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Probabilistic Simulation of Uncertainties in Composite Uniaxial Strengths

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PROBABILISTIC SIMULATION OF UNCERTAINTIES IN COMPOSITE UNIAXIAL STRENGTHS

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SUMMARY

Probabilistic composite micromechanics methods are developed that simulate uncertainties in unidirectional fiber composite strengths. These methods are in the form of computational procedures using composite mechanics with Monte Carlo simulation. The variables for which uncertainties are accounted include constituent strengths and their respective scatter. A graphite/epoxy unidirectional composite (ply) is studied to illustrate the procedure and its effectiveness to formally estimate the probable scatter in the composite uniaxial strengths. The results show that ply longitudinal tensile and compressive, transverse compressive and intralaminar shear strengths are not sensitive to single fiber anomalies (breaks, interfacial disbonds, matrix microcracks); however, the ply transverse tensile strength is.

INTRODUCTION

The analysis of composite structures requires reliable predictive models for material properties and strengths. However, the prediction efforts have been complicated by inherent scatter in experimental data. Since uncertainties in the constituent properties, fabrication variables, and internal geometry would lead to uncertainties in the measured composite properties, the question arises:

> How much of the "statistical" scatter of experimentally observed composite properties can be explained by reasonable statistical distribution of input parameters (primitive variables) in composite micromechanics and laminate theory predictive models?

In order to answer this question, a study was conducted to develop a computational simulation procedure for probabilistic composite micromechanics (ref. 1). Application of this approach for uniaxial thermal and mechanical properties is summarized in reference 2. The objective of the present paper is to describe this type of micromechanics for fiber composite uniaxial strengths and present typical results obtained therefrom. The computational simulation is performed using ply substructuring with an existing computer

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code (ref. 1) for composite mechanics and in conjunction with Monte Carlo simulation. The scatter in the constituent strengths is selected from anticipated respective probabilistic distributions.

COMPUTATIONAL SIMULATION FOR PROBABILISTIC COMPOSITE MICROMECHANICS

In this section, the formal approach to computationally simulate probabilistic composite micromechanics is summarized.

Deterministic/Probabilistic Model

The model commonly used for deterministic composite mechanics is based on the calculation of properties of the basic unit of an orthotropic ply. The layup geometry is then used in laminate equations to calculate composite properties as shown schematically in figure 1(a). In the probabilistic simulation however, the basic unit is taken as a subply (ply substructuring) which consists of only a single fiber-matrix. Deterministic micromechanics theory (refs. 3 and 4) is used to predict the properties of the assumed orthotropic subply. The probabilistic aspect is introduced by representing the scatter in the fabrication variables and constituent material properties. Probable fiber misalignment within the ply are then used in the laminate theory equations to predict ply properties. This substructuring of the ply represents a novel attempt at characterization of fiber composite material properties based on probabilistically distributed constituent properties, individual fiber misalignment and fabrication process (primitive) variables as shown schematically in figure 1(b).

Ply substructuring in conjunction with composite mechanics is particularly well-suited to the probabilistic description of fiber composite material properties. The micromechanics and laminate theory equations can be used to calculate ply properties at any number of points in a ply. This approach provides a rational procedure for composite material property assessment because it evaluates ply behavior as the result of a series of random events (uncertainties in the primitive variables) which occur at the intraply or micromechanics level.

Composite Mechanics

The probabilistic simulation is performed by considering the ply as an assembly (equivalent laminate) of 15 subplies. The composite mechanics used in the simulation is that available in the Integrated Composite Analyzer (ICAN) (ref. 5), which is a computer program for comprehensive linear analysis of multilevel fiber composite structures (ref. 5). The program contains the essential features required to effectively design structural components made from fiber composites. It now represents the culmination of research conducted since the early 1970's at the National Aeronautics and Space Administration (NASA) Lewis Research Center (LeRC), to develop and code reliable composite mechanics theories. This user friendly, publicly available code is depicted schematically in figure 2 and is described in detail in reference 5.

Probabilistic Simulation - Monte Carlo Methods

Complicated probabilistic events can be simulated by a variety of methods generally referred to as Monte Carlo methods (ref. 1). The term refers to that branch of mathematics concerned with numerical experiments on random numbers. Since the advent of high speed computers, they have found extensive use in most fields of science and engineering, in analyzing many physical processes of a probabilistic nature, or where physical experimentation is limited or not feasible.

A Monte Carlo simulation refers to the procedure where a single computational simulation is performed by randomly assigning a value to an independent random variable in a chosen model, and observing the dependent variable at the conclusion of the process being modeled. A Monte Carlo simulation is composed of n such independent simulations. When n is sufficiently large, the observations will yield a statistically meaningful description of the physical problem.

The form of Monte Carlo simulation used in the present investigation is as follows:

(1) Define the model by assuming that:

   (a) It represents the composite mechanics

   (b) It is formulated in terms of primitive variables

   (c) It has probability distributions for the scatter in each primitive variable

(2) Use the computer and random sampling techniques to select values of the primitive variables from their respective distributions.

