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# ANTIPROTON-MATTER INTERACTIONS IN ANTIPROTON APPLICATIONS

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## ABSTRACT

By virtue of the highly energetic particles released when they annihilate in matter, antiprotons have a variety of potentially important applications. Among others, these include remote 3-D density and composition imaging of the human body and also of thick, dense materials, cancer therapy, and spacecraft propulsion. Except for spacecraft propulsion, the required numbers of low-energy antiprotons can be produced, stored, and transported through reliance on current or near-term technology.

Paramount to these applications and to fundamental research involving antiprotons is knowledge of how antiprotons interact with matter. The basic annihilation process is fairly well understood, but the antiproton annihilation and energy loss rates in matter depend in complex ways on a number of atomic processes. The rates, and the corresponding cross sections, have been measured or are accurately predictable only for limited combinations of antiproton kinetic energy and material species. However, our knowledge has been improving in two areas: in energy loss and annihilation rates at low KeV energies and below, where adiabatic ionization, elastic scattering, and nuclear capture are important, and in differences between antiproton and proton atomic interactions with matter at high KeV and MeV energies. At present, estimates of annihilation and stopping rates adequate for planning purposes can be made in most aspects of the applications.

## I INTRODUCTION

Over the past several years it has been recognized that antiprotons, like positrons, have potentially important practical applications.[1] In each of these applications, some of which may be instituted within the next few years, the manner in which antiprotons interact with matter while moving through it, in terms of energy loss during slowing and energy production when they annihilate, is of considerable significance. Of particular significance is the fact that the slowing and annihilation rates are wholly determined or strongly influenced by atomic interactions.[2]

A description of the interaction between antiprotons and matter is the main purpose of this paper; it is discussed specifically in Sections III,V, and VI. The possible applications, and the means of achieving them, are briefly described in Section IV. The nature, significance, and properties of the various forms of antimatter, with emphasis on antiprotons, are summarized in the following section along with some historical notes.

## II BACKGROUND

The concept of antimatter, albeit in a gravitional context, goes back to the last century.[3] Antimatter in its modern form was first postulated by Dirac in 1931.[4] Anderson discovered the positron (antielectron) in 1933,[5] and Chamberlain et al. discovered the antiproton in 1955.[6] Around this time it was accepted that the already-discovered positive pion and positive muon were the antiparticles of their negative counterparts, and subsequently an increasing number of antiparticles were discovered, including the antineutron.

We now know that for every fundamental particle there is a corresponding antiparticle whose strong and electroweak internal quantum numbers (e.g. electric charge) are opposite in sign to those of the particle. Some antiparticles, like those of the photon and neutral pion, in a given state are each identical to their particle in another state, so particle and antiparticle are not distinguished. All antiparticles have the same lifetimes as the particles and, apparently, inertial masses, but there is reason to believe that some may have different gravitational masses.[7]

It is possible, at least in principle, to construct antinuclei, antiatoms, antimolecules, and even antisubstance from antiprotons, antineutrons, and positrons. Light antinuclei are observed in high energy accelerator experiments and in cosmic rays. The simplest antiatom, the antiprotium isotope of antihydrogen, is yet to be made although there are motivations[8] and plans[9] to do so.[10] Theoretical work concerning antihydrogen and other antiatoms began about 20 years ago.[11,12,13] Since antimatter, from particle through substantive form, is the mirror image of matter in a number of respects, it is also termed "mirror matter", a term promoted by Forward.[14]

The cosmological significance of antimatter was recognized in the 1950's and 1960's by Alfven, Klein, Harrison, Omnes, and others.[15] Models of the early universe include its presence at the initial or a very early stage, but definitive observations of cosmological antimatter or its consequences include only that which may have been or is being produced in later stages by matter interactions.[16] There is, however, a feature in the cosmic gamma ray background that suggests antiproton annihilation in the universe at a red shift of about 100, but this evidence of primordial antimatter is not conclusive.[17]

In addition to their role in improving our knowledge of physics and astrophysics, antiprotons may, as positrons do, have practical applications. Some years after Sänger suggested using positron-electron annihilation to propel interstellar spacecraft in 1953,[18] NASA reconsidered the issue of annihilation propulsion in the 1970's,[19] and Morgan developed the basic concepts that might allow use of antiproton annihilation for both interplanetary and interstellar propulsion.[20] That application may have to wait decades for the required amounts of antiprotons to be available (e.g. roughly one gram for a high performance interplanetary mission). More recently a number of individuals (in particular Kalogeropoulos[21]) and organizations have discovered and investigated practical applications for antiprotons that are more readily attainable. These include remote 3-D density and elemental composition mapping of the interior of materials[22] including the human body,[21] cancer therapy,[21] eqaution-of-state and opacity measurements,[23] and others mentioned in Section IV. Most of the applications require no more antiprotons than could be produced, captured, and transported with current and near-term attainable technology.[1]

## III BASIC ANNIHILATION PROCESSES

Most of the antiproton applications depend strongly on the energy and other characteristics of the annihilation products. These particles and gamma rays can deposit substantial amounts of energy in the matter around the point of annihilation, yet retain enough of their large energy to pass through substantial distances to exit the matter and be readily detected.

