NASA Technical Memorandum

NASA TM-100387

GLOBAL NONLINEAR OPTIMIZATION OF SPACECRAFT PROTECTIVE STRUCTURES DESIGN

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January 1990

National Aeronautics and
Space Administration
George C. Marshall Space Flight Center

Report Documentation Page

NASA

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-100387		

4. Title and Subtitle

Global Nonlinear Optimization of Spacecraft Protective Structures Design

5. Report Date

January 1990

6. Performing Organization Code

7. Author(s)

R.A. Mog,* J.N. Lovett, Jr.,† and S.L. Avans

8. Performing Organization Report No.

10. Work Unit No.

9. Performing Organization Name and Address

George C. Marshall Space Flight Center
Marshall Space Flight Center, Alabama 35812

11. Contract or Grant No.

13. Type of Report and Period Covered

Technical Memorandum

12. Sponsoring Agency Name and Address

National Aeronautics and Space Administration
Washington, D.C. 20546

14. Sponsoring Agency Code

15. Supplementary Notes

Prepared by Structures and Dynamics Laboratory, Science and Engineering Directorate.

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16. Abstract

The global optimization of protective structural designs for spacecraft subject to hypervelocity meteoroid and space debris impacts is presented. This nonlinear problem is first formulated for weight minimization of the space station core module configuration using the Nysmith impact predictor. Next, the equivalence and uniqueness of local and global optima is shown using properties of convexity. This analysis results in a new feasibility condition for this problem. The solution existence is then shown, followed by a comparison of optimization techniques. Finally, a sensitivity analysis is presented to determine the effects of variations in the systemic parameters on optimal design. The results show that global optimization of this problem is unique and may be achieved by a number of methods, provided the feasibility condition is satisfied. Furthermore, module structural design thicknesses and weight increase with increasing projectile velocity and diameter and decrease with increasing separation between bumper and wall for the Nysmith predictor.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Meteoroid-Debris Protection Structural Optimization Geometric Programming Space Station Freedom Module Structure	Unclassified – Unlimited

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	28	NTIS

NASA FORM 1626 OCT 86

For sale by the National Technical Information Service, Springfield, VA 22161-2171

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NOMENCLATURE

a	left endpoint of search interval
a_i	slope of line segment i
b	right endpoint of search interval
d	projectile diameter (cm)
h	separation between bumper and wall (cm)
N	number of line segments
S	constraint set
t_1	bumper thickness (cm)
$t_1{}^1$	initial value for bumper thickness (cm)
t_{1_i}	independent variable i for bumper thickness (cm)
t_2	wall thickness (cm)
$t_2{}^1$	initial value for wall thickness (cm)
V	projectile velocity (km/s)
W	weight objective function
W_1	linear objective function
W_L	objective function value at left search point
W_R	objective function value at right search point
x_L	left search point
x_R	right search point
ΔX	line segment interval length (cm)
Subscript O	optimal value

TECHNICAL MEMORANDUM

GLOBAL NONLINEAR OPTIMIZATION OF SPACECRAFT PROTECTIVE STRUCTURES DESIGN

1. INTRODUCTION

1.1 Problem Background

The space station core module configuration, which includes habitation and laboratory modules, will be subject to a number of harsh environs, including radiation, thermal, pressure, structural loadings at launch, and the topic of this study, meteoroid and space debris hypervelocity impacts [1,2,3]. For the space station, which has an extended orbital lifetime and large surface and projected areas, the pressure wall thicknesses of the core modules are often driven by these two environs [4,5]. Because this wall thickness contributes significantly to the structural weight of the module, it is important to minimize its effect at launch while maintaining adequate protection for the crew and equipment.

One method currently used in protective systems design to reduce the pressure wall thickness is to add a thin bumper spaced outboard from the wall. For many space debris and meteoroid projectile velocities, this bumper fragments the particle into smaller pieces which disperse behind the bumper, making their impact with the wall less severe [6,7].

The optimum number, thicknesses, and materials for the bumpers is the subject of ongoing research at NASA's Marshall Space Flight Center (MSFC). However, analytic predictor equations and models do exist for the single bumper/single wall design as envisioned for the space station [4,5,6,7]. These predictors, which vary greatly in form, provide ballistic limit information used to design the bumper and pressure wall.

For a given predictor, there are many combinations of bumper and pressure wall thicknesses that satisfy the model. In general, the thicker the bumper, the thinner the pressure wall, and vice versa. However, the optimal combination is that set of thicknesses which minimizes the module weight. The determination of these thicknesses is a nonlinear optimization process.

1.2 Study Goal

The goal of this study is to determine the uniqueness and existence of the globally optimal solution to the protective systems design problem formulated with the Nysmith impact predictor and its constraints. A secondary goal is to discover qualitative features of several nonlinear optimization techniques to determine their effectiveness in arriving at solutions to problems in this field.

1.3 Study Approach

The problem is first formulated as a nonlinear optimization problem in Section 2. In Section 3, the equivalence and uniqueness of local and global optimal solutions to this problem is proven using properties of convex sets and functions. Furthermore, an important feasibility condition which limits the usage of the Nysmith predictor is established. The existence of the optimal solution is shown in Section 4 using various solution techniques. Additionally, the analytical solution for this optimum is provided for most of the feasibility region. Section 4 concludes with a qualitative comparison of the optimization techniques considered. Finally, the sensitivity of the optimal design to the systemic parameters is presented in Section 5.

2. PROTECTIVE SYSTEMS DESIGN PROBLEM FORMULATION

2.1 Introduction

The formulation of the optimization problem is a key process, the importance of which cannot be overstated. Many of the assumptions made in this process have profound effects on the problem solution. The choice of objective function will be made first, followed by the manipulation of the problem constraints to proper form. Finally, the complete problem formulation is stated, along with some remarks concerning the degree of difficulty in obtaining optimal solutions.

The Nysmith equation [6] may be written

$$t_2 = \frac{5.08 V^{0.278} d^{2.92}}{t_1^{0.528} h^{1.39}} \quad , \tag{1}$$

with inequality constraints

$$\frac{t_1}{d} \le 0.5 \tag{2}$$

and

$$\frac{t_2}{d} \le 1.0 \quad . \tag{3}$$

Note that this predictor does not include parameters for bumper, wall, or projectile materials. This is due to the fact that the experimental data used to derive the Nysmith predictor was based on pyrex glass spheres impacting 2024-T3 aluminum bumpers and walls. Furthermore, note that d, h, and V are positive-valued parameters with nominal values discussed in Section 4. Finally, constraints (2) and (3) represent limitations on bumper and wall thicknesses in terms of projectile diameter.