(3) Calculate dependent response variables using the model.

(4) Replicate the experiment, each time with a new set of randomly sampled input values.

(5) Use appropriate statistical methods to calculate probability distribution of the properties of interest.

(6) Estimate regression parameters for the assumed model.

Computational Simulation Procedure

To perform the computational simulation, a computer code was developed to couple ICAN and an available statistical analysis code (ref. 1). The logic diagram for this code is shown in figure 3. The steps are as follows:

(1) Select values for the primitive variables for each subply from their respective assumed probabilistic distributions (fig. 4):

   (a) Normal - constituent elastic properties and fiber volume ratio, and fiber misalignment

(b) Weibull - constituent strengths

(c) Gamma - for void volume ratio

Fifteen different sets (one for each subply) are generated where the means and scatter ranges for the primitive variable were those typical for AS graphite/epoxy composite (tables I and II).

(2) Enter these values as inputs into ICAN.

(3) Run ICAN and retrieve and store ICAN output for desired ply properties.

(4) Repeat the process n-times where n is sufficiently large (50, herein) to provide data repeatability with an acceptable level of confidence.

(5) Process the stored output using statistical analysis for cumulative probability distributions, confidence levels and significance.

RESULTS AND DISCUSSION

Results obtained by using the computational procedure described previously are presented and discussed in this section. The results are for ply uniaxial strengths, longitudinal tension ($S_{\ell 11T}$), longitudinal compression ($S_{\ell 11C}$), transverse tension ($S_{\ell 22T}$), transverse compression ($S_{\ell 22C}$) and intralaminar shear ($S_{\ell 12S}$). The results are presented in graphical form where the ply uniaxial strength is plotted versus fiber volume ratio for ranges of scatter as represented by the Weibull shape parameter (α).

Longitudinal Tensile Strength ($S_{\ell 11T}$)

The influence of the scatter in fiber tensile strength on the ply longitudinal tensile strength is shown in figure 5. Recall that the greater the value of α, the smaller the scatter. It is seen that the spread in $S_{\ell 11T}$ increases with increasing fiber volume ratio. If we assume that the fiber strength scatter increases with increasing fiber volume ratio due to fiber contact (abrasion) damage during processing, the following is deduced from the figure: the scatter in the ply longitudinal strength will also increase. For example, we can see approximately from the figure, the anticipated range for $S_{\ell 11T}$ to be $132 \leq S_{\ell 11T}/S_{fT} \leq 144$ ksi for a mean fiber volume ratio of 0.5 and a mean fiber strength (S_{fT}) of 400 ksi (table I). This range is lower than $S_{\ell 11T} = 200$ ksi which is estimated using deterministic composite micromechanics (table III).

It is important to recall that the probabilistic ply strength was predicted by assuming that the ply fractures when the weakest fiber (subply) through its thickness fractures. Since the deterministic composite micromechanics value is considerably higher (about two times), the conclusions are that: (1) a single fiber break is not sufficient to fracture the ply, (2) fiber load redistribution must take place, and (3) several fibers through the ply thickness must break prior to ply fracture. This is consistent with what is common knowledge in the composites community. It indirectly demonstrates the substantial fracture toughness inherent in composite longitudinal tensile strength relative to isolated fiber breaks. These isolated fiber breaks have negligible influence on ply strength/fracture. As a side comment, the probable number of fiber breaks prior to ply fracture can be computationally simulated by accounting for fiber progressive fracture with simultaneous load redistribution. The computational procedure will be analogous to that in CODSTRAN (Composite Durability Structural Analysis) for deterministic progressive composite fracture (ref. 6) but applied to single fiber breaks.

Longitudinal Compressive Strength ($S_{\ell 11C}$)

The influence of the scatter in fiber compressive strength on ply $S_{\ell 11C}$ strength is shown in figure 6 for the same α ranges as for $S_{\ell 11T}$. The important observations from this figure are (1) the curves peak at about 0.5 FVR and then decrease, and (2) the curves for each α do not remain order-consistent but cross over. Both of these occur because of the four different fracture modes that are assumed to induce ply longitudinal compressive fracture (ref. 4). It is also observed that the spread in $S_{\ell 11C}$ changes for the different α's with increasing FVR. For example, the scatter between α equals 20 and 10 is greater for FVR 0.4 than for 0.6 and it is outside this range. One important conclusion from these observations is that it would be very difficult to make consistent conclusions from experimental data for $S_{\ell 11C}$.

The range for $S_{\ell 11C}$ at 0.5 FVR from figure 6 is estimated between 117 and 122 ksi which is lower compared to the micromechanics estimate of about 165 ksi. Again, the explanation is that a single fiber fracture does not cause ply fracture. And the ply is relatively insensitive to single fiber anomalies. This is a significant finding in view of the prevailing contention that single fiber anomalies are detrimental to ply longitudinal compressive strength.

