# PROTEUS Two-Dimensional Navier-Stokes Computer Code-Version 1.0 

## Volume 2-User's Guide

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Version 1.0
Volume 2 - User's Guide

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## PRINCIPAL NOTATION

## SYMBOLS

Unless specified otherwise, all variables are nondimensional.

| Symbol | Definition |
| :---: | :---: |
| $c_{p}, c_{v}$ | Specific heats at constant pressure and volume. |
| $c_{p}$ | Static pressure coefficient. |
| $E_{T}$ | Total energy per unit volume. |
| $g_{c}$ | Proportionality constant in 入ewton's second law. |
| $h_{r}$ | Stagnation enthalpy per unit mass. |
| $k$ | Effective thermal conductivity coefficient. |
| $L$, | Dimensional reference length. |
| M | Mach number. |
| $n$ | Time level. |
| $N_{1}, N_{2}$ | Number of grid points in the $\xi$ and $\eta$ directions. |
| $p$ | Static pressure. |
| Pr ${ }_{l}$ | Laminar Prandtl number. |
| Q | Vector of dependent variables in the Cartesian or cylindrical coordinate form of the governing equations. |
| R | Residual. |
| $R$ | Gas constant. |
| $R e_{r}$ | Reference Reynolds number. |
| $T$ | Static temperature. |
| $u, v$ | Velocities in the Cartesian $x$ and $y$ directions. |
| $u, v, w$ | Velocities in the cylindrical $x, r$, and swirl directions. |
| $x, r$ | Cylindrical axial and radial coordinates. |
| $x, y$ | Cartesian coordinates. |
| $\gamma$ | Ratio of specific heats, $c_{p} / c_{v}$. |
| $\varepsilon_{E}^{(2)}, \varepsilon_{E}^{(4)}$ | Second- and fourth-order explicit artificial viscosity coefficients in constant coefficient model. |
| $\varepsilon_{l}$ | Implicit artificial viscosity coefficient. |
| $\theta$ | Cylindrical circumferential coordinate. |
| $\theta_{1}, \theta_{2}, \theta_{3}$ | Parameters determining type of time differencing used. |
| $\kappa_{2}, \kappa_{4}$ | Constants in nonlinear coefficient artificial viscosity model. |
| $\mu$ | Viscosity coefficient. |
| $\xi, \eta$ | Computational coordinate directions. |

Symbol
$\rho$
$\tau$

## SIBSCRIPIS

Subscript
$i, j$
$n$
$r$
$T$
SUPERSCRIPTS

Definition
Denotes time level.
Overbar; denotes dimensional value.

# PROTELS TWO-DIMENSIONAL NAYIER-STOKES COMPLTER CODE - VERSION 1.0 

Volume 2 - User's Guide<br>Charles E. Towne, John R. Schwab, Thomas J. Benson<br>National Aeronautics and Space Administration<br>Lewis Research Center<br>Cleveland, Ohio<br>Ambady Suresh<br>Sverdrup Technology, Inc. NASA Lewis Research Center Group<br>Cleveland, Ohio


#### Abstract

SCMMARY A new computer code, called PROTEUS, has been developed to solve the two-dimensional planar or axisymmetric, Reynolds-averaged, unsteady compressible Navier-Stokes equations in strong conservation law form. The objective in this effort has been to develop a code for aerospace propulsion applications that is easy to use and easy to modify. Code readability, modularity, and documentation have been emphasized.

The governing equations are written in Cartesian coordinates and transformed into generalized nonorthogonal body-fitted coordinates. They are solved by marching in time using a fully-coupled alternating-direction-implicit solution procedure with generalized first- or second-order time differencing. The boundary conditions are also treated implicitly, and may be steady or unstcady. Spatially periodic boundary conditions are also available. All terms, including the diffusion terms, are linearized using second-order Taylor series expansions. Turbulence is modeled using an algebraic eddy viscosity model.


The program contains many operating options. The governing equations may be solved for twodimensional planar flow, or axisymmetric flow with or without swirl. The thin-layer or Euler equations may be solved as subsets of the Navier-Stokes equations. The energy equation may be eliminated by the assumption of constant total enthalpy. Explicit and implicit artificial viscosity may be used to damp preand post-shock oscillations in supersonic flow and to minimize odd-even decoupling caused by central spatial differencing of the convective terms in high Reynolds number flow. Several time step options are available for convergence acceleration, including a locally variable time step and global time step cycling. Simple Cartesian or polar grids may be generated internally by the program. More complex geometries require an externally generated computational coordinate system.

The documentation is divided into three volumes. Volume 1 is the Analysis Description, and presents the equations and solution procedure used in PROTEUS. It describes in detail the governing equations, the turbulence model, the linearization of the equations and boundary conditions, the time and space differencing formulas, the ADI solution procedure, and the artificial viscosity models. Volume 2, the current volume, is the User's Guide, and contains information needed to run the program. It describes the program's general features, the input and output, the procedure for setting up initial conditions, the computer resource requirements, the diagnostic messages that may be generated, the job control language used to run the program, and several test cases. Volume 3 is the Programmer's Reference, and contains detailed information useful when modifying the program. It describes the program structure, the Fortran variables stored in common blocks, and the details of each subprogram.


### 1.0 INTRODLCTION

Wuch of the effort in applied computational fluid dynamics consists of modifying an existing program for whatever geometries and flow regimes are of current interest to the researcher. Unfortunately, nearly all of the available nonproprietary programs were started as research projects with the emphasis on demonstrating the numerical algorithm rather than ease of use or ease of modification. The developers usually intend to clean up and formally document the program, but the immediate need to extend it to new geometries and flow regimes takes precedence.

The result is often a haphazard collection of poorly written code without any consistent structure. An extensively modified program may not even perform as expected under certain combinations of operating options. Fach new user must invest considerable time and effort in attempting to understand the underlying structure of the program if intending do anything more than run standard test cases with it. The user's subsequent modifications further obscure the program structure and therefore make it even more difficult for others to understand.

The PROTELS two-dimensional Navier-Stokes computer program is a user-oriented and easilymodifiable flow analysis program for aerospace propulsion applications. Readability, modularity, and documentation were primary objectives during its development. The entire program was specified, designed, and implemented in a controlled, systematic manner. Strict programming standards were enforced by immediate peer review of code modules; Kernighan and Plauger (1978) provided many useful ideas about consistent programming style. Every subroutine contains an extensive comment section describing the purpose, input variables, output variables, and calling sequence of the subroutine. With just two clearlydefined exceptions, the entire program is written in ANSI standard Fortran 77 to enhance portability. A master version of the program is maintained and periodically updated with corrections, as well as extensions of general interest (e.g., turbulence models.)

The PROTEUS program solves the unsteady, compressible, Reynolds-averaged Navier-Stokes equations in strong conservation law form. The governing equations are written in Cartesian coordinates and transformed into generalized nonorthogonal body-fitted coordinates. They are solved by marching in time using a fully-coupled alternating-direction-implicit (ADI) scheme with generalized time and space differencing (Briley and McDonald, 1977; Beam and Warming, 1978). The current turbulence model is based upon the algebraic eddy-viscosity model of Baldwin and Lomax (1978). All terms, including the diffusion terms, are linearized using second-order Taylor series expansions. The boundary conditions are treated implicitly, and may be steady or unsteady. Spatially periodic boundary conditions are also available.

The program contains many operating options. The governing equations may be solved for twodimensional planar flow, or axisymmetric flow with or without swirl. The thin-layer or Fuler equations may be solved as subsets of the Navier-Stokes equations. The energy equation may be climinated by the assumption of constant total enthalpy. Explicit and implicit artificial viscosity may be used to damp preand post-shock oscillations in supersonic flow and to minimize odd-even decoupling caused by central spatial differencing of the convective terms in high Reynolds number flow. Several time step options are available for convergence acceleration, including a locally variable time step and global time step cycling. Simple grids may be generated internally by the program; more complex geometries require extemal grid generation, such as that developed by Chen and Schwab (1988).

The documentation is divided into three volumes. Volume 1 is the Analysis Description, and presents the equations and solution procedure used in PROTEUS. It describes in detail the governing equations, the turbulence model, the linearization of the equations and boundary conditions, the time and space differencing formulas, the ADI solution procedure, and the artificial viscosity models. Volume 2, the current volume, is the L'scr's Guide, and contains information needed to run the program. It describes the program's gencral features, the input and output, the procedure for setting up initial conditions, the computer resource requirements, the diagnostic messages that may be generated, the job control language used to run

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the program, and several test cases. Volume 3 is the Programmer's Reference, and contains detailed information useful when modifying the program. It describes the program structure, the Fortran variables stored in common blocks, and the details of each subprogram.

The authors would like to acknowledge the significant contributions made by three co-workers in the development of the PROTELS program. Simon Chen did the original coding of the Baldwin-Lomax turbulence model, and consulted in the implementation of the nonlinear coefficient artificial viscosity model. William Kunik developed the original coding for computing the metrics of the generalized nonorthogonal grid transformation. Frank Molls made many debugging and verification runs, particularly for spatially periodic and unsteady flows.

### 2.0 GENERAL DESCRIPIION

In this section the basic characteristics and capabilities of the PROIFLS code are described in general. Vore detabed descriptions can be found in other sections of this manual or in Volumes 1 and 3.

### 2.1 ANAI.YSIS

PROTELS 2-D solves the two-dimensional planar or axisymmetric unsteady compressible NavierStokes equations. Swirl is allowed in axisymmetric flow. The planar equations are solved in fully conservative form. For turbulent flow the Reynolds time-averaged Navier-Stokes equations are used, with turbulence modeled using the algebraic eddy viscosity model of Baldwin and I omax (1978). As subsets of these equations, options are available to solve the Euler equations or the thin-layer Navier-Stokes equations. An option is also available to eliminate the energy equation by assuming constant total enthalpy. The governing equations and turbulence model are described in detail in Sections 2.0 and 3.0 of Volume 1.

The equations are solved by marching in time using the generalized time differencing of Beam and Warming (1978). The method may be either first- or second-order accurate in time, depending on the choice of time differencing parameters. Second-order central differencing is used for all spatial derivatives. The time and space differencing formulas are presented in Sections 4.0 and 6.0 of Volume 1 . . Vonlinear terms are linearized using second-order Taylor series expansions in time, as described in Section 5.0 of Volume 1. The resulting difference equations are solved using an alternating-direction implicit (ADI) technique, with Douglas-Gunn type splitting as written by Briley and McDonald (1977). The boundary conditions are also treated implicitly.

Artificial viscosity, or smoothing, is normally added to the solution algorithm to damp pre- and postshock oscillations in supersonic flow, and to prevent odd-even decoupling due to the use of central differences in convection-dominated regions of the flow. Implicit smoothing and two types of explicit smoothing are avalable in PROTIES. The implicit smoothing is second order with constant coefficients. For the explicit smoothing the user may choose a constant coefficient second-andor fourth-order model (Steger, 1978), or a nonlinear coefficient mixed second- and fourth-order model (Jameson, Schmidt, and Turkel, 1981). The nonlinear coefficient model was designed specifically for flow with shock waves. The artificial viscosity models are described in detail in Section 9.0 of Volume 1.

The equations are fully coupled, leading to a system of equations with a block tridiagonal coefficient matrix that can be solved using the block matrix version of the Thomas algorithm. Because this algorithm is recursive, the source code cannot be vectorized in the ADI sweep direction. However, it is vectorized in the non-sweep direction, leading to an efficient implementation of the algorithm. The solution algorithm is described in detail in Section 8.0 of Volume 1.

## 2.2 (BEOMEIRY AND GRID SYSTEM

The equations solved in PROTEUS were originally written in a Cartesian coordinate system, then transformed into a general nonorthogonal computational coordinate system as described in Section 2.3 of Volume 1. The code is therefore not limited to any particular type of geometry or coordinate system. The only requirement is that body-fitted coordinates must be used. In general, the computational coordinate system for a particular geometry must be created by a separate coordinate generation code and stored in an unformatted file that PROTELS can read. However, simple Cartesian and polar coordinate systems are built in.

The equations are solved at grid points that form a computational mesh within this computational coordinate system. Note that a distinction is being made between the terms computational coordinate system and computational mesh. The computational coordinate system refers to the $(\xi, \eta)$ system in which the goveming equations are written. It is determined by supplying a series of points whose Cattesian $(x, y)$ coor-
dinates are specified, either by reading them from a file or through one of the analytically defined coordinate systems built into subroutine GEOM. The computational mesh consists of grid points distributed along lines in the computational coordinate directions. These points may differ in number and location from those used to determine the computational coordinate system. The number of grid points in each direction in the computational mesh is specified by the user. The location of these grid points can be varied by packing them at either or both boundaries in any coordinate direction. The transformation metrics and Jacobian are computed using finite differences in a manner consistent with the differencing of the governing equations.

### 2.3 FLOW AND REFERENCE CONDITIONS

As stated earlier, the equations solved by PROTEUS are for compressible flow. Incompressible conditions can be simulated by running at a Mach number of around 0.1 . Lower Mach numbers may lead to numerical problems. The flow can be laminar or turbulent. The gas constant $\bar{R}$ is specified by the user, with the value for air as the default. The specific heats $c_{p}$ and $c_{v}$, the molecular viscosity $\mu$, and the thermal conductivity $k$ can be treated as constants or as functions of temperature. The empirical formulas used to relate these properties to temperature are contained in subroutine FTEMP, and can easily be modified if necessary. The perfect gas equation of state is used to relate pressure, density, and temperature. This equation is contained in subroutine EQSTAT, which could also be easily modified if necessary. All equations and variables in the program are nondimensionalized by normalizing values derived from reference conditions specified by the user, with values for sea level air as the default.

### 2.4 BOLNDARY CONDITIONS

The easiest way to specify boundary conditions in PROTEUS is by specifying the type of boundary (i.e., no-slip adiabatic wall, subsonic inflow, periodic, etc.). The program will then select an appropriate set of conditions for that boundary. For most applications this method should be sufficient. If necessary, however, the user may instead set the individual boundary conditions on any or all of the four computational boundaries.

A variety of individual boundary conditions are built into the PROTEUS code, including: (1) specified values and/or gradients of Cartesian velocities $u, v$, and $w$, normal and tangential velocities $V_{n}$ and $V_{t}$, pressure $p$, temperature $T$, and density $\rho$; (2) specified values of total pressure $p_{T}$, total temperature $T_{T}$, and flow angle; and (3) linear extrapolation. Another useful boundary condition is a "no change from initial condition" option for $u, v, w, p, T, \rho, p_{T}$, and/or $T_{T}$. Provision is also made for user-written boundary conditions using subroutines BCF and BCFLIN. Specified gradient boundary conditions may be in the direction of the coordinate line intersecting the boundary or normal to the boundary, and may be computed using two-point or three-point difference formulas. For all of these conditions, the same type and value may be applied over the entire boundary surface, or a point-by-point distribution may be specified. Lnsteady and time-periodic boundary conditions are allowed when applied over the entire boundary. The boundary conditions available in PROTEUS are described in detail in Section 3.1.7.

### 2.5 INITIAL CONDITIONS

Initial conditions are required throughout the flow field to start the time marching procedure. For unsteady flows they should represent a real flow field. A converged steady-state solution from a previous run would be a good choice. For steady flows, the ideal initial conditions would represent a real flow field that is close to the expected final solution.

The best choice for initial conditions, therefore, will vary from problem to problem. For this reason PROTELS does not include a general-purpose routine for setting up initial conditions. The user must supply a subroutine, called INIT, that sets up the initial starting conditions for the time marching procedure. Details on the Fortran variables to be specified by INIT may be found in Section 5.1.

A version of INIT is, however, built into PROTELS that specifies uniform flow with constant flow properties everywhere in the flow field. These conditions, of course, do represent a solution to the governing equations, and for many problems may help minimize starting transients in the time marching procedure. However, realistic initial conditions that are closer to the expected final solution should lead to quicker convergence.

### 2.6 TIMIE STEP SELECTION

Several different options are available for choosing the time step $\Delta \tau$, and for modifying it as the solution proceeds. $\Delta \tau$ may be specified directly, or through a value of the Courant-Friedrichs-I ewy (CFL) number. When specifying a CFL number, the time step $\Delta \tau$ may be either global (i.e., constant in space) based on the minimum CFL limit, or local (i.e., varying in space) based on the local CFL limit. For unsteady time-accurate flows global values should be used, but for steady flows using local values may lead to faster convergence. Options are available to increase or decrease $\Delta \tau$ as the solution proceeds based on the change in the dependent variables. An option is also available to cycle $\Delta \tau$ between two values in a logarithmic progression over a specified number of time steps. The various time step options are described in detail in Section 3.1.9.

### 2.7 COXVERGENCE

Five options are currently available for determining convergence. The user specifies a convergence criterion $\varepsilon$ for each of the governing equations. Then, depending on the option chosen, convergence is based on: (1) the absolute value of the maximum change in the conservation variables $\Delta \mathbf{Q}_{\text {max }}$ over a single time step; (2) the absolute value of the maximum change $\Delta Q_{\text {max }}$ averaged over a specified number of time steps; (3) the $L_{2}$ norm of the residual for each equation; (4) the average residual for each equation; or (5) the maximum residual for each equation. These criteria are defined in Section 4.1.5.

### 2.8 INPUT/OUTPUT

Input to PROTEUS is through a series of namelists' and, in general, an unformatted file containing the computational coordinate system. All of the input parameters have default values and only need to be specified by the user if a different value is desired. Reference conditions may be specified in either English or SI units. The namelist and coordinate system input are described in Section 3.0. A restart option is also available, in which the computational mesh and the initial flow field are read from unformatted restart files created during an earlier run. The use of the restart option is described in Sections 3.1.3 and 5.3.

The standard printed output available in PROTEUS includes an echo of the input, boundary conditions, normalizing and reference conditions, the computed flow field, and convergence information. The user controls exactly which flow field parameters are printed, and at which time levels and grid points. Several debug options are also available for detailed printout in various parts of the program. The printed output is described in Section 4.1.

In addition to the printed output, several unformatted files can be written for various purposes. The first is an auxiliary file used for post-processing, usually called a plot file, that can be written at convergence or after the last time step if the solution does not converge. Plot files can be written for the NASA Lewis plotting program CONTOLR or the NASA Ames plotting program PLOT3D. If PLOT3D is to be used, two unformatted files are created, an XYZ file containing the computational mesh and a Q file containing the computed flow field. The plot files are described in detail in Section 4.2. Another unformatted file written by PROTEUS contains detailed convergence information. This file is automatically incremented each time the solution is checked for convergence, and is used to generate the convergence history printout and with Lewis-developed post-processing plotting routines. The contents of the convergence history file are presented in Section 4.3. And finally, two unformatted files may be written at the end of a calculation that may be used to restart the calculation in a later run. One of these contains the computational mesh, and the other the computed flow field. The contents of the restart files are described in detail in Section 4.4.

[^0]
### 3.0 INPUT DESCRIPTION

The standard input to the two-dimensional version of PROTELS consists of a title line and several namelists. Additional input may be provided in the form of a pre-stored unformatted file containing the computational coordinate system. The calculation can also be started by reading the computational mesh and the initial flow field from restart files written during a previous run. This section describes only the standard input and the coordinate system file. The restart file contents and format are described in Section 4.4 .

### 3.1 STANDARD NPLT

All of the standard input parameters have default values and do not need to be specified by the user unless some other value is desired. The type (REAL or INTEGER) of the input parameters follows standard Fortran convention, unless stated otherwise (i.e., those starting with I, J, K, L, M, or N are INTEGER, and the remainder are REAL.) Note that in most, if not all, implementations of Fortran, namelist names and input start in character position 2 or higher in the input line. All of the input, except for namelist IC, is read in subroutine INPUT. Namelist IC is read in subroutine INIT.

### 3.1.1 Reference and Normalizing Conditions

Unless specified otherwise, all of the input parameters are specified in nondimensional form, with the appropriate reference condition as the nondimensionalizing factor. A few words explaning what we mean by reference conditions and normalizing conditions, and the differences between them, may be helpful at this point.

The normalizing conditions are, by definition, the conditions used in nondimensionalizing the governing equations, and are denoted by an $n$ subscript. (See Section 2.0 of Volume 1.) These normalizing conditions are defined by six basic reference conditions, for length, velocity, temperature, density, viscosity, and thermal conductivity, which are specified by the user. Reference conditions are denoted by an $r$ subscript. The normalizing conditions used in PROTEUS are listed in Table 3-1.

Note that for some variables, like pressure, the normalizing condition is dictated by the form of the governing equations once the six basic reference conditions are chosen. Unfortunately, some of these may not be physically meaningful or convenient for use in setting up input conditions. Therefore, some additional reference conditions are defined from the six user-supplied ones. The reference conditions are listed in Table 3-2.

To summarize, the normalizing conditions are used to nondimensionalize the governing equations. The average user need not be too concerned about these. The reference conditions are the ones used for nondimensionalization of all user-specified input and output parameters. ${ }^{2}$

### 3.1.2 Title

TITLE A descriptive title, used on the printed output and in the CONTOLR plot file, up to 72 characters long. This is a type CHARACTER variable.

2 Internal to the PROTEUS computer code itself, variables are generally nondimensionalized by the normalizing conditions. The reference conditions are used for input and output because they are usually more physically meaningful for the user.

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### 3.1.3 Namelist RSTRT

The parameters in this namelist control the use of the restart option. The contents of the restart files are described in Section 4.4.

IRIST $0 \quad$ if no restart files are to be read or written. The initial flow field will be generated in subroutine INIT.
1 to write restart files at the end of the calculation. The initial flow field will be generated in subroutine INIT.
2 to read restart files for the computational mesh and the initial flow field, and to write restart files at the end of the calculation. Note that only the initial flow field and the computational mesh are read from restart files. The usual namelist input must still be read in. Of course, some input parameters, such as the reference conditions or those specifying the grid, must not be changed during a restart.

The default value is 0 .
NRQIN Unit number for reading the restart flow field. The default value is 11 .
NRQOUT Unit number for writing the restart flow field. The default value is 12 .
NRXIN Unit number for reading the restart computational mesh. The default value is 13 .
NRXOUT Unit number for writing the restart computational mesh. The default value is 14 .

### 3.1.4 Namelist IO

## Printout Controls

The following parameters specify which variables are to be printed, and at what locations in both time and space.

IVOUT An array of up to 50 elements specifying which variables are to be printed. The variables currently available for printing are listed and defined in Table 3-3. ${ }^{3}$ The default values are $1,2,20,30,40,45^{*} 0$, corresponding to printout of $x$ and $y$-velocity, and static density, pressure, and temperature.

IDEBUG An array of up to 20 elements used to turn on additional printout, normally used for debugging purposes. Except where noted, set $\operatorname{IDEBUG}(\mathrm{I})=1$ for printout number I. For options 1 through 7, the input parameters IPRT1 and IPRT2, or IPRT1A and IPRT2A, determine the grid points at which the printout appears. Note that some of these options can generate a lot of output. Judicious use of the "IPRT" controls is recommended. The debug options currently available are as follows:

[^1]
## Number Printout

1 Coefficient block submatrices and source term subvectors at time level $n=\operatorname{IDEBLG}(1)$ if $\operatorname{IDEBLG}(1)>0$, or at time levels $n \geq|\operatorname{IDEBLG}(1)|$ if IDEBLG(1)<0. This printout is done after the elimination of any off-diagonal boundary condition submatrices (subroutine BCELIM) and after any artificial viscosity has been added (subroutine AVISCl or 2), but before any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)
2 Coefficient block submatrices and source term subvectors at time level $n=\operatorname{IDEBLG}(2)$ if $\operatorname{IDEBLG}(2)>0$, or at time levels $n \geq|\operatorname{IDEBUG}(2)|$ if IDEBLG(2)<0. This printout is done after the elimination of any off-diagonal boundary condition submatrices (subroutine BCEILIM), but before any artificial viscosity has been added (subroutine AVISCl or 2 ) and before any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)
3 Boundary condition coefficient block submatrices and source term subvectors at time level $n=\operatorname{IDEBUG}(3)$ if $\operatorname{IDEBUG}(3)>0$, or at time levels $n \geq|\operatorname{IDEBUG}(3)|$ if $\operatorname{IDEBLG}(3)<0$. This printout is done before the elimination of any off-diagonal boundary condition submatrices (subroutine BCELIM) and before any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)
4 Boundary condition coefficient block submatrices and source term subvectors at time level $n=\operatorname{IDEBUG}(4)$ if $\operatorname{IDEBUG}(4)>0$, or at time levels $n \geq|\operatorname{IDEBUG}(4)|$ if $\operatorname{IDEBUG}(4)<0$. This printout is done after the elimination of any off-diagonal boundary condition submatrices (subroutine BCELIM) and after any rearrangement of the elements in the boundary condition submatrices (subroutine FILTER.)
Intermediate solution $Q^{\bullet}$ after the first ADI sweep at time level $n=\operatorname{IDEBUG}(5)$ if $\operatorname{IDEBUG}(5)>0$, or at time levels $n \geq|\operatorname{IDEBUG}(5)|$ if $\operatorname{IDEBUG}(5)<0$.
6 Final solution $Q^{n}$ after the last ADI sweep at time level $n=\operatorname{IDEBUG}(6)$ if $\operatorname{IDEBUG}(6)>0$, or at time levels $n \geq|\operatorname{IDEBLG}(6)|$ if $\operatorname{IDEBUG}(6)<0$.
7 Cartesian coordinates, metric coefficients, and inverse of the grid transformation Jacobian computed in subroutine METS.

The default values are all 0 .
IUNITS $\quad 0$ for input and output in Einglish units.
1 for input and output in SI units.
The default value is 0 .
IPR'T Results are printed every IPRT'th time level. However, the initial and final flow fields are always printed. The default value is 1 .
IPRTA An array of up to 101 elements specifying the time levels at which results are to be printed. The initial conditions are at time level 1 . If the calculation converges, or if the pressure or temperature is non-positive, the results are printed regardless of the value of IPRTA. If this parameter is specified, it overrides the value of IPR'T. The default values are all 0 .

IPRT1 Results are printed at every IPRTI'th grid point in the $\xi$ direction. However, the results at the boundaries are always printed. The default value is 1 .

IPRT2 Results are printed at every IPRT2'th grid point in the $\eta$ direction. However, the results at the boundaries are always printed. The default value is 1 .

IPRT1A An array of up to Ni elements (see Namelist NUM) specifying the $\xi$ indices at which results are to be printed. If this parameter is specified, it overrides the value of IPRT1. The default values are all 0 .

IPRT2A
An array of up to $N 2$ elements (see Namelist NUM) specifying the $\eta$ indices at which results are to be printed. If this parameter is specified, it overrides the value of IPRT2. The default values are all 0 .

NHMAX Maximum number of time levels allowed in the printout of the convergence history file (not counting the first two, which are always printed.) The default value is 100 .

## Plot File Controls

In addition to the printed output, files called plot files may be written for use by various post-processing routines. The following parameters specify the type of plot files to be written, and at what locations in both time and space. These plot files are described in greater detail in Section 4.2.

IPLOT 0 for no plot file.
1 to write results into an auxiliary file, in CONTOUR format, for later postprocessing. If multiple time levels are to be written into the file, they will be stacked sequentially. The value of the time $\tau$ will not be written into the file. ${ }^{4}$
-1 to write results into an auxiliary file in CONTOUR format. For multiple time levels, $\tau_{i, j}$ will be stored in the $z$ slot. (The subscripts $i$ and $j$ represent grid point indices in the $\xi$ and $\eta$ directions.)
2 to write results into auxiliary files, in PLOT3D/WHOLE format. Multiple
3 time levels will be stacked sequentially, ${ }^{5}$ with $\tau_{t, 1}$ stored in the Q file header. ${ }^{6}$ o write results into auxiliary files, in PLOT3D/PLANES format. Multiple time levels will be stacked sequentially, ${ }^{5}$ with $\tau_{1,1}$ stored in the Q file header. ${ }^{6}$ Since PROTEUS 2-D is two-dimensional, the IPLOT $=3$ option creates $X Y Z$ and $Q$ files identical to those created using the IPLOT $=2$ option.
-3 to write results into auxiliary files, in PLOT3D/PLANES format. For multiple time levels, $\tau_{i, j}$ will be stored in the $z$ slot in the XYZ file.
4 to write results into auxiliary files, in PLOT2D format. Multiple time levels
will be stacked sequentially, ${ }^{5}$ with $\tau_{1,1}$ stored in the Q file header. ${ }^{6}$
The default value is 0 .
IPL'T Results are written into the plot file every IPLT'th time level. However, if IPLT $>0$, the initial and final flow fields are automatically included in the file. If $I P L T=0$, only the final flow field is written into the file. The default value is 0 .

IPLTA An array of up to 101 elements specifying the time levels at which results are to be written into the plot file. The initial conditions are at time level 1 . If the calculation converges, or if the pressure or temperature is non-positive, the results are written into the plot file regardless of the value of IPLTA. If this parameter is specified, it overrides the value of IPLT. The default values are all 0 .

[^2]
## Unit Numbers

The following parameters specify the Fortran unit numbers used for various input and output files. IIN, the unit number for reading the standard input file, is hardwired in the program as 5 .

NOUT Unit number for printing standard output. The default value is 6 .
NGRID Unit number for reading computational coordinate system file. The default value is 7 .
$\triangle$ PLOTX Unit number for writing XYZ file when using PLOT3D or PLOT2D plot file format. The default value is 8 .

NPLOT Unit number for writing CONTOLR plot file, or for writing $Q$ file when using

NHIST Unit number for writing convergence history file. The default value is 10 .
NSCR1 Unit number for scratch file used in subroutine PLOT when IPLOT $=-3$. The default value is 20 .

### 3.1.5 Namelist GMTRY

## Coordinate System Type

These parameters specify the type of flow domain being analyzed. Simple Cartesian or polar configurations can be done automatically. For more complex geometries, the configuration is determined by reading a pre-stored coordinate file. Note however, that the number of grid points and their distribution can be changed by the parameters in namelist NUM.

IAXI $\quad 0$ for a two-dimensional planar calculation.
1 for an axisymmetric calculation.
The default value is 0 .
NGEOM Flag used to specify type of computational coordinates. Currently coded are:
1 Cartesian ( $x-y$ ) computational coordinates.
2 Polar ( $r^{\prime}-\theta^{\prime}$ ) computational coordinates.'
10 Get computational coordinates from coordinate system file. The contents of this file are described in Section 3.2.

The default value is 1 .

[^3]
## Cartesian Compututional Coordinates

The following parameters specify the size of the flow domain for the Cartesian coordinate option ( $\mathrm{NGEOM}=1$ ). The computational $(\xi, \eta)$ domain for this option is shown in physical $(x, y)$ space in Figure 3.1.

XMIN Minimum $x$-coordinate for Cartesian coordinate option. The default value is 0.0 .
XMAX Maximum $x$-coordinate for Cartesian coordinate option. The default value is 1.0 .
YMIN Minimum $y$-coordinate for Cartesian coordinate option. The default value is 0.0 .
YMAX Maximum $y$-coordinate for Cartesian coordinate option. The default value is 1.0 .


Figure 3.1-Cartesian computational coordinates.

## Polar Computational Coordinates

The following parameters specify the size of the flow domain for the polar coordinate option ( $\mathrm{CGI} \mathrm{OM}=2$ ). The computational $(\xi, \eta)$ domain for this option is shown in physical $(x, y)$ space in Figure 3.2.

RMIN Minimum $r$-coordinate for polar coordinate option. The default value is 0.0 .
RMAX Maximum $r^{r}$-coordinate for polar coordinate option. The default value is 1.0 .
THMIN Minimum $\theta^{\prime}$-coordinate in degrees for polar coordinate option. The default value
THMAX Maximum $\theta^{\prime}$-coordinate in degrees for polar coordinate option. The default value is 90.0 .


Figure 3.2 - Polar computational coordinates.

### 3.1.6 Namelist FLOW

## Control Flags

The following parameters are flags that specify the type of equations to be solved, and which variables are being supplied as initial conditions.

IELLER $\quad 0$ for a full time-averaged Navier-Stokes calculation.
1 for an Euler calculation (i.e., neglecting all viscous and heat conduction terms.)
The default value is 0 .
ITHIN A 2-element array, specified as ITHIN(IDIR), indicating whether or not the thinlayer option is to be used in direction IDIR. The subscript IDIR $=1$ or 2 , corresponding to the $\xi$ and $\eta$ directions, respectively. Valid values of ITHIN(IDIR) are:

0 to include second derivative viscous terms in direction IDIR.
1 to use the thin-layer option in direction IDIR. This does not decrease the execution time much, but may be useful if the grid in direction IDIR is not sufficiently dense to resolve second derivatives in that direction.

The default values are both 0 .
IHSTAG $\quad 0$ to solve the energy equation.
1 to eliminate the energy equation by assuming constant stagnation enthalpy per unit mass. This significantly lowers the overall execution time.

The default value is 0 .
II.AMV 0 for constant laminar viscosity and thermal conductivity coefficients equal to MLR and KTR.
1 for variable laminar viscosity and thermal conductivity coefficients, computed as a function of local temperature using Sutherland's formula for air (White, 1974).

The default value is 0 .
ISWIRI ( for no swirl.
1 for a swirling calculation in axisymmetric flow.
The default value is 0 .
ICVARS Parameter specifying which variables are being supplied as initial conditions for the time marching procedure by subroutine INIT. Remember that the initial conditions must be nondimensionalized by the reference conditions listed in Table 3-2. (See Section 5.0 for details on defining initial conditions.) When the energy equation is being solved (IHSTAG=0), and the flow is two-dimensional, or axisymmetric without swirl $($ IAXI $=0$ or ISWIRL $=0)$, the allowed values are:

ICVARS Variables Supplied By INII

| 1 | $\rho, \rho u, \rho v, E_{T}$ |
| :--- | :--- |
| 2 | $\rho, u, v, T$ |
| 3 | $\rho, u, v, T$ |
| 4 | $p, u, v, \rho$ |
| 5 | $c_{p}, u, v, T$ |
| 6 | $p, M, \alpha_{v}, T$ |

When the energy equation is being solved (IHSTAG=0), and the flow is axisymmetric with swirl ( $\operatorname{IAXI}=1$ and ISWIRI $=1$ ), the allowed values are:

| ICVARS | Variables Supplied By INTT |
| :---: | :---: |
| 1 | $\rho, \rho u, \rho v, \rho w, E_{T}$ |
| 2 | $p, u, v, w, T$ |
| 3 | $\rho, u, v, w, T$ |
| 4 | $p, u, v, w, \rho$ |
| 5 | $c_{p}, u, v, w, T$ |
| 6 | $p, M, \alpha_{v}, \alpha_{w}, T$ |

When constant stagnation enthalpy is assumed (IHSTAG $=1$ ), and the flow is two-dimensional, or axisymmetric without swirl (IAXI $=0$ or ISWIRL $=0$ ), the allowed values are:

ICVARS Variables Supplied By INIT

| 1 | $\rho, \rho u, \rho v$ |
| :--- | :--- |
| 2 | $p, u, v$ |
| 3 | $\rho, u, v$ |
| 5 | $c, u, v$ |
| 6 | $p, M, x_{v}$ |

When constant stagnation enthalpy is assumed (IIISTAG $=1$ ), and the flow is axisymmetric with swirl (IAXI = 1 and ISWIRL $=1$ ), the allowed values are:

# ICVARS Variables Supplied By INIT 

| 1 | $\rho, \rho u, \rho v, \rho w$ |
| :--- | :--- |
| 2 | $p, u, v, w$ |
| 3 | $\rho, u, v, w$ |
| 5 | $c_{p}, u, v, w$ |
| 6 | $p, M, \alpha_{v}, \alpha_{w}$ |

In the above tables, $c_{p}, \alpha_{v}$, and $\alpha_{w}$ represent static pressure coefficient, flow angle in degrees in the $x-y$ (or $x-r$ ) plane, and flow angle in degrees in the $x-\theta$ plane, respectively. The default value is 2 .

## Reference Conditions

The following parameters specify the six basic reference conditions for length, velocity, temperature, density, viscosity, and thermal conductivity. These reference conditions are used, along with some additional reference conditions derived from them, as the nondimensionalizing factors for nondimensional input and output parameters. The dimensional reference conditions may be read in using either English or SI units, depending on the value of IUNITS.

| LR | Reference length $L$, in feet (meters). This is a type REAL variable. The default value is 1.0 . |
| :---: | :---: |
| UR | Reference velocity $u_{r}$ in $\mathrm{ft} / \mathrm{sec}$ ( $\mathrm{m} / \mathrm{sec}$ ). Fither LR or MACHR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. The default value is $a_{r}=\left(\gamma_{r} \bar{R} T_{r}\right)^{1 / 2}$, the speed of sound at the reference temperature. |
| MACHR | Reference Mach number, $M_{r}=u_{f} /\left(y_{r} \bar{R} T_{r}\right)^{1 / 2}$. Either MACIR or UR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. This is a type REAL variable. The default value is 0.0 . |
| TR | Reference temperature $T_{r}$ in ${ }^{\circ} \mathrm{R}(\mathrm{K})$. The default value is $519.0{ }^{\circ} \mathrm{R}(288.333 \mathrm{~K})$. |
| RIIOR | Reference density $\rho_{r}$ in $\mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$. The default value is $0.07645 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}$ $\left(1.22461 \mathrm{~kg} / \mathrm{m}^{3}\right)$. |
| MUR | Reference viscosity $\mu$, in $\mathrm{lb}_{\mathrm{m}} / \mathrm{ft}$-sec ( $\mathrm{kg} / \mathrm{m}$-sec). Either MUR or RER may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. This is a type REAL variable. The default value is the viscosity for air at the reference temperature TR. |
| RER | Reference Reynolds number, $R e_{r}=\rho_{r} u_{r} L_{r} / \mu_{r}$. Either RER or MUR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. The default value is 0.0 . |
| KTR | Reference thermal conductivity $k$, in $\mathrm{lb}_{\mathrm{m}}-\mathrm{ft} / \mathrm{sec}^{3}{ }^{\circ} \mathrm{R}\left(\mathrm{kg}-\mathrm{m} / \sec ^{3}-\mathrm{K}\right)$. Either KTR or PRLR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. This is a type REAL variable. The default value is the thermal conductivity for air at the reference temperature TR. |
| PRLR | Reference laminar Prandtl number $P r_{t_{r}}=c_{p}, \mu_{r} / k_{r}$. Either PRLR or KTR may be specified, but not both. The unspecified one will be computed from the remaining reference conditions. The default value is 0.0 . |

## Fluid Properties

The following parameters provide information about the fluid being used.

RG Gas constant $\bar{R}$ in $\mathrm{ft}^{2} / \mathrm{sec}^{2}{ }^{\circ} \mathrm{R}\left(\mathrm{m}^{2} / \mathrm{sec}^{2}-\mathrm{K}\right)$. The default value is $1716 \mathrm{ft}^{2} / \mathrm{sec}^{2}{ }^{\circ} \mathrm{R}$ (286.96 $\mathrm{m}^{2} / \mathrm{sec}^{2}-\mathrm{K}$ ).

GAMR Reference ratio of specific heats, $y_{r}=c_{p_{r}} / c_{\nu_{r}}$. This parameter acts as a flag for a constant specific heat option. If a non-zero value for GAMR is specified by the user, $c_{f_{s}}$ and $c_{v_{r}}$ are computed from GAMR and RG, and treated as constants. Otherwise they are computed locally as a function of temperature. The default value is 0.0 .

HSTAGR Stagnation enthalpy $h_{T}$ in $\mathrm{ft}^{2} / \mathrm{sec}^{2}\left(\mathrm{~m}^{2} / \mathrm{sec}^{2}\right)$. This parameter is only used with the constant stagnation enthalpy option (IHSTAG $=1$ ). The default value is computed from the reference conditions.

### 3.1.7 Namelist BC

The parameters in this namelist specify the boundary conditions to be used. NEQ conditions must be specified at each computational boundary, where $\triangle E Q$ is the number of coupled equations being solved. NEQ will be equal to 3,4 , or 5 depending on the values of IHSTAG and ISWIRL. (See Table 3-4.)

Note that the boundary conditions may be thought of as simply NEQ additional equations to be solved on the boundary. They do not necessarily have to be associated one-to-one with the governing differential equations or the dependent variables. They must, however, be functions of the dependent variables and sufficiently complete to set constraints on each of the dependent variables through their functional form. They must also, of course, be independent of one another and physically appropriate for the problem being solved.

Three different methods are available for setting boundary conditions for steady flow computations. The first, and easiest, way is to specify the type of boundary (i.e., solid wall, symmetry, etc.) using the KBC input parameters. These parameters act as "meta" flags, triggering the automatic setting of the necessary NEQ individual boundary conditions at the specified boundary.

Second, if more flexibility is needed, the NEQ individual boundary conditions may be set for each boundary using the JBC and GBC input parameters. The boundary condition type (specified value, specified gradient, etc.) is given by JBC, and the boundary condition value by GBC. With these parameters, the same conditions are applied over the entire surface.

And third, if even greater flexibility is needed, the NEQ individual boundary conditions may be set for each boundary using the IBC and FBC input parameters. These are analagous to the JBC and GBC parameters (i.e., the boundary condition type is given by IBC, and the value by FBC), but they allow a point-by-point distribution of type and value to be specified instead of using the same type and value over the entire surface. ${ }^{8}$

For a given boundary, boundary conditions specified via the KBC parameters override those specified using the JBC and GBC parameters, which in turn override those specified using the IBC and FBC parameters. However, the different methods may be used in combination as long as they don't conflict. For example, the KBC parameters may be used for two boundaries, the JBC and GBC parameters for the third boundary, and the IBC and FBC parameters for the fourth boundary. And, on a single boundary, the JBC and GBC parameters may be used for some of the NEQ boundary conditions, and the IBC and FBC parameters for the rest.

Unsteady boundary conditions may be used when individual boundary conditions are specified for the entire surface, but not when boundary conditions are specified point-by-point.

[^4]With one exception, the NEQ boundary conditions at each boundary may be specified in any order. The exception is any condition on one of the dependent conservation variables $\mathbf{Q}$. These must be specified in the order given in Table 3-4.

If a problem requires a boundary condition of the form $\Delta F=0, F=f, \partial F / \partial \phi=f$, or $\nabla F \cdot \vec{n}=f$, where $F$ is not one of the functions already built into PROTELS, the subroutines BCF and BCFLIN may be used. This requires that the user supply subroutine BCFLIN. A test case with a user-written version of BCFLIN is presented in Section 9.2. Subroutines BCF and BCFLIN are described in detail in Volume 3.

## Boundary Types with $K B C$

The following parameters set boundary conditions by specifying the type of boundary (i.e., solid wall, symmetry, etc.). These parameters act as "meta" flags, triggering the automatic setting of the necessary JBC and GBC values.
$\mathrm{KBCl} \quad$ An array, given as KBCl (IBOUND), specifying the types of boundaries in the $\xi$ direction. The subscript $1 B O L \backslash D=1$ or 2 , corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. The default values are both 0 .
$\mathrm{KBC} 2 \quad \mathrm{An}$ array, given as KBC 2 (IBOUND), specifying the types of boundaries in the $\eta$ direction. The subscript IBOUND $=1$ or 2 , corresponding to the $\eta=0$ and $\eta=1$ boundaries, respectively. The default values are both 0 .

The boundary types that may be specified are described briefly in the following table, and in greater detail in Table 3-5. For boundary types involving gradient boundary conditions, 2-point differencing is used if the input KBC value is positive, and 3 -point differencing is used if it is negative. For boundary types involving "no change from initial conditions"-type boundary conditions (e.g., $\Delta T=0$ ), the proper boundary values must be set in the initial conditions.

| KBC Value | Boundary Type |
| :---: | :---: |
| $\pm 1$ | No-slip adiabatic wall. |
| $\pm 2$ | No-slip wall, specified temperature. |
| $\pm 3$ | Inviscid wall. |
| 10 | Subsonic inflow, linear extrapolation. |
| $\pm 11$ | Subsonic inflow, zero gradient. |
| 20 | Subsonic outflow, linear extrapolation. |
| $\pm 21$ | Subsonic outflow, zero gradient. |
| 30 | Supersonic inflow. |
| 40 | Supersonic outflow, linear extrapolation |
| $\pm 41$ | Supersonic outflow, zero gradient. |
| $\pm 50$ | Symmetry. |
| 60 | Spatially periodic. |

Boundary conditions specified using the KBC parameter for a given boundary override any boundary conditions specified for that boundary using the JBC and GBC, or IBC and FBC, parameters. Note, however, that since the default values for the KBC parameters are all 0 , the default procedure for specifying boundary conditions is by using the JBC and GBC parameters.

## Surface Boundary Condition Types and Values with .JBC and GBC

The following parameters set the NEQ individual boundary condition types and values for each boundary using the JBC and GBC parameters. With these parameters, the same conditions are applied over the entire surface. Remember that the boundary condition values must be nondimensionalized by the reference conditions listed in Table 3-2. If boundary conditions are being specified using the KBC "meta" flags, none of the following parameters are used. If some of the boundary conditions are being specified using the IBC and FBC parameters, the appropriate JBC parameters must be set equal to -1 , as described below.
$\mathrm{JBCl} \quad$ A two-dimensional array, given as $\mathrm{JBCl}($ IEQ, IBOLND), specifying the type of boundary conditions to be used on the $\xi=0$ and $\xi=1$ boundaries. Here $\mathrm{IEQ}=1$ to NEQ corresponding to each equation, and $\mathrm{BBOL} \times \mathrm{D}=1$ or 2 corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. Setting $\mathrm{JBCl}=-1$ signals the code to use boundary conditions specified point-by-point, as given by the input arrays IBCl and FBC1. See Table 3-6 for a list of allowed boundary condition types. The default values are all 0 .
$J B C 2$ A two-dimensional array, given as JBC2(IEQ,IBOU\D), specifying the type of boundary conditions to be used on the $\eta=0$ and $\eta=1$ boundaries. Here $\mathrm{IEQ}=1$ to NE Q corresponding to each equation, and $\mathrm{IBOL} N \mathrm{D}=1$ or 2 corresponding to the $\eta=0$ and $\eta=1$ boundaries, respectively. Setting JBC2 $=-1$ signals the code to use boundary conditions specified point-by-point, as given by the input arrays IBC2 and FBC2. See Table 3-6 for a list of allowed boundary condition types. The default values are all 0 .

GBCl A two-dimensional array, given as GBCl (IEQ,IBOUND), specifying the values for the steady boundary conditions to be used on the $\xi=0$ and $\xi=1$ boundaries. Here IEQ $=1$ to NEQ corresponding to each equation, and IBOLND $=1$ or 2 corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. The default values are all 0.0 .

GBC2 A two-dimensional array, given as GBC2(IEQ,IBOUND), specifying the valucs for the steady boundary conditions to be used on the $\eta=0$ and $\eta=1$ boundaries. Here $\mathrm{IEQ}=1$ to NEQ corresponding to each equation, and $\mathrm{BOOUND}=1$ or 2 corresponding to the $\eta=0$ and $\eta=1$ boundaries, respectively. The default values are all 0.0 .

Note that boundary condition types $2,12,22$, etc., are specified values of the derivative with respect to the computational coordinate, not with respect to the physical distance in the direction of the computational coordinate. See Section 7.3 of Volume 1 for details.

Boundary conditions specified using the JBC and GBC parameters for given values of IEQ and IBOUND override any boundary conditions specified for those values of IEQ and IBOUND using the IBC and FBC parameters. Note that since the default values for the JBC parameters are all 0 , the default boundary conditions are "no change from initial conditions" for the conscrvation variables.

