GEOPOTENTIAL MODELS IN THE AUSTRALIAN REGION

N90-20519

A. H. W. Kearsley
R. D. Holloway

School of Surveying, University of New South Wales, Kensington, Australia

Abstract

We test the ability of three high-order geopotential models - OSU81, GPM2 and OSU86E - to recover the gravity anomaly field (Δg) in the Australian region. The region was divided into 2° x 2° blocks, and the mean and rms of the residual gravity ($\Delta g_{measured} - \Delta g_{modelled}$) found to estimate the fit of the model to the point gravity data. The results showed that OSU81 and GPM2 performed similarly, recovering the Δg with a mean value of $< \pm 5$ mGal in 63% and 70% of the blocks, respectively. However, both these models achieved a fit of worse that $\pm$ 13 mGal in 6 to 7% of cases. These were in areas either on or near the coast, or in the Central Australian region, inferring that for a precise geoid slope determination in these regions, a detailed analysis of Δg in region is needed. On the other hand, OSU86E produced a very good result, having a mean fit of $< \pm 5$ mGal in 80% of the blocks, and worse than $\pm$ 13 mGal in only 1% of cases. The rms values for this model were also improved over the other two models, indicating that for applications requiring highest precision, the preferred model is OSU86E.

1. Introduction

The growing interest in recovering orthometric heights from GPS has generated the need for models capable of recovering geoid features to finer detail. The past decade has seen the development of a series of models of increasing order, including two models taken to n_{max}= 180 (OSU79 and OSU81; Rapp, 1981), a model summed to n_{max}= 200 (GPM2; Wenzel (1985)), and most recently, two models taken to n_{max}= 360, OSU86E and OSU86F (Rapp, 1986). Geodesists have used these models extensively for geoid studies, mainly to determine the geoidal long to medium wavelength features and to provide the reference model for terrestrially-derived gravity anomalies used in, e.g., Stokes' integral, to find the short wavelength component of the geoid signal.

The ability of GPS to determine ellipsoidal height differences (Δh) over lines whose orthometric height differences (ΔH) are found by conventional levelling has provided, for the first time in history, high precision geometric determinations of ΔN, the geoid height change over the baseline. That is, for small deflections of the vertical

$$\Delta N = \Delta h - \Delta H \qquad (1)$$

ΔN will have a precision equivalent to $\sigma_{\Delta h}$, providing H is determined to 1st or 2nd order. This precision is thought to be of the order of 2 to 4 ppm of the line length. Networks in which Δh and ΔH have been precisely evaluated, therefore, provide valuable control data against which to compare gravimetric determinations of ΔN. We have recently performed a series of such comparisons, and have found that (Kearsley, 1988)

(i) the ability of a geopotential model to recover ΔN varies significantly, both with location and with n_{max},
(ii) the mean fit of ΔN from the geopotential model to the control ΔN constrains the precision obtainable from a full gravimetric solution

In the discussion which follows we test the fit of three models (OSU81, GPM2 and OSU86E) to the gravity anomaly field in the Australian region. Based upon earlier tests, we suggest how this mean fit may be used to estimate the ability of the model to recover ΔN in a particular region. Finally we recommend which model is most suitable for use as a reference in the Australian region.

2. Testing the Geopotential Models.

2.1 Description of Technique for Testing

In earlier tests we have compared the ΔN derived from OSU81 against control ΔN, derived from (1), where ΔH was found by conventional spirit levelling to 3rd-order or better, and Δh from GPS surveys (Kearsley, 1988, p. 6561). The comparisons showed that the ability of OSU81 to recover ΔN varied with both location, and with the upper limit of summation. The tests also showed that the best agreement does not necessarily occur when the geopotential model is taken to its maximum degree and order.

