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INVESTIGATIONS INTO A NEW ALGORITHM FOR CALCULATING H^∞ OPTIMAL CONTROLLERS

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ABSTRACT

A new algorithm for calculating H^∞ optimal controllers is investigated. The new algorithm is significantly simpler than existing approaches and yields much simpler controllers. The design equations are first presented. Special system transformations required to apply the new algorithm are then presented. The use of the new algorithm with sampled-data systems is outlined in detail.

Several constraints on the characteristics of the problem formulation are required for the application of the design equations. The consequences of these constraints are investigated by applying the algorithm to a simplified design for a subsystem of a large space structure ground test facility. The investigation of these constraints is continued by application of the design equations and constraints to an extremely simple tracking problem. The result of these investigations is the development of a frequency dependent weighting strategy that allows realistic control problems to be cast in a form compatible with the new algorithm.

Further work is indicated in the area of developing strategies for choosing frequency-dependent weights to achieve specific design goals. The use of the freedom in problem formulation to achieve robustness/performance tradeoffs should also be investigated.

It is not clear that the new algorithm always leads to simpler controllers. The more restrictive formulation may dictate that frequency-dependent weighting adds to the controller order disproportionately. This effect must also be investigated.

NOMENCLATURE

$\sigma_{min}(A)$	minimum singular value of a matrix A
$\sigma_{max}(A)$	maximum singular value of a matrix A
$\|\| F(s) \|\|_\infty$	infinity norm of a system transfer function matrix
IMC	image motion compensation
BET	base excitation table
LOS	line of sight
LQG	linear-quadratic Gaussian

INTRODUCTION

Until the recent work by Glover and Doyle[1], the design of H^∞ controllers promised to be a long and arduous ordeal for the designer. Moreover, the resulting controllers tended to be extremely complex, sometimes exceeding the order of the control model by a factor of five. Their publication of design equations for controllers of the same order as the control model is thus a significant advance in the state of the art. However, the question of whether H^∞ control techniques can be successfully applied to large space structure (LSS) control design problems is by no means answered.

The foremost question in the mind of any LSS control designer is that of applicability of H^∞ techniques to the usual goals of LSS control. It is known that H^∞ optimization can achieve (at least mathematically) any of the goals of disturbance rejection, command tracking, and robustness. What is not known is whether H^∞ design methods can be used to design controllers which simultaneously give acceptable performance <u>and</u> do not suffer from the known shortcomings of LQG techniques, e.g., lack of robustness. The purpose of the work presented here is to address the issues of the applicability of the new algorithm for calculating H^∞ controllers for large space structures.

The organization of the report is as follows. Section 1.0 contains a brief discussion of the H^∞ performance criterion and the design equations which must be solved in order to find the optimal H^∞ controller.

Section 2.0 describes a transformation required to satisfy constraints on using the design equations. The result is a set of equations that can be readily used to transform a given state space realization to one of the required form.

Section 3.0 outlines modifications to the state space formulation that are required in order to apply H^∞ design formulas to sampled data systems. The result is a set of state space formulas for applying the well known w-plane transformation to multivariable control problems. The equations, minus the derivation, can be found in Glover[2]. An outline of the derivation is included here for completeness.

Section 4.0 documents the application of H^∞ techniques to a simplified model of the ACES IMC subsystem, including BET disturbance effects. Section 5.0 uses a simple tracking problem to discuss weighting schemes which allow the new algorithm to be applied to realistic control design problems.

Section 6.0 contains conclusions and recommendations for further work. In particular, it is suggested that strategies for developing frequency-dependent weights be investigated.

1.0 H^∞ PERFORMANCE CRITERION AND DESIGN EQUATIONS

The H^∞ control problem can be stated as follows. The system equations are

$$\dot{x} = Ax + B_1 w + B_2 u$$

$$z = C_1 x + D_{11} w + D_{12} u$$

$$y = C_2 x + D_{21} w + D_{22} u$$

where w is in $\mathbf{R}^{m1}$, u is in $\mathbf{R}^{m2}$, z is in $\mathbf{R}^{p1}$, and y is in $\mathbf{R}^{p2}$. The signal w is an exogenous input which may be either disturbances or command signals; u is the control input vector; z is actually the performance related vector; and y is the vector of measurements that is actually available for feedback.

Although most calculations with this technique are done in state space form, the performance criterion is most easily stated in the frequency domain in terms of the closed loop transfer function matrix. The open loop transfer function matrix can be expressed in terms of appropriate partitions as

$$G_{11}(s) = [A,\ B_1,\ C_1,\ D_{11}]$$

$$G_{12}(s) = [A,\ B_2,\ C_1,\ D_{12}]$$

$G_{21}(s) = [A, B_1, C_2, D_{21}]$

$G_{22}(s) = [A, B_2, C_2, D_{22}]$.

