A DISCUSSION OF OBSERVATION MODEL, ERROR SOURCES AND SIGNAL SIZE FOR SPACEBORNE GRAVITATIONAL GRADIOMETRY

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Various space concepts have been discussed during the past 20 years for a global improvement of our knowledge of the Earth's gravity field. They reach from high-low and low-low satellite-to-satellite tracking via tethered satellite gradiometers to sophisticated super-conducting gradiometers, currently under discussion. The purpose of this article is to show that starting from one basic equation three criteria are sufficient to typify the various concepts and define the underlying observation model. Furthermore the different error sources, in particular the time varying part of self-gravitation, and the expected signal size of all six gravity gradient components shall be discussed.

Assume two proof masses A and B in free fall are observed from a moving orthonormal triad, see Figure 1. Then the relative acceleration $d\ddot{x}_i$, between A and B relative to their distance dx_j (components i and j = 1,2,3) obey the following conservation law:

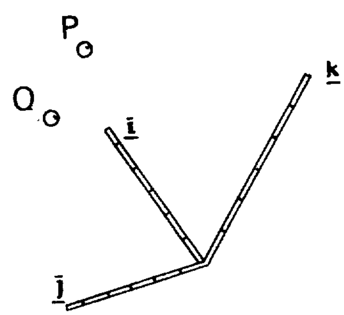

Figure 1.

$$\frac{d\ddot{x}_i}{dx_j} + 2\,\Omega_{ki}\,\frac{d\dot{x}_k}{dx_j} + \dot{\Omega}_{ij} + \Omega_{ik}\,\Omega_{kj} - V_{ij} - \frac{f_i(A)-f_i(B)}{dx_j} = 0 \qquad (1)$$

In eq.(1) it is $2\Omega_{ki}\frac{d\dot{x}_k}{dx_j}$, $\dot{\Omega}_{ij}$, and $\Omega_{ik}\Omega_{kj}$ the Coriolis, inertial rotation, and centrifugal term, respectively, with Ω_{ik} the angular velocities; $V_{ij} = \frac{\partial^2 V}{\partial x_i \partial x_j}$ are the gravitational gradients, and $f_i(A)$ and $f_i(B)$ non-gravitational accelerations acting on A and B. If the above experiment is carried out at satellite altitude and if the purpose is to determine V_{ij}, we speak of spaceborne gradiometry.

In order to derive V_{ij} as accurate as possible obviously the measurement precision has to be as high (10^{-2} to 10^{-4} E) and the satellite altitude as low as possible (preferably below 200 km). However, three criteria are sufficient to identify the various configurations. These are (1) the orientation of the instrument frame or triad, being either space stable or Earth pointing, (2) the motion of the proof masses, either free drifting or constrained to linear or rotational movement and (3) the shielding against non-gravitational forces, either by an active drag-free system, or by enclosing the proof masses in the satellite but the measurement triad rigidly fixed to its skin, or with no shielding at all. The choices on these three criteria decide about the form eq. (1) takes and what interpretation its terms acquire. Take two examples: In case the instrument frame is maintained space stable the three terms containing Ω and $\dot{\Omega}$ disappear. Or, for an active drag-free system and the proof masses constrained linearly to the triad e.g. by an electric spring, $d\ddot{x}$ and dx become zero and $f(A)$ and $f(B)$ the measured specific forces.

These choices decide as well what the observable accelerometer signal along the three axes will be from which the gradiometer components are derived. Take for example an Earth pointing gradiometer with no active drag-free control, with the x-axis along track, the y-axis cross track, and the z-axis radial and with the proof masses of the orthogonal set of accelerometers constrained to the axes. The dimension of the gradiometer is assumed to be 1 m and its center close to the center of mass of the spacecraft. Then the average accelerations (DC-part) listed in Table 1 shall be typically measured along the three axes. The variations in signal (AC-part) are less than $\frac{1}{100}$ of these values.

