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# ANALYTICAL OPTICAL SCATTERING IN CLOUDS

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# ABSTRACT

We report on the development of an analytical optical model for scattering of light due to lightning by clouds of different geometry. We use the self-consistent approach and the equivalent medium concept of Twersky to treat the case corresponding to outside illumination. Thus, the resulting multiple scattering problem is transformed with the knowledge of the bulk parameters, into scattering by a single obstacle in isolation. Based on the size parameter of a typical water droplet as compared to the incident wave length, the problem for the single scatterer equivalent to the distribution of cloud particles can be solved either by Mie or Rayleigh scattering theory. The super computing code of Wiscombe can be used immediately to produce results that can be compared to the Monte Carlo computer simulation for outside incidence.

A fairly reasonable inverse approach using the solution of the outside illumination case has been proposed to model analytically the situation for point sources located inside the thick optical cloud. Its mathematical details are still being investigated. When finished, it will provide scientists an enhanced capability to study more realistic clouds.

For testing purposes, the direct approach to the inside illumination of clouds by lightning is under consideration. Presently, we are on the verge of obtaining an analytical solution for the cubic cloud. For cylindrical or spherical clouds, we need preliminary results of scattering by bounded obstacles above or below a penetrable surface interface.

ACKNOWLEDGEMENT

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## LIST OF FIGURES

# I. INTRODUCTION

We consider the scattering of light (visible or infrared) due to lightning by cubic, cylindrical, and spherical clouds. In this report, a typical cloud is represented by a statistically homogeneous ensemble of configurations of N identical and aligned spherical water droplets whose centers are uniformly distributed in its volume V. The radius of a droplet varies from 5 to 15 microns. The incident light is from outside the penetrable cloud.

The optical effects of clouds on the light produced by lightning have received great interest for many years. Different techniques have been used in trying to explain the complicated nature of these effects. In particular, we mention the Monte Carlo method which is a computer simulated technique. In a Monte Carlo program, we follow the path of the photons emitted into the cloud by lightning. A photon is said to be scattered if it escapes from the cloud after colliding with the spherical droplets. Otherwise, it is considered as being absorbed by the cloud [1 to 8].

The two main limitations of the Monte Carlo method [9] are the limited number of photons which can be traced through an optically thick cloud with finite computer time, and the fact that a Monte Carlo program cannot be formally proven to be mathematically correct for a finite number of photons. Also, it is very difficult to obtain reliable statistics with the Monte Carlo program.

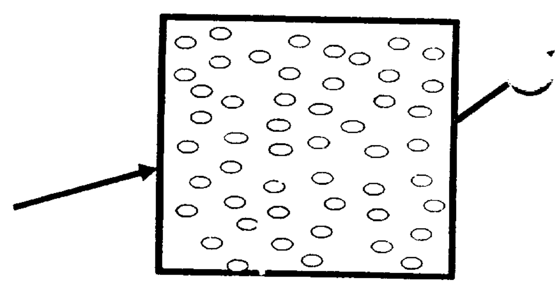
In this paper, we extend to cloud physics the work done by Twersky [10 to 12] for single and multiple scattering of electromagnetic waves. We solve the interior problem seperately to obtain the bulk parameters for the scatterer equivalent to the ensemble of spherical droplets. With the interior solution or the equivalent medium approach, the multiple scattering problem is reduced to that of a single scatterer in isolation. Hence, the computing methods of Wiscombe [13] specialized to Mie scattering can be used to generate numerical results to compare with those given by different literatures.

## II. OBJECTIVES

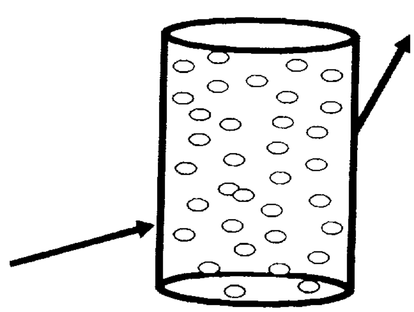
The primary objective of this project is to obtain an analytical optical model for the scattering of light due to lightning from different point sources located inside the cloud. We seek acceptable results in short computer time which will take into account the cloud physics and its boundary effects. To accomplish this objective, it was necessary to solve the multiple scattering problem with outside incidence. By varying the outside angle of incidence, we will obtain for a fixed point inside the cloud an orthogonal set of radiative solutions. From the same point source inside the cloud, we impose a uniformly radiative answer and expand it in terms of the orthogonal set of radiative solutions. The expansion coefficients will determine the outside incidence corresponding to the uniformly radiative imposed inside solution (see figure #1).