If a controller with transfer function matrix K(s) is connected from y to u, the closed loop transfer function matrix is given by

$$T(s) = G_{11} + G_{12}K(I - G_{22}K)^{-1}G_{21}.$$

The H^∞ control problem is to find a controller K which yields a stable closed loop system and for a prespecified real number Γ,

$$|| T ||_\infty < \Gamma$$

where

$$|| T ||_\infty = \sup_w \sigma_{max}(T(jw)).$$

In the special case of a scalar transfer function, the goal can be stated as simply insuring that the closed loop frequency response has a magnitude less than Γ. This is clearly important in disturbance rejection problems and can be used also for tracking type problems.

The H^∞ **optimal** control problem is that of finding the smallest such Γ such that a stabilizing controller exists. Note that once a method of satisfying a given Γ bound is identified, the job of obtaining an optimal solution is not difficult, although it is iterative. The significance of the recent work of Glover and Doyle is that their equations yield not only a controller that achieves the prespecified bound but is only the order of the original control model. The original factorization algorithms tended to yield controllers many times as large as the original plant.

Several constraints must be placed on the plant equations in order to apply the design equations. The first constraint is that the realization (A, B_2, C_2) be stabilizable and detectable, as is usual.

The second constraint is actually two constraints that are artifacts of the derivation procedure and are required for well-posedness. They are

$$\text{rank } D_{12} = m_2$$

and

$$\text{rank } D_{21} = p_2.$$

One of the consequences of the above rank conditions is that the control input u must appear in the performance oriented regulated variable z. This condition is similar to a condition required for well posedness of the LQG problem. These conditions also insure the calculation of a realizable controller. It will also be seen in subsequent sections that these requirements cause some difficulty in calculating a low order controller.

An assumption which is not independent of the above rank conditions is that

$$D_{12} = [0 \ I]^T$$

and

$$D_{21} = [0 \ I].$$

Transformations to achieve the required forms for D_{12} and D_{21} will be derived later.

The final constraints are necessary for the design equations to yield a solution. They are not necessary in the strict sense. However, it is not known how to calculate controllers to achieve the bound when these conditions are violated. Sufficient conditions for the design equations to work are

The realizations of G_{12} and G_{21} are minimal.

rank $G_{12}(jw) = m_2$ for all w

and

rank $G_{21}(jw) = p_2$ for all w.

The design equations can be presented in an abbreviated form by denoting the solution to the Riccati equation

$$Q + XA + A^T X - XPX = 0$$

by its Hamiltonian matrix

$$X = \text{Ric}\begin{bmatrix} A & -P \\ -Q & -A^T \end{bmatrix}.$$

D_{11} is also partitioned as

$$D_{11} = \begin{bmatrix} D_{1111} & D_{1112} \\ D_{1121} & D_{1122} \end{bmatrix}.$$

Two intermediate variables are defined:

$$D_{1x} = [D_{11} \ D_{12}]$$

$$D_{x1} = [D^T_{11} \ D^T_{12}]^T.$$

Then define

$$R = D^T_{1x} D_{1x} - \begin{bmatrix} \Gamma^2 I_{m1} & 0 \\ 0 & 0 \end{bmatrix}$$

$$\tilde{R} = D_{x1} D^T_{x1} - \begin{bmatrix} \Gamma^2 I_{p1} & 0 \\ 0 & 0 \end{bmatrix}.$$

$$X_\infty = \text{Ric} \begin{bmatrix} U & -P \\ -Q & -U^T \end{bmatrix}$$

where

$P = BR^{-1}B^T$, $U = A - BR^{-1}D^T_{1x}C_1$, and $Q = C^T_1 C_1 - C^T_1 D_{1x} R^{-1} D^T_{1x} C_1$.

Similarly,

$$Y_\infty = \text{Ric} \begin{bmatrix} U & -P \\ -Q & -U^T \end{bmatrix}$$

where now

$P = C^T \tilde{R}^{-1} C$, $U = A^T - C^T \tilde{R}^{-1} D_{x1} B^T_1$, and $Q = B_1 B^T_1 - B_1 D^T_{x1} \tilde{R}^{-1} D_{x1} B^T_1$.

Similarly to LQG design, matrices F and H are defined as

$$F = -R^{-1}[D^T_{1x}C_1 + B^T X_\infty] = [F^T_{11} \; F^T_{12} \; F^T_2]$$

$$H = -[B_1 D^T_{x1} + Y_\infty C^T]\tilde{R}^{-1} = [H_{11}\ H_{12}\ H_2].$$

At this point, it is necessary to perform tests to determine whether it is possible to achieve the specified Γ by designing a controller via the design equations. The first test has important practical consequences in terms of proper problem setup. It involves direct feed through of the disturbance input w in terms of D_{11}:

$$\Gamma > \max\{\sigma_{max}[D_{1111}\ D_{1112}],\ \sigma_{max}[D^T_{1111}\ D^T_{1121}]\}.$$

The last test involves the characteristics of the solutions to the two Riccati equations:

$$X_\infty > 0$$

$$Y_\infty > 0$$

and

$\mu_{max}(X_\infty Y_\infty) < \Gamma$, where μ is an eigenvalue.