## Point-by-Point Boundary Condition Types and Values with IBC and FBC

The following parameters set the XEQ individual boundary condition types and values for each boundary using the IBC and FBC parameters. With these parameters, point-by-point distributions are specified on the surface for the boundary condition types and values. Remember that the boundary condition values must be nondimensionalized by the reference conditions listed in Table 3-2. If boundary conditions are being specified using the KBC "meta" flags, none of the following parameters are used. Note that these parameters are activated by setting the appropriate JBC parameters equal to -1 , as described below.
$\mathrm{IBCl} \quad$ A three-dimensional array, given as $\mathrm{IBCl}(\mathrm{I} 2, \mathrm{IEQ}, \mathrm{IBOUND})$, specifying the type of boundary conditions to be used on the $\xi=0$ and $\xi=1$ boundaries. Here $12=1$ to $\ 2$ corresponding to each grid point on the boundary, IEQ $=1$ to NEQ corresponding to each equation, and $\mathrm{IBOU} N \mathrm{D}=1$ or 2 corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. JBCl (IEQ,IBOU $\sim \mathrm{D})$ must be set equal to -1. See Table 3-6 for a list of allowed boundary condition types. The default values are all 0 .

IBC2 A three-dimensional array, given as IBC2(II,IEQ,IBOUND), specifying the type of boundary conditions to be used on the $\eta=0$ and $\eta=1$ boundaries. Here $I 1=1$ to Ni corresponding to each grid point on the boundary, $\mathrm{IEQ}=1$ to NEQ corresponding to each equation, and $\mathrm{IBOL} \ \mathrm{D}=1$ or 2 corresponding to the
$\eta=0$ and $\eta=1$ boundaries, respectively: JBC2(IEQ,IBOUND) must be set equal to - 1 . See Table 3-6 for a list of allowed boundary condition types. The default values are all 0 .
$\mathrm{IBCl} \quad A$ three-dimensional array, given as $\mathrm{FBCl}(\mathrm{I} 2, \mathrm{IF}, \mathrm{Q}, \mathrm{IBOL} \mathrm{CD})$, specifying the values for the steady boundary conditions to be used on the $\xi=0$ and $\dot{\zeta}=1$ boundaries. Here $\mathrm{I} 2=1$ to $\ 2$ corresponding to each grid point on the boundary, $\mathrm{IEQ}=1$ to NEQ corresponding to each equation, and $\mathrm{IBOU} \times \mathrm{D}=1$ or 2 corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. The default values are all 0.0 .

FBC2 A three-dimensional array, given as $\mathrm{FBC} 2(I 1, I E Q, I B O L X D)$, specifying the values for the steady boundary conditions to be used on the $\eta=0$ and $\eta=1$ boundaries. Here $\mathrm{I}=1$ to Nl corresponding to each grid point on the boundary, $\mathrm{IEQ}=1$ to XEQ corresponding to each equation, and $\mathrm{IBOUND}=1$ or 2 corresponding to the $\eta=0$ and $\eta=1$ boundaries, respectively. The default values are all 0.0.

Note that boundary condition types $2,12,22$, etc., are specified values of the derivative with respect to the computational coordinate, not with respect to the physical distance in the direction of the computational coordinate. See Section 7.3 of Volume 1 for details.

## Unsteady Boundary Conditions

The following parameters are used to specify unsteady boundary conditions. The boundary condition type (specified value, specified gradient, etc.) is given by JBC, as described above, but the value is given by GTBC. The type of unsteadiness (general or periodic) is given by JTBC.

JTBC1 A two-dimensional array, given as $\mathrm{J} T \mathrm{CBC}(\mathrm{IEQ}, \mathrm{IBOUND})$, specifying the type of time dependency for the boundary conditions on the $\xi=0$ and $\xi=1$ boundaries. Here $\mathrm{IL} \mathrm{Q}=1$ to NEQ corresponding to each equation, and $\mathrm{IBOUND}=1$ or 2 corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. Valid values of JTBCI(IEQ,IBOUND) are:

0 for a steady boundary condition, whose value is given by GBCl.
1 for a general unsteady boundary condition, whose value is determined by linear interpolation in the input table of GTBC1 vs. NTBCA.
2 for a time-periodic boundary condition of the form $g_{1}+g_{2} \sin \left(g_{3} n+g_{4}\right)$, where $n$ is the time level and $g_{1}$ through $g_{4}$ are given by the first four values of GTBC1.

The default values are all 0 .
JHBC2 A two-dimensional array, given as $\mathrm{JTBC} 2(I E Q, I B O L N D)$, specifying the type of time dependency for the boundary conditions on the $\eta=0$ and $\eta=1$ boundaries. Here IEQ $=1$ to NEQ corresponding to each equation, and $\operatorname{IBOUND}=1$ or 2 corresponding to the $\eta=0$ and $\eta=1$ boundaries, respectively. Valid values of JTBC2 (IEQ,IBOUND) are:

0 for a steady boundary condition, whose value is given by GBC 2 .
1 for a general unsteady boundary condition, whose value is determined by linear interpolation in the input table of GTBC2 vs. NTBCA.
2 for a time-periodic boundary condition of the form $g_{1}+g_{2} \sin \left(g_{3} n+g_{4}\right)$, where $n$ is the time level and $g_{1}$ through $g_{4}$ are given by the first four values of GIBC2.

The default values are all 0 .
NTBC Number of values in the tables of GTBC1 and or GTBC2 vs. NIBCA for the general unsteady boundary condition option. The maximum value allowed is the value of the PARAMETER NTP. (See Section 6.2.) The default value is 0 .

NTBCA An array of NTBC time levels at which GTBC1 and or GTBC2 are specified for the general unsteady boundary condition option. The default values are all 0 .

GTBCl A three-dimensional array, given as GTBCI(ITBC,IEQ,IBOLND), used in the unsteady and time-periodic boundary condition options for the $\xi=0$ and $\xi=1$ boundaries. Here $\mathrm{IEQ}=1$ to NEQ corresponding to each equation, and IBOLND $=1$ or 2 corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. For general unsteady boundary conditions the subscript $I T B C=1$ to $\times I B C$, corresponding to the time levels in the array XIBCA, and GTBCI specifies the boundary condition value directly. For time-periodic boundary conditions the subscript ITBC $=1$ to 4 , and $G \Gamma B C 1$ specifies the four coefficients in the equation used to determine the boundary condition value. The default values are all 0.0 .

GIBC2 A three-dimensional array, given as GTBC2(ITBC,IEQ,IBOUND), used in the unsteady and time-periodic boundary condition options for the $\eta=0$ and $\eta=1$ boundaries. Here IEQ $=1$ to NEQ corresponding to each equation, and $I B O L N D=1$ or 2 corresponding to the $\eta=0$ and $\eta=1$ boundaries, respectively. For general unsteady boundary conditions the subscript ITBC $=1$ to NTBC, corresponding to the time levels in the array NIBCA, and GTBC2 specifies the boundary condition value directly. For time-periodic boundary conditions the subscript $I T B C=1$ to 4 , and GIBC2 specifies the four coefficients in the equation used to determine the boundary condition value. The default values are all 0.0 .

### 3.1.8 Namelist NL.

## Mesh Parameters

The following parameters specify the number of mesh points and the degree of packing.
N1 Number of grid points $N_{1}$ in the $\xi$ direction. For non-periodic boundary conditions in the $\xi$ direction, the maximum value allowed is the value of the PARAMETER NIP. For spatially periodic boundary conditions, the maximum is N1P-1. (See Section 6.2.) The default value is 5 .

N 2 Number of grid points $N_{2}$ in the $\eta$ direction. For non-periodic boundary conditions in the $\eta$ direction, the maximum value allowed is the value of the $P \Lambda R A M$ ETER N2P. For spatially periodic boundary conditions, the maximum is $\mathrm{N} 2 \mathrm{P}-1$. (See Section 6.2.) The default value is 5 .

IPACK A 2-element array, specified as IPACK(IDIR), indicating whether or not grid points are to be packed in direction IDIR. The subscript IDIR $=1$ or 2 , corresponding to the $\xi$ and $\eta$ directions, respectively. Valid values of IPACK(IDIR) are:

0 for no packing in direction IDIR.
1 to pack points in direction IDIR using a transformation due to Roberts (1971). The location and amount of packing are specified by the array SQ.

The default values are both 0 .
SQ A two-dimensional array controlling the packing of grid points near computational boundaries, specified as $\mathrm{SQ}(\operatorname{IDIR}, I P C)$. The subscript $\operatorname{IDIR}=1$ or 2 corresponding to packing in the $\xi$ and $\eta$ directions, respectively. The subscript $I P C=1$ or 2 , where $S Q(I D I R, 1)$ specifies the packing location, and $S Q(I D I R, 2)$ specifies the amount of packing.

Meaningful values for $\mathrm{SQ}($ IDIR, 1 ) are $0.0,0.5$, and 1.0 , where 0.0 corresponds to packing near the lower boundary only (i.e., at $\xi$ or $\eta=0$, depending on IDIR), 1.0
corresponds to packing near the upper boundary only, and 0.5 corresponds to equal packing at both boundaries.

Meaningful values for SQ (II)IR,2) are values above 1.0 , but generally 1.1 or below. The closer $\operatorname{SQ}(\operatorname{IDIR}, 2$ ) is to 1.0 , the tighter the packing will be.

The default values are $S Q(\operatorname{IDIR}, 1)=0.0$ and $S Q(I D I R, 2)=10000.0$ for $\operatorname{IDIR}=1$ and 2 .

## Artificial Viscosity Parameters

The following parameters specify the type and amount of artificial viscosity to be used.
IAV4E $\quad 0$ for no fourth-order explicit artificial viscosity.
1 to include fourth-order explicit artificial viscosity using the constant coefficient model of Steger (1978).
2 to include fourth-order explicit artificial viscosity using the nonlincar coefficient model of Jameson, Schmidt, and Turkel (1981).

The default value is 1 .
IAV2E $\quad 0$ for no second-order explicit artificial viscosity.
1 to include second-order explicit artificial viscosity using the constant coefficient model.
2 to include second-order explicit artificial viscosity using the nonlinear coefficient model of Jameson, Schmidt, and Turkel (1981).

The default value is 0 .
IAV2I 0 for no second-order implicit artificial viscosity.
1 to include second-order implicit artificial viscosity using the constant cocfficient model of Steger (1978).

The default value is 1 .
CAVS4E For the constant coefficient model, CAVS4E(ILQ) specities the fourth-order artificial viscosity coefficient $\varepsilon_{E}^{(4)}$ directly. For the nonlinear coefficient model it specifies the constant $\kappa_{4}$. The subscript IEQ varies from 1 to NEQ corresponding to each coupled equation. (See Table 3-4 for the order of the equations being solved.) Good values for a given application are usually determined by experience, but recommended starting values are 1.0 for the constant coefficient model, 0.005 for the nonlinear model when spatially varying second-order time differencing is used, and 0.0002 for the nonlinear model when a spatially constant first-order time differencing is used. The default values are all 1.0.

CAVS2E For the constant coefficient model, CAVS2E(IEQ) specifies the second-order artificial viscosity coefficient $\varepsilon_{E}^{2}$ directly. For the nonlinear coefficient model it specifies the constant $\kappa_{2}$. The subscript IEQ varies from 1 to XEQ corresponding to each coupled equation. (See Table 3-4 for the order of the equations being solved.) Good values for a given application are usually determined by experience, but recommended starting values are 1.0 for the constant coefficient model, 0.01 for the nonlinear model for flows without shocks, and 0.1 for the nonlinear model for flows with shocks. The default values are all 1.0.

CAVS2I Second-order implicit artificial viscosity coefficient, $\varepsilon_{I}$, specified as CAVS2I(IEQ). The subscript IEQ varies from 1 to NEQ corresponding to each coupled equation. (See Table $3-4$ for the order of the equations being solved.) Good values for a given application are usually determined by experience, but recommended starting
values are 2.0 for the constant coefficient model, and 0.0 for the nonlinear model. The default values are all 2.0.

## Time Difference Centering Parameters

The following parameters specify the type of time differencing scheme to be used. The generalized Beam and Warming (1978) time differencing formula is given by equation (4.1) of Volume 1.

THC A 2-element array specifying the time differencing centering parameters $\theta_{1}$ and $\theta_{2}$ to be used for the continuity equation. The default values are 1.0 and 0.0 .

TIIX A 3-element array specifying the time differencing centering parameters $\theta_{1}, \theta_{2}$, and $\theta_{3}$ to be used for the $x$-momentum equation. The default values are $1.0,0.0$, and 0.0 .

THY A 3-element array specifying the time differencing centering parameters $\theta_{1}, \theta_{2}$, and $\theta_{3}$ to be used for the $y$-momentum equation. The default values are $1.0,0.0$, and 0.0 .

THIZ A 3-element array specifying the time differencing centering parameters $\theta_{1}, \theta_{2}$, and $\theta_{3}$ to be used for the swirl momentum equation. The default values are $1.0,0.0$, and 0.0 .

THE A 3-element array specifying the time differencing centering parameters $\theta_{1}, \theta_{2}$, and $\theta_{3}$ to be used for the energy equation. The default values are $1.0,0.0$, and 0.0 .

The following table summarizes the time differencing schemes that may be used. The Euler implicit method is recommended for steady flows, and the 3-point backward implicit method is recommended for unsteady flows.

| $\underline{\theta}_{1}$ | $\underline{\theta}_{2}$ | $\underline{Q}_{3}$ | Method | Accuracy |
| :---: | :---: | :---: | :--- | :---: |
| 1 | 0 | 0 | Eulcr implicit | $O(\Delta \tau)$ |
| $1 / 2$ | 0 | $1 / 2$ | Trapezoidal implicit | $O(\Delta \tau)^{2}$ |
| 1 | $1 / 2$ | 1 | 3-point backward implicit | $O(\Delta \tau)^{2}$ |

## Spatial Difference Centering Parameters

The following parameters specify the type of spatial differencing scheme to be used. The general spatial differencing formula is given by equation (6.1) of Volume 1.

ALPHAI Spatial difference centering parameter $\alpha_{1}$ for the $\xi$ direction. The default value is 0.5 (central differencing.)

ALPHA2 Spatial difference centering parameter $\alpha_{2}$ for the $\eta$ direction. The default value is 0.5 (central differencing.)

### 3.1.9 Namelist TIME

## Time Step Selection Parameters

The following parameters determine the procedure used to set the time step size, and to change it as the solution proceeds.

IDTMOD The time step size $\Delta \tau$ is recomputed every IDTMOD'th step. The default value is 1 .

IDTAU $\quad 1$ for a global (i.e., constant in space) time step $\Delta \tau=(\mathrm{CFL}) \Delta \tau_{c f}$, where $\Delta \tau_{c f f}$ is the minimum of the allowable time steps at each grid point based on the CFL criteria for explicit methods.
2 for a global time step initially computed using the IDTAU $=1$ option, but adjusted as the solution proceeds based on $\Delta \mathbf{Q}_{\text {max }}$, the absolute value of the maximum change in the dependent variables. ${ }^{9}$ For any of the dependent variables, if $\Delta \mathbf{Q}_{\max }<\mathrm{CHGl}$, the CFL number is multiplied by DTF1. If $\Delta Q_{\text {max }}>C H G 2$, the CFL number is divided by DTF2. If $\Delta Q_{\max }>0.15$, the CFL number is cut in half. The CFL number will not be decreased below CFLMIN, or increased above CFLMAX.
3 for a global time step $\Delta \tau$ equal to the specified input DT.
4 for a global time step initially equal to the specified input DT, but adjusted as the solution proceeds based on $\Delta \mathbf{Q}_{\max }$, the absolute value of the maximum change in the dependent variables. ${ }^{9}$ For any of the dependent variables, if $\Delta Q_{\max }<\mathrm{CHG} 1, \Delta \tau$ is multiplied by DTF1. If $\Delta Q_{\max }>\mathrm{CHG} 2, \Delta \tau$ is divided by DTF2. If $\Delta \mathbf{Q}_{\max }>0.15, \Delta \tau$ is cut in half. $\Delta \tau$ will not be decreased below DTMIN, or increased above DTMAX.
5 for a local (i.e., varying in space) time step $(\Delta \tau)_{i, j}=(\mathrm{CFL})\left(\Delta \tau_{c f f}\right)_{i, j}$, where $\left(\Delta \tau_{c f f}\right)_{t, j}$ is the allowable time step at each grid point based on the CFL criteria for explicit methods
6 for a local time step initially computed using the IDTAU $=5$ option, but adjusted as the solution proceeds based on $\Delta Q_{\max }$, the absolute value of the maximum change in the dependent variables. ${ }^{9}$ For any of the dependent variables, if $\Delta \mathbf{Q}_{\text {max }}<C H G 1$, the CFL number is multiplied by DTF1. If $\Delta \mathbf{Q}_{\max }>\mathrm{CHG} 2$, the CFL number is divided by DTF2. If $\Delta \mathbf{Q}_{\max }>0.15$, the CFL number is cut in half. The CFL number will not be decreased below CFLMIN, or increased above CFLMAX.
7 for a global time step with cycling. $\Delta \tau$ will be cycled repeatedly between DTMIN and DTMAX using a logarithmic progression over NDTCYC time steps. For some problems this option has been shown to dramatically speed convergence. However, the choice of DTMIN, DTMAX, and NDTCYC is critical, and no method has been developed that assures a good choice. Poor choices may even slow down convergence, so this option should be used with caution.

If IDTAU $=7$, ICHECK and IDTMOD are both automatically set equal to 1 , and NITAVG is set equal to NDTCYC. In addition, if IDTAU $=7$ and ICTEST $=1$, ICTEST is changed to 2 . If IDTAU $=2,4$, or 6 , IDTMOD is automatically set equal to ICHECK. The default value is 5 .

The above parameters IDTAU and IDTMOD apply to every case. Which of the remaining parameters are needed depends on the value of IDTAU, as specified in the following table.

```
IDTAU Parameters Needed
1 CFL
2
| DT
4 DT, CHG1, CHG2, DTF1, DTF2, DTMIN, DTMAX
5 CFL
6 CFL, CHG1, CHG2, DTF1, DTF2, CFLMIN, CFLMAX
7 DTMIN, DTMAX, NDTCYC
```

[^5]CHL An array, given as CFL(ITSEQ), specifying the ratio $\Delta \tau / \Delta \tau_{c f}$, where $\Delta \tau$ is the actual time step used in the implicit calculation and $\Delta \tau_{c f f}$ is the allowable time step based on the CFL criteria for explicit methods. The subscript ITSEQ is the sequence number. For time steps 1 through NTIME(1), CFL(1) will be used. Then for steps NTIME(1) + 1 through NTIME(1) + NTIME(2), CFL(2) will be used, etc. ${ }^{10} \mathrm{CFL}$ is not used if IDTAU $=3,4$, or 7 . The default values are all 1.0 .

DT An array, given as DT(ITSEQ), specifying the time step size $\Delta \tau$. The subscript ITSEQ is the sequence number. For time steps 1 through NTIME(1), DT(1) will be used. Then for steps NГIME(1) + 1 through NTIME(1) + NTIME(2), DT(2) will be used, etc. ${ }^{11}$ DT is not used if IDTAU $=1,2,5,6$, or 7 . The default values are all 0.01 .

CHG1 Minimum change, in absolute value, that is allowed in any dependent variable before increasing $\Delta \tau$. CHGl is only used if IDTAU $=2,4$, or 6 . The default value is 0.04 .

CHG2 Maximum change, in absolute value, that is allowed in any dependent variable before decreasing $\Delta \tau$. CHG2 is only used if IDTAU $=2,4$, or 6 . The default value is 0.06 .

DTE $1 \quad$ Factor by which $\Delta \tau$ is multiplied if the solution changes too slowly. DTF1 is only used if IDTAU $=2,4$, or 6 . The default value is 1.25 .

DTF2 Factor by which $\Delta \tau$ is divided if the solution changes too quickly. DTF2 is only used if IDTAU $=2,4$, or 6 . The default value is 1.25 .

CFLMIN Minimum value that the CFL number is allowed to reach. CFIMIN is only used if IDTAU $=2$ or 6 . The default value is 0.5 .

CFLMAX Maximum value that the CFL number is allowed to reach. CFLMAX is only used if IDTAU $=2$ or 6 . The default value is 10.0 .

DTMIN Minimum value that $\Delta \tau$ is allowed to reach (IDTAU $=4$ ), or the minimum $\Delta \tau$ in the time step cycling procedure (IDTAU = 7.) The default value is 0.1 .

DTMAX Maximum value that $\Delta \tau$ is allowed to reach (IDTAU $=4$ ), or the maximum $\Delta \tau$ in the time step cycling procedure (IDTAU = 7.) The default value is 0.1 .

NDTCYC Number of time steps per time step cycle. NDTCYC is used only with IDTAU $=7$. The default value is 2 , which results in a constant $\Delta \tau$ if DTMIN = DTMAX.

## Time Marching Limits

These parameters determine the maximum number of time steps that will be taken.
NTSEQ The number of time step sequences being used. The maximum value allowed is the value of the PARAMETER NTSEQP. If NTSEQ $>1$, IDTAU must be equal to 1,3 , or 5 . (See Section 6.2.) The default value is 1 .

[^6]NTIME An array, given as XTIME(ITSEQ), specifying the maximum number of time steps to march. The subscript ITSEQ varies from 1 to NTSEQ, and allows a series of different time steps to be specified by the values of CFL or DT. NTIME(ITSEQ) specifies the number of time steps within sequence ITSEQ. If XTSEQ $=3$, for example, the total number of time steps taken will be $N_{\text {total }}=$ NTIME(1) + NTIME(2) + NTIME(3). The initial time level is level 1 , and the final computed time level will be level $N_{\text {total }}+1$. The default values are 10, 9*0.

## Convergence Testing Parameters

These parameters determine the convergence criteria to be used.
ICHECK Results are checked for convergence every ICHECK'th time level. The default value is 10 .

ICTEST $\quad 1$ to determine convergence based on the maximum change in absolute value of each of the conservation variables over a single time step, $\Delta Q_{\max }{ }^{12}$
2 to determine convergence based on the maximum change in absolute value of each of the conservation variables, averaged over the last NITAVG time steps, $\Delta \mathbf{Q}_{z v_{g}}{ }^{12}$
3 to determine convergence based on $\mathrm{R}_{L_{2}}$, the $L_{2}$ norm of the residual for each equation.
4 to determine convergence based on $\mathrm{R}_{\text {avg }}$, the average absolute value of the residual for each equation.
5 to determine convergence based on $\mathrm{R}_{\text {max }}$, the maximum absolute value of the residual for each equation.

Convergence is assumed when the maximum change or residual parameter is less than EPS. Note that the change in conservation variables over a time step is directly related to the size of the time step. Small time steps naturally yield small changes in conservation variables. With ICTEST $=1$ or 2 , therefore, convergence may be indicated prematurely.

If ICTEST $=2$, ICHECK and IDTMOD are automatically set equal to 1 . The default value is 3 .

EPS Level of convergence to be reached, specified as EPS(IVAR) where IVAR varies from 1 to NEQ, corresponding to each conservation variable or equation. The default values are all 0.001 .

NITAVG Number of time steps over which the maximum change in conservation variables is averaged to determine convergence. The maximum value allowed is the value of the PARAMETER NAMAX. (See Section 6.2.) NITAVG only applies to the ICTEST $=2$ option. The default value is 10 .

### 3.1.10 Namelist TLRB

## Model Type Controls

The following parameters determine the type of turbulence model that will be used.
ITURB 0 for laminar flow.
1 for turbulent flow, using the algebraic eddy viscosity model of Baldwin and Lomax (1978), as described in Section 3.0 of Volume 1.

[^7]The default value is 0 .
INNER $\quad 1$ to use the inner layer model of Baldwin and Lomax (1978).
2 to use the inner layer model of Spalding (1961) and Kleinstein (1967).
The default value is 1 .
ILDAMP $\quad 0$ to use the normal Baldwin-Lomax mixing length formula in the inner region. 1 to use the modified mixing length formula of Launder and Priddin (1973) in the inner region of the Baldwin-Lomax model.

The default value is 1 .
PRT If PRT $>0.0$, it specifies the turbulent Prandtl number, which will be treated as constant. If PRT $\leq 0.0$, the turbulent Prandtl number will vary, and be computed using the empirical formula of Wassel and Catton (1973). The default value is 0.91 .

## Control Parameters

These parameters specify which boundaries and directions are important in computing the turbulent viscosity coefficient.

IWALL1 A 2-element array, specified as IWALL1(IBOUND), specifying which $\xi$ boundaries are solid walls. The subscript IBOUND $=1$ or 2 , corresponding to the $\xi=0$ and $\xi=1$ boundaries, respectively. Valid values of IWALLI(IBOUND) are:

0 if the boundary is not a solid wall.
1 if the boundary is a solid wall.
IWALLI(IBOUND) is not needed if the boundary condition for boundary IBOUND is set using the KBCl (IBOUND) meta flag. The default values are both 0.

IWALL2 A 2-element array, specified as IWALL2(IBOUND), specifying which $\eta$ boundaries are solid walls. The subscript IBOUND $=1$ or 2 , corresponding to the $\eta=0$ and $\eta=1$ boundaries, respectively. Valid values of IWALL2(IBOUND) are:

0 if the boundary is not a solid wall.
1 if the boundary is a solid wall.
IWALL2(IBOUND) is not needed if the boundary condition for boundary IBOUND is set using the KBC2 (IBOUND) meta flag. The default values are both 0.

ITXI $\quad 0$ to bypass computation of turbulent viscosity on lines in the $\xi$ direction.
I to compute turbulent viscosity on lines in the $\xi$ direction (i.e., due to walls at $\eta=0$ and/or $\eta=1$, or due to a free turbulent flow in the $\xi$ direction.)

If $\operatorname{ITHIN}(1)=1$, ITXI is automatically set equal to 1 . The default value is 1 .
ITETA $\quad 0$ to bypass computation of turbulent viscosity on lines in the $\eta$ direction.
1 to compute turbulent viscosity on lines in the $\eta$ direction (i.e., due to walls at $\xi=0$ and $/$ or $\xi=1$, or due to a free turbulent flow in the $\eta$ direction.)

If $\operatorname{ITHIN}(2)=1$, ITETA is automatically set equal to 1 . The default value is 0 .

## Transition Parameters

These parameters are used in the laminar-turbulent transition model of Cebeci and Bradshaw (1984).
REXTl The Reynolds number at the beginning of the transition region. This parameter only applies to cases with flow predominantly in the $\xi$ direction, and with a leading edge at $\xi=0$. The Reynolds number is based on maximum total velocity and distance from $\xi=0$. The default value is 0.0 .

REXT2 The Reynolds number at the beginning of the transition region. This parameter only applies to cases with flow predominantly in the $\eta$ direction, and with a leading edge at $\eta=0$. The Reynolds number is based on maximum total velocity and distance from $\eta=0$. The default value is 0.0 .

## Constants

The following parameters are various constants used in the turbulence modeling procedure.
CCLAU The Clauser constant $K$ used in the Baldwin-Lomax outer region model. The default value is 0.0168 .

CCP The constant $C_{c_{p}}$ used in the Baldwin-I omax outer region model. The default value is 1.6 .

CWK The constant $C_{w k}$ used in the formula for $F_{w a k e}$ in the Baldwin-Lomax outer region model. The default value is 0.25 .

CKIEB The constant $C_{\text {Kieb }}$ used in the formula for the Klebanoff intermittency factor $F_{\text {Kieb }}$ in the Baldwin-Lomax outer region model. The default value is 0.3 .

APLUS The Van Driest damping constant $A^{+}$used in the Baldwin-Lomax outer and inner region models. The default value is 26.0 .

CB The constant $B$ used in the formula for the Klebanoff intermittency factor $F_{\text {Kleb }}$ in the Baldwin-Lomax outer region model, and in the Spalding-Kleinstein inner region model. The default value is 5.5 .

CVK The Von Karman mixing length constant $\kappa$ used in both the Baldwin-Lomax and Spalding-Kleinstein inner region models. The default value is 0.4 .

CNL The exponent $n$ in the Launder-Priddin modified mixing length formula. The default value is 1.7.

CNA The exponent $n$ in the formula used to average the two outer region $\mu_{t}$ profiles that result when both boundaries in a coordinate direction are solid surfaces. The default value is 2.0 .

### 3.1.11 Namelist IC

This namelist is used in subroutine INIT to read in parameters needed in setting up the initial conditions. The version of INIT built into PROTEUS specifies uniform flow with constant properties everywhere in the flow field. In general, however, the user will supply a version of INIT tailored to the problem being solved. This namelist, then, may be modified by the user to read in parameters different from those listed here.

P0 Initial static pressure $p_{0}$. The default value is 1.0 .

L0 Initial $x$-direction velocity $u_{0}$. The default value is 0.0 .
Initial $y$-direction velocity $v_{0}$. The default value is 0.0 .
W0 Initial swirl velocity $w_{0}$. The default value is 0.0 .

### 3.2 COORDINATE SYSTEM FILE

The type of computational coordinate sytem to be used is controlled by the input parameter NGl:OM in namelist GMIRY. Ior N( $\mathrm{SLOM}=10$, the coordinate system is read from a pre-stored file. This file may be created by any body-fitted coordinate system generator available to the user. The coordinates may be nonorthogonal.

The metric coefficients and Jacobian describing the nonorthogonal grid transformation are computed internally by PROTES. This calculation involves numerically computing first derivatives of the userspecified coordinates. Since PROTIDS solves the Navier-Stokes equations in fully conservative form, the metric coefficients themselves are factors in terms whose first and second derivatives are also computed numerically. In effect, then, third derivatives of the user-specified coordinates are used in the solution. Care should therefore be taken in ensuring that these coordinates are smooth. No coordinate smoothing is done by PROTl:S itself.

The Cartesian $(x, y)$ or cylindrical $(x, r)$ coordinates describing the computational coordinate system are read from an unformatted file as follows:

```
READ (NGRID) NG1,NG2
READ (NGRID) ((XC(J1,J2),Jl=1,NG1),J2=1,NG2),
$
    ((YC(J1,J2),J1=1,NG1),J2=1,NG2)
```

The parameters read from the file are defined as follows:

| NGl | Number of points in the $\xi$ direction. The maximum value allowed is the value of the PARAMETER NIP. (See Section 6.2.) |
| :---: | :---: |
| N(3) | Number of points in the $\eta$ direction. The maximum value allowed is the value of the PARAMETER N2P. (See Section 6.2.) |
| XC | Cartesian or cylindrical $x$-coordinate. |
| YC | Cartesian or cylindrical $y$ or $r$-coordinate. |

Note that the number of points NGI and NG2 used to specify the computational coordinate system need not be the same as the number of points N 1 and N 2 used in the computational mesh. The coordinates of the points in the computational mesh, which is the mesh used in the PROTEUS solution, will be found by interpolation among the points in the computational coordinate system.

TABLE 3-1. - NORMALIZING CONDITIONS

| Variable | Normalizing Value |
| :--- | :---: |
| Length | $L_{n}=L_{r}$ |
| Velocity | $u_{n}=u_{r}$ |
| Temperature | $T_{n}=T_{r}$ |
| Density | $\rho_{n}=\rho_{r}$ |
| Viscosity | $\mu_{n}=\mu_{r}$ |
| Thermal conductivity | $k_{n}=k_{r}$ |
| Pressure | $p_{n}=\rho_{r} u_{r}^{2}$ |
| Energy per unit volume | $e_{n}=p_{r} u_{r}^{2}$ |
| Gas constant | $R_{n}=u_{r}^{2} \mid T_{r}$ |
| Specific heat | $c_{p_{n}}=u_{r}^{2} \mid T_{r}$ |
| Enthalpy | $h_{n}=u_{r}^{2}$ |
| Time | $t_{n}=L_{r} u_{r}$ |

TABLE 3-2. - REFERENCE CONDITIONS

| Variable | Reference Value |
| :--- | :---: |
| Length | $L_{r}$ |
| Velocity | $u_{r}$ |
| Temperature | $T_{r}$ |
| Density | $\rho_{r}$ |
| Viscosity | $\mu_{r}$ |
| Thermal conductivity | $k_{r}$ |
| Pressure | $p_{r}=\rho_{r} \bar{R} T_{l} / g_{c}$ |
| Energy per unit volume | $e_{r}=\rho_{r} u_{r}^{2}$ |
| Enthalpy | $u_{r}^{2}$ |
| Specific heat | $u_{r}^{2} / T$, |
| Time | $L_{r} / u_{r}$ |

TABLE 3-3. - OLTPCT VARIABLES

| IVOUT | VARIABLE | DEFINITION |
| :---: | :---: | :---: |
| Velocities |  |  |
| 1 | $x$-velocity | $u$ |
| 2 | $y$ or $r$-velocity | $v$ |
| 3 | Swirl velocity | $w$ |
| 4 | Mach number | $M=\frac{\|V\|}{a}$ |
| 5 | Speed of sound | $a=\sqrt{\gamma R T}$ |
| 6 | Contravariant velocity normal to $\xi$ surface | $U=\xi_{t}+u \xi_{x}+\nu \xi_{y}$ |
| 7 | Contravariant velocity normal to $\eta$ surface | $V=\eta_{t}+u \eta_{x}+v \eta_{y}$ |
| 8 |  |  |
| 9 | Total velocity magnitude | $\|V\|=\left(u^{2}+v^{2}+w^{2}\right)^{1 / 2}$ |
| 10 | $x$-momentum |  |
| 11 | $y$ or $r$-momentum | $\rho v$ |
| 12 | Swirl momentum | $\rho w$ |
| 13 | $\xi$-velocity | $V_{\xi}=\left(\eta_{y} u-\eta_{x} v\right) /\left(\eta_{x}^{2}+\eta_{y}^{2}\right)^{1 / 2}$ |
| 14 | $\eta$-velocity | $V_{\eta}=\left(-\xi_{y} u+\xi_{x} v\right) /\left(\xi_{x}^{2}+\xi_{y}^{2}\right)^{1 / 2}$ |
| Densities |  |  |
| 20 | Static density | $\rho$ |
| 21 | Total density | $\rho_{T}=\rho\left(1+\frac{\gamma-1}{2} M^{2}\right)^{1 /(\gamma-1)}$ |


| IVOUT | VARIABLE | DEFINITION |
| :---: | :---: | :---: |
| Pressures |  |  |
| 30 <br> 31 <br> 32 <br> 33 <br> 34 <br> 35 | Static pressure <br> Total pressure <br> Static pressure coefficient <br> Total pressure coefficient <br> Pitot pressure <br> Dynamic pressure | $\begin{aligned} & p_{T}=p\left(1+\frac{\gamma-1}{2} M^{2}\right)^{\gamma /(\gamma-1)} \\ & c_{p}=\frac{\bar{p}-p_{r}}{\rho_{r} u_{r}^{2} / 2 g_{c}} \\ & c_{p_{T}}=\frac{\bar{p}_{T}-p_{T_{r}}}{\rho_{r} u_{r}^{2} / 2 g_{c}} \\ & p_{p}=p_{T} \quad \text { if } M \leq 1 \\ & p_{p}=p\left(\frac{\gamma+1}{2} M^{2}\right)^{\gamma /(\gamma-1)} \cdot \\ & \quad\left(\frac{2 \gamma}{\gamma+1} M^{2}-\frac{\gamma-1}{\gamma+1}\right)^{-1 /(\gamma-1)} \quad \text { if } M>1 \\ & \frac{1}{2} \rho\left(u^{2}+v^{2}+w^{2}\right) \frac{\rho_{r} u_{r}^{2}}{g_{c} p_{r}} \end{aligned}$ |
| Temperatures |  |  |
| $40$ $41$ | Static temperature <br> Total temperature | $\begin{aligned} & T \\ & T_{T}=T\left(1+\frac{y-1}{2} M^{2}\right) \end{aligned}$ |
| Energies |  |  |
| 50 <br> 51 <br> 52 <br> 53 | Total energy per unit volume <br> Total energy <br> Internal energy <br> Kinetic energy | $\begin{aligned} & E_{T} \\ & \frac{E_{T}}{\rho} \\ & e_{l}=c_{v} T \\ & e_{k}=\frac{1}{2}\left(u^{2}+v^{2}+w^{2}\right) \end{aligned}$ |
| Enthalpies |  |  |
| $\begin{aligned} & 60 \\ & 61 \end{aligned}$ | Static enthalpy <br> Total enthalpy | $\begin{aligned} & h=c_{p} T \\ & h_{T}=c_{P} T_{T} \end{aligned}$ |


| IVOUT | VARIABLE | DEFINITION |
| :---: | :---: | :---: |
| Vorticities |  |  |
| 70 | $x$-vorticity | $\Omega_{x}=\frac{\partial w}{\partial y} \quad\left(+\frac{w}{y} \text { if axisymmetric }\right)$ |
| 71 | $y$ or $r$-vorticity | $\Omega_{y}=-\frac{\partial w}{\partial x}$ |
| 72 | $z$ or $\theta$-vorticity | $\Omega_{z}=\frac{\partial v}{\partial x}-\frac{\partial u}{\partial y}$ |
| 73 | Total vorticity magnitude | $\|\Omega\|=\left(\Omega_{x}^{2}+\Omega_{y}^{2}+\Omega_{z}^{2}\right)^{1 / 2}$ |
| Entropies |  |  |
| 80 | Entropy | $s=\bar{c}_{v} \ln \left(\frac{\bar{p}}{p_{r}}\right)+\bar{c}_{p} \ln \left(\frac{\rho_{r}}{\bar{\rho}}\right)$ |
| Temperature-Dependent Parameters |  |  |
| 90 | Laminar viscosity coefficient | $\mu_{t}=\mu-\mu_{t}$ |
| 91 | Laminar second coefficient of viscosity | $\lambda_{i}=-\frac{2 \mu_{i}}{3}$ |
| 92 | Laminar thermal conductivity coefficient | $k_{l}=k-k_{t}$ |
| 93 | Specific heat at constant pressure |  |
| 94 | Specific heat at constant volume |  |
| 95 | Ratio of specific heats | $\gamma=\frac{c_{p}}{c_{v}}$ |
| Turbulence Parameters |  |  |
| 100 | Turbulent viscosity coefficient | $\mu_{t}$ |
| 101 | Turbulent second coefficient of viscosity | $\lambda_{t}=-\frac{2 \mu_{t}}{3}$ |
| 102 | Turbulent thermal conductivity coefficient | $k_{t}=\frac{\bar{c}_{p} \bar{\mu}_{t}}{P r_{t}} \frac{1}{k_{r}}$ |
| 103 | Effective viscosity coefficient |  |
| 104 | Effective second coefficient of viscosity | $\lambda$ |
| 105 | Effective thermal conductivity coefficient | $k$ |


| IVOLT | VARIABIE | DEFINTIGN |
| :---: | :---: | :---: |
| Coordinates |  |  |
| $\begin{aligned} & 200 \\ & 201 \end{aligned}$ | Cartesian $x$-coordinate Cartesian or cylindrical $y$ or $r$-coordinate | $\begin{aligned} & x \\ & y \text { or } r \end{aligned}$ |
| Metric Parameters |  |  |
| $\begin{aligned} & 210 \\ & 211 \\ & 212 \\ & 213 \\ & 214 \\ & 215 \\ & 216 \\ & 217 \\ & 218 \\ & 219 \\ & 220 \\ & 221 \\ & 222 \end{aligned}$ | Inverse of the grid transformation Jacobian <br> Metric coefficient <br> Metric coefficient <br> Metric coefficient <br> Metric coefficient <br> Metric coefficient <br> Metric coefficient | $\begin{aligned} & J^{-1} \\ & \xi_{t} \\ & \xi_{x} \\ & \xi_{y} \\ & \eta_{t} \\ & \eta_{x} \\ & \eta_{y} \end{aligned}$ |
|  |  | Times |
| $230$ | Time step size Iime | $\Delta \tau$ |

TABLE 3-4. - EQUATIONS SOLVED

| IIISTAG | ISWIRL | NEQ | Order of Equations | Order of <br> Dependent Variables |
| :---: | :---: | :---: | :--- | :--- |
| 0 | 0 | 4 | Continuity, $x$-momentum, <br> $y$ or $r$-momentum, energy <br> Continuity, $x$-momentum, <br> $y$ or $r$-momentum <br> Continuity, $x$-momentum, <br> $y$ or $r$-momentum, swirl momentum <br> Continuity, $x$-momentum, <br> $y$ or $r$-momentum, swirl momentum, <br> energy | $\rho, \rho u, \rho v, E_{T}$ |
| 1 | 0 | 3 | $\rho, \rho u, \rho v$ |  |
| 0 | 1 | 5 | $\rho u, \rho v, \rho w$ |  |

TABLE 3-5. - BOLNDARY TYPES

| $\begin{gathered} \text { KBC } \\ \text { VALUE } \end{gathered}$ | BOUNDARY TYPE | $\begin{gathered} \text { JBC } \\ \text { VALLES SET } \end{gathered}$ | EQUATIONS |
| :---: | :---: | :---: | :---: |
| $\pm 1$ | No-slip adiabatic wall | $\begin{aligned} & 11,21,31, \pm 43 \\ & \pm 53 \end{aligned}$ | $u=v=w=0, \partial p / \partial n=\partial T / \partial n=0$ |
| $\pm 2$ | No-slip wall, specified temperature | $\frac{11,21,31, \pm 43}{50}$ | $u=v=w=0, \partial p / \partial n=0, \Delta T=0$ |
| $\pm 3$ | Inviscid wall | $\frac{ \pm 33}{71,79} \pm 43, \pm 53$ | $\begin{aligned} & \partial w / \partial n=\partial p / \partial n=\partial T / \partial n=0 \\ & V_{n}=0, \partial^{2} V / \partial \phi^{2}=0 \end{aligned}$ |
| 10 | Subsonic inflow, linear extrapolation | $14,24,34,46,56$ | $\begin{aligned} & \partial^{2} u / \partial \phi^{2}=\partial^{2} v / \partial \phi^{2}=\partial^{2} w / \partial \phi^{2}=0 \\ & \Delta p_{T}=\Delta T_{T}=0 \end{aligned}$ |
| $\pm 11$ | Subsonic inflow, zero gradient | $\frac{ \pm}{46,56} \pm 22, \pm 32$ | $\begin{aligned} & \partial u / \partial \phi=\partial v / \partial \phi=\partial w / \partial \phi=0 \\ & \Delta p_{T}=\Delta T_{T}=0 \end{aligned}$ |
| 20 | Subsonic outflow, linear extrapolation | 14, 24, 34, 40, 54 | $\begin{aligned} & \partial^{2} u / \partial \phi^{2}=\partial^{2} v / \partial \phi^{2}=\partial^{2} w / \partial \phi^{2}=0 \\ & \Delta p=0, \partial^{2} T / \partial \phi^{2}=0 \end{aligned}$ |
| $\pm 21$ | Subsonic outflow, zero gradient | $\frac{ \pm 12}{40, \pm 52} \pm \pm 32$ | $\begin{aligned} & \partial u / \partial \phi=\partial v / \partial \phi=\partial w / \partial \phi=0, \\ & \Delta p=0, \partial T / \partial \phi=0 \end{aligned}$ |
| 30 | Supersonic inflow | 10, 20, 30, 40, 50 | $\Delta u=\Delta v=\Delta w=\Delta p=\Delta T=0$ |
| 40 | Supersonic outflow, linear extrapolation | $14,24,34,44,54$ | $\begin{aligned} & \partial^{2} u / \partial \phi^{2}=\partial^{2} v / \partial \phi^{2}=\partial^{2} w / \partial \phi^{2}=0, \\ & \partial^{2} p / \partial \phi^{2}=\partial^{2} T / \partial \phi^{2}=0 \end{aligned}$ |
| $\pm 41$ | Supersonic outflow, zero gradient | $\begin{aligned} & \pm 12, \pm 22, \pm 32 \\ & \pm 42, \pm 52 \end{aligned}$ | $\begin{aligned} & \partial u / \partial \phi=\partial v / \partial \phi=\partial w / \partial \phi=0, \\ & \partial p / \partial \phi=\partial T / \partial \phi=0 \end{aligned}$ |
| $\pm 50$ | Symmetry | $\frac{ \pm 33}{71}, \pm 43, \pm 53$ | $\begin{aligned} & \partial w / \partial n=\partial p / \partial n=\partial T / \partial n=0 \\ & V_{n}=0, \partial V / \partial n=0 \end{aligned}$ |
| 60 | Spatially periodic |  | $\mathbf{Q}_{1}=\mathbf{Q}_{N_{1}}$ or $\mathbf{Q}_{1}=\mathbf{Q}_{N_{2}}$ |

[^8]TABLE 3-6. - BOUNDARY CONDITION TYPES

| $\begin{gathered} \mathrm{JBC} \text { OR IBC } \\ \text { VALLE }^{\mathrm{a}} \end{gathered}$ | EQUATION | DESCRIPTION |
| :---: | :---: | :---: |
| Conservation Variable Boundary Conditions |  |  |
| $\begin{array}{r} 0 \\ 1 \\ \pm 2 \\ \pm 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array}$ | $\begin{gathered} \Delta \mathbf{Q}=0 \\ \mathbf{Q}=f \\ \partial \mathbf{Q} / \partial \phi=f \\ \partial \mathbf{Q} / \partial n=f \\ \partial^{2} \mathbf{Q} / \partial \phi^{2}=0 \end{gathered}$ | No change from initial conditions. Specified conservation variable. Specified coordinate direction gradient. Specified normal direction gradient. Linear extrapolation. |
| $x$-Velocity Boundary Conditions |  |  |
| $\begin{array}{r} 10 \\ 11 \\ \pm 12 \\ \pm 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \end{array}$ | $\begin{gathered} \Delta u=0 \\ u=f \\ \partial u \mid \partial \phi=f \\ \partial u / \partial n=f \\ \partial^{2} u \mid \partial \phi^{2}=0 \end{gathered}$ | No change from initial conditions. Specified $x$-velocity. <br> Specified coordinate direction gradient. Specified normal direction gradient. Linear extrapolation. |
| $y$ or $r$-Velocity Boundary Conditions |  |  |
| $\begin{array}{r} 20 \\ 21 \\ +22 \\ \pm 23 \\ 24 \\ 25 \\ 26 \\ 27 \\ 28 \\ 29 \end{array}$ | $\begin{gathered} \Delta v=0 \\ v=f \\ \partial v / \partial \phi=f \\ \partial v / \partial n=f \\ \partial^{2} v / \partial \phi^{2}=0 \end{gathered}$ $\tan ^{-1}(v / u)=f$ | No change from initial conditions. Specified $y$ or $r$-velocity. Specified coordinate direction gradient. Specified normal direction gradient. Linear extrapolation. <br> Specified flow angle in degrees. |
| Swirl Velocity Boundary Conditions |  |  |
| 30 31 +32 $\pm 33$ 34 35 36 37 38 39 | $\begin{gathered} \Delta w=0 \\ w=f \\ \partial w / \partial \phi=f \\ \partial w / \partial n=f \\ \partial^{2} w / \partial \phi^{2}=0 \\ \\ \\ \\ \tan ^{-1}(w / u)=f \end{gathered}$ | No change from initial conditions. Specified swirl velocity. <br> Specified coordinate direction gradient. <br> Specified normal direction gradient. <br> Linear extrapolation. <br> Specified flow angle in degrees. |


| $\begin{gathered} \text { JBC OR IBC } \\ \text { VALCE }^{2} \end{gathered}$ | EQUATION | DESCRIPTION |
| :---: | :---: | :---: |
| Pressure Boundary Conditions |  |  |
| $\begin{array}{r} 40 \\ 41 \\ \pm 42 \\ \pm 43 \\ 44 \\ 45 \\ 46 \\ 47 \\ 48 \\ 49 \\ \hline \end{array}$ | $\begin{gathered} \Delta p=0 \\ p=f \\ \partial p \mid \partial \phi=f \\ \partial p / \partial n=f \\ \partial^{2} p / \partial \phi^{2}=0 \\ \Delta p_{T}=0 \\ p_{T}=f \end{gathered}$ | No change from initial conditions. Specified static pressure. Specified coordinate direction gradient. Specified normal direction gradient. Lincar extrapolation. <br> No change from initial conditions. Specified total pressure. |
| Temperature Boundary Conditions |  |  |
| $\begin{array}{r} 50 \\ 51 \\ +52 \\ \pm 53 \\ 54 \\ 55 \\ 56 \\ 57 \\ 58 \\ 59 \end{array}$ | $\begin{gathered} \Delta T=0 \\ T=f \\ \partial T / \partial \phi=f \\ \partial T / \partial n=f \\ \partial^{2} T / \partial \phi^{2}=0 \\ \Delta T_{T}=0 \\ T_{T}=f \end{gathered}$ | No change from initial conditions. Specified static temperature. Specified coordinate direction gradient. Specified normal direction gradient. Linear extrapolation. <br> No change from initial conditions. Specified total temperature. |
| Density Boundary Conditions |  |  |
| $\begin{array}{r} 60 \\ 61 \\ \pm 62 \\ \pm 63 \\ 64 \\ 65 \\ 66 \\ 67 \\ 68 \\ 69 \\ \hline \end{array}$ | $\begin{gathered} \Delta \rho=0 \\ \rho=f \\ \partial \rho / \partial \phi=f \\ \partial / \partial n=f \\ \partial^{2} \rho / \partial \phi^{2}=0 \end{gathered}$ | No change from initial conditions. Specified static density. Specified coordinate direction gradient. Specified normal direction gradient. Linear extrapolation. |
| Normal and Tangential Velocity Boundary Conditions |  |  |
| $\begin{array}{r} 70 \\ 71 \\ \pm 72 \\ \pm 73 \\ 74 \\ 75 \\ 76 \\ \pm 77 \\ \pm 78 \\ 79 \\ \hline \end{array}$ | $\begin{gathered} V_{n}=f \\ \partial V_{n} \mid \partial \phi=f \\ \partial V^{n} \mid \partial n=f \\ \partial^{2} V_{n}^{n} \mid \partial \phi^{2}=0 \\ V_{1}=f \\ \partial V_{1} \mid \partial \phi=f \\ \partial V^{\prime} \mid \partial n=f \\ \partial^{2} V_{\\|} \mid \partial \phi^{2}=0 \end{gathered}$ | Specified normal velocity. <br> Specified coordinate direction gradient. <br> Specified normal dircction gradient. <br> Linear extrapolation. <br> Specified tangential velocity. <br> Specified coordinate direction gradient. <br> Specified normal direction gradient. <br> Linear extrapolation. |


| JBC OR IBC <br> VAILE | EQUATION | DESCRIPTION |
| :---: | :---: | :--- |
| User-Supplied Boundary Conditions |  |  |
| 90 | $\Delta F=0$ | Vo change from initial conditions. <br> 91 |
| $\pm 92$ | $F=f$ | Specified function. |
| $\pm 93$ | $\partial F / \partial \phi=f$ | Specified coordinate direction gradient. |
| 94 | $\partial F / \partial n=f$ | Specified normal direction gradient. |
| 95 | $\partial^{2} F / \partial \phi^{2}=0$ | Linear extrapolation. |
| 96 |  |  |
| 97 |  |  |
| 98 |  |  |
| 99 |  |  |

[^9]
### 4.0 OUTPCT DESCRIPTION

Several output files may be created during a PROTELS run. The standard output is a formatted file written to Fortran unit NOUT that is intended for printing. Additional unformatted files may be written for use as input by various post-processing programs. Unformatted restart files may also be written for use as input for a subsequent PROTEUS run.

