N90-20936

APPLICATION OF STOCHASTIC ROBUSTNESS TO AIRCRAFT CONTROL SYSTEMS

Laura E. Ryan
Department of Mechanical and Aerospace Engineering
Princeton University

INTRODUCTION

Guaranteeing robustness has long been an important design objective of control system analysis. *Stochastic robustness* is a simple numerical procedure that can be used to measure and gain insight into robustness properties associated with linear control systems. In the realm of aircraft control systems, problems such as the effects of flight condition perturbations and model-order uncertainties on robustness are easily and effectively analyzed using stochastic robustness. The concept of stochastic robustness will be reviewed and examples will be presented demonstrating its use in flight control system analysis.

- Summary of stochastic robustness
- Control system robustness with flight condition perturbations
- Control system robustness with model-order uncertainties

    Actuator dynamics
    Aeroelastic effects

- Summary of results

DEFINITIONS

Control system ***robustness*** is defined as the ability to maintain satisfactory stability or performance characteristics in the presence of all conceivable parameter variations. A good robustness measure is vital to guarantee and understand control system robustness. ***Stochastic robustness*** provides such a measure. It uses the statistics of a plant's variable parameters and Monte Carlo simulation to compute the probability distributions of closed-loop system characteristics. Present research has concentrated on stability robustness as characterized by the closed-loop eigenvalues, although the method can be extended to other closed-loop characteristics. Stochastic robustness is computationally simple. For a single Monte Carlo evaluation, random numbers are generated and shaped to match the parameter statistics, added to the mean parameter vector, and the closed-loop eigenvalues are computed using the modified parameters. Repeated Monte Carlo evaluations give rise to the ***stochastic root locus***, a plot of the probability distributions of the closed-loop eigenvalues. The ***probability of instability***, or probability that all of these eigenvalues lie in the open left-half s plane, is the scalar measure of robustness.

Robustness

The ability to maintain satisfactory stability/performance characteristics in the presence of all conceivable parameter variations.

Stochastic robustness

A robustness *measure* based on the probability distributions of closed-loop characteristics, given the statistics of a plant's variable parameters.

- Characteristics can be eigenvalues, performance, control authority, disturbance rejection
- Based on *Monte Carlo* simulation
- Not limited to Gaussian parameters

Stochastic root locus

Plot of the probability distributions of closed-loop eigenvalues.

Probability of instability

Probability that closed-loop system is unstable - a scalar measure of stability robustness.

STOCHASTIC ROBUSTNESS
APPLIED TO DEMONSTRATOR AIRCRAFT

As an example of the application of stochastic robustness, three controllers are applied to a fourth-order longitudinal model of an open-loop unstable aircraft. A ten-element parameter vector consisting of elements of the dynamic and control effect matrices is chosen. The three control designs are chosen to reflect increasingly robust controllers. The first two Cases are LQR controllers with low and high control weighting respectively, and the third controller multiplies the Case (b) controller by a factor of five to restore the closed-loop bandwidth to that of Case (a). These three cases have been chosen not to satisfy any particular flying qualities criteria, but merely to demonstrate the impact of differing generalized design criteria on stochastic robustness.

Fourth-order longitudinal dynamic model

$$\dot{\mathbf{x}} = \mathbf{F}(\mathbf{p})\,\mathbf{x} + \mathbf{G}(\mathbf{p})\,\mathbf{u}$$

$$\mathbf{u} = -\mathbf{C}\,\mathbf{x}$$

Ten-element parameter vector

$$\mathbf{p} = [f_{11}\ f_{12}\ f_{13}\ f_{22}\ f_{32}\ f_{33}\ g_{11}\ g_{12}\ g_{31}\ g_{32}]$$

f_{ij}, g_{ij} are elements of matrices **F** and **G**.

Control design (**C** *matrix) to demonstrate stochastic robustness*

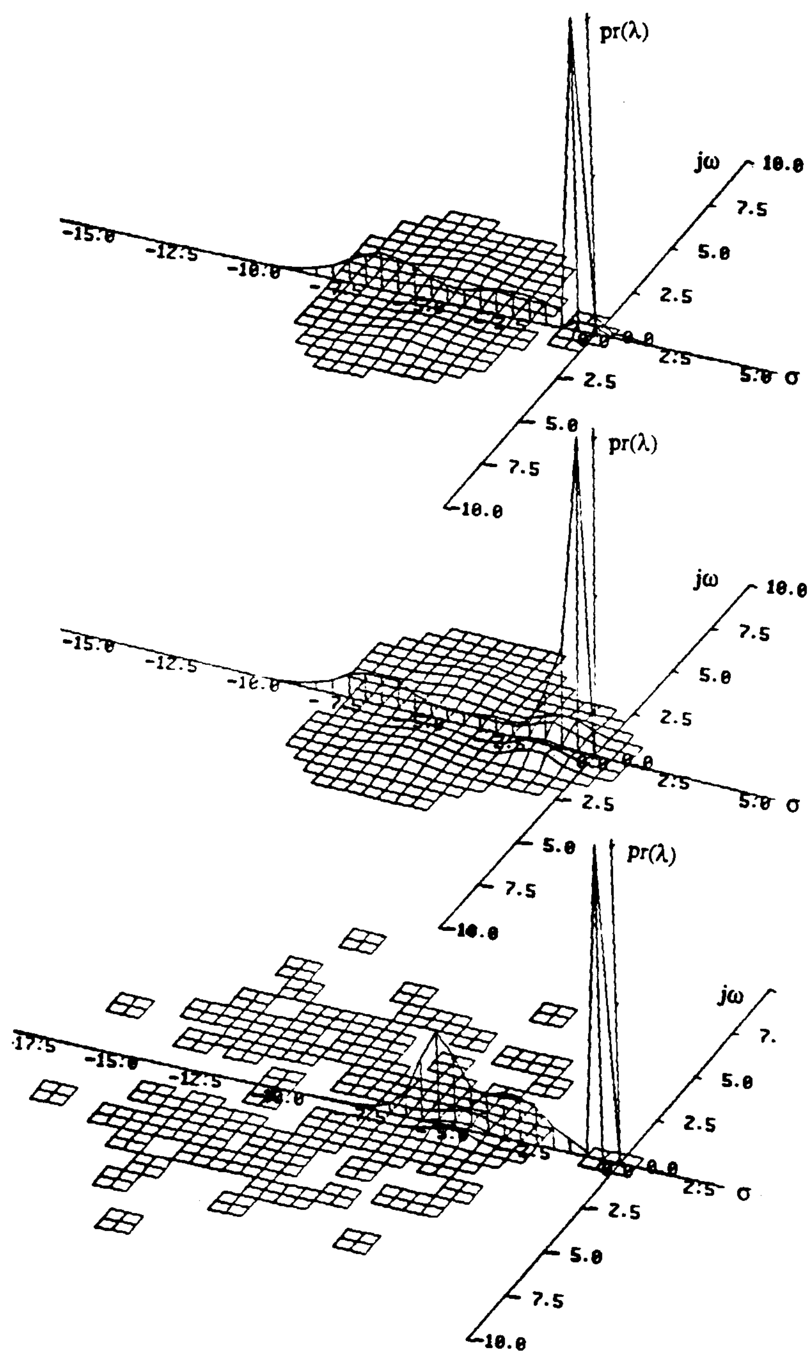
Case (a) LQR with low control weighting.