2.2 The Choice of Weight Objective Function

Detailed weight functions based on the applicable spacecraft configuration may be derived to any degree of representation and then minimized to reduce structural launch stresses and payload weight. However, detailed weight functions tend to limit the generality of the analysis while obscuring the mathematics. Furthermore, if the structural curvature of the spacecraft is relatively small, and if the bumper and wall materials are fixed and identical, an appropriate weight function is given simply by

$$W = t_1 + t_2 \quad . \tag{4}$$

Substituting equation (1) results in

$$W = t_1 + \frac{5.08V^{0.278}d^{2.92}}{t_1^{0.528}h^{1.39}} \quad . \tag{5}$$

Throughout this study, equation (5) will represent the spacecraft weight to be minimized with respect to the independent variable t_1.

2.3 Problem Constraints

The problem constraints must now be manipulated to proper form. Constraint (2) may be rewritten

$$t_1 \leq \frac{d}{2} \quad , \tag{6}$$

and substituting (1) into (3) and rearranging gives

$$t_1 \geq \frac{21.72V^{0.527}d^{3.636}}{h^{2.633}} \quad . \tag{7}$$

Equations (6) and (7) represent upper and lower bounds on the bumper thickness in terms of the systemic parameters V, d, and h. Note that since V, d, and h are positive, so is the bumper thickness.

2.4 Final Problem Formulation

The optimization problem may now be written:

Minimize: W from equation (5).

Subject to: Conditions (6) and (7), with independent variable, t_1. Note that because this is a constrained nonlinear optimization problem, traditional calculus techniques succeed only when a local minimum happens to satisfy the constraint set.

3. EQUIVALENCE AND UNIQUENESS OF LOCAL AND GLOBAL OPTIMA

3.1 Introduction

In optimization problems, it is important to determine whether solutions fall into the category of local or global optima. Simply put, a global optimal solution is optimal for all points in the constraint set, while a local optimal solution may be optimal in only a small neighborhood of itself. In this section, it will be shown that all local optimal solutions to the problem of Section 2.4 are global optimal solutions, and furthermore, that the global optimal solution to this problem is unique. Existence and computation of the actual solution is deferred to Section 4.

3.2 Condition for a Nonempty Feasibility Set

The first step in this analysis is to determine when the problem is feasible. This corresponds to the question: When is the constraint set defined by (6) and (7) nonempty? Clearly, this is the case if

$$\frac{d}{2} \geq \frac{21.72V^{0.527}d^{3.636}}{h^{2.633}} \tag{8}$$

or

$$d \leq \frac{0.239h}{V^{0.2}} \quad . \tag{9}$$

Thus, if equation (9) is not satisfied, then there is no feasible solution to this problem. Note that this feasibility condition places a restriction on the relative values of the systemic parameters associated with the physics of the problem. Thus, certain realistic physical problems are outside the realm of situations that may be modeled using the Nysmith predictor. Equation (9) represents an upper bound on the projectile diameter that may be considered in this analysis. However, this condition may be rewritten to find a lower bound on the separation between bumper and wall as

$$h \geq 4.184 d V^{0.2} \quad . \tag{10}$$

Equation (10) is a more useful form of the feasibility condition, since the structural designer generally has more control over the bumper/wall separation than over the projectile diameter (or velocity) that impacts the spacecraft.

3.3 Convexity of the Feasibility Set

When equation (10) holds, the feasibility set defined by (6) and (7) is nonempty. It is also convex, as shown in the following lemma.

Lemma 1: Consider the set S defined by (6) and (7). Provided (10) holds, S is convex.

Proof: Equation (10) provides the required nonemptiness of S. Recall that nonempty S is convex if for

$$t_{1_i} \in S \qquad i = 1, 2$$

then

$$\lambda t_{1_1} + (1-\lambda) t_{1_2} \in S \qquad \forall \lambda \in [0,1] \quad .$$

Suppose

$$t_{1_i} \in S \qquad \text{for } i = 1, 2$$

Then

$$t_{1_i} \leq \frac{d}{2}$$

and

$$t_{1_i} \geq \frac{21.72V^{0.527}d^{3.636}}{h^{2.633}}$$

for i = 1, 2 by (6) and (7). Suppose

$$\lambda \in [0,1] \quad .$$

Then

$$\lambda t_{1_1} + (1-\lambda)t_{1_2} \leq \lambda\left(\frac{d}{2}\right) + (1-\lambda)\left(\frac{d}{2}\right) = \frac{d}{2} \quad .$$

Similarly,

$$\lambda t_{1_1} + (1-\lambda)t_{1_2} \geq \frac{21.72V^{0.527}d^{3.636}}{h^{2.633}} \quad .$$

Therefore,

$$\lambda t_{1_1} + (1-\lambda)t_{1_2} \in S \quad ,$$

as desired. Thus, S is convex.

3.4 Strict Convexity of the Objective Function

Since the objective function W is a function of one independent variable, convexity may be proven using techniques from the calculus of a single variable.

Lemma 2: W from equation (5) is strictly convex on S.

Proof:

$$W'(t_1) = 1 - \frac{2.682V^{0.278}d^{2.92}}{t_1^{1.528}h^{1.39}} \quad , \tag{11}$$

$$W''(t_1) = \frac{4.098V^{0.278}d^{2.92}}{t_1^{2.528}h^{1.39}} > 0 \qquad , \tag{12}$$

since V, d, h, and t_1 are all positive. Thus, W is strictly convex on S.

3.5 Global Optimization Theorem

Theorem 1: Suppose equation (10) is satisfied. Then any local optimal solution to the problem of Section 2.4 is the unique global optimal solution to the problem.

Proof: If equation (10) is satisfied, then S defined by equations (6) and (7) is nonempty. Furthermore, S is convex from Lemma 1. Also, W is strictly convex on S from Lemma 2. Therefore, by Theorem 3.4.2 (part 2) of Reference 8, any local optimal solution to this problem is the unique global optimal solution.

Note that this theorem says nothing about the existence of an optimal solution to the problem or how to find it. This will be addressed in the next section.

4. EXISTENCE OF OPTIMUM AND COMPARISON OF OPTIMIZATION TECHNIQUES

4.1 Introduction

The conditions of existence of a local (and thus global) optimal solution to the problem will now be established.