Transverse Tensile Strength ($S_{\ell 22T}$)

The influence of the scatter of the matrix tensile strength on $S_{\ell 22T}$ is shown in figure 7 for the same values in α as for $S_{\ell 11T}$ and $S_{\ell 11C}$. The observations to be noted are: (1) the curves for the different α's remain order-consistent; that is, no cross-over occurs, (2) the curves indicate that $S_{\ell 22T}$ continuously decreases with increasing FVR, and (3) the spread in the scatter decreases as the FVR increases.

The range in $S_{\ell 22T}$ is between 9.0 and 10.4 ksi, which is lower than 12 ksi (table III) predicted by deterministic composite micromechanics. However, it is within the range of experimentally observed data for this strength (ref. 4). The authors consider this an important finding because it suggests that transverse tensile strength may be strongly influenced by single fiber disbonds or microcracks.

Transverse Compressive Strength ($S_{\ell 22C}$)

The influence of the scatter of the matrix compressive strength on $S_{\ell 22C}$ is shown in figure 8. The influence is similar to that for $S_{\ell 22T}$. The observations made for $S_{\ell 22T}$ are applicable to $S_{\ell 22C}$ as well except for the specific values comparisons. The scatter in $S_{\ell 22C}$ is between 20.6 and 23.8 ksi at 0.5 FVR which is considerably smaller than 27 ksi (table III) predicted by deterministic composite micromechanics which is also less than the range of experimental data of about 30 to 35 ksi (ref. 4).

The authors attribute this difference to indicate that the ply transverse compressive strength is not sensitive to single fiber anomalies as is the ply transverse tensile strength.

Intralaminar Shear Strength ($S_{\ell 12S}$)

The influence of the scatter of the matrix shear strength on $S_{\ell 11S}$ is shown in figure 9. The influence is similar to those for $S_{\ell 22T}$ and $S_{\ell 22C}$ as would be expected since all of these are matrix controlled properties. There is some difference in the variation of scatter with FVR. The greatest spread is at about 0.5 FVR for $S_{\ell 12S}$, whereas it progressively decreases with increasing FVR for the other two.

The range in scatter for $S_{\ell 12S}$ at 0.5 FVR is between 11.8 and 14.6 ksi compared to 10 ksi estimated by deterministic composite micromechanics. The authors interpret this good comparison to indicate the following two significant points (1) the in situ matrix shear strength is not influenced by the fabrication process and (2) the ply intralaminar shear strength is not sensitive to single fiber anomalies.

GENERAL DISCUSSION

The scatter in the ply uniaxial strengths described and discussed was limited to that influenced by respective scatter in fiber and matrix strengths. Other factors influence the scatter as described in references 1 and 2. These factors include scatter caused by single fiber anomalies in strength, interfacial disbonds and matrix microcracks.

The authors consider the probabilistic simulation described in this paper as an illustration of what can be done to quantify the uncertainties associated with the numerous factors that influence ply uniaxial strengths. They do not consider the simulation to be complete nor the graphical results presented and the respective numerical values discussed as absolute. The authors strongly believe, however, that this is a rational approach to formally represent uncertainties associated with various factors that influence ply uniaxial strengths. The results obtained to date demonstrate the authors' contention.

Regression results presented in reference 1, but not summarized here due to space limitation, indicate that factors influencing uniaxial different ply strengths are important in specific ranges of FVR and that no generalizations can be made at this time.

The authors hope that the description, results and discussion summarized herein, will stimulate other investigators to pursue probabilistic representation of composite uniaxial strength behavior beyond the longitudinal tensile strength which has extensively been investigated over the years.

SUMMARY OF RESULTS

The important results of an investigation to computationally simulate the probable scatter in composite uniaxial strengths as influenced by scatter in respective fiber and matrix strengths are summarized below.

1. A computational procedure has been described for probabilistic composite micromechanics for uniaxial strengths.

2. The scatter range in the uniaxial strengths is represented in terms of the Weibull shape function in the respective constituent material strengths (fiber and matrix) for different fiber volume ratios.

3. Comparisons with respective deterministic mean values and corresponding experimental data indicate that ply longitudinal tensile, longitudinal compressive, transverse compressive and intralaminar shear strengths are not sensitive to single fiber anomalies. However, the ply transverse tensile strength is.

REFERENCES

1. T.A. Stock, Probabilistic Composite Mechanics, Master's Thesis, Cleveland State University, Cleveland, OH, 1987.

2. T.A. Stock, P.X. Bellini, P.L.N. Murthy, and C.C. Chamis, "A Probabilistic Approach to Composite Micromechanics," NASA TM-101366, 1988.

3. C.C. Chamis, "Simplified Composite Micromechanics Equations for Hygral, Thermal, and Mechanical Properties," SAMPE Quarterly, Vol. 15, 1984, pp. 14-23.

4. C.C. Chamis, "Simplified Composite Micromechanics Equations for Strength, Fracture Toughness and Environmental Effects," SAMPE Quarterly, Vol. 15, 1984, pp. 41-55.