In contrast to the annihilation of a positron with an electron, which proceeds almost entirely through an electromagnetic interaction and produces two or more gamma rays, an antiproton annihilates predominantely through a strong interaction and initially produces other hadrons, mainly pions. When a slow antiproton ($p^-$), with an energy around a few MeV or less, annihilates with an individual proton ($p^+$) at rest, the result is

$$p^- + p^+ \rightarrow 1.5\,\pi^+ + 1.5\,\pi^- + 2.0\,\pi^0 + \text{kaons} + \text{others}\,, \qquad (1)$$

$$\pi^0 \rightarrow 2\gamma\,, \qquad (2)$$

$$\pi^\pm \rightarrow \mu^\pm + \nu_\mu, \qquad (3)$$

$$\mu^\pm \rightarrow e^\pm + \nu_e + \nu_\mu\,, \qquad (4)$$

$$e^+ + e^- \rightarrow 2\gamma\,, \qquad (5)$$

where $\pi$ denotes a pion, $\gamma$ denotes a gamma ray, $\mu$ denotes a muon, e denotes electrons and positrons, and $\nu$ denotes both neutrinos and antineutrinos.

In reaction 1 the average numbers of pions produced are given, there being a great number of possible outcomes to the reaction. About 4% of the annihilation energy goes into kaons (mainly) and other particles (including rarely two or more gammas). In reaction 2 the $\pi^0$ lifetime is so short that the $\pi^0$ travels only microns before decaying. In reaction 4 with the lower sign, the product electron will remain in the environment, and in reaction 5 the electron annihilating with the positron is a different (very likely) electron from that environment. Thus the end products of the annihilation are gamma rays and neutrinos. The energies and lifetimes of the pions and subsequent gammas and muons from the annihilation of $p^- + p^+$ are given in Table 1. The annihilation energy for reaction 1 is 1876.51 Mev, the mass energy of the proton plus the equal mass energy of the antiproton.

The relavent processes in most applications occur at a time after the neutral pions have decayed but before the charged pions (or in some cases, before the muons) have decayed. Because the relevant properties of kaons do not differ extremely from those of pions and because the neutral pions travel such a short distance before decaying, it is therefore usually adequate to assume

$$p^- + p^+ \rightarrow 1.6\,\pi^+ + 1.6\,\pi^- + 4.0\,\gamma\,, \qquad (6)$$

where, using the same mean energies as in Table 1, the artificial increase in the mean numbers of charged pions accounts for the 4% of energy going to other particles.

Table1. The energies and lifetimes of the pions and subsequent gammas and muons from the annihilation of $p^- + p^+$ at low relative speed with the center of mass at rest. It is assumed that the pions undergo no energy or number loss before decaying. Based on informayion from a variety of sources.

| Particle | Mean Number/Annihilation | Mass Energy /MeV | Mean Kinetic Energy /MeV | Lifetime / s |
|---|---|---|---|---|
| $\pi^+$ | 1.5 | 139.58 | 235 | $2.60 \times 10^{-8}$ |
| $\pi^0$ | 2.0 | 134.98 | 203 | $9. \times 10^{-16}$ |
| $\pi^-$ | 1.5 | 139.58 | 235 | $2.60 \times 10^{-8}$ |
| $\gamma$ | 4.0 | 0 | 169 | $\infty$ |
| $\mu^+$ | 1.5 | 105.66 | 189 | $2.20 \times 10^{-6}$ |
| $\mu^-$ | 1.5 | 105.66 | 189 | $2.20 \times 10^{-6}$ |

The annihilation of an antiproton with a neutron is about the same as annihilation with a proton, except that the mean number of negative pions is one greater than the mean number of positive pions and the number ratio of charged to neutral pions is somewhat greater. Depending on the nuclear environments of the neutron and proton, the cross section for annihilation with a neutron is about 0.75 times the cross section for annihilation with a proton.

When an antiproton annihilates with a proton or neutron in a nucleus other than that of protium ($^1$H), the above desrciption of the annihilation is altered. Some of the annihilation products interact with the remainder of the nucleus giving rise to additional products that, depending on the atomic number of the nucleus, may include light nuclear fragments consisting of individual neutrons and protons as well as deuterons, tritons, helions, alpha particles, etc. Details are given in Ref. 24 and 25 and in the references quoted there. The results of a slow antiproton annihilating in a uranium nucleus are given in Table 2. Likewise the details of the annihilation are altered when the antiproton has an appreciable kinetic energy (several MeV or greater). Besides the additional energy, the antiproton may annihilate within the nucleus, as opposed to on the surface, and the distribution of initial annihilation products is tilted toward the nucleus. These factors lead to more and more energetic secondary products. For uranium, the fraction of annihilation energy going into the kinetic energy of the charged nuclear fragments increases by roughly 35% as the incident kinetic energy increases from zero to 100 MeV.[25]