Theorem 2: If

$$d \leq 0.23hV^{-0.2} \qquad , \tag{13}$$

then the optimal solution to the problem of Section 2.4 exists and is given by

$$t_{1_0} = \frac{1.907V^{0.182}d^{1.91}}{h^{0.91}} \qquad , \tag{14}$$

$$t_{2_0} = \frac{3.613V^{0.182}d^{1.91}}{h^{0.91}} \qquad , \tag{15}$$

$$W_0 = \frac{5.520V^{0.182}d^{1.91}}{h^{0.91}} \quad . \tag{16}$$

Proof: Note first that equation (14) satisfies equation (11). Also, substituting equation (14) into equation (1) results in equation (15). Inserting equations (14) and (15) into (4) gives (16). Thus, equations (14), (15), and (16) define the local optimal solution for the unconstrained problem. Note, also, that since inequality (13) is satisfied, then so are inequalities (6) and (7), as well as feasibility condition (10). Thus, by Theorem 1, equations (14), (15), and (16) define the globally optimal solution under condition (13).

Note that the ratio of optimal bumper thickness to total thickness is 0.345. The corresponding ratio for the wall is 0.655. Thus, provided the values of the systemic parameters satisfy equation (13), these ratios are constant.

Finally, notice that Theorem 2 provides optimality conditions for most of the feasibility region. In fact, it is now only necessary to determine the existence of optimal solutions in the interval

$$0.23hV^{-0.2} \le d \le 0.24hV^{-0.2} \quad . \tag{17}$$

This existence will be shown using various optimization methods.

The baseline systemic parameters for these analyses are determined from existing environment curves and data on mission risk and duration and velocity probability distributions for the space station core module configuration [2,5]. The dominant environment for this application is the space debris environment. The corresponding parameters are

$$V = 10 \text{ km/s} , \quad d = 0.84 \text{ cm} , \quad \text{and} \quad h = 10 \text{ cm} \quad . \tag{18}$$

Note that these baseline parameters satisfy (13) and thus Theorem 2. Thus, the optimal baseline solution as given by (14), (15), and (16) is

$$t_{1_0} = 0.256cm \qquad t_{2_0} = 0.484cm \qquad W_0 = 0.740cm \quad . \tag{19}$$

These results will be confirmed with a number of optimization methods to follow. Detailed variations in the systemic parameters will be discussed in Section 5.

4.2 Problem Graph

Figure 1 shows the problem graph for the baseline case defined by equation (18). Note that the maximum total thickness occurs at an interior point of the feasible solution region. Also, note that the region is convex, and the objective function is strictly convex, as shown in Lemmas 1 and 2.

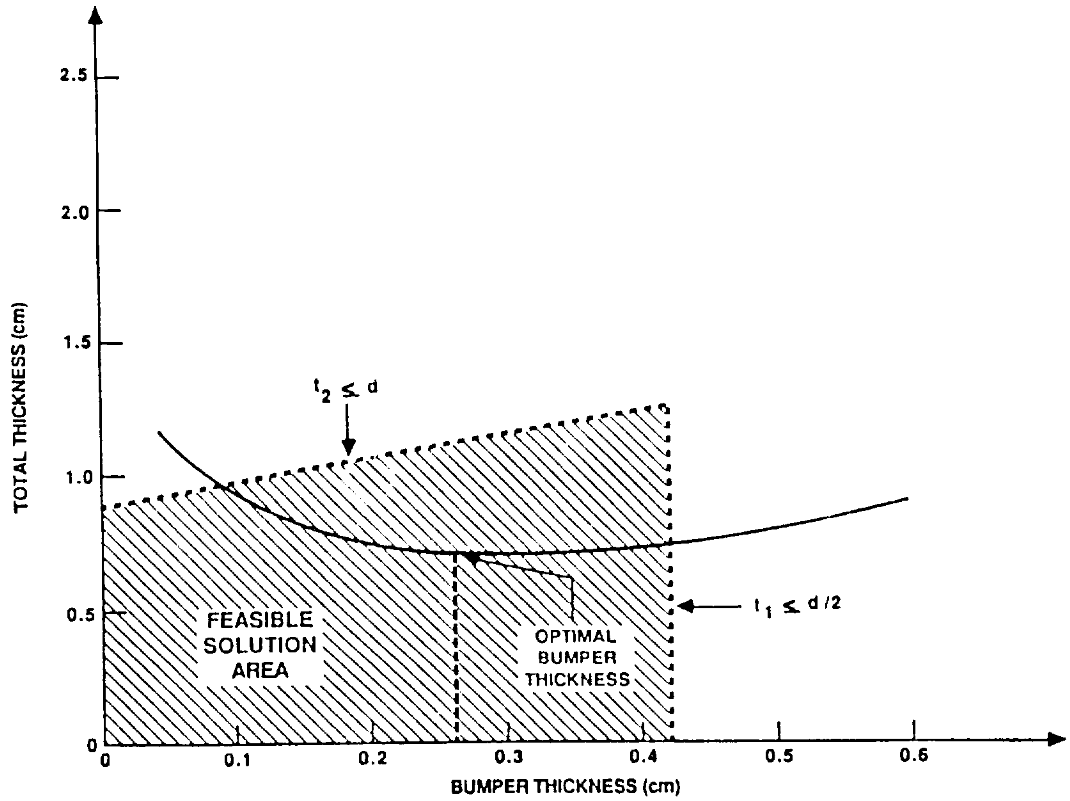

Figure 1. Determination of optimal bumper thickness.

4.3 Six-Point Fibonacci Search

The Fibonacci search is a nonlinear, one-dimensional optimization technique which does not require gradient calculations. This technique successively reduces the length of an initial interval of uncertainty by logically updating search points located equidistant from the interval endpoints. A practical discussion of this technique is found in Reference 9, and a more formal description is found in Reference 8. The success of this technique depends on the objective function being unimodal on the interval of interest. Since our objective function, W, is strictly convex on S, this requirement is satisfied. The Fibonacci search is typically employed as an unconstrained technique. However, constraints (6) and (7) in our study provide an initial interval of uncertainty from which to proceed. Substituting conditions (18) into constraints (6) and (7) gives this interval as

$$0.09cm \leq t_1 \leq 0.42cm \quad . \tag{20}$$

A six-point Fibonacci search uses the sixth and seventh numbers of the Fibonacci sequence to determine the initial search points. Recall that these numbers are 8 and 13. If the final two search points are required to be separated by 0.1 cm or less, the right search point is given by (see Reference 6)

$$x_R = 8\frac{(0.42-0.09)}{13} + (-1)^6\frac{(0.1)}{13} + 0.09 = 0.3008 \quad .$$

Since the left search point must be equidistant from the midpoint of the interval, it is given by

$$x_L = 0.2092 \quad .$$

The objective function values are then computed and the less desirable subinterval is discarded. Table 1 shows at each iteration the values of the interval endpoints, left and right search points, and left and right objective function values. The approximate solutions are given by

$$t_{1_0} \sim 0.2644cm \qquad t_{2_0} \sim 0.4764cm \qquad W_0 \sim 0.7408cm.$$

Thus, a six-point Fibonacci search results in a relative error in bumper thickness of about 3 percent.