<u>Geometry For Points Sources Inside The Cloud</u>

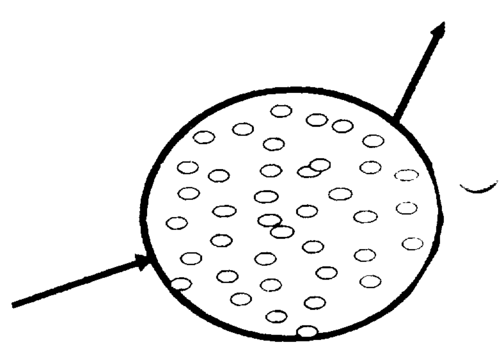
(i). The Cubic Cloud



(ii). The Cylindrical Cloud



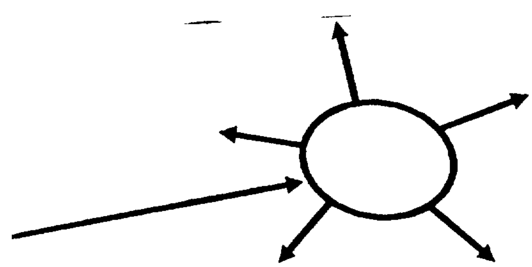
(iii). The Spherical Cloud



## III. SOLUTION INSIDE THE CLOUD FOR OUTSIDE INCIDENCE

The solution inside the cloud for outside illumination corresponds to the multiple scattering of a plane electromagnetic wave by an ensemble of configurations of N identical and aligned spherical water droplets. To obtain the solution inside the cloud, we first consider (see figure #2):

(i). The Single Scatterer In Isolation



For an incidence plane electromagnetic wave $\vec{\phi} = \hat{\mathbf{a}} e^{i\vec{\kappa}_1 \cdot \mathbf{r}}, \kappa_1 = k\eta'$,

and $\eta'$ being the complex relative index of refraction for the host medium inside the cloud, the total outside solution

$$\vec{\psi} = \vec{\phi} + \mathbf{u}_o \tag{1}$$

satisfied the following differential equation obtained from Maxwell's equations after suppressing the harmonic time dependence factor $e^{-i\omega t}$

$$\left[\vec{\nabla} \times \vec{\nabla} \times +\kappa_1^2\right] \vec{\psi} = 0, \vec{\nabla} \cdot \vec{\psi} = 0. \tag{2}$$

The solution inside the single spherical water droplet in isolation $\vec{\psi}_{in}$ satisfied

$$\left[\vec{\nabla} \times \vec{\nabla} \times +\kappa_2^2\right] \vec{\psi}_{in} = 0, \vec{\nabla} \cdot \vec{\psi}_{in} = 0. \tag{3}$$

Here,

$$\kappa_2 = \kappa_1 \eta'' = k\eta'\eta'', \tag{4}$$

with $\eta''$ being the complex relative index of refraction for the medium inside the spherical water droplet.

Similar to Twersky [11], we have from (1)

$$\vec{\psi} = \hat{\mathbf{a}} e^{i\vec{\kappa}_1 \cdot \mathbf{r}} + \left\{ \tilde{h}\left(\kappa_1 \left|\mathbf{r} - \mathbf{r}'\right|\right), \mathbf{u}_o\left(\mathbf{r}'\right) \right\}$$

$$\text{with}$$

$$\mathbf{u}_o\left(\mathbf{r}'\right) = \left\{ \tilde{h}, \mathbf{u}_o \right\} \tag{5}$$

$$\equiv -\frac{\kappa_1}{i4\pi} \int \left[ \left( \tilde{h} \times \hat{\mathbf{n}} \right) \cdot \left( \vec{\nabla} \times \mathbf{u}_o \right) - \left( \vec{\nabla} \times \tilde{h} \right) \cdot \left( \hat{\mathbf{n}} \times \mathbf{u}_o \right) \right] dS\left(\mathbf{r}'\right).$$

Here,

$$\tilde{h} = \left( \tilde{\mathbf{I}} + \frac{\vec{\nabla}\vec{\nabla}}{\kappa_1^2} \right) h\left(\kappa_1 \left|\mathbf{r} - \mathbf{r}'\right|\right),$$

$$\text{and} \tag{6}$$

$$h(x) = \frac{e^{ix}}{ix},$$

and $\tilde{\mathbf{I}}$ being the identity dyadic.

