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Screening Actuator Locations for Static Shape Control

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Screening Actuator Locations

for Static Shape Control

by

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Abstract

This paper deals with correction of shape distortion due to zero-mean normally distributed errors in structural sizes which are random variables. A bound on the maximum improvement in the expected value of the root-mean-square shape error is obtained. The shape correction associated with the optimal actuators is also characterized. An actuator effectiveness index is developed and shown to be helpful in screening actuator locations in the structure. The results are specialized to a simple form for truss structures composed of **nominally identical members**. The bound and effectiveness index are tested on a 55-m radiometer antenna truss structure. It is found that previously obtained results for optimum actuators had a performance close to the bound obtained here. Furthermore, the actuators associated with the optimum design are shown to have high effectiveness indices. Since only a small fraction of truss elements tend to have high effectiveness indices, the proposed screening procedure can greatly reduce the number of truss members that need to be considered as actuator sites.

Introduction.

Manufacturing errors are recognized as a major cause of shape distortion in large space

structures (e.g. Ref. 1). These errors, as well as many other sources of distortion (such as variations in the coefficient of thermal expansion), are random in nature, and a statistical analysis needs to be carried out to estimate the resulting shape distortion (e.g. Refs. 1-3). When actuators are used to reduce shape distortion, the corrected shape is also random in nature and requires a more complex and costly statistical analysis (Ref. 3).

Trusses are one candidate for large space structures. When these trusses are used as backup structures for electromagnetic reflectors, maintaining accurate surface shape is important. Optimizing locations of actuators used to reduce shape distortion on large trusses is a formidable problem because of the discrete nature of the problem. Actuators are typically located in truss elements, and given m truss elements and n actuators we have $\binom{n}{m}$ possibilities of locating them. Several ad hoc methods for tackling this combinatorial problem were proposed (e.g. Refs 4-6). However, these methods are not guaranteed to converge even to a local optimum, and therefore the quality of the actuator configuration they produce is difficult to evaluate.

The present paper has two objectives. The first objective is to show that under some reasonable conditions it is possible to obtain a lower bound to the expected value of the root-mean-square (rms) of the corrected distortion. The second objective is to develop an effectiveness index for truss members that can be used to screen them as potential sites for actuators. This is accomplished by characterizing the correction shape associated with optimal actuators, and comparing the correction shape associated with an actuator located in a given truss member with the optimum shape. Results obtained by a heuristic optimization algorithm in Ref. 3 are used to validate the bound and effectiveness index.

Linear Distortion Correction.

The structure is assumed to have a set of n_s sensors that measure n_s components of the distortion field, and a set of n actuators used to correct the distortion. Denoting the vector

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of sensor-measured components of the distortion as $\hat{\psi}$, we assume that the actuators seek to minimize a weighted root-mean-square (rms) measure of the distortion ψ_{rms} defined as

$$\psi_{rms}^2 = \hat{\psi}^T B \hat{\psi} \tag{1}$$

where B is a positive semidefinite weighting matrix.

It is convenient to work in a transformed (modal) coordinate system where the matrix B is the unit matrix. That is we write $\hat{\psi}$ as a linear combination of modes ψ_i $i = 1, ..., n_s$

$$\hat{\psi} = \Phi \psi \tag{2}$$

where the matrix Φ is composed of columns ϕ_i which are orthonormal with respect to B. That is

$$\Phi^T B \Phi = I \tag{3}$$

where I denotes the unit matrix. Then from Eqs. (1) - (3)

$$\psi_{rms}^2 = \psi^T \psi \tag{4}$$

We assume that the behavior of the structure and actuators is linear, so that if a unit action of the ith actuator produces a displacement vector u_i, then the corrected shape vector δ is given as

$$\delta = \psi + \sum_{i=1}^{n} u_i \theta_i = \psi + U\theta \tag{5}$$

where θ is a vector of actuator outputs (with components θ_i) and U is a matrix with columns u_i. Note that the components of all of these vectors are modal (amplitude) coordinates with the physical vector (denoted with a hat) obtained by the modal transformation, e.g.,

$$\hat{u}_i = \Phi u_i \tag{6}$$

The rms of the corrected distortion is

$$\delta_{rms}^2 = \hat{\delta}^T B \hat{\delta} = \delta^T \delta \tag{7}$$

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The optimum vector θ which minimizes δ_{rms} is easily shown (e.g., Ref. 4) to be the solution of the system

$$A\theta = r \tag{8}$$

where

$$A = U^T U \tag{9a}$$

$$r = -U^T \psi \tag{9b}$$

and then the corrected shape δ is

$$\delta = \psi - U A^{-1} U^T \psi \tag{10}$$

and

$$\delta^2_{rms} = \psi^2_{rms} - \psi^T U A^{-1} U \psi \tag{11}$$

Statistical Analysis.

