

20197
N90-16449

ESTIMATING WATER FLOW THROUGH A HILLSLOPE USING THE MASSIVELY
PARALLEL PROCESSOR

J. E. Devaney
Science Applications Research
Lanham, Maryland

P. J. Camillo and R. J. Gurney
NASA/Goddard Space Flight Center
Greenbelt, Maryland

ABSTRACT

A new two-dimensional model of water flow in a hillslope has been implemented on the Massively Parallel Processor at GSFC. Flow in the soil both in the saturated and unsaturated zones, evaporation and overland flow are all modelled, and the rainfall rates are allowed to vary spatially. Previous models of this type had always been very limited computationally. This model takes less than a minute to model all the components of the hillslope water flow for a day. The model can now be used in sensitivity studies to specify which measurements should be taken and how accurate they should be to describe such flows for environmental studies.

INTRODUCTION

One important part of the global hydrological system is a catchment, which separates rainfall into evaporation, overland flow, and infiltration. For a heavy rain, infiltration excess reaches the stream first as overland flow. Part of the infiltrated water may then flow rapidly below the surface to re-emerge downslope or enter the stream. This is usually referred to as saturated subsurface flow. The rest reaches the unsaturated zone. The flow there is vertical and horizontal, and the latter component may eventually contribute to the stream flow. Another component which can contribute to the stream flow is horizontal flow in a perched water table above the bedrock.

The primary output of catchment models is the hydrograph, in which the rainfall and fluxes to the stream from each of the above processes are plotted as a function of time. The rainfall rate and the sum of all the output fluxes are the usual data from a catchment, and a primary goal of catchment modelling is to understand the sensitivity of the output to the physical characteristics of the catchment, such as topography, cover type, soil characteristics, and antecedent moisture.

Ref. 13 define catchment models as being of three basic types, but with overlapping characteristics so they may be considered a continuum. The first is stochastic. These models are statistical, in which time series of measured hydrographs (output) are correlated to rainfall (input) using classical time series analysis techniques. This leads quite naturally to parametric models, their second class, in which the parameters of the stochastic models are related empirically to the physical properties

of the catchment. The third class contains deterministic models based on the laws of conservation of energy, mass, and momentum, usually expressed as time and space dependent differential equations. As these almost always contain non-measurable parameters which must be calibrated, deterministic models are partly parametric.

There are many deterministic catchment models, but none of them includes all of the processes in the hydrological cycle. In part this is because we don't even know what they all are, due to the extreme complexity and variability of natural catchments. However, no existing model even includes all the processes previously described, because no serial computer can model them with a reasonable amount of computer time for a spatially variable catchment and for a long enough time period (Ref. 1,7,8,13,15).

The concept of partial (or contributing) areas is one basis of our understanding of how catchments distribute rainfall (Ref. 17). Due to the spatial variability of catchment characteristics (soils, cover, topography), different areas handle the rain in different ways. For example, if the rain rate exceeds the infiltration capacity for a particular area, then the excess rain becomes overland flow. Once the soil is saturated, the water can flow rapidly below the surface and parallel to it. This process is referred to as saturated subsurface flow. The water will re-emerge somewhere downslope, adding to overland flow. The areas change over time, so the saturated partial area which contributes to overland flow varies in time as well as in space.

We have tried to overcome the computing limitations by developing a model on the Massively Parallel Processor (MPP). The model consists of a set of partial differential equations, solved in parallel, and so adapts naturally to a parallel architecture. The MPP hillslope model includes the following components:

- Surface retention
- A complete surface energy balance (temperature and moisture) with separate evaporation rates from the soil, plants (with water extraction from the unsaturated zone), and surface retention
- Overland flow
- Saturated subsurface flow parallel to the surface

PRECEDING PAGE BLANK NOT FILMED

-- Horizontal and vertical flow in the unsaturated zone

-- Horizontal flow in an unconfined aquifer

Our model is a vertical slice of a hillslope, so it is basically a two-dimensional model. It may be considered three-dimensional only if the gradients are all downslope, not across the slope. It is based on a catchment model of Ref. 11, which is simply a series of uncoupled one-dimensional soil columns placed side by side. We have improved their design by allowing for horizontal flow in the unsaturated zone between the columns, and including the soil and surface temperatures.

