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STABILIZED DETERMINATION OF GEOPOTENTIAL COEFFICIENTS  
BY THE MIXED HOM-BLUP APPROACH

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1. INTRODUCTION

For the determination of geopotential coefficients we can use data from rather different sources, e.g. satellite tracking, gravimetry or altimetry. As each data type is particularly sensitive to certain wavelengths of the spherical harmonic coefficients it is of essential importance how they are treated in a combination solution. For example the longer wavelengths are well described by the coefficients of a model derived by satellite tracking, while other observation types such as gravity anomalies  $\Delta g$  and geoid heights  $N$  from altimetry contain only poor information for these long wavelengths. Therefore, the lower coefficients of the satellite model should be treated as being superior in the combination. In our contribution we present a new method which turns out to be highly suitable for this purpose due to its great flexibility combined with robustness.

2. METHODS

In *B. Middel/B. Schaffrin (1987)* we introduced a method based on "robust collocation", which according to *B. Schaffrin (1985,1986)* is the Best homogeneously Linear (weakly) Unbiased Prediction (hom-BLUP), as a promising technique for the combination of terrestrial gravity data with spherical harmonic coefficients from satellite tracking. With this method we obtain the predicted coefficients, collected in the vector  $\tilde{x}$ , by

$$\tilde{x} = (P_S + P_T)^{-1} (P_S \xi_S + P_T \xi_T a) \quad (2.1)$$

where  $\xi_S$  contains the satellite coefficients and  $P_S$  is the corresponding weight matrix. Vector  $\xi_T$  contains coefficients which we obtained by a least-squares adjustment within a Gauß-Markov Model from terrestrial gravity data and  $P_T$  is again the corresponding weight matrix. The terrestrial coefficient set  $\xi_T$  is taken to be inferior with respect to the lower coefficients and by comparison with the satellite coefficient set  $\xi_S$  we obtain the scalar factor  $a$  to fit  $\xi_T$  to  $\xi_S$ . When the fitting factor is defined as  $a=1$  we obtain the weighted mean of both data sets being, of course, the "geodetic collocation" according to *H. Moritz (1973)*, which is the Best inhomogeneously Linear Prediction (inhom-BLIP).

However hom-BLUP turned out to be robust against inconsistencies in  $\xi_T$  and therefore, in this sense, superior to inhom-BLIP as we showed by applying statistical tests in *B. Middel/B. Schaffrin (1988)*. Nevertheless we can make this approach more flexible by splitting up the weaker coefficient set  $\xi_T$  into groups of a special character. We allow them specific fitting factors collected in a vector  $\underline{a}$  and name it "Mixed hom-BLUP", thus leading to the following solution:

$$\tilde{x} = (P_S + P_T)^{-1} (P_S \xi_S + P_T \Xi_T \underline{a}) . \quad (2.2)$$

In this formulation the vector  $\xi_T$  is modified to a matrix  $\Xi_T$  where each column contains one group of coefficients at the respective places with zero entries otherwise.

### 3. NUMERICAL EXAMPLE

After this short description we now present some results of combination solutions up to degree 36 where we merged coefficients  $\xi_T$ , derived either from gravity anomalies or geoid heights or alternatively from a combination of them both, with GEM-L2 coefficients  $\xi_S$  up to degree 20, as described in *F.J. Lerch et al (1982)*. For each combination we split up the weaker coefficient set  $\xi_t$  by degree and by order when using the Mixed hom-BLUP technique. In figures 3.1 to 3.3 we plotted the components of the vector  $\underline{a}$  with respect to the group (i.e. degree or order) together with their confidence intervals of a significance level of 95%. In addition, the scalar factors  $a$  of inhom-BLIP and hom-BLUP were added.

Figures 3.1 show that we obtained with all methods different fitting factors  $a$  and therefore different results when we merge the GEM-L2 coefficients with coefficients  $\xi_T$  adjusted from gravity anomalies  $\Delta g$ . But it has to be mentioned that when using Mixed hom-BLUP and splitting  $\xi_T$  up by order most of the components of  $\underline{a}$  are not significantly different from the scalar  $a$  of ordinary hom-BLUP since these values lie inside the 95% confidence interval.

The situation changes when we introduce coefficients  $\xi_T$  computed with geoid heights  $N$ . This data set is fully compatible with the GEM-L2 coefficients and therefore all fitting factors are very close to 1 as illustrated in figures 3.2. In this case we obtained nearly identical solutions with all the methods.