Case (b) LQR with high control weighting.

Case (c) Gain matrix of Case (b) is multiplied by 5 to restore bandwidth.

Stochastic root loci for demonstrator aircraft with 30% standard-deviation Gaussian parameters

The results for 30% Gaussian parameters and 10,000 Monte Carlo evaluations reflect the expected increase in robustness between control designs. The stochastic root locus shows the extent to which eigenvalues can vary. The eigenvalue near the origin is least affected by the parameter changes, and its peak dominates the distribution. In Cases (a) and (c), the left-most eigenvalue (not shown) has an enormous variance along the real axis. Interaction of roots around the origin causes instability. Robustness improves from Case (a) to (b) as control usage is restrained by high control weighting, and the ad hoc robustness recovery technique used in Case (c) gives additional improvement.



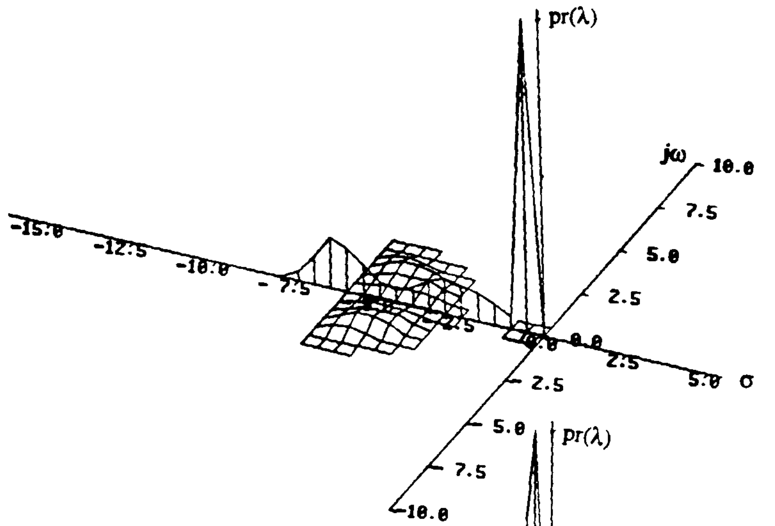
Case (a)
$\mathbb{P} = 0.0711$

Case (b)
$\mathbb{P} = 0.0169$

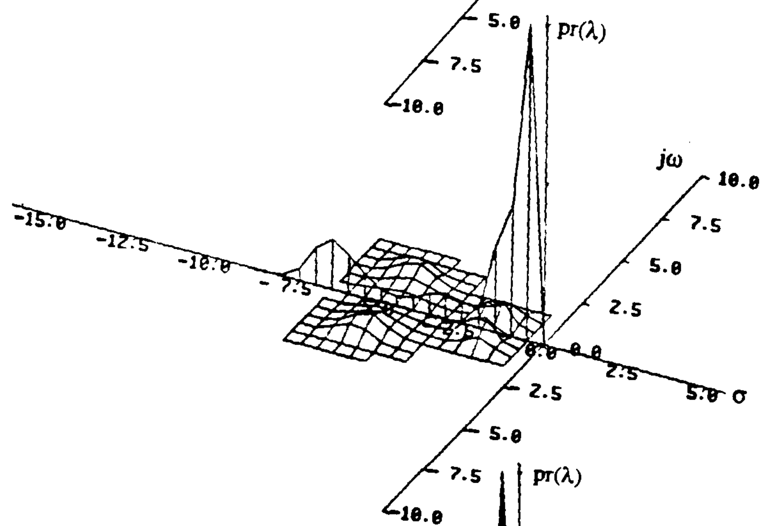
Case (c)
$\mathbb{P} = 0.0033$

Stochastic root loci for demonstrator aircraft with 30% uniformly distributed parameters

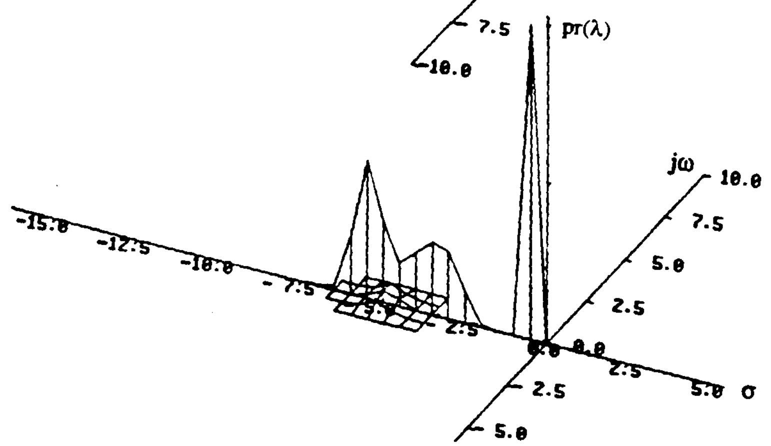
For 30% uniformly distributed parameters and 10,000 Monte Carlo evaluations, the probability of instability for all three cases is zero. The stochastic root locus gives a good indication of the effects of Gaussian "tails" on the eigenvalue probability densities.



Case (a)
$\mathbb{P} = 0.0$



Case (b)
$\mathbb{P} = 0.0$



Case (c)
$\mathbb{P} = 0.0$

CONTROL SYSTEM ROBUSTNESS WITH FLIGHT CONDITION PERTURBATIONS

Demonstrator Aircraft with Flight Condition Effects

Dynamic pressure variations can be considered separately and included in the parameter vector. Although velocity (V) and air-density (ρ) are essentially deterministic, including them as separate parameters gives the ability to look at flight condition perturbations around the nominal and eliminates correlation of the remaining parameters. A twelve-element parameter vector results. ρ and V are modeled as uniform parameters, giving an indication of stochastic robustness over a range of flight conditions.

Fourth-order longitudinal dynamic model

$$\dot{\mathbf{x}} = \mathbf{F}(\boldsymbol{p})\,\mathbf{x} + \mathbf{G}(\boldsymbol{p})\,\mathbf{u}$$

$$\mathbf{u} = -\mathbf{C}\,\mathbf{x}$$

Twelve-element parameter vector

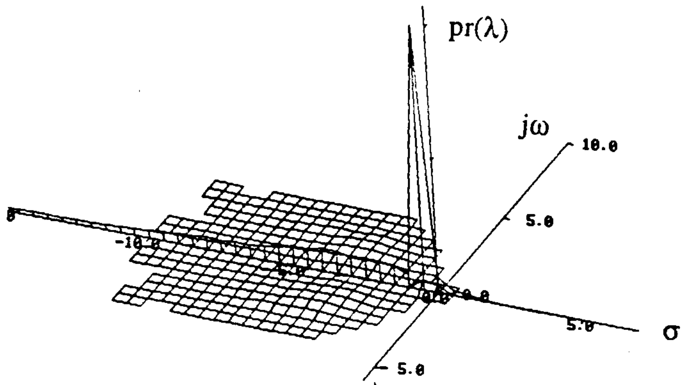
$$\boldsymbol{p} = [\rho \; V \; f_{11} \; f_{12} \; f_{13} \; f_{22} \; f_{32} \; f_{33} \; g_{11} \; g_{12} \; g_{31} \; g_{32}]$$

ρ is the air density (nominal value 0.00152 s/ft^3)
V is the velocity (nominal value 670 ft/sec)
f_{ij}, g_{ij} are elements of **F** and **G** with ρ and V considered separately.