Under the constraints listed and subject to passage of the tests an n^{th} order stabilizing controller which achieves the inequality $||T||_\infty < \Gamma$ is given by

$$D_{11} = -D_{1121}D^T_{1111}(\Gamma^2 I - D_{1111}D^T_{1111})^{-1}D_{1112} - D_{1122}$$

D_{12} and D_{21} satisfy

$D_{12}D^T_{12} = I - D_{1121}(\Gamma^2 I - D^T_{1111}D_{1111})^{-1}D^T_{1121}$ and

$$D^T_{21}D_{21} = I - D^T_{1112}(\Gamma^2 I - D_{1111}D^T_{1111})^{-1}D_{1112}$$

from which

$$B_2 = (B_2 + H_{12})D_{12}$$

$$C_2 = -D_{21}(C_2 + F_{12})Z$$

$$B_1 = -H_2 + B_2 D^{-1}{}_{12} D_{11}$$

$$C_1 = F_2 Z + D_{11} D^{-1}{}_{21} C_2$$

$$A = A + HC + B_2 D^{-1}{}_{12} C_1$$

and

$$Z = (I - \Gamma^{-2} Y_\infty X_\infty)^{-1}.$$

An n^{th} order controller which achieves the norm bound is then given by the realization (A, B_1, C_1). There exist other controllers which satisfy the norm bound; however, they are not necessarily of n^{th} order. The controller design equations for this case are omitted. These more complex design equations may be found in Glover and Doyle[1].

2.0 Transformation to the Standard Form

In Section 1.0, it was stated that the use of the design equations place constraints on the form of D_{12} and D_{21}. This section outlines in detail a method for achieving the required form for these matrices. The first step is to write them in terms of their singular value decompositions as is shown for D_{12}:

$$D_{12} = [U_{121}\ U_{122}][\Sigma_{12}\ 0]^T V^T{}_{12}$$

$$D_{12} = [U_{122}\ U_{121}][0\ \Sigma_{12}]^T V^T{}_{12}$$

$$D_{12} = [U_{122} \ U_{121}][0 \ I]^T \Sigma^{-1}_{12} V^T_{12}.$$

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Similarly,

$$D_{21} = U_{21}[\Sigma_{21} \ 0][V_{211} \ V_{212}]$$

$$D_{21} = U_{21}[0 \ I][V_{212} \ V_{211}]^T.$$

Now let

$$X_{12} = \Sigma^{-1}_{12} V^T_{12}$$

$$X_{21} = U_{21}\Sigma^{-1}_{21}$$

$$U'_{12} = [U_{122} \ U_{121}]$$

$$V'_{21} = [V_{212} \ V_{211}].$$

Then

$$D_{12} = U'_{12}[0 \ I]^T X_{12}$$

$$D_{21} = X_{21}[0 \ I](V'_{21})^T.$$

The equations for z and y are

$$z = C_1 x + D_{11} w + D_{12} u$$

$$y = C_2 x + D_{21} w + D_{22} u$$

or

$$z = C_1 x + D_{11} w + U'_{12}[0 \ I] X_{12}$$

$y = C_2x + X_{21}[0\ I](V'_{21})^T w + D_{22}$

or

$(U'_{12})^T z = (U'_{12})^T C_1 x + (U'_{12})^T D_{11} w + [0\ I]^T X_{12} u$

$X^{-1}_{21} y = X^{-1}_{21} C_2 x + [0\ I](V'_{21})^T w + X^{-1}_{21} D_{22} u.$

Letting

$z' = (U')^T z$

$u' = X_{12} u$

$y' = X^{-1}_{21} y$

$w' = (V'_{21})^T w$

gives

$A' = A$

$B'_1 = B_1 V'_{21}$

$B'_2 = B_2 X^{-1}_{12}$

$C'_1 = (U'_{12})^T$

$D'_{11} = (U'_{12})^T D_{11} V'_{21}$

$D'_{12} = [0\ I]^T$

$C'_2 = X^{-1}_{21}C_2$

$D'_{21} = [0 \; I]$

$D'_{22} = X^{-1}_{21}D_{22}X^{-1}_{12}$

The significance of the transformation is that the norms of z and w are preserved. This means that designing a controller to achieve a particular $||T||_\infty$ for the transformed system is equivalent to designing a controller to achieve the same goal for the untransformed system, once the reverse transformation is applied to the controller.

3.0 Modifications for Sampled-Data Systems

More traditional controller design techniques must be developed separately for sampled-data and continuous-time systems. Fortunately, this is not necessary for H^∞ designs due to the fact that the performance criterion has a relatively simple frequency domain representation. The approach is equivalent to "w-plane" design for single-input, single-output systems. The open loop system is assumed to have the form

$$x(k+1) = Ax(k) + B_1w(k) + B_2u(k)$$

$$z(k) = C_1x(k) + D_{11}w(k) + D_{12}u(k)$$

$$y(k) = C_2x(k) + D_{21}w(k) + D_{22}u(k).$$

This discrete time representation can be obtained using standard techniques such as those found in Kuo[3]. The transfer function matrix is given by

$$G(z) = D + C(zI - A)^{-1}B$$

where for simplicity B, C, D are appropriate concatenations of the open loop system matrices. The bilinear transform