Asymptotically, for $\kappa_1 r >> 1$, we can write

$$\mathbf{u}_o(\mathbf{r}) = h(\kappa_1 r)\mathbf{g}(\hat{\mathbf{r}}, \widehat{\kappa}_1 : \hat{\mathbf{a}}), \hat{\mathbf{r}} \cdot \mathbf{g} = 0$$

$$\text{and}$$

$$\mathbf{g}(\mathbf{r}) = \tilde{\mathbf{I}}_t \cdot \mathbf{g}(\mathbf{r}), \tag{7}$$

$$\tilde{\mathbf{I}}_t = \left( \tilde{\mathbf{I}} - \hat{\mathbf{r}}\hat{\mathbf{r}} \right).$$

The scattering amplitude

$$\mathbf{g}(\hat{\mathbf{r}}, \widehat{\kappa}_1 : \hat{\mathbf{a}}) = \left\{ \tilde{\mathbf{I}}_t e^{-i\vec{\kappa}_1 \cdot \mathbf{r}'}, \mathbf{u}_o\left(\mathbf{r}'\right) \right\} \tag{8}$$

can also be evaluated from Mie scattering theory.

The spectral representation of the scattered wave for a single spherical water droplet is

$$\mathbf{u}_o(\mathbf{r}) = \frac{1}{2\pi} \int_c e^{i\vec{\kappa}_{1c} \cdot \mathbf{r}'} \mathbf{g}(\hat{\mathbf{r}})\, d\Omega(\theta_c, \varphi_c)$$

$$\text{with} \tag{9}$$

$$r > \left( \hat{\mathbf{r}} \cdot \mathbf{r}' \right), \vec{\kappa}_{1c} = \kappa_{1c} \hat{\mathbf{r}}_c(\theta_c, \varphi_c).$$

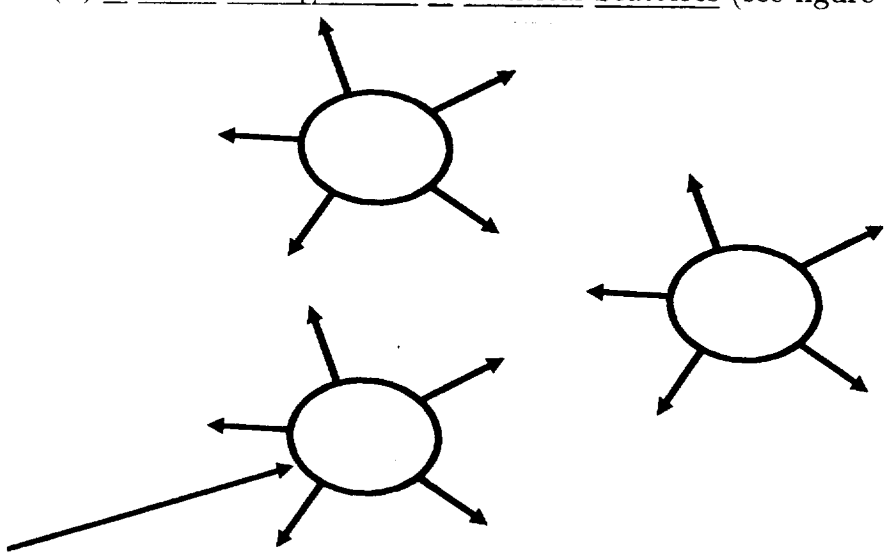
From general reciprocity relations, we have Twersky [11]

$$
\begin{gathered}
-\Re\left[\hat{\mathbf{a}}\cdot\mathbf{g}\left(\hat{\kappa}_1,\hat{\kappa}_1:\hat{\mathbf{a}}\right)\right]=\frac{\sigma_a+\sigma_s}{\sigma_o}\\
\textit{with}\\
\sigma_o=\frac{4\pi}{\kappa_1^2},\ \frac{\sigma_a}{\sigma_o}=\frac{1}{2}\left\{\vec{\psi}^\star,\vec{\psi}\right\}\\
\textit{and}\\
\frac{\sigma_s}{\sigma_o}=\frac{1}{2}\left\{\mathbf{u}_o^\star,\mathbf{u}_o\right\}=\frac{1}{4\pi}\int\mathbf{g}^\star\cdot\mathbf{g}\,d\Omega\left(\hat{\mathbf{r}}\right)\\
\equiv\mathsf{M}\left|\mathbf{g}\left(\hat{\mathbf{r}},\hat{\kappa}:\hat{\mathbf{a}}\right)\right|^2.
\end{gathered}
\tag{10}
$$

Here, $\sigma_a$ and $\sigma_s$ are the absorption and scattering cross sections. The operator $\mathsf{M}$ is the mean over all directions of observation.