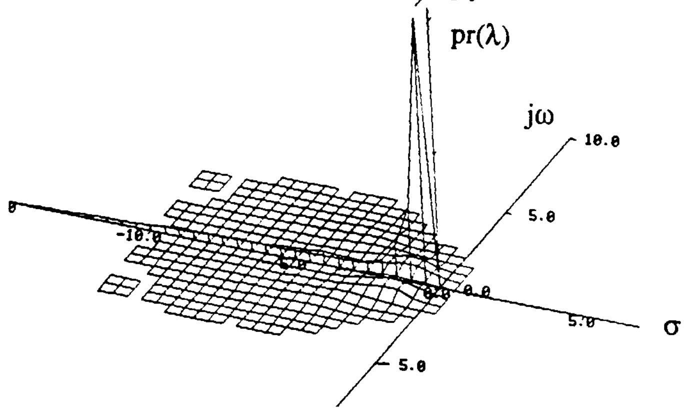
Model ρ and V as uniform parameters and apply stochastic robustness using the same three control designs as previous example.

Stochastic root loci for demonstrator aircraft with 30% Gaussian parameters and 30% uniform ρ and V

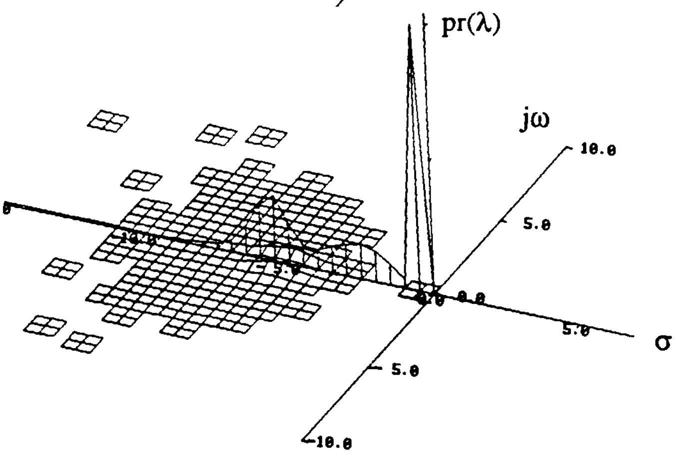
Next, we examine the stochastic root loci for these three cases: for 30% uniform ρ and V, and 30%-standard-deviation Gaussian uncertainty on each of the remaining elements of the parameter vector. The shapes of the root loci are similar to the case with correlated parameters.



Case (a)
$\mathbb{P} = 0.0736$



Case (b)
$\mathbb{P} = 0.0166$



Case (c)
$\mathbb{P} = 0.006$

SUMMARY OF STABILITY ROBUSTNESS RESULTS FOR DEMONSTRATOR AIRCRAFT FOURTH-ORDER MODEL

Considering a case with non-varying flight condition along with the above results, the probabilities of instability seem to indicate that instability in Case (a) is a stronger function of uncertainties in individual parameters or stability derivatives than 30% V and ρ variations, while the remaining two cases are sensitive to flight condition variations.

	Controller		
	Case (a)	Case (b)	Case (c)
Correlated parameters 30% Gaussian variations	0.0711	0.0169	0.0033
Uncorrelated parameters 30% Gaussian variations 30% uniform ρ and V	0.0736	0.0166	0.0060
Uncorrelated parameters 30% Gaussian variations no ρ and V variations	0.0746	0.0162	0.0030

CONTROL SYSTEM ROBUSTNESS WITH ACTUATOR DYNAMICS

Stochastic robustness can be used to quantify the effects on robustness of actuator dynamics. First-order actuator dynamics are added for each control, resulting in a 14-element parameter vector. A controller is designed with LQR weighting specifications intended to approximate the controller of Case (a), while not pushing the actuator dynamics to unrealistic frequencies.

Fourth-order longitudinal dynamics and first-order actuator dynamics for each control

$$\dot{\mathbf{x}} = \mathbf{F}'(\boldsymbol{p}')\,\mathbf{x} + \mathbf{G}'(\boldsymbol{p}')\,\mathbf{u}$$

$$\mathbf{u} = -\mathbf{C}\,\mathbf{x}$$

14-element parameter vector

$$\boldsymbol{p}' = [\rho\ V\ f_{11}\ f_{12}\ f_{13}\ f_{22}\ f_{32}\ f_{33}\ g_{11}\ g_{12}\ g_{31}\ g_{32}\ \tau_c\ \tau_t\,]$$

τ_c = canard time constant (nominal value 0.1 sec)

τ_t = thrust time constant (nominal value 1.0 sec)

- *Redesign Case (a) controller such that closed-loop longitudinal eigenvalues are the same as previous example and actuator dynamics are reasonable.*

- *Apply stochastic robustness using new controller.*

PROBABILITY OF INSTABILITY FOR VARIOUS GAUSSIAN CONTROL PARAMETER UNCERTAINTIES

Stochastic robustness is applied for different values of the variance associated with each time constant, in order to detail the separate effects of each control lag. As indicated by the first line in the Table, simply including actuator dynamics increases the probability of instability, even if the associated parameters are known perfectly. This is a reasonable result because actuator dynamics are no longer infinitely fast but are allowed to interact with the rigid-body states. Qualitatively, bringing actuator dynamics in from infinity pushes the root-locus closer to instability. Stochastic robustness quantifies the effect. The thrust time constant has a small effect on the probability of instability, while a large increase in the probability of instability is seen as the canard time-constant standard-deviation increases from 30% to 150%.