Table 2 shows a case where equation (17) holds and Theorem 2 does not apply. Here,

$$V = 10 \text{ km/s} \;, \qquad h = 10 \text{ cm} \;, \qquad \text{and} \quad d = 1.46 \text{ cm} \quad . \tag{21}$$

Thus, by (6) and (7), we have

$$0.67cm \le t_1 \le 0.73cm \quad . \tag{22}$$

This results in the solution

$$t_{1_0} = 0.728cm \qquad t_{2_0} = 1.401cm \qquad W_0 = 2.129cm \quad . \tag{23}$$

Note that the optimal bumper thickness satisfies (6) and (7). Thus, there do exist solutions in the region defined by (17). However, there is not a defined analytic form for these solutions.

TABLE 1. SIX-POINT FIBONACCI SEARCH WITH CONDITION (13) SATISFIED

a	b	x_L	x_R	W_L	W_R
0.0900	**0.4200**	**0.2092**	**0.3008**	**0.7483**	**0.7458**
0.2092	**0.4200**	**0.3008**	**0.3284**	**0.7458**	**0.7532**
0.2092	**0.3284**	**0.2368**	**0.3008**	**0.7417**	**0.7458**
0.2092	**0.3008**	**0.2368**	**0.2732**	**0.7417**	**0.7414**
0.2368	**0.3008**	**0.2644**	**0.2732**	**0.7408**	**0.7414**

TABLE 2. SIX-POINT FIBONACCI SEARCH WITH CONDITION (13) NOT SATISFIED

a	b	x_L	x_R	W_L	W_R
0.6700	**0.7300**	**0.6854**	**0.7146**	**2.1320**	**2.1297**
0.6854	**0.7300**	**0.7008**	**0.7146**	**2.1305**	**2.1297**
0.7008	**0.7300**	**0.7146**	**0.7162**	**2.1297**	**2.1296**
0.7146	**0.7300**	**0.7162**	**0.7284**	**2.1296**	**2.1292**
0.7162	**0.7300**	**0.7178**	**0.7284**	**2.1295**	**2.1292**

4.4 Linearization

One promising technique for solving nonlinear optimization problems is to approximate the nonlinear portion of the objective function using line segments [9]. Conventional linear programming techniques are then applied to solve the resulting problem. This technique works particularly well for the problem at hand [10].

The relationship between bumper and wall thickness for the Nysmith equation may be estimated linearly [9] by

$$t_2 = t_2^1 + \sum_{i=1}^{N} a_i t_{1_i} \quad , \tag{24}$$

$$t_1 = t_1^1 + \sum_{i=1}^{N} t_{1_i} \quad , \tag{25}$$

where the a_i's represent the slopes of the N line segments used to approximate the wall thickness as a function of the bumper thickness. Thus, we may rewrite the formulation in Section 2.4 as

Minimize $\quad W = t_1^1 + t_2^1 + \sum_{i=1}^{N} (1 + a_i) t_{1_i}$ (26)

with respect to t_{1_i}

subject to $\quad t_2^1 + \sum_{i=1}^{N} a_i t_{1_i} \leq d$ (27)

$$t_1^1 + \sum_{i=1}^{N} t_{1_i} \leq \frac{d}{2} \tag{28}$$

$$0 \leq t_{1_i} \leq \Delta X \quad . \tag{29}$$

Note that the initial values for the bumper and wall are constant and may be removed from the objective function. Furthermore, by picking the initial wall thickness as the projectile diameter, and the initial bumper thickness corresponding to that choice of wall thickness, constraint (27) becomes redundant, since the wall thickness is a monotonically decreasing function of the bumper thickness. Thus, constraint (3) actually simplifies the linear programming process by providing a set of initial conditions. Similarly, a check for ending the iteration should be given by constraint (2) which corresponds to constraint (28). However, this constraint may not be removed since, as will be seen, there is no guarantee that the number of intervals between the initial bumper thickness and d/2 is an integer. Thus, the final linear programming problem formulation may be written

$$\text{Minimize} \quad W_1 = \sum_{i=1}^{N} (1 + a_i) t_{1_i} \tag{30}$$

with respect to t_{1_i}

$$\text{subject to} \quad \sum_{i=1}^{N} t_{1_i} \le \frac{d}{2} - t_1^1 \tag{31}$$

$$0 \le t_{1_i} \le \Delta X \quad , \tag{32}$$

and based on this, the final solution is given by

$$W = W_1 + t_1^1 + t_2^1 \tag{33}$$

$$t_2 = t_2^1 + \sum_{i=1}^{N} a_i t_{1_i} \qquad t_1 = t_1^1 + \sum_{i=1}^{N} t_{1_i} \quad . \tag{34}$$

Note that this problem has N linearly independent variables and N + 1 constraints. This linear programming problem is solved using a revised simplex algorithm as a subroutine in Protective Systems Design – Linear Program (PSDLP). Note, however, that when the objective function coefficients become positive (that is, when the slopes of the approximating line segments become greater than −1) there is no longer incentive for selecting nonzero decision variables, since this is a minimization problem.

Figure 2 depicts a four-segment linearization of the relationship between bumper and wall thickness for the Nysmith predictor. Note, that in this case, the last line segment extends beyond the constraint on bumper thickness. Also, the initial values for bumper and wall thicknesses are given by the wall thickness constraint (3). Recall that the optimal solution is found at an extreme point of the linearized model. This explains why increasing the number of segments improves the accuracy of the solution: the probability of finding the optimal solution near an extreme point increases with decreasing line segment interval length. Figure 3 shows how decreasing interval length improves the accuracy of the optimal solution. Finally, the absolute error in optimal bumper thickness as a function of the ratio of optimal bumper thickness to interval length is shown in Figure 4. Nearly exact correlation between the two methods is found for an interval length of 0.01 cm, which corresponds to a 33 line segment approximation. Since problems with under 100 variables and constraints are considered "small" in the linear programming sense, the effectiveness of this method appears to be quite good.