We decided at the beginning of this research effort to create one-, two-, and then three-dimensional models in succession. The one-dimensional model (Ref. 6) was compared to a similar one which runs on a serial machine (Ref. 5,10) to make sure the equations are solved correctly on the MPP, and as a timing benchmark. After the two-dimensional model is completely tested, we plan to develop a three-dimensional version.

Our use of a parallel processor significantly reduces the execution time. Typically a 24 hour period may be modeled in about one CPU minute. Ref. 11 state that their model does not use excessive computer time on a serial machine, but they only present results from 6 hour simulations.

THE TWO-DIMENSIONAL MODEL

The specifications for each of the components of the model given in the first section are described here as flux and continuity partial differential equations. The method of solution is also briefly described.

Unsaturated Zone

Moisture flow is modeled as described in Ref. 5, except we now have a horizontal component in the soil moisture flux. The surface temperature is modeled by the force-restore method.

Boundary value fluxes must be specified for moisture at the top and bottom of the hillside (vertical direction) and at the hillslope divide and surfaces (horizontal direction). The top boundary flux is the infiltration or evaporation rate, computed from the surface energy balance. The horizontal flux into the hillslope at the divide is zero. The horizontal flux at the hillslope surface depends on whether that cell is saturated. If it is and the sum of the vertical fluxes plus the horizontal flux into the cell from the interior of the hillslope would cause soil moisture to exceed saturation, then the flux onto the surface is set to whatever value is needed to keep moisture just as saturation. Otherwise, it is zero. This is the mechanism which allows subsurface return flow.

Saturated Zone

The water table height in each column is H_B . The horizontal flux is Q_B , and the vertical flux is Q_Z . The fluxes and vertical boundary conditions are calculated by the one dimensional Boussinesq equation (Ref. 14). The flux into the water table from the unsaturated zone is modeled as the vertical hydraulic conductivity of the layer, and the bottom boundary condition is an input parameter representing an impervious layer or upward or downward seepage.

The flux at the catchment divide is set to zero. At the seepage face the height H_B is a fixed input parameter. Therefore the time derivation of H_B is zero for the last column, and the discretized form of this derivative may be solved for the horizontal flux at the seepage face. This is the saturated zone flux which contributes to the hydrograph.

Overland Flow

If the surface water height is larger than a critical value, the overland flow flux is determined by Manning's equation (Ref. 7).

The infiltration rate is basically the Green-Ampt model (Ref. 9), with the usual modification which replaces the depth of the wetting front with the cumulative infiltration:

$$I(t) = a + b \int_0^t I(t') dt' \quad (1)$$

Surface Energy Balance

The energy balance equation provides the surface fluxes:

$$G = R + LE + LH \quad (2)$$

All fluxes are positive downward. G is the heat absorbed by the soil, R is the net radiation flux, LE is the evapotranspiration energy flux, and H is the sensible heat. After finding the solution, the surface moisture flux q_0 is set equal to the soil evaporation rate, and G is used in the force-restore model. The surface temperature needed to evaluate the fluxes is known from the force-restore equation. The latent and sensible heat fluxes are the usual resistance formulations. We imagine the soil and vegetation as one surface with the temperature T_s . We also allow for some surface water storage. ^s This affects the evaporation rates, because the surface resistance is zero for the fraction of the evaporation which comes from the stored water.

Method of Solution

The soil moisture and temperature continuity equations are solved by calculating the spatial derivatives of the moisture fluxes and then computing the time integral using numerical models.

The soil is divided into cells by creating a grid of N layers and M columns of varying widths Δz_i and Δx_j respectively, which are input parameters. ¹ At a specified time the fluxes at the interior

boundaries are calculated. The surface energy balance equations are evaluated and all boundary conditions applied. The continuity equations are of the form:

$$\frac{dy}{dt} = f(t,y) \quad (3)$$

The vector y represents the state of the system in the unsaturated zone and $f(t,y)$ represents the model equations. This is solved with an Adams-Bashforth predictor-corrector method (Ref. 3,16). This solution is described in detail in Ref. 5. Since double precision is not available on the MPP, the form of the predictor-correction equations with the calculations done with the derivatives instead of the backward differences was used. New values of the state vector, $y(p)(t+\Delta t)$ are predicted in terms of the previous derivatives. The derivatives are recalculated from the model equations, and then the corrected value of the state vector, $y(c)(t+\Delta t)$, is obtained.