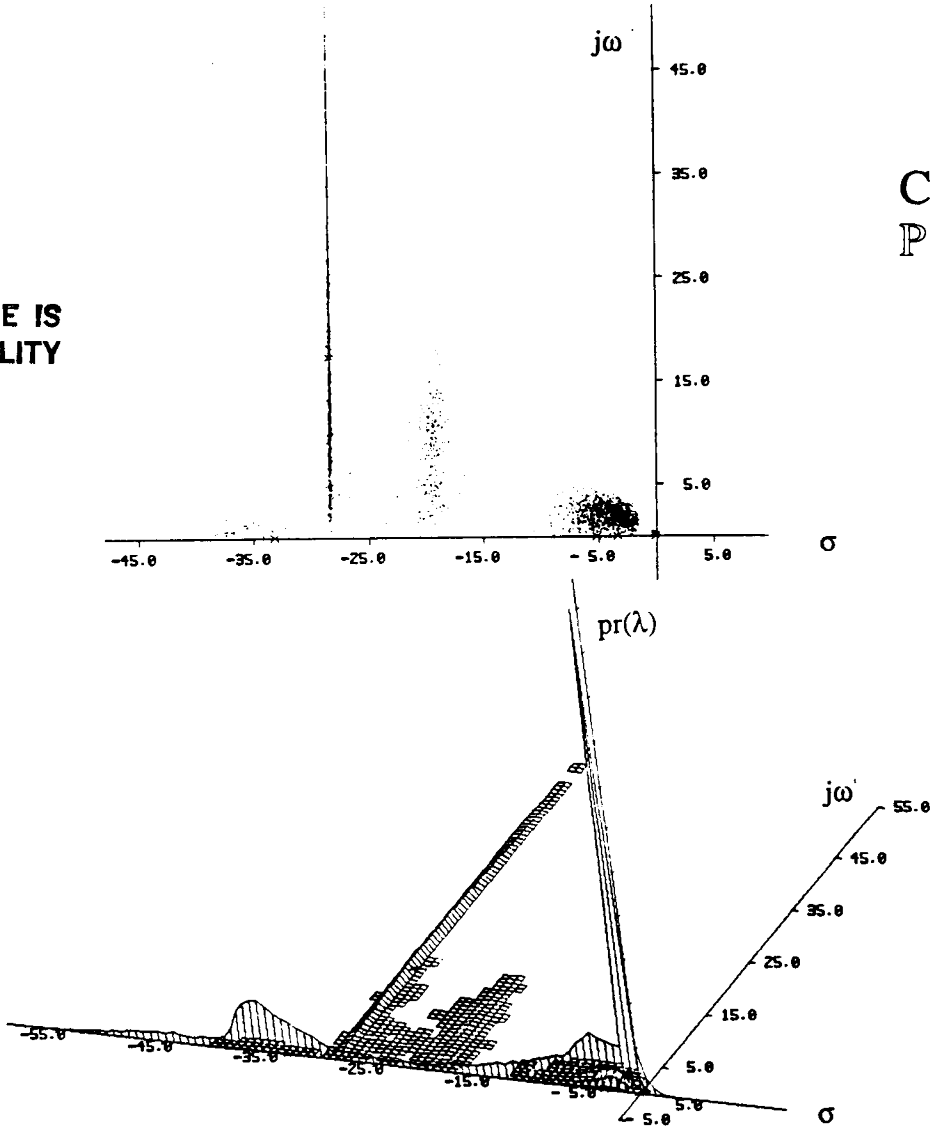
ρ and V are 30% uniform parameters.
f_{ij}, g_{ij} are 30% Gaussian parameters.

standard-deviation of τ_c	standard-deviation of τ_t	$\mathbb{P}$
0	0	0.092
0	30	0.0988
30	30	0.101
30	150	0.1014
150	30	0.1474
No control dynamics		0.0736

Stochastic root loci for demonstrator aircraft with 30% Gaussian parameters, 30% uniform ρ and V, and non-varying τ_c and τ_t

The stochastic root loci show that a strong coupling due to uncertainties can occur between the control and dynamic states, which tends to push more eigenvalues towards the right-half plane.

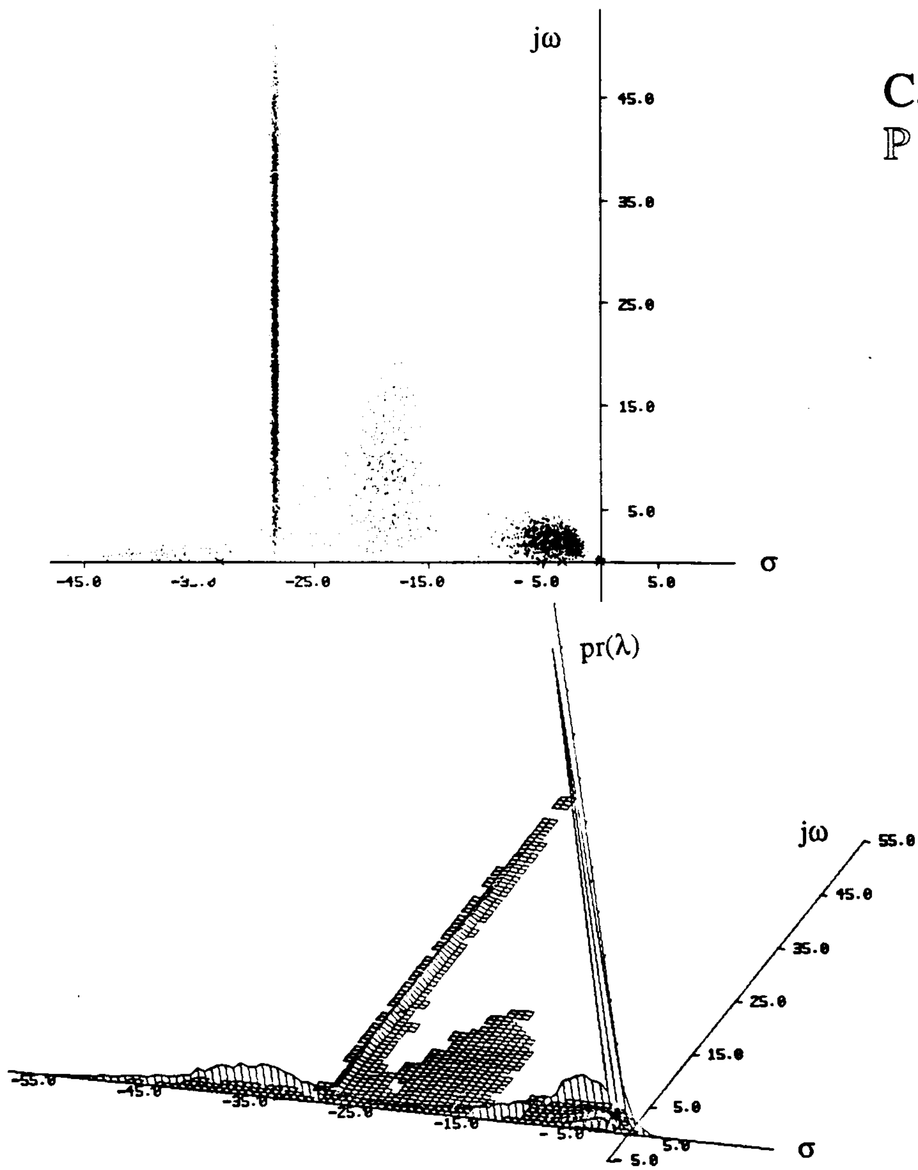
ORIGINAL PAGE IS OF POOR QUALITY



Case (a)
$\mathbb{P} = 0.092$

Stochastic root loci for demonstrator aircraft with 30% Gaussian parameters, 30% uniform ρ and V, and 30% Gaussian τ_c and τ_t