Let the distortion field be due to a set of errors or disturbances in the structure characterized by their amplitude parameters ϵ_i $i = 1, ..., N$. Often the statistical properties of the ϵ_i's are known and we want to obtain the statistics of ϕ and δ. Since the behavior of the structure is linear the total distortion due to the N errors, ψ, is given as

$$\psi = \sum_{i=1}^{N} \epsilon_i \psi_i = \Psi e \tag{12}$$

where ψ_i is the shape distortion due to a unit ϵ_i, Ψ is a matrix with ψ_i as its ith column, and e is the vector of ϵ_i's.

We assume that the ϵ_i's are zero-mean, normally distributed random variables with a covariance matrix Σ. From Eq. (12) the covariance matrix of ψ is

$$C_{\psi\psi} = E(\psi\psi^T) = \Psi E(ee^T)\Psi = \Psi\Sigma\Psi^T \tag{13}$$

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where E denotes the expected value. Using Eq. (4) we get that the expected value of ψ^2_{rms} is

$$E(\psi^2_{rms}) = \sum_{i=1}^{n_s} (C_{\psi\psi})_{ii} \qquad (14)$$

that is, the trace of $C_{\psi\psi}$. The expected value of δ^2_{rms} is obtained by rewriting Eq. (11) as

$$\delta^2_{rms} = \psi^2_{rms} - W^T W \qquad (15)$$

where

$$W = L^T U^T \psi \qquad (16)$$

and L is a square factor of A^{-1} (i.e., $A^{-1} = LL^T$, e.g., the Cholesky factor). The covariance matrix of W is then given as

$$C_{WW} = L^T U^T C_{\psi\psi} U L \qquad (17)$$

and

$$E(\delta^2_{rms}) = \sum_{i=1}^{n_s} (C_{\psi\psi})_{ii} - \sum_{i=1}^{n} (C_{WW})_{ii} \qquad (18)$$

The effectiveness of the actuators is measured by the distortion ratio, g, defined as

$$g^2 = \frac{E(\delta^2_{rms})}{E(\psi^2_{rms})} \qquad (19)$$

Optimum Actuators

The problem of optimizing the properties of the actuators (such as location) requires the minimization of $E(\delta^2_{rms})$ which is the same as the maximization of the trace of C_{WW}. We rewrite Eq. (17) as

$$C_{WW} = U^{*T} C_{\psi\psi} U^* \qquad (20)$$

where

$$U^* = UL \qquad (21)$$

First we note that U^* consists of a set of orthonormal vectors. Indeed

$$(U^{*T} U^*)^{-1} = (L^T U^T U L)^{-1} = (L^T A L)^{-1} = L^{-1} A^{-1} L^{-T} = L^{-1} L L^T L^{-T} = I \qquad (22)$$

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It is shown in Ref. 7 (theorem 1) that where λ_j are the eigenvalues of $C_{\psi\psi}$ arranged as

$$\max_{U^*} \sum_i (C_{WW})_{ii} = \sum_{j=n_s-n+1}^{n_s} \lambda_j \tag{23}$$

where λ_j are the eigenvalues of $C_{\psi\psi}$ arranged as

$$0 \leq \lambda_1 \leq \lambda_2 \ldots \leq \lambda_{n_s} \tag{24}$$

We are able to realize this maximum by taking the columns of U^* to be the eigenvectors v_j, $j = n_s - n + 1, ..., n$ corresponding to these largest eigenvalues.[+] From Eq. (21) we see that this is equivalent to taking U as a set of linearly independent combinations of these eigenvectors. Altogether we have that

$$E(\delta^2_{rms})_{opt} = \sum_{i=1}^{n_s} (C_{\psi\psi})_{ii} - \sum_{j=n_s-n+1}^{n_s} \lambda_j \tag{25}$$