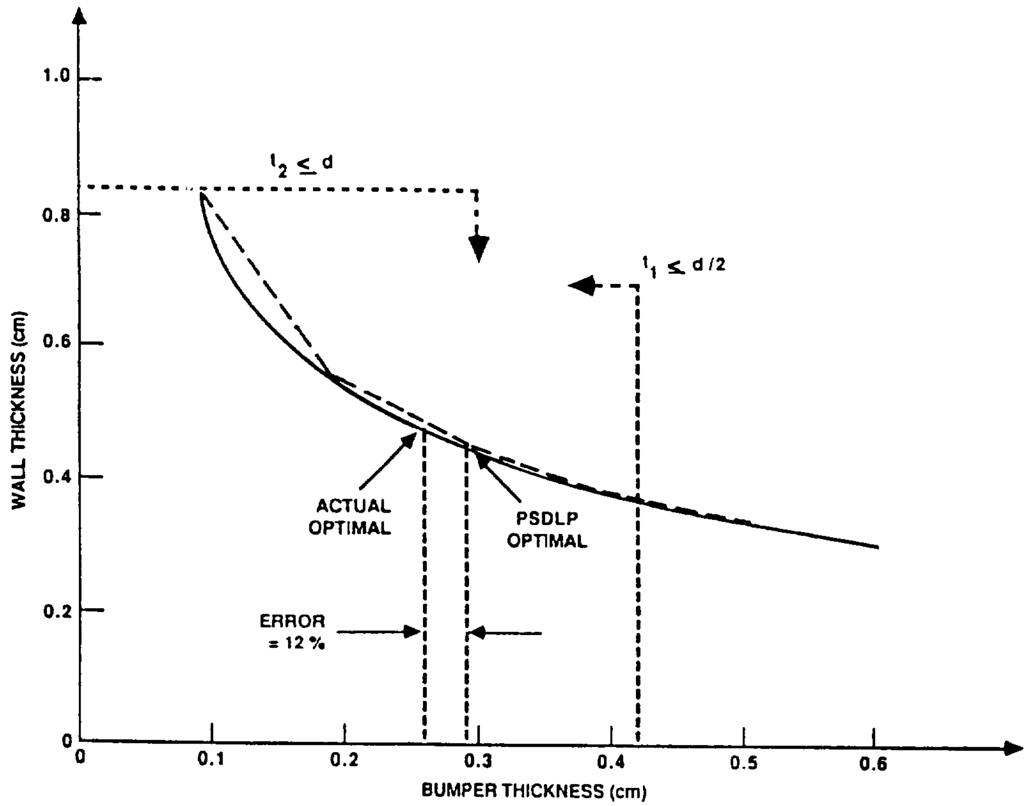

Figure 2. Linearization of the Nysmith predictor using four segments.

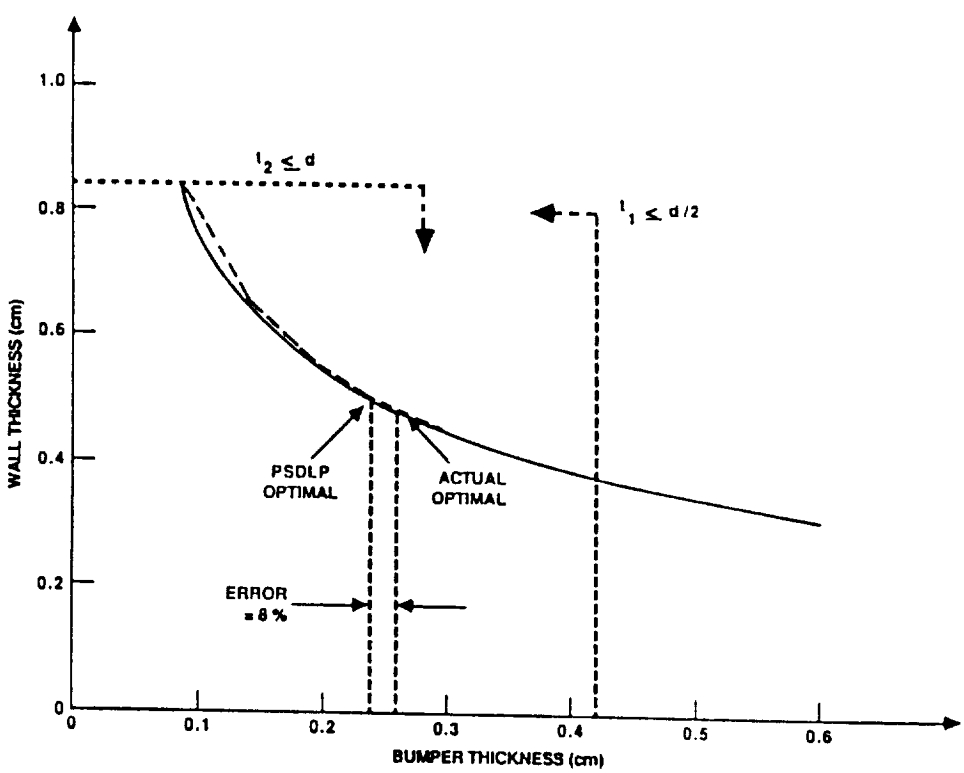

Figure 3. Linearization of the Nysmith predictor using seven segments.

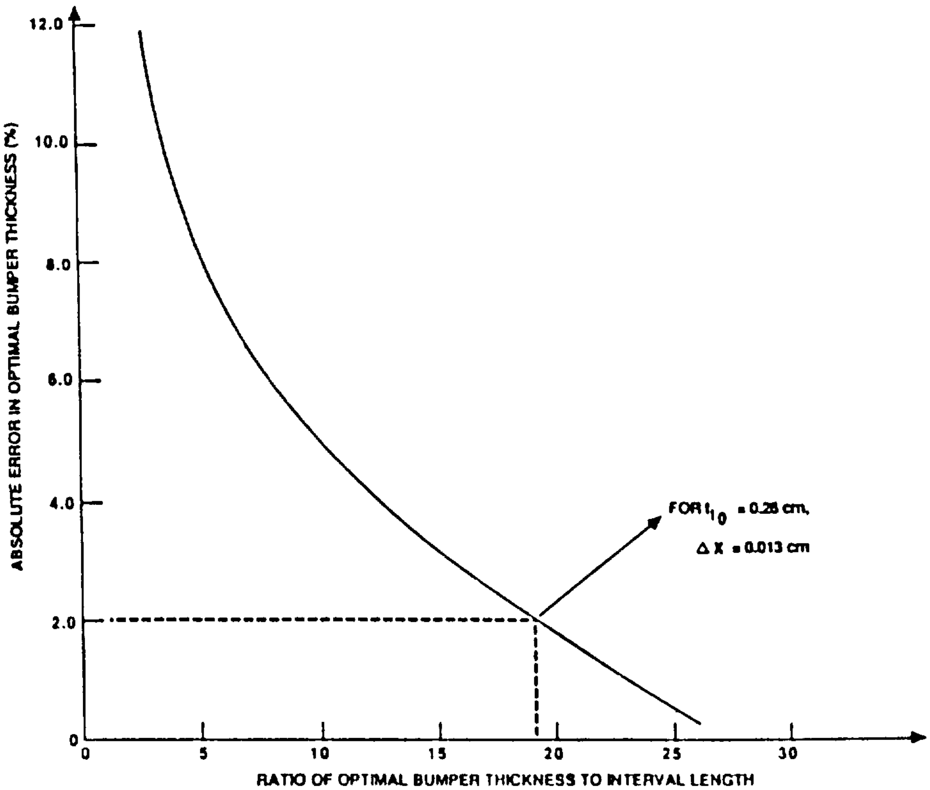

Figure 4. Error sensitivity to interval length.