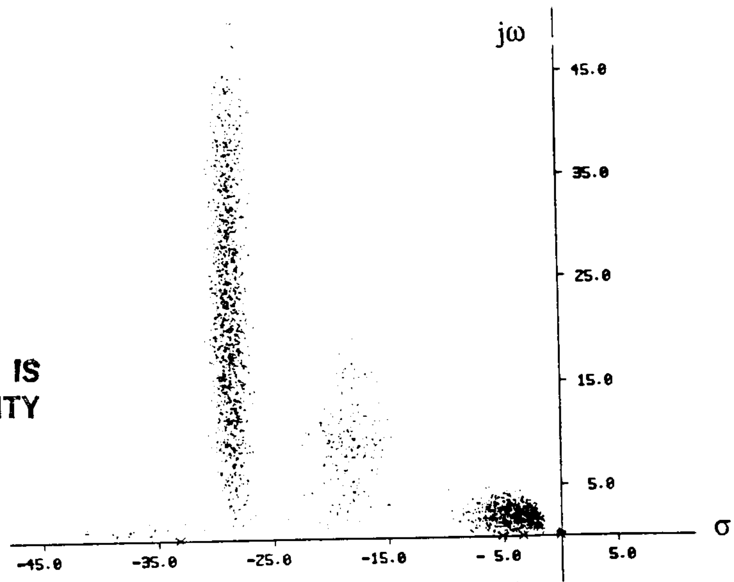
Increasing the standard-deviations associated with the time constants shows that the complex pair of eigenvalues has a small "variance" in the σ-direction, and a large variance in the $j\omega_d$ direction.



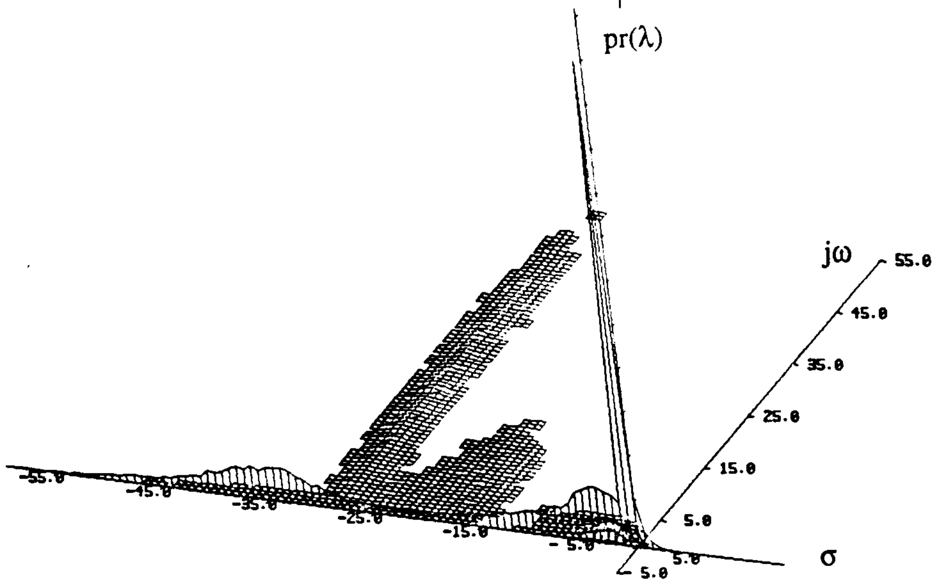
Case (a)
$\mathbb{P} = 0.101$

Stochastic root loci for demonstrator aircraft with 30% Gaussian parameters, 30% uniform ρ and V, 30% Gaussian τ_c, and 150% Gaussian τ_t

The σ-direction variance is largely due to the uncertainty associated with the thrust time constant, as illustrated by increasing the standard-deviation on this parameter to 150%. Increasing the uncertainty of τ_t has little effect on the probability of instability because it does not cause significant coupling with the dynamic modes.

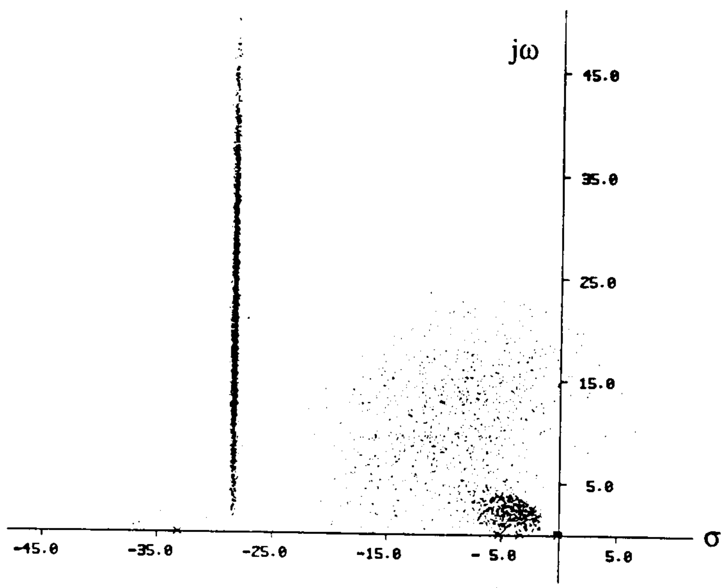


Case (a)
$\mathbb{P} = 0.1014$

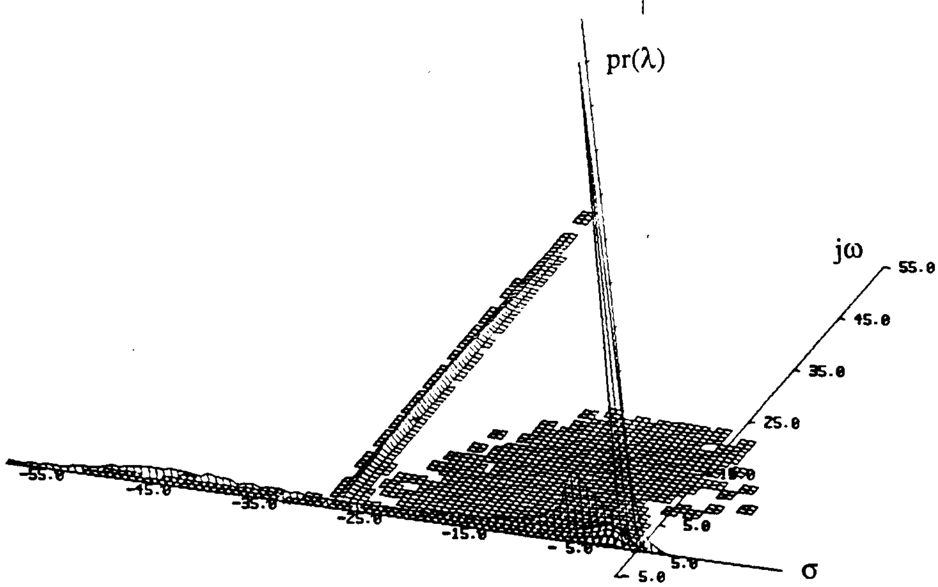


Stochastic root loci for demonstrator aircraft with 30% Gaussian parameters, 30% uniform ρ and V, 150% Gaussian τ_c, and 30% Gaussian τ_t

The variation at constant σ and coupling of controller and dynamic modes is largely due to variation of the canard time constant. Uncertainty in τ_c causes eigenvalues to migrate to the real axis and split off to form the complex "cloud" of eigenvalues that reaches instability.

