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Methods for Local Gravity Field Approximation

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ABSTRACT

The most widely known modern method for estimating gravity field values from observed data is least-squares collocation. Its advantages are that it can make estimates at arbitrary locations based on irregularly spaced observations, and that it makes use of statistical information about errors in the input data while providing corresponding information about the quality of the output estimates. Disadvantages of collocation include the necessity of inverting square matrices of dimension equal to the number of data values and the need to assume covariance models for the gravity field and the data errors. Fourier methods are an important alternative to collocation. They have the advantage of greater computational efficiency, but require data estimates to be on a regular grid and do not use or provide statistical accuracy information.

The GEOFAST algorithm is an implementation of collocation that achieves high computational efficiency by transforming the estimation equations into the frequency domain where an accurate approximation may be made to reduce the workload. The forward and inverse Fast Fourier Transforms (FFT) are utilized. We have demonstrated the accuracy and computational efficiency of the GEOFAST algorithm using two sets of synthetic gravity data: marine gravity for an ocean trench region including wavelengths longer than 200 km; and local land gravity containing wavelengths as short as 5 km. We discuss these results along with issues such as the advantages of first removing reference field models before carrying out the estimation algorithm.

1. INTRODUCTION

Algorithms for estimating gravity field quantities should be theoretically sound, accurate, and computationally efficient. In characterizing and describing such algorithms, the issues we consider include the specific geodetic quantity or quantities to be computed or estimated, the general approach (deterministic or statistical), the type or types of measurement data to be used, the geographic distribution of points at which geodetic quantities are observed and are to be computed, and the approximations (if any) made in the computation algorithm. The class of *least-squares collocation methods* is *statistical* in nature; *general* in terms of the quantities to be estimated, the measured data to be used, and the geographical distribution of data and computation points; and, in principle, *exact* if assumed covariance models are reasonable.

GEOFAST is an approach to implementing collocation with high efficiency by carrying out computations in the spatial frequency domain. It assumes the availability of two-dimensional gridded data; it uses stationary statistical gravity field models; it computes minimum-variance estimates of gravity field quantities; and it achieves high computational efficiency of the order of N log N operations, where N is the number of measurement points. This efficiency depends on the use of a critical approximation which, in practice, introduces little loss of accuracy. Furthermore, the user has control of the tradeoff between accuracy and computational efficiency.

2. THE GEOFAST ALGORITHM

The GEOFAST algorithm provides an efficient computational solution to the minimum-variance estimation equations of collocation:

$$\underline{x} = C_{xz} (C_{zz})^{-1} \underline{z} \tag{1}$$

where $\underline{z}$ is a data vector of dimension N, C_{zz} (of dimension N by N) is the sum of its autocovariance plus measurement noise, $\underline{x}$ is a vector of estimates (of dimension M), and C_{xz} is the cross-covariance matrix (of dimension M by N) between the estimated and measured quantities. Direct solution of equation (1) involves on the order of $N^3 + MN$ operations, limiting the feasible applications of collocation methods where data sets of thousands of measurements are involved.

This workload can be reduced to the order of N log N, if the following assumptions are valid: measured values are given, and estimates are required, at the same points on a rectangular grid, and covariances are a function of relative position only (shift invariance). In practice, shift invariance is equivalent to the assumption of stationary statistics. These assumptions impose a special structure on the matrices appearing in equation (1): they are block Toeplitz. The special properties of block Toeplitz matrices are essential to the GEOFAST algorithm.