5. P.L.N. Murthy and C.C. Chamis, "ICAN: Integrated Composites Analyzer," Journal of Composites Technology and Research, Vol. 8, No. 1, Spring 1986, pp. 8-17.

6. C.C. Chamis, "Computational Simulation of Progressive Fracture in Fiber Composites," NASA TM-87341, 1986.

TABLE I. - COMPUTATIONAL PROCEDURE INPUT

DATA-FIBER PROPERTIES

Input	Deterministic	Probabilistic	
	Case 1	Case 2	Case 3
Theta, deg	0.0	------	------
μ	------	0.0	0.0
α	------	5.0	10.0
FVR	0.50	------	------
μ	------	0.5	0.5
α	------	0.1	0.2
VVR	0.01	------	------
λ	------	0.3	0.3
κ	------	0.03	0.05
E_{f11}, ksi	31 000	------	------
μ	------	31 000	31 000
α	------	1 500	3 000
E_{f22}, ksi	2 000	------	------
μ	------	2 000	2 000
α	------	100	200
G_{f12}, ksi	2 000	------	------
μ	------	2 000	2 000
α	------	100	200
G_{f23}, ksi	1 000	------	------
μ	------	1 000	1 000
α	------	50	100
ν_{f12}	0.2	------	------
μ	------	0.2	0.2
α	------	0	0
α_{f11}, ppm/°F	0.2	------	------
μ	------	0.2	0.2
α	------	0	0
α_{f22}, ppm/°F	5.6	------	------
μ	------	5.6	5.6
α	------	0	0
S_{fT}, ksi	400	------	------
β	------	400	400
α	------	20	10
S_{fC}, ksi	400	------	------
β	------	400	400
α	------	20	10

TABLE II. - COMPUTATIONAL PROCEDURE INPUT DATA

MATRIX PROPERTIES

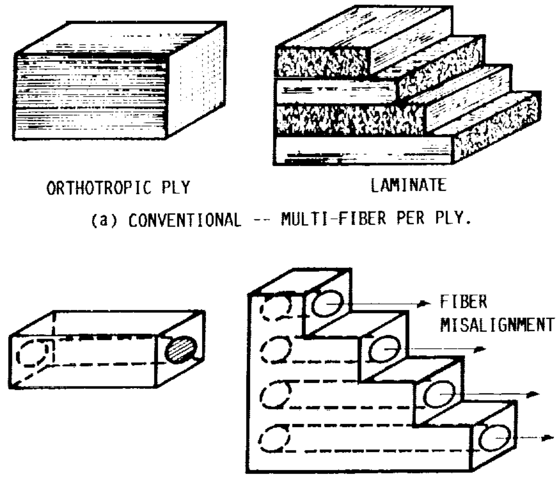
Input	Deterministic	Probabilistic	
	Case 1	Case 2	Case 3
E_m, ksi	500	----	----
μ	---	500	500
α	---	25	50
ν_m		0.35	----
μ	---	0.35	0.35
α	---	0	0
α_m, ppm/°F	36	----	----
μ	---	36	36
α	---	0	0
S_{mT}, ksi	15	----	----
β	---	15	15
α	---	20	10
S_{mC}, ksi	35	----	----
β	---	35	35
α	---	20	10
S_{mS}, ksi	13	---	---
β	---	13	13
α	---	20	10

TABLE III. - DETERMINISTIC

PLY PROPERTIES

[0.5 Fiber volume ratio.]

Property	Value
$E_{\ell 11}$, mpsi	15.8
$E_{\ell 22}$, mps	1.06
$G_{\ell 12}$, mpsi	0.52
$\nu_{\ell 12}$	0.28
$\nu_{\ell 21}$	0.02
$\alpha_{\ell 11}$, ppm	0.08
$\alpha_{\ell 22}$, ppm	18.4
$\alpha_{\ell 12}$	0
$S_{\ell 11T}$, ksi	203
$S_{\ell 11C}$, ksi	165
$S_{\ell 22T}$, ksi	12
$S_{\ell 22C}$, ksi	27
$S_{\ell 12S}$, ksi	10



(a) CONVENTIONAL -- MULTI-FIBER PER PLY.

(b) PLY SUBSTRUCTURING - SINGLE FIBER PER PLY.

FIGURE 1. - PLY SUBSTRUCTURING ANALOGOUS TO LAMINATE DECOMPOSITION, SUBSTRUCTURING.