Table 2. Annihilation Energy Partition when a slow antiproton annihilates in a uranium nucleus at rest, compared to annihilation with a proton. The fission energy includes the energies of fission Gamma rays (0.4 %) and the kinetic energies of the fission neutrons (0.5 %) and the daughter nuclei. Based mainly on information in Ref. 24.

| Partition Category | Percent of Proton-Antiproton Annihilation Energy In Uranium Nucleus | With Proton |
|---|---|---|
| Fission Energy | 10 | 0 |
| Neutrons (Non-Fission) (Kinetic Energy) | 18 | 0 |
| Charged Fragments (Kinetic Energy) | 16 | 0 |
| Charged Pions (Kinetic Energy) | 28 | 38 |
| Neutral Pions (Kinetic Energy) | 10 | 22 |
| All Pions (Mass Energy) | 26 | 36 |
| Other (Kinetic and Mass Energy) | 2 | 4 |
|  | — | — |
| Total | 110 | 100 |

The direct annihilation cross section for antiprotons on protons is the cross section for annihilation when there is no intervening state between the antiproton's state of free motion and its annihilation. It is known experimentally for antiproton kinetic energies from about 20 MeV to about 10 GeV.[26] At the lower end of this range, the scattering is predoninately s-wave, so it may be extrapolated to lower energies with the 1/v law, where v is the speed of the antiproton relative to the nucleus. Below 10 MeV it is necessary to include the coulomb correction factor, which represents the enhancement of the cross section due to the attraction between the antiproton and proton. The result is[11]

$$\sigma = 0.19\,(c/v)\pi r_0^2\gamma/(1-e^{-\gamma})\ , \qquad (7)$$

$$\text{with}\quad \gamma = 2\pi\alpha c/v\ ,$$

where $\gamma/(1-e^{-\gamma})$ is the coulomb correction factor, c is the speed of light (2.998 x $10^{10}$ cm/s), $r_0$ is the classical electron radius (2.82 x $10^{-13}$ cm), and $\alpha$ is the fine structure constant (1/137.0). The cross section is shown in Figure 1. Eq. 7 does not apply for energies around 20 eV and below where $p^-$ capture in the $p^-$ + H (hydrogen atom) rearrangement reaction (Section VI) becomes important. Radiative capture is unimportant at essentially all energies.[11]

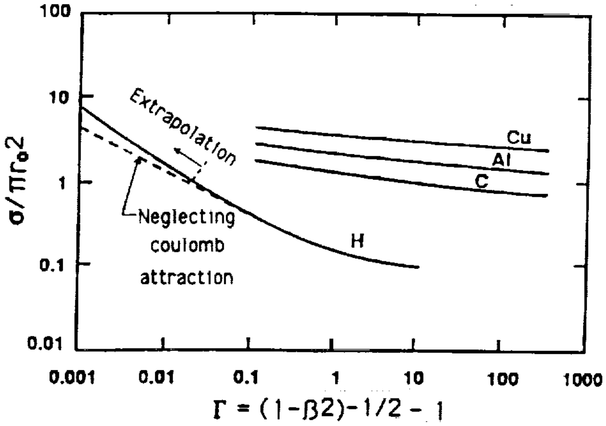

Fig. 1. Cross section ($\sigma$) for annihilation of antiprotons in hydrogen, carbon, aluminium, and copper. $r_0$ is the classical electron radius, $\Gamma$ is the ratio of antiproton kinetic energy to rest mass energy, and $\beta = v/c$.

Cross sections for direct antiproton annihilation in carbon, aluminum, and copper nuclei have been measured for energies from about 100 MeV to about 200 GeV.[27] They are also shown in Fig. 1. A formula that fits these data below about 10 GeV and may allow extrapolation to heavier nuclei is:[25]

$$\sigma = \pi(1.35\ A^{1/3} + 0.83)^2 \times (p/\ 600\ \mathrm{MeV/c})^{-B} \times 10^{-26}\ \mathrm{cm}^2\ , \qquad (8)$$

$$\text{with } B = 0.5A^{-0.4}\ ,$$

where A is the atomic weight of the nucleus and $p$ is the antiproton momentum. Data are needed for a wider variety of nucleil and energies.

## IV APPLICATIONS

The number of antiprotons required in each application event in the applications mentioned below varies from roughly $10^6$ to $10^{12}$. The larger of these figures is also the approximate, current upper limit on the number that probably can be transported in a storage device on a truck.[28] That device might be a storage ring, a Penning-like trap, or another kind of ion trap. For currently achievable vacuums, the lifetime of the antiprotons could be a few weeks to several months.