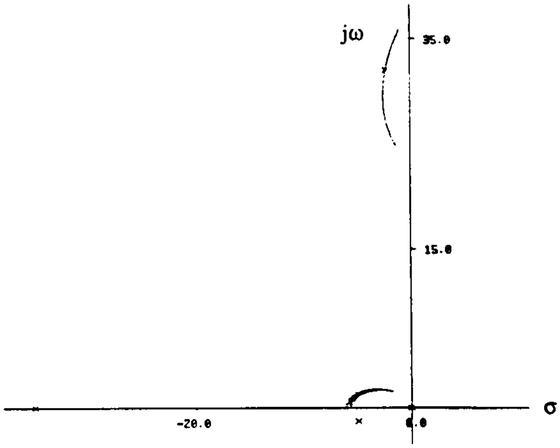


Case (a)
$\mathbb{P} = 0.1474$

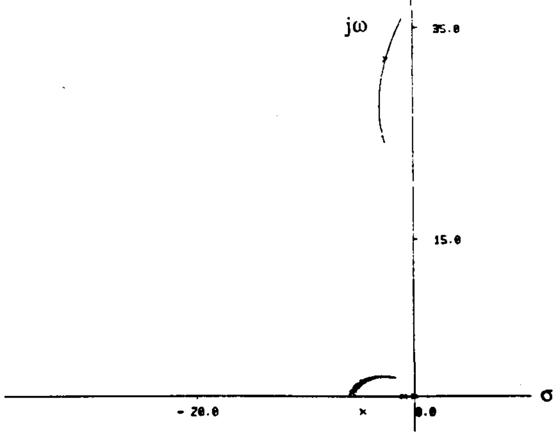


Stochastic root loci for demonstrator aircraft with aeroelastic effects for 30% uniform ρ and V

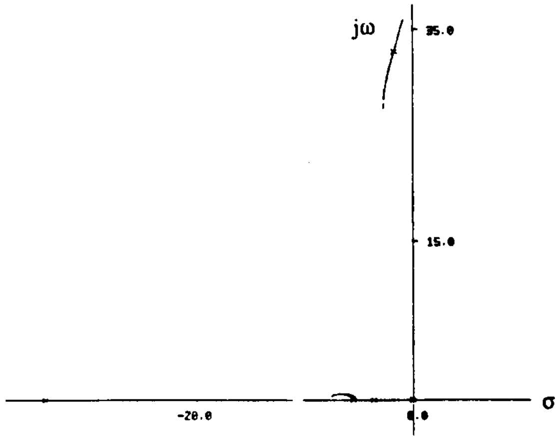
For 30% uniform variations in velocity and density alone, using the reduced-order gains, the probabilities of instability are zero. As expected, the closed-loop torsion eigenvalues at s= -0.1 ± 212.5j (not shown in figures) do not change with velocity and do not effect the probabilities of instability. Bending mode eigenvalues show a definite velocity trend, migrating towards instability as velocity increases. The closed-loop mean eigenvalues of rigid-body modes shift from the reduced-order case because of the presence of the added dynamics. In each case, the open-loop bending mode eigenvalues (-2.95 ± 32.03j) shift towards instability.



Case (a)
$\mathbb{P} = 0.0$



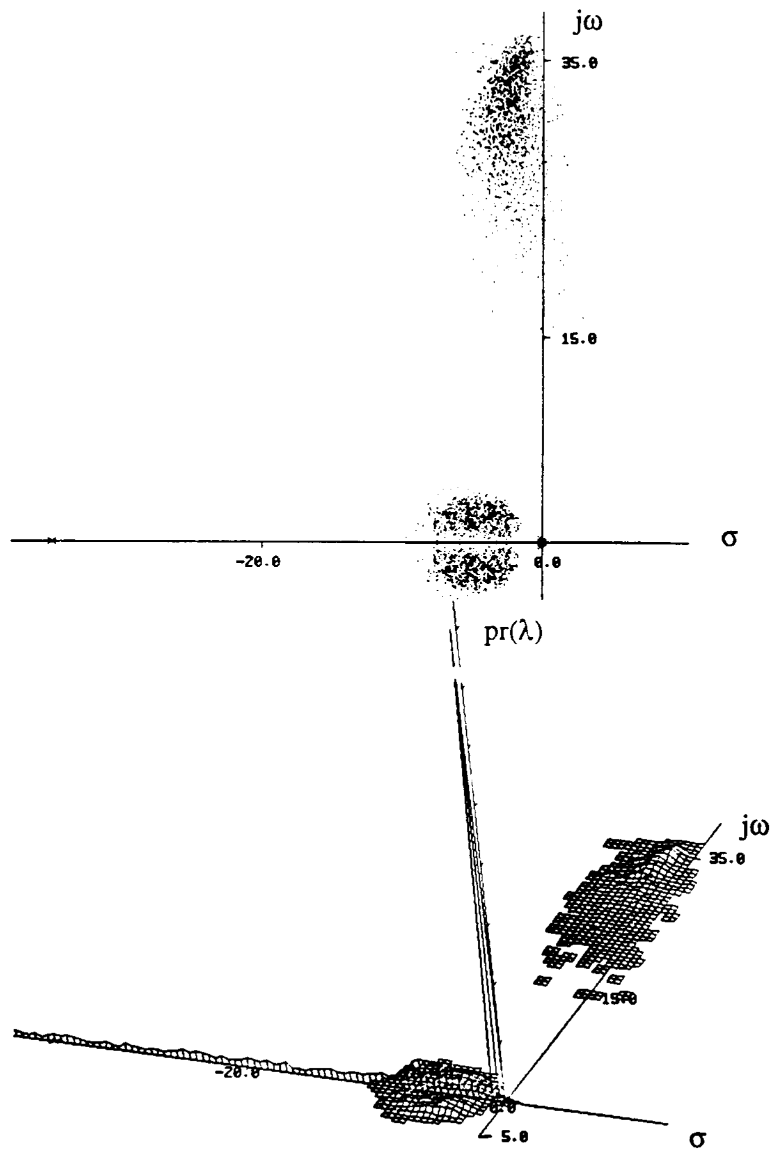
Case (b)
$\mathbb{P} = 0.0$



Case (c)
$\mathbb{P} = 0.0$

Stochastic root loci for demonstrator aircraft with aeroelastic effects for 30% Gaussian parameters and 30% uniform ρ and V

Next, the stochastic root loci for 30% uniform variations in ρ and V and 30% Gaussian variations of the parameters are evaluated. The peak near the origin is magnified to bring out the distribution associated with the bending mode eigenvalues.