$$z = \frac{1 + w}{1 - w}$$

is applied to G(z) to obtain

$$G(w) = D + C[(I+wI)(I-wI)^{-1} - A]^{-1}B$$

$$= D + C(I-wI)[w(I+A) - (A-I)]^{-1}B$$

$$= D + C(I-wI)[wI - (I+A)^{-1}(A-I)]^{-1}(I+A)^{-1}B.$$

Using the identity

$$[I-wI][wI - (I+A)^{-1}(A-I)]^{-1} = -I + [I - (I+A)^{-1}(A-I)] \times [wI - (I+A)^{-1}(A-I)]^{-1}$$

$$G(w) = D - C(I+A)^{-1}B + C(I+A)^{-1}[wI - (I+A)^{-1}(A-I)]^{-1}(I+A)^{-1}B$$

so that the w-plane state space representation of G(w) is

$$D_s = D - C(I+A)^{-1}B$$

$$C_s = 2C(I+A)^{-1}$$

$$B_s = (I+A)^{-1}$$

$$A_s = (I+A)^{-1}(A-I).$$

The controller is then designed using this representation to obtain K(w), which is represented as a set of continuous time state equations with matrices A_k, B_k, C_k, D_k. The inverse transform

$$w = \frac{z - 1}{z + 1}$$

is applied to obtain K(z). State space formulas for this transformation are

$$A_z = -(A_k+I)(A_k-I)^{-1}$$

$$B_z = (I+A_z)B_k$$

$$C_z = C_k(I+A_k)$$

$$D_z = D_k + C_k(I+A_k)^{-1}B_k.$$

4.0 Problem Setup for IMC Controller Design

The ACES configuration of the Marshall Space Flight Center Large Space Structure Ground Test Facility is shown in schematic form in Figure 1. The image motion compensation (IMC) subsystem is comprised of the line-of-sight (LOS) detectors and the IMC pointing gimbals. The base excitation table (BET) is the excitation device.

The natural setup for IMC controller design to minimize the effects of BET excitation is outlined here. The equations are obtained from a FEM model of the LSS ACES ground test facility. The first step in the problem setup is to define the z, w, y, and u. Since the actuators and sensors are limited to the IMC components, y and u are the x and y axis detector signals and pointing gimbal torques, respectively:

$$u = [IMC_x \ IMC_y]^T$$

$$y = [DET_x \ DET_y]^T.$$

The disturbance vector is most naturally chosen to be the x and y axis BET excitation table forces:

$$w = [BET_x \ BET_y].$$

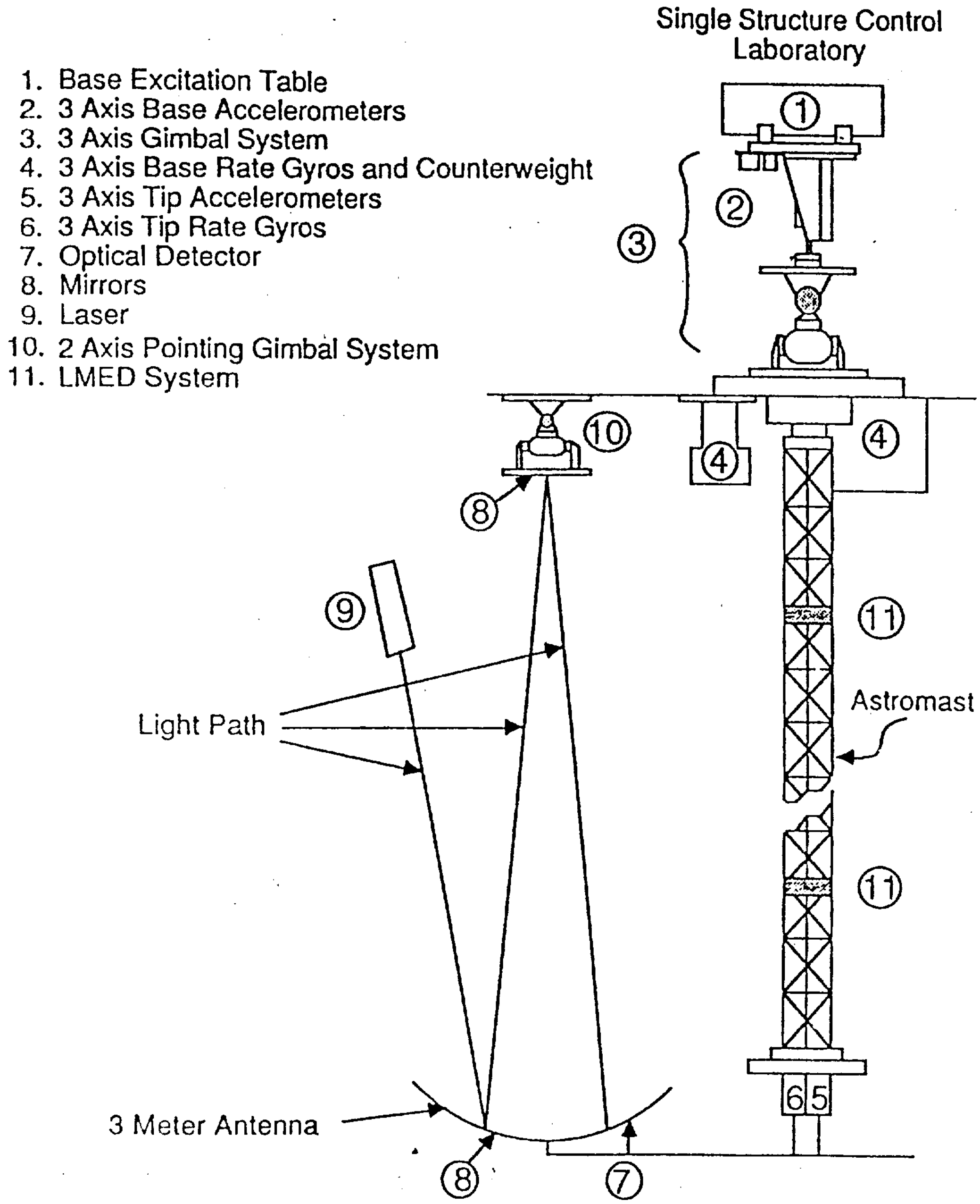

Figure 1. ACES Configuration of the Marshall Space Flight Center Large Space Structure Ground Test Facility

The first elements of the performance oriented controlled variable vector are

$$z_1 = [DET_x \ DET_y]^T$$

since the goal of this system is to reduce the displacement of the line-of-sight of the laser beam from its equilibrium position on the detector. However, this is not sufficient to insure that the conditions of the design equations are met. The additional requirement of control input weighting is achieved by letting

$$z_2 = [IMC_x \ IMC_y]^T.$$

This satisfies the rank condition on D_{12}. The remainder of the parameters of the problem can be chosen as follows.

B_1 is comprised of the appropriate modal gains at the BET actuators.

B_2 is comprised of the appropriate modal gains at the IMC actuators.

C_1 is comprised of the LOS gains at the detector

C_2 is also derived from the LOS gains at the detector.

Also,

A is in block 2 x 2 diagonal form and

$D_{11} = [0 \ 0]^T$, $D_{12} = [0 \ I]^T$

$D_{21} = 0$, $D_{22} = 0$.

The design equations still cannot be applied due to the rank condition on D_{21}. This condition is equivalent to requiring that the same disturbance enters at two physically separated points in the system. The reason for the condition is a mathematical technicality. Unfortunately, it places actual constraints on the formulation of the problem.

A possible solution to this rank problem is to define D_{21} to be a very small constant with respect to the norm of the transfer function matrix G_{12} at all frequencies of interest.

However, it turns out that this is not the only theoretical problem with the above formulation. Another difficulty is the minimality condition on the G_{12} realization. The difficulty in the present setup is that the requirement is equivalent to the requirement that the disturbance (in this case the modes ehich can be excited by the BET) must be controllable at the IMC pointing gimbals. Unfortunately, this is not the case. In fact, the pointing gimbals have significant authority over only four or five modes. This is especially troublesome if the technique is used without regard to the minimality condition, as the design equations will yield a controller without regard to the satisfaction of the requirement. In this case, however, the controller will not satisfy the norm bound.

The minimality condition is an artifact of the particular procedure used to derive the design equations. In the usual factorization approach to H^{∞} control, the minimality condition is not required since it is possible to carry along completely unrelated realizations for each of the four transfer functions matrices. It is interesting to note that an equivalent requirement for an LQG approach would be that the disturbance states be controllable as well as observable.

The consequences of violating the minimality condition are illustrated in Figures 2 and 3. Figure 2(a) is the magnitude of the open loop frequency response from the x-axis BET force input to the x-axis detector. The mode at .15 hertz is the AGS hinge point pendulum mode and is uncontrollable at the IMC gimbals. The other modes are controllable. Figure 2(b) is the magnitude of the closed loop frequency response from the x-axis BET force input to the x-axis detector. It is apparent that the modes which are controllable at the IMC gimbals have been effectively suppressed. However, the AGS hinge point pendulum mode continues to predominate, although the low frequency baseline has been reduced. Figure 3 illustrates the fact that the closed loop norm is not improved. In Figure 3(a) the open loop frequency response from the y-axis BET force input to the y-axis detector has a maximum of roughly -3 decibels, as does the closed loop response of Figure 3(b). In each case, the values are reliable indicators of the infinity norm, since the the x and y axes are only slightly coupled. All frequency responses are in transformed input/output coordinates, as discussed in Section

As these problems were uncovered in the attempt to obtain an IMC controller design, it was decided to investigate the properties of the design equations using an extremely simple model. The next section documents the findings of this investigation.

