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# OPTIMIZATION OF THE STRUCTURAL AND CONTROL SYSTEM FOR LSS WITH REDUCED-ORDER MODEL

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## INTRODUCTION

The objective of this study is the simultaneous design of the structural and control system for space stuctures. The minimum weight of the structure is the objective function, and the constraints are placed on the closed-loop distribution of the frequencies and the damping parameters. The controls approach used is linear quadratic regulator with constant feedback. In the present investigation a reduced-order control system is used. The effect of uncontrolled modes is taken into consideration by the model error sensitivity suppression (MESS) technique which modifies the weighting parameters for the control forces. For illustration, an ACOSS-FOUR structure is designed for a different number of controlled modes with specified values for the closed-loop damping parameters and frequencies. The dynamic response of the optimum designs for an initial disturbance is compared.

## OBJECTIVES

- MINIMUM WEIGHT DESIGN
- SIMULTANEOUS STRUCTURAL AND CONTROL DISCIPLINES
- CLOSED-LOOP DAMPING AND EIGENVALUE REQUIREMENTS
- REDUCED ORDER CONTROL MODEL
- EFFECT OF NUMBER OF MODES CONTROLLED ON THE DESIGN
- DYNAMIC RESPONSE OF OPTIMUM DESIGNS

## OPTIMIZATION PROBLEM

Minimize $W$, the weight of the structure, such that the constraints on the closed-loop frequencies, $\tilde{\omega}_i$, and the closed-loop damping, $\bar{\xi}_i$, are satisfied. This optimization problem was solved by using the NEWSUMT-A program which is based on the extended interior penalty function method with Newton's method of unconstrained minimization.

# Structure/Control Optimization Problem

Minimize weight

$$W = \sum \rho_i A_i l_i \quad (1)$$

Such that

$$g_j(\tilde{\omega}_i) \leq 0 \quad (2)$$

$$g_j(\xi_i) = 0 \quad (3)$$

$$g_j(A_i) \geq 0 \quad (4)$$

Where

$$g_j(\tilde{\omega}_i) = \tilde{\omega}_i - \overline{\tilde{\omega}}_i \quad (5)$$

$$g_j(\xi_i) = \xi_i - \overline{\xi}_i \quad (6)$$

$$g_j(A_i) = A_i - \overline{A}_i(\text{min}) \quad (7)$$

## MODEL ERROR SENSITIVITY SUPPRESSION

The control problem is defined in Eqs. 1 and 2, where $\{x\}_c$ and $\{x\}_s$ are the controlled and suppressed states. The model error sensitivity suppression technique involves setting a singular perturbation on the $\dot{x}$ system which implies that the derivatives $\dot{x}$ be set identically to zero. This condition when applied to the suppressed states yields Eq. 3. This algebraic equation now can be solved for the suppressed states as given in Eq. 4. Using Eqs. 1 and 4 a new performance index can be written as given in Eq 5.

## REDUCED ORDER MODEL

$$PI = \int_0^{\infty}(\{x\}_c^T[Q]_c\{x\}_c + \{x\}_s^T[Q]_s\{x\}_s + \{f\}^T[R]\{f\})dt \qquad (1)$$

Subject to

$$\begin{bmatrix} \dot{x}_c \\ \dot{x}_s \end{bmatrix} = \begin{bmatrix} A_c & 0 \\ 0 & A_s \end{bmatrix} \begin{bmatrix} x_c \\ x_s \end{bmatrix} + \begin{bmatrix} B_c \\ B_s \end{bmatrix} \{f\} \qquad (2)$$