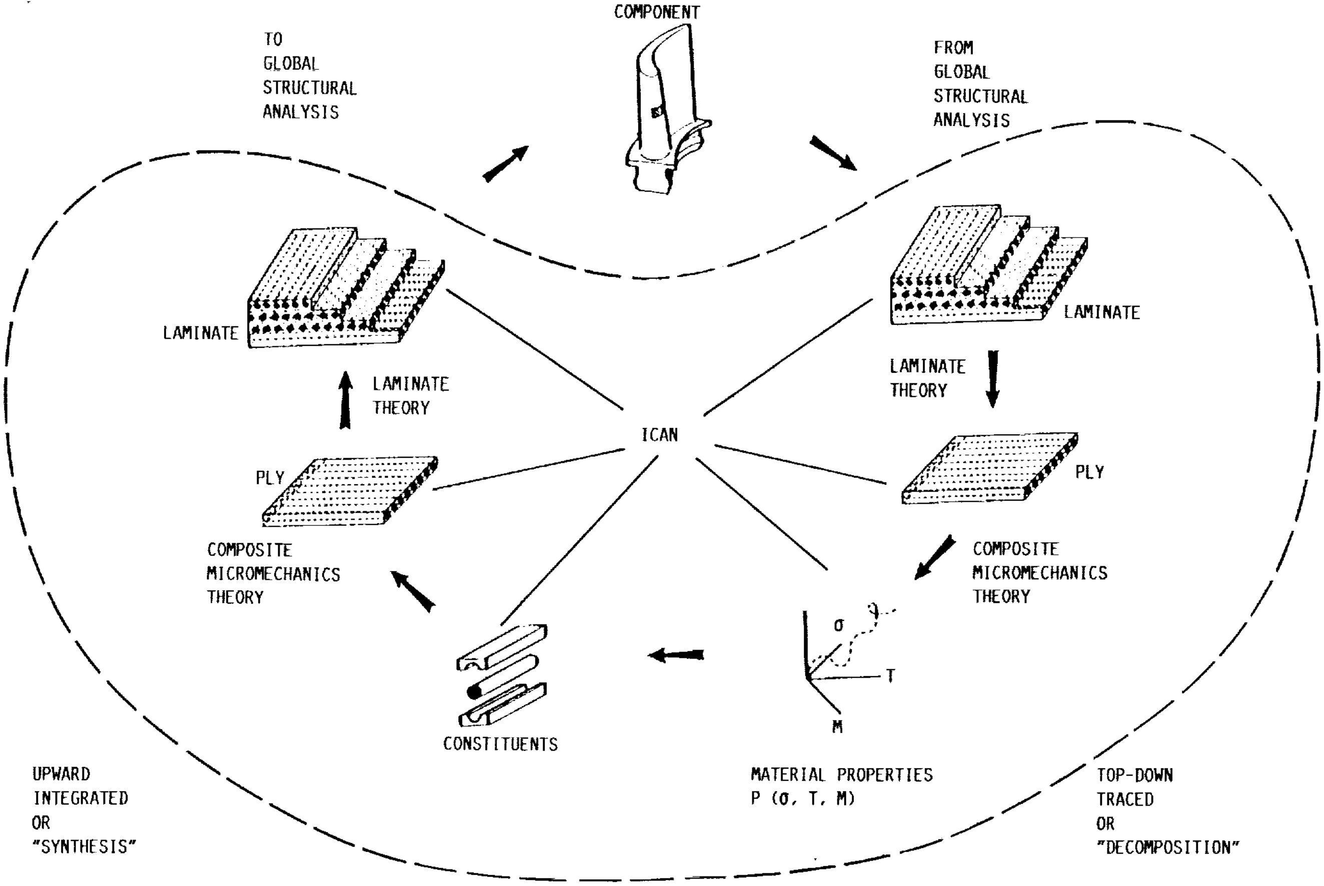


FIGURE 2. - ICAN: INTEGRATED COMPOSITIES ANALYZER.