4.5 Geometric Programming

Geometric programming (GP) is a mathematically elegant and powerful optimization technique. It is based on the arithmetic-geometric inequality, and it transforms any problem which fits its form into a convex programming problem. Thus, any solution found using this technique is guaranteed to be the globally optimal solution. This fact means that the convexity analysis of Section 2 is unnecessary for any problem falling into the GP form. Since not all problems suitable for this form are convex, this property of GP typically provides increased confidence with less effort in the optimal solution. The general form that GP accommodates is

$$\text{Minimize} \quad \sum_{i=1}^{n} c_i x_1^{a_{i_1}} x_2^{a_{i_2}} \ldots x_k^{a_{i_k}} \tag{35}$$

$$\text{subject to} \quad 1 \geq g_l = \sum_{i=1l}^{ml} c_{i_l} x_1^{a_{i_{1l}}} x_2^{a_{i_{2l}}} \ldots x_k^{a_{i_{kl}}} \quad , \tag{36}$$

where l = 1,...,p, k is the number of independent variables, n is the number of polynomial terms in the objective function, ml is the number of polynomial terms in constraint l, p is the number of constraints, and all coefficients and independent variables are positive.

Form (35) has been denoted a polynomial by Zener and Duffin, two developers of this method. Note that equations (5), (6), and (7) may be put in this form, where

$$k = 1\ ,\quad n = 2\ ,\quad m1 = 1 = m2\ ,\quad p = 2\ , \tag{37}$$

$$c_1 = 1, \qquad c_2 = \frac{5.08V^{0.278}d^{2.92}}{h^{1.39}},$$

$$a_1 = 1, \qquad a_2 = -0.528,$$

$$c_{1_1} = \frac{2}{d}, \qquad c_{1_2} = \frac{21.72V^{0.527}d^{3.636}}{h^{2.633}},$$

$$a_{1_1} = 1, \qquad a_{1_2} = -1 \qquad "".""$$

The general problem is then converted to the dual problem:

$$\text{Maximize} \quad \nu(\delta) = \Pi_{i=1}^{n}\left(\frac{c_i}{\delta_i}\right)^{\delta_i}\left(\Pi_{l=1}^{p}\mu_l^{\mu_l}\left(\left(\frac{c_{1l}}{\delta'_{1l}}\right)^{\delta'_{1l}}\left(\frac{c_{2l}}{\delta'_{2l}}\right)^{\delta'_{2l}}\cdots\left(\frac{c_{ml}}{\delta'_{ml}}\right)^{\delta'_{ml}}\right)\right) \tag{38}$$

with

$$\Sigma_{i=1}^{n}\delta_i a_{ij} + \Sigma_{l=1}^{p}(\Sigma_{i=1l}^{ml}\delta'_{il}a_{ijl}) = 0 \qquad j=1,2,\ldots,k \tag{39}$$

and

$$\Sigma_{i=1}^{n}\delta_i = 1 \tag{40}$$

$$\mu_l = \Sigma_{i=1l}^{ml}\delta'_i \qquad l=1,2,\ldots,p. \tag{41}$$

Substituting our problem variables into this form yields

$$\text{Maximize} \quad \nu(\delta) = \left(\frac{1}{\delta_1}\right)^{\delta_1}\left(\frac{c_2}{\delta_2}\right)^{\delta_2}\left(\frac{2}{d}\right)^{\delta'_{11}}(c_{12})^{\delta'_{12}} \tag{42}$$

subject to $$\delta_1 - 0.528\delta_2 + \delta'_{11} - \delta'_{12} = 0 \tag{43}$$

$$\delta_1 + \delta_2 = 1 \tag{44}$$

$$\delta_1, \quad \delta_2, \quad \delta'_{11}, \quad \delta'_{12} > 0 \quad . \tag{45}$$

This is a 2 degree-of-difficulty problem, since there are two equations and four unknowns. Performing a two-dimensional search over the dual (prime) variables gives

$$\begin{gathered} \delta'_{11} = \delta'_{12} \sim 0 \\ \delta_1 \sim 0.346 \qquad \delta_2 \sim 0.654 \\ v(\delta) \sim 0.7413 \quad . \end{gathered} \tag{46}$$

Furthermore, since

$$t_{1_0} = \delta_1 v(\delta) \qquad t_{2_0} = v(\delta) - t_{1_0} \quad , \tag{47}$$

we have

$$t_{1_0} = 0.256cm \qquad t_{2_0} = 0.485cm \qquad W_0 = 0.741cm$$

as found approximately in equation (19).

4.6 Relative Merits of the Techniques

For a one-dimensional problem such as this one, the six-point Fibonacci search is computationally efficient. However, like the graphic method, it provides no general analytical information about the solution, as in Theorem 2. This is also true of the linearization method. However, it fits nicely into standard linear programming packages. On the other hand, GP provides analytical information about the form of the optimal solution, and gives the same results when condition (13) (and thus, the Nysmith constraints) is satisfied. However, it too suffers in the case presented because it transforms an optimization problem with one independent variable into a problem involving a two-dimensional search for the dual variables. Hence, no single method is unconditionally superior for this problem. Fortunately, for this particular problem, the results of Theorem 2 suffice for most feasible sets of the systemic values, V, d, and h.

5. OPTIMAL DESIGN SENSITIVITY TO SYSTEMIC VARIABLES

5.1 Introduction

The existence and uniqueness of the globally optimal solution to the problem of Section 2.4 has been shown. We now consider the effect of changes in the systemic parameters, V, d, and h, on this solution. These changes affect solution feasibility [see equation (10)] and optimality (see Theorem 2).

Figure 5 depicts the feasibility condition (10) in terms of the minimum separation between bumper and wall versus projectile diameter for various projectile velocities. The region above each line segment denotes feasibility. This condition must be checked prior to calculating the optimal solution.