The difference between $y(p)$ and $y(c)$ is a reliable estimate of the discretization error, and the software determines if each element of this difference lies within a user-specified window. If all differences are smaller than this window, the integration step size (Δt) is doubled, leading to increased computational efficiency and reduced roundoff errors. If any difference is too large, the step size is halved. Doubling of the time step was accomplished by saving the previously calculated derivatives and using them. Thus, maximum accuracy could be retained. Where the time step could be doubled because the errors are small enough but there were insufficient back derivatives, doubling was postponed until there were sufficient back data. When the error window checks required that the time step be halved, three of the required derivatives for the predictor-corrector were available, and two were missing. The Runge-Kutta method was used to calculate these needed derivatives. The continuity equations for surface and saturated flow are solved using a Runge-Kutta method throughout.

UTILIZING THE MPP ARCHITECTURE FOR SPEED

Since identical calculations were needed at each soil cell, the mapping of the two dimensional model of the hillslope was accomplished by assigning an individual processing element to each soil cell (see Fig. 1). Thus, the local memory of each processor contains the values which belong to that cell, i.e. moisture, position, thickness, depth, conductivities, etc. Surface temperature, deep soil temperature, cumulative infiltration, overland flow, and saturated flow were all stored as vectors in the same array as the moisture values since they were part of the state vector.

The first step in the solution required calculation of the fluxes at the interior boundaries of the

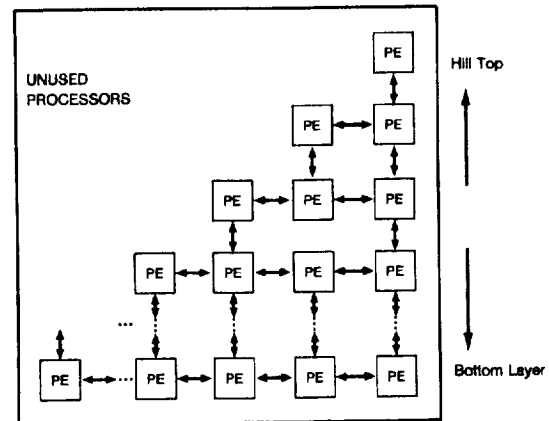


Figure 1. One processing element is assigned to one soil cell

soil cells. These calculations involved only array arithmetic and nearest neighbor (in one direction for horizontal fluxes and in the other direction for vertical fluxes) calculations. Since the interconnect scheme of the MPP is a nearest neighbor network, all of the array arithmetic and nearest neighbor calculations could be done in parallel. The next step in the solution required the surface energy balance equations be evaluated and the boundary conditions applied. These all involved vector calculations. Numerous input vectors were required to do these calculations over the course of a model run. Some were time dependent vectors such as the air temperature across the surface of the hillslope throughout the day and some were static throughout the model run, such as surface slope, surface roughness, and surface vegetation properties. These vectors were packed into array columns. To get the vector data to a convenient place to do calculations, the row and column broadcast capability of the MPP was used. This allows fast broadcast of one element from each row (column) to the other processor memories in the same row (column) (see Fig. 2).

It is not necessary that the broadcast row (column) be composed only of elements in a horizontal (vertical) direction but merely that one element per column (row) be selected. The MPP's capability to select arbitrary areas of an array for calculation via boolean masks allowed the completed vector calculation results to be placed for example into the processor memories of only the surface of the hillslope. This combination of data movement via broadcast and boolean selection enabled the vector calculations to be done simply. In addition, since many of the vector calculations were similar, it was possible to do more than one set at a time.

Once the derivatives were calculated, the predictor-corrector equations were used and the differences between them found. The tests on the

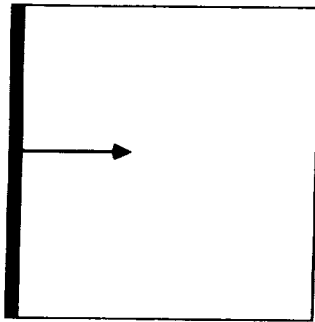


Figure 2. The row and column broadcasting feature of the MPP allows quick movement of data for vector calculations

halving (doubling) converted to a hardware instruction on the MPP and could thus be done in parallel. This global testing ability of the MPP was also used to decide if whole blocks of code needed to be executed or could be skipped. This occurred for example with the infiltration calculations under surface saturation. If no part of the surface was saturated, then these calculations could be skipped entirely. This also contributed to the overall speed of execution.