### 4.1 STANDARD OLTPCT

The standard PROTELS output is a formatted file written to Fortran unit NOUT, and is intended for printing. Actual examples of typical standard output files are presented in Section 9.0. Lnless specified otherwise, all of the output parameters in the standard output are nondimensional, with the appropriate reference condition from Table 3-2 as the nondimensionalizing factor.

### 4.1.1 Title Page and Namelists

The standard PROTEUS output begins with a title page, ${ }^{13}$ which identifies the version of PROTEUS being run and lists the user-specified title for the run. This is followed by a printout of the contents of the input namelists RSTRT, IO, GMTRY, FLOW, BC, NUM, TIME, and TURB. Note that, for variables not specified by the user in the input namelists, the values in this printout will be the default values.

### 4.1.2 Boundary Conditions

The next page is a printout of the boundary conditions being used. The boundary condition parameters JBC and GBC are printed in a box-like manner, with the values for the $\xi=0$ and $\xi=1$ surfaces on the left and right, and the values for the $\eta=0$ and $\eta=1$ surfaces on the bottom and top. If time-dependent boundary conditions are being used, this is followed by a listing of the input tables of GTBC vs. NTBCA.

### 4.1.3 Normalizing and Reference Conditions

The dimensional values for the normalizing and reference conditions are printed on the next page, with the appropriate units as set by the input parameter IUNITS. The normalizing conditions are the parameters used to nondimensionalize the governing equations. The reference conditions are used during input and output for nondimensionalization of various parameters and for specifying various flow conditions. The distinction between normalizing and reference conditions is described in greater detail in Section 3.1.1. They are listed in Tables 3-1 and 3-2.

After the printout of the normalizing and reference parameters comes anything written to unit NOUT by the user-supplied subroutine INIT. For the default version of INIT supplied with PROTELS, this consists only of the contents of namelist IC.

### 4.1.4 Computed Flow Field

The bulk of the standard PROTELS output consists of printout of the computed flow field. The input array IVOUT determines which variables are printed, as described in Section 3.1.4. The variables currently available for printing are listed and defined in Table 3-3. The printout for each variable at a given time level will begin on a separate page. The header for each variable will include the time level $n$, and, for global time steps (IDTAU $=1-4,7$ ), the time $t$ and time increment $\Delta t$ in seconds. In the flow field printout, each

[^10]column corresponds to a $\xi$ location, and each row to an $\eta$ location. The columns and rows are numbered with the ${ }_{5}^{5}$ and $\eta$ indices.

Flow field results are printed at time levels and grid points specified by the user through parameters in namelist IO. Since this printout can be very lengthy, the user is encouraged to minimize the amount of prites doutput by making judicious use of these parameters. Usually, the computed results can be examined mos afficiently using post-processing graphics routines like COXTOUR or PLOT3D. (See Section 4.2).

Whe the feld frintout if the run ends normally a message is printed indicating whether or not the ator: anged

### 4.1.5 Comergence listory

In cvaluating the results of a steady PROTEUS calculation, it's important to consider the level of conwreme. This may be done by examining one of the forms of the residual for each equation. The residual abrily the rumber resulting from evaluating the steady form of the equation at a specific grid point and time (or iteration) level. Ideally, the residuals would all approach zero at convergence. In practice, however, Win "rat" problems they often drop to a certain level and then level off. Continuing the calculation beyond this point will not improve the results.

A decrease in the $L_{2}$ norm of the residual of three orders of magnitude is sometimes considered sufficient. Convergence, however, is in the eye of the beholder. The amount of decrease in the residual necessary for convergence will vary from problem to problem. For some problems, it may even be more appropriate to measure convergence by some flow-related parameter, such as the lift coefficient for an airfoil. Determining when a solution is sufficiently converged is, in some respects, a skill best açuired through experience.

At the end of a PROTELS calculation, if first-order time differencing and steady boundary conditions were used, a summary of the convergence history is printed for each governing equation. ${ }^{14}$ The parameters in this printout are defined as follows:

IEVEL $\quad$ Time level $n$.

RESL 2

RISAVG

Maximum change in absolute value of the dependent vanables from time level
$n-1$ to $n n^{15}$

$$
\Delta \mathbf{Q}_{\max }=\max \left|\Delta \mathbf{Q}_{i, j}^{n-1}\right|
$$

Maximum change in absolute value of the dependent variables, averaged over the last NITAVG time steps. ${ }^{15}$

$$
\Delta \mathbf{Q}_{\text {avg }}=\frac{1}{\text { NITAVG }} \sum_{m=n-\text { NITAVG }}^{n} \Delta \mathbf{Q}_{\max }^{m-1}
$$

The $L_{2}$ norm of the residual at time level $n$.

$$
\mathrm{R}_{L_{2}}=\left(\sum\left(\mathrm{R}_{i, j}^{n}\right)^{2}\right)^{1 / 2}
$$

The average absolute value of the residual at time level $n, \mathrm{R}_{\text {arg }}$.

[^11]RESMAX The maximum absolute value of the residual at time level $n, \mathrm{R}_{\text {max }}$.

$$
\text { LRMAX The grid indices }(i, j) \text { corresponding to the location of } \mathrm{R}_{m a x} \text {. }
$$

In computing the residuals, the summations, maximums, and averages are over all interior grid points, plus points on spatially periodic boundaries.

To avoid undesirably long tables, the convergence parameters are printed at an interval that limits the printout to NHMAX time levels. NHMAX can be specified by the user in namelist IO. However, the residuals are always printed at the first two time levels. This is done because the residuals at time level 1 (the initial condition level) may not be truly representative of the degree of convergence. For instance, if the initial conditions are zero velocity and constant pressure and temperature at every interior point, the computed residuals will be exactly zero. When the time marching procedure begins, however, the flow field will start developing in response to the boundary conditions, and the residuals will reach a maximum in the first few time steps. Note that, in the printout, CHGMAX will be zero until time level $n=$ ICHECK. CHGAVG will only be computed when ICTEST $=2$, and will be zero until time level $n=$ NITAVG.

As noted in Section 9.1 of Volume 1, adding artificial viscosity changes the original governing partial differential equations. For cases run with artificial viscosity, therefore, the residuals are printed both with and without the artificial viscosity terms included. This may provide some estimate of the overall error in the solution introduced by the artificial viscosity. Convergence is determined by the residuals with the artificial viscosity terms included.

### 4.1.6 Additional Output

In addition to the output discussed above, various types of additional printout can be generated by the IDEBUG options, as discussed in Section 3.1.4. Various diagnostics may also appear in the standard output file. These are discussed in greater detail in Section 7.0.

### 4.2 PLOT FILES

The amount of flow field data generated by a Navier-Stokes code is normally much too large to efficiently comprehend by examining printed output. The computed results are therefore generally examined graphically using various post-processing plotting routines. These plotting routines require as input a file or files, generally called plot files, that are written by the flow solver and contain the coordinates and computed flow field data.

Various types of unformatted plot files may be written by PROTEUS, as controlled by the input parameter IPI OT discussed in Section 3.1.4. These files are designed for use by either the CONTOUR or PLOT3D plotting programs. ${ }^{16}$

CONTOLR is a three-dimensional plotting program developed at NASA Lewis using internal Lewisdeveloped graphics routines. It currently can be used only at NASA Lewis on the Amdahl 5860 computer using the VM operating system, or from the Scientific VAX Cluster using the VMS operating system. Originally designed for use with three-dimensional Parabolized Navier-Stokes ( $\mathrm{P} N \mathrm{~S}$ ) codes, it can be used to generate various types of contour and velocity vector plots in computational planes.

PLOT3D (Walatka and Buning, 1990) is a sophisticated three-dimensional plotting program specifically designed for displaying results of computational fluid dynamics analyses. It is widely used in government, industry, and universities for interactive visualization of complex flow field data generated by CFD analyses. The computational grid is stored in one file, called an XYZ file, and the computed flow field is stored in another file, called a Q file. There are several options within PLOT3D concerning the format of these files. At $\operatorname{NASA}$ Lewis, PLOT3D is available on the Silicon Graphics IRIS workstations and on the Scientific VAX Cluster.

[^12]It should be noted that, in Fortran, unformatted files contain information at the beginning and or end of each record about the record length, file type, etc. The way this information is stored with the record varies from computer to computer. Unformatted files are therefore not generally transportable. If the plot files written by PROTELS are to be used on some other computer (e.g., a graphics workstation), a separate conversion program will normally be required.

### 4.2.1 CONTOLR Plot File (IPLOT $=1$ )

With the IPLOT $=1$ option in PROTELS, a plot file is generated for use by CONTOUR using the following Fortran statements:

```
        WRITE (NPLOT) TITLE
        WRITE (NPLOT) MACHR,RER,LR,UR,PR,TR,RHOR,RG,GAMR
        WRITE (NPLOT) LEVEL,N1,N2,ISYM,SYSTEM
        DO 10 II = 1,N1
        WRITE (NPLOT) ((F(IVAR,I1,I2),IVAR=1,14),I2=1,N2)
1 0
CONTINUE
```

All of the above WRITE statements are executed for each time level written into the file. The plot file thus consists of multiple sets of data, each containing the computed results at a single time level. Note that with this option, the value of the time $\tau$ is not written into the file. ${ }^{17}$

Unless specified otherwise, all of the parameters written into the CONTOUR plot file are nondimensional, with the appropriate reference condition as the nondimensionalizing factor. The parameters are defined as follows:

| TITLE | A descriptive title for the problem. |
| :---: | :---: |
| MACIIR | Reference Mach number, $M_{r}=u_{r} / \sqrt{\gamma_{r} \bar{R} T_{r}}$. This is a type REAL variable. |
| RER | Reference Reynolds number, $R e_{r}=\rho_{r} u_{r} L_{r} / \mu_{r}$. |
| LR | Reference length $L_{r}$, in feet (meters). This is a type REAL variable. |
| UR | Reference velocity $u_{\text {, }}$ in $\mathrm{ft} / \mathrm{sec}(\mathrm{m} / \mathrm{sec})$. |
| PR | Reference static pressure $p_{r}=\rho_{r} \bar{R} T_{r} / g_{c}$ in $\mathrm{lb}_{\mathrm{f}} / \mathrm{ft}^{2}\left(\mathrm{~N} / \mathrm{m}^{2}\right)$. |
| TR | Reference temperature $T_{\text {, }}$ in ${ }^{\circ} \mathrm{R}(\mathrm{K})$. |
| RHOR | Reference density $\rho$, in $1 \mathrm{~b}_{\mathrm{m}} / \mathrm{ft}^{3}\left(\mathrm{~kg} / \mathrm{m}^{3}\right)$. |
| RG | Gas constant $\bar{R}$ in $\mathrm{ft}^{2} / \mathrm{sec}^{2}{ }^{\circ} \mathrm{R}\left(\mathrm{m}^{2} / \mathrm{sec}^{2}-\mathrm{K}\right)$. |
| GAMR | Reference ratio of specific heats, $\gamma_{r}=c_{p_{r}} / c_{v_{r}}$. |
| IIVVEL | Time level $n$. |
| N1 | Number of grid points $N_{1}$ in the $\xi$ direction. |
| N2 | Number of grid points $N_{2}$ in the $\eta$ direction. |
| ISYM | A symmetry parameter used in CONTOUR, set equal to 1 . |
| SYSTE:M | A coordinate system parameter used in CONTOUR, set equal to 0. |
| $F(1,1)$ | Set equal to 0. |
| $\left[\left(2,{ }^{\prime}\right.\right.$ | Cartesian $x$ coordinate. |
| $\mathrm{F}\left(3_{1}\right)$ | Cartesian $y$ coordinate or cylindrical $r$ coordinate. |
| $\mathrm{F}(4$, ) | Set equal to 0. |

[^13]| $\mathrm{F}(5,)$, | Velocity in the $\xi$ direction, $V_{\xi}$. |
| :--- | :--- |
| $\mathrm{F}(6,)$, | Velocity in the $\eta$ direction, $V_{\eta}$. |
| $\mathrm{F}(7,)$, | Static pressure $\bar{p} / p_{r}$. |
| $\mathrm{F}(8,)$, | Static temperature $T$. |
| $\mathrm{F}(9,)$, | Mach number $M=\sqrt{u^{2}+v^{2}+w^{2}} / \sqrt{y R T}$ |
| $\mathrm{~F}(10,)$, | $x$-velocity $u$. |
| $\mathrm{F}(11,)$, | $y$ or $r$-velocity $v$. |
| $\mathrm{F}(12,)$, | Swirl velocity $w$. |
| $\mathrm{F}(13,)$, | Static density $\rho$. |
| $\mathrm{F}(14,)$, | Total energy per unit volume $E_{T}$. |

### 4.2.2 CONTOLR Plot File (IPLOT $=-1$ )

The CONTOUR plot file generated using the IPLOT $=-1$ option is essentially the same as the one discussed in the previous section. There are just two differences. First, the first two records, containing the title and the reference conditions, are written only once, at the beginning of the file, and not at each time level. And second, the time $\tau_{i, j}$ is written in the $\mathrm{F}(1$,$) position. (The subscripts i$ and $j$ represent grid point indices in the $\xi$ and $\eta$ directions.)

As noted in the previous section, CONTOUR was originally designed for use with three-dimensional Parabolized Navier-Stokes codes. With PNS codes, the streamwise marching coordinate was written into the $F(1$,$) position, and contours or velocity vectors could be plotted at different streamwise stations.$

With the IPLOT $=-1$ option in PROTEUS, the resulting CONTOUR plot file is analogous to the one produced for PNS codes, but with the streamwise marching coordinate replaced by the time. Contours and velocity vectors can thus be easily plotted at different time levels.

### 4.2.3 PLOT3D/WHOLE Plot Files (IPLOT = 2)

With the IPLOT $=2$ option in PROTEUS, the $X Y Z$ and $Q$ files are written in PLOT3D/WHOLE format. With this option, the XYZ file is written using the following Fortran statements:

```
WRITE (NPLOTX) N1,N2,N3
    WRITE (NPLOTX) ((X(I1,I2),I1=1,N1),I2=1,N2),
$
$ ((ZDUM, ,I I=1,N1),I2=1,N2)
```

The Q file is written using the following Fortran statements:
WRITE (NPLOT) N1,N2,N3
WRITE (NPLOT) MACHR,AOA,RER,TAU(1,1)
WRITE (NPLOT) (( (QPLOT(I1,I2,IVAR), I $1=1, N 1), I 2=1, N 2), I V A R=1,5)$
The above WRITE statements for the $Q$ file are executed for each time level written into the file. The $Q$ file thus consists of multiple sets of data, each containing the computed results at a single time level. The XYZ file is written only once. ${ }^{18}$

The parameters written into the file are defined as follows:
$N 1 \quad$ Number of grid points $N_{1}$ in the $\xi$ direction.
N 2 Number of grid points $N_{2}$ in the $\eta$ direction.

[^14]| $N 3$ | In PLOT3D, the number of grid points in the $\zeta$ direction. Set equal to 1 for 2-D PROTELS. |
| :---: | :---: |
| X | Cartesian $x$ coordinate. |
| Y | Cartesian $y$ coordinate or cylindrical $r$ coordinate. |
| 7DCM | In PLOT3D, the Cartesian $z$ coordinate. Set equal to 0. in 2-D PROTELS. |
| MACHR | Reference Mach number, $M_{r}=u_{r} / \sqrt{\gamma} \bar{R} T$. This is a type REAL variable. |
| AOA | In PLOT3D, the angle of attack. Set equal to 0 . in PROTELS. |
| RER | Reference Reynolds number, $R e_{r}=\rho_{r} u_{r} L_{r} / \mu_{r}$. |
| TAL( 1,1 ) | The time $\tau_{1,1}{ }^{19}$ |
| QPLOT $(, 1)$ | Static density $\rho$. |
| QPLOT (, 2) | $x$-momentum $\rho u M_{r}$. |
| QPLOT $(, 3)$ | $y$ or $r$-momentum $\rho v . M_{r}$. |
| QPLOT $(,, 4)$ | Swirl momentum $\rho w M_{r}$. |
| QPLOT $(,, 5)$ | Total energy per unit volume $E_{T} M_{r}^{2}$. |

All of the parameters written into the XYZ and Q files are nondimensional. However, PLOT3D assumes that velocity is nondimensionalized by the reference speed of sound $a_{r}=\left(\gamma, \bar{R} T_{r}\right)^{1 / 2}$, and that energy is nondimensionalized by $\rho_{r} a_{r}^{2}$. In PROTEUS these variables are nondimensionalized by $u_{r}$ and $\rho_{r} u_{r}^{2}$. That is why $M_{r}$ appears in the definitions of QPIOT $(,, 2)$ through QPLOT $(,, 5)$.

### 4.2.4 PLOT3D/PLANES Plot Files (IPLOT = 3)

Since PROTEUS 2-D is two-dimensional, the IPLOT $=3$ option creates XYZ and Q files identical to those created using the IPLOT $=2$ option described in the previous section.

### 4.2.5 PLOT3D/PLANES Plot Files (IPIOT $=-3$ )

This option is similar to the IPI OT $=3$ option, except that the time $\tau$ is written into the $z$ slot in the XY 7 . file. The $\mathrm{XY} Z$ file is written using the following Fortran statements:

```
    WRITE (NPLOTX) N1,N2,N3
    DO 10 I3 = 1,N3
    WRITE (NPLOTX) (( X(I1,I2),Il=1,N1),I2=1,N2),
$
(( Y(I1,I2),Il=1,N1),I2=1,N2),
1 0
CONTINUE
```

The $Q$ file is written using:

```
WRITE (NPLOT) N1,N2,N3
WRITE (NPLOT) MACHR,AOA,RER,TDUM
DO 20 I3 = 1,N3
WRITE (NPLOT) (((QPLOT(I1,I2,IVAR),I1=1,N1),I2=1,N2),IVAR=1,5)
CONTINUE
```

The parameters written into the file that have not yet been defined are:
N3
Number of time levels written into the XYZ and Q files. ${ }^{20}$

[^15]\[

$$
\begin{array}{ll}
\text { TAU } & \text { The time } \tau_{i, \theta} \\
\text { TDUM } & \text { Set equal to } 0 .
\end{array}
$$
\]

Even though a time-dependent version of PIOT3D is not yet available, the IPLOT $=-3$ option allows plots to be generated at different time levels by plotting at different PLOT3D "z" stations.

### 4.2.6 PIOT2D Plot Files (IPLOT = 4)

This option generates $\mathrm{XY} /$ and Q files in PLOI3D's 2D format. The XYZ file is written using the following Fortran statements:

```
WRITE (NPLOTX) N1,N2
    WRITE (NPLOTX) ((X(11,I2),I I=1,N1),I2=1,N2),
$
```

The Q file is written using:

```
WRITE (NPLOT) N1,N2
WRITE (NPLOT) MACHR,AOA,RER,TAU(1,1)
WRITE (NPLOT) (((QPLDT(I1,I2,IVAR),I1=1,N1),I2=1,N2),IVAR=1,4)
```

As in the IPIOT $=2$ option, the above WRITE statements for the $Q$ file are executed for each time level written into the file. The $Q$ file thus consists of multiple sets of data, each containing the computed results at a single time level. The $\mathrm{XY} /$ file is written only once. ${ }^{18}$

All of the parameters written into the files are the same as previously defined, except:
QPIOT(.,4) Total energy per unit volume $E_{T} M_{r}^{2}$.
Note that with this option, the swirl momentum $\rho w$ is not written into the $Q$ file.

### 4.3 CONVERGENCE HISTORY FILE

In Section 4.1.5, the convergence history printout is described. The information in this printout is read from an unformatted convergence history file that is updated whenever convergence is checked during a PROTEUS calculation. Plotting routines are also available at NASA Lewis to plot any of the convergence parameters as a function of time level.

The file is written in subroutine RESID using the following Fortran statements. At the first time step of the run,

```
    WRITE (NHIST) N1,N2,NEQ,IDTAU,ICTEST,NITAVG,ISWIRL,IHSTAG,
$
    IAV2E,IAV4E,UR,LR,(EPS(IEQ),IEQ=1,NEQ)
```

Then, at each time level that convergence is being checked,

```
WRITE (NHIST) IT,TAU(1,1),ICHECK
    WRITE (NHIST) (CHGMAX(IEQ,l),CHGAVG(IEQ),RESL2(IEQ,1),
$
RESAVG(IEQ,1),RESMAX(IEQ,1),
    LRMAX(1,IEQ,1),LRMAX(2,IEQ,1),IEQ=1,NEQ)
```

Finally, again at each time level that convergence is being checked, but only for cases run with explicit artificial viscosity,

WRITE (NHIST) (CHGMAX(IEQ,1), CHGAVG(IEQ),RESLZ(IEQ,2),
\$ RESAVG(IEQ,2),RESMAX(IEQ,2),
$\$ \quad \operatorname{LRMAX}(1, I E Q, 2), L R M A X(2, I E Q, 2), I E Q=1, N E Q)$
PROTELS run. Therefore, the results are actually written to a scratch file. At the end of the calculation, when X 3 is known, the scratch file is read and the XYZ and Q files are writuen.

The parameters written into the file are defined as follows:

| N | Number of grid points $N_{1}$ in the $\xi$ direction. |
| :---: | :---: |
| N 2 | Number of grid points $N_{2}$ in the $\eta$ direction. |
| NEQ | The number of coupled governing equations $V_{e q}$ being solved. |
| IDTAU | Flag for time step selection method. |
| ICTEST | Flag for type of convergence test. |
| NITAVG | Number of time steps used in the moving average convergence test. |
| ISWIRI. | Flag for swirl in axisymmetric flow. |
| IISTAG | Flag for constant stagnation enthalpy option. |
| LAV2E | Flag for second-order explicit artificial viscosity. |
| IAV4E | Flag for fourth-order explicit artificial viscosity. |
| UR | Reference velocity $u_{r}$. |
| LR | Reference length $L_{r}$. This is a type REAL variable. |
| EPS(ILQ) | Value $\varepsilon$ used to determine convergence. |
| IT | Current time level $n$. |
| TAL(1,1) | Current value of the time marching parameter $\tau_{1,1}$ at $\xi=\eta=0$. |
| ICIIECK | Results are checked for convergence every ICIIECK'th time level. |
| CHGMAX (IEQ,1) | Absolute value of the maximum change in the dependent variables from time level $n-1$ to $n$. |
| CHGAVG(IEQ) | Average of the absolute value of the maximum change in the dependent variables for the last NITAVG time steps. |
| RESL 2(IEQ,IAVR) | The $L_{2}$ norm of the residual at time level $n$. |
| RESAVG(IEQ,IAVR) | The average absolute value of the residual at time level $n$. |
| RESMAX (IEQ,IAVR) | The maximum absolute value of the residual at time level $n$ |
| LRMAX(IDIR,IEQ,IAVR) | The grid indices ( $i, j$ ) corresponding to the location of $\mathrm{R}_{\max }$. |

In the above definitions, the subscript IEQ $=1$ to $N_{e q}$, corresponding to the $N_{e q}$ governing equations, $\operatorname{IAVR}=1$ or 2 , corresponding to residuals computed without and with the artificial viscosity terms, and $\operatorname{IDIR}=1$ or 2 , corresponding to the $\xi$ and $\eta$ directions.

### 4.4 RESTART FILES

It's often necessary or desirable to run a given case in a series of steps, stopping and restarting between each one. This may be done because of limitations in computer resources, or to change an input parameter. This capability is provided in PROTECS through a restart option. With this option, the computational mesh and the computed flow field are written as unformatted output files at the end of one run, saved on a magnetic disk or tape, and read in as input files at the beginning of the next run. This process is controlled by the input parameters in namelist RSTRT. (See Section 3.1.3).

The restart files are written and read in subroutine REST. The computational mesh is written using the following Fortran statements:

```
N3 = 1
WRITE (NRXOUT) N1,N2,N3
WRITE (NRXOUT) (( X(I1,I2),Il=1,N1),I2=1,N2),
$ $ ((YY(I1,I2),Il=1,N1),I2=1,N2),
```

The computed flow field is written using:

```
N3 = l
WRITE (NRQOUT) N1,N2,N3
WRITE (NRQOUT) MACHR,AOA,RER,TLEVEL
WRITE (NRQOUT) (((Q (I1,I2,IVAR),I I=1,N1),I2=1,N2),IVAR=1,5)
WRITE (NRQOUT) (((QL(I1,I2,IVAR),I1=1,N1),I2=1,N2),IVAR=1,5)
```

The parameters written into these files are defined as follows:

| N1 | Number of grid points $N_{1}$ in the $\xi$ direction. |
| :--- | :--- |
| N 2 | Number of grid points $N_{2}$ in the $\eta$ direction. |
| X | Cartesian $x$ coordinate. |
| Y | Cartesian $y$ coordinate. |
| TAU | Computational time $\tau$. |
| MACHR | Reference Mach number, $M_{r}=u_{r} / \sqrt{\gamma_{r} \bar{R} T_{r}}$. This is a type REAL variable. |
| AOA | Set equal to 0. |
| RER | Reference Reynolds number, $R e_{r}=\rho_{r} u_{r} L_{r} / \mu_{r}$. |
| TLEVEL | The current time level $n$. |
| $\mathrm{Q}(,, 1)$ | Static density $\rho$ at time level $n$. |
| $\mathrm{Q}(,, 2)$ | $x$-momentum $\rho u M_{r}$ at time level $n$. |
| $\mathrm{Q}(,, 3)$ | $y$ or $r$-momentum $\rho v M_{r}$ at time level $n$. |
| $\mathrm{Q}(,, 4)$ | Swirl momentum $\rho w M_{r}$ at time level $n$. |
| $\mathrm{Q}(,, 5)$ | Total energy per unit volume $E_{r} M_{r}^{2}$ at time level $n$. |
| $\mathrm{QL}(,, 1-5)$ | Same as Q(,,1-5), except at time level $n-1$. |

Note that, except for the QL variables, these files have the same format as the XYZ and Q files created using the IPLOT $=2$ and 3 options. These restart files can thus also be used as XYZ and Q files for the PLOT3D plotting program. Since $\mathrm{N} 3=1$, the QL variables will not be read by PLOT3D. Note also, however, that the reverse is not true. The XYZ and Q files created using the IPLOT $=2$ or 3 option may not be used as restart files, since they do not include the QL variables. ${ }^{21}$

[^16]
### 5.0 NITTLAL CONDITIONS

Initial conditions are required throughout the flow field to start the time marching procedure. Although the best choice for initial conditions will be problem-dependent, some general comments can be made. lirst, for unsteady flows they should represent a real flow field. A converged steady-state solution from a previous run would be a good choice. Second, to minimize the number of iterations required for steady flows, the initial conditions should be reasonably close to the expected final solution. And third, to minimize problems with starting transients it is important that they represent a physically realistic flow.

Initial conditions may be supplied by a user-written subroutine, called INIT, by a default version of INIT that specifies uniform flow with constant flow properties everywhere in the flow field, or by restart mesh and flow field files written during a previous run.

### 5.1 LSER-WRITTE.X INITIAL CONDITIONS

As stated above, the best choice for initial conditions will be problem-dependent. For this reason PROTEUS does not include a general-purpose routine for setting up initial conditions. Instead, provision is made for a user-written subroutine, called INIT, that sets up the initial conditions.

The time-marching algorithm used in PROTEUS requires initial conditions for $\rho, u, v, w, E_{T}, p$, and $T$. ${ }^{22}$ These variables may, of course, be computed from many different combinations of known parameters. To make this process reasonably flexible, the user may choose from several combinations exactly which variables subroutine INIT will supply. This choice is determined by the input parameter ICVARS in namelist FLOW. The following tables list the flow field variables to be supplied by subroutine INIT for the various ICVARS options, along with the PROTELS Fortran variables into which they should be loaded. ${ }^{23}$ Remember that the initial conditions must be nondimensionalized by the reference conditions listed in Table 3-2. The default value for ICVARS is 2.

When the energy equation is being solved (IHSTAG $=0$ ), and the flow is two-dimensional, or axisymmetric without swirl ( $\mathrm{IAXI}=0$ or ISWIRL $=0$ ), the allowed values are:

| ICVARS | Variables Supplied By INIT | Fortran Variables |
| :---: | :---: | :---: |
| 1 | $\rho, \rho u, \rho v, E_{T}$ | RHO, U, V, ET |
| 2 | $p, u, v, T$ | P, L, V, T |
| 3 | $\rho, u, v, T$ | RHO, U, V, T |
| 4 | $p, u, v, \rho$ | P, U, V, RHO |
| 5 | $c_{p}, u, v, T$ | P, U, V, T |
| 6 | $p, M, \alpha_{\nu}, T$ | P, U, V, T |

When the energy equation is being solved (IHSTAG $=0$ ), and the flow is axisymmetric with swirl ( $\operatorname{IAXI}=1$ and $\operatorname{ISWIRL}=1$ ), the allowed values are:

[^17]| ICVARS | Variables Supplied By INIT | Fortran Variables |
| :---: | :---: | :---: |
| 1 | $\rho, \rho u, \rho v, \rho w, E_{T}$ | RHO, U, V, W, ET |
| 2 | $p, u, v, w, T$ | P, U, V, W, T |
| 3 | $\rho, u, v, w, T$ | RHO, U, V, W, T |
| 4 | $p, u, v, w, \rho$ | P, U, V, W, RHO |
| 5 | $c_{p}, u, v, w, T$ | P, U, V, W, T |
| 6 | $p, M, \alpha_{v}, \alpha_{w}, T$ | $P, U, V, W, T$ |

When constant stagnation enthalpy is assumed (IHSTAG $=1$ ), and the flow is two-dimensional, or axisymmetric without swirl (IAXI $=0$ or ISWIRL $=0$ ), the allowed values are:

| ICVARS | Variables Supplied By INIT | Fortran Varia |
| :---: | :---: | :---: |
| 1 | $\rho, \rho u, \rho v$ | RHO, U, V |
| 2 | $p, u, v$ | P, U, V |
| 3 | $\rho, u, v$ | RHO, U, V |
| 5 | $c_{p}, u, v$ | P, U, V |
| 6 | $p, M, \alpha_{v}$ | $P, U, V$ |

When constant stagnation enthalpy is assumed (IHSTAG =1), and the flow is axisymmetric with swirl ( $\mathrm{IAXI}=1$ and $\mathrm{ISWIRL}=1$ ), the allowed values are:

| ICVARS | Variables Supplied By INIT |  | Fortran Variables |
| :---: | :---: | :--- | :--- |
|  |  |  |  |
| 1 | $\rho, \rho u, \rho v, \rho w$ |  | RHO,U,V,W |
| 2 | $p, u, v, w$ |  | P,U,V,W |
| 3 | $\rho, u, v, w$ |  | RHO,U,V,W |
| 5 | $c_{p}, u, v, w$ |  | P,U,V,W |
| 6 | $p, M, \alpha_{v}, \alpha_{w}$ |  | P,U,V,W |

In the above tables, $c_{p}, \alpha_{v}$, and $\alpha_{w}$ represent static pressure coefficient, flow angle in degrees in the $x-y$ (or $x-r$ ) plane, and flow angle in degrees in the $x-\theta$ plane, respectively. These parameters are defined as:

$$
\begin{aligned}
& c_{p}=\frac{\bar{p}-p_{r}}{\rho_{r} u_{r}^{2} / 2 g_{c}} \\
& \alpha_{v}=\tan ^{-1} \frac{v}{u} \\
& \alpha_{w}=\tan ^{-1} \frac{w}{u}
\end{aligned}
$$

The PROTELS subroutine INITC will use the variables supplied by subroutine INIT to compute $\rho$, $u, v, w$, and $E_{T}$, using perfect gas relationships if necessary. From these variables, the pressure and temperature will then be recomputed using the equation of state in subroutine EQSTAT, overwriting any values specified by the user in INIT. This ensures a consistent set of initial conditions for the time marching procedure.

Subroutine INIT is called once, immediately after the input has been read, the reference and normalizing conditions have been set, and the geometry and mesh have been defined. The user may do anything he or she desires in the subroutine, such as reading files, reading additional namelist input, making computations, etc. Any of the defined Fortran variables in common blocks may be used. ${ }^{24}$ (Of course, they should not be changed.) The only requirement is that subroutine INIT return to the calling program (which is INITC) the combination of variables specified by ICVARS, defined at every grid point.

[^18]Subroutine INIT is also a convenient place to specify point-by-point boundary condition types and values. It's often easier to do this using Fortran coding rather than entering each value into the namelist input file. See Section 9.2 for a test case with a user-written version of subroutine IXITP.

### 5.2 DEFALLT INTHAL COXDHIONS

A default version of subroutine INIT is built into FROTELS that specifies uniform flow with constant flow properties everywhere in the flow field. It uses the ICVARS $=2$ option (the default value) and reads initial flow field values of $p, u, v, w$, and $T$ from namelist IC. The defaults for these parameters are $1.0,0.0$, $0.0,0.0$, and 1.0 , respectively, resulting in an initial flow field with $\bar{p}=p_{r}, u=v=w=0$, and $\bar{T}=T$, If a value for ICVARS other than 2 is set in the input, a warning message is generated and ICVARS is reset to 2.

### 5.3 RESTART INITLAL CONDITIOXS

If restart mesh and flow field files were created during a previous run by setting IRIST $>0$ in namelist RSTRT, these files can be used to continue the calculation from the point where the previous run stopped. In this case no subroutine INIT is needed. The restart case is run by linking the existing restart mesh and flow field files to lortran units XRXIN and NRQIN, respectively, and setting IRES ${ }^{\circ}=2$ in the input. New restart files will also be written to units NRXOUT and NRQOUT.

When a restart case is being run, the usual namelist input described in Section 3.1 must still be read in. The following input parameters must have the same values as during the original run: 10 NIIS, IAXI, IHSTAG, ILAMV, ISWIRL, IR, UR, MACHR, TR, RHOR, MLR, RIR, KTR, PRIR, GAMR, RG, HSTAGR, N1, N2, IPACK, and SQ. The remaining input parameters either may be changed during a restart, or do not apply to a restart case. Note, however, that for many of the input parameters, such as those specifying the boundary conditions, changing values during a restart may result in temporary restarting" transients or even cause the calculation to blow up.

### 6.0 RESOURCE REQUIREMENIS

PROTLLS was developed on the Cray X-MP computer at NASA Lewis Research Center. Changes that may be necessary when porting the code to another system are described in Section 6.1. Sections 6.2 and 6.3 discuss the memory and CPU time required to run the code. The values presented in these sections were derived from tests run on the NASA Lewis Cray X-MP in September, 1989. At that time the Cray was running UNICOS Version 4.0, CFT 1.15 BF 2 , and CFT77 2.0. UNICOS, CFT, and CFT77 are described in the UNICOS Lser Commands Reference Manual (Cray Research, Inc., 1988c), the Fortran (CIT) Reference Manual (Cray Research, Inc., 1986), and the CFT77 Reference Manual (Cray Research, Inc., 1988a), respectively. Section 6.4 describes the size and format of the various input and output files used in the code.

### 6.1 COMPLTER

PROTELS should be transportable to other computer systems with minimal changes. With just two known exceptions, the code is written entircly in ANSI standard Fortran 77. The first exception is the use of namelist input. With namelist input, it's relatively easy to create and/or modify input files, to read the resulting files, and to program default values. Since most Fortran compilers allow namelist input, its use is not considered a serious problem. The second exception is the use of *CALL statements to include COMDECKs, which contain the labeled common blocks, in most of the subprograms. This is a Cray UPDATE feature, and therefore the source code must be processed by UPDATE to create a file that can be compiled. ${ }^{25}$ UPDATE is described in the UPDATE Reference Manual (Cray Rescarch, Inc., 1988d). Since using the ${ }^{*}$ CALL statements results in cleaner, more readable code, and since many computer systems have an analogous feature, the *CALL statements were left in the program.

Six library subroutines are called by PROTEUS. ISAMAX, SASUM, and SNRM2 are Cray Basic I inear Algebra Subprograms (BLAS). ISAMIN is a Cray extension to the BLAS routines. SGEFA and SGESL are Cray versions of LINPACK routines. These or similar routines may be available on other systems. If not, equivalent routines will have to be coded. All of these routines are described in detail in Section 4.0 of Volume 3, and in the Programmer's Library Reference Manual (Cray Research, Inc., 1988b).

### 6.2 MEMORY

The sizes of the dimensioned arrays in PROTELS, and hence the amount of memory required to run the program, are set using PARAMETERs. These PARAMETERs are set in COMDECK PARAMS1. Larger or smaller dimensions may be set for the entire program simply by changing the appropriate PARAMETER, and then recompiling the entire program. The PARAMETERs are defined as follows:

| NIP | Maximum number of grid points in the $\xi$ direction. The current value is 51. |
| :--- | :--- |
| N2P | Maximum number of grid points in the $\eta$ direction. The current value is 51. |
| NMAXP | Maximum of N1P and N2P. |
| NTOTP | Total storage required for a single two-dimensional array (i.e., N1P $\times N 2 \mathrm{P}$ ). |
| NEQP | Maximum number of coupled equations that can be solved. The current value is |
| NAMAX | Maximum number of time steps allowed in the moving average convergence test <br> (the ICTEST $=2$ option). The current value is 10. |
| NBC | Number of boundary conditions per equation. The current value is 2. |

NIP Maximum number of entries in the table of time-dependent boundary condition values. The current value is 10 .

NTSEQP Maximum number of time step sequences in the time step sequencing option. The current value is 10 .

The total amount of memory in computer words required to run PROIELS 2-D, compiled using CFI77, is listed in the following table for various combinations of the PARAMETERs N1P and \2P. On the Cray X-MP, each word is 64 bits long. If CFT is used to compile the code, the memory required to run is slightly less than the amount listed in the table. The figures in the table include approximately 283,000 words used by the system software.

| $\triangle 1 \mathrm{P}$ | Х2P | ME.MORY |
| :---: | :---: | :---: |
| 26 | 26 | 479,744 |
| 51 | 51 | 692,736 |
| 101 | 101 | 1,561,088 |
| 26 | 51 | 659,968 |
| 25 | 101 | 1,331,712 |
| 51 | 101 | 1,413,632 |

### 6.3 CPU TIME

Compilation of PROTLTS 2-I) using Release 2.0 of the ClT77 compiler requires about 142 seconds of CPU time. Based on the results of the test cases in Section 9.0 , the CP U time required for execution ranges from about $3 \times 10^{5}$ seconds per grid point per time step for runs using first-order time differencing without the energy equation, to $6 \times 10^{s}$ seconds per grid point per time step for runs using second-order time differencing with the energy equation.

The code can be compiled using Release 1.15 BF 2 of the CFI compiler in only 41 seconds. Experience has shown, however, that the CPL time required for execution then increases by a factor of 2 to 3 .

### 6.4 INPCT/OLIPLT FILES

Several files are used in PROTEUS for various types of input and output. The contents of these files have been described previously in Sections 3.0 and 4.0. This section describes the characteristics of the files themselves. The files are identified by the lortran variable representing the unit number. The unit numbers have default values, but all of them except NIN may be read in by the user.

Table 6-1 lists the files used in PROTELS, giving the default unit number, briefly describing the contents of the file, and indicating when it is used. Table 6-2 summarizes the computational resources needed for each file. In this table, the record length is specified in bytes for units NIN and NOUT, and in computer words for the remaining units. The total file size is specified in printed pages for unit NOUT, and in computer words for the remaining units. Several symbols and Fortran variables are used in Table 6-2, and are defined as:

| $\gamma$ | 0 if explicit artificial viscosity is not being used, 1 if it is. |
| :---: | :---: |
| $N_{4}$ | The number of coupled equations being solved. |
| $N$, | The number of time levels written into the plot file. This is determined by the input parameters IPLI and IPITA. |
| $V_{11}, V_{2}$ | The time level at the beginning and at the end of the calculation. |
| $N_{n}, N_{\square}$ | The number of grid points in the $\eta$ and $\xi$ directions at which output is being printed. This is determined by the input parameters IPRT1, IPRT2, IPRT1A, and IPRT2A. |
| $N_{1}, N_{2}$ | The number of grid points in the $\xi$ and $\eta$ directions. |
| ICHECK | Input parameter specifying frequency for checking convergence. |

IPLOT Input flag specifying type of plot file being written.
NGI, NG2 Number of points in the $\xi$ and $\eta$ directions in the coordinate system file.
The typical record length and total size values listed in the table were computed assuming $N_{a v}=1, N_{e q}=4$, $N_{\mathrm{t}}=1, N_{t 1}=1, N_{12}=2000, N_{n}=N_{\xi}=26, N_{1}=N_{2}=51, \mathrm{ICHECK}=10$, and $\mathrm{NG} 1=\mathrm{NG} 2=51$.

TABLE 6-1. - IO FILE TYPES

| LNIT | $\begin{aligned} & \text { DEFALLT } \\ & \text { NAT NO. } \end{aligned}$ | $\begin{aligned} & \text { RECORD } \\ & \text { FORMAT } \end{aligned}$ | CONTENTS | WHEN USED |
| :---: | :---: | :---: | :---: | :---: |
| NIN | 5 | Formatted | Standard input | Always |
| NOUT | 6 | Formatted | Standard output | Always |
| NGRID | 7 | Unformatted | Coordinate system input | NGEOM $=10$ |
| NPLOTX | 8 | Unformatted | PLOT3D XYZ file output | $\mathrm{IPLOT}=2,3,-3,4$ |
| NPLOT | 9 | Unformatted | CONTOUR plot file or PLOT3D Q file output | IPLOT $\neq 0$ |
| NHIST | 10 | Unformatted | Convergence history output | Always |
| NRQIN | 11 | Unformatted | Restart flow field input | IREST $=2$ |
| NRQOUT | 12 | Unformatted | Restart flow field output | $\operatorname{IREST}=1,2$ |
| NRXIN | 13 | Unformatted | Restart computational coordinates input | IRESI $=2$ |
| NRXOUT | 14 | Unformatted | Restart computational coordinates output | $\operatorname{IREST}=1,2$ |
| NSCR1 | 20 | Unformatted | Scratch | IPLOT $=-3$ |

TABLE 6-2. - IO FILE SIZES

| L.NIT | $\begin{gathered} \text { MAXIMEM } \\ \text { RECORD LENGTH } \end{gathered}$ |  | TYPICAL <br> MAXIMLM RECORD LENGTH ${ }^{\text {a }}$ | TOTAL SIZE ${ }^{\text {b }}$ |  | TYPICAL <br> TOTAL SIZE ${ }^{\text {b }}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| NIN | 80 |  | 80 | Variable |  | Variable |
| NOLT | 132 |  | 132 | $\begin{aligned} & \simeq 2 N_{e f}+3 \text { pages }+ \\ &\left\{\left(N_{n}+3\right)\left[\left(N_{\xi}-1\right) / 10+1\right]\right. \\ &+2\} / 55 \text { pages per variable } \\ & \text { per time level } \end{aligned}$ |  | 11 pages + 2 pages per variable per time level |
| \GRID | 2(NG1)(NG2) |  | 5,202 | $2(\mathrm{NGL})(\mathrm{NG} 2)+2$ |  | 5,204 |
| NPLOTX | IPIOT | $\begin{aligned} & \\ & 3 \mathrm{~N}_{1} \mathrm{~N}_{2} \\ & 3 \mathrm{~N}_{1} \mathrm{~N}_{2} \\ & 2 \mathrm{~N}_{1} \mathrm{~N}_{2} \end{aligned}$ | $\begin{aligned} & 7,803 \\ & 7,803 \\ & 5,202 \end{aligned}$ | IPLOT |  |  |
|  | 2,3 -3 4 |  |  | 2,3 -3 4 | $\begin{aligned} & 3 N_{1} N_{2}+3 \\ & 3 V_{1} N_{2} N_{r}+3 \\ & 2 N_{1} N_{2}+2 \end{aligned}$ | $\begin{aligned} & 7,806 \\ & 7,806 \\ & 5,204 \end{aligned}$ |
| NPLOT | IPIOT | $\begin{aligned} & 14 V_{2} \\ & 14 . V_{2} \\ & 5 N_{1} V_{2} \\ & 4 N_{1} N_{2} \end{aligned}$ | $\begin{aligned} & 714 \\ & 714 \end{aligned}$ | IPIOT | $\begin{aligned} & \left(14 N_{1} N_{2}+32\right) N_{t}+5 \\ & \left(14 N_{1} N_{2}+5\right) N_{t} \\ & +27 \\ & \left(5 N_{1} N_{2}+7\right) N_{t} \\ & \left(4 N_{1} N_{2}+6\right) N_{t} \end{aligned}$ |  |
|  | $\begin{gathered} 1 \\ -1 \\ 2,3,-3 \\ 4 \end{gathered}$ |  |  | $\begin{gathered} 1 \\ -1 \\ 2,3,-3 \\ 4 \end{gathered}$ |  | $\begin{aligned} & 36,446 \\ & 36,446 \\ & 13,012 \\ & 10,410 \\ & \hline \end{aligned}$ |
| NHIST | $7{ }^{7}$ |  | 28 | $\begin{gathered} \left(N_{a v}+1\right)\left(7 V_{e g}\right)\left(N_{22}-N_{11}+1\right) \\ 1 \text { ICHECK }+N_{e q}+12 \\ \hline \end{gathered}$ |  | 11,216 |
| NRQIN, <br> NRQOUT | $5 N_{1} N_{2}$ |  | 13,005 | $10 N_{1} N_{2}+7$ |  | 26,017 |
| $\begin{aligned} & \text { NRXIN, } \\ & \text { NRXOUT } \end{aligned}$ | $3 N_{1} N_{2}$ |  | 7,803 | $3 N_{1} N_{2}+3$ |  | 7,806 |
| NSCR1 | $6 N_{1} N_{2}$ |  | 15,606 | $6 N_{1} N_{2} N_{t}$ |  | 15,606 |

a In bytes for units NIN and NOUT, and in computer words for the remaining units.
${ }^{\circ}$ In pages for units NIN and NOUT, and in computer words for the remaining units.