Equations (1) to (10) solve the single scattering problem.

(ii).<u>A Fixed Configuration N Identical Scatteres</u> (see figure #3)



We consider a fixed configuration of N identical scatterers with centers located by $\mathbf{r}_{m(m=1,2,3,\ldots,N)}$.

The total outside field $\Psi(\mathbf{r})$ for an incoming wave $\vec{\phi}$ is

$$\begin{aligned}\Psi(\mathbf{r}) &= \vec{\phi}(r) + \sum_{m=1}^{N} \mathbf{U}_m(\mathbf{r}-\mathbf{r}_m) \\ \mathbf{U}_m(\mathbf{r}-\mathbf{r}_m) &= \mathbf{U}_m(\mathbf{r}-\mathbf{r}_m; \mathbf{r}_1, \mathbf{r}_2, ..., \mathbf{r}_N),\end{aligned} \tag{11}$$

with

$$\mathbf{U}_m(\mathbf{r}-\mathbf{r}_m) \sim h(\kappa_1 |\mathbf{r}-\mathbf{r}_m|)\, \mathbf{G}_m \tag{12}$$

for $|\mathbf{r}-\mathbf{r}_m| \to \infty$. The multiple configurational scattering amplitude is given by

$$\mathbf{G}_m(\hat{\mathbf{r}}) = \left\{ \vec{\mathbf{I}}_t e^{-i\vec{\kappa}_1 \cdot \mathbf{r}'}, \mathbf{U}_m \right\}. \tag{13}$$

Equivalently, for the scatterer located at t, we use the self-consistent approach of Twersky [10] to obtain the total outside configurational field

$$\begin{aligned}\Psi_t(\mathbf{r}) = \vec{\phi}(\mathbf{r}) + {\sum}'_m \mathbf{U}_m(\mathbf{r}-\mathbf{r}_m) \\ +\mathbf{U}_t(\mathbf{r}-\mathbf{r}_t), \\ \textit{with} \\ {\sum}'_m = \sum_{m \neq t}.\end{aligned} \tag{14}$$

Using (14) and the general reciprocity relation

$$\left\{ \Psi, \vec{\psi}_{\mathbf{a}} \right\}_t = 0 \tag{15}$$

for any arbitrary direction of incidence, we derive as in [11] the self-consistent integral equation for the multiple configurational scattering amplitude

$$\begin{aligned}\mathbf{G}_t(\hat{\mathbf{r}}) = \tilde{\mathbf{g}}_t(\hat{\mathbf{r}}, \hat{\kappa}_1) \cdot \hat{\mathbf{a}} e^{i\vec{\kappa} \cdot \mathbf{r}_t} + {\sum}'_m \int_c \tilde{\mathbf{g}}_t(\hat{\mathbf{r}}, \hat{\mathbf{r}}_c) \cdot \mathbf{G}_m(\hat{\mathbf{r}}_c) e^{i\vec{\kappa}_{1c} \cdot \mathbf{R}_{tm}}, \\ \textit{with} \\ \mathbf{R}_{\mathbf{tm}} = \mathbf{r}_t - \mathbf{r}_m, \textit{and} \int_c = \frac{1}{2\pi} \int d\Omega_c.\end{aligned} \tag{16}$$

(iii).Ensemble of Configurations

We take the average of (16) over a statistically homogeneous ensemble of configurations of N identical and aligned scatterers whose centers are uniformly distributed in the volume of the thick cloud. Using the equivalent medium approach and Green's theorems, we obtain [11] the dispersion relation determining the coherent parameters

$$\mathcal{G}\left(\vec{\kappa}_1|\vec{\mathrm{K}}\right) = -\frac{\rho}{c_o\left(\mathrm{K}^2 - \kappa_1^2\right)}\left\{\left[e^{-i\vec{\mathrm{K}}\cdot\mathbf{R}}, \mathsf{U}\right]\right\} + \rho \int\limits_{V_\infty - v} \left[f(\mathbf{R}) - 1\right] e^{-i\vec{\mathrm{K}}\cdot\mathbf{R}} \mathsf{U} d(\mathbf{R}). \tag{17}$$