Singular perturbation of suppressed system

$$0 = [A]_s\{x\}_s + [B]_s\{f\} \qquad (3)$$

Solve for

$$\{x\}_s = -[A]_s^{-1}[B]_s\{f\} \qquad (4)$$

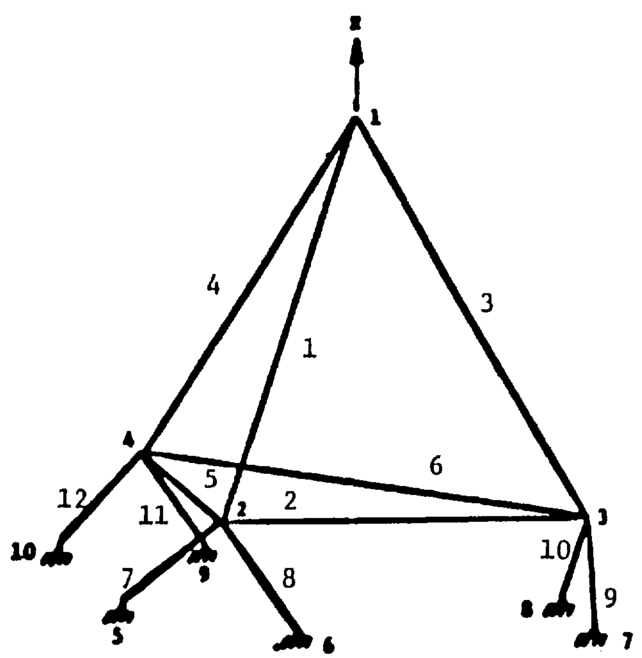
Substitute for $\{x\}_s$ in $PI$

$$PI = \int_0^{\infty} \left(\{x\}_c^T[Q]_c\{x\}_c + \{f\}^T\left[R + [B]_s^T[A]_s^{-T}[Q]_s[A]_s^{-1}[B]_s\right]\{f\}\right) dt \qquad (5)$$

## PROBLEM DESCRIPTION

The finite element model of the ACOSS-FOUR is shown in this figure. The edges of the tetrahedron are 10 units long. The structure has twelve degrees of freedom and four nonstructural masses of 2 units each are attached at nodes 1 through 4. The dimensions of the structure and the elastic properties are defined in unspecified consistent units. The collocated actuators and sensors are located in six bipods. The objective of the control system is to control the line of sight (LOS) error which is the displacement of node 1 in the $X - Y$ plane due to some initial disturbance.

### ACOSS FOUR (ELEMENT NUMBERS)



## CONSTRAINTS

The nominal design was used as the initial design for optimization. The cross-sectional areas of this design are given in the second table. The weight of the structure for this design was 43.69 units. The imaginary parts of the closed-loop eivenvalues and the damping parameters associated with the lowest two frequencies are given below on the left side. The constraints imposed on the optimum design are given below on the right side. In the optimum design the specified damping parameters are twice those of the nominal design. The weighting matrix $[Q]$ for the state variables is a function of the square of the structural frequencies. The weighting matrix $[R]$ is the identity matrix.

| NOMINAL DESIGN | CONSTRAINTS ON OPTIMUM DESIGN |
|---|---|
| weight $= 43.69$ | |
| $\bar{\omega}_1 = 1.341$ | $\bar{\omega}_1 \geq 1.341$ |
| $\bar{\omega}_2 = 1.666$ | $\bar{\omega}_2 \geq 1.6$ |
| $\bar{\xi}_1 = 0.061169$ | $\bar{\xi}_1 = 0.122$ |
| $\bar{\xi}_2 = 0.07822$ | $\bar{\xi}_2 = 0.156$ |

## NUMERICAL RESULTS

This table gives the closed-loop damping parameter associated with different modes. The numbers under the first column are for the initial nonoptimum design. The second column contains the damping parameters for an optimal design where all the twelve modes were controlled. Subsequent columns contain damping parameters for different optimum designs with the number of controlled modes given in the first row. It is seen that the damping parameters associated with the first two modes for all optimum designs are the same. These were the constraints on the optimum design.