### 7.0 DIAGNOSIIC MESSAGES

Varions diagnostic messages may be printed by PROTlSS as part of its standard output. ${ }^{26}$ Most of these concern inconsistent or invalid input, although some describe problems encountered during the calculation itself. Two types of messages may appear - errors and warnings. lirror mesages are preceded by the characters $* * * *$ ERROR, and indicate either a serious problem or an input error that PROTVES cannot resolve. Waming messages are preceded by the characters $*^{*} * *$ WARNING, and indicate cither a potential problem or a non-standard combination of input parameters. Errors catuse the calculation to stop, while warnings do not.

The various error and waming messages are listed and explained in the following two subsections. I ower case letters, like value, are used to indicate variable values.

### 7.1 ERRORS

```
BOTH ITXI AND ITETA = 0.
```

A turbulent flow is being computed, with flags set in namelist I IRB to bypas the turbulent viscosity computation in both coordinate directions. This makes no sense.

```
BOTH MACHR AND UR SPECIFIED.
MACHR = value, UR = value
```

Fither the reference Mach number or velocity may be specified in namelist IIOW, but not both.

```
BOTH PRLR AND KTR SPECIFIED.
```

PRLR = value, $K T R=$ value

Either the reference laminar Prandtl number or thermal conductivity may be specified in namelist EI OW, but not both.

```
BOTH RER AND MUR SPECIFIED.
```

$R E R$ = value, $M U R$ = value

Iither the reference Reynolds number or viscosity may be specified in namelist FIOW, but not both.

```
COORDINATE SYSTEM FILE HAS NGI AND/OR NG2 > MAX ALLOWED.
```

NG1 = value, NG2 = value

A coordinate system file has been read in, using the $\mathrm{NGIO} O=10$ option, with more grid points than allowed. The maximum allowed values of $\mathrm{X} G 1$ and NG 2 are the values of the Cray PARAMEILRs N1P and N2P, respectively.

GRID TRANSFORMATION JACOBIAN CHANGES SIGN OR $=0$.
The nonorthogonal grid transformation Jacobian $J$ must be cither everywhere positive or everywhere negative. This error indicates that the computational mesh contains crossed or coincident grid lines. The error message is followed by a printout of the Cartesian coordinates, the Jacobian, and the metric coefficients.

[^19]```
ILLEGAL OPTION FOR COMPUTATIONAL COORDINATES REQUESTED.
NGEOM = value
```

An illegal value of NGEOM has been specified in namelist GMTRY. The legal values are 1,2 , and 10 , and are described in Section 3.1.5.

```
ILLEGAL PLOT FILE OPTION REQUESTED.
IPLOT = value
```

An illegal value of IPLOT has been specified in namelist IO. The legal values are $0, \pm 1,2, \pm 3$, and 4, and are described in Section 3.1.4.

```
ILLEGAL TIME STEP SELECTION OPTION REQUESTED.
IDTAU = value
```

An illegal value of IDTAU has been specified in namelist TIME. The legal values are 1 to 7 , and are described in Section 3.1.9.

```
IlLEGAL VALUE FOR ICVARS.
ICVARS = value
```

An illegal value of ICVARS has been specified in namelist FLOW. The legal values are 1 to 6 , and are described in Section 3.1.6.

```
IlLEGAL VALUE FOR ILAMV.
ILAMV = value
```

An illegal value of ILAMV has been specified in namelist FLOW. The legal values are 0 and 1 , and are described in Section 3.1.6.

```
INVALID BOUNDARY CONDITION TYPE REQUESTED.
JBCl(ieq,ibound) OR IBCl(j,ieq,ibound) = value
INVALID BOUNDARY CONDITION TYPE REQUESTED.
JBC2(ieq,ibound) OR IBC2(i,ieq,ibound) = value
```

These messages result from an invalid boundary condition type being specified in namelist BC for the $\xi$ and or $\eta$ direction. Here ieq is the boundary condition equation number; ibound $=1$ or 2 , corresponding to the $\xi=0$ or 1 surface, or the $\eta=0$ or 1 surface; and $i$ and $j$ are the indices in the $\xi$ and $\eta$ directions. The valid boundary conditions are listed in Table 3-6.

```
INVALID BOUNDARY TYPE REQUESTED.
KBCl(ibound) = value
INVALID BOUNDARY TYPE REQUESTED.
KBC2(ibound) = value
```

These messages result from an invalid boundary type being specified in namelist $B C$, for the $\xi$ and or $\eta$ direction, when the KBC meta flags are used. Here ibound $=1$ or 2 , corresponding to the $\xi=0$ or 1 surface, or the $\eta=0$ or 1 surface. The valid boundary types are listed in Section 3.1.7.

INVALID DEBUG OPTION SPECIFIED.
IDEBUG(i)
An invalid debug option, number $i$, has been specified in namelist IO. The valid IDEBCG options are 1 to 7 , and are described in Section 3.1.4.

```
INVALID GRID PACKING LOCATION FOR ROBERTS FORMULA.
SQ(idir,l) = value
```

An invalid grid packing location, given by the value of $S Q(I D I R, 1)$ in namelist NU.M, has been specified. Here idir $=1$ or 2 , corresponding to the $\xi$ and $\eta$ directions, respectively. The valid values are $0.0,0.5$, and 1.0 .

INVALID GRID PACKING PARAMETER FOR ROBERTS FORMULA.
SQ(idir,2) = value
An invalid grid packing parameter, given by the value of $\operatorname{SQ}(\operatorname{IDIR}, 2)$ in namelist NUM, has been specified. Mere idir $=1$ or 2 , corresponding to the $\xi$ and $\eta$ directions, respectively. The valid values are $>1$.

```
INVALID TIME STEP SELECTION METHOD FOR TIME STEP SEQUENCING OPTION.
IDTAU = value, NTSEQ = value
```

A time step selection option that adjusts $\Delta \tau$ as the solution proceeds has been specified in namelist TIME in conjunction with the time step sequencing option. If the time step sequencing option is being used, IDTAU must be 1,3 , or 5 .

```
INVALID TYPE OF UNSTEADINESS FOR BOUNDARY CONDITION REQUESTED.
JTBCl(ieq,ibound) = value
INVALID TYPE OF UNSTEADINESS FOR BOUNDARY CONDITION REQUESTED.
JTBC2(ieq,ibound) = value
```

These messages result from an invalid type of unsteadiness being specified in namelist BC for the boundary conditions in the $\xi$ and/or $\eta$ direction. Here ieq is the boundary condition equation number, and $\mathbf{i}$ bound $=1$ or 2 , corresponding to the $\xi=0$ or 1 surface, or the $\eta=0$ or 1 surface. The valid values for $\mathrm{JTBC1}$ and JTBC2 are 0,1 , and 2, and are described in Section 3.1.7.

```
MESH SIZE REQUESTED > MAX ALLOWED.
Nl = value, N2 = value
```

More grid points have been requested in namelist NUM than are allowed. For non-periodic boundary conditions, the maximum allowed values of N1 and N2 are the values of the Cray PARAMETERs N1P and N2P, respectively. For spatially periodic boundary conditions, the maximum values are $\mathrm{N} 1 \mathrm{P}-1$ and $\mathrm{N} 2 \mathrm{P}-1$.

```
MORE TIME STEP SEQUENCES REQUESTED THAN ALLOWED.
```

NTSEQ = value

For the time step sequencing option, the number of time step sequences, specified in namelist TIME, cannot exceed the value of the Cray PARAMETER NTSEQP.

```
NON-EXISTENT TURBULENCE MODEL REQUESTED
ITURB = value
```

A non-zero value for ITURB has been specified in namelist FLOW that does not correspond to one of the turbulence models currently available in PROTEUS. The only valid non-zero value for ITLRB is 1 , corresponding to the algebraic Baldwin-Lomax turbulence model.

NON-POSITIVE PRESSURE AND/OR TEMPERATURE AT TIME LEVEL $n$

| Il | I2 | P |
| :---: | :---: | :---: |
| value | value | ${ }^{\top}$ |
| value |  |  |

During the solution, a non-positive value for pressure and/or temperature has been computed in subroutine EQSTAT. L'p to 50 values will be printed. These values, of course, are non-physical and indicate a failure of the solution. Although the calculation will stop, the standard output and plot file will include this time level, if that is consistent with the "IPRT" and "IPLT" type parameters in namelist IO. The restart files will not be written. This failure may be caused by bad initial or boundary conditions, or by too large a time step.

```
NUMbER OF VALUES IN UNSTEADY bOUNDARY CONDITION TABLE > MAX ALLOWED.
NTBC = value
```

For unsteady boundary conditions, the number of values in the tables of GTBC1 and/or GTBC2 vs. NTBCA, specified in namelist TIME, cannot exceed the value of the Cray PARAMETER NTP.

PERIODIC BOUNDARY CONDITION REQUESTED IN RADIAL DIRECTION IN AXISYMMETRIC FLOW. IAXI = value, $K B C 2(1)=$ value, $K B C 2(2)=$ value

A spatially periodic boundary condition in the radial direction in axisymmetric flow does not make sense physically.

```
SINGULAR block matrix for b. C. AT LOWER BOUNDARY, SWEEP n
```

SINGULAR BLOCK MATRIX FOR B. C. AT UPPER BOUNDARY, SWEEP n
When boundary conditions are specified using the JBC and or IBC input parameters, zero values may appear on the diagonal of the block tridiagonal coefficient matrix. Subroutine FILTER attempts to tearrange the elements of the boundary condition block submatrices to climinate any of these fero values. These messages indicate it was unable to do so for the boundary and sweep indicated. This means the diagonal submatrix B is singular, which in turn means the specified houndary conditions are not independent of one another.

```
SURFACE AND POINT-BY-POINT BOUNDARY CONDITIONS BOTH SPECIFIED.
JBCl(ieq,ibound) = value, IBCl(j,ieq,ibound) = value
SURFACE AND POINT-BY-POINT BOUNDARY CONDITIONS BOTH SPECIFIED.
JBC2(ieq,ibound) = value, IBC2(i,ieq,ibound) = value
```

These messages indicate both surface and point-by-point boundary conditions were specified in the $\xi$ and/or $\eta$ directions. Each boundary condition on each boundary may be specified for the entire surface using the JBC and GBC parameters, or on a point-by-point basis using the IBC and FBC parameters, but not both. Here ieq is the boundary condition equation number; ibound $=1$ or 2 , corresponding to the $\xi=0$ or 1 surface, or the $\eta=0$ or 1 surface; and $\mathbf{i}$ and $j$ are the indices in the $\xi$ and $\eta$ directions. A likely cause of this error message is specifying point-by-point boundary conditions without setting the appropriate JBC parameter equal to -1 . See the discussion of boundary condition input in Section 3.1.7.

### 7.2 WARNINGS

CHGMAX > 0.15, CFL CUT IN HALF.
CHGMAX > 0.15, DTAU CUT IN HALF.
With the IDTAU $=2,4$, and 6 options, the time step is adjusted gradually as the solution proceeds based on the absolute value of the maximum change in the dependent variables. Onc of these messages may occur if the solution changes very rapidly. (The first message applies to the IDTAU $=2$ and 6 options, and the second to the IDTAU $=4$ option.) Cnder these conditions the time step is arbitrarily cut in half, and the solution continues. This may stabilize the calculation, but consideration should be given to rerunning the problem with a smaller time step, especially for unsteady flows.

## ICVARS RESET TO 2 FOR DEFAULT INIT.

The default version of subroutine INIT is set up assuming ICVARS $=2$. If another value of ICVARS is read in, it is automatically reset to 2 and the calculation will continue.

## IlLEGAL CONVERGENCE TESTING OPTION REQUESTED. <br> ICTEST = value, RESET TO ICTEST $=3$

An illegal value of ICTEST has been specified in namelist TIME. ICTEST will be reset to 3 , corresponding to convergence based on the $L_{2}$ norm of the residual, and the calculation will continue. The legal values are 1 to 5, and are described in Section 3.1.9.

## ILLEGAL OUTPUT REQUESTED. <br> $\operatorname{IVOUT}(n)=$ value

An illegal value of IVOLT has been specified in namelist IO. It will be ignored and the calculation will continue. The legal values of IVOUT are listed in Table 3-3.

IMPLICIT \& NONLINEAR EXPLICIT ARTIFICIAL VISCOSITY BOTH REQUESTED.
IAV2I, IAV2E, IAV4E = value value value
CAVS2I = value value value value value
Normally, the nonlinear artificial viscosity model, specified by setting IAV2E and IAV4E $=2$, is explicit only. This message is printed when implicit artificial viscosity is requested at the same time. PROTEUS will assume that you know what you are doing and the implicit artificial viscosity will be included.
non-standard time difference centering requested.
THC = value value
THX = value value value
THY = value value value
THZ = value value value
THE = value value value
The Beam-Warming type time differencing used in PROTEUS includes three standard implicit schemes - Euler, trapezoidal, and three-point backward. This message indicates that a combination of time differencing parameters $\theta_{1}, \theta_{2}$, and $\theta_{3}$ has been specified for at least one of the governing equations that does not correspond to any of the three standard schemes. PROTELS will assume that you know what you are doing and the specified time differencing parameters will be used.
spatially varying time step requested with time-accurate differencing scheme.
IDTAU = value
THC = value value
THX = value value value
THY = value value value
THZ = value value value
THE = value value value
For steady flows, a spatially varying time step may be used to enhance convergence, and first-order time differencing is recommended. Using a second-order time-accurate differencing scheme should not give wrong results, but is wasteful. For unsteady flows, second-order time-accurate differencing should be used, but only a global (i.e., constant in space) time step makes sense.
TIME LEVEL MAY FALL OUTSIDE RANGE OF INPUT TABLE FOR UNSTEADY B. C.
ITSTRT = value, ITEND = value, NTBCA(1) = value, NTBCA(n) = value
General unsteady boundary conditions are being used, and the time levels in the input table of GTBC1 and/or GTBC2 vs. NTBCA do not cover the time levels that will occur during the time marching loop. Here ITSTRT and ITEND are the first and last time levels in the time marching loop, and $n$ is the value of the input parameter NTBC. If the time level $n<$ NTBCA(1), the boundary condition value will be set equal to the first value in the table. Similarly, if $n>\operatorname{NTBCA}(2)$, the value will be set equal to the last value in the table.

### 8.0 JOB CONTROL LANGLAGE

At DASA Lewis, PROTELS is currently being run on the Cray X-MP computer, with UNICOS 4.0 as the operating system. In this section several general examples are presented showing the UNICOS job control language ( JCL ) that may be used as starting points when setting up specific cases. ${ }^{27}$ The individual UNICOS commands are described in detail in the UNICOS User Commands Reference Manual (Cray Research, Inc., 1988c). These examples are written for the Bourne shell. Some changes may be necessary if the C shell is being used. These examples also use the Amdahl 5860 computer running VM as the front end to the Cray. It is assumed that the user is familiar with the procedures used to submit and receive Cray jobs through the V.M Cray station.

Each example is given with reference line numbers, which are not part of the actual JCL statement, followed by a line-by-line explanation. Note that in LNICOS, the case (upper or lower) of the letters in commands and arguments is significant. In this respect, the examples should be followed exactly.

### 8.1 COMPILING THE CODE

In this example, the PROTEUS code is fetched from the front end and compiled on the Cray. The object code is then stored in the user's home directory on the Cray. It is assumed that the source code is in Cray UPDATE format, as described in the LPDATE Reference Manual (Cray Research, Inc., 1988d).

```
1. #USER=yourid PW=yourpW
2. QSUB -eo -1M 1.0MW -1T 300
3. QSUB -r EXAMPLE
4. set -x
5. fetch upinput -muX -t'fn=filename,ft=filetype'
6. update -i upinput -n SHOME/p2dlo.u -c p2dlO -f
7. cft777-b $HOME/p2dl0.o -d pq p2di0.f
```

I ines 1 through 3 are actually Cray Network Queueing System (NQS) commands, not UNICOS commands. They must appear first in the runstream, and begin with a \# sign. This first line contains your userid and password, in lower case letters.
Line 2 tells the Cray to put any system error messages into the standard output file (the eeo option), and sets the memory and CPU time limits for the job at 1.0 million words and 300 seconds.

I line 3 gives the name of the job as EXAMPLE.
line 4 causes your LNICOS commands to be printed as part of the output runstream.
Line 5 fetches the PROTEUS source code from VM, and stores it in a temporary Cray file called upinput. ${ }^{28}$ Filename and fletype are the file name and file type of the file stored on V.M.
Line 6 uses the Cray LPDATE facility to create a temporary file, $p 2 d 10 . f$, which contains the complete compilable Fortran code for PROTELS, and a permanent file, p2d/0.u, which contains the PROTELS update program library and is stored in the user's home directory. The update command uses as input the file upinput.
Line 7 compiles the program $p 2 d / 0 . f$, storing the object code in the file $p 2 d / 0.0$, in the user's home directory.

[^20]
### 8.2 RLNNING THE MASTER FILE

The simplest way to run PROTELS is shown in this example. The existing master file is being used, without making any changes.

```
1. USER=yourid PW=yourpw
2. QSUB -eo -1M 1.0mW -1T 60
* QSUB -r EXAMPLE
set -x
ja
fetch input -mUX -t'fn=filename,ft=filetype'
touch plotx
touch plotq
touch chist
touch rqout
touch rxout
touch scrl
ln plotx fort. }
In plota fort.9
ln chist fort.10
ln rqout fort.l2
ln rxout fort.14
ln scrl fort. }2
In $HOME/casename/coords fort. }
segldr -o p2dl0.e 今HOME/p2dlo.o
p2dlo.e < input
CP rqout $HOME/casename/rqin
cp rxout $HOME/casename/rxin
bintran -mUX -v plotq
bintran -mUX -v plotx
bintran -mUX -v chist
ja-cslt
```

Lines 1 through 3 are actually Cray Network Qucueing System (NQS) commands, not UNICOS commands. They must appear first in the runstream, and begin with a \# sign. This first line contains your userid and password, in lower case letters.
Line 2 tells the Cray to put any system error messages into the standard output file (the -eo option), and sets the memory and CPU time limits for the job at 1.0 million words and 60 seconds. On the Cray under UNICOS, the standard output is preconnected to unit 6 . Any system errors will thus be part of the normal PROTELS output, as long as the input parameter NOUT $=6$.
Line 3 gives the name of the job as EXAMPLE.
Line 4 causes your UNICOS commands to be printed as part of the output runstream.
Line 5 tells the Cray to begin keeping accounting information for later printing.
Line 6 fetches the standard PROTEUS input file, containing the job title and the namelist data, from VM, and stores it in a temporary Cray file called input. Filename and filetype are the file name and file type of the file stored on V.M.

Lines $7-12$ create empty temporary files with the file names as shown.
Lines 13-18 link these temporary files with the indicated Fortran unit numbers. The files are thus the PLOT3D XYZ file, the PLOT3D Q file or CONTOUR plot file, the convergence history file, the restart flow field and mesh files, and the scratch file used with the IPLOT $=-3$ option. These lines implicitly open the files for input and output. Fortran OPEN statements are not used in PROTELS. If the PROTEUS input is such that any of these files are unnecessary (see Table 6-1), then the touch and in for those files can be eliminated.
Line 19 links an existing computational coordinate system file with Fortran unit 7. In this example, this file is assumed to be stored in your home directory, under the subdirectory casename. If the input parameter NGEOM $\neq 10$, this line should be eliminated. If a restart case is being run (IREST $=2$ ), this line should be replaced by the following two lines:

```
ln $HOME/casename/rxin fort.l3
```

The above lines link existing restart flow field and computational mesh files with Fortran units 11 and 13. In this example, these files are assumed to be stored in your home directory, under the subdirectory casename.
Line 20 creates a temporary executable file, $p 2 d / 0 . e$, from the existing object file $p 2 d / 0.0$ in your home directory.
Line 21 actually runs the program, getting the standard PROTELS input from the temporary file input.
Lines 22-23 save the temporary output restart flow ficld and mesh files by copying them into the files rain and rxin in the subdirectory casename in your home directory. Note that this will overwrite the existing files with the same names. Use different file names if you need to keep the existing files also. If restart files are not written by your job, these lines should be eliminated.
Lines 24-26 convert the indicated UNICOS unformatted files to VM unformatted files and dispose them to VM. Bintran is a local NASA Lewis command, not a standard Cray UNICOS command. The files will appear in your VM reader, along with the standard PROTELS output. If these files are ultimately to be used on some other computer (e.g., a graphics workstation), a different procedure or additional conversion steps may be required. If these files were not created by your job, or if you have no use for them on the front end, then these lines should be eliminated.
Line 27 causes various accounting information to be printed at the end of your output.

### 8.3 MODIFYING THE MASTER FILE, FLLL UPDATE

This example shows how to run with a temporarily modified version of the master file. In this particular case, the existing master file is modified to increase the Cray PARAMETERS N1P and N2P, thus allowing more mesh points to be used. Since this affects almost every subroutine, the entire program is updated and recompiled.

```
1. USER=yourid PW=yourpw
2. QSUB -eo -1M 1.0mW -1T }30
3. #SUB - EXAMPLE
4. set -x
5. ja
6. fetch input -mUX -t'fn=filename,ft=filetype'
7. touch plotx
8. touch plota
9. touch chist
10. touch rqout
11. touch rxout
12. touch scrl
13. In plotx fort.8
14. In plota fort.9
15. In chist fort.l0
16. In rqout fort.l2
17. In rxout fort.14
18. In scrl fort. }2
19. In $HOME/casename/coords fort.7
20. cat > mods << EOF
    *ID TEMP
*D PARAMSI. }1
                            PARAMETER (N1P = 81, N2P = 81)
21. EOF
22. update -p $HOME/p2dl0.u -i mods -c temp -f
23. cft77 -d pq temp.f
24. segldr -o temp.e temp.o
25. temp.e < input
26. cp rqout $HOME/casename/rqin
27. CP rxout $HOME/casename/rxin
28. bintran -mUX - v plotq
29. bintran -mUX -v plotx
30. bintran -mUX -v chist
31. ja -cslt
```

I ines 1 through 3 are actually Cray Network Queueing System (NQS) commands, not UNICOS commands. They must appear first in the runstream, and begin with a \# sign. This first line contains your userid and password, in lower case letters.

I ine 2 tells the Cray to put any system error messages into the standard output file (the -eo option), and sets the memory and CPL time limits for the job at 1.0 million words and 60 seconds. On the Cray under UNICOS, the standard output is preconnected to unit 6 . Any system errors will thus be part of the normal PROTLLS output, as long as the input parameter $\mathrm{NOLT}=6$.
I ine 3 gives the name of the job as EXA.ITPLE.
I ine 4 causes your UNICOS commands to be printed as part of the output runstream.
I ine 5 tells the Cray to begin keeping accounting information for later printing.
$I$ ine 6 fetches the standard PROTELS input file, containing the job title and the namelist data, from VM, and stores it in a temporary Cray file called input. Filename and filetype are the file name and file type of the file stored on VM.
lines 7.12 create empty temporary files with the file names as shown.
I ines 13-18 link these temporary files with the indicated Fortran unit numbers. The files are thus the PIOT3D XY\% file, the PIOT3D Q file or CONTOUR plot file, the convergence history file, the restart flow field and mesh files, and the seratch file used with the IPLOT $=-3$ option. These lines implicitly open the files for input and output. Fortran OPEN statements are not used in PROTEUS. If the PROTEUS input is such that any of these files are unnecessary (see Table 6-1), then the touch and in for those files can be eliminated.

Line 19 links an existing computational coordinate system file with Fortran unit 7. In this example, this file is assumed to be stored in your home directory, under the subdirectory casename. If the input parameter $\mathrm{NGIOM} \neq 10$, this line should be eliminated. If a restart case is being run (IREST $=2$ ), this line should be replaced by the following two lines:

```
ln $HOME/casename/rqin fort.ll
ln $HOME/casename/rxin fort.l3
```

The above lines link existing restart flow hield and computational mesh files with Fortran units 11 and 13. In this example, these files are assumed to be stored in your home directory, under the subdirectory casename.

I ine 20 creates a temporary Cray file called mods. The file will consist of all the records between line 20 and line 21 , which contains the marker "EOF". Your Cray UPDATE directives and new code should therefore be inserted between lines 20 and 21. The LPDATE directives and new code could also be kept in a tile on the front end machine, and fetched just as the input data file was in line 6 . In that case, lines 20 and 21 should be eliminated.

I ine 22 uses the Cray CPDAII facility to create a temporary file, temp.f, which contains the complete fortran code for the modified version of PROTEUS. The update command uses as input the existing PROTILS program library $p^{2} d /(0 . u$, which is stored in your home directory, and the temporary file mods containing the (PDATE directives and new code.
line 23 compiles the modified program temp.f, storing the object code in the temporary file temp.o.
I ine 24 creates a temporary executable file, temp.e, from the temporary object file temp.o.
I ine 25 actually runs the program, getting the standard PROTELS input from the temporary file input.
I ines $26-27$ save the temporary output restart flow field and mesh files by copying them into the files rqin and rxin in the subdirectory casename in your home directory. Note that this will overwrite the existing files with the same names. Use different file names if you need to keep the existing files also. If restart files are not written by your job, these lines should be eliminated.
I ines $28-30$ convert the indicated LWICOS unformatted files to VM unformatted files and dispose them to V.M. Bintran is a local NASA Lewis command, not a standard Cray UNICOS command. The files will appear in your VM reader, along with the standard PROTECS output. If these files are ultimately to be used on some other computer (e.g., a graphics workstation), a different procedure or additional conversion
steps may be required. If these files were not created by your job, or if you have no use for them on the front end, then these lines should be eliminated.
Line 31 causes various accounting information to be printed at the end of your output.

### 8.4 MODIFYIXG THE MASTER FILE. PARTIAL IPDATE

This example shows how to run with temporarily modified versions of just a few routines. In this particular case, the default version of subroutine INIT is replaced by a user-supplied version, and an additional user-supplied geometry option is added to subroutine GEOM. Since these changes affect only INIT and GEOM, only these subroutines are updated and recompiled.

```
ll
```

user-supplied version of INIT goes here
*I GEOM. 128
new user-supplied geometry option goes here

```
21. EOF 
22. update -p $HOME/p2d
23. cft7% -d Pq temp. temp.o $HOME/p2dl0.o
25. temp.e< input
26. cp rqout $HOME/casename/rqin
27. cp rxout $HOME/casename/rxin
28. bintran -mUX - v plotq
29. bintran -mUX -v plotx
30. bintran -mUX -v chist
31. ja -cslt
```

Lines 1 through 3 are actually Cray Network Queueing System (NQS) commands, not UNICOS commands. They must appear first in the runstream, and begin with a \# sign. This first line contains your userid and password, in lower case letters.
Line 2 tells the Cray to put any system error messages into the standard output file (the -eo option), and sets the memory and CPU time limits for the job at 1.0 million words and 60 seconds. On the Cray under LXICOS, the standard output is preconnected to unit 6 . Any system errors will thus be part of the normal PROTELS output, as long as the input parameter $\mathrm{NOUT}=6$.
Line 3 gives the name of the job as EXAMPLE.
Line 4 causes your LNICOS commands to be printed as part of the output runstream.
Line 5 tells the Cray to begin keeping accounting information for later printing.
line 6 fetches the standard PROTEUS input file, containing the job title and the namelist data, from VM, and stores it in a temporary Cray file called input. Filename and filetype are the file name and file type of the file stored on VM.
Lines $7-12$ create empty temporary files with the file names as shown.
lines 13-18 link these temporary files with the indicated Fortran unit numbers. The files are thus the PLOT3D XYZ file, the PLOT3D Q file or CONTOUR plot file, the convergence history file, the restart flow field and mesh files, and the scratch file used with the IPLOT $=-3$ option. These lines implicitly open the files for input and output. Fortran OPEN statements are not used in PROTELS. If the PROTELS input is such that any of these files are unnecessary (see Table 6-1), then the touch and $\ln$ for those files can be climinated.

Line 19 links an existing computational coordinate system file with Fortran unit 7. In this example, this file is assumed to be stored in your home directory, under the subdirectory casename. If the input parameter $\therefore$ GEOM $\neq 10$, this line should be eliminated. If a restart case is being run (IREST $=2$ ), this line should be replaced by the following two lines:

```
ln $HOME/casename/rqin fort.ll
In $HOME/casename/rxin fort.l3
```

The above lines link existing restart flow ficld and computational mesh files with Fortran units 11 and 13. In this example, these files are assumed to be stored in your home directory, under the subdirectory casename.
Line 20 creates a temporary Cray file called mods. The file will consist of all the records between line 20 and line 21, which contains the marker "EOF". Your Cray UPDATE directives and new code should therefore be inserted between lines 20 and 21. The UPDATE directives and new code could also be kept in a file on the front end machine, and fetched just as the input data file was in line 6. In that case, lines 20 and 21 should be eliminated.
Line 22 uses the Cray UPDATE facility to create a temporary file, temp.f, which contains the Fortran code for the modified versions of subroutines GEOM and INIT. The update command uses as input the existing PROTELS program library $p^{2} d / 0 . u$, which is stored in your home directory, and the temporary file mods containing the UPDATE directives and new code.
Line 23 compiles the modified versions of GEOM and INIT, contained in the temporary file iemp.f, storing the object code in the temporary file temp.o.
I ine 24 creates a temporary executable file, temp.e, from the temporary object file temp.o containing the modified versions of GEOM and INII, and from the existing object file p2d/0.o in your home directory.
Line 25 actually runs the program, getting the standard PROTEUS input from the temporary file input.
Lines $26-27$ save the temporary output restart flow field and mesh files by copying them into the files rain and rxin in the subdirectory casename in your home directory. Note that this will overwrite the existing files with the same names. Lise different file names if you need to keep the existing files also. If restart files are not written by your job, these lines should be eliminated.
I ines 28-30 convert the indicated UNICOS unformatted files to VM unformatted files and dispose them to VM. Bintran is a local NASA Lewis command, not a standard Cray UNICOS command. The files will appear in your VM reader, along with the standard PROTEL'S output. If these files are ultimately to be used on some other computer (e.g., a graphics workstation), a different procedure or additional conversion steps may be required. If these files were not created by your job, or if you have no use for them on the front end, then these lines should be eliminated.
Line 31 causes various accounting information to be printed at the end of your output.

### 9.0 TEST CASES

In this section, three test cases are described in detail. The first, developing Couette flow, is a timeaccurate calculation of laminar flow generated in a channel by a moving wall. The second case is flow past a circular cylinder. Both inviscid and laminar viscous flow are computed. The third case is transonic turbulent flow in a converging-diverging channel. The discussion of each test case includes a description of the problem, listings of the PROTELS input file and the JCI , a description of the standard PROTELS output, and figures illustrating the computed results. All the cases were run on the NASA Lewis Cray X-MP running UNICOS 4.0 and using the Bourne shell. The code was compiled using Release 2.0 of CFT77.

### 9.1 DEVELOPING COLETTE FLOW

## Problem Description

Couette flow is incompressible laminar flow between two infinite parallel walls, one at rest and one moving with velocity $u_{w}$. For $\partial p / \partial x=0$, the steady-state velocity profile is linear, as shown in Figure 9.1. In this test case the time-development of this flow was computed by starting with $u=0$ everywhere, and suddenly accelerating the top wall to $u=u_{w}$.


Figure 9.1 - Steady Couette flow with $\partial p / \partial x=0$.

## Reference Conditions

When setting up a problem for PROTELS, it's usually convenient to first fix the reference conditions. For this case, an obvious choice for the reference length $L$, was the distance $\bar{h}$ between the two walls, which we set equal to 1 ft . Standard sea level conditions of $519^{\circ} \mathrm{R}$ and $0.07645 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}$ were used for the reference temperature and density. Since PROTEUS is a compressible code, incompressible conditions must be
simulated by running at a low Mach number. We therefore set the reference Mach number $M_{r}=0.1$. The reference velocity $u$, was then computed by PROTEUS from $M_{r}$. In setting up the boundary conditions, described below, the velocity at the top wall was set equal to 1 . Therefore, $u_{r}=\bar{u}_{w}=0.1 \bar{a}$, where $\bar{a}$ is the dimensional speed of sound. In order to reach steady state within a relatively small number of time steps, the reference Reynolds number $R e$, was set equal to 100.0 .

## Computational Coordinates

For this problem a simple Cartesian computational coordinate system can be used. As shown in Figure 9.1, the physical $(x-y)$ and computational $(\xi-\eta)$ coordinates are thus in the same directions. Since $L_{r}=\bar{h}$, the coordinate limits in the $y$ direction were $y_{\text {min }}=0$ and $y_{\text {max }}=1$. The solution does not depend on $x$, so any limits could have been used in the $x$ direction. Convenient values were $x_{\min }=0$ and $x_{\max }=1$.

## Initial Conditions

Constant stagnation enthalpy was assumed, so only three initial conditions were required. They were simply $u=v=0$ and $p=1$ everywhere in the flow field.

## Boundary Conditions

Similarly, only three boundary conditions were required at each computational boundary. Since the solution is independent of $x$, constant pressure and zero velocity gradient conditions were set at $\xi=0$ and $\xi=1$. At both walls, the $y$-velocity and the normal pressure gradient were set equal to zero. The $x$-velocity was set equal to zero at the lower wall and 1 at the top wall. These conditions are summarized in the following table.

| Boundary | Boundary Conditions |
| :---: | :--- |
| $\xi=0$ | $\partial u / \partial \xi=\partial v / \partial \xi=0, p=1$ |
| $\xi=1$ | $\partial u / \partial \xi=\partial v / \partial \xi=0, p=1$ |
| $\eta=0$ | $u=v=0, \partial p / \partial \eta=0$ |
| $\eta=1$ | $u=1, v=0, \partial p / \partial \eta=0$ |

## PROTELS Input File

The namelist input file for this case was called CFORM P2DIN0 and is listed below, along with a brief explanation of each line. The contents of this listing should be compared with the detailed input description in Section 3.0.

```
UNSTEADY DEVELOPING COUETTE FLOW
    &RSTRT
    &END
    &IO
        IVOUT=1,2,32,47*0,
        IPRTA =1,3,9,33,73,129,
        IPRT1A =1,6,11, IPRT2=1,
        IPLOT=-1,
        IPLTA =3,9,33,73,129,
    &END
    &GMTRY
        NGEOM=1,
        XMIN=0.0, XMAX=1.0,
    YMIN=0.0, YMAX=1.0,
    &END
    &FLOW
        IHSTAG=1,
        MACHR=.1, RER=100.,
    GAMR=1.4,
    &END
    &BC
    JBCl(1,1)=41, JBCl(1,2)=41, GBCl(1,1)=1., GBCl(1,2)=1., p=1 at \xi=0,1.
```

```
    \(\operatorname{JBCl}(2,1)=12, \operatorname{JBCl}(2,2)=12, \operatorname{GBCl}(2,1)=0 ., \operatorname{GBCl}(2,2)=0 ., \quad \partial u / \partial \xi=0\) at \(\xi=0,1\).
    \(\operatorname{JBCl}(3,1)=22, \operatorname{JBCl}(3,2)=22, \operatorname{GBCl}(3,1)=0 ., \operatorname{GBCl}(3,2)=0 ., \quad \partial v / \partial \xi=0\) at \(\xi=0,1\).
    \(J B C 2(1,1)=42, J B C 2(1,2)=42, \quad \operatorname{GBC} 2(1,1)=0 ., \quad G B C 2(1,2)=0 ., \quad \partial p / \partial \eta=0\) at \(\eta=0,1\).
    \(J B C 2(2,1)=11, J B C 2(2,2)=11, \operatorname{GBC} 2(2,1)=0 ., \operatorname{GBC} 2(2,2)=1 ., \quad u=0,1\) at \(\eta=0,1\).
    \(J \operatorname{BC} 2(3,1)=21, J \operatorname{BC2}(3,2)=21, \operatorname{GBC} 2(3,1)=0 ., \operatorname{GBC} 2(3,2)=0 ., \quad v=0\) at \(\eta=0,1\).
\&END
\& NUM
    \(\mathrm{N} 1=11, \mathrm{~N} 2=21, \quad\) Use an \(11 \times 21\) mesh.
    IAV4E \(=0\), IAV2E=0, IAV2I \(=0, \quad\) No artificial viscosity.
    THC=1.0,0.5,
    \(\mathrm{THX}=1.0,0.5,1.0, \quad\) Second-order time differencing.
    \(T H Y=1.0,0.5,1.0\),
\&END
\& IIME
    IDTMOD \(=1000\),
    IDTAU \(=3\), DT \(=.1953125, \quad\) Constant global \(\Delta \tau\).
    NTIME \(=1000\), Limit of 1000 time steps.
\&END
\&TURB
\& END
\&IC
\& END
Don't recompute \(\Delta \tau\).
```

L.aminar flow.

Use default initial conditions.

Note that since the defaults for IVOUT(4) and IVOUT(5) are 30 and 40 , they are set to zero in namelist 10 to avoid printout of the static pressure and temperature. The time levels specified for the printout and plot files correspond to the time values used in a plot of an exact solution to this problem given by Schlichting (1968). Since the solution should not depend on $x$, only 11 points are used in the $\xi$ direction, and results are printed at only three $\xi$ indices. In namelist FIOW, the only specified reference conditions are MACHR and RER, since the desired values for the remaining ones are the same as the default values. IL $\Lambda$ MV is defaulted, resulting in constant viscosity $\mu$ and thermal conductivity $k$. In namelist BC, the JBC values corresponding to derivative boundary conditions are positive, specifying that two-point one-sided differences are to be used. In namelist NUM, the parameters IPACK and SQ are defaulted, which results in an evenly spaced mesh in both directions. The artificial viscosity is turned off because of the low Reynolds number. The second-order three-point backward-implicit time differencing scheme is being used because we want an accurate unsteady solution. ${ }^{29}$ The time step size DT in namelist TIME is simply half of the first time value in the plot of the exact solution mentioned above, and corresponds to a CFL number of about $44 .{ }^{30}$

## JCL

The Cray UNICOS job control language used for this case is listed below.

```
# USER=yourid PW=
    * QSUB -eo -1M 1.0mW -1T 60
    # QSUB -r CFORM
set -x
ja
touch plot
touch chist
ln plot fort.9
ln chist fort.10
fetch input -mUX -t'fn=cform,ft=p2din0'
segldr -o temp.e $HOME/p2dl0.o
temp.e < input
bintran -mUX -v plot
```

[^21]```
bintran -mUX -v chist
ja-cslt
```

This JCL is essentially the same as the example presented in Section 8.2, but with lines eliminated that are not applicable to this case.

## Standard PROTELS Output

The output listing is shown below. In the flow field printout, only the last time level is included. Note that a converged solution is obtained at time level 350 , and that this level is automatically included in the standard output and in the plot file.

> NASA LEWIS RESEARCH CENTER
> INTERNAL FLUID MECHANICS DIVISION
> 2-D PROTEUS VERSION 1.0
> SEPTEMBER 1989

UNSTEADY DEVELOPING CDUETTE FLDN


NORMALIZING CONDITIONS

boundary condition parameters


```
    10
Y OR R VELOCITY
    AT TIME LEVEL
    350, TIME =6.1045E-01 SEC, DTIME = 1.7491E-03 SEC
    IXI= l
    21 0.0000E+00
    20-1.7459E-08 1.9710E-07 -1.5651E-07
    19-8.5587E-08 5.3278E-07-3.3265E-07
    18-1.1763E-07 7.3490E-07 -4.7745E-07
    17 -1.3305E-07 6.3559E-07 -5.4990E-07
    16 -1.2631E-07 3.0707E-07 -5.7216E-07
    15-1.1410E-07-1.7391E-07 -5.1893E-07
    14 -7.8428E-08-7.0845E-07 -4.1569E-07
    -3.2256E-08 -1.2344E-06 -2.7445E-07
    2.4906E-08 -1.6908E-06-1.3457EE-07
    1.1215E-07 -2.03888E-06 -1.2009E-08
    1.333EE-07 -2.2514E-06 7.3234E-08
    1.4573E-07 -2.3171E-06 1.2360E-07
    1.4912E-07 -2.0330E-06 1.3171E-07
    1.5303E-07 -1.7189E-06 ,0759E-07
        1. E.08IE-07-1.3164E-06 7.4553E-08
        4102E-07 -8.7928E-07 3.5631E-08
        0:25E~07 -4.4715E-07 -4.8431EE-09
    4.0214E-08-1.3386E-07 -1.6290E-08
    0.0000F-00 0.0000E+00 0.0000E+OD
STATIC PRESS. COEFF. AT TIME LEVEL 350, TIME = 6.1045E-01 SEC, DTIME = 1.7491E-03 SEC
    IM1 = 1 1 11
    21-2.0258E-02-1.9536E-02 -2.0259E-02
    20-1.8190E-12 -1.9536E-02 -9.0949E-13
    9-6.3665E-12 -1.9536E-02 -6.3665E-12
    18-1.3642E-11 -1.9536E-02 -1.4552E-111
    17-2.2737E-11 -1.9536E-02 -2.2737E-111
    16 -3.2742E-11 -1.9536E-02 -3.1832E-111
    15 -4.2746E-11 -1.9536E-02 -4.2746E-11
    13-5.9117E-11 -1.9536E-02 -5.9117E-11
    12 -6.3665E-11-1.9536E-02 -6.2755E-11
    11 -6.4574E-11 -1.9536E-02 -6.4574E-11
    0-6.2755E-11 -1.9536E-02 -6.3665E-11
    9-6.0027E-11 -1.9536E-02 -5.9117E-11
    8-5.184[E-11 -1.9537E-02 -5.2751E-11
    7 -4.2746E-11 -1.9527E-02 -4.3656E-11
    6 - 3.3651E-11-1.9537E-02 - J.2742E-11
    5 -2.3647E-11 -1.9537E-02 -2.1828E-11
    4-1.3042E-11-1.9537E-02 -1.3642E-11
    3-6.3665E-12-1.9537E-02-7.2760E-12
    2-1.8190E-12-1.9537E-02 -9.0949E-13
*** CONVERGED SOLUFION AT TIME LEVEL 350
    STOP
```

This case used 4.8 seconds of CPU time.

## Computed Results

As noted earlier, an exact solution exists for this problem (Schlichting, 1968). The solution is in the form of a series of complementary error functions, and is given by ${ }^{31}$

$$
\begin{aligned}
\frac{u}{u_{w}} & =\sum_{n=0}^{\infty} \operatorname{erfc}\left[(2 n+1) \phi_{1}-\phi\right]-\sum_{n=0}^{\infty} \operatorname{erfc}\left[(2 n+1) \phi_{1}+\phi\right] \\
& =\operatorname{erfc}\left(\phi_{1}-\phi\right)-\operatorname{erfc}\left(\phi_{1}+\phi\right)+\operatorname{erfc}\left(3 \phi_{1}-\phi\right)-\operatorname{erfc}\left(3 \phi_{1}+\phi\right)+\cdots-\cdots
\end{aligned}
$$

where

[^22]\[

$$
\begin{aligned}
\phi & =\frac{y}{2 \sqrt{v t}} \sqrt{R e_{r}} \\
\phi_{1} & =\frac{h}{2 \sqrt{v t}} \sqrt{R e_{r}}
\end{aligned}
$$
\]

The results computed using PROTELS are compared with the exact solution in Figure 9.2. The results are plotted at times corresponding to $4 \sqrt{v t} / h \sqrt{R e_{r}}=0.25,0.5,1.0,1.5$, and 2.0 , plus the steady state solution.


Figure 9.2 - Computed and exact solutions for developing Couette flow.

### 9.2 FLOW PAST A CIRCLLAR CYLINDER

## Problem Description

In the second test case, steady flow past a two-dimensional circular cylinder was investigated. Both Euler and viscous flow were computed. The geometric configuration (not to scale) is shown in Figure 9.3.


Figure 9.3 - Flow past a circular cylinder.

## Reference Conditions

The cylinder radius was used as the reference length $L_{r}$, and was set equal to 1 ft . Standard sea level conditions of $519^{\circ} \mathrm{R}$ and $0.07645 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}$ were used for the reference temperature and density. In order to allow comparison of the PROTELS results with incompressible experimental data and with potential flow results, the reference Mach number $M_{r}$, was set to the low value of 0.2 . The reference velocity $u_{r}$ was then computed by PROTEUS from $M_{r}$. The experimental data were taken at a Reynolds number based on cylinder diameter of 40 . Since our reference length was the cylinder radius, the reference Reynolds number $R e$, was 20.

## Computational Coordinates

For this problem a polar computational coordinate system was the obvious choice. Figure 9.3 shows the relationship between the physical Cartesian $(x-y)$, physical polar $(r-\theta)$, and computational ( $\xi-\eta)$ coordinates. The coordinate limits in the $r$ direction were $r_{\min }=1$ and $r_{\max }=30$. Since the flow is symmetric about the $x$ axis, only the top half of the flow field was computed. The limits in the $\theta$ direction were thus $\theta_{\min }=0^{\circ}$ and $\theta_{\max }=180^{\circ}$. The $\xi=0$ and $\xi=1$ boundaries thus correspond to the $\theta=\theta^{\circ}$ and $\theta=180^{\circ}$ boundaries, respectively. The $\eta=0$ and $\eta=1$ boundaries correspond to the $r=1$ and $r=30$ boundaries.

## Initial Conditions

Constant stagnation enthalpy was assumed, so only three initial conditions were required. For the Euler flow case, uniform flow with $u=1, v=0$, and $p=1$ was used.

For the viscous flow case, the exact potential flow solution was used to set the initial conditions at all the non-wall points. Thus, with nondimensional free stream conditions of $\rho_{\infty}=u_{\infty}=T_{\infty}=p_{\infty}=1$,

$$
\begin{gathered}
u=1-\frac{1}{r^{2}} \cos (2 \theta) \\
v=-\frac{1}{r^{2}} \sin (2 \theta) \\
p=\left(p_{T}\right)_{\infty}-\frac{1}{2} \rho_{\infty}\left(u^{2}+v^{2}\right)
\end{gathered}
$$

where

$$
\left(p_{T}\right)_{\infty}=p_{\infty}+\frac{1}{2} \frac{\rho_{\infty} u_{\infty}^{2}}{R}
$$

Note that the nondimensional gas constant $R$ appears in the above equation. This is a result of nondimensionalizing the initial condition for pressure by the reference pressure $p_{r}=\rho_{r} \bar{R} T_{r}$. (See Section 3.1.1). At the cylinder surface, we set the velocities $u$ and $v$ equal to zero, and the pressure $p$ equal to the pressure at the grid point adjacent to the surface. Thus, with two-point one-sided differencing, $\partial p / \partial n=0$ at the surface.