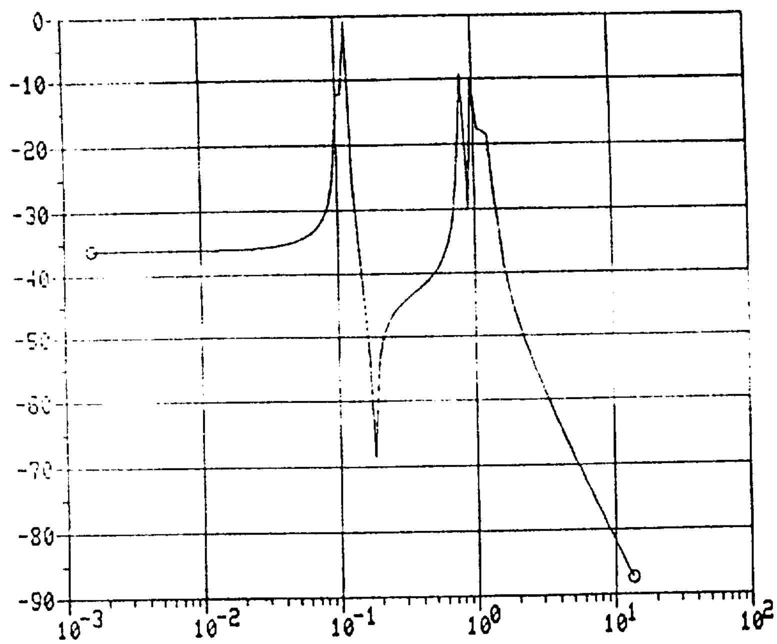

Figure 2(a). Open Loop Frequency Response Magnitude (BET_x to LOS_x)

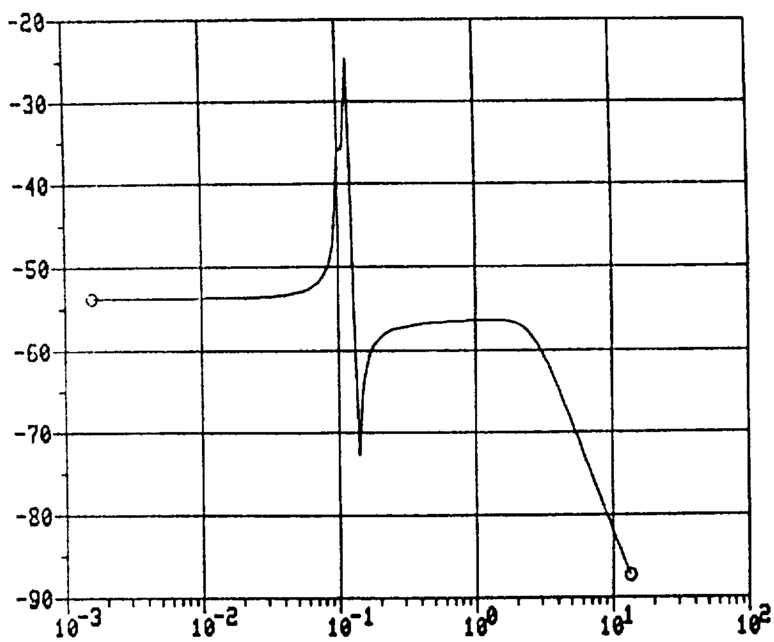

Figure 2(b). Closed Loop Frequency Response Magnitude (BET_x to LOS_x)

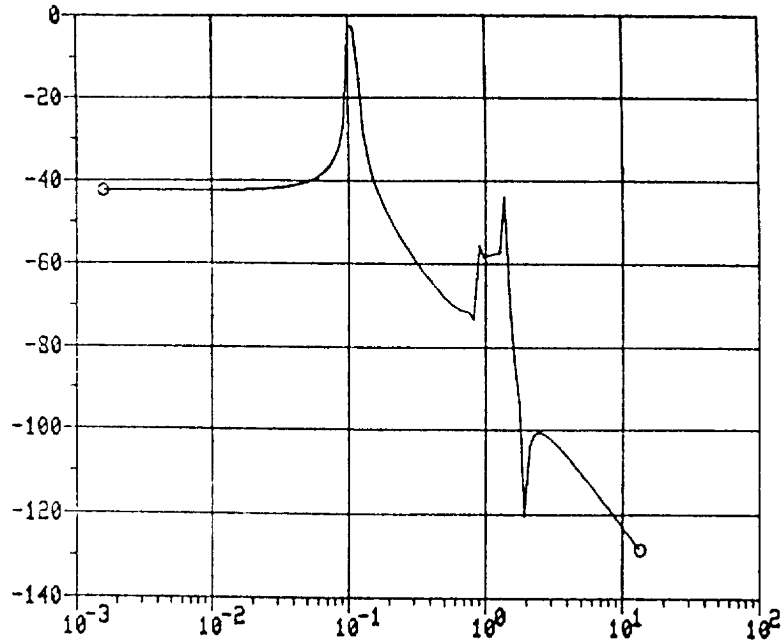

Figure 3(a). Open Loop Frequency Response Magnitude (BET_y to LOS_y)