Antiprotons are currently produced in particle accelerators by bombarding nuclei with protons with energies of a few tens of GeV to a few hundred GeV. The potential production rate of storable antiprotons (i.e. slowed to KeV energies) for each of a few current or planned accelerators, as they are or with well understood modifications, is about $10^{16}$ per year.[29] With current and foreseeable technology, it may be possible to construct an accelerator specifically for antiproton production giving 1 mg ($6 \times 10^{20}$) per year.[30] Reasonable speculations exist on means to produce gram or even kilogram amounts per year.[31]

Antiprotons can be used to obtain three dimensional "x-rays" of materials,[22] including the human body,[21] by directing a narrow beam of them into the material. The annihilation products can pass through a meter of condensed materials of light elements or at least a few centimeters of those of heavy elements and remain detectable. Reconstruction of their paths allows the coordinates of annihilation points to be determined as functions of beam direction and energy. The depth of penetration up to the anihilation point is a function of the energy and integrated density of the material, with some dependence on elemental composition. Thus one may obtain a density map of the interior of the material. Resolutions of about 1 mm or less appear obtainable. The radiation dosage is about one tenth the value resulting from procedures using x-rays that accomplish the same quality of density map. By combining this technique with measurements of the x-ray spectrum given off by the antiprotons captured by nuclei prior to annihilation, one might also obtain a 3-D map of the elemental composition.[21]

Antiprotons may be valuable in cancer therapy[21,32] and in healing defects within a material[22,33] because, in a condensed material, a significant portion of their combined kinetic and annihilation energy is released close to the point of annihilation. The Bragg peak in the energy loss as a function of distance travelled is narrow, most annihilations occur only after the antiprotons have stopped, and the flux of annihilation products decreases as the inverse square of the distance from the annihilation point. Additionally, for most relevant materials, a significant fraction of the annihilation products are protons that usually stop within a few centimeters. For an antiproton beam in water, roughly 100 MeV per particle is deposited within about 5 mm of the aim point. The remainder is spread thinly over the surroundings or escapes.

For equation-of-state and opacity measurements in a small laboratory, transportable antiprotons may allow pressures and temperatures comparable to those available in a large facility.[22] The antiprotons are used to induce fissions in fissile material placed next to the material sample under investigation. Essentially one fission results per annihilation,[24,34] and the exploding fissile material (a very small amount) compresses and heats the sample. To assess feasability it is important to know how short in duration the pulse of antiprotons could be made and how the fractions of stopped antiprotons and deposited fragment energy depend on fissile material size.

Roughly speaking, antiproton annihilation propulsion of spacecraft could make exploration of the solar system like exploration of the earth by steamship, and its near-light-speed capability over longer distances could make travel to nearby stars a reality. Reasonable concepts exist for annihilation rocket engines,[35-38] for means of producing solid antihydrogen (required storage form of antiprotons),[39] for antihydrogen storage, [36,37] for extracting antiprotons from solid hydrogen,[36] and for other necessary processes and hardware. However, the amounts of antiprotons required per mission, milligrams (earth to orbit) through tonnes (interstellar), are well beyond current means to produce. Nevertheless, available amounts of antiprotons allow many worthwhile experiments that can explore and validate these concepts.[35,40]

For most annihilation engine concepts, the basic problem is to get the antiprotons well into the annihilation/propellant medium without annihilations occuring elsewhere, while containing (e.g. with magnetic fields) a large fraction of the annihilation products as they transfer their energy to the medium. Hence the importance to propulsion of the slowing and annihilation rates of antiprotons in matter. Knowledge of antiproton interactions with matter are likewise important in other aspects of annihilation propulsion.

Two means have been suggested in which antiproton annihilation is coupled to nuclear fusion processes. In one, antiproton annihilation initiates a deuterium/tritium fusion reaction in a bomb configuration.[41] The minimum number of antiprotons required is probably on the order of $10^{17}$ or $10^{18}$, which will be very expensive to produce, at the least, for some time to come. In the other, deuterium and tritium are introduced into the combustion chamber of an annihilation engine in which the annihilation/propellant medium is in a gas or plasma state.[42] The muons from pion decay then induce fusion of the deuterium and tritium nuclei through the muon-catalyzed-fusion process.[43] The extra energy might double or triple the energy output of the engine for little additional mass. [35] In the "plasma-core" engine,[36] the temperature and density are sufficiently high that deuterium-tritium fusion would occur without the presence of muons. Whether such an engine could sustain a fusion reaction if the antiprotons were turned off is not known.

## V SLOWING AND ANNIHILATION RATES

In most applications and many physics experiments, it is important to know how antiproton kinetic energy and annihilation probability depend on initial energy, distance traveled through a material, and composition of the material.

For antiprotons in hydrogen at energies above 20 eV, Eq. 1 may be multiplied by the atomic number density, n, to give an approximation for the annihilation probability per increment of the distance, x, traveled (the annihilation rate):

$$dP/dx = 0.19\, n(c/v)\pi r_0^2 \gamma/(1-e^{-\gamma}) \;, \qquad (9)$$

$$\text{with}\quad \gamma = 2\pi\alpha c/v \;.$$

For substances with atomic numbers equal to carbon and above and antiproton energies from 100 MeV to 10 GeV, dP/dx may be obtained similarly from Eq. 8. The result cannot be extrapolated to lower energies without knowledge of how higher angular momentum waves and the coulomb correction factor are involved in the data used in Eq. 8. A very rough approximation at low energies for other materials may be obtained by multiplying the right side of Eq. 9 by the two thirds power of the mean atomic number, a rough measure of the ratio of an effective nuclear cross sectional area to that of the proton. Annihilation and slowing rates below 20 eV in unionized or partially ionized media are dealt with in the following section.