Here, $\mathcal{G}$ is the equivalent scattering amplitude and $\mathsf{U}$ is the radiative function defined by

$$\left[\vec{\Delta}_{|\mathbf{R}|} + \kappa_1^2\right]\mathsf{U} = 0,$$

$$\mathsf{U} = \int\limits_c \tilde{\mathbf{g}}\left(\hat{\mathbf{r}}, \hat{\mathbf{r}}_c\right)\cdot\mathcal{G}\left(\vec{\kappa}_{1c}|\vec{\mathrm{K}}\right) e^{i\vec{\kappa}_{1c}\cdot\mathbf{R}}, \tag{18}$$

$$\rho = \frac{N}{V}$$

and the bulk propagation parameter is

$$\mathrm{K} = \kappa_1 \eta \tag{19}$$

with $\eta$ being the bulk index of refraction.

In equation (17), $\left[f(\mathbf{R}) - 1\right]$ is the total correlation function, and $(V_\infty - v)$ represents the depleted volume ( the volume of all space less that of the exclusion region ). The operator

$$\{[f, g]\} = \int\limits_S \left[f\partial_n g - g\partial_n f\right] dS \tag{20}$$

is the Green exclusion surface operator with outward unit normal from $v$.

Equation (17) solves formally the interior problem for the cloud. To obtain numerical results, one can apply stationary phase method [12] on

(18) and reduce (17) to

$$\mathrm{K} - \kappa_1 \sim -\frac{ig\sigma_o}{2\eta}\mathfrak{L}^{-1}$$
*and*
$$\mathfrak{L} = \left\{ 1 - \rho\frac{g\sigma_o}{2\eta}\int_0^\infty [f(\mathbf{R}) - 1]\, e^{i(\kappa_1 - \mathrm{K})\mathrm{R}} d(\mathrm{R}) \right\}. \tag{21}$$

To obtain equation (21), we neglect back scattering in the forward direction since the spherical droplets are large tenuous scatterers and the magnitude of the single scattering amplitude is evaluated in the forward direction. Equation (21) can be solved numerically, iteratively or directly subject to explicit restriction on

$$g = \hat{\mathbf{e}} \cdot \mathbf{g}\left(\hat{\mathrm{K}}, \hat{\mathrm{K}}\right). \tag{22}$$

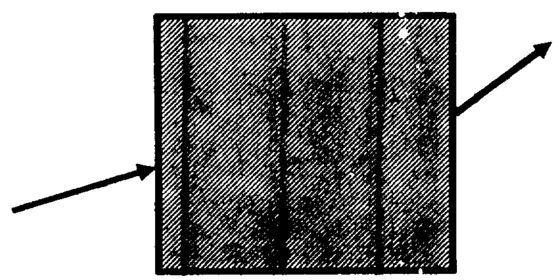
It is important to stress that equation (21) can be subjected to different approximations procedures [14] depending on the cloud composition and the nature of its boundary. In particular, we mention the asymptotic leading term approximation, the generalized Rayleigh's approximation, and the two-space scatterer formalism leading term approximation [11].

The two-space scatterer formalism can be used directly in cloud with a slab geometry, and it will preserve the self-consistency of the solution. The scatterers in the slab geometry will be excited by a coherent wave traveling in the equivalent medium but will radiate into free space.

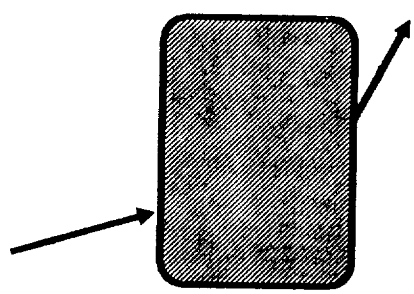
## IV. THE EQUIVALENT MEDIUM APPROACH

Now, we can use the results of part (III) to solve the outside illumination for clouds with different geometry since the net behavior of the interior is known. The multiple scattering problem becomes the scattering by a single obstacle (see figure #4). To produce numerical results, we can apply Wiscombe's advanced super computer code specialized to Mie scattering
Geometry of The Equivalent Medium Approach

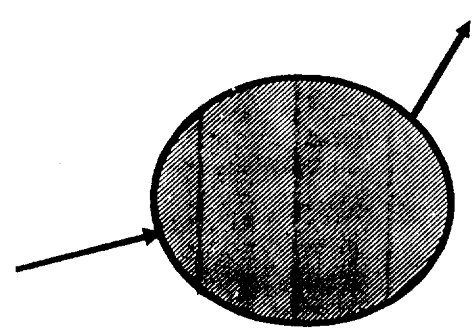
(i). The Cubic Cloud (front view)