In summary, the program's speed was achieved through array arithmetic (masked and unmasked), parallel data movement through nearest neighbor communication and row and column broadcasting, and global testing of conditions using 'any' or 'all' for the purpose of choice in the next set of calculations. All of these fitted naturally with the MPP architecture and the computational requirements of the model. A comparison of the times (see Table 1) for the model as it has evolved from a 14 layer, one-dimensional limited flow model to the current two dimensional model shows that a single day of data run through the model requires only about a minute of CPU time.

Table 1. Timing measurements comparing MPP and a serial processor for 24 hours of data processed.

One Dimensional Model	
(14 soil layers, no rain, vertical flows only)	
IBM (Full processing capability):	4 sec
MPP (14/16384 processors):	10 sec
Two Dimensional Model	
(102 soil layers, 102 soil columns, horizontal and vertical unsaturated flows, saturated flow, overland flow, one hour of rain)	
MPP:	57 sec

MODEL OUTPUT

We have not yet completed unit testing of all the processes in the model. Here we present the results of one test, which includes the surface energy balance of and infiltration into an initially very dry sandy loam soil.

The hillslope is divided into 102 columns of width .5 meters each. The first column has 100 soil layers of thickness .1 m and the bottom two layers .5 meters. The last column has only the bottom two layers. The slope is a line drawn from the top of the first column to the top of the last, so the area modeled is a right triangle with height 11 meters and base 60 meters. These soil cells plus the additional cells for temperature, infiltration, overland flow and saturated flow use approximately one-third of the Array Unit Processor capacity.

The initial volumetric moisture in the unsaturated zone is set to $.05 \text{ m}^3 \text{ m}^{-3}$ everywhere. To model a sandy loam we have set the parameters in the hydraulic conductivity and matric potential models to $\theta^s = .375$, $K^s = 2.8 \times 10^{-5} \text{ m s}^{-1}$, $\psi^s = -.43 \text{ m}$, and $b^s = 5$. These values were derived from fits to the characteristic curves measured during an experiment near Phoenix in 1972 (Ref. 12). They were reused for each of the 6 days modeled here. Ref. 4 show how these data were fitted to the surface energy balance model. The rainfall rate was 1.6 cm h^{-1} for the first 3 hours.

Perhaps the most important result is that the simulations took approximately 1 minute of CPU time per 24 hour period, or 6 minutes for the entire 6 day run. In numerical simulations on earth science problems, computer runs of an hour or more are not uncommon. In such a time, it is feasible to simulate 2 months or more of model time on the MPP. This will allow for simulations of many storms and inter-storm periods.

Figure 3a and 3b show the force restore solutions to the surface and deep soil temperatures as functions of time and column number. Time zero is the start of the simulation, which here is midnight. Column 1 is at the hillslope divide and column 102 is at the seepage face. It is difficult from these plots to project the daily maximum value onto the time axis, but for each day this occurs at 2 p.m. The temperatures range from 22 to 40 ($^{\circ}\text{C}$), increasing as the soil surface dries. The temperatures in the last three columns show some problems, which we are examining.

Figure 4a shows soil moisture in the top soil layer as a function of time and position. The rapid rise as the initially dry soil absorbs all the rain and the subsequent decline over the next 5 days are physically realistic.

Figure 4b shows the soil moisture profile in column 50 (halfway down the hillslope) as a function of time. This shows that the moisture never penetrates deeper than the top 5 layers, or .5 meters. It also shows that after 2 days the surface exhibits small oscillations about a value of .05 (same as in Fig. 4a), increases to a value