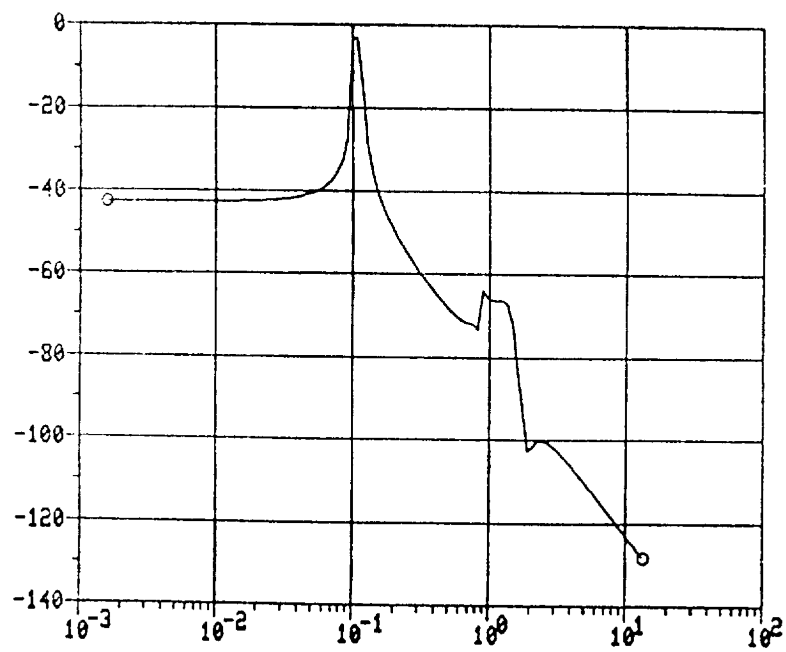

Figure 3(b). Closed Loop Frequency Response Magnitude (BET_y to LOS_y)

5.0 SIMPLE TRACKING PROBLEM

The usual way to introduce design flexibility in the factorization approach to H^∞ optimal control is via frequency dependent weightings. To see the effect of frequency dependent weighting, the usual parametrization of the closed loop transfer function is useful. For a stable plant, the closed loop transfer function can be written as

$$T_1 - T_2 Q T_3$$

where

$$T_1 = G_{11}$$

$$T_2 = G_{12}$$

$$T_3 = G_{21}.$$

Any stable transfer function Q generates a stable closed loop transfer function and a controller which achieves that transfer function. In fact, every stable closed loop transfer function is generated by some stable Q.

The usual requirement for a solution to exist is that T_2 and T_3 have constant rank on the extended jw axis. No minimality condition is required. Weighting is introduced by solving the modified problem of minimizing the infinity norm of

$$T_1 - T_2 W_2 Q W_3 T_3$$

where W_2 and W_3 are chosen to satisfy the constant rank condition and to define frequency ranges over which optimality is emphasized. The major difference in the general H^∞ problem and the problem solved by the new design equations in question is the presence of the two minimality conditions. The effects of the constraints are most easily seen by examining the block diagram of Figure 4.

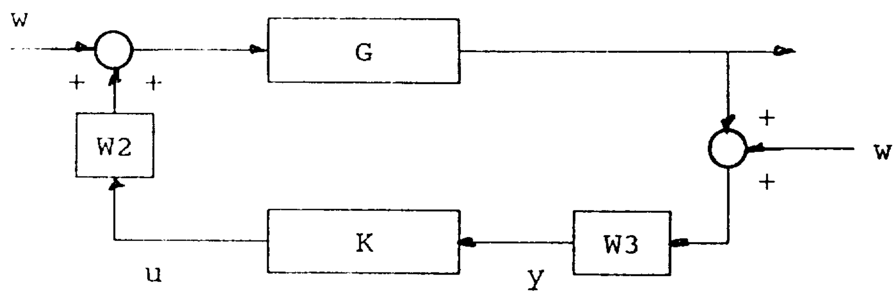

Figure 4. Block Diagram Used for Examining the Effects of Frequency Dependent Weighting.

The minimality constraints imply the following:

> The system realization is simultaneously completely controllable by u and observable by z.
>
> The system realization is simultaneously completely controllable by w and observable by y.

The first can be satisfied by including the signal y in the definition of z. A consequence of this is that T_1 is also weighted by W_3. The second condition can only be satisfied if W_2 has zero dynamical order and is a fundamental limitation of the new algorithm. It should be pointed out that if W_2 is derivative in nature, an appropriate realization can be obtained, but it will generally increase the order of the plant, and hence the controller. Also, the appropriate modifications to account for nonzero D_{22} must be used. The actual details of combining the weightings into an appropriate realization remain to be worked out.

Although no results are available using the weighting scheme of Figure 4, the alternate weighting scheme of Figure 5 was used with an extremely simple plant to illustrate what is possible when all of the constraints of the new algorithm are satisfied.

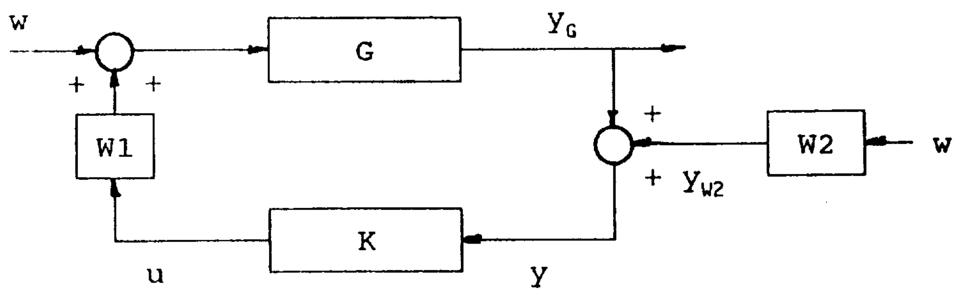

Figure 5. Alternate Weighting Scheme Used for a Simple Tracking Problem

The plant transfer function is

$G(s) = 1/(s + 1)$.

