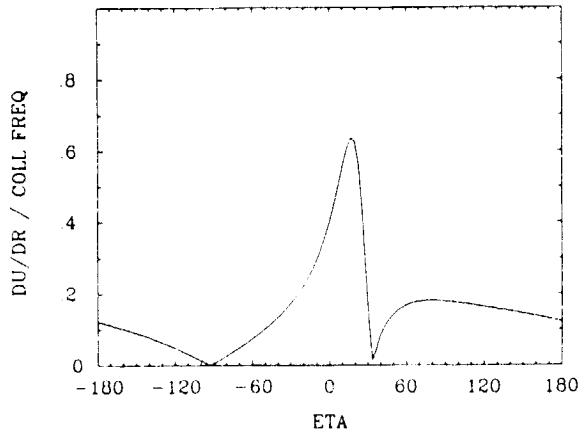SPIRAL DENSITY-WAVE GAS PROFILE

FIGURE 2

SPIRAL DENSITY-WAVE   0.0% GAS

SPIRAL DENSITY-WAVE 14.3% GAS

SPIRAL DENSITY-WAVE 14.3% GAS

FIGURE 3