5.2 Projectile Velocity

Figure 6 shows the design sensitivity to projectile velocity for various projectile diameters and a fixed bumper/wall separation of 10 cm. Note that in the high velocity region (10 to 16 km/s), the optimal design does not vary significantly. Thus, optimal design increases with increasing projectile velocity for the Nysmith predictor.

5.3 Projectile Diameter

Figure 7 shows the sensitivity of optimal design to projectile diameter for various bumper/wall separations and a fixed projectile velocity of 10 km/s. Optimal design is sensitive to and increases with projectile diameter. The stopping point on each curve represents the limitation on projectile diameter given by (9).

5.4 Separation Between Bumper and Wall

Figure 8 shows the sensitivity of optimal design to bumper/wall separation for various projectile diameters and a fixed projectile velocity of 10 km/s. The shaded region to the left represents the infeasibility area as determined from equation (10). Note that optimal design decreases with increasing separation.

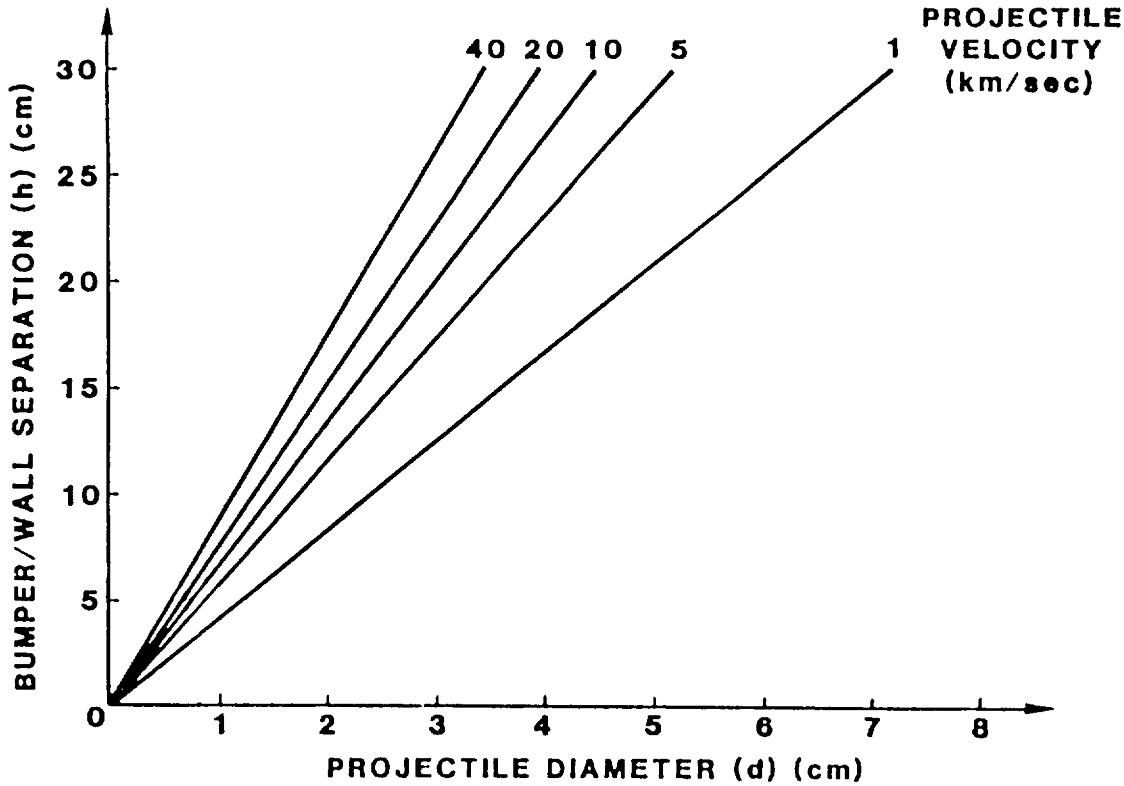

Figure 5. Feasibility condition for Nysmith predictor usage.

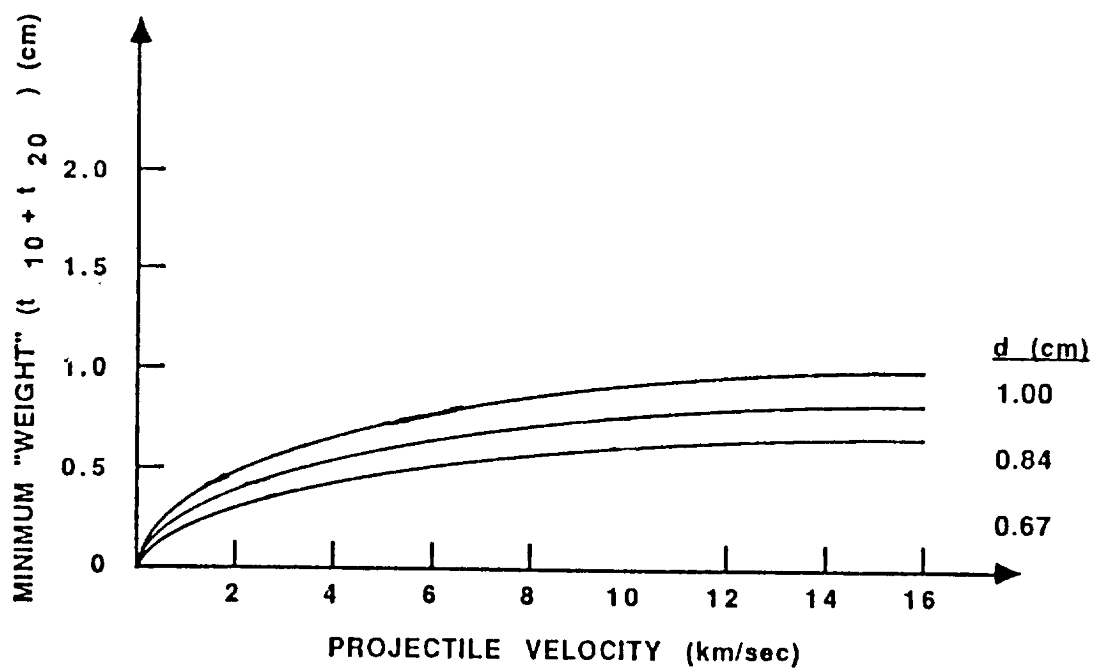

Figure 6. Optimal design sensitivity to projectile velocity.