ORIGINAL PAGE
BLACK AND WHITE PHOTOGRAPH

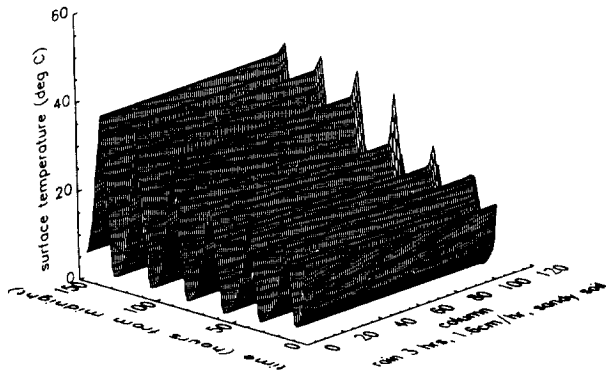


Figure 3a. Force-restore solution for the surface temperature as a function of time and position on the hillslope

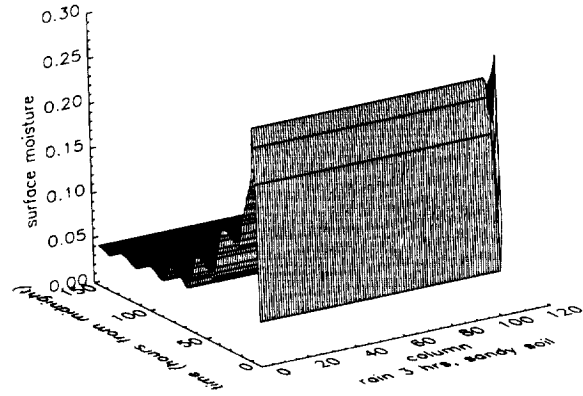


Figure 4a. Surface soil moisture as a function of time and position on the hillslope

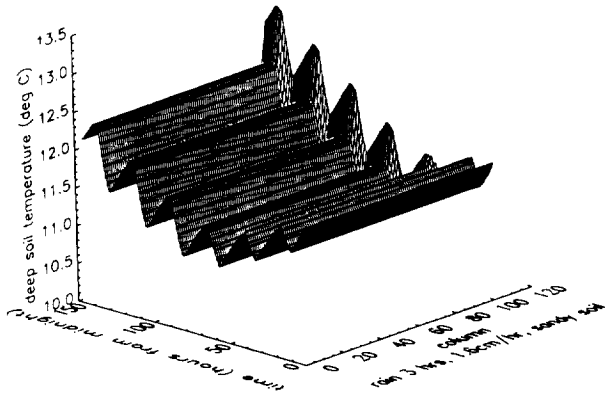


Figure 3b. Force-restore solution for the deep soil temperature as a function of time and position on the hillslope

of about .12 at about .3 meters then decreases to an unchanging value of .05 below .5 meters. Thus, the dynamic zone seems to be the top .5 meters.

Figure 4c shows the variation of the top cell soil moisture as a function of time. The effects of infiltration and evaporation, as well as of capillary action, can be seen.

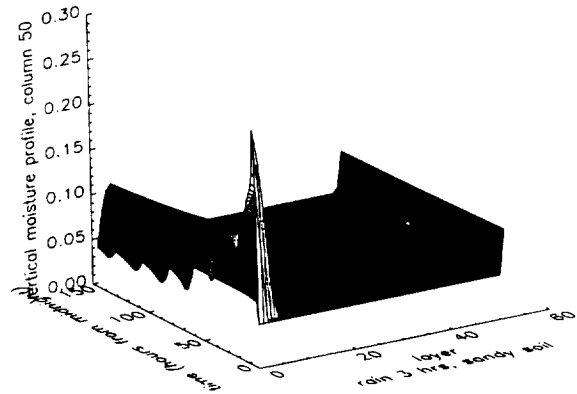


Figure 4b. Soil moisture profile for column 50 as a function of time

Figure 5a shows the infiltration rate as a function of time and position. The maximum rate shown here ($4.4 \times 10^{-4} \text{ cm s}^{-1}$) equals the rain rate, 1.6 cm h^{-1} . Figure 5b shows the cumulative evaporation everywhere as 4.8 cm, exactly equal to the cumulative rainfall. For this simulation, then, all the rain immediately infiltrated into the soil surface. Figure 5b also shows that the cumulative infiltration calculation is correct. There is no surface retention.