To test a model for its "fitness" - its ability to recover ΔN, in areas where observed ΔN are either non-existent or sparse, we use the statistics of residual gravity δg, where

$$\delta g = \Delta g - \Delta g_L \quad (2)$$

where Δg is the free air gravity anomaly from gravimetric survey and Δg_L is the gravity anomaly generated from the geopotential model.For these tests Δg_L was generated on 0.1° mesh across the Australian region. A value of Δg_L was estimated by interpolation at each gravity point in the Australian Gravity Data Base, and δg obtained by (2). The data set was then analysed in 2° x 2° blocks, this approximating the area used in a spherical cap of integration in a full gravimetric evaluation. The δg_i were then analysed to obtain the mean ($m_{\delta g}$) and root mean square ($rms_{\delta g}$) for the population in the block.

This analysis was repeated for each of three recent geopotential models - OSU81, GPM2 and OSU86E. Each refers to GRS80 (Moritz, 1980) and their maximum degree and order are 180, 200 and 360 respectively.

2.2 Inferences to be drawn from statistics

From the few tests in Australia which compared ΔN_{Grav} against control ΔN a trend has appeared which relates $m_{\delta g}$ to $m_{\delta N}$. This trend is shown in Table 1, where $m_{\delta N}$ is the mean fit of ΔN for GPS lines in the region, in ppm of the line length, and $m_{\delta g}$ is the mean (bias) of the δg field in mGal, analysed over the 2° block containing the control data.

On the basis of this evidence we have inferred across Australia the likely value of $m_{\delta N}$ from the $m_{\delta g}$. This is of particular importance to precise geoid studies because, as we noted above, the geopotential model will constrain the potential precision of the full gravimetric solution if it is unable to sense the geoidal undulations at, say, the 4 to 6 ppm level or better.

3. Discussion of Results

The mean and rms of the δg population for the 256 2° x 2°blocks across the Australian continent were calculated for each of the three models tested.

3.1 Mean fit of Geopotential models to gravity data

The values of $m_{\delta g}$ have been placed into 4 bins, as shown in Table 1 . As is seen from this table, the bin limits were chosen because they appeared to be equivalent to the 5 ppm divisions in $m_{\delta N}$.

The results have been summarised in the histograms in Figure 1, allowing a direct comparison between the three models.

Table 1 : Comparison of $m_{\delta g}$ with m_{dN}

Bin	$\pm m_{\delta g}$ (mGal)	$\pm m_{\delta N}$ (ppm)
1	0-5	0-5
2	5-13	5-10
3	13-21	10-15
4	>21	>15