For most problems it will be impossible to select actuators that produce displacement fields which are exact linear combinations of the eigenvectors v_j corresponding to the largest eigenvalues of $C_{\psi\psi}$. However, it may be possible to come close. In particular, we can judge the suitability of an actuator by checking how close is its displacement field to a linear combination. If the eigenvectors v_j of $C_{\psi\psi}$ are normalized to unit length

$$v_j^T v_j = 1 \tag{26}$$

then an effectiveness index of the ith actuator (which produces the displacement field u_i) is q_i

$$q_i = \left(\sum_{j=n_s-n+1}^{n_s} v_j^T u_i / u_i^T u_i \right)^{1/2} \tag{27}$$

which is the cosine of the angle between u_i and the hyperplane spanned by the last n eigenvectors. Only actuators with an effectiveness index close to one are likely to be good actuators.

[+] From the derivation in [7] it is also clear that any linear combination of these eigenvectors will do.

Applications to Truss Structures.

Consider a truss structure composed of a large number (N) of members of similar lengths, and where the shape distortion is due mostly to length errors (typically manufacturing errors). We assume that the length errors are uncorrelated and have the same standard deviation σ_ϵ so that the covariance matrix Σ is

$$\Sigma = \sigma_\epsilon^2 I \tag{28}$$

To obtain the modes ϕ_j we expand the matrix B to the full set of finite-element degrees of freedom by adding rows and columns of zeroes denote it as $\bar{B}$, treat it as a mass matrix and solve for the vibration modes from

$$(K - \omega_j^2 \bar{B})\bar{\phi}_j = 0 \tag{29}$$

where $\bar{\phi}_j$ denotes the jth B-vibration mode (excluding rigid body modes). The sensed components of $\bar{\phi}_j$ are extracted as

$$\phi_j = T\bar{\phi}_j \tag{30}$$

where T is a Boolean matrix. Note that

$$B = T^T \bar{B} T \tag{31}$$

Because the rank of $\bar{B}$ is smaller than or equal to n_s, only n_s or less of the frequencies ω_j have finite values. The infinite frequencies correspond to zero-rms modes satisfying

$$\bar{\phi}^T \bar{B} \bar{\phi}_j = \phi_j^T B \phi_j = 0 \tag{32}$$

To calculate the distortion associated with length error Δl_i in the ith member (corresponding to $\epsilon_i = 1$) we apply a force vector F_i to the truss and solve

$$K\bar{\psi}_i = \Delta l_i F_i \tag{33}$$

where F_i consists of a pair of forces colinear with the ith member of magnitude

$$F_i = (EA)_i / l_i \tag{34}$$

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where $(EA)_i$ and l_i denote the axial rigidity and length, respectively, of the ith member. We can then extract the sensed components of $\bar{\psi}_i$

$$\hat{\psi}_i = T\bar{\psi}_i \tag{35}$$

The displacement vector can be written in terms of modal coordinates

$$\bar{\psi}_i = \sum_{j=1}^{n_s} \psi_{ij}\bar{\phi}_j \tag{36}$$

where

$$\psi_{ij} = \bar{\phi}_j^T \bar{B}\bar{\psi}_i = \phi_j^T B\hat{\psi}_i \tag{37}$$

Equation (37) implies that the modal coordinates of $\bar{\psi}_i$ and of $\hat{\psi}_i$ are the same. This is a result of the fact that the modes beyond n_s do not contribute to the rms

$$(\psi_i^2)_{rms} = \hat{\psi}_i^T B\hat{\psi}_i = \bar{\psi}_i^T \bar{B}\bar{\psi}^i = \sum_{j=1}^{n_s} \psi_{ij}^2 \tag{38}$$

The modal coordinates can be calculated without solving first Eq. (33) by premultiplying it by $\bar{\phi}_j^T$ and using Eq. (36) to obtain

$$\bar{\phi}_j^T K \sum_{\ell=1}^{n_s} \psi_{i\ell}\bar{\phi}_\ell = \Delta l_i \bar{\phi}_j^T F_i \tag{39}$$