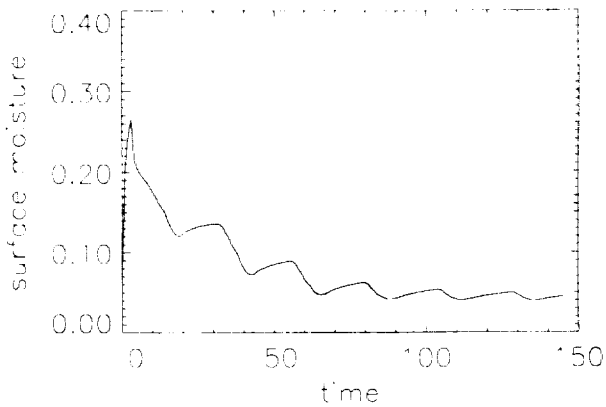


Figure 4c. Surface moisture for column 50 as a function of time

radiation, a rather large value. The problem is not in the values for thermal conductivity and heat capacity, as may be seen in Figure 7. These vary with soil moisture as they should.

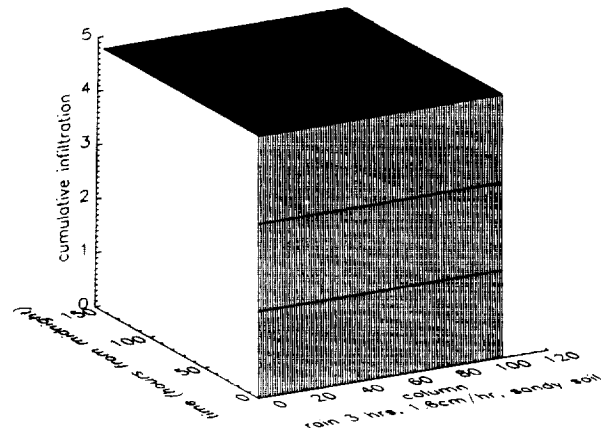


Figure 5b. Cumulative infiltration as a function of position and time

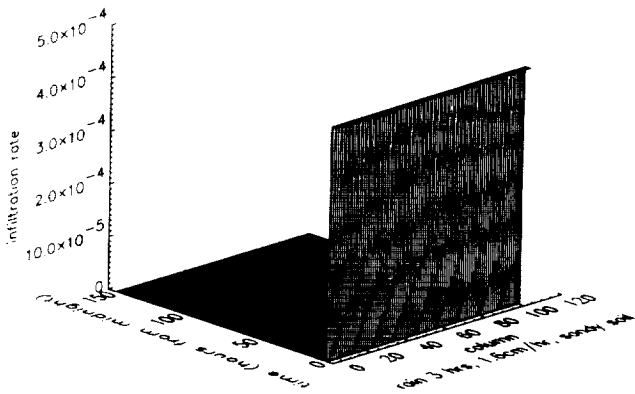


Figure 5a. Infiltration rate as a function of position and time

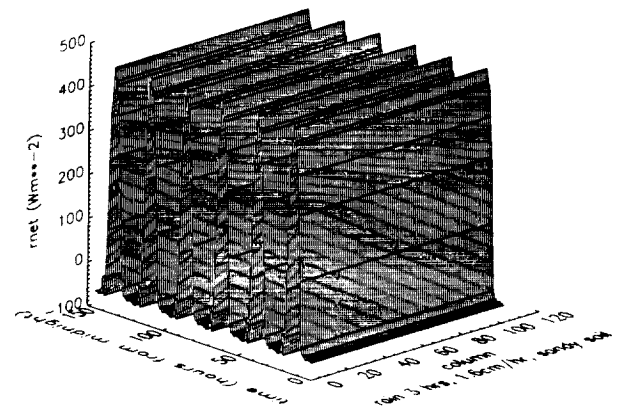


Figure 6a. Surface net radiation as a function of time and position

The surface energy balance fluxes are plotted in Figs. 6a-6d. The net radiation (Fig. 6a) is the data used to drive the energy balance model. These are the same every day, as we simply reused the 24 hour data set each day. The latent heat flux (Fig. 6b) decreases each day as the soil dries out. The sensible heat flux (Fig. 6c) exhibits peculiar behavior, being predominantly positive (towards the soil in the sign convention of Eq. 2) for the first 4 days and negative thereafter. Finally, Figure 6d shows the soil heat flux. It is positive during the day as it should be for a soil surface which is getting warmer every day, but it is also 50% of net

These peculiarities in the surface fluxes are most likely due to the use of the same net radiation every day, which cannot be representative of all the surface conditions modeled here. This is being checked out by using modeled instead of measured radiation.