## Boundary Conditions

Again, since we assumed constant stagnation enthalpy, only three boundary conditions were required at each computational boundary. For the Euler flow case, symmetry conditions were used at $\xi=0$ and $\xi=1$. At $\eta=0$, the cylinder surface, the radial velocity and the radial gradient of the circumferential velocity were set equal to zero. The radial gradient of pressure was computed from the polar coordinate form of the incompressible radial momentum equation written at the wall. The equation is (IIughes and Gaylord, 1964)

$$
\rho v_{r} \frac{\partial v_{r}}{\partial r}+\frac{\rho v_{\theta}}{r} \frac{\partial v_{r}}{\partial \theta}-\rho \frac{v_{\theta}^{2}}{r}=-\frac{\partial p}{\partial r}
$$

where $v_{r}$, and $v_{\theta}$ are the radial and circumferential velocities, respectively. At the cylinder surface, $v_{r}=0$. Thus, at $\eta=0$,

$$
\frac{\partial \rho}{\partial r}=\rho \frac{v_{\theta}^{2}}{r}=\rho \frac{u^{2}+v^{2}}{r}
$$

This must be transformed into computational coordinates using

$$
\frac{\partial p}{\partial r}=\frac{\partial p}{\partial \eta} \frac{\partial \eta}{\partial r}+\frac{\partial p}{\partial \xi} \frac{\partial \xi}{\partial r}
$$

With the computational coordinate system being used for this problem, $\partial \xi / \partial r=0$. The boundary condition in computational coordinates was thus

$$
\frac{\partial p}{\partial \eta}=\frac{\rho v_{\theta}^{2}}{r}\left(\frac{\partial \eta}{\partial r}\right)^{-1}
$$

where, since $x=r \cos \theta$ and $y=r \sin \theta$,

$$
\begin{aligned}
\frac{\partial \eta}{\partial r} & =\frac{\partial \eta}{\partial x} \frac{\partial x}{\partial r}+\frac{\partial \eta}{\partial y} \frac{\partial y}{\partial r} \\
& =\frac{\partial \eta}{\partial x} \cos \theta+\frac{\partial \eta}{\partial y} \sin \theta
\end{aligned}
$$

And finally, at $\eta=1$ the free stream conditions were specified as boundary conditions. These conditions are summarized in the following table.

$$
\begin{array}{cl}
\text { Boundary } & \text { Boundary Conditions } \\
=0 & \partial u / \partial \xi, v=0, \partial p / \partial \xi=0 \\
\xi=1 & \partial u / \partial \xi, v=0, \partial p / \partial \xi=0 \\
\eta=0 & V_{r}=0, \partial V_{\theta} / \partial \eta=0, \partial p / \partial \eta=\rho v_{\theta}^{2} /(r \partial \eta / \partial r) \\
\eta=1 & u=1, v=0, p=1
\end{array}
$$

For the viscous flow case, symmetry conditions were again used at $\xi=0$ and $\xi=1$. At $\eta=0$, the cylinder surface, no-slip conditions were used for the velocity, and the radial pressure gradient was set equal to zero. The outer boundary, at $\eta=1$, was split into an inlet region and wake region. The split was made, somewhat arbitrarily, at $\theta=45^{\circ}$. In the inlet region, the boundary values of $u, v$, and $p$ were kept at their initial values, which were the potential flow values. In the wake region, the boundary values of $p$ were kept at their initial values, and the radial gradients of $u$ and $v$ were set equal to zero. These conditions are summarized in the following table.

## Boundary Boundary Conditions

$$
\begin{array}{cl}
\xi=0 & \partial u / \partial \xi, v=0, \partial p / \partial \xi=0 \\
\xi=1 & \partial u / \partial \xi, v=0, \partial p / \partial \xi=0 \\
\eta=0 & u=v=0, \partial p / \partial \eta=0 \\
\eta=1 \text { (inlet) } & \Delta u=\Delta v=\Delta p=0 \\
\eta=1 \text { (wake) } & \partial u / \partial \eta=\partial v / \partial \eta=0, \Delta p=0
\end{array}
$$

## PROTEUS Input File for Euler Flow Case

The namelist input file for the Euler flow case was called CYIPF P2DIN0 and is listed below, along with a brief explanation of each line. The contents of this listing should be compared with the detailed input description in Section 3.0.

```
EULER FLOW PAST A CIRCULAR CYLINDER
    &RSTRT Not a restart case.
    &END
    &IO
        IVOUT=1,2,32,47*0,
    IPRT=10000,
    IPRT1=1, IPRT2=2,
    IPLOT=-1,
&END
&GMTRY
    NGEOM=2, Polar computational coordinates.
    RMIN=1.0, RMAX=30.0, r-coordinate limits.
    THMIN=0.0, THMAX=180.0, 0-coordinate limits.
    &END
&FLOW
    IEULER=1,
    IHSTAG=1,
    MACHR=.2, RER=20.,
    GAMR=1.4,
&END
&BC
    JBCl(1,1)=42, JBCl(1,2)=42, GBCl(1,1)=0., GBCl(1,2)=0., 
    JBCl}(2,1)=12, JBCl(2,2)=12, GBCl(2,1)=0., GBCl(2,2)=0., 讪/\partial\xi=0 at \xi=0,1.
```

```
JBCl(3,1)=21, JBC1(3,2)=21, GBCl(3,1)=0., GBCl(3,2)=0., v=0 at 5=0,1.
```



```
    p=1 at }\eta=0,1
V
    at }\eta=0,1
JBC2(3,1)=77, JBC2(3,2)=21, GBC2(3,1)=0., GBC2(3,2)=0.,
aV, }\mp@subsup{\sigma}{n}{
    at \eta}=0,1
&END
    N1=21, N2=51, Use a 21\times51 mesh.
    IPACK(2)=1, Pack in \eta direction.
    SQ(2,1)=0., SQ(2,2)=1.01, Pack moderately tighly near }\eta=0
&END
&TIME
    IDTAU=5, CFL=10., Spatially varying \Delta\tau.
    NTIME=1000, Limit of 1000 time steps.
&END
&TURB Laminar flow.
&END
&IC
    UO=1., Uniform flow initial conditions.
&END
```

In namelist IO, setting IPRT equal to a number larger than the number of time steps to be taken results in a printout at the initial and final time levels only.

## PROTELS Input File for Viscous Flow Case

The namelist input file for the viscous flow case was called CYLV P2DIN0 and is listed below, along with a brief explanation of each line. The contents of this listing should be compared with the detailed input description in Section 3.0.

```
VISCOUS FLOW PAST A CIRCULAR CYLINDER
    &RSTRT
    &END
    &IO
        IVOUT=1,2,32,47*0,
        IPRT=10000,
        IPRT1=2, IPRT2=2,
        IPLOT=-1,
    &END
    &GMTRY
        NGEOM=2,
        RMIN=1.0, RMAX=30.0,
        r-coordinate limits.
        0-coordinate limits.
    &END
    &FLOW
        IHSTAG=1,
        MACHR=.2, RER=20.,
        GAMR=1.4,
    &END
    &BC
        JBCl(1,1)=42, JBCl(1,2)=42,
        JBCl(2,1)=12, JBCl}(2,2)=12,
        JBCl(3,1)=21, JBCl(3,2)=21,
        JBC2(1,1)=42,
        JBCC2(2,1)=11,
        JBC2(1,1)=11,
    &END
    &NUM
    N1=51, N2=51, Use a 51\times51 mesh.
    IPACK(2)=1,
    SQ(2,1)=0., SQ(2,2)=1.001,
GBCl(1,1)=0., GBCl(1,2)=0., }\quad\partialp/\partial\xi=0\mathrm{ at }\xi=0,1
GBCl(2,1)=0., GBCl(2,2)=0., ould\xi=0 at \xi=0.,i.
GBC1(3,1)=0., GBC1(3,2)=0., v=0 at }\xi=0,1
GBC2(1,1)=0., }\quad\partialp/\partial\eta=0\mathrm{ at }\eta=
GBC2(2,1)=0., u=0 at \eta=0.
GBC2(3,1)=0., v=0 at }\eta=0
Constant stagnation enthalpy.
Set \(M_{r}\) and Re, .
Constant specific heats.
\& END
\& BC
```

$\operatorname{JBCl}(2,1)=12, \operatorname{JBCl}(2,2)=12$, $\mathrm{JBCl}(3,1)=21, \quad \mathrm{JBCl}(3,2)=21$,

Use a $51 \times 51$ mesh.
Pack in $\eta$ direction.
Pack fairly tightly near $\eta=0$.
\& END
\&TIME

```
    IDTAU=5, CFL=10.,
```

    NTIME \(=1000\),
    \&END
\&TURB
\&END

Spatially varying $\Delta \tau$
Limit of 1000 time steps.
Laminar flow.

Vote that in namelist BC , boundary conditions are not specified at the outer, or $\eta=1$, boundary. These conditions will be set in subroutine INIT, as described below.

## JCL for Euler Flow Case

The Cray UNICOS job control language used for the Euler flow case is listed below.

```
* USER=yourid PW=.
# QSUB -eo -1M l.0mW -1T 60
* QSUB -r cyl2
set -x
ja
touch plot
touch chist
ln plot fort.9
ln chist fort.10
fetch input -mUX -t'fn=cylpf,ft=p2dinO'
cat > mods << EOM
*IDENT MODS
*PURGEDK BCFLIN
*DK BCFLIN
    SUBROUTINE BCFLIN (IBC,IEQ,IBOUND,IMIN,IMAX,F,DFDRHO,DFDRU,DFDRV,
        $
    DFDRW,DFDET,FBC
C
THIS IS A USER-SUPPLIED SUBROUTINE USED IN CONJUNCTION
WITH THE GENERAL BOUNDARY CONDITION ROUTINE BCF (JBC
                                    OR IBC OPTIONS 90-99). IT COMPUTES THE VALUES NEEDED
                                    FOR LINEARIZATION OF THE BOUNDARY CONDITION (I.E., THE
                                    VALUES OF THE FUNCTION F AND ITS DERIVATIVES WRT THE
                                    DEPENDENT VARIABLES). NOTE THAT DIFFERENT USER-SUPPLIED
                                    BOUNDARY CONDITIONS CAN BE USED AT DIFFERENT BOUNDARIES
                                    THROUGH USE DF THE VALUES OF ISWEEP, IEQ, AND IBOUND.
                                    THIS VERSION SETS THE PRESSURE BOUNDARY CONDITION FOR
                                    INVISCID FLOW OVER A CIRCULAR CYLINDER. THE PRESSURE
                                    GRADIENT IN THE RADIAL DIRECTION IS SET EQUAL TO THE
                                    CURVATURE TERM, RHO*VTHETA**2/R, IN THE RADIAL MOMENTUM
                                    EQUATION. IN COMPUTATIONAL COORDINATES, THE B. C. IS
                                    DP/DETA = RHO*VTHETA**2/(R*DETA/DR).
C-----CALLED BY: BCF
```

```
DPDRHO, DPDRU, - DERIVATIVES OF PRESSURE WITH RESPECT TO DPDRV, DPDET, RHO, RHO※U, RHO※V, ET, AND RHO*W
DPDRW
ETAX, ETAY - METRICS OF GRID TRANSFORMATION (I.E., DERIVATIVES OF ETA WRT \(X\) AND \(Y\) )
IBASE, ISTEP - BASE INDEX AND STEP FACTOR FOR I-D INDEXING OF 2-D ARRAYS
IBC - BOUNDARY CONDITION TYPE FOR CURRENT SWEEP DIRECTION
IBOUND - FLAG SPECIFYING BOUNDARY; 1 FOR LOWER, 2 FOR UPPER
IEQ - BOUNDARY CONDITION EQUATION NUMBER (1 TO NEQ)
```

```
C
C-----COMMON BLOCKS
*CALL PARAMSI
*CALL DUMMY1
*CALL FLOW1
*CALL METRICI
*CALL NUMI
    DIMENSION IBC(NEQP,NBC),FBC(NEQP,NBC)
    DIMENSION F(3),DFDRHO(3),DFDRU(3),DFDRV(3),DFDRW(3),DFDET(3)
    DIMENSION IIW(3),JJW(3)
C
C-----SET 1-D INDICES FOR WALL AND ADJACENT POINTS
    IF (IBOUND .EQ. 1) THEN
        IIW(1) = IBASE + ISTEP*(IMIN-1)
        IIW(2) = IBASE + ISTEP*(IMIN )
            IIW(3) = IBASE + ISTEP*(IMIN+1)
            JJW(1) = IMIN
            JJW(2) = IMIN + 1
            JJW(3) = IMIN + 2
            ELSE
            IIW(1) = IBASE + ISTEP*(IMAX-1)
            IIW(2) = IBASE + ISTEP*(IMAX-2)
            IIW(3) = IBASE + ISTEP*(IMAX-3)
            JJW(1) = IMAX
            JJW(2) = IMAX - 1
            JJW(3) = IMAX - 2
            ENDIF
C
C-----SET F, DFDRHO, ETC., EQUAL TO PRESSURE AND ITS DERIVATIVES WRT
C----- DEPENDENT VARIABLES
    DO 10 IW = 1,3
    II = IIW(IW)
    JJ = JJW(IW)
    F(IW) = Pl(II)
    DFDRHO(IW) = DPDRHO(JJ)
    DFDRU(IW) = DPDRU(JJ)
    DFDRV(IW) = DPDRV(JJ)
    DFDRW(IW) = DPDRW(JJ)
    DFDET(IW) = DPDET(JJ)
10 CONTINUE
C-----SET FBC EQUAL TO B. C. VALUE
    II = IIW(I)
```

```
        RADIUS = SQRT(X1(II)**2 + YI(II)**2)
        THETA = ATAN2(Y1(II),X1(II))
        DETADR = ETAXI(II)*COS(THETA) + ETAYI(II)*SIN(THETA)
        VTHSQ = Ul(II)**2 + VI(II)**2
        FBC(IEQ,IBOUND)= RHOI(II)*VTHSQ/RADIUS/DETADR
C----AT BEGINNING OF SWEEP, FILL DUMMY IF SPECIFYING DF/DN
    IF (IABS(IBC(IEQ,IBOUND)) .NE. 93) RETURN
    IF (ISWEEP.EQ.1 .AND. I2.GT.2) RETURN
    IF (ISWEEP.EQ.2 .AND. II.GT.2) RETURN
    DO 20 J = 1,N2
    DO 20 I = l,Nl
    DUMMY(I,J) = P(I,J)
    CONTINUE
    RETURN
    END
EOM
update -p $HOME/p2dlO.u -i mods -c temp -q BCFLIN
cft77 -d pq temp.f
segldr -o temp.e temp.o $HOME/p2dl0.o
temp.e < input
bintran -muX -v plot
bintran -mUX -v chist
ja -cslt
```

I his JCI is very similar to the example presented in Section 8.4. In this test case, we are making a temporary change to the code by supplying a new version of subroutine BCFLIN to implement the boundary condition for pressure at the cylinder surface. BCFIIN is used in conjunction with subroutine BCF for writing boundary conditions that are not among those already built into the code. These routines are described in detail in Volume 3. For the current case, we set $F=p, \partial F / \partial \rho=\partial p / \rho_{\rho}$, cte. FBC, the boundary condition value, is set equal to $\rho v_{i}^{2}\left(r_{y} / / \frac{r}{2}\right)$.

## JCL for Viscous Flow Case

The Cray LNCOS job control language used for the viscous flow case is listed below.

```
# USER=yourid PW=.
# QSUB -eo -1M 1.0mW -1T 60
# QSUB -r cyll
set -x
ja
touch plot
touch chist
ln plot fort.9
ln chist fort.10
fetch input -mUX -t'fn=cylv,ft=p2din0'
cat > mods << EOM
*IDENT NEWINIT
*PURGEDK INIT
*DECK INIT
    SUBROUTINE INIT
C
C----PURPOSE: SET UP INITIAL FLOW FIELD FOR VISCOUS FLOW PAST A
C CIRCULAR C'HINDER. THE EXACT POTENTIAL FLOW SOLUU
                                    IS USED AS THE INITIAL FLOW FIELD, EXCEPT AT THE
                                    PRESSURE GRADIENT ARE SET EQUAL TO ZERO. THIS ROUTINE
                                    IS ALSO USED TO SET POINT-BY-POINT BOUNDARY CONDITIONS
                                    AT THE OUTER (ETA = 1) BOUNDARY.
C-----CALLED BY: INITC
C----CALLS:
C
ICVARS - FLAG SPECIFYING WHICH VARIABLES ARE BEING SUPPLIED
                                    AS INITIAL CONDITIONS
```



```
        DO 200 IEQ = 1, NEQ
        JBC2(IEQ,2) = -1
200 CONTINUE
C
    NWAKE = Nl/4
C-----IN WAKE REGIDN
    DO 210 II = l,NWAKE
    IBC2(I1,1,2) = 40
    IBC2(Il,2,2)=12
    IBC2(Il,3,2) = 22
    FBC2(Il,1,2)=0.
    FBC2(I1,2,2)=0.
    FBC2(11,3,2)=0.
210 CONTINUE
C-----IN INLET REGION
    DO 220 Il = NWAKE+1,N1
    IBC2(I1,1,2) = 40
    IBC2(I1,2,2) = 10
    IBC2(I1,3,2) = 20
    FBC2(Il,1,2)=0
    FBC2(I1,2,2)=0.
    FBC2(11,3,2) = 0.
220 CDNTINUE
    RETURN
    END
EOM
update -p $HOME/p2dl0.u -i mods -c temp -q INIT
cft77 -d pq temp.f
segldr -o temp.e temp.o $HOME/p2dl0.o
temp.e < input
bintran -mUX -v plot
bintran -mUX -v chist
ja -cslt
```

This JCl , is also very similar to the example presented in Section 8.4. In this case, we are making a temporary change to the code by supplying a new version of subroutine INIT to set the initial conditions described earlier. The procedure for using user-written initial conditions is described in Section 5.1.

Note that in this case we are also using subroutine INIT to set point-by-point boundary condition types and values at the outer $(\eta=1)$ boundary. It's often easier to set point-by-point boundary conditions in Fortran rather than in the namelist input file.

## Standard PROTEUS Output for Euler Flow Case

The output listing for the Euler flow case is shown below. In the flow field printout, only the last time level is included. Note that a converged solution is obtained at time level 210, and that this level is automatically included in the standard output and in the plot file.

```
                                    NASA LEWIS RESEARCH CENTER
INTERNAL FLUID MECHANICS DIVISION
    2-D PROTEUS VERSION 1.0
        SEPTEMBER 1989
```

EULER FLOW PAST A CIRCULAR CYLINDER




```
    IUNITS = 0, IVOUT = 1, 2, 32, 47*D, NGRID = 7, NHIST = 10, NHMAX = 100, NOUT = 6, NPLOT = 9, NPLOTX = 8,
8GMTRY IAXI = 0, NGEOM = 2,
    YMAX = 1.O, YMIN = O.O, &END , HS, ICVARS = 2, IEULER = 1, IHSTAG = 1, ILAMV = O, ISWIRL = 0, ITHINN = 2*O,
```



```
    KTR = 0.0, LR = 1.0, MACHR = 0.2, MUR = 0.0, PRLR = 0.0, RER = 20.0, RG = 1716.0, RHOR = 7.645E-02, TR = 519.0,
```





```
    IPACK = 0, 1, N1 = 21, N2 = 51, SO = 2*0.0, 10000.0, 1.01, THC = 1.0, 2*O.0, THE = 1.0, 2*0.0, THX = 1.0,
```



NORMALIZING CONOITITR＊：


REFERENCE CONOLIIUN：

| REYNULDS NUMEER，RER | $\therefore 0000 \mathrm{E} \cdot 01$ |
| :---: | :---: |
| MACH NUMBER，MACHR | $2.0000 \mathrm{E}-01$ |
| SPECIFIC HEAT RATIO．GAMR | ．．h000e－00 |
| LAMINAR PRANDTL NUMEER，PRLP． | $=5.050 \mathrm{OE} \cdot 04$ |
|  | 4．0484Eas |
|  |  |
| SFPECIFIE litat at tomel vil |  |
| GAS CONSTANT，KG |  |
| PRESSURE．FȦ | $=\therefore$－ioctoc ieffote |
| Stagnation enthaipy．bごTatr | 3．14．1E＊OD FTZ／SEC2 |



| $x$－vElo | OCITr | al 1／ame | E： 20 |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IXI | $=$ | 2 | $\Sigma$ | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
| IETA |  |  |  |  |  |  |  |  |  |  |
| 51 | 1．00000．00 | 1．0000E＊00 | 1．0000F－00 | 1．0000E +00 | 1．0000F＋00 | 1．0000E＋00 | 1．0000E－00 | $1.0000 E+00$ | 1．0000E＊00 | $1.00000 \cdot 00$ |
| 50 | $1.0008 \mathrm{E}+00$ | 1．0008E +00 | $1.00085+00$ | 1．O0022E＋00 | 9．9977E－01 | $9.9953 E-01$ | $9.9953 \mathrm{E}-01$ | $9.9973 \mathrm{E}-01$ | $1.0000 E+00$ | 1．0003E＊00 |
| 48 | 1．0006E +00 | 1．0006E．00 | 1．0004E＋00 | 1．0001E＋00 | $9.9085 E 01$ | $9.9983 \mathrm{E}-01$ | 1．0001E＋00 | 1．0005E 000 | $1.0009 \mathrm{E}+00$ | 1．0013E－00 |
| 66 | 1．0008E－00 | 1．0008E +10 | 1． $000 \mathrm{bE}+00$ | 1．0002e +00 | 9．9932E－01 | $9.999 \mathrm{EE}-01$ | 1．0003E＊00 | $1.0009 \mathrm{E} \cdot 00$ | 1．0015E＋00 | 1．0020E＊90 |
| 44 | 1．a010E＋00 | 1．Bototeno | 1．E0006 +30 | －．0001E＋00 | 9． $9+80 \mathrm{E}-61$ | $9.9993 E-01$ | 1．0005E＋00 | $1.0013 \mathrm{E}-00$ | 1．0021E＋00 | 1．9027E 90 |
| 42 | 1．0011E．00 | 1．0011200 | 1．000ter 0 | 9.7990701 | 9.9955 E －01 | 9．9982E－01 | 1．0007E 20 | 1．0018E＊00 | 1．00こ8E＋00 | 1．0036E．00 |
| 40 | $1.0010 \mathrm{E} \cdot 00$ | 1．0010E＋00 | 1．0004E＋00 | 9．945．2t 01 | $9.9917 \mathrm{E}-01$ | $9.9969 \mathrm{E}-01$ | $1.0009 E+00$ | 1．0025E＋00 | $1.0039 \mathrm{E} \cdot 00$ | 1．004BE－00 |
| 38 | $1.0008 \mathrm{E}+00$ | 1．0008E－00 | 9．990．2e Ol | 9．9880t－01 | 9.9800 E － 01 | 9.995 EE－01 | $1.0014 \mathrm{E}+00$ | $1.0035 E+00$ | 1．005 5 ［ +00 | $1.0065 \mathrm{E} \cdot 00$ |
| 56 | 1．0003E－00 | 1．9003E－00 | 9.4904501 | 9．9779［－01 | $9.9775 \mathrm{E}-01$ | 9．993JE－01 | $1.0020 E+00$ | 1．0049E－00 | 1．0074E－00 | 1． $0090 \mathrm{E} \cdot 00$ |
| 34 | $9.9905 E 01$ | 4.1405 E －01 | 4，उंदTE d | ？． C －03E－01 | $9.90532-01$ | 9． $9911 \mathrm{E}-01$ | $1.0030 \mathrm{E}+00$ | 1．0071E＋00 | $1.0104 \mathrm{E}+00$ | $1.0126 \mathrm{E}+00$ |
| 52 | 9．9084E OI | 9.7084501 | 9． 0.685 E －1 | 9．ajust 01 | 9，94TOE O1 | 9．9895 ${ }^{\text {a }}$－ 01 | $1.0046 \mathrm{E}+00$ | 1．0102E＋00 | 1．0149E－00 | 1．0177E +00 |
| 30 | $9.9305 E-191$ | 9．0305E 01 | 9.9003501 | 9． P ＊5， $3 \mathrm{E}-01$ | 9.9255 E 01 | $9.9 B 82 E-01$ | $1.0069 \mathrm{E}+00$ | $1.0148 \mathrm{E}+00$ | $1.0210 \mathrm{E}+00$ | $1.0251 \mathrm{E}+00$ |
| 28 | 9.8689 f－01 | 9.8089 El | 9．HGOPE－？1 |  | 9．8900E－01 | 9．9879E－01 | 1.010 SE +00 | 1．0212E +00 | $1.0298 \mathrm{E}+00$ | 1． $03555 \mathrm{E}+00$ |
| 26 | 9．7728E－01 |  | 9.7431201 | 7． $3503 \mathrm{E}-01$ | $9.8448 \mathrm{EE}-01$ | 9．9891E－01 | 1．0151E＋00 | $1.0301 \mathrm{E}+00$ | 1．0422E＋00 | 1．0501E +00 |
| 24 | $9.6283 E-01$ | 9．0．883t－01 | 9． 60 ［汭－01 | 9．CuS－E－01 | 9． $7848 \mathrm{E}-01$ | 9．9925E－01 | $1.0218 \mathrm{E}+00$ | 1．0425E＋00 | $1.0592 \mathrm{t}+00$ | $1.0701 \mathrm{E}+00$ |
| 22 | 9.4184 E － 01 | 9．4184E－01 | 9.6022 El | 7．\％9112 OI | $9.7067 \mathrm{E}-\mathrm{61}$ | 9．9991E－01 | 1．0309E＋00 | 1．0594E＋00 | $1.0823 E+00$ | $1.0972 \mathrm{E}+00$ |
| 20 | 9．12こ1E－01 | 9．1231E－01 | 9．1319E－01 | a． a $^{\text {a }}$－5E－01 | $9.0071 E-01$ | 1．0010E＋00 | $1.0431 E+00$ | 1．0817［－00 | 1．1159E＋00 | 1．1331E－00 |
| 18 | 8．72071－01 | 8．7．07E 01 | 8．7784E 01 | －ก¢¢18F－01 | $9.4829 E 61$ | 1．0025E．00 | $1.0590 \mathrm{E}+00$ | 1．1108E +00 | $1.1520 \mathrm{E}+00$ | 1．1797E 00 |
| 16 | 8．1892E 0： | 8．18921－01 | 8.3340501 | 6．7372E－01 | 9．3さ19E．01 | $1.0047 \mathrm{E} \cdot 00$ | 1．0792E＋00 | $1.1478 \mathrm{E}+00$ | 1．20300．00 | 1．2388E－00 |
| 14 | 7．50885－01 | 7．5088E 01 | 7．79705－01 | 3．3810t－01 | $9.1545 E \cdot 01$ | 1．0．976E＊00 | 1．1044E＋00 | 1．1935E＋00 | 1．2651E＋00 | 1． $3115 \mathrm{E}+\mathrm{CO}$ |
| 12 | －． 6 656E－01 | 0.0 cober－01 | フ． $1087 \mathrm{~F}-01$ | 7．984n5－11 | 8．9575E－01 | 1．0113E＋00 | $1.1346 E+00$ | 1． $2484 \mathrm{E}+00$ | 1． $3396 \mathrm{E}+00$ | 1． $3486 \mathrm{E}+00$ |
| 10 | 5．05U5E－01 | 5．0．0．05t－01 | b．4．415－01 |  | 8．70．8E－01 | 1．0165E＋00 | 1．1693E＋00 | 1．3122E＋00 | $1.4203 E+00$ | 1．4999E＋00 |
| 8 | $4.4814 \mathrm{E}-01$ | 4．4816E－01 | 5．5148s－11 | tarabt 01 | 8．6031E－01 | 1．0275E＋00 | 1．2094E＋00 | 1．3834E＋00 | 1．5236E＋00 | 1．6138E＋00 |
|  | 3.216 .401 | 3． 10 7E－01 | 4.5459 F 31 | $0.618 \pm 01$ | $8.4315 E-01$ | 1．0497E＋00 | 1．2630E＋00 | 1．4002E＊00 | $1.6316 \mathrm{E}+00$ | 1． $738 \mathrm{oE}+00$ |
| 4 | 1．a／5ak 01 | 1．9759E 01 | 3.5050 CJ － 1 |  | 7.9503 EL | 1． $1.034 E+00$ | $1.3259 \mathrm{E}+00$ | 1，5059E．00 | 1．7589E•00 | 1．8827E．09 |
|  | 8．8400t 2 | $8.389090:$ | 2.0555 E 01 | 6．140tr 01 | b．95305 01 | 1． $0138 \mathrm{E}+00$ | 1．3292E＋00 | $1.6149 \mathrm{E}+00$ | $1.8440 \mathrm{E}+00$ | 1．9929E 000 |
|  | 3.7540502 | 2．9471f a： | 1．5509F－0．1 | $3.62+8 E 01$ | $6.5092 \mathrm{E} \cdot 01$ | $9.8023 E-0$. | $1.3070 \mathrm{E}+00$ | $1.6018 \mathrm{E}+00$ | $1.8278 E+00$ | 1．97IEE 00 |
| $1 \times 1$ | 11 | 12 | 4 | 14 | 15 | 16 | 1.7 | 18 | 19 | 20 |
| IETA |  |  |  |  |  |  |  |  |  |  |
| 51 | $1.00002+00$ | 1．0000E•00 | 1．0000t－00 | 1．0000e 00 | $1.0000 \mathrm{t} \cdot 00$ | 1．0000E +00 | $1.0000 \mathrm{E}+00$ | 1．0000e．00 | 1．0000E＊00 | 1．0000e＋00 |
| 50 | 1． $00000 \mathrm{E}+00$ | 1．0007E＊00 | 1．00nste 1.00 | 1．000＞E 00 | $1.0000 \mathrm{E}+00$ | 1．0004E＋00 | $1.0003 E+00$ | 1．0001E＊00 | 1．0000E +00 | $1.0000 t+00$ |
| 48 | 1．00ise．00 | $1.0015 \mathrm{E}+00$ | 1．0014E－i0］ | 1．0012F－00 | $1.0009 F+00$ | 1．0006E +00 | 1．0003E＊00 | 1．0001E＊00 | 9．9998E－01 | 9．9994E－01 |
| 46 | 1．ロッãe +00 | 1．0022e 000 | 1．0020E＊00 | 1．001tto0 | 1．0012E＋00 | 1．0008E +00 | $1.0004 E+00$ | 1．0000E +00 | 9． $9985 \mathrm{E}-01$ | $9.9979 \mathrm{E}-01$ |
| 44 | 1．0030E＋00 | 1．0050E＊00 | 1．0027E＊00 | 1．0022E＊00 | $1.0016 E+00$ | $1.0009 \mathrm{E}+00$ | $1.0003 E+00$ | 9．9989E－01 | 9．9962E－01 | $9.9953 \mathrm{E}-01$ |
| 42 | $1.0039 E+00$ | 1．0059E－00 | 1．0035E＊00 | 1．0028E－00 | $1.0019 E+00$ | 1．0010E＋00 | 1．0002E 00 | 9．9960E－01 | 9．9922E－01 | 9．9909E－01 |
| 40 | $1.0052 \mathrm{E}+00$ | 1．0051E．00 | $1.0045 E+00$ | $1.003 \mathrm{bE} \cdot 00$ | 1．0024E＋00 | 1．0012E－00 | $1.0000 \mathrm{E}+00$ | $9.9913 \mathrm{E}-01$ | $9.9858 \mathrm{E}-01$ | 9．9838F－01 |
| 38 | 1．0071E＋00 | 1．0009E，00 | $1.0000 E+00$ | 1．0047E＋00 | 1．00SIE +00 | 1．0013E＋00 | $9.9967 E-01$ | $9.9838 \mathrm{E}-01$ | 9．97585－01 | 9．9728E－01 |
| 56 | 1．0097E＋00 | $1.00945+00$ |  | 1．000 $56+00$ | $1.0059 E+30$ | 1．0014E＋C0 | $9.9910 E-01$ | 9．9723E－01 | $9.9605 \mathrm{E}-01$ | 9．95006 01 |
| 34 | 1．0134E＊00 | 1．0150E＊00 | $1.9113 E+10$ | 1．0080E＋00 | 1．0052E +00 | 1．0016E＊00 | $9.9821 E-01$ | 9．9550E－01 | 9.9310 E 01 | 9．9308E－01 |
| 32 | 1．0189E＋00 | 1．0182E－00 | 1．0157E＊00 | 1．0118E．00 | $1.0069 E+00$ | $1.0017 \mathrm{E}+00$ | $9.9685 E-01$ | 9．9292E－01 | $9.9036 \mathrm{E}-01$ | $9.8933 E-01$ |
| 30 | 1．0207E＊00 | 1．0350E－00 | $1.0219 \mathrm{E}+00$ | 1．0163E＋00 | 1．0093E－00 | $1.0018 \mathrm{E}+00$ | 9．9481E－01 | 9．8912E－01 | 9.85 こ̧E－01 | 9．8382E－01 |
| 28 | 1．0370E＋00 | 1．0360E＊00 | 1．0507E +00 | 1．022EE＋00 | 1．0125E＋00 | 1．0019E＋00 | $9.9183 E 01$ | $9.8353 E-01$ | 9．1816E－01 | $9.7583 \mathrm{E}-01$ |
| 26 | 1．0529E＊00 | 1．0504E．00 | 1．0429E＋00 | 1．0313E＋00 | 1．0170E＊00 | $1.0019 \mathrm{E}+00$ | $9.8756 \mathrm{E}-01$ | $9.7580 \mathrm{E}-01$ | $9.6788 \mathrm{E}-01$ | 9．6441E－01 |
| 24 | 1．0740E－00 | 1．0703E＊00 | $1.0590 \mathrm{E}+00$ | 1．0432E＋00 | 1．0231E＋00 | $1.0018 \mathrm{E}+00$ | $9.8156 \mathrm{E}-01$ | $9.6484 E-01$ | $9.5347 E-01$ | 9．48こ7E－01 |
| 22 | 1．1023E－00 | $1.0971 E+00$ | 1．0821E＋00 | 1．0593E＋00 | 1．0314E＋00 | $1.0016 \mathrm{E}+00$ | $9.7332 \mathrm{E}-01$ | 9．4980E－01 | 9．33b7E－01 | 9．2626E－01 |
| 20 | 1．1400E．00 | 1．15こ7E＊00 | 1．1121E．00 | $1.0808 \mathrm{~F}+00$ | 1．0425E＋00 | $1.0014 E+00$ | $9.6232 \mathrm{E}-01$ | $9.2967 \mathrm{E}-01$ | $9.0705 E-01$ | 8．9645E 01 |
| 18 | 1．1888E＊00 | 1．11896＊00 | 1．1511E＋00 | 1．1088E 000 | $1.0589 \mathrm{E}+00$ | $1.0013 \mathrm{E}+00$ | 9．4805F－01 | $9.0350 \mathrm{E}-01$ | 8．7215E－al | 8．515ct－01 |
| 16 | $1.2508 \mathrm{E}+00$ | 1．3370E＋00 | 1．2007E＋00 | 1．14C3E 000 | 1．0755E＋00 | 1．0014E＋00 | $0.3016 E-01$ | 8．6982E－01 | 8．2765E－01 | 8．0094E－01 |
| 14 | 1．3271E＊00 | $1.3099 \mathrm{E}+00$ | 1．2020E＋00 | 1．1889E－00 | 1．0989E＋00 | 1．0020E＊00 | 9．0857E－01 | 8． 3 －370E－01 | 7．726．E－01 | 7．4431E－01 |
| 12 | 1．4185E．00 | 1．3967E－00 | 1．3357t＋00 | 1．2455E＋00 | 1．1277E00 | 1．0037E＋00 | 8．8361E－01 | 7．6010E－01 | 7．0670E－01 | $6.6858 E \cdots 01$ |



```
42 0.0000E +00
0.0000E+00
38
0.0000E+00
.0000E +00
.0000E+00
0.0000E+00
0.0000E+00
0.0000E +00
0.0000E +00
0.0000E*00
0.0000EE+00
0.0000E*00
0.0000E*OD
0.0000E+00
O 0.0000E*00
6 0.0000E+00
4 0.0000E+00
2 0.0000E*00
```

static press. Coeff. at time level 210

| - | $=1$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IETA |  |  |  |  |  |  |  |  |  |  |
| 51 | $3.8829 \mathrm{E}-03$ | 2.2737E-13 | 2.2737E-13 | 37E-13 | $2.2737 \mathrm{E}-13$ | 2.2731を-3 | 7E-13 | 2.2737E-13 | 2.2737E-13 | $2.2737 \mathrm{E}-13$ |
| 50 | 6.1637E-03 | $6.1637 E-03$ | 5.1405E-03 | $2.8359 \mathrm{E}-03$ | -3.1985E-04 | -3.6459E-03 | -6.5574E-03 | -8.7151E-03 | -1.0008E-02 | $1.0468 \mathrm{E}-02$ |
| 48 | 3.9352E-03 | 3.9352E-03 | 3.2633E-03 | 1.9545E-03 | 3.0544E-04 | -1.3614E-03 | -2.8247E-03 | -3.9875E-03 | -4.8289E-03 | 3456E-03 |
| 46 | 5.5038E-03 | $5.5038 \mathrm{EE}-03$ | 4.5401E-03 | $2.7119 \mathrm{E}-03$ | $4.9782 \mathrm{E}-04$ | -1.6302E-03 | -3.3906E-03 | $4.6973 \mathrm{E}-03$ | $5.5700 \mathrm{E}-03$ | 62E-03 |
| 44 | 7.2408E-03 | 7.2408E-03 | 5.9716E-03 | $3.6298 \mathrm{E}-03$ | $8.7716 \mathrm{E}-04$ | -1.6998E-03 | 3.8057E-03 | -5.3889E-03 | -6.493!E-03 | $1448 \mathrm{E}-03$ |
| 42 | $9.6240 \mathrm{E}-03$ | $9.6240 \mathrm{E}-03$ | 7.8992E-03 | $4.8486 \mathrm{E}-03$ | 1.4004E-03 | -1.7303E-03 | -4.2573E-03 | -6.1841E-03 | -7.5800E-03 | -8.4489E-03 |
| 40 | $1.2939 \mathrm{E}-02$ | $1.2939 \mathrm{E}-02$ | 1.0555E-02 | -. $5175 \mathrm{E}-03$ | 2.1224E-03 | -1.7806E-03 | -4.9471E-03 | -7.4452E-03 | -9.3441E-03 | -1.0584E-02 |
| 38 | 1.7444E-02 | $1.7444 \mathrm{E}-02$ | 1.4141E-02 | B. $7786 \mathrm{E}-03$ | $3.1433 \mathrm{E}-03$ | -1.7972E-03 | -5.8876E-03 | -9.2618E-03 | -1.1944E-02 | -1.3752E-02 |
| 36 | 2.3562E-02 | 2.3562E-02 | $1.8986 \mathrm{E}-02$ | 1.1862E-02 | $4.5986 \mathrm{E}-03$ | -1.7673E-03 | -7.2236E-03 | -1.1941E-02 | -1.5817E-02 | -1.8466E-02 |
| 34 | 3.1867E-02 | 3.1867E-02 | 2.5543E-02 | 1.6095E-02 | $6.6882 \mathrm{E}-03$ | -1.6723E-03 | -9.1616E-03 | -1.5908E-02 | -2.1554E-02 | -2.5407E-02 |
| 32 | $4.3148 \mathrm{E}-02$ | 4.3148E-02 | 3,4440E-02 | $2.1943 \mathrm{E}-02$ | $9.6851 \mathrm{E}-03$ | -1.5193E-03 | -1.2034E-02 | -2.1790E-02 | -2.9998E-02 | -3.5537E-02 |
| 30 | $5.8479 \mathrm{E}-02$ | 5.8479E-02 | 4.6524E-02 | 3.0042E-02 | $1.39408-02$ | -1.3697E-03 | -1.6345E-02 | $-3.0434 \mathrm{E}-02$ | -4.2313E-02 | 5.0172E-02 |
| 28 | 7.9276E-02 | $7.9276 \mathrm{E}-02$ | $6.2903 \mathrm{E}-02$ | $4.1223 E-02$ | $1.9865 \mathrm{E}-02$ | -1.3909E-03 | $-2.2843 \mathrm{E}-02$ | -4.3190E-02 | -6.0079E-02 | 7.1103E-02 |
| 26 | $1.0734 \mathrm{E}-01$ | $1.0734 \mathrm{E}-01$ | 8.4937E-02 | $5.6498 \mathrm{E}-02$ | $2.7874 \mathrm{E}-02$ | -1.9369E-03 | - $3.2598 \mathrm{E}-02$ | -6.1603E-02 | -8.5409E-02 | -1.0073E-01 |
| 24 | 1.4481E-01 | 1,4481E-01 | $1.1416 \mathrm{E}-01$ | $7.6980 \mathrm{E}-02$ | $3.8255 E-02$ | -3.6703E-03 | -4.7113E-02 | -8.7934E-02 | -1.2109E-01 | -1.4222E-01 |
| 22 | 1.9402E-01 | 1.9402E-01 | $1.5210 \mathrm{E}-01$ | $1.0371 \mathrm{E}-01$ | 5.0953E-02 | -7.7344E-03 | -6.8469E-02 | -1.2510E-01 | -1.7073E-01 | $1.9966 \mathrm{E}-01$ |
| 20 | $2.5710 \mathrm{E}-01$ | $2.5710 \mathrm{E}-01$ | $1.9991 \mathrm{E}-01$ | $1.3732 \mathrm{E}-01$ | 6.5234E-02 | -1.5970E-02 | -9.9492E-02 | -1.7685E-01 | -2.3889E-01 | 7819E-01 |
| 18 | $3.3525 \mathrm{E}-01$ | 3.3525E-01 | $2.5795 \mathrm{E}-01$ | $1.7755 \mathrm{E}-01$ | $7.9306 E-02$ | -3.1148E-02 | -1.4392E-01 | -2,4790E-01 | -3.3114E-01 | 1 |
| 16 | $4.2766 \mathrm{E}-01$ | $4.2766 \mathrm{E}-01$ | $3.2538 \mathrm{E}-01$ | $2.2267 \mathrm{E}-01$ | $9.0033 \mathrm{E}-02$ | -5.7156E-02 | -2.0650E-01 | -3.4391E-01 | -4.5399E-01 | 1 |
| 14 | $5.3032 \mathrm{E}-01$ | 5.3032E-01 | $3.9969 \mathrm{E}-01$ | $2.6913 \mathrm{E}-01$ | $9.2947 E-02$ | -9.9040E-02 | -2.9293E-01 | -4.7126E-01 | -6.1445E-01 | 1 |
| 12 | 6.3512E-01 | 6.3512E-01 | 4.7622E-01 | $3.1156 \mathrm{E}-01$ | 8.2777E-02 | -1.6271E-01 | -4.0951E-01 | -6.3652E-01 | -8.1932E-D1 | 1 |
| 10 | 7.3009E-01 | 7.3009E-01 | $5.4725 E-01$ | 3.4332E-01 | 5.4450E-02 | -2.5398E-01 | -5.6244E-01 | -8.4557E-01 | $-1.0741 \mathrm{E}+00$ | 0 |
| 8 | $8.0183 \mathrm{E}-01$ | 8.0183E-01 | $6.0183 E-01$ | $3.5659 \mathrm{E}-01$ | 3.6111E-03 | -3.7701E-01 | -7.5632E-01 | $-1.1024 E+00$ | , | 5602E+00 |
| 6 | 8.4144E-01 | 8.4144E-01 | $6.2904 \mathrm{E}-01$ | 3.4253E-01 | -7.5060E-02 | -5.3440E-01 | -9.9296E-01 | -1.4080E*00 | 7417E+00 | 9551E+00 |
| 4 | 8.5138E-01 | $8.5138 \mathrm{E}-01$ | $6.2659 E-01$ | 2.9588E-01 | -1.8766E-01 | -7.2944E-01 | -1.2732E+00 | 7616E+00 | 517E*00 | 4004E + 00 |
| 2 | 8.4242E-01 | 8.4242E-01 | $6.0524 E-01$ | 2.2448E-01 | -3.2813E-01 | 5365E-01 | 5851E+00 | $1508 \mathrm{E}+00$ | $2.6004 \mathrm{E}+00$ | 8865E-00 |
| 1 | $9.4053 \mathrm{E}-01$ | 8.3848E-01 | 5.8972E-01 | $1.8552 \mathrm{E}-01$ | -4.0220E-01 | 0 | O | 3383E+00 | 8148E+00 | 184E-00 |
| IKI | 11 | 2 | 13 | 4 | 5 | 16 | 7 | 18 | 19 | 20 |
| IETA |  |  |  |  |  |  |  |  |  |  |
|  | 2.2737E-13 | 2.2737E-13 | 2.2737E-13 | 2.2737E-13 | 2.2737E-13 | 2.2757E-13 | 2.2737E-13 | 2.2737E-13 | 2.2737E-13 | 2.2737E-13 |
|  | -1.0195E-02 | -9.3111E-03 | -7.9694E-03 | -6.3578E-03 | -4.6034E-03 | -2.9598E-03 | -2.5869E-03 | -6.2181E-04 | -1.1495E-04 | 1.7802E-05 |
| 48 | -5.5242E-03 | $-5.3499 E-03$ | -4.8306E-03 | -4.0154E-03 | -2.9986E-03 | -1.9142E-03 | -9.2088E-04 | -1.7096E-04 | 2.4389E-04 | 3.5799E-04 |
|  | -6.1360E-03 | -5.8386E-03 | -5.1692E-03 | -4.1833E-03 | -2.9835E-03 | $\rightarrow 1.7143 \mathrm{E}-03$ | -5.4932E-04 | 3.3978E-04 | 8.4368E-04 | 9247E-04 |
|  | -7. $3253 \mathrm{E}-03$ | -7.0045E-03 | -6.1871E-03 | -4.9401E-03 | -3.3952E-03 | -1.7386E-03 | -1.9351E-04 | $1.0138 \mathrm{E}-03$ | 1.7272E-03 | 3 |
|  | -8.7259E-03 | -8.3449E-03 | 7.3054E-03 | -5.6995E-03 | -3.7028E-03 | -1.5515E-03 | 4.7735E-04 | $2.0987 \mathrm{E}-03$ | 3.0975E-03 | 3 |
| 40 | -1.1020E-02 | -1.0536E-02 | -9.1336E-03 | -6.9543E-03 | -4.2497E-03 | $-1.3369 \mathrm{E}-03$ | 1.4292E-03 | 3.6829E-03 | 5.1269E-03 | 899E-03 |
|  | -1.4420E-02 | $-1.3770 \mathrm{E}-02$ | -1.1820E-02 | -8.7923E-03 | -5.0518E-03 | -1.0330E-03 | $2.8010 \mathrm{E}-03$ | $5.9801 \mathrm{E}-03$ | $8.0964 \mathrm{E}-03$ | 8.9879E-03 |
|  | -1.9453E-02 | -1.8532E-02 | $-1.5763 \mathrm{E}-02$ | -1.1490E-02 | -6.2423E-03 | -6.2299E-04 | $4.7520 \mathrm{E}-03$ | $9.2747 \mathrm{E}-03$ | 1.2395E-02 | 3808E-02 |
|  | -2.6814E-02 | -2.5464E-02 | -2.1494E-02 | -1.5423E-02 | -8.0034E-03 | -8.0952E-05 | 7.5059E-03 | 1.3962E-02 | $1.8564 \mathrm{E}-02$ | 2.0793E-02 |
|  | -3.7484E-02 | -3.5477E-02 | -2.9776E-02 | -2.1135E-02 | -1.0611E-02 | $6.1229 E-04$ | $1.1364 \mathrm{E}-02$ | 2.0583E-02 | 2.7338E-02 | 0825E-02 |
|  | -5.2815E-02 | -4.9842E-02 | -4.1684E-02 | -2.9407E-02 | -1.4474E-02 | $1.4531 \mathrm{E}-03$ | 1.6715E-02 | $2.9856 \mathrm{E}-02$ | 3.9699E-02 | 4.5089E-02 |
|  | -7.4660E-02 | -7.0315E-02 | -5.8716E-02 | -4.1357E-02 | -2.0198E-02 | 2.3816E-03 | 2,4034E-02 | $4.2697 \mathrm{E}-02$ | 5.6905E-02 | 6.5129E-02 |
|  | -1.0552E-01 | -9.9286E-02 | -6.2922E-02 | -5.8451E-02 | -2.8665E-02 | 3.2177E-03 | 3.3840E-02 | 6.0217E-02 | 8.0499E-02 | 9.2867E-02 |
|  | -1.4871E-01 | -1.3993E-01 | -1.1705E-01 | -8.2839E-02 | -4.1159E-02 | 3.5477E-03 | 4.6601E-02 | 8.3648E-02 | 1225E-01 | $.3057 E-01$ |
|  | -2.0851E-01 | -1.9635E-01 | -1.6470E-01 | -1.1732E-01 | -5.9542E-02 | 2.5402E-03 | 6.2544E-02 | 1.1417E-01 | 1.5396E-01 | $.8067 E-01$ |
|  | -2.9023E-01 | -2.7366E-01 | -2.3044E-01 | -1.6562E-01 | -8.6477E-02 | -1.3181E-03 | 8.1339E-02 | 1.5259E-01 | 2.0717E-01 | 2.4550E-01 |
|  | -4.0022E-01 | -3.7802E-01 | -3.1990E-01 | -2.3253E-01 | -1.2567E-01 | -1.0551E-02 | 1.0167E-01 | 1.9890E-01 | $2.7260 \mathrm{E}-01$ | 3.2674E-01 |
|  | -5.4567E-01 | -5.1647E-01 | -4.3965E-01 | -3.2392E-01 | -1.8207E-01 | -2.9015E-02 | $1.2074 \mathrm{E}-01$ | $2.5162 \mathrm{E}-01$ | $3.4948 \mathrm{E}-01$ | $4.2479 \mathrm{E}-01$ |
| 14 | -7.3408E-01 | -6.9643E-01 | -5.9686E-01 | $-4.4653 \mathrm{E}-01$ | -2.6179E-01 | -6.2067E-02 | $1.3388 \mathrm{E}-01$ | 3.0732E-01 | 4.3490E-01 | 5. $3806 \mathrm{E}-01$ |
| 12 | -9.7245E-01 | -9.2496E-01 | -7.9858E-01 | -6.0735E-01 | -3.7179E-01 | -1.1639E-01 | 1. $3450 \mathrm{E}-01$ | $3.6019 \mathrm{E}-01$ | 5.2329E-01 | $6.6251 E-01$ |
|  | -1.2661E•00 | -1.2075E+00 | -1.0506E+00 | -8.1265E-01 | -5.1888E-01 | -1.9932E-01 | 1.1468E-01 | $4.0228 \mathrm{E}-01$ | 6.0652E-01 | $6.62515-01$ $7.9143 E-01$ |
|  | -1.6173E400 | -1.5466E*00 | $-1.3560 E+00$ | $-1.0665 E+00$ | -7.0851E-01 | -3.1761E-01 | $6.6339 \mathrm{E}-02$ | $4.2434 \mathrm{E}-01$ | $6.7465 \mathrm{E}-01$ | $9.1588 \mathrm{E}-0.1$ |
|  | -2.0239E*00 | $-1.9405 E+00$ | -1.7142E+00 | $-1.3698 \mathrm{E}+00$ | -9.4328E-01 | -4.7598E-01 | -1.7139E-02 | $4.1731 E-01$ | 7.1732E-01 | 1.0256E+00 |
|  | $-2.4809 E+00$ | -2.3840E 000 | $-2.1207 E+00$ | -1.7196E+00 | -1.2222E+00 | -6.7590E-01 | -1.3935E-01 | $3.7416 \mathrm{E}-01$ | $7.2548 \mathrm{E}-01$ | 1.1104E*00 |
|  | $-2.9798 \mathrm{E}+00$ | -2.8702E*00 | -2.5703E+00 | $-2.1124 E+00$ | $-1.5438 \mathrm{E}+00$ | -9.1801E-01 | -3.0254E-01 | $2.8938 \mathrm{E}-01$ | $6.9190 E-01$ | 1.1606E+00 |
|  | $-3.2181 E * 00$ | $-3.1035 \mathrm{E} \cdot 00$ | -2.7878E*00 | $-2.3049 E+00$ | $-1.7043 \mathrm{E}+00$ | $-1.0428 \mathrm{E}+00$ | -3.9124E-01 | $2.3459 E-01$ | 6.64 55E-01 | $1.1537 E+00$ |