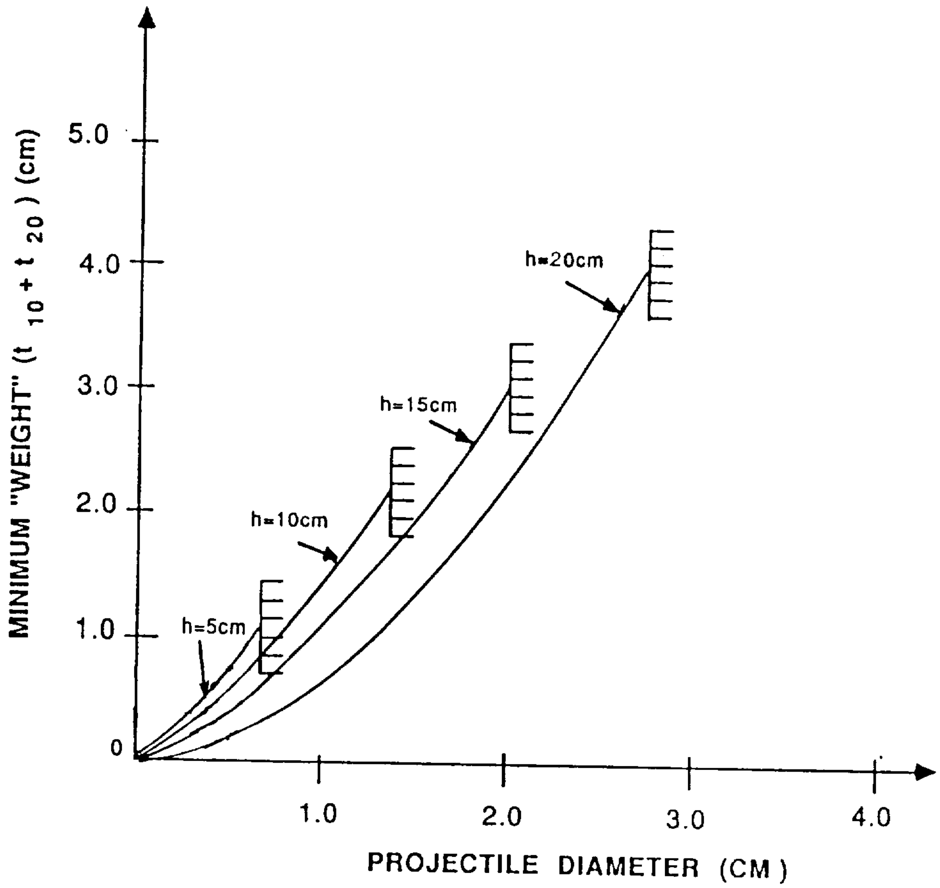

Figure 7. Optimal design sensitivity to projectile diameter.

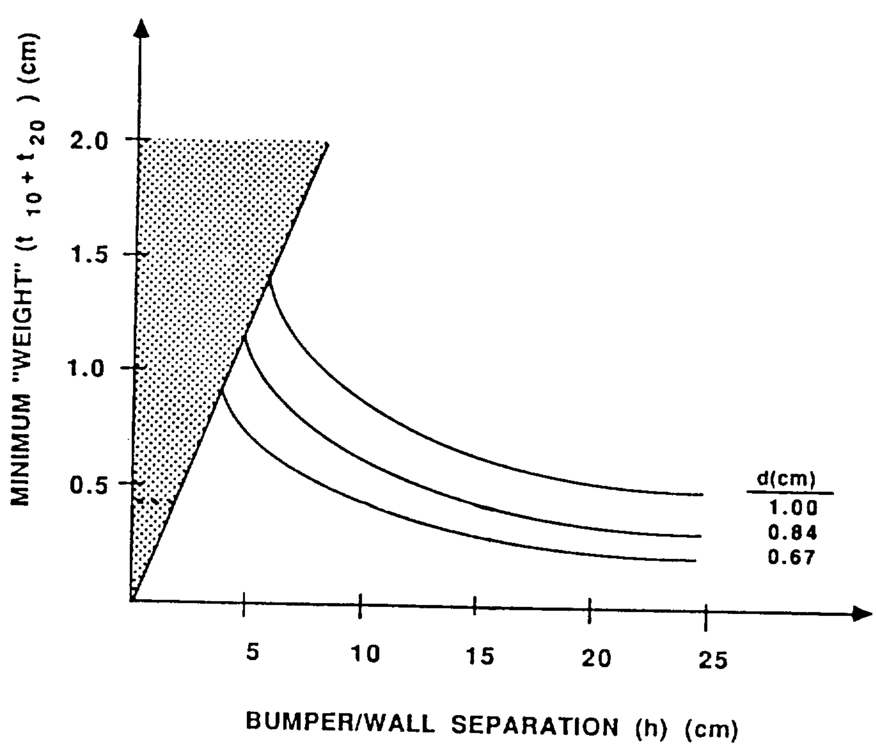

Figure 8. Optimal design sensitivity to bumper/wall separation.

6. SUMMARY, CONCLUSIONS, AND RECOMMENDATIONS

6.1 Summary

The protective systems design problem was formulated as a nonlinear, single variable, optimization problem with two constraints with the goal of minimizing the sum of the bumper and wall thicknesses. A feasibility condition which defines the limitations on the usage of the Nysmith predictor was developed in Section 3.2. It was then shown, using set and function convexity attributes, that any local minimum to this problem is the unique global minimum solution. In Section 4, the existence of this minimum was shown (for problems which satisfy the feasibility condition, of course), and several techniques were used to compare their relative effectiveness in finding the solution. A theorem was also presented which provides the analytical solution for the global minimum over most of the feasibility set. Finally, the effect of changes in the systemic parameters on the optimal design was presented in Section 5.

6.2 Conclusions

The problem defined in Section 2.4 has a unique globally optimal solution, provided the nonempty feasibility set condition (10) is satisfied. When condition (13) is satisfied, this optimal solution may be expressed analytically. The six-point Fibonacci search provides the least computations in achieving the optimal solution, while the GP technique (and Theorem 2 when it applies) provides the most insight into the general form of the solution. The optimal design increases with increasing projectile velocity and diameter and decreases with increasing bumper/wall separation for the Nysmith predictor. The optimal thickness distribution for the Nysmith predictor is approximately 35 percent bumper and 65 percent wall.

6.3 Recommendations

A logical next step would be to determine the optimal design for a weight objective function expressed in terms of specific space station core module configuration parameters and compare the results with this analysis. It is also important to perform design optimization and sensitivity analyses for other available impact predictors to see how these differ from the Nysmith model. Furthermore, the results should be correlated with current test data to determine regions of disagreement. Finally, these design optimization methodoologies should be applied to other space station components.

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APPROVAL

GLOBAL NONLINEAR OPTIMIZATION OF SPACECRAFT PROTECTIVE STRUCTURES DESIGN

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The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

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