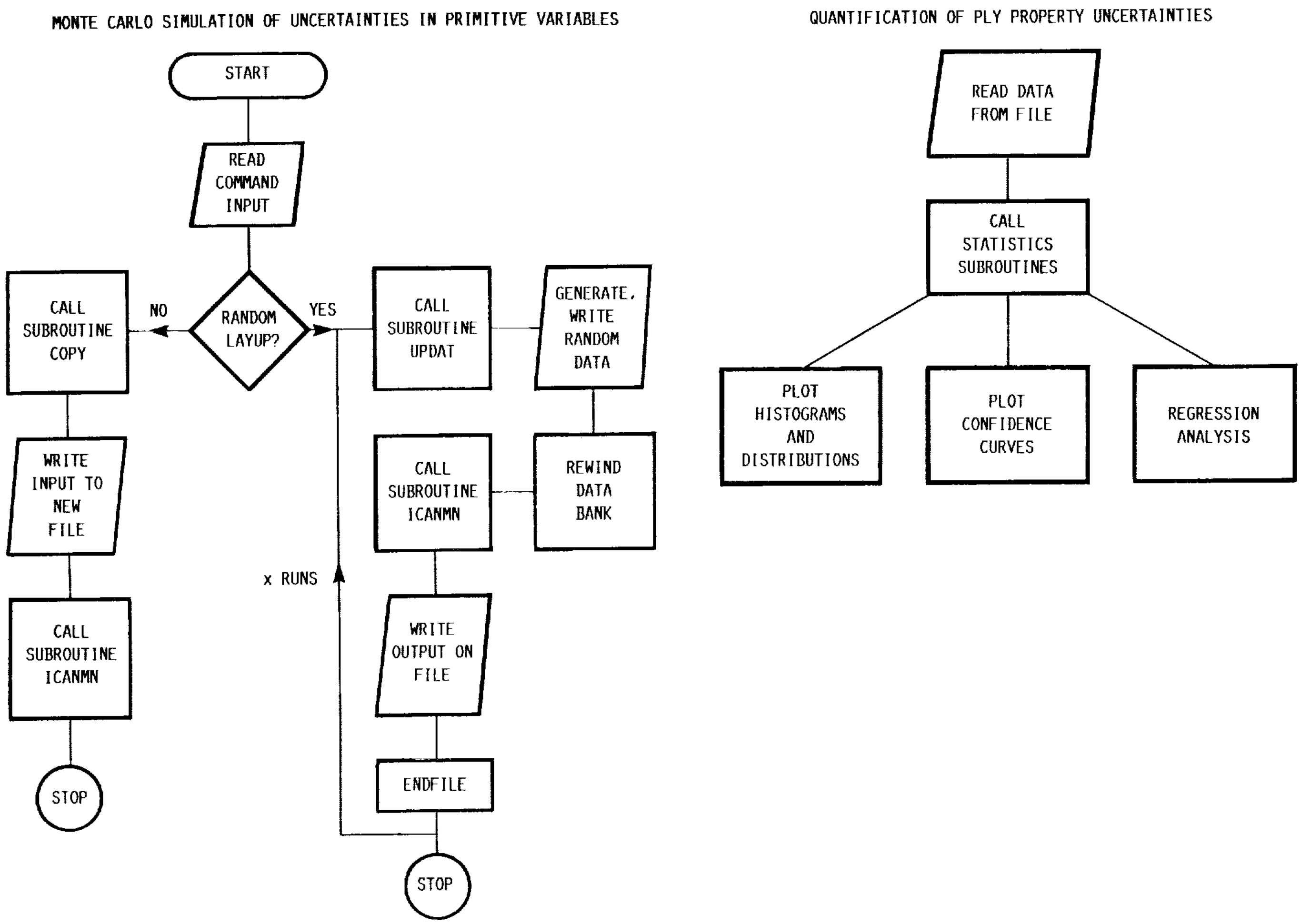


FIGURE 3. - PROBABILISTIC COMPOSITE MICROMECHANICS COMPUTATIONAL PROCEDURE LOGIC DIAGRAMS.