But

$$\bar{\phi}_j^T K \bar{\phi}_\ell = \omega_j^2 \delta_{jl} \tag{40}$$

where $\delta_{j\ell}$ is the Kronecker delta. Altogether we get

$$\psi_{ij} = \Delta l_i \phi_j^T F_i / \omega_j^2 \tag{41}$$

We can simplify expressions by rewriting Eq. (40) as

$$\bar{\phi}_j^T K \bar{\phi}_\ell = \sum_{i=1}^{N} \bar{\phi}_j^T K_i \bar{\phi}_\ell \tag{42}$$

where K_i is the stiffness matrix of the ith element. Then

$$\bar{\phi}_j^T K \bar{\phi}_\ell = \sum_{i=1}^{N} (\phi_j^T F_i)(\phi_\ell^T F_i)l_i / (EA)_i = \omega_j^2 \delta_{j\ell} \tag{43}$$

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Finally using Eqs. (13), (28) and (41) we get

$$(C_{\psi\psi})_{j\ell} = \sigma_\epsilon^2 \sum_{i=1}^{N} \psi_{ij}\psi_{i\ell} = \frac{\sigma_\epsilon^2}{\omega_j^2\omega_\ell^2} \sum_{i=1}^{N} \Delta l_i^2 (\phi_j^T F_i)(\phi_\ell^T F_i) \tag{44}$$

Using Eqs. (14), (19), (25) and (44) we can calculate the optimum distortion ration (g^2) opt. However, further simplification is possible since in comparing Eqs. (43) and (44) we see that if

$$\Delta l_i^2 (EA)_i / l_i = c \tag{45}$$

then

$$(C_{\psi\psi})_{j\ell} = \frac{c\sigma_\psi^2}{\omega_\ell^2}\delta_{j\ell} \tag{46}$$

This happens, in particular, when all the members of the truss have identical nominal properties. In this case since $C_{\psi\psi}$ is diagonal its eigenvalues are equal to the diagonal elements and Eq. (25) becomes

$$E(\delta_{rms}^2)_{opt} = \sum_{j=n+1}^{n_s} \lambda_j = c\sigma_\psi^2 \sum_{j=n+1}^{n_s} \frac{1}{\omega_j^2} \tag{47}$$

and the distortion ratio of Eq. (12) becomes

$$(g^2)_{opt} = \frac{\displaystyle\sum_{j=n+1}^{n_s} \frac{1}{\omega_j^2}}{\displaystyle\sum_{j=1}^{n_s} \frac{1}{\omega_j^2}} \tag{48}$$

Note that in this case the eigenvectors of $C_{\psi\psi}$ can be taken to be the unit coordinate vectors. That is, for the case of a truss satisfying Eq. (45) the optimum actuators produce a deformation state which is a linear combination of the first n B-vibration modes.

Application to 55m Antenna Truss.

The truss support structure for a 55-m radiometer antenna shown in Figure 1 was used as an example. The reflector is built up from tetrahedral truss modules, and consists of 420 members connected at 109 joints. The upper surface and the r.m.s. of the vertical motion of

this surface was used as a measure of the error. This corresponds to the vector $\hat{\psi}$ consisting of the vertical displacements of the 61 joints on the upper surface, and the matrix B equal to $(1/\sqrt{61})I$.

The truss elements of the upper and lower surface have similar lengths of about 315 in. with nominal length variations of about one percent. The diagonal elements connecting the lower and upper surface are about 180 inches long with nominal length variations of about 3-4 percent. This antenna structure was analyzed in Ref. 3 assuming that shape errors are introduced due to manufacturing length errors from the nominal values. The length errors were assumed to be uncorrelated and of equal standard deviation σ_ℓ corresponding to all Δl_i being equal, and to Σ given by Eq. (28). The actuators were assumed to be mechanisms (such as screws) embedded in some elements which can be used to change the length of these elements.

Two methods were used to obtain optimum actuator positions on the upper surface of the antenna in Ref. 3. First an approximate procedure that permitted actuators to be located anywhere in space(that is not restricted to truss member location) was used. This procedure employed the conjugate gradient method to minimize $E(\delta_{rms}^2)$. The second procedure was heuristic integer programming approach called ESPS (Exhaustive Single-Point-Substitution, Ref. 4).