TABLE 1 : Acceleration Signal (units $10^{-5} ms^{-2}$)

	gravitational	centrifugal	drag
x (along)	0.15	0.15	2
y (across)	0.15	0	0
z (radial)	0.31	0.15	0

We observe that the along track component is heavily affected by the drag, whereas the cross-track component remains largely free from non-gravitational perturbations. This is one of the main reasons, why for the ARISTOTELES mission a plane (y-z)-two dimensional gradiometer is considered.

Once a decision is made about a specific gradiometer design, it is important to develop a realistic error model. In order to get some structure into the various error sources, we divide them into (1) instrument errors, (2) satellite related errors and (3) geodetic gravity recovery model errors. The instrument errors depend largely on the chosen design. Adequate models can only be developed in cooperation with the instrument designer. At this point we refer to (Reinhardt et al.,1982), (Balmino et al., 1985), (Paik & Richard, 1986), or (Sepers, 1986). Satellite related errors are e.g. thermal, electro-magnetic or vibrational effects, deviations from common mode rejection of drag effects due to non-linearities (Barlier & Berger, 1988), self-gravitation, or attitude related errors. We studied the time-varying self-gravitation effect due to fuel consumption. Assuming 1000 kg fuel consumed over half a year the main effect is - depending on the symmetry of the tank configuration - a drift of about 50E per half year. Additional sloshing effects could reach 2-5 E and are to be avoided. Error sources related to the gravity field recovery model reach from the proper modelling of the sampled signal, via the effects of induced symmetries in the adjustment models to stability and convergency problems of downward continuation. Their study requires more attention in the forthcoming years.

In order to get an impression of the size of the gravitational signal, all six gradient components were generated on a global $1^{o} \times 1^{o}$

grid, with the OSU-180 field (Rapp, 1986) at an altitude of 200 km. Then a spherical harmonic analysis was carried out on each of the components separately and the degree variances c_n and degree-order variances c_{nm} were computed. The degree-order variance is defined as $c_{nm} = c_n/(2n+1)$ with n degree and m order and represents the square of the expected average size of an individual spherical harmonic coefficient. The result up to degree 180 is given in terms of the r.m.s. values of c_{nm} in Figure 2. As expected, the (zz)-component is roughly half an order of magnitude greater than the (xx), (yy), (xz), and (yz) component, which are in turn half an order of magnitude greater than (xy). This implies among others that most emphasis should be put on a precise recovery of the (zz) component.

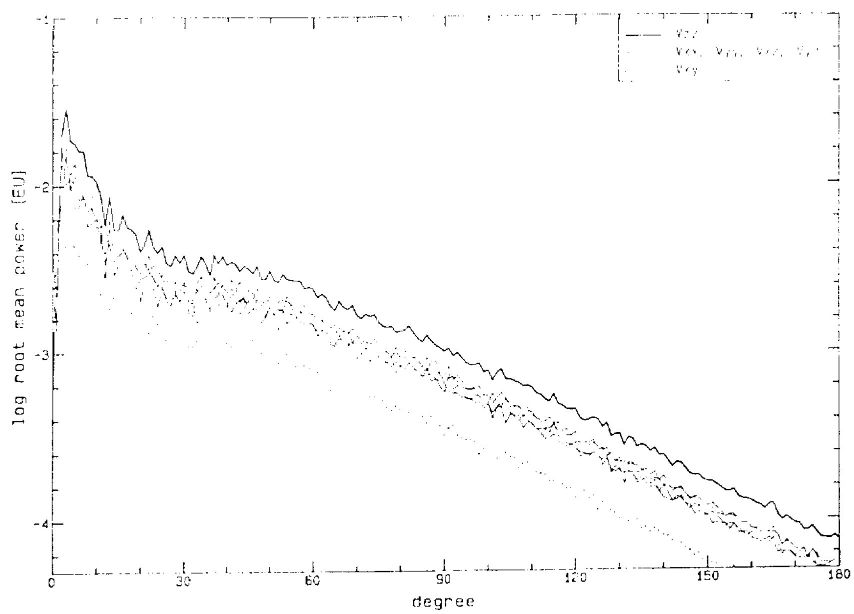

Figure 2.

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