Case (a)
$\mathbb{P} = 0.043$

ORIGINAL PAGE IS
OF POOR QUALITY

CONTROL SYSTEM ROBUSTNESS WITH AEROELASTIC EFFECTS

With second-order, dynamic-pressure-dependent aeroelastic effects representing the wing's first bending and torsional modes, the (8x8) and (8x2) system matrices can be partitioned as shown, where $\mathbf{F_r} = \mathbf{F}$ and $\mathbf{G_r} = \mathbf{G}$ represent dynamic and control effects for rigid body modes, $\mathbf{F_{ra}}$ and $\mathbf{F_{ar}}$ couple the rigid and flexible modes, and $\mathbf{F_a}$, $\mathbf{G_a}$ represent aeroelastic dynamic and control effects. To separate effects of material properties, the four parameters K_b, M_b, K_t, and M_t are assumed to be known perfectly. The aeroelastic matrices introduce 32 additional parameters, and the resulting 44-element parameter vector includes separate ρ and V effects. For preliminary analysis, 40 parameters were used, although concern for statistical significance and limits on the computational facilities used to date calls for modification of the number of parameters for future studies.

Fourth-order longitudinal dynamics coupled with fourth-order aeroelastic effects

$$\dot{\mathbf{x}} = \mathbf{F}'(p')\,\mathbf{x} + \mathbf{G}'(p')\,\mathbf{u}$$

$$\mathbf{u} = -\mathbf{C}\,\mathbf{x}$$

$$\mathbf{F}' = \begin{bmatrix} \mathbf{F_r} & \mathbf{F_{ra}} \\ \mathbf{F_{ar}} & \mathbf{F_a} \end{bmatrix} \qquad \mathbf{G}' = \begin{bmatrix} \mathbf{G_r} \\ \mathbf{G_a} \end{bmatrix}$$

$\mathbf{F_r} = \mathbf{F}$, $\mathbf{G_r} = \mathbf{G}$ represent rigid body modes

$\mathbf{F_a}$, $\mathbf{G_a}$ represents coupled second-order bending and torsion modes

$\mathbf{F_{ra}}$ and $\mathbf{F_{ar}}$ couple rigid and aeroelastic modes

44-element parameter vector

$p' = [\rho\ V\ f_{11}\ f_{12}\ f_{13}\ f_{22}\ f_{32}\ f_{33}\ g_{11}\ g_{12}\ g_{31}\ g_{32}$ + uncertain terms from $\mathbf{F_{ra}}$, $\mathbf{F_{ar}}$, $\mathbf{F_a}$, and $\mathbf{G_a}$]

CONTROL SYSTEM ROBUSTNESS WITH AEROELASTIC EFFECTS (CONTINUED)

In terms of structural-dimensional derivatives, $\mathbf{F_a}$ can be represented as given. Material properties dominate the torsional mode, which varies little with dynamic pressure variations. Stochastic robustness is applied to the new system using the reduced-order gains. Coupling of the systems through $\mathbf{F_{ra}}$ and $\mathbf{F_{ar}}$ causes the closed-loop system to be sensitive to aeroelastic terms.

• *Assume generalized mass and stiffness of bending and torsion modes are known perfectly.*

$$\mathbf{F_a} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ (S^1_{\eta_b} - K_b)/M_b & S^1_{\dot{\eta}_b}/M_b & S^1_{\eta_t}/M_b & S^1_{\dot{\eta}_t}/M_b \\ 0 & 0 & 0 & 1 \\ S^2_{\eta_b}/M_t & S^2_{\dot{\eta}_b}/M_t & (S^2_{\eta_t} - K_t)/M_t & S^2_{\dot{\eta}_t}/M_t \end{bmatrix}$$

K_b, M_b, K_t and M_t are generalized stiffness and mass for each mode.

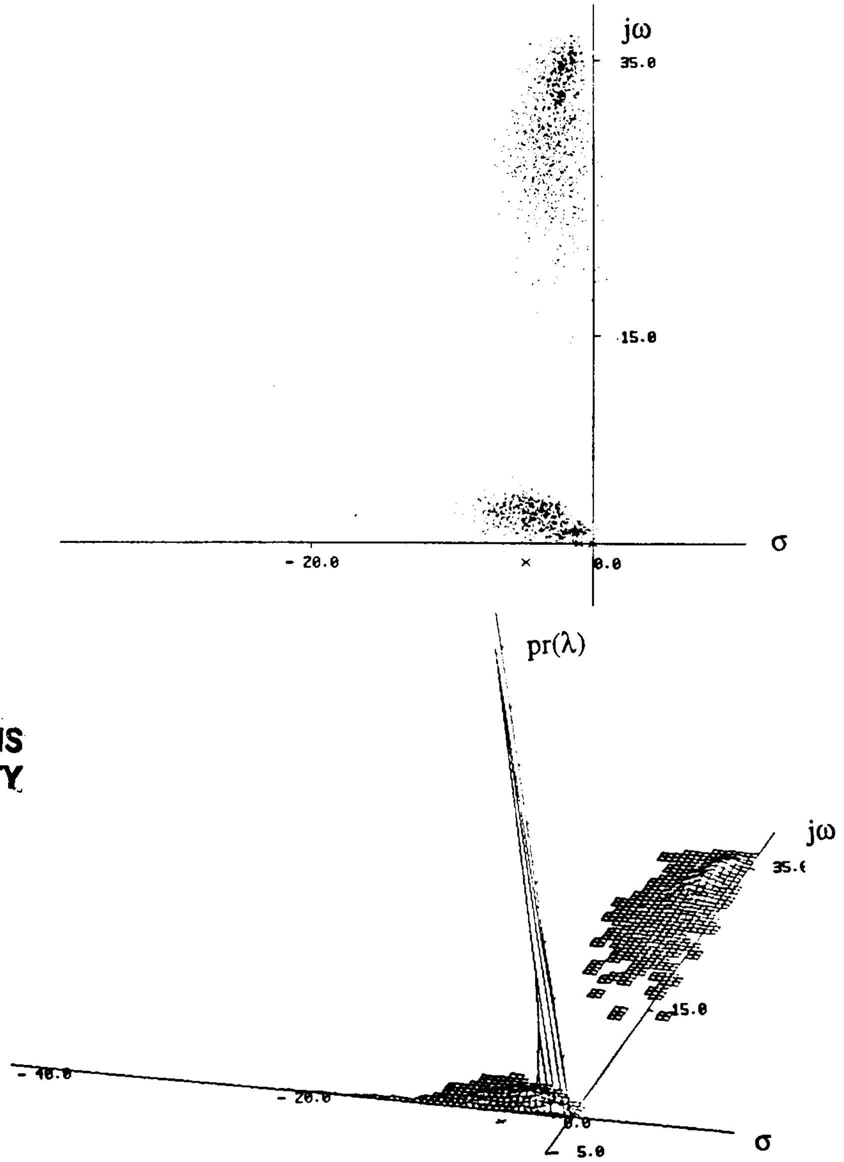
$S_{\dot{\eta}}^{i}$ are structural dimensional derivatives.