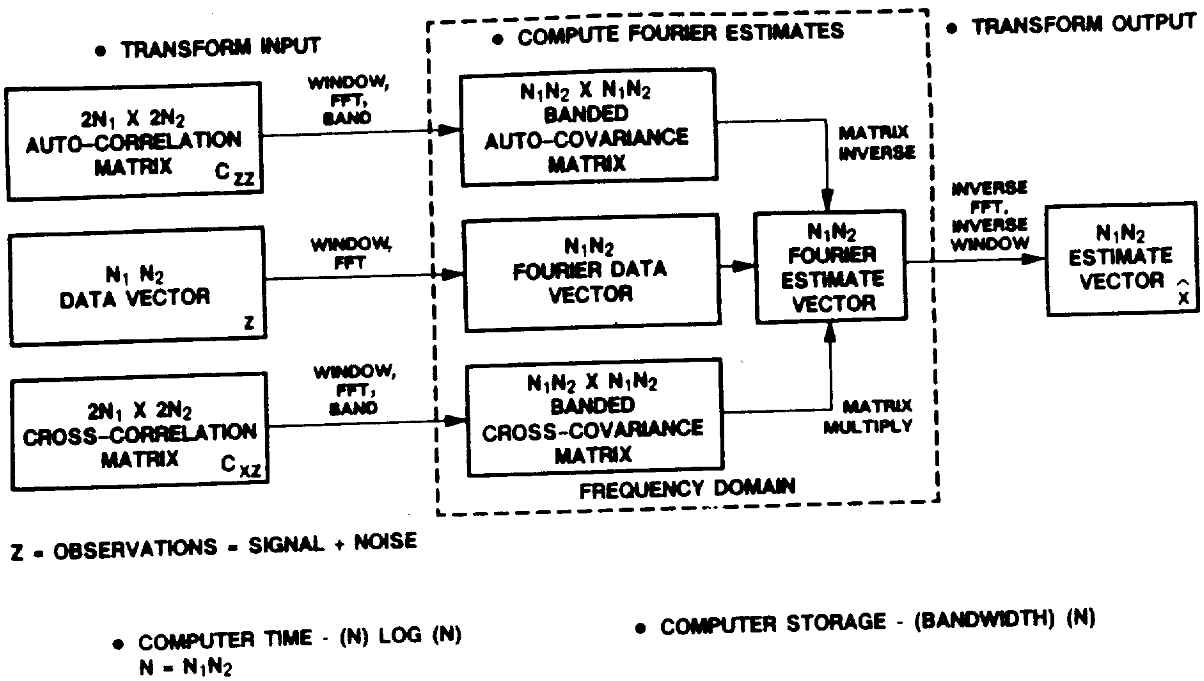

Figure 1. Outline of the GEOFAST algorithm.

Transformation of equation (1) to the spatial frequency domain is carried out efficiently through the use of the Fast Fourier Transform (FFT), leading to an equivalent estimation equation in the transformed variables $\underline{x}'$ and $\underline{z}'$:

$$\underline{x}' = C'_{xz}\,(C'_{zz})^{-1}\,\underline{z}' \tag{2}$$

In practice, the first step of the GEOFAST algorithm, as illustrated in the left-hand column of Figure 1, is the transformation of the data vector $\underline{z}$ and the matrices C_{zz} and C_{xz} into the frequency domain. The transforma- tions, however, are not *exact*; approximations essential to the computational economy of the GEOFAST algorithm are incorporated. In effect, the C_{zz}' and C_{xz}' matrices that would be the exact transforms are replaced by banded approximations, and a compensating data windowing is applied in the computation of $\underline{z}'$. The second stage of the GEOFAST algorithm, shown in the central box of Figure 1, is the approximate solution of the banded version of equation (2) through the use of an iterative technique. In effect, this is equivalent to the computation of $\underline{x}'$. The final step (right-hand column of Figure 1) is the inverse FFT from $\underline{x}'$, the frequency-domain solution, to $\underline{x}$, the spatial domain solution, incorporating the inverse of the windowing operation that was part of the transformation from $\underline{z}$ to $\underline{z}'$.

User control of tradeoffs between accuracy and computing time in the GEOFAST algorithm involves the selection of parameters governing the *bandwidth* retained in the approximations to the matrices C_{zz}' and C_{xz}', and the number of iterations carried out in the iterative solution of the approximate frequency-domain equations. In actual applications, reasonable parameter choices result in an accurate and highly efficient algorithm.

3. FIRST TEST DATASET — MARINE GRAVITY

For the first set of tests, gridded values of gravity anomaly and deflection of the vertical were generated for a 36 deg by 36 deg area in the western Pacific that includes a significant ocean trench, using the Rapp (1981) worldwide spherical harmonic expansion to degree and order 180. The GEOFAST algorithm is used to compute components of the deflection from the gravity anomaly data; comparison with the deflection components computed originally from the spherical harmonic model (regarded as truth data) provides a measure of algorithm performance. To quantify the effects of using high-order reference fields, the same spherical harmonic model can be used to generate a set of anomalies and deflections, but for degrees 91 through 180 only. Using GEOFAST with this high-frequency field is equivalent to removing a degree-and-order-90 reference field from the data prior to processing. Other parameters explored in the testing include data extent, grid spacing, GEOFAST bandwidth, choice of covariance models, and measurement noise level.