For antiprotons in the low KeV range and above, the slowing rate ($-dE/dx$, where E is the antiproton kinetic energy in the rest frame of the medium) is almost entirely due to transfer of energy to atomic electrons in binary collisions. A good approximation is provided by the "Bethe formula"[44] which is based on the Born approximation and applies to charged particles in an unionized medium. For antiprotons it is

$$-dE/dx \;=\; 8\pi e^4 (Nn/fE) \ln(fE/I_m) \quad , \quad (10)$$

$$\text{with} \quad f = 4 m_e m/(m+m_e)^2 \equiv 4 m_e/m,$$

where e (= $-4.8 \times 10^{-10}$ esu) is the electron charge, N and $I_m$ are the mean atomic number and ionization energy of the medium, and $m_e$ (= $9.11 \times 10^{-28}$g) and m (= $1.673 \times 10^{-24}$ g) are the electron and antiproton masses. For most elements, $I_m/N \equiv 13$ eV. Some exceptions are helium, beryllium, nitrogen, and calcium for which $I_m/N$ is 24, 16, 11, and 11 eV respectively. More accurate versions of Eq. 10 exist that include relativistic corrections, important around 1 GeV and above, shell corrections, Barkas-effect corrections, which lead to different slowing rates for particles of equal mass but opposite sign, and the block correction.[45] There is recent experimental confirmation of the Barkas effect for antiprotons (vs. protons) at low MeV energies, and it has been confirmed that at least some single and multiple ionization cross sections are different for protons and antiprotons.[46]

Eq. 10 is probably accurate to within a few tens of percent or better for E » $I_m/f$ ( ≡ 5 to 500 KeV for hydrogen to uranium) but less than 1 GeV. Such accuracy is consistent with the magnitudes of the above theoretical corrections. In addition, calculations of ionization and excitation cross sections for antiprotons in hydrogen[47,48] at such energies are in good agreement for these energies with the cross sections which, along with the final energies of the ionized electrons, give Eq. 10.

For E ≡ 2.7 $I_m/f$, $-dE/dx$ has a maximum (the "Bragg peak"), similar to that seen experimentally for other charged particles. In lieu of experimental information on antiprotons at such energies, the accuracy, if not the form, of Eq. 10 and the more accurate versions is questionable, however, for a number of reasons. First, the perturbative nature of the calculation may not be valid. Second, the electrons may not be treatable as free particles for energy transferral, as is assumed. Last, the equations are certainly inaccurate for E ≡ $I_m/f$, since the fact that they give $-dE/dx = 0$ at $E = I_m/f$ is not true. That energy is the cutoff below which the antiproton (or any other particle with the same mass) cannot transfer an energy equal to $I_m$ to a free electron in a binary collision. An alternate formula for Eq. 10 employs a more realistic, distributed ionization energy.[2] It may obviate the need for shell corrections, and within the Born approximation it gives a lower, more realist cutoff energy since that cutoff can be based on the minimum ionization (or excitation) energy.

Eq. 10 is used to give the rightmost, nearly straight portion of the curve in Fig. 2 of $-dE/d(\rho x)$ for antiprotons slowing in hydrogen, where $\rho$ is the mass density of the hydrogen. In spite of the incorporation of $\rho$, $-dE/d(\rho x)$ is still dependent on the properties of the slowing medium. Both the factor, N/A (A = medium's mean atomic mass), which appears in Eq. 10 after division by $\rho$, and $I_m^{-1}$, which appears in the argument of the logarithm, decrease (on the average) as N and A increase. Thus materials composed of heavier elements usualy have lower values of $-dE/d(\rho x)$ than materials composed of lighter elements.

Under most circumstances, knowledge of slowing below the Bethe formula cutoff is inconsequential. In condensed media or gasses at normal pressures, charged particles are then moving so slowly that even small subsequent energy loss leads them to thermal energies in a very short distance (a few mm or less) if they have not already decayed (if unstable) or been captured (if negative). However, if slowing is purposely used as a means to produce antiprotons at low and sub KeV energies or if rocket engines with very low density annihilation media should seem worth considering, then knowledge of slowing mechanisms around and below the cutoff can be necessary.

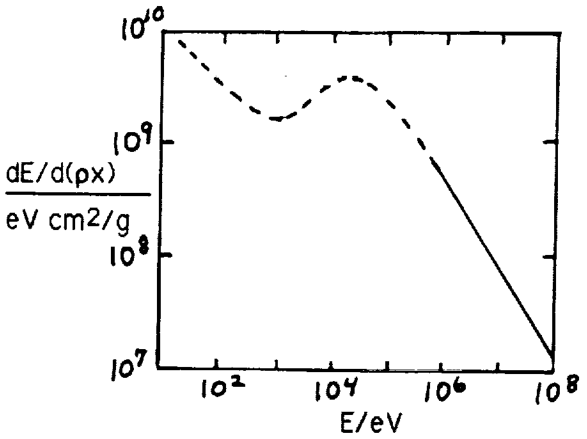

Fig. 2. Energy loss rate (per unit distance-times-density) for antiprotons slowing in unionized hydrogen. ρ is the mass density of hydrogen and E is the antiproton kinetic energy in the lab frame.