# CLOSED-LOOP DAMPING PARAMETERS

| # modes | 12† | 12‡ | 10‡ | 8‡ | 5‡ | 3‡ |
|---|---|---|---|---|---|---|
| | 0.062 | 0.122 | 0.122 | 0.122 | 0.126 | 0.122 |
| | 0.078 | 0.156 | 0.156 | 0.156 | 0.143 | 0.156 |
| | 0.097 | 0.164 | 0.148 | 0.165 | 0.143 | 0.164 |
| | 0.106 | 0.123 | 0.146 | 0.123 | 0.159 | |
| | 0.112 | 0.056 | 0.127 | 0.054 | 0.144 | |
| | 0.117 | 0.077 | 0.082 | 0.077 | 0.124 | |
| | 0.105 | 0.079 | 0.083 | 0.082 | | |
| | 0.099 | 0.047 | 0.073 | 0.049 | | |
| | 0.048 | 0.040 | 0.038 | | | |
| | 0.041 | 0.046 | 0.036 | | | |
| | 0.029 | 0.028 | | | | |
| | 0.009 | 0.037 | | | | |

† Non–Optimum
‡ Optimum

## NUMERICAL RESULTS (CONT)

This table gives the cross-sectional areas of the members and the weights of all the designs. The initial weight or the weight of the nominal design was 43.69 units while the optimum design weights varied between 32.89 to 36.92. Even though there is not too much variation in the weights of the optimum designs, the relative values of the cross-sectional areas of the members are not the same.

### AREA OF MEMBERS

| ELE | 12† | 12‡ | 10‡ | 8‡ | 5‡ | 3‡ |
|---|---|---|---|---|---|---|
| 1 | 1000 | 607 | 614 | 588 | 654 | 572 |
| 2 | 1000 | 652 | 804 | 652 | 214 | 637 |
| 3 | 100 | 155 | 206 | 184 | 667 | 175 |
| 4 | 100 | 680 | 770 | 688 | 337 | 669 |
| 5 | 1000 | 192 | 175 | 168 | 780 | 174 |
| 6 | 1000 | 748 | 852 | 748 | 392 | 727 |
| 7 | 100 | 45 | 118 | 44 | 929 | 46 |
| 8 | 100 | 517 | 625 | 524 | 129 | 511 |
| 9 | 100 | 41 | 42 | 42 | 45 | 43 |
| 10 | 100 | 448 | 41 | 406 | 49 | 407 |
| 11 | 100 | 168 | 57 | 155 | 52 | 128 |
| 12 | 100 | 46 | 67 | 45 | 58 | 46 |
| wt | 43.69 | 33.94 | 36.92 | 33.74 | 34.06 | 32.89 |

† Initial Design
‡ Number of Controlled Modes

## NUMERICAL RESULTS (CONT)

This table gives the square of the structural frequencies for all designs. The band of frequencies for an optimum design with twelve modes controlled is minimum. The frequencies associated with the first and second modes are nearly equal for all the designs. This is due to the constraints imposed on the closed-loop frequencies.

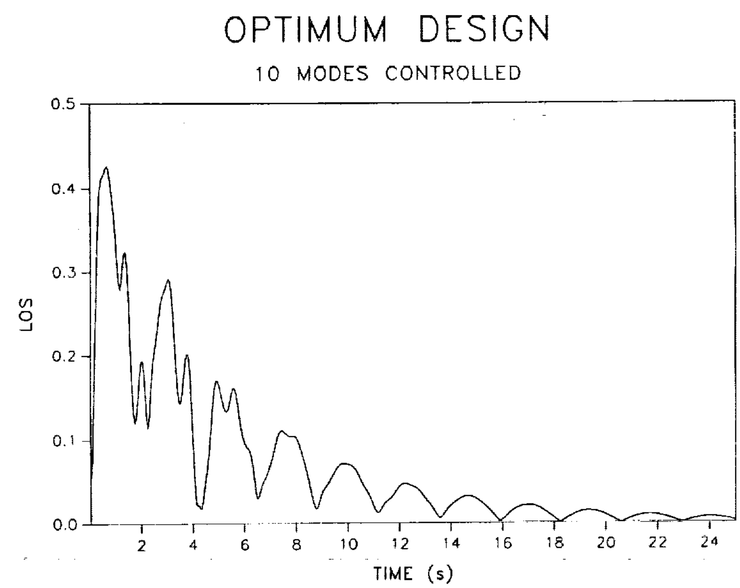
## STRUCTURAL FREQUENCIES $\left(\omega_j^2\right)$