(ii). The Cylindrical Cloud



(iii). The Spherical Cloud



# V. CONCLUSIONS AND RECOMMENDATIONS

The analytical solution for the outside illumination of clouds by lightning using the equivalent medium approach has been completed. The multiple scattering problem has been reduced to that of a single scatterer in isolation. Depending on the size parameter of the cloud particles as compared to the wave length of the incident light, either Rayleigh or Mie scattering technique can be used to determine Qext, Qscat, and Qbacs.

With the bulk parameters, we can use Wiscombe's computer code to obtain in short computer time, acceptable numerical results for a medium with a complex relative index of refraction which is an improvement of Bohren [15]. The equivalent medium approach gives naturally the polarizations and the angular distributions of photons which escape the cloud surface.

Due to the complexity of the problem and the time constraint, the mathematical details for points sources inside the cloud are still being worked out. They will be the subject of more studies in the future. To the best of my knowledge, no analytical solution of the problem has yet been published. Even partial results will play a major role in describing the transport of radiation produced by lightning from point sources located inside the cloud.

To test the analytical solution, we intend to solve the cubic case directly since we know the results of scattering by a penetrable plane interface. For cylindrical and spherical clouds however, one has to establish preliminary results for the scattering of light by an obstacle above or below a penetrable surface interface.

With the completion of the analytical model, we will have the ability and the capability to study the transfer of radiation through realistic clouds. That is to say, we will be able to consider clouds made up of water and ice particles, of inhomogeneous distribution of diversified constituents; we will be able to study the fluctuational changes of polarization of light through the cloud due to sudden geometrical deformations of its boundary and the frequent realignment of its particles (e.g, needle shape ice particles aligned in a strong electric field).

## VI. REFERENCES

1. Aida, M., 1977: Scattering of solar radiation as a function of cloud dimensions and orientation. J. Quant. Spectrosc. Radiat. Transfer, 17, 303–310.

2. Bucher, E. A., 1973: Computer simulation of light pulse propagation for communication through thick clouds. Appl. Opt., 12, 2391–2400.

3. Danielson, R. E., D. R. Moore and H. C. van de Hulst, 1969: The transfer of visible radiation through clouds. J. Atmos. Sci., 26, 10778–1087.

4. Davies, R., 1978: The effect of finite geometry on the three dimensional transfer of solar irradiance in clouds. J. Atmos. Sci., 35, 1712–1725.

5. Davis, J. M., S. K. Cox and T. B. McKee, 1979: Vertical and horizontal distributions of solar absorption in finite clouds. J. Atmos. Sci., 36, 1976–1984.

6. McKee, T. B., and S. K. Cox, 1974: Scattering of visible radiation by finite clouds. J. Atmos. Sci., 31, 1885–1892.

7. Plass, G. N., and G. W. Kattawar, 1968: Monte Carlo calculations of light scattering from clouds. Appl. Opt., 1, 126–130.

8. Van Blerkom, D., 1969: Diffuse reflection from clouds with horizontal inhomogeneities. Astrophys. J., 166, 235–242.

9. Thomason, L. W., and E. P. Krider, 1982: The effects of clouds on the light produced by lightning. J. Atmos. Sci., 39, 2051–2065.

10. Twersky, V., 1977: Coherent scalar field in pair-correlated random distributions of aligned scatterers. J. Math. Phys., 18, 2468–2486.

11. Twersky, V., 1978: Coherent electromagnetic waves in pair-correlated random distributions of aligned scatterers. J. Math. Phys., 19, 215–230.

12. Twersky, V., 1983: Propagation in correlated distributions of large-spaced scatterers. J. Opt. Soc. Am., 73, 313–320.

13. Wiscombe, W. J., 1979. Mie scattering calculations: Advances in technique and fast, vector-speed computer codes, NCAR/TN-140 +STR, National Center for Atmospheric Research, Boulder, Colo.

14. Twersky, V., 1983: Wavelength-dependent refractive and absorptive terms for propagation in small-spaced correlated distributions. J. Opt. Soc. Am., 73, 1562–1567.

15. Twomey, S., and C. F. Bohren, 1980: Simple approximations for calculations of absorption in clouds. J. Atmos. Sci., 37, 2086–2094.