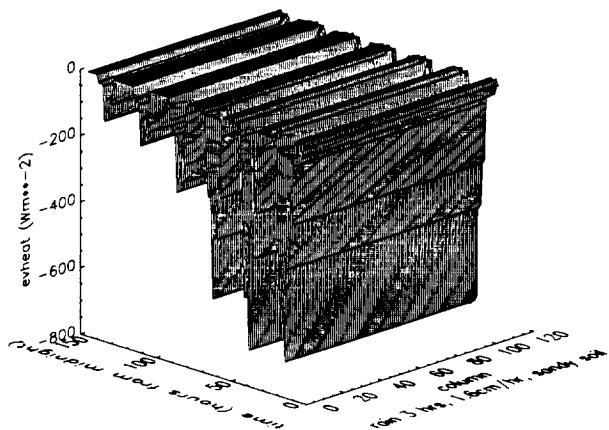


Figure 6b. Latent heat flux as a function of time and position

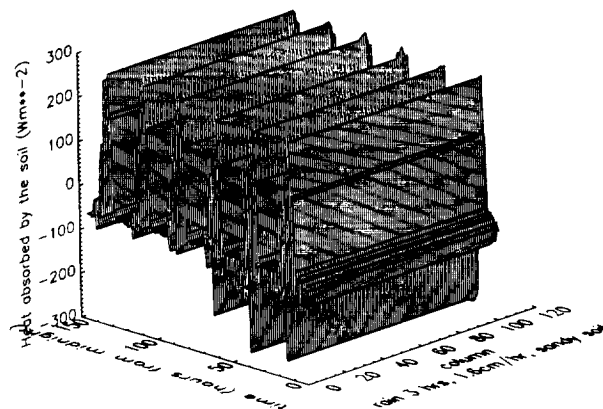


Figure 6d. Soil heat flux as a function of time and position

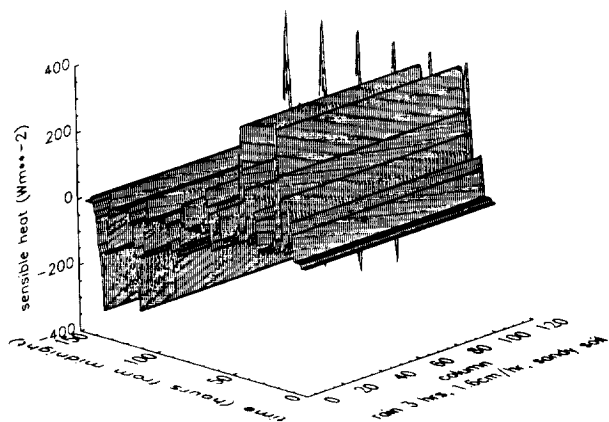


Figure 6c. Sensible heat flux as a function of time and position

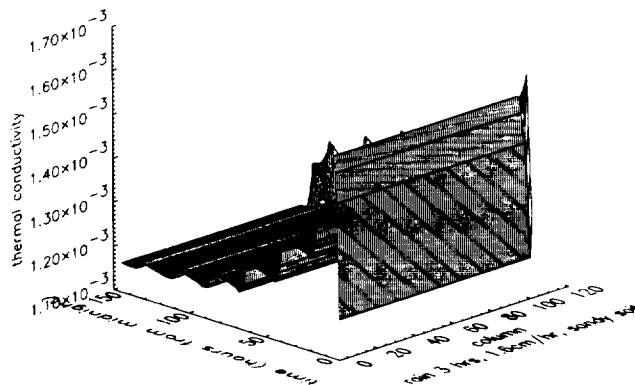


Figure 7a. Thermal conductivity of the top soil layer as a function of time and position

SUMMARY

We have presented a new model of the hydrological response of a hillslope to rain. It runs on a SIMD parallel architecture computer, the Massively Parallel Processor, at Goddard Space Flight Center. Its major advantage over other models of its type is its much reduced execution times (due to the parallel architecture of the MPP) from what one gets on a serial machine. This allows the model to include more of the hydrological processes than any other model has been able to, including saturated subsurface flow and a sophisticated surface energy balance.