| # modes | 12† | 12‡ | 10‡ | 8‡ | 5‡ | 3‡ |
|---|---|---|---|---|---|---|
| | 1.80 | 1.80 | 1.79 | 1.79 | 1.98 | 1.79 |
| | 2.77 | 2.56 | 2.56 | 2.56 | 2.56 | 2.56 |
| | 8.35 | 7.63 | 5.15 | 6.34 | 5.68 | 6.40 |
| | 8.74 | 9.31 | 6.59 | 8.42 | 6.56 | 8.21 |
| | 11.55 | 13.19 | 12.10 | 10.64 | 11.83 | 10.40 |
| | 17.68 | 26.41 | 18.67 | 24.84 | 20.63 | 22.78 |
| | 21.73 | 27.78 | 21.64 | 26.35 | 29.51 | 25.33 |
| | 22.61 | 34.33 | 31.84 | 51.68 | 33.86 | 50.62 |
| | 72.92 | 40.32 | 69.89 | 66.42 | 47.05 | 64.25 |
| | 85.57 | 44.70 | 81.73 | 93.66 | 72.19 | 92.60 |
| | 105.8 | 46.32 | 124.9 | 109.1 | 110.8 | 105.9 |
| | 166.5 | 50.10 | 133.7 | 116.6 | 185.9 | 113.4 |

† Non–Optimum
‡ Optimum

## TRANSIENT RESPONSE

These two figures show the dynamic response of the designs with ten modes and three modes controlled. The transient response was simulated for a period of 25 seconds at a time interval t = 0.05 secs. The magnitude of the LOS is given by the square root of the sum of the squares of the $X$ and $Y$ components of the displacements at node 1. The dash line is for the case where unmodeled modes are also included in the calculation of the transient response. For the design with ten modes controlled the two curves coincide. In the case of 3 modes controlled a small difference in the response is observed.

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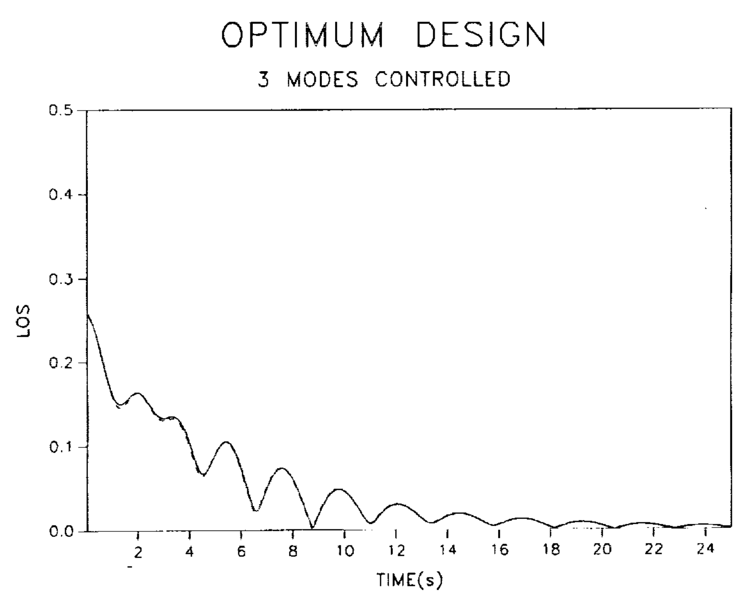

## CONCLUSIONS

This presentation included the results of an investigation to design a minimum weight structure by taking into consideration a reduced order control system. The reduced order approach was based on the model error sensitivity suppression technique. It was found that the weights of the structures with a different number of modes controlled were not substantially different. The work done by the actuators was found to be reduced with a less number of contolled modes. The transient response of the different designs was not the same. There was not much difference in the LOS when unmodelled modes were included in calculating the response.

- Simultaneous structural and control with closed–loop damping and eigenvalue requirements
- NEWSUMT — An optimizer for solving the problem
- Control design based on reduced order model
- The transient response for designs with different number of modes controlled was not the same