IKI=
$=21$
$-4.4916 E-05$
$1.7802 \mathrm{E}-05$
$3.5799 \mathrm{E}-04$
$9.9247 \mathrm{E}-04$
$1.9607 \mathrm{E}-03$
$3.4559 \mathrm{E}-03$
$5.6899 \mathrm{E}-03$
$8.9879 \mathrm{E}-03$
$1.3808 \mathrm{E}-02$
$2.0793 \mathrm{E}-02$
$3.0825 \mathrm{E}-02$
$4.5089 \mathrm{E}-02$
$6.5129 \mathrm{E}-02$
$9.2867 \mathrm{E}-02$
$1.3057 \mathrm{E}-01$
$1.8067 \mathrm{E}-01$
$2.4550 \mathrm{E}-01$
$3.2674 \mathrm{E}-01$
$4.2479 \mathrm{E}-01$

```
14 5.3806E-01
12 6.6251E-01
10 7.9143E-01
8 9.1588E-01
6 1.0256EE+00
4 1.1104E+00
1 1.3839E*00
```

**** CONVERGED SOLUTION AT Time LEVEL 210
CONVERGENCE HISTORY FOR CONTINUITY EQUATICN


CONVERGENCE HISTORY FOR K-MOMENTUM EQUATION


CONVERGENCE MISTORY FOR Y-MOMENTUM EQUATION

| LEVEL | ChGmax | chang | **** WITHOUT ARTIFICIAL VISCOSITY *m** |  |  |  |  | *n*m | WITH ARTIFICIAL |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | RESL2 | resavg | RESMAX | RESMAX | LOC. | RESL2 | RESAVO | RESMAX | RESMA | . |
|  |  |  | 2.679E-09 | 4.900E-11 | $6.109 \mathrm{E}-10$ | 115. | 461 | 2.679E-09 | 4.900E-11 | $6.109 \mathrm{E}-10$ | ( 15. | 46) |
| 2 | 0.000E+00 | $0.000 \mathrm{E} \cdot 00$ | 3.649E.02 | $6.258 E+00$ | 5.732E-01 | ( 16. | 21 | 3.637E*02 | 6.257E-00 | 5.826E*01 | 116. | 2) |
| 10 | 4.443E-02 | 0.000E+00 | 2.749E*02 | 5.418E+00 | 7.392E*01 | (14. | 501 | 2.124E-02 | $5.159 \mathrm{E}+00$ | 2.592E-01 | 8. | 50) |
| 20 | 6.572E-03 | $0.000 E+00$ | 1.652E*02 | 2.501E+00 | 5.891E+01 | (10. | 49) | 1.127E 02 | 1.849E-00 | 2.254E*01 | 1 | 50 |
| 30 | 9.466E-03 | $0.000 \mathrm{E}+00$ | 8.003E+01 | 1.183E+00 | 2.426E*01 | 110. | $50)$ | 2.965E*01 | 5.691E-01 | $6.716 \mathrm{E}+00$ |  | $50)$ |
| 40 | 1.120E-02 | $0.000 E+00$ | 3.611E+01 | 6.420E-01 | 1.055E*01 | 10. | 501 | $1.714 \mathrm{E}+01$ | 3.868E-01 | 0 |  | 48) |
| 50 | 1.026E-02 | $0.000 \mathrm{E}+00$ | $3.8315+01$ | 5.167E-01 | $1.242 E+01$ | $\bigcirc 10$. | 501 | 1.220E901 | 2.284E-01 | $1.653 E .00$ $1.287 E .00$ | 4. | 493 |
| 60 | $6.902 \mathrm{E}-03$ | $0.000 E+00$ | 3.395E-01 | 4.190E-01 | 1.160E+01 | 10. | 501 | B. $298 \mathrm{E}+00$ | 1.400E-01 | 1.287E-00 |  |  |
| 70 | $3.666 \mathrm{E}-03$ | $0.000 E+00$ | 3.294E+01 | $3.903 \mathrm{E}-01$ | 1.117E*01 | (10. | $50)$ | 4.955E+00 | $8.192 E-02$ $4.512 E-02$ | B.381E-01 $4.502 \mathrm{E}-01$ | 3. | 7) |
| 80 | 1.808E-03 | 0.000E*00 | 3.192E*01 | 3.793E-01 | 1.077E+01 | 10. | $50)$ | 1.300E+00 | 4.51cE-02 | 2.181E-01 | 3. | 7 |
| 90 | 9.717E-04 | $0.000 E \cdot 00$ | 3.115EP01 | 3.789E-01 | 1.047E01 | 10, | 501 | 1.300E*00 | 2.171E-02 | 1.099E-01 | 3 , | $6)$ |
| 100 | 5.048E-04 | $0.000 E+00$ | 3.086EP01 | 3.791E-01 | 1.038E+01 | 10 | 50) | 6.622E-01 | 5.827E-03 | 5.686E-02 | 3. | 6) |
| 110 | 2.473E-04 | $0.000 E+00$ | 3.085E-01 | 3.797E-01 | $1.037 E+01$ | 10. | $50)$ | 3.333E-01 |  | 2.907E-02 | 3. | $6)$ |
| 120 | 1.240E-04 | $0.000 E+00$ | 3.085E401 | 3.808E-01 | 1.035E+01 | 10. | 501 | 1.703E-01 | 2.854E-03 | 2.907E-02 | 3. |  |
| 130 | 6.170E-05 | $0.000 \mathrm{E}+00$ | 3.082E-01 | 3.812E-01 | 1.035E*O1 | 10. | 501 | 8.763E-02 | 1.312E-03 | 1.955E-03 | 3. |  |
| 140 | 3.022E-05 | 0.000E+00 | 3.080E401 | $3.813 \mathrm{E}-01$ | 1.033E*01 | (10, | $50)$ | 4.175 E 02 | 7.334E-04 | 6.981E-03 | 13. |  |
| 150 | 1.551E-05 | $0.000 E+00$ | 5.079E-01 | 3.814E-01 | 1.032E*O! | 10. | 501 | 1.953E-02 | 3.383E-04 | 1.809E-03 | 3. |  |
| 160 | 7.682E-06 | $0.000 E+20$ | 3.079E+01 | 3.814E-01 | 1.032E*O1 | ( 10, | 501 | $9.856 \mathrm{E}-03$ | 1.683E-04 | 8.711E-04 | 3. |  |
| 170 | 3.632E-06 | $0.000 E+00$ | 3.079E-01 | 3.814E-01 | 1.032E+01 | ( 10. | 501 | 4.932E-O3 | 8.4.4E-05 | 4.189E-04 | 3. | 6) |
| 180 | 1.861E-06 | $0.000 \mathrm{E}+00$ | 3.079E-01 | 3.814E-01 | 1.032E+01 | ( 10. | 501 | $2.399 \mathrm{E}-03$ | 4.108E-05 | 2. 2 OBE-04 | (3. |  |
| 190 | 9.284E-07 | 0.000E*00 | 3.079E+01 | 3.814E-01 | 1.052E-01 | $(10$. | 50) | $1.265 \mathrm{E}-03$ | 2.109E-05 | 2.608E-04 | ( 3 , |  |
| 200 | 4.299E-07 | $0.000 E+00$ | 3.079E-01 | 3.814E-01 | 1.032E+01 | 10. | 50) | 7.242 E | 1.207E-05 | 1.456E-04 | ( 3 , |  |
| STOP |  |  |  |  |  |  |  |  |  |  |  |  |

This case used 6.5 seconds of CPU time.

## Standard PROTELS Output for Viscous Flow Case

The output listing for the viscous flow case is shown below. In the flow field printout, only the last time level is included. Note that a converged solution is obtained at time level 360 , and that this level is automatically included in the standard output and in the plot file.

NASA LEWIS RESEARCH CENTER<br>INTERNAL FLUID MECHANICS DIVISION

2-D PROTEUS VERSION 1.0
SEPTEMBER 1989
VISCOUS FLOW PAST A CIRCULAR CYLINDER

```
    &RSTRT IREST = 0, NRQIN = 11, NRDOUT = 12, NRXIN = 13, NRXOUT = 14, &END
    8.0 IDEBUG = 20*0, IPLOT = -1, IPLT = 0, IPLTA = 10I*0, IPRT = 10000, IPRTI = 2, IPRT2 = 2, IPRTA = 101*O,
    IPRTIA = 1, 2, 4, 6, 8, 10, 12, 14, 16, 10, 1B, 20, 2, 22, 24, 24, 26, 28, 30, 32, 3, 34, 36, 38, 40, 42, 4, 4, 4, 4, 4, 48, 
42.
    44, 46, 48, 50, 51, 24*D, IUNITS = 0, IVOUT = 1, 2, 3PLOTX = 32, 4, 4*0, NGRID = 7, NHIST = 10, NHMAX = 100, NOUT = 6,
    8GMTRY IAXI = D, NGEOM = 2, RMAX = 30.0, RMIN = 1.0, THMAX = 180.0, TMMIN = 0.0, XMAX = 1.0, XMIN = 0.0,
    YMAX = 1.0, YMIN = 0.0, BEND
    &FLOW GAMR = 1.4, MSTAGR = 0.0, ICVARS = 2, IEULER = 0, IHSTAG = 1, ILAMV = O, ISWIRL = 0, ITHIN = 2MO,
```



```
    UR = 0.0, 8END
    8BC IBCI= 510*0, IBC2 = 510*0, JBC1 = 42, 12, 21, 2*0, 42, 12, 21, 2*0, JBC2 = 42, 11, 21, 7*0,
```



```
    GGFC1 = 100*0.0, GTBC2 = 100*0.0, KBC1 = 2*0, KBC2 = 2*0, BEND
```




```
    ZIME, THYY = 1.0, 2*0.0. THZ = 1.0, 2*0.0, 8END
1.25, CFL = 10.0, 9*1.0. CFLMAX = 10.0, CFLMIN = 0.5, CHGI = 4.OE-02, CHG2 = 6.OE-02, DT = 10M1.0E-02, DTF1 =
    25,
    DTF2 = 1.25, DTMAK = 1.OE-02, DTMIN = 1.OE-02, EPS = 5*1.OE-03, ICHECK = 10, ICTEST = 3, IDTAU = 5, IDTMOD = 1,
    NDTCYC = 2, NITAVG = 10, NTIME = 1000, 9*0, NTSEO = 1. BEND
```




```
NORMALIZING CONDITIONS
LENGTH, LR =1,0000E+00 FT
VELOCITY, UR 
TEMPERATURE, TR 
IEMPERATURE, RHR 
VISCOSITY, MUR 
THERMAL CONDUCTIVITY, KTR = 1.0192E-OI LBM-FT/SECS-DEG R
PRESSURE, RHOR*UR**2,KTR=3.8129E*OS LBM/FT-SEC2
ENERGY/VOL,. RHOR*UR**2 }=\mathbf{= 3.8129E*03 LBM/FT-SEC2
GAS CONSTANT, UR**2/TR 
SPECIFIC HEAT, UR**2/TR =9.6096E+01 FT2/SEC2-DEG R 
```



```
REFERENCE CONDITIONS
\begin{tabular}{|c|c|c|}
\hline REYNOLDS NUMBER, RER & 2.0000E•01 & \\
\hline MACH NUMBER, MACHR & \(=2.0000 \mathrm{E}-01\) & \\
\hline SPECIFIC HEAT RATIO, GAMR & \(=1.4000 \mathrm{E}+00\) & \\
\hline LAMINAR PRANDTL NUMBER, PRLR & \(=5.0306 \mathrm{E}+04\) & \\
\hline "REFERENCE" PRANDTL NUMBER, PRR & \(=8.0489 \mathrm{E}+02\) & \\
\hline SPECIFIC HEAT AT CONST. PRESS. & \(=6.0060 \mathrm{E}+03\) & FT2/SEC2-DEG R \\
\hline SPECIFIC HEAT AT CONST, VOL. & \(4.2900 \mathrm{E} \cdot 03\) & FT2/SEC2-DEG R \\
\hline gas constant, rg & \(1.7160 \mathrm{E} \cdot 03\) & FT2/SEC2-DEG R \\
\hline PRESSURE, PR & 2.1162 E -03 & LBF/FT2 \\
\hline STAGNATION ENTHALPY, HSTAGR & 3.1421E*06 & FT2/SEC2 \\
\hline
\end{tabular}
```



$T \times I=\quad \therefore 0$

|  |  |
| :---: | :---: |
| 51 | 1． $0008 \mathrm{E}+00$ |
| 50 | 1．0009E＋00 |
| 49 | $1.0165 E+00$ |
| 46 | 1．02ここE＋00 |
| 4 | 1．0303E +00 |
| $4 ?$ | 1．0403E．00 |
| 40 | 1．0544E＋00 |
| 38 | 1．0732E＋00 |
| 25 | 1．0908E＋00 |
| 34 | 1．12S2E＋00 |
| \％ | 1．1583E＋00 |
| 30 | 1．1969E＋00 |
| H | 1．2．22E＋80 |
| 20 | 1．2021E＋30 |
| 26 | 1．0898E＋30 |
| $\because$ | 6．9253E－61 |
| 20 | 6．7017F．01 |
| 18 | 的，ictese－id |
| 16 | 3．．3う6E 0： |
| 14 | 2.1582501 |
| 12 | 1．3971E 01 |
| 10 | 8.8671 E .02 |
| 8 | $5.4200^{\circ} \mathrm{E}-02$ |
| 6 | $3.0898 \mathrm{E}-02$ |
| 4 | 1．50こ4E O2 |
| ？ | $4.1085 \mathrm{E}-35$ |
| 1 | 0．0000E＋00 |

22
$1.0010 E+00$
$1.0066 E+00$
$1.0153 E+00$
$1.0216 E+00$
$1.0283 E+00$
$1.0374 E+00$
$1.0498 E+00$
$1.0662 E+00$
$1.0868 E+00$
$1.1117 E+00$
$1.1409 E+00$
$1.1748 E+00$
$1.2097 E+00$
$1.2237 E+00$
$1.1720 E+00$
$1.0391 E+00$
$8.2765 E-01$
$0.2151 E-01$
$4.4527 E-01$
$3.1078 E-01$
$2.1101 E-01$
$1.3924 E-01$
$8.8104 E-02$
$5.1726 E-02$ 42


$4.7725 \mathrm{~F}-03$
$-1.8184 \mathrm{E}-02$
$2.025 \mathrm{E}-02$
$-2.6257 \mathrm{E}-02$
$-2.2570 \mathrm{E}-02$
$-1.7787 E-02$
$1.3170 \mathrm{E}-02$
$-9.2435 \mathrm{E}-03$
$-6.1124 \mathrm{E}-03$
$-3.7040 \mathrm{E}-03$
$-1.8889 E-03$
$-5.3052 \mathrm{E}-04$
$0.0000 \mathrm{E}+00$
26 $1.0011 \mathrm{E}+00$
$1.0053 \mathrm{E}+00$
$1.0120 \mathrm{E}+00$
$1.0170 \mathrm{E}+00$
$1.0222 \mathrm{E}+00$
$1.0288 \mathrm{E}+00$
$1.0374 \mathrm{E}+00$
$1.0485 \mathrm{E}+00$
$1.0024 \mathrm{E}+00$
$1.0795 \mathrm{E}+00$
$1.1003 \mathrm{E}+00$
$1.1259 \mathrm{E}+00$
$1.1572 \mathrm{E}+00$
$1.1910 \mathrm{E}+00$
$1.2054 \mathrm{E}+00$
$1.1648 \mathrm{E}+00$
$1.0447 \mathrm{E}+00$
$8.6940 \mathrm{E}-01$
$6.8103 \mathrm{E}-01$
5.1099 E
$3.0897 \mathrm{E}-01$
$2.5625 \mathrm{E}-01$
$1.6908 \mathrm{E}-01$
$1.0267 \mathrm{E}-01$
$5.2557 \mathrm{E}-02$
$1.4966 \mathrm{E}-02$
$0.0000 \mathrm{E}+00$

46




 $9.5247 E-01$
$7.1912 E-01$
$28 \quad 30$

$1.00038+00$ $1.0018 E+00$
$1.00550+00$ $1.0055 \mathrm{E}+00$
$1.0079 \mathrm{E}+0 \mathrm{D}$ $1.9098 \mathrm{E}+00$

34

 $1.0002 E+90$
9.98985
1.0008

1． $100016+00$ 9． $9 / 59 \mathrm{C}-01$ $9.9881 \mathrm{E}-01$
$9.9893 \mathrm{E}-01$
$9.9893 E-01$
$9.934 \subset E$
9.91
 $\begin{array}{ll}1.0164 E+00 & 1.005 E E+00 \\ 1.0189 E+00 & 1.0039 E-00\end{array}$

Q． 946 FE－01
$9.9094 E-01$
$9.8469 E-01$
$9.8932 \mathrm{E}-01 \quad 9.7502 \mathrm{E}-01$
$\begin{array}{ll}.8186 E 01 & 9.6313 E-01 \\ 9.7235 E-01 & 9.4691 E-01 \\ 0.0137 E-01 & 9.055 E E 01\end{array}$
$\begin{array}{cc}0.0137 E-01 & 9.2711 E-01 \\ 9.5009 E-01 & 9.0452 E-01\end{array}$

$\begin{array}{ll}9.256 F-01 & 8.5: 94 E-01 \\ 3.9550 \mathrm{E} & 01 \\ 7.9048 E-01\end{array}$
$\begin{array}{ll}3.0550 E-01 & 7.9048 E-01 \\ \text { 3．} 2516 E-01 & 7.3697 E-01\end{array}$
$\begin{array}{ll}7.3536 E-01 & 6.4851 E-01 \\ 0.1035 E-01 & 5.3949 E-01\end{array}$
$\begin{array}{cc}0.1035 \mathrm{E}-01 & 5.3949 \mathrm{E}-01 \\ 4.7858 \mathrm{E}-01 & 4.2625 \mathrm{E}-01 \\ 3.5437 \mathrm{E}-01 & 3.1491 \mathrm{E}-01\end{array}$
$\begin{array}{ll}2.4550 \mathrm{E}-01 & 2.1850 \mathrm{E}-01 \\ 1.5464 \mathrm{E} & 01 \\ 8.1353 \mathrm{E}-02 & 7.3585 \mathrm{E}-01 \\ 2.3642 \mathrm{E}-02 & 2.1507 E-02\end{array}$
$\begin{array}{ll}2.3642 E-02 & 2.11 .07 E-02 \\ 0.0000 E+00 & 0\end{array}$
48
$9.9897 E-01$
$9.9295 E-01$
$9.9200 E-01$
$9.9023 E-01$
$9.8550 E-01$
$9.7877 E-01$
$9.0864 E-01$
$9.5382 E-01$
$9.3269 E-01$
$9.0337 E-01$
$8.6388 E-01$
$8.1245 E-01$
$7.4744 E-01$
$6.7040 E-01$
$5.8154 E-01$
$4.8440 E-01$
$3.8440 E-01$
$2.8856 E-01$
$2.0445 E-01$
$1.3715 E-01$
$8.3525 E-02$
$5.3294 E-02$
$3.0783 E-02$
$1.6447 E-02$
$7.4874[-03$
$1.9352 E-03$

50


y or r veladity

$0.0000 \mathrm{E}+00$
2 4
$-1.2705 \mathrm{E}-02$
$-1.2705 \mathrm{E}-02$
$-2.0706 \mathrm{E}-02$
$-3.3519 \mathrm{E}-02$
$-4.8785 \mathrm{E}-02$
$-6.7387 \mathrm{E}-02$
$-8.7461 \mathrm{E}-02$
$-1.0364 \mathrm{E}-01$
$-1.0856 \mathrm{E}-01$
$-9.7546 \mathrm{E}-02$
$-7.3745 \mathrm{E}-02$
$-4.6526 \mathrm{E}-02$
$-2.3259 \mathrm{E}-02$
$-5.4739 \mathrm{E}-03$
$7.9844 \mathrm{E}-03$
$1.7979 \mathrm{E}-02$
$2.4220 \mathrm{E}-02$
$2.6999 \mathrm{E}-02$
$2.6614 \mathrm{E}-02$
$2.3935 \mathrm{E}-02$
$1.9928 \mathrm{E}-02$
$1.5441 \mathrm{E}-02$
$1.1069 \mathrm{E}-02$
$7.1531 \mathrm{E}-03$
$3.8349 \mathrm{E}-03$
$1.1299 \mathrm{E}-03$
0.0000 E
6
$-1.2575 \mathrm{E}-02$
$-1.2575 \mathrm{E}-02$
$-1.9766 \mathrm{E}-02$
$-3.0432 \mathrm{E}-02$
$-4.2409 \mathrm{E}-02$
$-5.7922 \mathrm{E}-02$
$-7.7579 \mathrm{E}-02$
$-9.8697 \mathrm{E}-02$
$-1.1411 \mathrm{E}-01$
$-1.1441 \mathrm{E}-01$
$-9.5863 \mathrm{E}-02$
$-6.5744 \mathrm{E}-02$
$-3.5295 \mathrm{E}-02$
$-1.0020 \mathrm{E}-02$
$9.8257 \mathrm{E}-03$
$2.4673 \mathrm{E}-02$
$3.4301 \mathrm{E}-02$
$3.8607 \mathrm{E}-02$
$3.8224 \mathrm{E}-02$
$3.4423 \mathrm{E}-02$
$2.8666 \mathrm{E}-02$
$2.2206 \mathrm{E}-02$
$1.5915 \mathrm{E}-02$
$1.0282 \mathrm{E}-02$
$5.5125 \mathrm{E}-03$
$1.6244 \mathrm{E}-03$
$0.0000 \mathrm{E}-00$

| 8 | 10 |
| :---: | :---: |
| $-9.6042 \mathrm{E}-03$ | $-9.5844 \mathrm{E}-03$ |
| $-9.6042 \mathrm{E}-03$ | $-9.5844 \mathrm{E}-03$ |
| $-1.3684 \mathrm{E}-02$ | $-1.2556 \mathrm{E}-02$ |
| $-1.9942 \mathrm{E}-02$ | $-1.7983 \mathrm{E}-02$ |
| $-2.6890 \mathrm{E}-02$ | $-2.4020 \mathrm{E}-02$ |
| $-3.6637 \mathrm{E}-02$ | $-3.1828 \mathrm{E}-02$ |
| $-5.0225 \mathrm{E}-02$ | $-4.0557 \mathrm{E}-02$ |
| $-6.6900 \mathrm{E}-02$ | $-4.9317 \mathrm{E}-02$ |
| $-8.3498 \mathrm{E}-02$ | $-5.4072 \mathrm{E}-02$ |
| $-9.3675 \mathrm{E}-02$ | $-5.8122 \mathrm{E}-02$ |
| $-9.0209 \mathrm{E}-02$ | $-6.0118 \mathrm{E}-02$ |
| $-7.1592 \mathrm{E}-02$ | $-5.6616 \mathrm{E}-02$ |
| $-4.4998 \mathrm{E}-02$ | $-4.4370 \mathrm{E}-02$ |
| $-1.8082 \mathrm{E}-02$ | $-2.5667 \mathrm{E}-02$ |
| $4.8435 \mathrm{E}-03$ | $-5.5728 \mathrm{E}-03$ |
| $2.2812 \mathrm{E}-02$ | $1.2240 \mathrm{E}-02$ |
| $3.4953 \mathrm{E}-02$ | $2.5476 \mathrm{E}-02$ |
| $4.0900 \mathrm{E}-92$ | $3.3006 \mathrm{E}-0.3$ |
| $4.1294 \mathrm{E}-\mathrm{JZ}$ | $3.5046 \mathrm{E}-02$ |
| $3.7592 \mathrm{E}-02$ | $3.2840 \mathrm{E}-02$ |
| $3.1501 \mathrm{E}-02$ | $2.8014 \mathrm{E}-02$ |
| $2.4493 \mathrm{E}-02$ | $2.2033 \mathrm{E}-02$ |
| $1.7593 \mathrm{E}-02$ | $1.5946 \mathrm{E}-02$ |
| $1.1382 \mathrm{E}-02$ | $1.0369 \mathrm{E}-02$ |


| 12 | 14 | 10 | 18 |
| :---: | :---: | :---: | :---: |
| O110E－02 | 3 | 3 | 3814E－04 |
| 1．0110E－02 | －7．7230E－03 | －5．8948E－03 | －6． $2839 \mathrm{E}-03$ |
| 1．4016E－02 | －1．3327E 0？ | －1．0317E－02 | －7．2649E－03 |
| 1．8682E－02 | －1．5503E－02 | －1．0906E－02 | －6．9941E－03 |
| $2.3483 E-02$ | －1．7892E－02 | －1．1428E－02 | －6．1580E－03 |
| ．9341E－02 | －2．0940E－02 | －1．2101E－02 | －4．7814E－03 |
| 4804E－02 | －2． $34536-0$. | $-1.1650 \mathrm{E}-02$ | －1．5724E－03 |
| 7317E－02 | －2．3310t－02 | 8．3721E－03 | 4.974 1E－03 |
| $4732 \mathrm{E}-02$ | －1．7841E－02 | －3．3365E－04 | 1．6219E－02 |
| 7641E－02 | －5．3268E－03 | 1．43：4E－02 | 3．3265E－02 |
| 141．85－0． | $1.0761 \mathrm{E}-02$ | 3．5180E－0？ | 5．6393E－02 |
| 2．1521E－0： | 1．47120－02 | 3．4641E－0？ | 8．2163E－02 |
| $\therefore .68176-02$ | 1．3375E－02 | 5．8161F－0？ | $9.8540 E-02$ |
| 2． $3101 \mathrm{E}-02$ | －1．2ア6E－05 | 3．968BE－02 | $8.9925 \mathrm{E}-02$ |
| 1．4414E－02 | －1．1122E．02 | 1．2629E－02 | 5．7096E－02 |
| 2．3987E－03 | －1．2275F－02 | －7．754：2E－03 | 1．9672E－02 |
| 8．9496E－03 | －1．90．35 03 | 1．6122E－0？ | －7．0129E－03 |
| 1．7049E－0．2． | －2．01／1E－03 | －1．7158E－02 | －2．0498E－02 |
| 2．1020E－02 | $2.80595-05$ | －1．4406E－02 | $-2.4331 \mathrm{E}-02$ |
| $2.1328 \mathrm{E}-0$. | 5．6173E－03 | $-1.0629 \mathrm{E}-02$ | －2．2751E－02 |
| 1．9092E－02 | 6．5278E－03 | －7．1939E－03 | －1．8793E－02 |
| 1.5493502 | $6.0920 E-03$ | －4．5500E－03 | －1．4231E．02 |
| 1．1452E－02 | 4．9043E－03 | －2．6942E－03 | －9．9435E－03 |
| 7．5572E－03 | 3．42ここE－03 | －1．4617E－03 | －6．2753E－03 |
| $4.1119 \mathrm{~F}-03$ | 1．9346E－03 | －6．6984E－04 | －3．2962E－03 |
| 1．2255E－03 | 5．9346E－04 | －1．6946E．04 | －9．5127E－04 |
| $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 E+00$ | $0.0000 \mathrm{E}+00$ |


| IXI | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IETA |  |  |  |  |  |  |  |  |  |  |
| 51 | -7.6061E-04 | -5.3528E-04 | -2.7632E-04 | -2.9665E-11 | 2.7632E-04 | 5.3528E-04 | 7.6061E-04 | $9.3814 E-04$ | 1.0567E-03 | 1.1089E-03 |
| 50 | -2.8382E-03 | -1.3612E-03 | 1.4306E-04 | 1.5865E-03 | $2.8946 \mathrm{E}-03$ | 4.0027E-03 | 4.8556E-03 | 5.4162E-03 | 5.6682E-03 | 5.6138E-03 |
| 48 | $-4.4659 \mathrm{E}-03$ | -1.7638E-03 | 7.4539E-04 | $2.9463 \mathrm{E}-03$ | $4.7950 \mathrm{E}-03$ | $6.2607 E-03$ | 7.3152E-03 | 7.9447E-03 | $8.1523 \mathrm{E}-03$ | $7.9560 \varepsilon-03$ |
| 46 | -3.3593E-03 | 2.0068E-04 | $3.4345 \mathrm{E}-03$ | 6.1916E-03 | 8.4278E-03 | $1.0112 \mathrm{E}-02$ | 1.1224E-02 | $1.1768 \mathrm{E}-02$ | 1.1767E-02 | 1.1259E-02 |
| 44 | -1.2404E-03 | $3.4756 \mathrm{E}-03$ | $7.6766 \mathrm{E}-03$ | $1.1226 \mathrm{E}-02$ | $1.4069 \mathrm{E}-02$ | $1.6155 \mathrm{E}-02$ | $1.7457 \mathrm{E}-02$ | $1.7980 \mathrm{E}-02$ | 1.7757E-02 | 1.6840E-02 |
| 42 | $2.0114 \mathrm{E}-03$ | B.3097E-03 | $1.3803 \mathrm{E}-02$ | $1.8381 \mathrm{E}-02$ | $2.1979 \mathrm{E}-02$ | $2.4541 \mathrm{E}-02$ | 2.6041E-02 | 2.6492E-02 | 2.5937E-02 | 2.4446E-02 |
| 40 | 7.5799E-03 | 1.5846E-02 | $2.2973 \mathrm{E}-02$ | $2.8851 \mathrm{E}-02$ | $3.3397 E-02$ | $3.6546 \mathrm{E}-02$ | 3.8272E-02 | 3.8587E-02 | 3.7545E-02 | 3.5236E-02 |
| 38 | 1.6855E-02 | $2.7398 \mathrm{E}-02$ | 3.6447E-02 | $4.3869 \mathrm{E}-02$ | 4.9554E-02 | 5.3413E-02 | $5.5396 \mathrm{E}-02$ | 5.5504E-02 | 5.3786E-02 | $5.0350 \mathrm{E}-02$ |
| 36 | 3.1051E-02 | $4.4140 \mathrm{E}-02$ | 5.5388E-02 | 6.4635E-02 | 7.1717E-02 | 7.6482E-02 | 7.8822E-02 | 7.8690E-02 | $7.6111 \mathrm{E}-02$ | 7.1192E-02 |
| 34 | 5.0882E-02 | $6.6712 \mathrm{E}-02$ | 8.0503E-02 | $9.1996 E-02$ | $1.0091 \mathrm{E}-01$ | 1.0697E-01 | J.0994E-01 | 1.0967E-01 | $1.0613 \mathrm{E}-01$ | 9.9379E-02 |
| 32 | 7.6344E-02 | $9.4951 \mathrm{E}-02$ | 1.1172E-01 | 1.2612E-01 | $1.3762 \mathrm{E}-01$ | $1.4569 \mathrm{E}-01$ | $1.4990 \mathrm{E}-01$ | 1.4988E-01 | $1.4546 E-01$ | 1.3663E-01 |
| 30 | 1.0585E-01 | 1.2778E-01 | $1.4816 E^{\text {c }}$-01 | $1.6639 \mathrm{E}-01$ | $1.8160 \mathrm{E}-01$ | $1.9287 \mathrm{E}-01$ | 1.9936E-01 | $2.0038 \mathrm{E}-01$ | 1.9550E-01 | $1.8456 \mathrm{E}-01$ |
| 28 | $1.3219 E-01$ | 1.6126E-01 | 1.8754E-01 | 2.1134E-01 | $2.3191 \mathrm{E}-01$ | 2.4801E-01 | 2,5830E-01 | 2.6162E-01 | 2.5710E-01 | 2.4429E-01 |
| 26 | $1.3920 \mathrm{E}-01$ | $1.8324 \mathrm{E}-01$ | 2.2201E-01 | 2.5620E-01 | $2.8567 \mathrm{E}-01$ | $3.0937 E-01$ | $3.2573 \mathrm{E}-01$ | 3.3312E-01 | $3.3018 \mathrm{E}-01$ | 3.1611E-01 |
| 24 | $1.1460 \mathrm{E}-01$ | 1.7568E-01 | 2.3405E-01 | $2.8681 \mathrm{E}-01$ | $3.3250 \mathrm{E}-01$ | 3.6971E-01 | $3.9667 \mathrm{E}-01$ | 4.1149E-01 | 4.1245E-01 | 3.9844E-01 |
| 22 | $6.8999 E-02$ | 1.3494E-01 | 2.0892E-01 | $2.8310 \mathrm{E}-01$ | $3.5164 \mathrm{E}-01$ | $4.1017 \mathrm{E}-01$ | 4.5522E-01 | $4.8383 \mathrm{E}-01$ | $4.9369 \mathrm{E}-01$ | $4.8334 E-01$ |
| 20 | $2.5039 \mathrm{E}-02$ | 8.0584E-02 | 1.5503E-01 | $2.4045 \mathrm{E}-01$ | $3.2792 \mathrm{E}-01$ | $4.0909 \mathrm{E}-01$ | $4.7685 \mathrm{E}-01$ | 5.2552E-01 | 5.5088E-01 | $5.5035 \mathrm{E}-01$ |
| 18 | -4.9260E-03 | $3.3922 E-02$ | 9.6242E-02 | $1.7766 \mathrm{E}-01$ | $2.7021 \mathrm{E}-01$ | $3.6411 \mathrm{E}-01$ | 4.4937E-01 | 5.1711E-01 | 5.6030E-01 | $5.7419 \mathrm{E}-01$ |
| 16 | -2.0351E-02 | 3.1911E-03 | $4.9340 \mathrm{E}-02$ | 1.1722E-01 | $2.0171 \mathrm{E}-01$ | 2.943:E-01 | $3.8467 \mathrm{E}-01$ | $4.6229 E-01$ | 5.1803E-01 | 5.4502E-01 |
| 14 | -2.5311E-02 | -1.2866E-02 | $1.8663 \mathrm{E}-02$ | 7.0511E-02 | $1.4015 \mathrm{E}-01$ | 2.2133E-01 | 3.0511E-01 | 3.8136E-01 | $4.4043 E-01$ | 4.7447E-01 |
| 12 | -2.4128E-02 | -1.8621E-02 | $1.7228 \mathrm{E}-03$ | 3.9001E-02 | $9.2361 E-02$ | $1.5765 \mathrm{E}-01$ | 2.2792E-01 | 2.9458E-01 | 3.4888E-01 | $3.8326 \mathrm{E}-\mathrm{Dl}$ |
| 10 | -1.9983E-02 | -1.8280E-02 | -5.7635E-03 | 1.9726E-02 | 5.8211E-02 | $1.0710 \mathrm{E}-01$ | 1.6138E-01 | $2.1442 \mathrm{E}-01$ | 2.5912E-01 | $2.8906 E-01$ |
| 8 | -1.4847E-02 | -1.4904E-02 | -7.5897E-03 | 8.8991E-03 | 3.4920E-02 | 6.8946E-02 | 1.0760E-01 | $1.4618 \mathrm{E}-01$ | $1.7946 \mathrm{E}-01$ | 2.0257E-01 |
| 6 | -9.7921E-03 | -1.0411E-02 | -6.4539E-03 | 3.3604E-03 | $1.9418 \mathrm{E}-02$ | $4.0888 \mathrm{E}-02$ | $6.5698 \mathrm{E}-02$ | 9.0835E-02 | $1.1286 \mathrm{E}-01$ | $1.2853 \mathrm{E}-01$ |
| 4 | -5.3125E-03 | -5.8674E-03 | -4.0428E-03 | 8.9056E-04 | $9.1996 \mathrm{E}-03$ | 2.0499E-02 | $3.3720 \mathrm{E}-02$ | $4.7261 \mathrm{E}-02$ | 5.9262E-02 | 6.7934E-02 |
| 2 | $-1.5714 \mathrm{E}-03$ | -1.7807E-03 | $-1.3052 \mathrm{E}-03$ | 7.8695E-05 | 2.4622E-03 | 5.7445E-03 | $9.6201 \mathrm{E}-03$ | 1.3620E-02 | 1.7192E-02 | 1.9803E-02 |
| 1 | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 E+00$ | 0.0000E+00 | $0.0000 E+00$ | 0.0000E+00 | $0.0000 \mathrm{E} \cdot 00$ | $0.0000 E+00$ | 0.0000E+00 |
| IXI | $=40$ | 42 | 44 | 46 | 48 | 50 | 51 |  |  |  |
| IETA 51 |  |  |  |  |  |  |  |  |  |  |
| 51 | 1.0914E-03 | 1.0054E-03 | 9.5613E-04 | 6.5309E-04 | $4.0903 \mathrm{E}-04$ | 1.3926E-04 | 0.0000E 000 |  |  |  |
| 50 | $5.27115-03$ | $4.6697 \mathrm{E}-03$ | $3.8470 \mathrm{E}-03$ | 2.8462E-03 | 1.7224E-03 | 5.6384E-04 | 0.0000E-00 |  |  |  |
| 48 | 7.3864E-03 | $6.4851 E-03$ | $5.3013 \mathrm{E}-03$ | $3.8941 \mathrm{E}-03$ | 2.3423E-03 | 7.6128E-04 | 0.0000E•00 |  |  |  |
| 46 | 1.0295E-02 | 8.9348E-03 | 7.2452E-03 | 5.3022E-03 | $3.1968 \mathrm{E}-03$ | 1.0445E-03 | $0.0000 \mathrm{E} \cdot 00$ |  |  |  |
| 44 | 1.5300E-02 | 1.3220E-02 | 1.0694E-02 | 7.8235E-03 | $4.7269 \mathrm{E}-03$ | 1.5492E-03 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 42 | 2.2115E-02 | 1.9056E-02 | 1.5391E-02 | 1.1258E-02 | $6.811 \mathrm{BE}-03$ | $2.2366 \mathrm{E}-05$ | 0.0000E*00 |  |  |  |
| 40 | 3.1783E-02 | $2.7336 \mathrm{E}-02$ | 2.2060E-02 | $1.6137 \mathrm{E}-02$ | $9.7758 \mathrm{E}-03$ | $3.2156 \mathrm{E}-03$ | $0.0000 E+00$ |  |  |  |
| 38 | $4.5348 \mathrm{E}-02$ | $3.8971 \mathrm{E}-02$ | 3.1444E-02 | $2.3012 \mathrm{E}-02$ | 1.3958E-02 | $4.5994 \mathrm{E}-03$ | 0.0000E+00 |  |  |  |
| 36 | $6.4111 E-02$ | 5.5112E-02 | $4.4493 \mathrm{E}-02$ | $3.2592 \mathrm{E}-02$ | 1.9794E-02 | $6.5343 \mathrm{E}-03$ | 0.0000E +00 |  |  |  |
| 34 | 8.9620E-02 | $7.7156 \mathrm{E}-02$ | $6.2380 \mathrm{E}-02$ | $4.5759 \mathrm{E}-02$ | 2.7831E-02 | 9.2044E-03 | 0.0000E+00 |  |  |  |
| 32 | 1.2558E-01 | $1.0668 \mathrm{E}-01$ | 8.6451E-02 | 6.3540E-02 | 3.8708E-02 | 1.2826E-02 | 0.0000E+00 |  |  |  |
| 30 | $1.6768 \mathrm{E}-01$ | 1.4531E-01 | $1.1812 \mathrm{E}-01$ | $8.7023 E-02$ | 5.3109E-02 | 1.7631E-02 | 0.0000E*00 |  |  |  |
| 28 | 2.2322E-01 | ].9435E-01 | $1.5858 \mathrm{E}-01$ | $1.1716 \mathrm{E}-01$ | 7.1640E-02 | 2.3825E-02 | $0.0000 E+00$ |  |  |  |
| 26 | 2.9071E-01 | $2.5447 \mathrm{E}-01$ | $2.0851 \mathrm{E}-01$ | 1.5452E-01 | 9.4672E-02 | 3.1536E-02 | $0.0000 E+00$ |  |  |  |
| 24 | 3.6912E-01 | 3.2501E-01 | $2.6755 \mathrm{E}-01$ | $1.9896 \mathrm{E}-01$ | 1.2217E-01 | $4.0761 \mathrm{E}-02$ | $0.0000 E+00$ |  |  |  |
| 22 | 4.5235E-01 | $4.0145 \mathrm{E}-01$ | $3.3250 \mathrm{E}-01$ | $2.4841 \mathrm{E}-01$ | $1.5303 \mathrm{E}-01$ | 5.1167E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 20 | $5.2303 \mathrm{E}-01$ | $4.6968 \mathrm{E}-01$ | 3.9258E-01 | $2.9539 \mathrm{E}-01$ | $1.8296 \mathrm{E}-01$ | 6.1403E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 18 | $5.5653 \mathrm{E}-01$ | 5.0746E-01 | $4.2928 \mathrm{E}-01$ | $3.2612 E-01$ | $2.0358 \mathrm{E}-01$ | 6.8713E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 16 | 5.3928E-01 | $4.9978 \mathrm{E}-01$ | $4.2825 E-01$ | $3.2874 E-01$ | $2.0705 \mathrm{E}-01$ | 7.0396E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 14 | $4.7832 \mathrm{E}-01$ | $4.4987 \mathrm{E}-01$ | $3.9001 \mathrm{E}-01$ | 3.0225E-01 | 1.9200E-01 | $6.5806 \mathrm{E}-02$ | $0.0000 \mathrm{E}+00$ |  |  |  |
| 12 | 3.9235E-01 | $3.7350 \mathrm{E}-01$ | 3.2691 E -01 | 2.5532E-01 | $1.6338 \mathrm{E}-01$ | 5.6452E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 10 | $2.9948 \mathrm{E}-01$ | $2.8777 E-01$ | $2.5372 \mathrm{E}-01$ | 1.9933E-01 | 1.2829E-01 | 4.4671E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 8 | 2.1177E-01 | 2.0492E-01 | $1.8165 E-01$ | $1.4334 \mathrm{E}-01$ | $9.265 \mathrm{CE}-02$ | 3.2487E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 6 | 1.3527E-01 | 1.3157E-01 | 1.1709E-01 | 9.2680E-02 | 6.0094E-02 | 2.1200E-02 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 4 | 7.1851E-02 | 7.0145E-02 | 6.2604E-02 | 4.9661E-02 | 3.2269E-02 | $1.1443 \mathrm{E}-02$ | $0.0000 E+00$ |  |  |  |
| 2 | 2.1018 E-02 | $2.0574 \mathrm{E}-02$ | 1.8399E-02 | 1.4618E-02 | $9.5121 \mathrm{E}-03$ | 3.3874E-03 | $0.0000 \mathrm{E}+00$ |  |  |  |
| 1 | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 E+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 E+00$ |  |  |  |