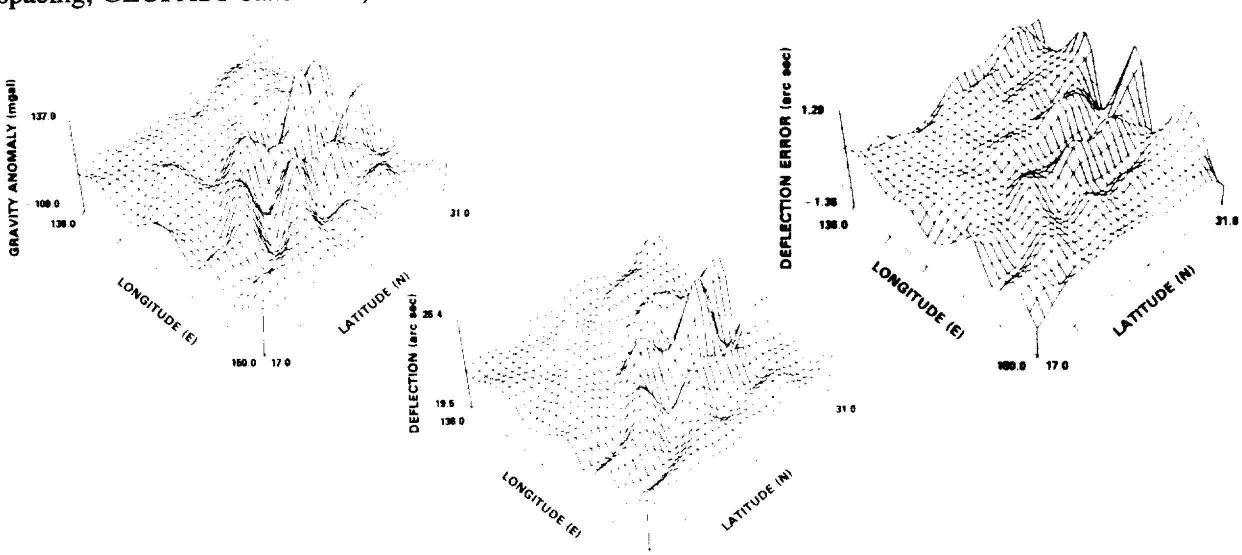

Figure 2. Input data and results for GEOFAST test. Input gravity anomalies are shown at the left; synthetic deflection truth data are shown in the center; errors in the estimated deflection appear on the right.

A typical test is illustrated in Figure 2. The gravity anomaly data (in this case with the degree-90 reference field removed) at 0.5 deg grid spacing are shown on the left; these are used as input to GEOFAST. Deflection data computed by GEOFAST for the same grid, assuming an attenuated white noise covariance model (with correlation distance of 0.5 deg), will be compared to the synthetic "truth data" shown in the middle. The resulting deflection error field appears on the right. Some test results are summarized in Table 1, which shows the effects of the bandwidth parameter, m_B, and the grid spacing (0.25 or 0.50 deg) on accuracy and computing time. Table 2 summarizes the effects of measurement noise on estimation accuracy. To a first approximation, measurement noise (in this case, white noise) contributes additively to estimation error variance. The

Table 1. Summary of GEOFAST Test Results.

Resolution		Bandwidth		Error (arcsec)		
Grid (deg)	Points	MB	Time* (min)	Mean	Sigma	Maximum
0.25	3600	0	4	0.001	0.15	0.70
0.25	3600	4	30	0.003	0.09	0.31
0.50	900	0	1	0.003	0.17	0.59

Table 2. Effects of Measurement Error on GEOFAST Performance.

Grid (deg)	Measurement Error rms (mgal)	Noise-Induced Error rms (arcsec)	Total Estimation Error rms (arcsec)
0.25	0	0.00	0.15
	1	0.03	0.15
	2	0.07	0.17
	5	0.16	0.22
0.50	0	0.00	0.17
	1	0.04	0.17
	2	0.08	0.19
	5	0.20	0.26

effects of removing a reference field from the measured data were explored in another set of tests. A typical result is that the error standard deviation is reduced by a factor of more than five by removing the reference field.

4. SECOND TEST DATASET — LAND GRAVITY

The second set of tests was designed to investigate the performance of GEOFAST on local data sets containing high-frequency information. Synthetic data were generated, using a mass dipole model originally formulated for the gravity gradiometer testing program. The statistical properties of the data are consistent with those for an actual test area in north Texas. As in the marine gravity tests, synthetic truth data were generated in the form of gravity anomalies and deflections of the vertical, for each point of a 5 km grid covering a square 500 km on a side. GEOFAST is used to estimate the east component of the deflection from the anomaly data; comparison with the directly generated deflection values provides a measure of the algorithm's performance. These tests use a covariance model with a correlation distance of 10 km. Estimation regions ranging from 40 to 150 km on a side are selected from the interior of a 150 km square located centrally within the original 500 km area, with resulting error standard deviations between 0.4 and 1.9 arcsec. The estimation accuracy would be improved by using a finer grid spacing.

5. SUMMARY

The GEOFAST algorithm, described briefly above, is used to estimate deflection of the vertical from gravity anomaly to an rms accuracy of better than 0.2 arcsec with modest computer cost. Conventional implementations of collocation require considerably more computing effort. GEOFAST provides a user tradeoff between computing cost and estimation accuracy. In a typical application reducing the accuracy requirement from 0.1 to 0.2 arcsec reduces computing time by a factor of eight. Other test results confirm the importance of removing a high-order reference field from the data before applying the algorithm. For example, removing a degree-90 reference field from the synthetic marine gravity data reduces estimation errors by a factor of five.

The GEOFAST algorithm can be applied to current real-world problems involving the reduction of gradiometer or altimeter data. Potential application areas include the production of accurate maps for marine crustal and lithospheric studies.