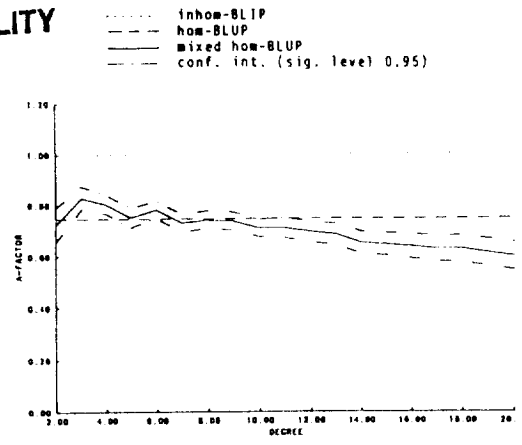
In figures 3.3 and 3.4 we present results which we obtained by a combination of GEM-L2 coefficients with coefficients which we adjusted from both data sets  $\Delta g$  and  $N$ . Figures 3.3 show that the scalar  $a$  is very close to 1 but with the Mixed hom-BLUP we obtained, in both cases of splitting,  $a$ -components far away from this value. Therefore, the solution with ordinary hom-BLUP leads to different results in both cases of splitting up the coefficients  $\xi_t$ . This can be very clearly seen in figure 3.4 where the degree variances of the different solutions have been plotted.

### 4. CONCLUSIONS

The use of "Mixed hom-BLUP", which we present in this contribution, leads to different solutions compared with "geodetic collocation" (inhom-BLIP) and "robust collocation" (hom-BLUP) if the combined data sets are not compatible. Due to the great flexibility and robustness of this method we expect that it is highly suitable for estimating geopotential coefficients when combining heterogeneous data sets.

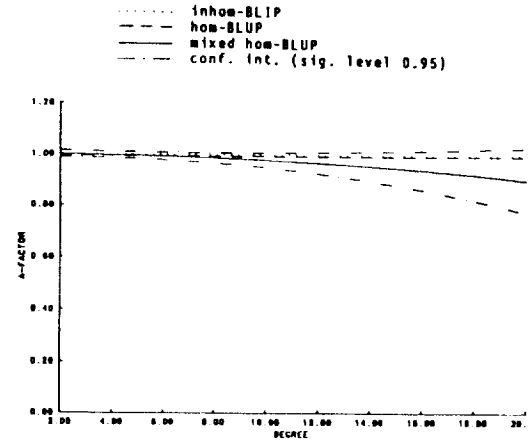
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a-factors for combination of GEM-L2 with  $ag$  ( $i=1$ )



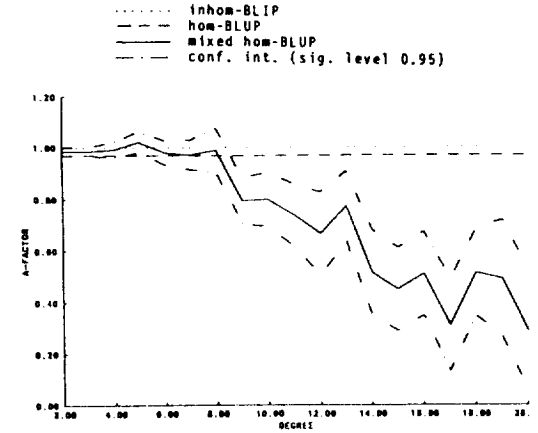
Figures 3.1

a-factors for combination of GEM-L2 with  $N$  ( $i=2$ )



Figures 3.2

a-factors for combination of GEM-L2 with  $ag$  and  $N$  ( $i=3$ )



Figures 3.3

degree variances for combination of GEM-L2 with  $ag$  and  $N$  ( $i=3$ )

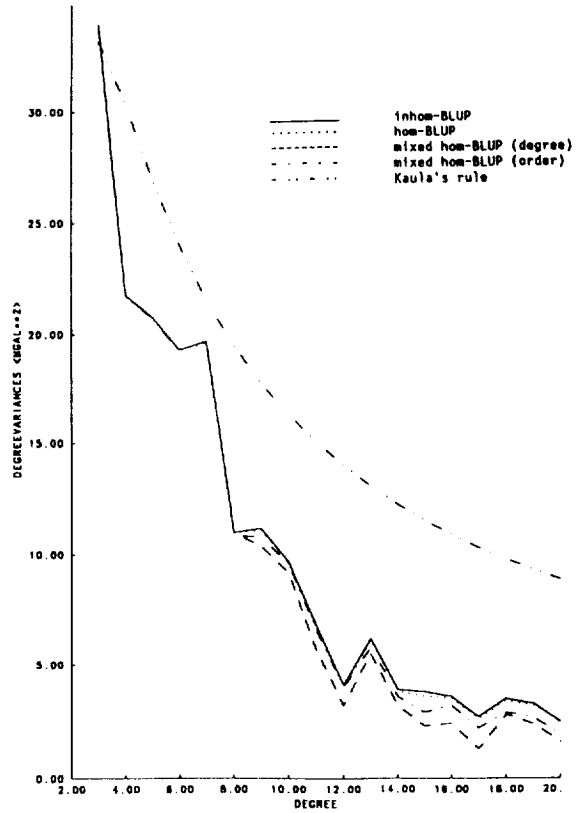


Figure 3.4

## ACKNOWLEDGEMENT

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