Static press. Coeff. at time level 360

| IXI | $=1$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IETA |  |  |  |  |  |  |  |  |  |  |
| 51 | 3.1342E-02 | 2.1952E-03 | $2.0584 \mathrm{E}-03$ | 1.7918E-03 | 1.4097E-03 | 9.3822E-04 | $4.0870 \mathrm{E}-04$ | -1.4077E-04 | -6.8794E-04 | -1.1920E-03 |
| 50 | 4.9751E-02 | 4.9751E-02 | 3.8739E-02 | 2.3334E-02 | 9.3175E-03 | -3.9895E-03 | -1.6026E-02 | -1.1178E-02 | -7.1565E-03 | -6.5283E-03 |
| 48 | 3.0001E-02 | $3.0001 \mathrm{E}-02$ | 2.3861E-02 | $1.8691 \mathrm{E}-02$ | $9.7681 \mathrm{E}-03$ | -6.5430F-05 | -4.1677E-03 | -9.9676E-03 | -1.3437E-02 | -1.4008E-02 |
| 46 | 2.5651E-02 | $2.5651 \mathrm{E}-02$ | 2.0005E-02 | $1.6516 \mathrm{E}-02$ | 5.9261E-03 | -7.0283E-03 | -1.4620E-02 | -1.8600E-02 | -2.1611E-02 | -2.3033E-02 |
| 44 | $2.1240 \mathrm{E}-02$ | $2.1240 \mathrm{E}-02$ | $1.4995 \mathrm{E}-02$ | $1.0992 \mathrm{E}-02$ | -4.1048E-03 | -2.1266E-02 | -3.0326E-02 | -3.4493E-02 | -3.7193E-02 | -3.7933E-02 |
| 42 | $4.5056 \mathrm{E}-03$ | $4.5056 \mathrm{E}-03$ | $-1.7540 \mathrm{E}-03$ | -5.9929E-03 | -2.4812E-02 | -4.5085E-02 | -5.4872E-02 | -5.8912E-02 | -6.0976E-02 | -6.0387E-02 |
| 40 | -3.0853E-02 | -3.0853E-02 | -3.7323E-02 | -4.2153E-02 | -6.2384E-02 | -8.5802E-02 | -9.3540E-02 | -9.6260E-02 | -9.6110E-02 | -9.2792E-02 |
| 38 | -9.6693E-02 | -9.6693E-02 | -1.0333E-01 | -1.0813E-01 | -1.2462E-01 | -1.4256E-01 | -1.5010E-01 | -1.4960E-01 | -1.4518E-01 | -1.3733E-01 |
| 36 | -1.9747E-01 | -1.9747E-01 | -2.0412E-01 | -2.0850E-01 | -2.1661E-01 | -2.2508E-01 | -2.2660E-01 | -2.2048E-01 | -2.0964E-01 | -1.9535E-01 |
| 34 | -3.2054E-01 | -3.2054E-01 | -3.2706E-01 | -3.3224E-01 | -3.3285E-01 | -3.2933E-01 | -3.2112E-01 | -3.0727E-01 | -2.8877E-01 | -2.6688E-01 |
| 32 | -4.3946E-01 | -4.3946E-01 | -4.4557E-01 | -4.5293E-01 | -4.5253E-01 | -4.4267E-01 | -4.2609E-01 | -4.0477E-01 | -3.7937E-01 | -3.5051E-01 |
| 30 | -5.3337E-01 | -5.3337E-01 | -5.3861E-01 | -5.4741E-01 | -5.5079E-01 | -5.4401E-01 | -5.2748E-01 | -5.04]0E-01 | - $4.7589 \mathrm{E}-01$ | , |
| 28 | -5.9800E-01 | -5.9800E-01 | -6.0227E-01 | -6.1093E-01 | -6.1756E-01 | -6.1773E-01 | -6.0949E-01 | -5.9311E-01 | -5.6999E-01 | -5.4076E-01 |
| 26 | -6.3896E-01 | -6.3896E-01 | -6.4268E-01 | -6.5075E-01 | $-6.5889 E-01$ | -6.6418E-01 | -6.6530E-01 | -6.6129E-01 | -6.5119E-01 | -6.3392E-01 |
| 24 | -6.6330E-01 | -6.6330E-01 | -6.6699E-01 | -6.7504E-01 | -6.8435E-01 | -6.9297E-01 | -7.0067E-01 | -7.0757E-01 | -7.125 1E-01 | -7.1267E-01 |
| 22 | -6.7658E-01 | -6.7658E-01 | -6.8070E-01 | -6.8947E-01 | -7.0050E-01 | -7.1226E-01 | -7.2491E-01 | -7.3945E-01 | -7.5598E-01 | -7.7219E-01 |
| 20 | -6.8274E-01 | -6.8274E-01 | -6.8763E-01 | -6.9769E-01 | -7.1099E-01 | -7.2622E-01 | -7.4350E-01 | $-7.6399 \mathrm{E}-01$ | -7.8850E-01 | -8.1592E-01 |
| 18 | -6.8445E-01 | -6.8445E-01 | -6.9031E-01 | -7.0197E-01 | -7.1780E-01 | -7.3660E-01 | -7.5836E-01 | -7.8407E-01 | -8.1464E-01 | -8.4956E-01 |
| 16 | -6.8348E-01 | -6.8348E-01 | -6.9039E-01 | -7.0373E-01 | -7.2203E-01 | -7.4417E-01 | -7.7001E-01 | -8.0032E-01 | -8.3590E-01 | -8.7635E-01 |
| 14 | $-6.8105 E-01$ | -6.8105E-01 | -6.8901E-01 | -7.0391E-01 | -7.2444E-01 | -7.4947E-01 | -7.7877E-01 | -8.1298E-01 | -8.5277E-01 | -8.9763E-01 |
| 12 | -6.7800E-01 | -6.7800E-01 | -6.8692E-01 | -7.0321E-01 | -7.2564E-01 | $-7.5303 \mathrm{E}-01$ | -7.8512E-01 | -8.2249E-01 | -8.6574E-01 | -9.1424E-01 |
| 10 | -6.7491E-01 | -6.7491E-01 | -6.8464E-01 | -7.0209E-01 | -7.2609E-01 | $-7.5537 E-01$ | -7.8963E-01 | -8.2947E-01 | -8.7548E-01 | -9.2696E-01 |
|  | -6.7207E-01 | -6.7207E-01 | -6.8247E-01 | -7.0088E-01 | -7.2613E-01 | $-7.5689 \mathrm{E}-01$ | -7.9279E-01 | -8.3451E-01 | -8.8270E-01 | -9.3659E-01 |
|  | -6.6963E-01 | -6.6963E-01 | -6.8057E-01 | -6.9973E-01 | -7.2599E-01 | -7.5787E-01 | -7.9501E-01 | -8.3816E-01 | -8.8803E-01 | -9.4384E-01 |
|  | -6.6759E-01 | -6.6759E-01 | -6.7895E-01 | -6.9872E-01 | -7.2577E-01 | -7.5852E-01 | -7.9660E-01 | -8.4083E-01 | -8.9200E-01 | -9.4935E-01 |
| 2 | -6.6584E-0. | -6.6584E-01 | -6.7754E-01 | -6.9781E-01 | -7.2552E-01 | -7.5899E-01 | -7.9783E-01 | -8.4291E-01 | -8.9516E-01 | $5378 \mathrm{E}-01$ |
| 1 | -6.6255E-01 | -6.6584E-01 | -6.7754E-01 | $-6.9781 \mathrm{E}-01$ | -7.2552E-01 | -7.5899E-01 | -7.9783E-01 | -8.4291E-01 | -8.9516E-01 | $5378 \mathrm{E}-01$ |
|  | 20 | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 |
| IETA 36 |  |  |  |  |  |  |  |  |  |  |
|  | -1.6212E-03 | -1,9486E-03 | -2.1536E-03 | -2.2235E-03 | -2.1536E-03 | -1.9486E-03 | -1.6212E-03 | -1.1920E-03 | -6.8794E-04 | 1.4077E-04 |
| 50 | -5.5059E-03 | -3.1127E-03 | 5.9640E-04 | $5.3046 E-03$ | 1.0676E-02 | $1.6393 \mathrm{E}-02$ | 2.2162E-02 | $2.7740 \mathrm{E}-02$ | $3.2936 \mathrm{E}-02$ | $3.7612 \mathrm{E}-02$ |
| 48 | -1.3322E-02 | -1.1383E-02 | -8.3231E-03 | -6.2553E-03 | $6.4334 E-04$ | $6.1399 \mathrm{E}-03$ | $1.1983 E-02$ | $1.7925 \mathrm{E}-02$ | $2.3733 \mathrm{E}-02$ | $2.9201 \mathrm{E}-02$ |
| 46 | -2.2404E-02 | -2.0102E-02 | -1.6510E-02 | -1.1855E-02 | -6.3712E-03- | -3.1466E-04 | 6.0552E-03 | $1.2490 \mathrm{E}-02$ | $1.8758 \mathrm{E}-02$ | $2.4652 \mathrm{E}-02$ |
| 44 | -3.6276E-02 | -3.2591E-02 | -2.7238E-02 | -2.0505E-02 | -1.2719E-02 | -4.2281E-03 | $4.6243 \mathrm{E}-03$ | 1.3509E-02 | 2.2120E-02 | $3.0186 \mathrm{E}-02$ |
| 42 | -5.6908E-02 | -5.1046E-02 | -4.3203E-02 | -3.3724E-02 | -2.3006E-02 | -1.1464E-02 | $4.8646 \mathrm{E}-04$ | $1.2437 \mathrm{E}-02$ | 2.4004E-02 | 3.4840E-02 |
| 40 | -8.6202E-02 | -7.6851E-02 | -6.5180E-02 | -5.1605E-02 | -3.6592E-02 | -2.0633E-02 | -4.2363E-03 | $1.2092 \mathrm{E}-02$ | 2.7863E-02 | $4.2626 \mathrm{E}-02$ |
| 38 | -1.2602E-01 | -1.1167E-01 | -9.4684E-02 | -7.5515E-02 | -5.4664E-02 | -3.2691E-02 | -1.0205E-02 | $1.2163 \mathrm{E}-02$ | 3.3782E-02 | $5.4052 \mathrm{E}-02$ |
| 36 | -1.7770E-01 | -1.5685E-01 | -1.3308E-01 | -1.0678E-01 | -7.8421E-02 | -4.8626E-02 | -1.8109E-02 | $1.2337 \mathrm{E}-02$ | $4.1881 \mathrm{E}-02$ | $6.9702 \mathrm{E}-02$ |
| 34 | -2.4184E-01 | -2.1351E-01 | -1.8190E-01 | -1.4717E-01 | -1.0974E-01 | -7.0229E-02 | -2.9481E-02 | 1.1494E-02 | 5.1574E-02 | 8.9602E-02 |
| 32 | -3.1840E-01 | -2.8271E-01 | -2.4303E-01 | $-1.9924 \mathrm{E}-01$ | $-1.5154 \mathrm{E}-01$ | -1.0054E-01 | -4.7227E-02 | 7.0925E-03 | $6.0874 E-02$ | $1.1245 \mathrm{E}-01$ |
| 30 | -4.0679E-01 | $-3.6552 \mathrm{E}-01$ | -3.1889E-01 | -2.6639E-01 | -2.0795E-01 | -1.4408E-01 | -7.5937E-02 | -5.2308E-03 | $6.5901 \mathrm{E}-02$ | $1.3506 E-01$ |


|  | -5.0513E-01 | -4.6245E-01 | -4.1181E-01 | $-3.5238 \mathrm{E}-01$ | -2.8378E-01 | -2.0641E-01 | -1.2160E-01 | -3.1549E-02 |  | $1.5203 \mathrm{E}-01$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | -6.0784E-01 | -5.7109E-01 | -5.2209E-01 | -4.5964E-01 | $-3.8312 \mathrm{E}-01$ | $-2.9286 \mathrm{E}-01$ | -1.9042E-01 | -7.8653E-02 | $3.8492 \mathrm{E}-02$ | $1.5625 \mathrm{E}-01$ |
|  | $-7.0411 \mathrm{E}-01$ $-7.8313 \mathrm{E}-01$ | $-6.8244 E-01$ $-7.8207 E-01$ | $-6.4377 \mathrm{E}-01$ $-7.6190 \mathrm{E}-01$ | -5.8538E-01 | -5.0593E-01 | -4.0563E-01 | $-2.8640 \mathrm{E}-01$ | -1.5193E-01 | -7.4925E-03 | 1.4043E-01 |
| 20 | -8.4213E-01 | -8.5995E-01 | $-7.619 \mathrm{E}-01$ $-8.6012 \mathrm{E}-01$ | $-7.1661 E-01$ $-8.3330 E-01$ | $-6.4245 E-01$ $-7.7210 E-01$ | $-5.3847 E-01$ $-6.7269 E-01$ | $-4.0654 E-01$ $-5.3553 E-01$ | $-2.5125 \mathrm{E}-01$ | -7.9483E-02 | 1.0015E-01 |
| 18 | -8.8569E-01 | -9.1651E-01 | -9.3245E-01 | -9.2234E-01 | -8.7582E-01 | -7.8581E-01 | $-5.3553 E-01$ $-6.5024 E-01$ | $-3.6529 \mathrm{E}-01$ $-4.7259 \mathrm{E}-01$ | -1.7020E-01 | $3.8875 E-02$ $-2.9292 E-02$ |
|  | -9.1891E-01 | -9.5764E-01 | -9.8327E-01 | -9.8416E-01 | -9.4848E-01 | -8.6694E-01 | -7.5024E-01 | $-4.7259 E-01$ $-5.5502 E-01$ | $-2.6146 E-01$ $-3.3454 E-01$ | $\begin{aligned} & -2.9292 \mathrm{E}-02 \\ & -8.6850 \mathrm{E}-02 \end{aligned}$ |
|  | -9.4481E-01 | -9.8848E-01 | -1.0195E-00 | -1.0262E*00 | -9.9635E-01 | -9.1947E-01 | -7.8991E-01 | -6.0834E-01 | -3.8214E-01 | $1.2453 \mathrm{E}-01$ |
| 12 | $-9.6504 E-01$ $-9.8074 E-01$ | -1.0121E*00 | -1.0463E+00 | $-1.0558 \mathrm{BE}+00$ $-1.0776 \mathrm{*}+00$ | -1.0281E+00 | -9.5252E-01 | -8.2278E-01 | -6.3901E-01 | -4.0826E-01 | -1.4375E-01 |
| B | $\begin{aligned} & -9.8074 E-01 \\ & -9.9282 E-01 \end{aligned}$ | $-1.0305 \mathrm{E}+00$ $-1.0448 \mathrm{E}+00$ | $-1.0668 E+00$ $-1.0827 E+00$ | -1.0776E*00 | $-1.0503 \mathrm{E}+00$ $-1.0667 E+00$ | -9.7411E-01 | -8.4263E-01 | -6.5587E-01 | -4.2085E-01 | -1.5090E-01 |
|  | -1.0021E•00 | $-1.0559 \mathrm{E}+00$ | -1.0951E+00 | -1.10943E+00 | -1.0667E+00 | -9.8926E-01 | $-8.5550 E-01$ $-8.6470 E-01$ | $-6.6554 E-01$ $-6.7177 E-01$ | -4.2652E-01 | -1.5201E-01 |
|  | $-1.0092 \mathrm{E}+00$ | $-1.0645 \mathrm{E}+00$ | -1.1049E-00 | $-1.1176 \mathrm{E}+00$ | $-1.0896 E+00$ | -1.0097E+00 | -8.7188E-01 | $-6.7177 E-01$ $-6.7642 \mathrm{E}-01$ | -4.2924E-01 | $\begin{aligned} & -1.5086 E-01 \\ & -1.4927 E-01 \end{aligned}$ |
|  | $-1.0150 \mathrm{E}+00$ | $-1.0717 \mathrm{E}+00$ | -1.1131E*00 | $-1.1264 E+00$ | $-1.0982 E+00$ | -1.0175E*00 | -8.7805E-01 | -6.8042E-01 | $-4.3229 \mathrm{E}-01$ | $\begin{aligned} & -1.4927 E-01 \\ & -1.4785 E-01 \end{aligned}$ |
|  | $-1.0150 E+00$ | $-1.0717 E+00$ | -1.1131E+00 | -1.1264E+00 | -1.0982E+00 | -1.0175E*00 | -8.7805E-01 | -6.8042E-01 | -4.3229E-01 | $\begin{aligned} & -1.4785 E-01 \\ & -1.4785 E-01 \end{aligned}$ |
| IXI | $=40$ | 42 | 44 | 46 | 48 | 50 | 51 |  |  |  |
| IETA |  |  |  |  |  |  |  |  |  |  |
| 51 | 4.1517E-04 | 9.4494E-04 | $1.4153 \mathrm{E}-03$ | $1.7966 \mathrm{E}-03$ | 2.0649E-03 | 2.2035E-03 | $2.5053 \mathrm{E}-02$ |  |  |  |
| 50 | $4.1677 \mathrm{E}-02$ | $4.5073 \mathrm{E}-02$ | $4.7762 \mathrm{E}-02$ | $4.9710 \mathrm{E}-02$ | 5.0870E-02 | 5.1218E-02 | 5.1218E-02 |  |  |  |
| 48 | 3.4152E-02 | $3.8443 \mathrm{E}-02$ | 4.1967E-02 | $4.4642 \mathrm{E}-02$ | 4.6438E-02 | $4.7280 \mathrm{E}-02$ | 4.7280E-02 |  |  |  |
| 46 | 2.9995E-02 | $3.4641 E-02$ | 3.8481E-02 | 4.1427E-02 | 4.3427E-02 | $4.4419 E-02$ | $4.4419 \mathrm{E}-02$ |  |  |  |
| 44 | $3.7477 \mathrm{E}-02$ | 4.3802E-02 | 4.9017E-02 | 5.3006E-02 | $5.5683 \mathrm{E}-02$ | 5.7021E-02 | 5.7021E-02 |  |  |  |
| 42 | $4.4642 \mathrm{E}-02$ | 5.3157E-02 | 6.0184E-02 | 6.5572E-02 | 6.9188E-02 | 7.1045E-02 | 7.1045E-02 |  |  |  |
| 40 | $5.5981 \mathrm{E}-02$ | $6.7589 \mathrm{E}-02$ | $7.7173 \mathrm{E}-02$ | 8.452BE-02 | 8.9465E-02 | $9.2041 \mathrm{E}-02$ | 9.2041E-02 |  |  |  |
| 38 | 7.2424E-02 | B.8426E-02 | 1.0166E-01 | 1.1184E-01 | 1.1868E-01 | 1.2228E-01 | 1.2228E-01 |  |  |  |
| 36 | $9.5027 \mathrm{E}-02$ | $1.1717 \mathrm{E}-01$ | 1.3555E-01 | 1.4970E-01 | 1.5926E-01 | 1.6431E-01 | $1.6431 \mathrm{E}-01$ |  |  |  |
| 34 | $1.2445 \mathrm{E}-01$ | $1.5511 \mathrm{E}-01$ | $1.8067 \mathrm{E}-01$ | 2.0042E-01 | 2.1382E-01 | 2.2090E-01 | $2.2090 \mathrm{E}-01$ |  |  |  |
| 32 | $1.6017 \mathrm{E}-01$ | $2.0246 \mathrm{E}-01$ | $2.3794 \mathrm{E}-01$ | $2.6548 E-01$ | 2.8427E-01 | 2.9416E-01 | 2.9416E-01 |  |  |  |
| 30 | 1.9977E゙-01 | $2.5766 \mathrm{E}-01$ | 3.0659E-0. | 3.4478E-01 | 3.7095E-01 | $3.8467 \mathrm{E}-01$ | 3.8467E-01 |  |  |  |
| 28 | 2.3851E-01 | $3.1669 \mathrm{E}-01$ | $3.8331 \mathrm{E}-01$ | 4.3562E-01 | $4.7163 \mathrm{E}-01$ | $4.9045 \mathrm{E}-01$ | $4.9045 \mathrm{E}-01$ |  |  |  |
| 26 | 2.6945E-01 | 3.7293E-01 | $4.6188 \mathrm{E}-01$ | 5.3221E-01 | 5.8082E-01 | $6.0617 E-01$ | $6.0617 \mathrm{E}-01$ |  |  |  |
| 24 | 2.8471E-01 | 4.1811E-01 | 5.3384E-01 | $6.2598 \mathrm{E}-01$ | 6.8999E-01 | 7.2333E-01 | $7.2333 \mathrm{E}-01$ |  |  |  |
| 22 | $2.7810 \mathrm{E}-01$ | $4.4460 \mathrm{E}-01$ | $5.9042 \mathrm{E}-01$ | $7.0744 \mathrm{E}-01$ | $7.8927 \mathrm{E}-01$ | $8.3194 E \sim 01$ | B.3194E-01 |  |  |  |
| 20 | $2.4965 E-01$ | $4.4946 \mathrm{E}-01$ | $6.2626 E-01$ | $7.6943 \mathrm{E}-01$ | $8.7039 \mathrm{E}-01$ | $9.2339 \mathrm{E}-01$ | 9.2339E-01 |  |  |  |
| 18 | 2.0914E-01 | $4.3832 \mathrm{E}-01$ | $6.4333 E-01$ | B. $1093 \mathrm{E}-01$ | $9.3035 \mathrm{E}-01$ | $9.9386 \mathrm{E}-01$ | $9.9386 \mathrm{E}-01$ |  |  |  |
| 16 | 1.7164E-01 | $4.2317 \mathrm{E}-01$ | $6.5039 \mathrm{E}-01$ | 8.3777E-01 | $9.7267 E-01$ | 1.0458E+00 | 1.0458E*00 |  |  |  |
| 14 | 1.4722E-01 | 4.1393E-01 | $6.5656 E-01$ | $8.5794 \mathrm{E}-01$ | 1.0042E+00 | $1.0851 E+00$ | $1.0851 \mathrm{E}+00$ |  |  |  |
| 12 | 1.3680E-01 | 4.1342E-01 | $6.6605 \mathrm{E}-01$ | 8.7654E-01 | 1.0303E+00 | 1.1172E+00 | 1.0851E+00 |  |  |  |
| 10 | 1.3596E-01 | 4.1928E-01 | $6.7846 \mathrm{E}-01$ | 8.9479E-01 | 1.0534E+00 | 1.1448E-00 | 1.1448E+00 |  |  |  |
| 8 | 1.3974E-01 | $4.2796 \mathrm{E}-01$ | 6.9170E-01 | 9.1197E-01 | 1.0738E+00 | $1.1687 E+00$ | 1.1687E+00 |  |  |  |
| 6 | 1.4485E-01 | $4.3688 \mathrm{E}-01$ | 7.0405E-01 | 9.2719E-01 | $1.0914 \mathrm{E}+00$ | 1.1890E +00 | $1.1890 \mathrm{E}+00$ |  |  |  |
| 4 | 1.4971E-01 | $4.4483 \mathrm{E}-01$ | 7.1474E-01 | $9.4014 \mathrm{E}-01$ | 1.1060E +00 | $1.2060 \mathrm{E}+00$ | $1.2060 \mathrm{E}+00$ |  |  |  |
| 2 | 1.5397E-01 | $4.5178 \mathrm{E}-01$ | 7.2405E-01 | 9.5137E-01 | $1.1187 \mathrm{E}+00$ | $1.2206 \mathrm{E}+00$ | 1.2206E+00 |  |  |  |
| 1 | $1.5397 \mathrm{E}-01$ | $4.5178 E-01$ | 7.2405E-01 | 9.5137E-01 | 1.1187E +00 | $1.2206 E+00$ | $1.2502 \mathrm{E}+00$ |  |  |  |

CONVERGENCE HISTORY FOR CONTINUITY EOUATION

| LEVEL | hgmax | CHGAVG | **** WITHOUT ARTIFICIAL VISCOSITY **** |  |  |  |  |  | *** WITH ARTIFICIAL VISCOSITY **** |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  |  | RESL2 | RESAVG | RESMAX |  |  | L LOC. | RESL2 | RESAVG | RESmax |  | ESMAX LOC. |
|  | $0.000 \mathrm{E} \cdot 00$ | $0.000 E+00$ | 1.139E+01 | $1.733 \mathrm{E}-01$ | 6.048E-01 |  |  |  |  |  |  |  |  |
| 2 | $0.000 \mathrm{E}+00$ | $0.000 \mathrm{E}+00$ | 9.262E*01 | $7.788 \mathrm{E}-01$ | $1.695 \mathrm{E}+01$ |  |  |  | $9.299 \mathrm{E}+01$ | 1.734 | $6.049 E-01$ $1.711 E+01$ |  |  |
| 10 | 4.162E-03 | $0.000 E+00$ | $4.151 \mathrm{E}+01$ | $4.636 \mathrm{E}-01$ | $6.635 E+00$ |  | 46 , | 50) | 3.125E+01 | $4.447 \mathrm{E}-01$ | $1.711 E+01$ $2.470 E+00$ |  |  |
| 20 | $1.870 \mathrm{E}-03$ | $0.000 E+00$ | 2.423E-01 | 3.103E-01 | $3.967 E+00$ |  |  | 50) | 2.249 E -01 | 2.972E-01 | $2.465 E+00$ |  |  |
| 30 | 1.226E-03 | $0.000 \mathrm{E}+00$ | 5.494E*01 | $3.189 \mathrm{E}-01$ | $9.735 \mathrm{E}+00$ |  | 45. | 501 | 1.483E+01 | 1.661E-01 | 3.803E +00 |  | 13, 50) |
| 40 | 5.444E-04 | $0.000 \mathrm{E} \cdot 00$ | 4.486E+01 | $2.549 E-01$ | B. $389 \mathrm{E}+00$ |  | 48. | 50) | $6.417 \mathrm{E}+00$ | $7.958 \mathrm{E}-02$ | 1.145E-00 |  | 12, 50) |
| 50 | 5.562E-04 | $0.000 \mathrm{E} \cdot 00$ | $5.271 E+01$ | $2.589 \mathrm{E}-01$ | $1.005 \mathrm{E}+01$ | ( | 48. | $50)$ | $5.862 \mathrm{E}+00$ | $5.413 \mathrm{E}-02$ | $1.340 \mathrm{E}+00$ |  | 13, 50) |
| 60 | $3.111 \mathrm{E}-04$ | $0.000 \mathrm{E}+00$ | $5.524 E+01$ | 2.657E-01 | $1.069 \mathrm{E}+01$ | ¢ | 48. | 501 | $4.370 E+00$ | 3.661E-02 | $1.176 \mathrm{E}+00$ |  | 12, 50) |
| 70 | 2.473E-04 | $0.000 \mathrm{E}+00$ | $5.492 E+01$ | $2.585 \mathrm{E}-01$ | $1.079 \mathrm{E}+01$ | 1 |  | 5a) | 3. $\mathbf{B C 4 E}+00$ | 3.163E-02 | 8.702E-01 |  | 12, 50) |
| 80 | $1.768 \mathrm{E}-04$ | $0.000 \mathrm{E}+00$ | $5.720 E+01$ | $2.618 \mathrm{E}-01$ | $1.179 E+01$ | ( |  | 50) | $3.421 E+00$ | 2.528E-02 | $9.025 \mathrm{E}-01$ |  | 12, 501 |
| 90 | $1.259 \mathrm{E}-04$ | $0.000 E+00$ | $5.688 \mathrm{E}+01$ | $2.556 \mathrm{E}-01$ | $1.265 E+01$ | ( |  | 50) | 2.857E+00 | $2.057 \mathrm{E}-02$ | 7.172E-01 |  | 12, 50) |
| 100 | $9.474 \mathrm{E}-05$ | $0.000 E+00$ | 5.762E-01 | $2.533 \mathrm{E}-01$ | $1.339 E+01$ | 6 | 2. | $50)$ | $2.380 \mathrm{E}+00$ | $1.564 E-02$ | 6.322E-01 |  | 12, 50) |
| 110 | 6.529E-05 | 0.000E*00 | 5.772E*01 | $2.500 E-01$ | $1.403 \mathrm{E}+01$ | 5 | 2, | 501 | $1.878 \mathrm{E}+00$ | $1.221 \mathrm{E}-02$ | $4.996 \mathrm{E}-01$ |  | 12, 50) |
| 120 | 4.640E-05 | 0.000E-00 | 5.788E+01 | 2.475E-01 | $1.454 E+01$ | 5 | 2. | 503 | $1.425 E+00$ | B. $634 \mathrm{E}-03$ | 3.827E-01 |  | 12, 50) |
| 130 | $\begin{aligned} & 3.016 \mathrm{E}-05 \\ & 1.889 \mathrm{E}-05 \end{aligned}$ | $0.000 E * 00$ $0.000 E+00$ | $5.796 E+01$ $5.794 E+01$ | 2.459E-01 | $1.494 \mathrm{E}+01$ | ' | 2. | 501 | $1.034 \mathrm{E}+00$ | $6.303 \mathrm{E}-03$ | $2.848 \mathrm{E}-01$ |  | 12, 50) |
| 150 | 1.106E-05 | $0.000 E+00$ $0.000 E+00$ | E + +1 | 1 | $1.523 \mathrm{E}+01$ | $($ | 2, | 50) | 7.084E-01 | $4.216 \mathrm{E}-03$ | $1.948 \mathrm{E}-01$ |  | 12, 50) |
| 160 | 6.477E-06 | $0.000 \mathrm{E}+00$ | 5.782E+01 | $2.432 \mathrm{E}-01$ | $1.555 E+01$ |  |  |  | $4.632 \mathrm{E}-01$ | 2.814E-03 | 1.299E-01 |  | 12, 50) |
| 170 | 4.252E-06 | $0.000 E+00$ | 5.774E+01 | $2.427 \mathrm{E}-01$ | 1.562E+01 | ( |  |  | 1 |  | 2 |  | 2.50) |
| 180 | 2.799E-06 | 0.000E +00 | $5.767 \mathrm{E} \cdot 01$ | 2.423E-01 | 1.565E+01 |  |  |  | 9.648E-01 | 49E-04 | 510E-02 |  | 12, 50] |
| 190 | 1.775E-06 | $0.000 \mathrm{E}+00$ | 5.761E+01 | 2.420E-01 | $1.566 \mathrm{E}+01$ | ( |  |  | 5.380E-02 | 3.829E-04 | 2.308E-02 |  | 2, 43] |
| 200 | 1.131E-06 | $0.000 \mathrm{E}+00$ | $5.756 \mathrm{E}+01$ | $2.417 E-01$ | $1.567 \mathrm{E}+01$ | ( |  |  | $3.227 \mathrm{E}-02$ | 2.366E-04 | 6.114E-03 |  | 2, 38 ) |
| 210 | 6.984E-07 | $0.000 \mathrm{E}+00$ | $5.753 \mathrm{E} * 01$ | $2.416 \mathrm{E}-01$ | $1.566 \mathrm{E}+01$ |  |  | 503 | 2.062E-02 | 1.465E-04 | $4.296 \mathrm{E}-03$ |  | 2, 39) |
| 220 | 4.313E-07 | $0.000 \mathrm{E} \cdot 00$ | $5.751 \mathrm{E}+01$ | $2.415 E-01$ | $1.566 E+01$ | ' |  | $50)$ | 1.380E-02 | 9.386E-05 | 3.282E-03 |  | 13, 50) |
| 230 | 2.902E-07 | $0.000 \mathrm{E} \cdot 00$ | $5.750 E+01$ | $2.414 \mathrm{E}-01$ | $1.566 \mathrm{E}+01$ | 1 |  |  | $9.118 \mathrm{E}-03$ | 5.782E-05 | 2.337E-03 |  | 13, 50) |
| 240 | 1.912E-07 | $0.000 \mathrm{E}+00$ | $5.749 \mathrm{E}+01$ | $2.414 \mathrm{E}-01$ | $1.566 E+01$ | ¢ |  |  | $5.812 \mathrm{E}-03$ | 3.491E-05 | $1.453 \mathrm{E}-03$ |  | 13, 501 |
| 250 260 | 1.236E-07 | $0.000 \mathrm{E}+00$ | $5.749 \mathrm{E}+01$ | $2.414 \mathrm{E}-01$ | $1.565 E+01$ | ( |  |  | $3.580 \mathrm{E}-03$ | 2.080E-05 | 8.041E-04 |  | 2, 42) |
| 260 270 | 8.020E-08 | $0.000 E * 00$ | $5.749 \mathrm{E}+01$ | 2.414E-01 | $1.565 E+01$ | ¢ |  |  | $2.168 \mathrm{E}-03$ | $1.238 \mathrm{E}-05$ | 4.735E-04 |  | 2, 43 ${ }^{\text {d }}$ |
| 270 | 5.164E-08 | 0.000E*OO | $5.749 \mathrm{E}+01$ | $2.414 E-01$ | 1.565E+01 | ¢ |  | 501 | 1.342E-03 | 7.781E-06 | 2.700E-04 |  | 2, 44) |
| 280 | $3.313 E-08$ | $0.000 \mathrm{E}+00$ | $5.749 \mathrm{E}+01$ | 2.414E-01 | 1.565E*01 | ( |  | 50) | $8.733 \mathrm{E}-04$ | $4.992 \mathrm{E}-06$ | 1.731E-04 |  | 2, 38) |
| 290 300 | $2.074 \mathrm{E}-08$ $1.250 \mathrm{E}-08$ | $0.000 \mathrm{E}+00$ $0.000 \mathrm{E}+00$ | $5.749 E+01$ $5.749 \mathrm{E}+01$ | $2.414 \mathrm{E}-01$ | $1.565 \mathrm{E}+01$ | ( | 2,5 | $50)$ | 5.895E-04 | 3.330E-06 | 1.268E-04 |  | 2, 39) |
| 310 | 7.394E-09 | 0.000E 0.000 | $5.749 \mathrm{E}+01$ $5.749 \mathrm{E}+01$ |  | $1.565 E+01$ | ( |  | 50) | $3.975 \mathrm{E}-04$ | 2.216E-06 | 8.819E-05 |  | 2, 40) |
| 320 | 5.014E-09 | $0.000 \mathrm{E}+00$ | $5.749 \mathrm{E}+01$ | $2.414 \mathrm{E}-01$ | $1.565 E+01$ | ( |  | 501 501 | $2.596 \mathrm{E}-04$ $1.621 \mathrm{E}-04$ | 1.476E-06 | 5.800E-05 |  | 2. 40 ) |
| 330 | 3.334E-09 | $0.000 E+00$ | $5.749 \mathrm{E}+01$ | $2.414 \mathrm{E}-01$ | $1.565 \mathrm{E}+01$ |  |  |  |  |  |  |  |  |
| 340 | 2.212E-09 | $0.000 E+00$ | $5.749 \mathrm{E} \cdot 01$ | $2.414 E-01$ | $1.565 E+01$ | ( |  | $50)$ | 5.801 E -05 | $3.863 \mathrm{E}-07$ | 1.262E-05 |  |  |
| 350 | 1.465E-09 | $0.000 E+00$ | $5.749 \mathrm{E}+01$ | $2.414 E-01$ | $1.565 \mathrm{E}+01$ | 1 | 2, 5 | $50)$ | 3.627E-05 | $2.501 \mathrm{E}-07$ | 6.887E-06 |  | 2, 44) |

CONVERGENCE HISTORY FDR X-MOMENTUM EQUATION


| 90 | 3．593k－03 | $0.000 E \cdot 00$ | 3．231E＋02 | 1．777E +00 | $6.328 E+01$ | （ | 2，411 | $3.970 \mathrm{E} \cdot 01$ | 3．097E－01 | 5．742E＋00 | （ | 2． 382 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 100 | $2.543 \mathrm{E}-03$ | $0.000 E+00$ | $3.377 E+02$ | $1.810 \mathrm{E}+00$ | $6.755 \mathrm{E} \cdot 0 \mathrm{~L}$ | 1 | 2，41） | 3．204E＊01 | $2.378 E-01$ | $4.690 E+00$ |  | 2． 393 |
| 110 | 1．826E－03 | $0.000 E \cdot 00$ | $3.4915+02$ | $1.838 \mathrm{E}+00$ | 7．10bE＊01 | （ | 2，41） | 2．472E＊01 | 1．747E－01 | 3．647E＋00 |  | 2，40） |
| 120 | 1．304E－03 | $0.000 E \cdot 00$ | $3.580 E+02$ | $1.867 E+00$ | $3.354 E \cdot 01$ | ， | 2，41） | 1．832E－01 | 1．244E－01 | 2．703E＊00 |  | 2，40］ |
| 130 | 9．210E－04 | $0.000 E * 00$ | $3.644 E+02$ | $1.893 E+00$ | 7．518E－01 | 1 | 2，41） | $1.284 E+01$ | 8．358E－02 | 1．940E +00 |  | 2，41） |
| 140 | b． $173 \mathrm{E}-04$ | $0.000 E+00$ | $3.686 E+02$ | $1.914 \mathrm{E} \cdot 00$ | $7.623 E+01$ | （ | 2．42） | $8.532 \mathrm{E}+00$ | 5．434E－02 | 1．285E－00 |  | 2，421 |
| 150 | 3．911E－04 | $0.000 E+00$ | 3．712E＋02 | 1．929E＊00 | 7．697E＊01 | （ | 2，42） | 5．34be•00 | 3．390E－02 | 8．158E－01 |  | 421 |
| 160 | $2.598 \mathrm{E}-04$ | $0.000 E+00$ | 3．726E＋02 | $1.939 E \cdot 00$ | 7．731E＋01 | （ | 2，423 | $3.167 E+00$ | 2．036E－02 | 4．777E－01 |  | 6．37） |
| 170 | $1.42 \mathrm{bE}-04$ | $0.000 E \cdot 00$ | 3．732E 022 | $1.944 E+00$ | 7． $742 \mathrm{E}+01$ | 1 | 2，42） | $1.793 \mathrm{E} \cdot 00$ | 1．192E－0．2 | 2．066E－01 |  | 6，37\％ |
| 180 | $8.568 \mathrm{E}-05$ | $0.000 E+00$ | $3.7345+02$ | 1． $946 E \cdot 00$ | $7.740 \mathrm{E}+01$ | （ | 2，42） | 1．DODE +00 | $6.897 \mathrm{E}-03$ | 1．501E－01 |  | 2，37） |
| 190 | 6． $21: 1 \mathrm{E}-05$ | $0.000 \mathrm{E}+00$ | 3.73 JE＊02 | 1． $947 \mathrm{E}+00$ | 7．734E＋01 | （ | 2，42） | $5.817 \mathrm{E}-01$ | $4.053 \mathrm{E}-03$ | $1.0315-01$ |  | 2． 381 |
| 200 | $4.274 \mathrm{E}-05$ | $0.000 \mathrm{E}+00$ | $3.732 E+02$ | 1． $946 \mathrm{E}+00$ | 7．728E＊01 | （ | 2，42） | 3．692E－01 | $2.549 \mathrm{E}-03$ | 3．774E－07 |  | 2，39） |
| 210 | $2.809 \mathrm{E}-05$ | $0.000 E+00$ | $3.7318+02$ | $1.946 E \cdot 00$ | $7.723 \mathrm{E}+01$ | （ | 2，42） | $2.683 E-01$ | $1.636 \mathrm{E}-03$ | $4.435 E-02$ |  | 2， 40 ） |
| 220 | 1．784E－05 | $0.000 \mathrm{E}+00$ | 3．730E＋02 | $1.940 E+00$ | $7.719 \mathrm{E}+01$ | 1 | 2，42） | 1．670E－01 | 1．056E－03 | 2．852E－02 |  | 2． $41 \%$ |
| 230 | $1.099 E-05$ | $0.000 \mathrm{E} * 00$ | $3.729 \mathrm{E}+02$ | 1． $945 \mathrm{E}+00$ | 7．717E＋01 | $($ | 2，42） | 1．084E－01 | $6.776 \mathrm{E} \cdot 04$ | 1．768E－02 | 1 | 2，42） |
| 240 | 6．643E－06 | $0.000 \mathrm{E}+00$ | 3．729E＋02 | 1． $945 \mathrm{E}+00$ | $7.717 \mathrm{E}+01$ | （ | 2，423 | $6.757 E-02$ | 4．188E－04 | 1．034E－02 |  | 2，4，${ }^{\text {a }}$ |
| 250 | $3.966 \mathrm{E}-06$ | $0.000 E+00$ | $3.729 \mathrm{E}+02$ | 1，945E＋00 | 1．716E＋01 | （ | 2，42） | 4．112E－02 | $2.590 \mathrm{E}-04$ | －．072E－03 | ， | 6． 381 |
| 260 | 2．357E－06 | $0.000 \mathrm{E}+00$ | 3．729E＋02 | 1．945E－00 | 7．717E＊01 | 1 | 2，423 | 2．52．7E－02 | 1．64．2E－04 | $4.161 E-02$ |  | 2，37） |
| 270 | 1．605E－06 | $0.000 E \cdot 00$ | 3．729E＋02 | $1.945 \mathrm{E}+00$ | 7．717E＋01 | （ | 2，421 | 1．628E－02 | 1．052E－04 | $\therefore 860 \mathrm{E}-03$ |  | 2， 381 |
| 280 | 1．125E－06 | $0.0005+00$ | 3．729E＋02 | $1.945 E+00$ | 7．717E＋01 | （ | 2，421 | 1．09EE－02 | $6.838 \mathrm{E}-05$ | 1．947E－03 |  |  |
| 290 | 7．539E－0\％ | $0.000 \mathrm{E}+00$ | $3.729 \mathrm{E}+02$ | 1．94SE＋00 | 7．717E＋01 | （ | 2．423 | 7．388E－03 | $4.437 \mathrm{E}-05$ | 1．298E－03 |  | 2，40） |
| 300 | $4.869 \mathrm{E}-07$ | $0.000 E+00$ | $3.729 \mathrm{E}+02$ | 1． $945 \mathrm{E}+00$ | 7．717E＊01 | （ | 2，421 | 4.856 E ． $0 \%$ | 2．812E－05 | 8．326E－04 |  | 2，4，！ |
| 310 | $3.050 \mathrm{E}-07$ | $0.000 \mathrm{E} \cdot 00$ | 3．729E＋02 | 1．945E－00 | $7.717 \mathrm{E}+01$ | （ | 2， 421 | 3．048E－03 | 1．772E－05 | 5．04EE－04 |  | 2，42］ |
| 320 | $1.871 \mathrm{E}-07$ | $0.000 \mathrm{E} \cdot 00$ | $3.729 E+02$ | 1． $945 \mathrm{E}+00$ | 7．717E＋01 | 1 | 2，42） | 1．827E－03 | 1．124E－05 | 2．878E－04 |  | 2，42） |
| 330 | 1．133E－07 | $0.000 \mathrm{E} \cdot 00$ | 3．729E＋02 | 1．94SE +00 | 7．717E＊01 | 1 | 2，421 | $1.072 \mathrm{E}-03$ | 7．047E－06 | 1．671E－0．4 |  | 2，36） |
| 340 | $6.849 \mathrm{E}-08$ | 0．000E＊00 | $3.729 \mathrm{E}+02$ | 1．945E＊00 | 7．717E＋01 | （ | 2，421 | 6．529E－04 | $4.528 \mathrm{E}-06$ | 1．163E－04 | （ | 2，37） |
| 350 | $4.325 \mathrm{E}-08$ | $0.000 \mathrm{E}+00$ | $3.729 \mathrm{E}+0$ | 1． $945 E+00$ | 7．717E＊01 | （ | 2． 421 | $4.3715-04$ | 3．043E－06 | 8．111E－05 | ， | 2， 38 ） |

CONVERGENCE HISTORY FOR Y－MOMENTUM EQUATION

|  |  |  | ＊＊＊＊WITHOUT ARTIFICIAL VISCOSITY＊＊＊＊ |  |  |  |  | ＊＊＊＊WITH ARTIFICIAL VISCOSITY＊＊＊＊ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Level | chamax | chgavg | RESL2 | RESAVG | RESMAX | RESMAX | LOC． | RESL2 | resavg | RESMAX |  | SMAX | LOC． |
| 1 | 0．000E＊00 | 0．000E＋00 | 8．313E＊03 | 2．209E＊01 | $1.660 E+03$ | （ 16， | 2） | 8．317E 03 | 2．212E＋01 | $1.601 E+03$ | 6 | 14. | 2） |
| 2 | 0．000E＊00 | $0.000 E+00$ | 1．932E＊03 | 8．514E＋00 | $4.013 E+02$ | （ 10 ， | 2） | 1．934E＋03 | 8．537E＊00 | $4.015 E+02$ | ¢ | 10. | 2） |
| 10 | 5．181F－02 | $0.0005+00$ | 1．344E＊02 | 1．005E＋00 | $2.139 E+01$ | 133. | 50） | 1．118E＋02 | 1．609E＊00 | 7．824E＋00 | ， | 34. | 491 |
| 20 | $2.340 \mathrm{E}-02$ | $0.000 \mathrm{E}+00$ | 9．622E＊01 | 1．311E＋00 | $1.056 E+01$ | （ 15. | 50） | $8.934 \mathrm{E}+01$ | 1．227E 00 | $1.292 \mathrm{E}+01$ | C | 2. | 50） |
| 30 | 1．750E－02 | $0.000 E+00$ | $1.733 \mathrm{E}+02$ | 1．449E＊00 | $3.263 E+01$ | 31 | 501 | 8．236E＊01 | $9.646 \mathrm{E}-01$ | $9.482 \mathrm{E}+00$ | （ | 13, | 503 |
| 40 | 1．089E－02 | $0.000 E+00$ | $1.479 \mathrm{E}+02$ | $1.287 \mathrm{E}+00$ | 2．512E＋01 | 134. | 50） | 7．754E＋01 | 7．628E－01 | $9.728 E+00$ | ¢ |  | 273 |
| 50 | 7．577E－03 | $0.000 \mathrm{E}+00$ | 1．661E＋02 | 1．307E－00 | 3．016E＋01 | （ 34， | 50） | 7．542E＋01 | $6.961 \mathrm{E}-01$ | $1.062 \mathrm{E}+01$ | $\stackrel{1}{6}$ | 5. | 9） |
| 60 | 5．192E－03 | $0.000 E+00$ | 1．690E＋02 | 1．211E＋00 | 3．088E＋01 | 135 | 50） | $6.774 \mathrm{E}+01$ | 5． $3595-01$ | $1.077 \mathrm{E}+01$ | ［ |  | 30） |
| 70 | 3．734E－03 | 0．000E +00 | 1．650E 02 | 1．157E＊00 | $3.015 E+21$ | （ 35. | 503 | $6.015 \mathrm{E}+01$ | $4.650 \mathrm{E}-01$ | $1.043 E+01$ | （ |  | 21） |
| 80 | 2．72こE－03 | $0.000 \mathrm{E} \cdot 00$ | 1．085E＊02 | 1．114E＊00 | $3.099 \mathrm{E}+01$ | （ 35 | 50） | 5．312E－01 | $3.777 E-01$ | $9.873 \mathrm{E}+00$ | ¢ | 5. |  |
| 90 | 1．988E－03 | $3.000 \mathrm{E}+00$ | 1．665E＋02 | $1.074 \mathrm{E}+00$ | $3.016 E+01$ | （ 36 | 50） | $4.6035+01$ | 3．1445－01 | $9.973 \mathrm{E}+00$ | ¢ |  |  |
| 100 | 1．400E－0．3 | $0.000 E+00$ | $1.675 \mathrm{E}+02$ | 1．045E +00 | 3．014E＋01 | （ 36， | 50） | $3.879 \mathrm{E}+0$. | $2.507 \mathrm{E}-01$ | $7.924 E+00$ | ， |  | 4） |
| 110 | 9．740E－04 | c． $0000 \mathrm{E}+00$ | $1.675 E+02$ | $1.019 \mathrm{E}+00$ | $2.969 E+01$ | （ 30 | 50］ | 3．129E＊01 | 1.930 E 01 | $6.478 E+00$ | （ | 5. | 35） |
| 120 | $6.430 \mathrm{E}-04$ | $0.000 \mathrm{E}+00$ | $1.678 \mathrm{E}+02$ | 1．000E +00 | $2.939 E+01$ | （ 36， | 50） | $2.400 \mathrm{E} * 01$ | $1.426 \mathrm{E}-01$ | $5.181 E+00$ | （ |  | 363 |
| 130 | 4．109E－04 | $0.000 \mathrm{E}+00$ | $1.683 \mathrm{E}+02$ | $9.901 \mathrm{E}-01$ | $2.909 E+01$ | （ 36， | 50） | $1.760 E+01$ | 9．995E－02 | 3．862E 00 | ¢ |  |  |
| 140 | 2．843E－04 | $0.000 \mathrm{E}+00$ | $1.686 \mathrm{E}+02$ | $9.883 \mathrm{E}-01$ | $2.881 E+01$ | （ 36 | 50） | $1.219 \mathrm{E}+01$ | $6.738 \mathrm{E}-02$ | $2.781 E+00$ | ！ |  | $37)$ |
| 150 | 1．901E－04 | $0.000 \mathrm{E}+00$ | 1．689E－02 | 9．911E－01 | $2.860 E+01$ | 36. | 501 | 8．011E＊00 | $4.305 E-02$ | $1.863 \mathrm{E}+00$ | （ |  |  |
| 160 | 1．200E－04 | $0.000 \mathrm{E}+00$ | $1.691 \mathrm{E}+02$ | $9.935 \mathrm{E}-01$ | $2.841 E+01$ | （ 36， | 501 | $5.022 E+00$ | $2.671 \mathrm{E}-02$ | $1.151 \mathrm{E}+00$ | ， |  |  |
| 170 | 8．078E－05 | $0.000 E+00$ | 1．692E＋02 | 9．942E－01 | $2.828 E+01$ | （ 36 | 501 | $3.040 E+00$ | $1.637 \mathrm{E}-02$ | $6.930 \mathrm{E}-01$ | t | 4. | 39） |
| 180 | 3．018E－05 | $0.000 \mathrm{E}+00$ | 1．692E＋02 | $9.938 \mathrm{E}-01$ | $2.819 E+01$ | （ 36. | $50]$ | $1.812 E+00$ | $9.946 \mathrm{E}-03$ | 3．958E－01 |  |  |  |
| 190 | 2．047E－05 | $0.000 \mathrm{E}+00$ | $1.691 \mathrm{E}+02$ | 9．931E－01 | $2.813 \mathrm{E}+01$ | （ 36， | 50 J | $1.091 E+00$ | $6.006 \mathrm{E}-03$ | $2.365 E-01$ | 1 |  |  |
| 200 | 1．853E－05 | $0.000 \mathrm{E}+00$ | $1.691 \mathrm{E}+02$ | 9．921E－01 | $2.809 E+01$ | t 36， | $50)$ | $6.785 \mathrm{E}-01$ | $3.707 \mathrm{E}-03$ | 1．587E－01 | 1 |  |  |
| 210 | 1．184E－05 | $0.000 \mathrm{E}+00$ | $1.690 \mathrm{E}+02$ | 9．911E－01 | $2.806 \mathrm{E}+01$ | （ 36， | 50 J | 4．342E－01 | 2．325E－03 | $1.048 \mathrm{E}-01$ | （ |  |  |
| 220 | 7．755E－0． | $0.000 E+00$ | $1.690 E+02$ | $9.904 E-01$ | $2.805 E+01$ | （ 36， | 501 | $2.806 \mathrm{E}-01$ | $1.493 \mathrm{E}-0 \mathrm{~S}$ | $6.807 \mathrm{E}-02$ | 1 |  | 37） |
| 230 | 5．010E－06 | $0.000 E+00$ | $1.690 \mathrm{E}+02$ | 9．899E－01 | $2.804 E+01$ | （ 36. | 501 | $1.796 E-01$ | $9.463 \mathrm{E}-04$ | $4.309 \mathrm{E}-02$ | （ |  |  |
| 240 | 3．ここ退－06 | $0.000 \mathrm{E}+00$ | $1.689 \mathrm{E}+02$ | 9．896E－01 | $2.804 E+01$ | （ 36， | 501 | 1．132E－01 | 5．904E－04 | $2.668 \mathrm{E}-02$ |  |  |  |
| 250 | 2．078E－06 | 0．000E +00 | $1.689 \mathrm{E} \cdot 02$ | 9．895E－01 | $2.804 \mathrm{E}+01$ | （ 36， | 503 | 7．073E－02 | 3．675E－04 | $1.629 \mathrm{E}-02$ | ！ |  |  |
| 260 | 1． $324 \mathrm{E}-06$ | 0．000E 00 | $1.689 \mathrm{E} \cdot 02$ | $9.894 \mathrm{E}-01$ | $2.804 E+01$ | （ 36， | 50） | $4.453 \mathrm{E}-02$ | 2．330E－04 | $9.662 \mathrm{E}-03$ | $t$ |  |  |
| 270 | 8．324E－07 | $0.000 \mathrm{E}+00$ | 1． $689 \mathrm{E}+02$ | 9．894E－01 | 2．804E＊01 | （ 36． | 501 | 2．867E－02 | 1．SODE－04 | $6.006 \mathrm{E}-03$ | ， |  |  |
| 280 | $5.134 \mathrm{E}-07$ | $0.000 \mathrm{E}+00$ | $1.689 \mathrm{E} \cdot 02$ | 9．895E－01 | $2.804 E+01$ | （ 36， | 501 | $1.885 E-02$ | 9．854E－05 | $4.2435-03$ | ！ |  |  |
| 290 | 3．148E－07 | $0.000 \mathrm{E}+00$ | 1．689E＋02 | 9．895E－01 | 2．804E－01 | （ 36， | 503 | $1.246 \mathrm{E}-02$ | $6.511 \mathrm{E}-05$ | $2.916 \mathrm{E}-03$ | 1 |  |  |
| 300 | 2．079E－07 | 0．000E－00 | 1．$-: 9 E \cdot 02$ | $9.895 E-01$ | $2.804 E+01$ | （ 36. | 501 | 8．120E－03 | $4.225 E-05$ | $1.929 \mathrm{E}-03$ | 1 |  | 371 |
| 310 | 1．358E－07 | $0.000 \mathrm{E}+00$ | 1．689E＋02 | 9．895E－01 | $2.804 \mathrm{E}+01$ | （ 36. | 50） | $5.156 \mathrm{E}-03$ | $2.660 \mathrm{E}-05$ | $1.218 \mathrm{E}-03$ | 1 |  |  |
| 320 | 8．785E－08 | 0．000E－00 | $1.689 \mathrm{E} \cdot 02$ | 9．895E－01 | $2.804 E+01$ | 36. | 501 | $3.188 \mathrm{E}-03$ | $1.656 \mathrm{E}-05$ | 7．634E－04 | ， |  |  |
| 330 | 5．75bE－08 | $0.000 \mathrm{E}+00$ | 1．689E＋02 | 9．895E－01 | 2．804E＊01 | （ 36， | 50） | $1.946 \mathrm{E}-03$ | $1.035 \mathrm{E}-05$ | 4．567E－04 | （ |  | 391 |
| 340 | 3．730E－08 | $0.000 \mathrm{E}+00$ | 1．689E＋02 | 9．895E－01 | $2.804 E+01$ | （ 36． |  | $1.204 E-03$ | $6.586 \mathrm{E}-06$ | $2.643 \mathrm{E}-04$ | 1 |  | 401 |
| 350 | 2．386E－08 | $0.000 E+00$ | 1． $689 \mathrm{E}+02$ | 9．895E－01 | 2．804E＊01 | （ 36. | 501 | 7．772E－04 | $4.301 E-06$ | 1．651E－94 | （ |  | 34） |
| STOP |  |  |  |  |  |  |  |  |  |  |  |  |  |

This case used 24.1 seconds of CPU time．

## Computed Results

In Figure 9.4 the computed static pressure coefficient，defined as $\left(\bar{p}-p_{r}\right) /\left(p_{r} u_{r}^{2} / 2 g_{c}\right)$ is plotted as a func－ tion of $\theta$ for both the Euler and viscous flow cases．Also shown are the experimental data of Grove，Shair， Petersen，and Acrivos（1964），and the exact solution for potential flow．


Figure 9.4 - Pressure coefficient for flow past a circular cylinder.