There are at least two mechanisms that lead to energy loss around and below the cutoff but still above about 20 eV where antiproton capture begins to become likely: adiabatic excitation and ionization of atomic electrons by the antiprotons and elastic scattering of the antiprotons by whole atoms (or molecules). In the former, the presence of the antiproton near a nucleus lowers its effective charge, so an electron may move to a higher state, or be ionized, and remain in the altered configuration when the antiproton, having therefore lost energy, leaves. In the latter, the antiproton loses energy in the lab frame, on the average, as long as its kinetic energy is above the mean thermal energy of the medium.

An approximate, but complicated formula for energy loss by heavy negative particles due to elastic scattering by atoms is given in Ref. 2 and 35. It assumes the atoms are free and separated by at least a few Bohr radii, so it applies to gas media and with less accuracy to some condensed media. For antiprotons in hydrogen this formula gives the leftmost, fairly straight portion of the curve of -dE/d(ρx) in Fig. 2. A consequence of the model employed there for the particle-atom potential energy, V, (a raised coulomb potential, cutoff when V=0) is that the scattering (classical) is exactly backwards for a particular particle energy in the center-of-mass system for all impact parameters for which $V \neq 0$. If a negative particle at this energy encounters an atom of equal mass it stops dead in its tracks in the rest frame of the atom (lab frame approximately); if it encounters a more massive atom it reverses direction in that same frame. Hence the term, "brick wall" scattering, for this process. The particular energy is about 10 eV for antiprotons in hydrogen, so the capture process may dominate, but for media with heavier atoms it occurs at higher energies (60 eV in carbon) for which capture may be less important. There are apparently no experiments or more accurate calculations that bear on the reality of this possible phenomenon.

A rough consideration of adiabatic excitation/ionization[35] indicates that it may be important for antiprotons in hydrogen at energies around 1 KeV but that energy loss by elastic collisions is more important around and below a few hundred eV. Other loss mechanisms at these energies may be vibrational and rotational excitation of molecules and the creation of phonons and similar entities.

Division of the energy loss process into particular mechanisms operating over particular energy ranges is in part a consequence of a need to find relatively simple, pictureable, and tractable means of describing and calculating the process. The particle-electron collision mechanism, that leads to the Bethe formula for the slowing rate, and adiabatic excitation/ionization are perhaps better described as high and low energy approximate views of a single process. This is borne out by Ermolaev's recent calculation of excitation and ionization cross sections for antiprotons on hydrogen atoms.[47] The total excitation/ionization cross section is smooth and roughly constant from 2 to 50 KeV (2.3 to $1.5\times10^{-16}$ cm$^2$). This range includes the Bethe formula cutoff at about 6 KeV and the Bragg peak at about 16 KeV. If it assumed that the mean energy loss per collision in that range varies from 10 eV at the low end to 30 eV at the high end, then the portion of the curve of -dE/d(ρx) in Fig. 2 around the local maximum results. Portions of the curve around 1 KeV and around 200 KeV are interpolations between the elastic scattering result on the left, the Ermolaev-based results in the middle, and the simple-Bethe-formula results on the right.

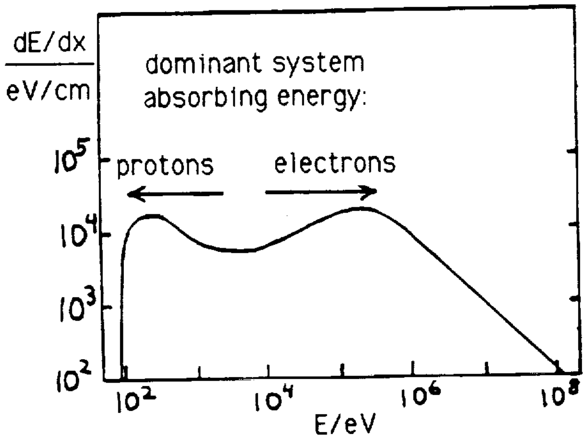

Fig. 3. Energy loss rate (per unit distance) for antiprotons slowing in a fully ionized hydrogen plasma at a temperature of $10^6$K and an atomic number density of $3.6\times10^{18}$ / cm$^3$. E is the antiproton kinetic energy in the lab frame.

For antiproton slowing in fully ionized plasmas one may use Langmire's formula.[49] Application to a hydrogen medium with a temperature of $10^6$ K and an atomic number density of $3.6\times10^{18}$ (conceivable conditions within a plasma-core annihilation engine) yields the result for -dE/dx shown in Fig. 3. Here the slowing rate is not exactly proportional to the density so ρ cannot be factored out; results must be given for particular densities. Such factorization is approximately correct at high energies for which, interestingly, the slowing rates in Fig. 2 and 3 are about the same.