• *Apply stochastic robustness to new system using gains established previously.*

$$\mathbf{C'} = [\,\mathbf{C}\ \mathbf{0}\,]$$

$$\mathbf{F}_{\text{closed-loop}} = \begin{bmatrix} \mathbf{F_r} - \mathbf{G_r C} & \mathbf{F_{ra}} \\ \mathbf{F_{ar}} - \mathbf{G_a C} & \mathbf{F_a} \end{bmatrix}$$

Stochastic root loci for demonstrator aircraft with 30% Gaussian parameters and 30% uniform ρ and V

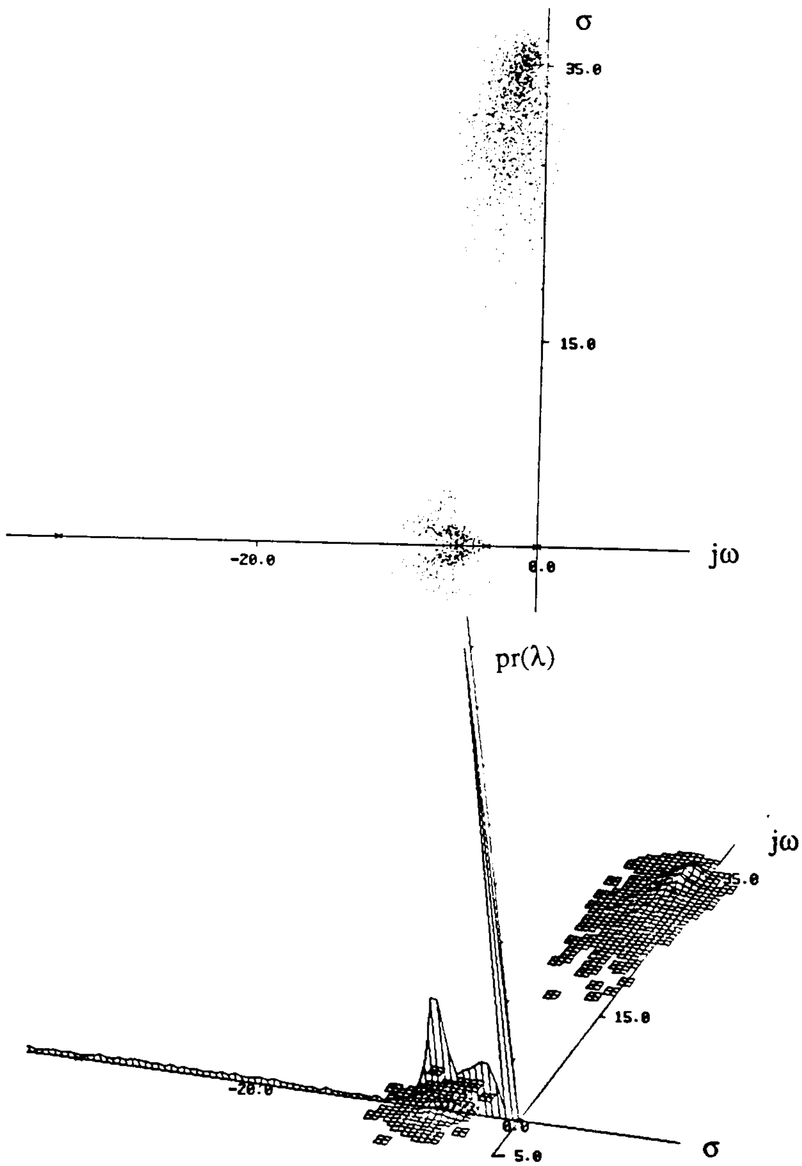


Case (b)
$\mathbb{P} = 0.017$

ORIGINAL PAGE IS
OF POOR QUALITY

Stochastic root loci for demonstrator aircraft with 30% Gaussian parameters and 30% uniform ρ and V

Case (c)
$\mathbb{P} = 0.0415$



SUMMARY OF ROBUSTNESS OF DEMONSTRATOR AIRCRAFT WITH AEROELASTIC EFFECTS

While definite conclusions cannot be reached because of the small sample space, this example illustrates the type of analysis possible using stochastic robustness. Certain trends are evident. The disparity in robustness between Cases (a) and (c) is reduced, and Case (c) shows a considerable decrease in robustness, while the robustness of the first two cases is at least retained or possibly improved. Application of a reduced-order controller to a higher order system does not guarantee that the robustness margins of the original system are retained, but the robustness of the system does not always go in the adverse direction. (This is somewhat analogous to the loss of guaranteed stability margins when applying LQG). Stochastic robustness again provides an excellent framework to quantify the effects of applying a reduced-order controller to a higher-order system. In each of these examples, questions concerning the selection of the number of Monte Carlo simulations, confidence limits, and statistical significance of results are issues of future research.

Probability of instability:

	without aeroelastic effects	with aeroelastic effects
Case (a)	0.0736	0.0430
Case (b)	0.0166	0.017
Case (c)	0.006	0.0415

Closed-loop eigenvalues:

	without aeroelastic effects	with aeroelastic effects
Case (a)	-0.02 -3.32, -5.14 -35.0	-0.02 $-4.96 \pm 1.27j$ -35.0 $-2.3 \pm 32.0j$
Case (b)	-0.02 -1.09 -3.36, -5.15	-0.02 -1.01 $-4.8 \pm 1.38j$ $-2.53 \pm 32.0j$
Case (c)	-0.01 -3.44, -5.15 -32.21	-0.02 -3.6, -5.53 -34.1 $-1.74 \pm 32.88j$

SUMMARY

Stochastic robustness offers a rigorous yet straightforward alternative to current metrics for control system robustness that is simple to compute and is unfettered by normally difficult problem statements, such as non-Gaussian statistics, products of parameter variations, and structured uncertainty. The approach answers the question, "How likely is the closed-loop system to fail, given limits of parameter uncertainty?" It makes good use of modern computational and graphic tools, and it is easily related to practical design considerations.

The examples presented here illustrate the use of stochastic robustness and its advantage in studying aircraft control systems. The parameters of aircraft stability and control effect matrices (stability derivatives and nominal flight condition parameters) lend themselves to this type of analysis tool. The stochastic robustness of different control system designs can be directly compared. Stochastic robustness can be used to study stability with flight condition variations. The method is also easily applied to model-order uncertainties in aircraft control systems by adding the uncertain dynamics to the system and assigning appropriate statistics to the new parameters. Quantitative effects of individual parameters or combinations of parameters on robustness can be measured in terms of the probability of instability. The principal difficulty in applying this method to control systems is that it is computationally intensive; however, requirements are well within the capabilities of existing computers. The principal advantage of the approach is that it is easily implemented, and results have direct bearing on engineering objectives.

1. Stochastic robustness can be used to study effects of flight condition perturbations on robustness.

- By considering flight condition parameters separately, parameters are uncorrelated.
- Can separate flight condition effects on robustness from parameter uncertainty effects.

2. Stochastic robustness can be used to study effects of model-order uncertainties on robustness.

- Shows magnitude of actuator dynamics effect on robustness.
- Reveals instability or robustness degredation due to neglected dynamics.