The optimum actuator locations for the cases of three and six actuators, are shown in Figures 2 and 3, respectively. The corresponding optimal values of the distortion ratio g, Eq. (12), are compared in Table 1 with the lower bound on g predicted by Eq. (48). It is seen that the approximate optimum (which permits actuators even where there are no truss elements) comes close to the lower bound, with the ESPS optimum not too far behind. This good agreement is achieved in spite of the fact that the assumption of Eq.(45) is not met exactly (with small variations for surface elements and a large discrepancy for the diagonal

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elements).

A check on the effectiveness index of the individual actuators used in the ESPS solution was performed next. For the three actuator case q_i ranged between 0.130 and 0.974 for the 420 elements of the truss. The elements selected by the ESPS procedure as actuators, on the other hand, had effectiveness indices q_i beween 0.961 and 0.974, indicating that they produce a displacement field overwhelmingly represented by the highest 3 modes of $C_{\psi\psi}$. It is interesting to note that only 29 of the 420 members had q_i larger than 0.95 so that the selection of actuator locations could have been greatly simplified. For the six actuator case q_i ranged between 0.164 and 0.980 with the ESPS procedure selecting actuators in the range 0.966 to 0.980. Of the 420 members 36 had q_i larger than 0.960 and 45 had q_i larger than 0.95. It should be noted that it is not reasonable to simply select the actuators with the highest effectiveness indices. It is important that the actuators produce displacement field that are not linearly dependent or close to being linearly dependent (see Ref. 8).

These results have three implications. First they validate the procedures used in Ref. 3 for obtaining good actuator positions. Second they indicate that Eq. (48) can be used to assess the degree of shape control possible with a given structure and a given number of actuators. Therefore, it can be employed in a structural redesign of the truss to improve shape controllability. Third, they show that the effectiveness index of Eq. (27) can be used to screen actuator locations and greatly reduce the number of actuator locations that need to be considered in the search for the optimum locations. For example, for the six actuator case, if we consider only the 45 members with q_i greater than 0.95, the number of possible configurations drops from 7×10^{12} to 12×10^{6}.

Concluding Remarks.

This work investigated the correction of shape distortion due to errors n structural sizes which are zero-mean normally distributed random variables. A bound on the maximum

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improvement in the expected value of the root-mean-square shape error was obtained as well an effectiveness index of a given actuator. The results were specialized to a simple form for truss structures composed of nominally identical members. The bound was tested on a 55-m radiometer antenna truss structure. It was found that previously obtained results for optimum actuators had a performance close to the bound obtained here. These results raise the possibility that the bound obtained here will be useful in antenna truss design for improved shape controllability. Furthermore, it was verified that the optimum actuators found previously rank very high in terms of the effectiveness index developed here. The effectiveness index can therefore be used to screen potential actuator locations and reduce drastically the number of truss members that need to be considered in the selection for actuator placement.

Acknowledgement

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Table 1

Comparison of lower bound of distortion ratio with values achieved by optimization of actuator locations

Number of actuators	lower bound Eq. (48)	ESPS optimum	approximate optimum (Ref.3)
3	0.6384	0.6919	0.6595
6	0.5346	0.5967	0.5510
9	0.4617	0.5271	
12	0.4049	0.4710	
15	0.3531	0.4061	
18	0.3044	0.3574	

Figure 1: Side and Top Views of Tetrahedral Truss
            Antenna Reflector

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Continuous optimum, g=0.6595