Once an antiproton reaches thermal energy in a plasma (usually doing so before annihilating), it diffuses and eventually annihilates. For the above plasma, annihilation will occur within a few millimeters and within several microseconds of the point and time at which it thermalizes.[50]

## VI REARRANGEMENT AND CAPTURE

At low- and sub-eV energies in media that are not largely ionized, antiproton slowing and annihilation rates are thoroughly dominated by a rearrangement reaction in which the antiproton loses energy and becomes bound to (captured by) a nucleus, while the electrons absorb that energy, most likely through ionization.[2,11,12,51-53] Once captured, the antiproton cascades to lower energy levels, while emitting x-rays, and eventually annihilates in the nucleus.[54] For antiprotons that have not annihilated before slowing to energies around 1 eV and below (normally most of them), the cross section for this process is so high ( > $20\pi a_0^2$, $a_0$ = Bohr radius = $5.29\times10^{-9}$cm) that final stopping and annihilation occur within lengths that can be measured in interatomic distances in all media but dilute gasses.

For antiproton energies around or below 20 eV relative to the medium, the antiproton's speed will be less than 0.03 of the mean speed of the least bound electrons (slowest) of an atom of the medium. Thus the abiabatic approximation applies to the response of all of the atomic electrons to the influence of an incoming antiproton as long as the mean speed of any electron does not decrease considerably. In this approximation, also called the Born-Oppenheimer approximation, the wave function of the electrons at any instant is taken to be the wave function the electrons would have if the antiproton were stationary at its location at that instant. In addition, an antiproton at 20 eV or less can transfer no more than a small amount (0.04 eV) of its energy to an electron in a binary encounter. Thus, any significant transferral of energy will occur adiabatically.

As the antiproton passes by or within the atom, the electrons will adiabatically reconfigure themselves into a state of higher energy, since they are repelled by the antiproton. That extra energy is taken from the kinetic energy of the antiproton. There are then three possibilities: (1) the antiproton leaves, gaining back the energy from the electrons as they return to their initial configuration, (2) the antiproton leaves, having lost energy to to the electrons, which are left in a higher energy state (adiabatic excitation/ionization spoken of above at higher energies), or (3) the antiproton becomes permanently bound to the nucleus with the electrons left in a negative-ion state (possibly excited) of an atom with atomic number one less, or one or more electrons ionize while the remainder are in a neutral or positive-ion state of that new atom (possibly excited).

The first possibility is always possible; it will certainly occur for sufficiently high angular momentum waves of the incoming antiproton (i.e. large impact parameters classically). Indeed, if the adiabatic condition were perfectly satisfied, it is the only possibility that could occur. The second possibility can occur, in particular at the higher antiproton energies (and therefore speeds) considered earlier, because at least one of the electrons will slow down to a speed more comparable to that of the antiproton as it moves to a less bound or an unbound state. Under that condition, its portion of the wave function can no longer change rapidly enough to follow the changing influence of the antiproton to satisfy adiabaticity. It is this latter fact that allows the excitation or ionization to be permanent. The third possibility, of main concern here, similarly requires a breakdown in the adiabatic approximation.

In the second and third possibilities, the low energy of the antiproton for the range under consideration requires that at least one electron slow down considerably and/or the antiproton speed up considerably for the adiabatic approximation to break down. The former can occur if an electron moves to a state of nearly zero energy, either highly excited or ionized. The latter can occur if the antiproton gets close enough to the nucleus for its attraction to increase its kinetic energy considerably.

For an antiproton to excite or ionize a hydrogen atom (essentially same mass as the antiproton) without being captured requires that there be at least 10.2 eV (minimum excitation energy) of kinetic energy in the center-of-mass (c.m.) frame, so the antiproton must have an energy of at least 20.4 eV in the rest frame of the atom (lab frame approximately). This is consistent with earlier statements that adiabatic excitation/ionization (without capture) becomes unimportant toward the lower end of the eV energy range. Capture of the antiproton with the electron remaining in a negative ion is impossible for hydrogen. Capture into a state of protonium (bound $p^+$-$p^-$) that is just barely bound, with ionization of the electron to a state of zero energy, requires that the antiproton energy (lab frame) be no more than 27.2 eV (kinetic energy in c.m. frame = 13.6 eV = ionization energy). If the antiproton had any additional energy, it would would have to be carried off by the electron, but with reference to statements above, it appears unlikely that this additional energy could be more than a relatively small amount. Capture into protonium states of greater binding energy requires that the antiproton energy not exceed progressively smaller amounts.

For hydrogen, therefore, rearrangement (i.e. capture and ionization) is unlikely for energies above about 20 eV, but as will be seen, it is very likely at lower energies. This dividing line is probably roughly the same for other atoms and for molecules because it is the outer electrons that will respond most strongly to the antiproton.