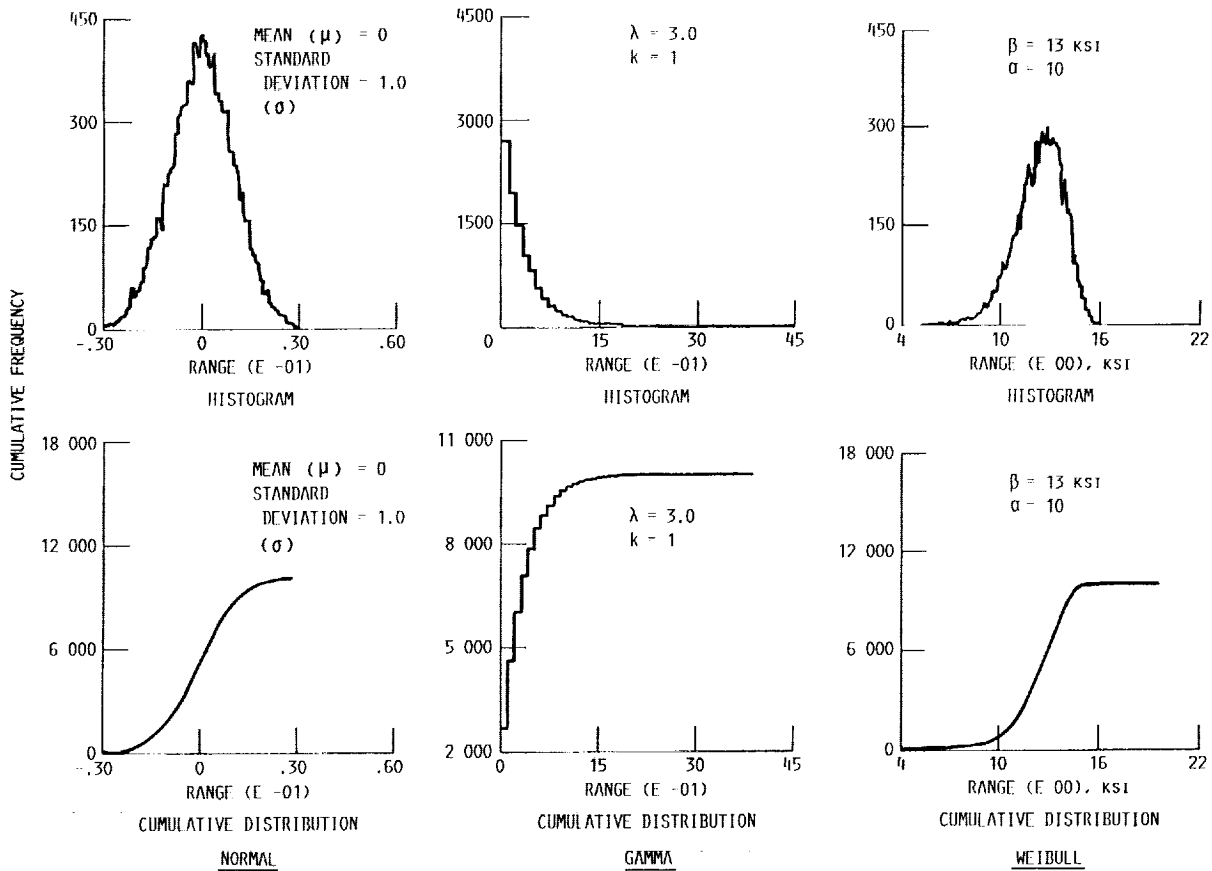


FIGURE 4. - ASSUMED PROBABILITY DENSITY FUNCTIONS FOR UNCERTAINTIES IN PRIMITIVE VARIABLES.

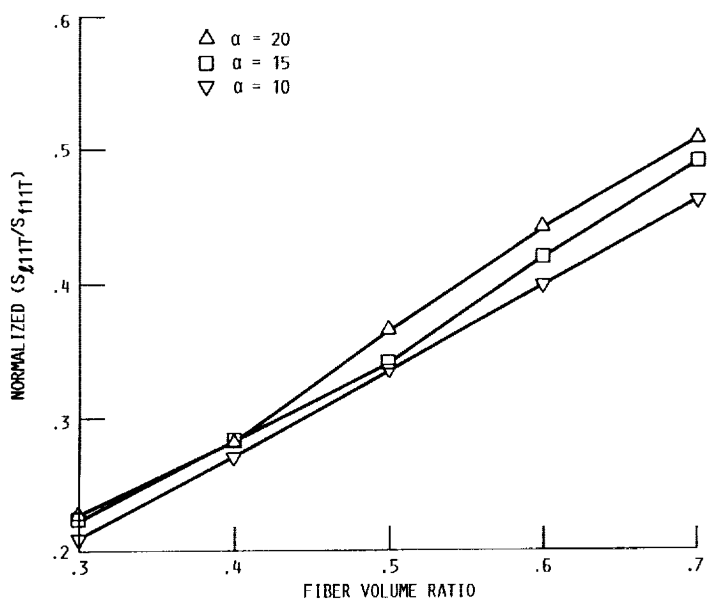


FIGURE 5. - FIBER TENSILE STRENGTH SCATTER EFFECTS ON LONGITUDINAL TENSILE STRENGTH.

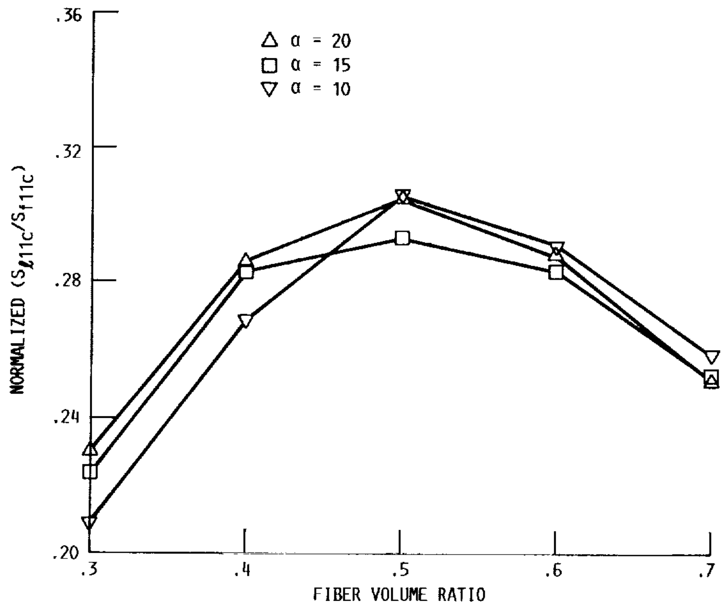


FIGURE 6. - FIBER COMPRESSIVE STRENGTH SCATTER EFFECTS ON LONGITUDINAL COMPRESSIVE STRENGTH.