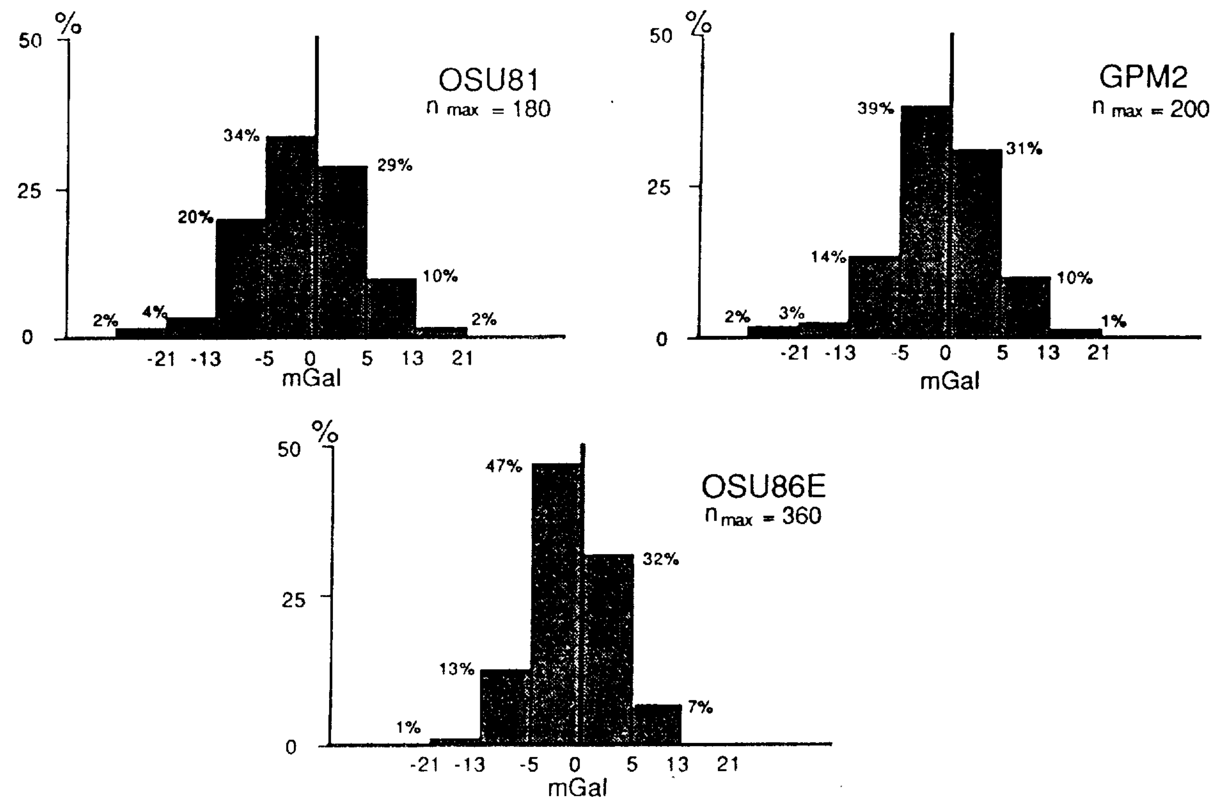
(a) OSU81 and GPM2

For OSU81, 63% of blocks lay in bin 1, 30% in bin 2, 6% in bin 3, with 2% being worse than ± 21 mGal in their mean fit. From this we infer this model can recover ΔN, on average, to 10 ppm or better in 93%, or nearly

240, of the 2° blocks. The areas of poorest fit are near the south western coast, and in the centre of Australia. In the first location, the model may reflect the fact that for the ocean regions, the Δg field used in the model was collocated from geoid undulations derived from radar altimetry. This fact may also explain the high correlation between bin 2 and 3 areas with the coastline. The bin 3 and 4 results in Central Australia are located of the Officer and Amadeus Basins and the McDonnell Ranges (about 1500 m elevation), an area noted for the unusually high signal variations in the Bouger anomaly field. These results suggest the 180 degree model insufficiently sensitive to the gravity signal in this region.
The results for GPM2 were slightly improved over those for OSU81, reflecting the higher order of the former model.

Figure 1: Distribution of Mean Fit of Potential Model to Terrestrial Gravity on 2° x 2° blocks

Number of 2° blocks = 256



(b) OSU86E

The results of this analysis are impressive. From Figure 1 we see that 79% of the blocks fell into bin 1, 20% into bin 2, with 1% in bin 3. No blocks had a fit worse than ± 21 mGal. It appears that, for nearly 80% of Australia OSU86E is capable of recovering ΔN to, on average, 5 ppm. In only three cases will the fit to ΔN be worse than 10 ppm. It is strange that, in one of these cases OSU81 recovers Δg better than does OSU86E. There still appears to be some correlation between bin 2 results and coastal regions, but the poor bin 4 results of OSU81 on the south western coast, and in Central Australia, have disappeared. These trouble spots now lie in bin 2.

It is obvious from this analysis that the mean fit, or bias , of this model with respect to terrestrial gravity has improved greatly over both OSU81 and GPM2. This may be explained by the increased order of the model, enabling it to capture shorter wavelength features in the gravity field. The numerical analysis adopted in this solution, which used quadratures with a desmoothing procedure suggested by Colombo (1981) may also be partly responsible, although the fact that OSU86E summed to 180 replicates almost exactly OSU81 taken to the same order in the South Australian and West Australian test areas tends to discount this factor.