ACKNOWLEDGEMENT

One of the authors, J. E. Devaney, was partially supported by NASA Contract NAS5-28200.

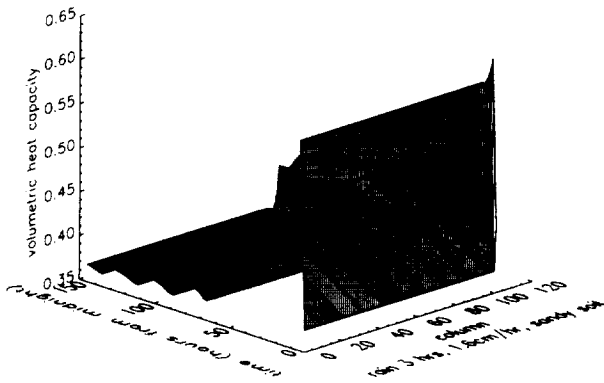


Figure 7b. Heat capacity of the top soil layer as a function of time and position

REFERENCES

1. Bathurst, J. C., "Physically-based Distributed Modeling of an Upland Catchment Using the Systeme Hydrologique Europeen," Institute of Hydrology, Wallingford, Oxon, UK.
2. Becker, F., P. J. Camillo and B. Choudhury, "Review of ET Estimation by Means of Satellite Data," Proceedings of the ISLSCP Conference on Satellite Data Algorithms, Pasadena, CA, January 1987.
3. Devaney, J. E., P. J. Camillo and R. J. Gurney, "A SIMD Implementation of a One-Dimensional Energy-Moisture Balance Model," submitted to Proceedings of the ISLSCP Conference on Satellite Data Algorithms, Pasadena, CA, January 1987.
4. Booth, A. D., Numerical Models, Academic Press, New York, 1957.
5. Camillo, P. J. and R. J. Gurney, "A Resistance Parameter for Bare Soil Evaporation Models," Soil Science, 141, 1986, 95-105.
6. Camillo, P. J., R. J. Gurney and T. J. Schmugge, "A Soil and Atmospheric Boundary Layer Model for Evapotranspiration and Soil Moisture Studies," Water Resources Research, 19, 1983, 371.
7. Devaney, J. E., P. J. Camillo and R. J. Gurney, "A SIMD Implementation of a Distributed Watershed Model," Proceedings of the Second International Conference on Supercomputing, Santa Clara, California, May 3-8, 1987.
8. Eagleson, P. S., Dynamic Hydrology, McGraw-Hill, New York, NY, 1970.
9. Freeze, R. A., "Three-Dimensional, Transient, Saturated-Unsaturated Flow in a Groundwater Basin," Water Resources Research, 7, 1971, 929-941.
10. Green, W. A. and H. A. Ampt, Studies on Soil Physics, 1. The Flow of Air and Water Through Soils, Journal of Agricultural Soils, 4, 1911, 1-24.
11. Gurney, R. J. and P. J. Camillo, "Modelling Daily Evaporation Using Remotely Sensed Data," Journal of Hydrology, 69, 1984, 305-324.
12. Hillel, D. and G. M. Hornberger, "Physical Model of the Hydrology of Sloping Heterogeneous Fields," Soil Science Soc. Am. J., 43, 1979, 434-439.
13. Jackson, R. D., "Diurnal Changes in Soil-Water Content During Drying," in Field Soil Water Regime, R. R. Bruce et al. (eds.), Soil Sci. Soc. Am. Proc, Special Publ. 5, 1973.
14. Kirkby, M. J. (ed.), Hillslope Hydrology, Wiley-Interscience, Chichester, UK, 1972.
15. Pikul, M. F., R. L. Street, I. Remson, "A Numerical Model Based on Coupled One-Dimensional Richard and Boussinesq Equations," Water Resources Research, 10, 1974, 295-302.
16. Smith, R. E. and D. A. Woolhiser, "Overland Flow on an Infiltrating Surface," Water Resources Research, 7, 1971, 899-913.
17. Teddington, A., Modern Computing Methods, Philosophical Library, New York, 1958.
18. van de Griend, A. A. and E. T. Engman, "Partial Area Hydrology and Remote Sensing," Journal of Hydrology, 81, 211-251.