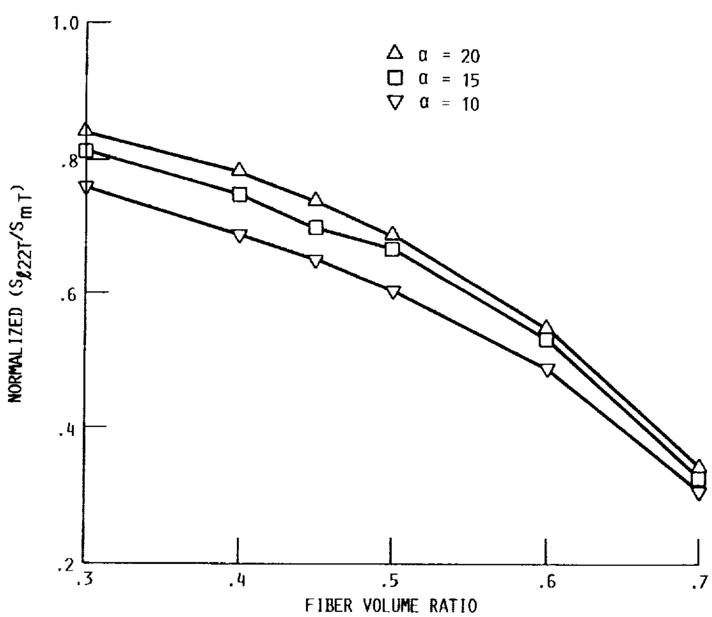


FIGURE 7. - MATRIX TENSILE STRENGTH SCATTER EFFECTS ON TRANSVERSE TENSILE STRENGTH.

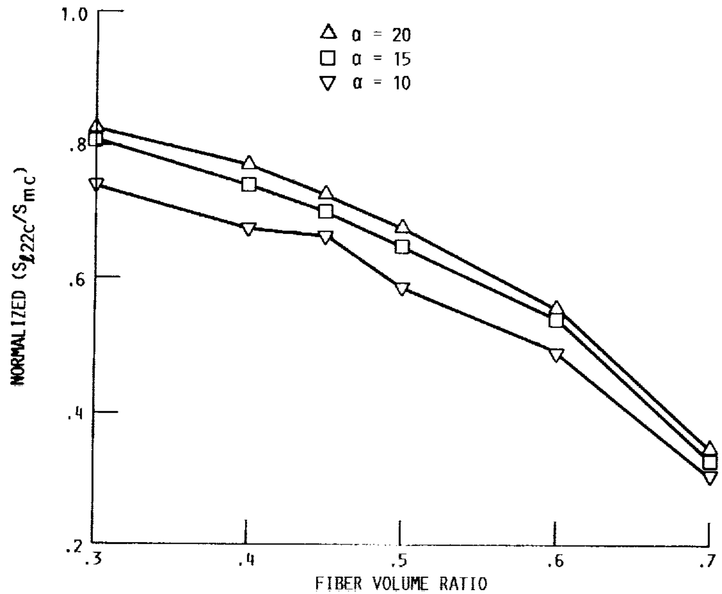


FIGURE 8. - MATRIX TRANSVERSE COMPRESSIVE SCATTER STRENGTH EFFECTS ON TRANSVERSE COMPRESSIVE STRENGTH.

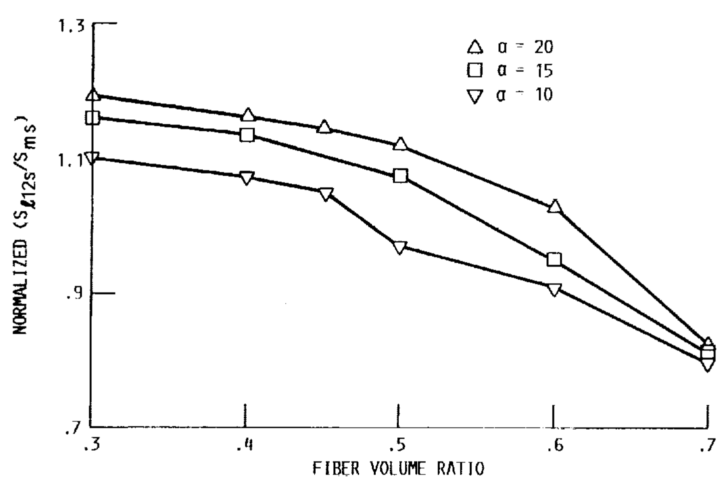


FIGURE 9. - MATRIX SHEAR STRENGTH SCATTER EFFECTS ON IN-PLANE SHEAR STRENGTH.

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16. Abstract

Probabilistic composite micromechanics methods are developed that simulate uncertainties in unidirectional fiber composite strengths. These methods are in the form of computational procedures using composite mechanics with Monte Carlo simulation. The variables for which uncertainties are accounted include constituent strengths and their respective scatter. A graphite/epoxy unidirectional composite (ply) is studied to illustrate the procedure and its ffectiveness to formally estimate the probable scatter in the composite uniaxial strengths. The results show that ply longitudinal tensile and compressive, transverse compressive and intralaminar shear strengths are not sensitive to single fiber anomalies (breaks, interfacial disbonds, matrix microcracks); however, the ply transverse tensile strength is.

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