Best near feasible design, g=0.7028

ESPS Solution g = 0.6919

Figure 3: Optimum Upper Surface Actuator Configurations - 6 Actuators
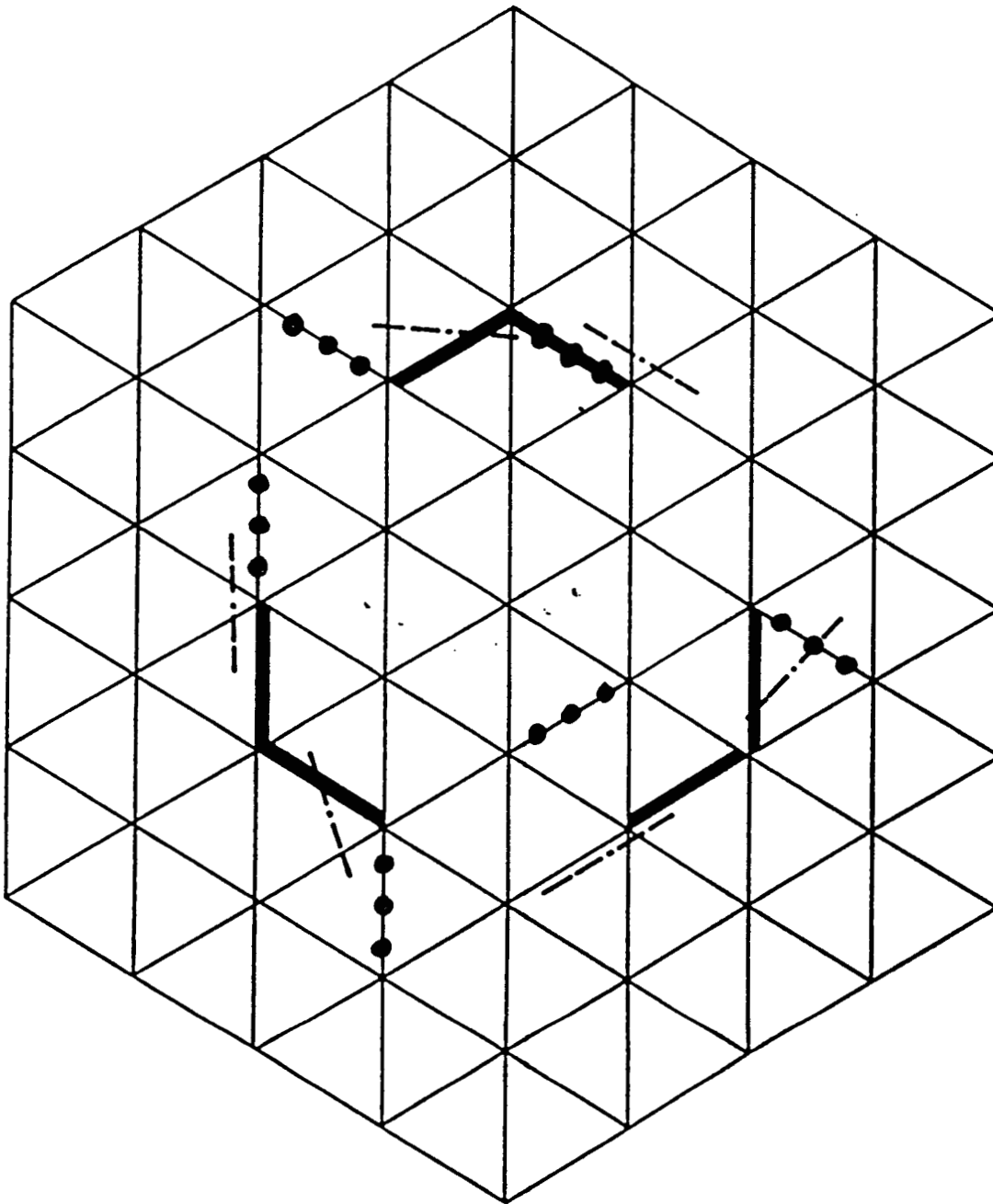
——·—— Continuous optimum
g=0.5510

▬▬▬▬ Best near by feasible design,
g=0.6039

•-•-•-• ESPS Solution g = 0.5967

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16. Abstract

This paper deals with correction of shape distortion due to zero-mean normally distributed errors in structural sizes which are random variables. A bound on the maximum improvement in the expected value of the root-mean-square shape error is obtained. The shape correction associated with the optimal actuators is also characterized. An actuator effectiveness index is developed and shown to be helpful in screening actuator locations in the structure. The results are specialized to a simple form for truss structures composed of nominally identical members. The bound and effectiveness index are tested on a 55-m radiometer antenna truss structure. It is found that previously obtained results for optimum actuators had a performance close to the bound obtained here. Furthermore, the actuators associated with the optimum design are shown to have high effectiveness indices. Since only a small fraction of truss elements tend to have high effectiveness indices, the proposed screening procedure can greatly reduce the number of truss members that need to be considered as actuator sites.