The weights are given by

$W_1 = .99(s/10 + 1)$

$W_2 = .001$.

The z vector is defined by

$z_1 = y_G - y_{W2}$

$z_2 = 10^{-5}u$.

It is interesting to note that although the plant is scalar, the two-dimensional nature of the z vector means that H^∞ control is inherently a multivariable problem. The z_2 element is chosen small enough to simplify the process of obtaining an approximate solution. Another important point to note is that the .99 multiplier in W_1 is necessary to achieve a high gain controller.

Figure 7 is the closed loop frequency response when the controller is implemented in the block diagram of Figure 6. The important point to be made here is that an H^∞ approach can be used to design a simple tracking system with specified closed loop bandwidth (by choosing the break frequency of W_1) and specified steady-state error constants (in this case less than .01 error to a unit step input).

CONCLUSIONS AND RECOMMENDATIONS

The design equations and appropriate constraints and assumptions for a new simplified algorithm for designing H^∞ optimal controllers have been reviewed. A transformation required to satisfy an important constraint has been derived. The use of the design equations with sampled-data systems is outlined in detail and the required state space transformations are summarized.

The requirement of using frequency-dependent weights is illustrated via two examples. One of the examples is a simplified but realistic design for a subsystem of a large space structure ground test verification facility. The other is used to illustrate the efficacy of a particular weighting scheme.

Further work is indicated in the area of developing stategies for finding weights to achieve particular design goals. It is also suggested that possible tradeoffs between performance and robustness be investigated by studying various problem formulations.

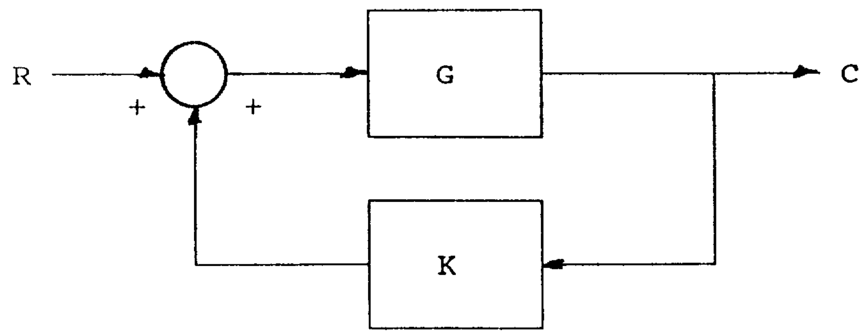

Figure 6. Block Diagram for Implementation of Controller for Simple Tracking Problem.

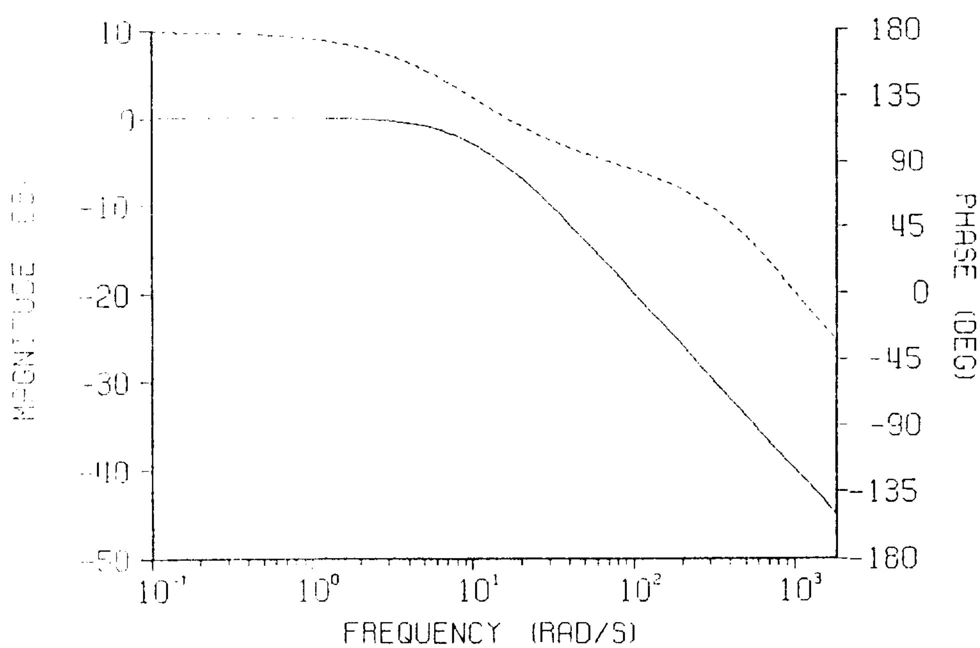

Figure 7. Closed Loop Frequency Response of Simple Tracking Problem of Figure 6.

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