Using a semiclassical method, Morgan and Hughes calculated the cross section for antiproton - hydrogen atom rearrangement for energies from a few eV down to about 1 meV.[11,12] An approximation that may be applicable for any neutral atom, accurate below about 2 eV for hydrogen, is

$$\sigma = \pi(2(1+m/M)e^2a/E)^{1/2} \quad , \qquad (11)$$

where $\sigma$ is the rearrangement cross section and thence the annihilation cross section, M is the mass of the atom, and a is its polarizability (4.502 $a_0^3$ for hydrogen). They assumed, as had others in reference to negative pions and muons,[55] that the rearrangement takes place whenever the antiproton passes closer than a certain distance, $R_c$, to the proton. $R_c$ is called the critical radius; it is the maximum distance ($0.639a_0$ [55]) between the proton and antiproton for which there are no bound states for the electron in the adiabatic approximation.

A feature of the interaction that helps with the accuracy of their results is that the inner turning point of the antiproton orbit relative to the proton is a discontinuous function of the impact parameter. For energies below a few eV, the inner turning point is well outside of $R_c$ (no rearrangement) or it is well inside it (100% rearrangement probability assumed). Their more acurate calculation is based on the exact antiproton - hydrogen atom interatomic potential, while Eq. 11 is based on the long range, induced dipole part of that potential energy, $-e^2a/(2R^4)$, where R is the proton-antiproton separation. It is the long range part of the potential energy that principally determines the value of impact parameter at which the discontinuity occurs. Since the energy-dependent impact parameter at which the discontinuity occurs is typically several Bohr radii, the rearrangement cross section is quite large.

As the antiproton approaches to within a short distance of $R_c$, the adiabatic approximation breaks down as the antiproton speeds up and the electron, whose wave function has expanded considerably as its energy approaches zero from below, is slowing down. At this time the electron motion becomes decoupled from the motion of the antiproton. Considering this process in detail, Morgan has made an estimate of the probability that the electron will reattach itself to the proton as the antiproton returns to the vicinity of $R_c$.[52] The probability is 20% for $E \leq 1$ eV that rettachment will occur with the antiproton proceeding away from the atom and the electron returning to its initial state, so Eq. 11 and the "more accurate" results might be more correct if the cross section were multiplied by 0.80. For energies of a few eV and less the rearrangement cross sections can be so high that the separate cross sections overlap within each layer of molecules in a solid or liquid.[56] This means that antiprotons at these energies will be captured and will annihilate within the first few molecular layers of the substance, with the actual values of the cross sections under these circumstances being less than given by Eq. 11.

## VII ROCKET ENGINE INJECTION ENERGIES

Information on the annihilation and energy loss rates of antiprotons in hydrogen is adequate to determine the antiproton injection energy required to center the annihilation region within the engine when the annihilation/ propellant medium is hydrogen. Additionally one may determine the fraction of annihilations that occur away from the center.[50] Results for two engine types will be sumarized.

In a gas-core engine the hydrogen medium is heated by the charged pions and subsequent muons and electrons which are confined by a magnetic field as they lose their energy. By design the antiproton injection rate and consequent heating rate relative to the mass flow rate of hydrogen is insufficient to produce significant ionization. A typical density within such an engine is about $10^{-3}$ g/cm$^3$ and a typical combustion chamber radius is roughly 1 m. Under these conditions the approximate nature of -dE/dx (as shown in Fig. 1) below about 500

KeV is inconsequential since any reasonable values of -dE/dx will make distance traveled from 500 KeV to stopping be a very small fraction of engine size. Additionally the rearrangement-capture process is so strong and the subsequent annihilation process so fast that when the antiprotons come close to a stop they annihilate before moving any significant distance. Thus one may employ Eq. 9 and 10 for annihilation and slowing rates and assume that reaching 500 KeV is tantamount to stopping. Thereby the required injection energy that leads to the antiprotons stopping at the center of the engine is 14 MeV, and the fraction of annihilations that occur before the antiprotons reach the center is only 0.025.

In a particular concept for a plasma-core engine, the medium is fully ionized hydrogen at a temperature of $10^6$K and a number density of $3.6 \times 10^{18}$ ionized atoms per $cm^3$. The results shown in Fig. 2 and Eq. 9 combine to give 1.3 MeV for the injection energy if the engine radius is 1 m, and the fraction of annihilations occurring before the antiprotons thermalize is only 0.003. Once they thermalize, they undergo direct annihilation before moving but a small fraction of the size of the engine.

## VIII CONCLUSIONS AND COMMENTS

There a number of potentially important and feasible practical applications of antiprotons. In these, knowledge of the interactions of antiprotons with matter is necessary, and in particular, formulae for the annihilation and slowing rates of antiprotons in matter are required.

The annihilation rate in hydrogen appears to be known with fair accuracy for all important antiproton energies, but the values for energies in the low MeV range and throughout the KeV and eV ranges need experimental confirmation. The annihilation rates in other substances are known experimentally for only a limited number of cases and only for energies of a few hundred MeV and above. Experiments and/or experimentally confirmed formulae are needed for other substances and for lower energies for all substances.

Formulae for the slowing rate are accurate around and above an energy that is in the high KeV range for hydrogen up to the low or mid MeV range for substances with higher atomic number. Experiments and/or experimental confirmation of these or other formulae are needed at lower energies.

Present information on the interactions is often adequate for estimates in planning the applications, but improved knowledge is required for the actual design of procedures and equiptment and for the interpretation of the resultant information coming from each application.

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