3.2 Root mean square of residual gravity anomalies

The other statistic of significance in this analysis is the rms, which gives some measure of the fluctuations of the δg field from the geopotential model. The results of the computations are summarised in Table 2.

Table 2 Distribution of rms (Population 256)

Bin	Rms Range (mGal)	OSU81	GPM2	OSU86E
1	0-10	64 (25%)	62 (24%)	143 (56%)
2	10-20	142 (55%)	151 (59%)	96 (37%)
3	20-30	38 (15%)	32 (13%)	15 (6%)
4	>30	12 (5%)	11 (4%)	2 (1%)

As is seen from these figures and the summary, OSU81 and GPM2 perform nearly equally well, with 80% and 83% of blocks with rms of less than 10 mGal. As with the mean fit, the blocks of poorest representation are generally in the south western corner and in the centre, and along the east coast of Australia. Table 2 also shows how much the 360 degree model recovers the variations in the gravity field more faithfully. Over half the area is recovered with an rms of less than 10 mGal, while all but 7% of the blocks have an rms less than 20 mGal. Only 2 blocks modeled by OSU86E have an rms > 30 mGal.

Combining mean and rms for OSU86E

The above results confirm the integrity of OSU86E, already demonstrated in the test on the mean fit, in its ability to recover the gravity field across Australia. Indeed, when combining the mean and the rms results for OSU86E, we find that a large portion of the region have both small bias and rms values. Over 40% of the blocks analysed have a mean and an rms both of which lie in bin 1. Obviously the shorter wavelength features in these areas are well modeled by OSU86E. From Table 1 we infer that, in these regions, OSU86E will recover ΔN to better than 5 ppm, and that there may be little benefit in incorporating the short wavelength signal in ΔN obtained from the detailed analysis of surface gravity. Predictably, however, most of these areas lie across the inland, sparsely developed region of Australia. The areas which contain most development activity, the coastal regions, still exhibit less favourable recovery by this model and will apparently still require a full gravimetric solution for highest precision.

4. Conclusions

From these tests we see how the ability of a geopotential model to fit the gravity field across Australia improves with the increased order of the model Iit appears that this improvement is due almost entirely to the higher order of the model, and not to the different numerical technique used to solve for the potential coefficients. Finally, it appears that for 40% of the region, OSU86Ewill serve in the recovery of ΔN for all but the most exacting purposes. However for the coastal regions where most development activity occurs, a full gravimetric solution involving a detailed analysis of detailed gravity will be required for most higher-order surveying tasks.

References

Colombo, O.L., Numerical methods for harmonic analysis on the sphere, *Report No. 310*, Dept. Geod. Sci., The Ohio State University, Colombus, 1981.

Kearsley, A. H. W., Tests on the recovery of precise geoid height differences from gravimetry, *J. Geophys. Res.*, *93*(B6), 6559-6570, 1988

Moritz, H., Geodetic Reference System 1980, *Bull. Geod.*, *54*.(3), 395-405, 1980.

Rapp, R. H., The earth's gravity field to degree and order 180 using Seasat altimeter data, terrestrial gravity data and other data, *Rep. 322*, Dep. of Geod. Sci. and Surv., Ohio State.

Rapp, Richard H. and Cruz, Jaime, Y. Spherical harmonic expansions of the earth's gravitational potential to degree 360 using 30' mean anomalies, *Report No. 376*, Dept Geod. Sci. and Surv., The Ohio State University, Columbus, Ohio, 1986.

Wenzel, H.-G. Hochauflösende Kugelfunktionsmodelle für das Gravitationspotential der Erde, *Wissenschaftliche Arbeiten der Fachrichtung Vermessungswessen der Universität Hannover*, Hannover, 1985.