### 9.3 TRAXSOXIC DIFFCSER FLOW

In this test case, transonic turbulent flow was computed in a converging-diverging duct. The flow entered the duct subsonically, accelerated through the throat to supersonic speed, then decelerated through a normal shock and exited the duct subsonically. Extensive experimental data are available for flow through this duct (Chen, Sajben, and Kroutil, 1979; Bogar, Sajben and Kroutil, 1983; Salmon, Bogar, and Sajben, 1983; Sajben, Bogar, and Kroutil, 1984; Bogar, 1986). The computational domain is shown in Figure 9.5.


Figure 9.5-Computational domain for transonic diffuser flow.

## Reference Conditions

The throat height of 0.14435 ft . was used as the reference length $L_{r}$. The reference velocity $u_{r}$ was 100 ft sec. The reference temperature and density were $525.602^{\circ} \mathrm{R}$ and $0.1005 \mathrm{lb}_{\mathrm{m}} / \mathrm{ft}^{3}$, respectively. These values match the inlet total temperature and total pressure used in other numerical simulations of this flow (Hsieh, Bogar, and Coakley, 1987).

## Computational Coordinates

The $x$ coordinate for this duct runs from -4.04 to +8.65 . The Cartesian coordinates of the bottom wall are simply $y=0$ for all $x$. For the top wall, the $y$ coordinate is given by (Bogar, Sajben, and Kroutil, 1983).

$$
y= \begin{cases}1.4144 & \text { for } x \leq-2.598 \\ \alpha \cosh \zeta /(\alpha-1+\cosh \zeta) & \text { for }-2.598<x<7.216 \\ 1.5 & \text { for } x \geq 7.216\end{cases}
$$

where the parameter $\zeta$ is defined as

$$
\zeta=\frac{C_{1}\left(x / x_{i}\right)\left[1+C_{2} x / x_{l}\right]^{C_{3}}}{\left(1-x / x_{l}\right)^{C_{4}}}
$$

The various constants used in the formula for the top wall height in the converging ( $-2.598 \leq x \leq 0$ ) and diverging ( $0 \leq x \leq 7.216$ ) parts of the duct are given in the following table.

| Constant | Converging | Diverging |
| :---: | :---: | :---: |
| $\alpha$ | 1.4114 | 1.5 |
| $x_{t}$ | -2.598 | 7.216 |
| $C_{1}$ | 0.81 | 2.25 |
| $C_{2}$ | 1.0 | 0.0 |
| $C_{3}$ | 0.5 | 0.0 |
| $C_{4}$ | 0.6 | 0.6 |

A body-fitted coordinate system was generated for this duct and stored in an unformatted file to be read by PROTELS. The Cray runstream used to generate this file is listed below. Note that the Fortran program is included in the JCL as a CNICOS "here document".

```
* USER=yourid PW=.
# QSUB -eo -1T 60
QSUB -r SAJBEN$
set -x
ja
touch coords
ln coords fort. }
cat > temp.f<< EOF
l
```



```
C-----OUTPUT COMPUTED:
X(I,J) - X COORDINATES AT GRID POINTS
Y(I,J) - Y COORDINATES AT GRID POINTS
    DIMENSION P(150),Q(150)
    DIMENSION X(150,150),Y(150,150)
C
C----- SECTION, "P" FOR DIVERGING SECTION
    DATA ALFAM,DLM,C1M,C2M,C3M,C4M /1.4114,-2.598,0.81,1.0,0.5,0.6/
    DATA ALFAP,DLP,C1P,C2P,C3P,C4P/1.5 , 7.216,2.25,0.0,0.0,0.6/
C
    XMIN = -4.04
    XMAX = 8.650
    XP1 = 0.0
    XP2 = 4.0
    NX = 81
    NY = 51
    YMIN = 0.0
    YMAX = 1.0
    NP1 = 20
    NP2 = 57
    XINC1 = (XP1 - XMIN)/(NP1 - 1)
    XINC2 = (XP2 - XP1)/(NP2 - NP1)
    XINC3 = (XMAX - XP2) /(NX - NP2)
    YINCR = (YMAX - YMIN)/(NY - 1)
    NIT = 8
C
    P(1) = XMIN
    DO 10 I = 2,NX
    IF (I .LE.NP1) XINCR = XINCI
    IF (I .GT. NP1 .AND. I .LE.NP2) XINCR = XINC2
    IF (I .GT. NP2) XINCR = XINC3
    P(I) = P(I-1) + XINCR
10 CONTINUE
C-----SmOoth mesh by avERAGING between NPl and NP2
    DO 30 ITER = 1,NIT
    DO 20 I = 2,NX-1
```

```
    P(I) = (P(I-1) + P(I+1))*0.5
20 CONTINUE
30 CONTINUE
C
C-----GENERATE COORDINATES OF TOP WALL
C
    DO 100 I = 1,NX
    IF (P(I) .LT. O.0) THEN
C--------CONVERGING SECTION
            XBAR = P(I)/DLM
            XD = 1. - XBAR
            IF (XD .LE. O.0) XD = 1.E-3
            XI = ClM MXBAR*((1.+C2M*XBAR)**C3M)/(XD**CGM)
            Q(I) = ALFAM*COSH(XI)/((ALFAM-1.0) + COSH(XI))
            ELSE IF (P(I) .GE. O.0) THEN
C--------DIVERGING SECTION
            XBAR = P(I)/DLP
            XD = 1. - XBAR
            IF (XD .LE. O.0) XD = 1.E-3
            XI = ClP*XBAR*((1.+C2P*XBAR)**C3P)/(XD**C4P)
            Q(I) = ALFAP*COSH(XI)/((ALFAP-1.0) + COSH(XI))
            ENDIF
100 CONTINUE
C
C-----GENERATE X AND Y ARRAYS
C
            DO 200 I = 1,NX
            DO 200 J = 1,NY
            X(I,J) = P(I)
            YTEMP = (J-1) EYINCR + YMIN
            Y(I,J) = Q(I)*YTEMP/YMAX
200 CONTINUE
C
C-----WRITE UNFORMATTED COORDINATE SYSTEM FILE
C
            WRITE (7) NX,NY
            WRITE (7) ((X(I,J),I=1,NX),J=1,NY),
            $
            STOP
            END
EOF
cft77 -b temp.o -d pq temp.f
segldr -o temp.e temp.o
temp.e
cP coords $HOME/sajben.coords
ja -cslt
```

The resulting body-fitted coordinate system is shown in Figure 9.5. For clarity, the grid points are thinned by factors of 2 and 10 in the $x$ and $y$ directions, respectively. Note that for good resolution of the flow near the normal shock, the grid defining the computational coordinate system is denser in the $x$ direction in the region just downstream of the throat. In the $y$ direction, the computational coordinates are evenly spaced. The computational mesh, however, was tightly packed near both walls to resolve the turbulent boundary layers. ${ }^{32}$

## Initial Conditions

The initial conditions were simply zero velocity and constant pressure and temperature. Thus, $u=v=0$ and $p=T=1$ everywhere in the flow field.

[^23]
## Boundary Conditions

This calculation was performed in three separate runs. In the first run, the exit static pressure was gradually lowered to a value low enough to establish supersonic flow throughout the diverging portion of the duct. The pressure was lowered as follows:

$$
p(t)= \begin{cases}0.99 & \text { for } 1<n \leq 100 \\ -2.1405 \times 10^{-3} n+1.20405 & \text { for } 101 \leq n \leq 500 \\ 0.1338 & \text { for } 501 \leq n \leq 3001\end{cases}
$$

where $n$ is the time level. The equation for $p$ for $101 \leq n \leq 500$ is simply a linear interpolation between $p=0.99$ and $p=0.1338$. In the second run, the exit pressure was gradually raised to a value consistent with the formation of a normal shock just downstream of the throat. Thus,

$$
p(t)= \begin{cases}3.4327 \times 10^{-4} n-0.89636 & \text { for } 3001<n \leq 5000 \\ 0.82 & \text { for } 5001 \leq n \leq 6001\end{cases}
$$

Again, the equation for $p$ for $3001<n \leq 5000$ is simply a linear interpolation between $p=0.1338$ and $p=0.82$. In the third run, the exit pressure was kept constant at 0.82 .

The remaining boundary conditions were the same for all runs. At the inlet, constant total pressure and temperature were specified, and the $y$-velocity and the normal gradient of the $x$-velocity were both set equal to zero. At the exit, the normal gradients of temperature and both velocity components were set equal to zero. At both walls, no-slip adiabatic conditions were used, and the normal pressure gradient was set equal to zero.

These boundary conditions are summarized in the following table.

$$
\begin{array}{cl}
\text { Boundary } & \text { Boundary Conditions } \\
\xi=0 & \partial u / \partial n=0, v=0, p_{T}=T_{T}=1 \\
\xi=1 & \partial u / \partial n=\partial v / \partial n=0, p=p(t), \partial T / \partial n=0 \\
\eta=0 & u=v=0, \partial p / \partial n=\partial T / \partial n=0 \\
\eta=1 & u=v=0, \partial p / \partial n=\partial T / \partial n=0
\end{array}
$$

## PROTELS Input File

The namelist input file for the first run was called SAJBENA P2DIN0 and is listed below, along with a brief explanation of each line. The contents of this listing should be compared with the detailed input description in Section 3.0.

```
TRANSONIC DIFFUSER FLOW, RUN I
    &RSTRT
        IREST=1, Write restart files.
    &END
    &IO
        IVOUT=1,4,30,47*0,
    IPRT=1000,
    IPRT1=2, IPRT2=1,
    IPLOT=-1,
    &END
&GMTRY
    NGEOM=10, Get computational coordinates from file.
&END
&FLOW
    LR=0.14435,
```

Write restart files.

Print $u, M, p$.
Print every 1,000 time levels.
Print at every other $\xi$ index, every $\eta$ index.
Write CONTOUR plot file.

Get computational coordinates from file.

Set $L_{r}$.

```
    UR=100.,
    TR=525.602,
    RHOR=0.1005,
    GAMR=1.4,
&END
&BCD
    JBCl(1,1)=57, JBCl(1,2)=53, GBCl(1,1)=1., GBCl(1,2)=0., T
```



```
    JBCl}(3,1)=21,\operatorname{JBCl}(3,2)=23,\operatorname{GBCl}(3,1)=0., GBCl(3,2)=0.,
    JBCl(4,1)=47, JBCl(4,2)=41, GBCl(4,1)=1.,
    JBC2(1,1)=53, JBC2(1,2)=53, GBC2(1,1)=0., GBC2(1,2)=0., 
    JBC2(2,1)=11, JBC2(2,2)=11, GBC2(2,1)=0., GBC2(1,2)=0.,
    JBC2(3,1)=21, JBC2(3,2)=21,}\operatorname{GBC
    JBC2(4,1)=43, JBC2(4,2)=43, GBC2(4,1)=0., GBC2(4,2)=0., }\quad\partialp/\partialn=0 at \eta=0,1
    JTBC1(4,2)=1, 4th. b. c. at }\xi=1\mathrm{ is general unsteady.
    NTBC=2,
    NTBCA=100,500, Time levels in b.c. table.
    GTBCl(1,4,2)=0.99, }\operatorname{GTBCl}(2,4,2)=0.1338,\quadValues in b.c.table.
&END
&END
    Nl=81, N2=51, Use an 81\times51 mesh.
    IPACK(2)=1, Pack in \eta direction.
    SQ(2,1)=0.5, SQ(2,2)=1.002, Pack fairly tightly near both \eta boundaries.
    IAV4E=2, IAV2E=2, IAV2I=0, Use nonlinear coefficient artificial viscosity.
    CAVS4E=5*0.005, CAVS2E=5*0.1, CAVS2I =5*0., Artificial viscosity coefficients.
    THC=1.0,0.5,
    THX=1.0,0.5,1.0, Second-order time differencing.
    THY=1.0,0.5,1.0,
    THE=1.0,0.5,1.0,
&END
&TIME
    IDTMOD=1, Recompute }\Delta\tau\mathrm{ every time step.
    IDTAU=5, CFL=0.5, Spatially varying }\Delta\tau
    NTIME=3000, Limit of 3000 time steps.
    ICTEST=1, EPS=4*.000001, Use }\Delta\mp@subsup{Q}{max}{\prime}=1\mp@subsup{0}{}{-6}\mathrm{ as convergence criteria.
&END
&TURB
    ITURB=1,
    IWALL2=1,1, Walls at }\eta=0\mathrm{ and }1
&END
&IC
&END
Set ur.
Set Tr.
Set \rhor.
Constant specific heats.
    at }\xi=0,1
    \partialu/\partialn=0 at \xi=0,1.
    v=0,\partialv/\partialn=0
    at }\xi=0,1
    JBC1(4,1)=47.JBC1(4,2)=41, GBC1(4,1)=1..,
    JBC2(1,1)=53, JBC2(1,2)=53, GBC2(1,1)=0, GBC2(1,2)=0, at \xi=0,1.
    2 values in b. c. table.
    Turbulent flow.
Use default initial conditions.
```

In namelist BC , note that at $\xi=1$ the fourth boundary condition type is specified by $\operatorname{JBCl}(4,2)$ as $p=f$, and that the value $f$ is specified by the table of GTBC1 $(, 4,2)$ vs. NTBCA. In namelist NUM, three-point second-order time differencing is being used with a spatially varying time step. This is somewhat unusual, and causes a warning message to be printed, but may be beneficial in some cases. The values of CAVS2E and CAVS4E are based on experience for internal flows with normal shocks. The optimum values for these parameters seem to be a function of the type of flow, the type of time differencing, and the size of the time step. In namelist TIME, a small CFL value is being used. This minimized starting transients and enhanced stability when using an unsteady pressure boundary condition. Note that the time level will run from 1 to 3001 , but that the time levels in the unsteady boundary condition table only run from 100 to 500 . This causes a warning message to be printed. For $n<100$ the boundary condition value will be set equal to the first value in the table, and for $n>500$ it will be set equal to the last value in the table.

The input file for the second run, SAJBENB P2DIN0, was similar to that for the first run. The only differences were:

1. In the TITLE, RUN 1 was changed to RUN 2 .
2. In namelist RSTRT, IREST $=2$, to read and write restart files.
3. In namelist BC , the time levels NTBCA in the boundary condition table were 3001 and 5000 , and the values $\operatorname{GTBCl}(1,4,2)$ and $\operatorname{GTBCl}(2,4,2)$ were 0.1338 and 0.82 .

The input file for the third run, SAJBENC P2DIN0, was similar to that for the second run. The only differences were:

1. In the TITLE, RUN 2 was changed to RUN 3.
2. In namelist $\mathrm{BC}, \mathrm{GBCl}(4,2)$ was specified as 0.82 . The parameters $\operatorname{JTBCl}(4,2), \mathrm{NTBC}, \mathrm{NTBCA}$, and GTBCl were defaulted.
3. In namelist NUM, CAVS4E was lowered to $5 * 0.0004$.
4. In namelist TIME, CFL was raised to 5.0.

## JCL

The Cray UNICOS job control language used for the first run is listed below.

```
* USER=yourid PW=.
* QSUB -eo -1M 2.0Mw -1T 1800
* QSUB -r SAJBENA
set -x
ja
touch plot
touch chist
touch rqout
touch rxout
In plot fort.9
ln chist fort.lo
ln rqout fort.l2
ln rxout fort.l4
ln $HOME/sajben.coords fort.7
fetch input -mUX -t'fn=sajbena,ft=p2din0'
cat > mods << EOM
*ID TEMP
*D PARAMSI.l9
    PARAMETER (N1P = 81, N2P = 51)
EOM
update -p $HOME/p2dl0.u -i mods -c temp -f
cft77 -d pq temp.f
segldr -o temp.e temp.o
temp.e < input
cp rqout SHOME/sajbenb.rqin
cp rxout SHOME/sajbenb.rxin
bintran -mUX -v plot
bintran -mUX -v chist
ja -cslt
```

This JCL is essentially the same as the example presented in Section 8.3, but with lines eliminated that are not applicable to this case.

The JCL for the second run was similar to that for the first run. The only differences were:

1. The following in commands were added to those used in the first run.

In $\$$ HOME/sajbenb.rqin fort. 11
ln \$HOME/sajbenb.rxin fort. 13
2. In the fetch of the input file from the front end, the file name was sajbenb.
3. In the $c p$ commands used to save the restart files at the end of the run, the names of the output files were SHOME/sajbenc.rqin and SHOME/sajbenc.rxin.

The JCL for the third run was similar to that for the second run. The only differences were:

1. In the In commands for the restar files at the beginning of the run, the names of the restar files were SHOMF sujbene rqin and SHOME sajbenc.rxin.
$\therefore$ In the fetch of the input file from the front end, the file name was sajbenc.
2. In the cp commands used to save the restart files at the end of the run, the names of the output files were SHOME/sajbend rqin and SHOME sajbendrxin.

Qote that restart files were written and saved at the end of the third run in case a fourth run was necessary. A fourth run was not used for this case, however.

## Shamand Prelles Output

The output listing for the third run is shown below. In the flow field printout, only the Wach number at the list time level is included.

NASA LEWIS RESEARCH CENTER<br>INTERNAL FLUID MECHANICS DIVISION<br>2-D PROTEUS VERSION 1.0<br>SEPTEMEER 1989




## MACH NUMBER

| XI $=$ | $=1$ | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| IETA |  |  |  |  |  |  |  |  |  |  |
| 51 | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E} \cdot 00$ | $0.0000 E+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.00000+00$ | 0.0000E+00 | 0.0000E*00 |
| 50 | 1.1224E-01 | 1.1418E-01 | 1.0697E-01 | 8.4799E-02 | 7.2660E-02 | 1.1059E-01 | 1.5130E-01 | $1.9280 \mathrm{E}-01$ | 1.9240E-01 | $1.8602 \mathrm{E}-01$ |
| 49 | 2.1777E-01 | 2.2081E-01 | 2.0805E-01 | $1.7531 \mathrm{E}-01$ | $1.4563 E-01$ | $2.0730 \mathrm{E}-01$ | $2.8389 \mathrm{E}-01$ | $3.7016 \mathrm{E}-01$ | $3.7633 \mathrm{E}-01$ | 3.6923E-01 |
| 48 | 2.9665E-01 | $2.9971 \mathrm{E}-01$ | 2.7726E-01 | $2.4031 E-01$ | 2.0004E-01 | $2.7685 \mathrm{E}-01$ | $3.7954 \mathrm{E}-01$ | $4.9235 \mathrm{E}-01$ | $5.0906 \mathrm{E}-01$ | $5.0716 \mathrm{E}-01$ |
| 47 | $3.4551 \mathrm{E}-01$ | $3.4813 \mathrm{E}-01$ | 3.2157E-01 | $2.7694 \mathrm{E}-01$ | $2.3589 \mathrm{E}-01$ | 3.2413E-01 | $4.4907 \mathrm{E}-01$ | 5.6935E-01 | $6.0172 \mathrm{E}-01$ | 5.9827E-01 |
| 45 | $3.7432 \mathrm{E}-01$ | 3. $7649 \mathrm{E}-01$ | $3.5332 \mathrm{E}-01$ | $3.0169 \mathrm{E}-01$ | $2.5861 \mathrm{E}-01$ | $3.5733 \mathrm{E}-01$ | 5.0034E-01 | $6.2116 \mathrm{E}-01$ | 6.7070E-01 | $6.6458 \mathrm{E}-01$ |
| 45 | 3.9646E-01 | 3.9818E-01 | $3.7968 \mathrm{E}-01$ | 3.2520E-01 | $2.7205 \mathrm{E}-01$ | $3.79318-01$ | $5.3735 \mathrm{E}-01$ | $6.6057 \mathrm{E}-01$ | $7.2525 \mathrm{E}-01$ | $7.2431 \mathrm{E}-01$ |
| 44 | 4.1624E-01 | $4.1748 \mathrm{E}-01$ | $4.0191 \mathrm{E}-01$ | 3.5124E-01 | $2.8256 \mathrm{E}-01$ | $3.9167 \mathrm{E}-01$ | $5.6563 \mathrm{E}-01$ | $6.9277 \mathrm{E}-01$ | 7.6862E-01 | $7.82408-01$ |
| 43 | 4.3312E-01 | 4.3389E-01 | $4.1866 \mathrm{E}-01$ | $3.7919 \mathrm{E}-01$ | $2.9701 \mathrm{E}-01$ | $4.0075 \mathrm{E}-01$ | $5.8705 \mathrm{E}-01$ | $7.1915 \mathrm{E}-01$ | $7.9932 \mathrm{E}-01$ | 8.3399E-01 |
| 42 | $4.4637 E-01$ | $4.4673 E-01$ | $4.3173 \mathrm{E}-01$ | $4.05512-01$ | $3.1816 \mathrm{E}-01$ | $4.1173 \mathrm{E}-01$ | $6.0262 \mathrm{E}-01$ | 7.4177E-01 | 8.2017E-01 | 8.7266E-01 |
| 41 | $4.5524 E-01$ | $4.5532 \mathrm{E}-01$ | $4.4245 E-01$ | $4.2638 E-01$ | $3.4363 \mathrm{E}-01$ | $4.2479 \mathrm{E}-01$ | $6.1385 E-01$ | 7.6192E-01 | $8.3559 \mathrm{E}-01$ | $8.9510 \mathrm{~L}-01$ |
| 40 | $4.6050 \mathrm{E}-01$ | $4.6039 \mathrm{E}-01$ | $4.5108 \mathrm{E}-01$ | $4.3960 \mathrm{E}-01$ | $3.6836 E-01$ | $4.3666 E-01$ | $6.2210 \mathrm{E}-01$ | 7.7897E-01 | $8.4747 \mathrm{E}-01$ | 9.0611E-01 |
| 39 | $4.6102 \mathrm{E}-01$ | $4.6089 \mathrm{E}-01$ | $4.5639 \mathrm{E}-01$ | $4.4528 E-01$ | $3.8726 \mathrm{E}-01$ | $4.4454 E-01$ | $6.2908 \mathrm{E}-01$ | $7.8998 \mathrm{E}-01$ | 8.5777E-01 | $9.1550 \mathrm{E}-01$ |
| 38 | $4.5950 \mathrm{E}-01$ | $4.5942 E-01$ | $4.5729 \mathrm{E}-01$ | $4.4586 \mathrm{E}-01$ | $3.9813 E-01$ | $4.4899 \mathrm{E}-01$ | $6.3592 \mathrm{E}-01$ | $7.9143 \mathrm{E}-01$ | $8.6850 \mathrm{E}-01$ | $9.2646 \mathrm{E}-01$ |
| 37 | $4.5851 \mathrm{E}-01$ | $4.5847 \mathrm{E}-01$ | $4.5642 \mathrm{E}=01$ | $4.4509 \mathrm{E}-01$ | $4.0332 \mathrm{E}-01$ | $4.5247 \mathrm{E}-01$ | $6.4030 \mathrm{E}-01$ | $3.8660 \mathrm{E}-01$ | $8.7779 \mathrm{E}-01$ | $9.3510 \mathrm{E}-01$ |
| 36 | $4.5810 \mathrm{E}-01$ | $4.5806 \mathrm{E}-01$ | 4.5602E-01 | $4.4497 \mathrm{E}-01$ | 4.0698E-01 | $4.5555 E-01$ | $6.3934 E-01$ | $7.8293 \mathrm{E}-01$ | $8.7849 \mathrm{E}-01$ | $9.3734 \mathrm{E}-01$ |
| 35 | $4.5799 \mathrm{E}-01$ | $4.5796 \mathrm{E}-01$ | $4.5607 \mathrm{E}-01$ | $4.4546 \mathrm{E}-01$ | $4.1094 \mathrm{E}-01$ | $4.5926 \mathrm{E}-01$ | $6.3801 \mathrm{E}-01$ | $7.8011 \mathrm{E}-01$ | 8.7608E-01 |  |
| 34 | $4.5800 \mathrm{E}-01$ | $4.5796 \mathrm{E}-01$ | $4.5623 E-01$ | $4.4624 E-01$ | $4.1578 \mathrm{E}-01$ | $4.6454 \mathrm{E}-01$ | $6.3684 \mathrm{E}-01$ | $7.7688 \mathrm{E}-01$ | $8.7334 \mathrm{E}-01$ | $9.3522 \mathrm{E}-01$ |
| 33 | $4.5808 \mathrm{E}-01$ | $4.5803 \mathrm{E}-01$ | $4.5644 \mathrm{E}-01$ | $4.4724 E-01$ | $4.2151 \mathrm{E}-01$ | $4.7107 \mathrm{E}-01$ | $6.3534 \mathrm{E}-01$ | 7.7275E-01 | $8.6972 \mathrm{E}-01$ | $9.3318 \mathrm{E}-01$ |
| 32 | $4.5814 \mathrm{E}-01$ | $4.5808 \mathrm{E}-01$ | $4.5667 \mathrm{E}-01$ | $4.4850 \mathrm{E}-01$ | $4.2809 \mathrm{E}-01$ | $4.7844 \mathrm{E}-01$ | $6.3353 \mathrm{E}-01$ | $7.6747 \mathrm{E}-01$ | $8.6514 \mathrm{E}-01$ | $9.3029 \mathrm{E}-01$ |
| 31 | $4.5819 \mathrm{E}-01$ | $4.5811 \mathrm{E}-01$ | $4.5692 \mathrm{E}-01$ | $4.5007 \mathrm{E}-01$ | $4.3541 E-01$ | $4.8661 \mathrm{E}-01$ | $6.3156 \mathrm{E}-01$ | $7.6152 \mathrm{E}-01$ | $8.5978 \mathrm{E}-01$ | $9.2682 \mathrm{E}-01$ |
| 30 | $4.5832 \mathrm{E}-01$ | $4.5822 \mathrm{E}-01$ | $4.5735 \mathrm{E}-0$. | $4.5216 \mathrm{E}-01$ | $4.4346 E-01$ | $4.9531 \mathrm{E}-01$ | $6.2960 E-01$ | $7.5517 \mathrm{E}-01$ | 8.5392E-01 | $9.2297 E-01$ |
| 29 | $4.5872 \mathrm{E}-0 \mathrm{~L}$ | 4.5858E-01 | $4.5816 \mathrm{E}-01$ | 4.5491E-01 | $4.5207 \mathrm{E}-01$ | 5.0426E-01 | $6.2791 \mathrm{E}-01$ | 7.4891E-01 | 8.4789E-01 | $9.1896 \mathrm{E}-01$ |
| 28 | $4.5941 \mathrm{E}-01$ | 4.5924E-01 | $4.5940 \mathrm{E}-01$ | $4.5828 \mathrm{E}-01$ | $4.6088 \mathrm{E}-01$ | 5.1293E-01 | $6.2661 E-01$ | $7.4304 \mathrm{E}-01$ | 8.4207E-01 | $9.1506 \mathrm{E}-01$ |
| 27 | द.6027E-01 | 4.6007E-01 | $4.6089 \mathrm{E}-01$ | $4.6191 \mathrm{E}-01$ | $4.6922 \mathrm{E}-01$ | $5.2078 \mathrm{E}-01$ | $6.2561 \mathrm{E}-01$ | 7.3773E-01 | 8.3658E-01 | 9.1127 E -01 |
| 26 | $4.6108 \mathrm{E}-01$ | $4.6089 \mathrm{E}-01$ | $4.6240 E-01$ | $4.6542 \mathrm{E}-01$ | 4.7662E-01 | $5.2743 \mathrm{E}-01$ | $6.2478 \mathrm{E}-01$ | $7.3310 E-01$ | $8.3160 E-01$ | $9.0773 \mathrm{E}-01$ |
| 25 | $4.6179 \mathrm{E}-01$ | $4.6162 \mathrm{E}-01$ | $4.6378 \mathrm{E}-01$ | $4.6853 \mathrm{E}-01$ | $4.8275 \mathrm{E}-01$ | 5.3277E-01 | $6.2411 \mathrm{E}-01$ | $920 \mathrm{E}-01$ | 8.2729E-01 | 1 |
| 24 | $4.6235 \mathrm{E}-01$ | $4.6220 E-01$ | $4.6495 E-01$ | $4.7113 \mathrm{E}-01$ | $4.8761 \mathrm{E}-01$ | $5.3685 \mathrm{E}-01$ | $6.2357 \mathrm{E}-01$ | 7.2607E-01 | 8.2375 | 0194E-01 |
| 23 | $4.6281 E-01$ | $4.6268 E-01$ | $4.65948-01$ | $4.7321 \mathrm{E}-01$ | $4.9129 E-01$ | 5.3991E-01 | 6.2322E-01 | 7.2371E-01 | $8.2100 \mathrm{E}-01$ | B. $9985 \mathrm{E}-01$ |
| 22 | 4.6320E-01 | $4.6310 \mathrm{E}-01$ | $4.6678 \mathrm{E}-01$ | $4.7487 \mathrm{E}-01$ | $4.9406 \mathrm{E}-01$ | $5.4219 \mathrm{E}-01$ | $6.2303 \mathrm{E}-01$ | 7.2207E-01 | $8.1902 \mathrm{E}-01$ | B. $9834 \mathrm{E}-01$ |
| 21 | $4.6358 \mathrm{E}-01$ | $4.6350 \mathrm{E}-01$ | $4.6750 \mathrm{E}-01$ | $4.7618 \mathrm{E}-01$ | $4.9609 \mathrm{E}-01$ | 5.4392E-01 | $6.2304 \mathrm{E}-01$ | $7.2103 \mathrm{E}-01$ | 8.1770E-01 | 8.9734E-01 |
| 20 | $4.6394 \mathrm{E}-01$ | $4.6386 \mathrm{E}-01$ | $4.6814 \mathrm{E}-01$ | 4.7724E-01 | $4.9761 \mathrm{E}-01$ | $5.4526 E-01$ | $6.2322 E-01$ | $7.2050 E-01$ | $8.1692 \mathrm{E}-01$ | $8.9678 \mathrm{E}-01$ |
| 19 | $4.6430 \mathrm{E}-01$ | $4.6422 E-01$ | $4.6867 \mathrm{E}-01$ | $4.7806 \mathrm{E}-01$ | $4.9874 \mathrm{E}-01$ | $5.4636 \mathrm{E}-01$ | $6.2359 E-01$ | $7.2040 \mathrm{E}-01$ | 8.1666E-01 | $8.9660 \mathrm{E}-01$ |
| 18 | $4.6456 \mathrm{E}-01$ | $4.6448 \mathrm{E}-01$ | $4.6906 \mathrm{E}-01$ | $4.7863 \mathrm{E}-01$ | $4.9951 \mathrm{E}-01$ | $5.4716 \mathrm{E}-01$ | $6.2403 E-01$ | $7.2058 \mathrm{E}-01$ | B. $1672 \mathrm{E}-01$ | $8.9673 \mathrm{E}-01$ |
| 17 | $4.6475 \mathrm{E}-01$ | $4.6468 \mathrm{E}-01$ | $4.6926 \mathrm{E}-01$ | $4.7892 \mathrm{E}-0$. | $4.9987 \mathrm{E}-01$ | $5.4765 \mathrm{E}-01$ | 6.2430E-01 | $7.2065 \mathrm{E}-01$ | 8.1667E-01 | $8.9669 \mathrm{E}-01$ |
| 16 | $4.6492 \mathrm{E}-01$ | $4.6484 E-01$ | $4.6940 \mathrm{E}-01$ | $4.7910 \mathrm{E}-01$ | 5.0010E-01 | 5.4798E-01 | $6.2458 \mathrm{E}-0.1$ | $7.2058 \mathrm{E}-01$ | 8.1649E-01 | 8.9651E-01 |
| 15 | $4.6520 \mathrm{E}-01$ | $4.6511 \mathrm{E}-01$ | $4.6963 E-01$ | $4.7947 \mathrm{E}-01$ | $5.0046 \mathrm{E}-01$ | 5.4841E-01 | $6.2516 \mathrm{E}-01$ | $7.2070 \mathrm{E}-01$ | $8.1673 \mathrm{E}-01$ | $8.9639 \mathrm{E}-01$ |
| 14 | $4.6575 \mathrm{E}-01$ | $4.6564 E-01$ | $4.7021 \mathrm{E}-01$ | $4.8010 \mathrm{E}-01$ | $5.0134 E-01$ | 5.4828E-01 | $6.2407 \mathrm{E}-01$ | $7.2079 \mathrm{E}-01$ | $8.1607 E-01$ | 8.9441E-01 |
| 13 | $4.6701 \mathrm{E}-01$ | $4.6686 \mathrm{E}-01$ | 4.7002E-01 | $4.7945 \mathrm{E}-01$ | 5.0137E-01 | 5.4458E-01 | $6.1920 \mathrm{E}-01$ | $7.1946 E-01$ | 8.1085E-01 | $8.8969 \mathrm{E}-01$ |
| 12 | $4.6742 \mathrm{E}-01$ | $4.6726 \mathrm{E}-01$ | $4.6630 E-01$ | $4.7526 \mathrm{E}-01$ | $4.9612 \mathrm{E}-01$ | 5.3656E-01 | $6.1223 E-01$ | 7.1308E-01 | 8.0204E-01 | $8.8274 E-01$ |
| 11 | $4.6300 \mathrm{E}-01$ | $4.6302 \mathrm{E}-01$ | 4.5882E-01 | $4.6694 \mathrm{E}-01$ | $4.8571 \mathrm{E}-01$ | $5.2550 \mathrm{E}-01$ | $6.0399 \mathrm{E}-01$ | $7.0119 \mathrm{E}-01$ | $7.9192 \mathrm{E}-01$ | 1 |
| 10 | $4.5482 \mathrm{E}-01$ | $4.5512 \mathrm{E}-01$ | $4.4845 E-01$ | $4.5336 \mathrm{E}-01$ | $4.7067 \mathrm{E}-01$ | $5.1270 \mathrm{E}-01$ | $5.9343 \mathrm{E}-01$ | $6.8693 \mathrm{E}-01$ | 1 | 1 |
| 9 | 4.4282E-01 | $4.4349 \mathrm{E}-01$ | $4.3535 \mathrm{E}-01$ | $4.3368 \mathrm{E}-01$ | $4.5232 \mathrm{E}-01$ | $4.9890 \mathrm{E}-01$ | 5.7948E-01 | $6.6957 E-01$ | 1 | $8.1516 \mathrm{E}-01$ |
| 8 | $4.2694 \mathrm{E}-01$ | $4.2808 \mathrm{E}-01$ | $4.1871 \mathrm{E}-01$ | $4.0827 E-01$ | $4.3198 \mathrm{E}-01$ | $4.8310 \mathrm{E}-01$ | $5.6087 \mathrm{E}-01$ | 6.4470E-01 | 7.2691E-01 | 1 |
| 7 | $4.0809 \mathrm{E}-01$ | $4.0970 \mathrm{E}-01$ | $3.9630 \mathrm{E}-01$ | 3.7919E-01 | $4.1093 \mathrm{E}-01$ | $4.6134 \mathrm{E}-01$ | 5.3352E-01 | $6.1027 E-01$ | $6.8806 E-01$ | $7.1210 \mathrm{E}-01$ |
| 6 | $3.8650 \mathrm{E}-01$ | 3.8859E-01 | $3.6911 \mathrm{E}-01$ | $3.4922 E-01$ | 3.8731E-01 | $4.3084 \mathrm{E}-01$ | $4.9304 \mathrm{E}-01$ | 5.6803E-01 | $6.3566 \mathrm{E}-01$ | $6.5218 \mathrm{E}-01$ |
| 5 | $3.5725 E-01$ | 3.5985E-01 | $3.3615 \mathrm{E}-01$ | $3.1683 \mathrm{E}-01$ | 3.5428E-01 | $3.9115 \mathrm{E}-01$ | $4.4291 \mathrm{E}-01$ | $5.1655 \mathrm{E}-01$ | 5.6805E-01 | $5.8525 E-01$ |
| 4 | $3.0678 \mathrm{E}-01$ | 3.0990E-01 | 2.9033E-01 | 2.7234E-01 | 3.0401E-01 | 3. $3658 \mathrm{E}-01$ | 7785E-01 | $4.3969 \mathrm{E}-01$ | $4.8195 \mathrm{E}-01$ | $4.9545 \mathrm{E}-01$ |
| 3 | 2.2679E-01 | $2.2996 E-01$ | $2.1908 E-01$ | $2.0155 E-01$ | $2.2776 \mathrm{E}-01$ | $2.5383 \mathrm{E}-01$ | 8297E-01 | $3.2852 \mathrm{E}-01$ | $3.5739 \mathrm{E}-01$ | $3.6304 \mathrm{E}-01$ |
| 2 | 1.1801E-01 | 1.2008E-0. | $1.1403 \mathrm{E}-01$ | $1.0328 \mathrm{E}-01$ | $1.1883 \mathrm{E}-01$ | 3387E-01 | 1.4944E-01 | $1.7256 \mathrm{E}-01$ | $1.8527 E-01$ | $1.8514 \mathrm{E}-01$ |
| 1 | $0.0000 E+00$ | $0.0000 \mathrm{E} \cdot 00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | 0.a000etao | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E}+00$ |
| $=$ | $=20$ | 22 | 24 | 26 | 28 | 30 | 32 | 34 | 36 | 38 |
| ETA |  |  |  |  |  |  |  |  |  |  |
| 51 | $0.0000 \mathrm{E}+00$ | $0.0000 \mathrm{E} \cdot 00$ | $0.0000 \mathrm{E} \cdot 00$ | $0.0000 E+00$ | 000E+00 | 0000E+00 |  | $E+00$ |  |  |
| 50 | 1.8673E-01 | 1.8602E-01 | 1.9090E-01 | $1.9711 \mathrm{E}-01$ | 2.1293E-01 | $2.2201 E-01$ | $2.2959 E-01$ | 2.3111E-01 | $1.4466 \mathrm{E}-01$ | 4.3245E-02 |
| 49 | 3.7350E-01 | 3.7346E-01 | $3.8334 \mathrm{E}-01$ | $3.9873 \mathrm{E}-01$ | $4.3301 \mathrm{E}-01$ | 4.3645E-01 | 4.5885E-01 | $4.7190 \mathrm{E}-01$ | $3.3541 \mathrm{E}-01$ | 1.2786E-01 |
| 48 | 5.1537E-01 | 5.1678E-01 | $5.3099 \mathrm{E}-01$ | $5.5335 \mathrm{E}-01$ | $6.0511 \mathrm{E}-01$ | 5.9867E-01 | $6.3503 \mathrm{E}-01$ | $6.5807 \mathrm{E}-01$ | $4.7931 \mathrm{E}-01$ | 1.8333E-01 |
| 47 | $6.1078 \mathrm{E}-01$ | $6.1833 \mathrm{E}-01$ | $6.3580 \mathrm{E}-01$ | 6.5837E-01 | 7.1981E-01 | 7.1506E-01 | 7.5810E-01 | $7.8226 E-01$ | 5.6147E-01 | 2.2302E-01 |
| 46 | $6.7562 E-01$ | $6.8657 \mathrm{E}-01$ | $7.0802 \mathrm{E}-01$ | 7.3319E-01 | $7.9994 E-01$ | 8.0314E-01 | 8.4855E-01 | B.7615E-01 | 6.2364E-01 | $2.6169 \mathrm{E}-01$ |
| 45 | 7.3057E-01 | $7.4164 E-01$ | $7.6574 \mathrm{E}-01$ | 7.9716E-01 | $8.6556 \mathrm{E}-01$ | 8.7735E-01 | 9.2294E-01 | 9.6185E-01 | $6.8913 \mathrm{E}-01$ | 3.0299E-01 |
| 44 | $7.8750 \mathrm{E}-01$ | $7.9812 \mathrm{E}-01$ | $8.2310 \mathrm{E}-01$ | 8.6059E-01 | $9.2223 E-01$ | 9.4458E-01 | $9.9183 \mathrm{E}-01$ | $1.0407 \mathrm{E}+00$ | 7.6877E-01 | $3.4891 \mathrm{E}-01$ |
| 43 | 8-4501E-01 | 8.5666E-01 | $8.8309 \mathrm{E}-01$ | 9.2475E-01 | 9.7100E-01 | $1.0063 \mathrm{E}+00$ | $1.0549 \mathrm{E}+00$ | 1. $1018 \mathrm{E}+00$ | 8.6676E-01 | $4.0262 \mathrm{E}-01$ |
| 42 | 8.9564E-01 | $9.1151 \mathrm{E}-01$ | 9.4052E-01 | 9.8176E-01 | $1.0143 \mathrm{E}+00$ | $1.0588 \mathrm{E}+00$ | $1.1033 \mathrm{E}+00$ | 1.1396E+00 | $9.7703 E-01$ | $4.6995 E-01$ |
| 41 | $9.3276 \mathrm{E}-01$ | $9.5594 \mathrm{E}-01$ | 9.8775E-01 | $1.0265 \mathrm{E}+00$ | $1.0524 E+00$ | 1.0985E*00 | 1.1408E+00 | 1.1655E+00 | $1.0837 E+00$ | 5.5458E-01 |
| 40 | 9.5291E-01 | $9.8459 \mathrm{E}-01$ | $1.0197 \mathrm{E}+00$ | $1.0572 \mathrm{E}+00$ | 1.0851E 00 | 1.1262E+00 | 1.1722E+00 | $1.2018 \mathrm{E}+00$ | $1.1659 E+00$ | 6.5426E-01 |
| 399 | 9.6261E-01 | $9.9806 \mathrm{E}-01$ | $1.0371 \mathrm{E}+00$ | $1.0755 \mathrm{E}+00$ | $1.1106 \mathrm{E}+00$ | 1.1464E+00 | $1.1959 E+00$ | 1.2364E+00 | 1.2103E+00 | 7.6235E-01 |
| 389 | $9.7201 E-01$ | $1.0071 E+00$ | $1.0481 \mathrm{E}+00$ | 1.0864E +00 | 1.1297E+00 | 1.1623E 00 | $1.2098 \mathrm{E}+00$ | 1.2418E +00 | $1.2117 \mathrm{E}+00$ | B.6904E-01 |
| 379 | $9.8157 E-01$ | $1.0165 E+00$ | $1.0579 \mathrm{E}+00$ | $1.0953 \mathrm{E}+00$ | 1.142こE•00 | 1.1756E-00 | $1.2174 E+00$ | 1.2389E +00 | 1.2108E+00 | 9.5821E-01 |
| 36 | $9.8497 E-01$ | 1.0197E +00 | $1.0605 E+00$ | 1.0972E-00 | 1.1431E+00 | 1.1814E+00 | $1.2236 E+00$ | 1.2580E +00 | 1.2377E+00 | 1.0210E+00 |
| 35 | $9.8456 \mathrm{E}-01$ | $1.0194 E+00$ | $1.0603 \mathrm{E} \cdot 00$ | $1.0974 \mathrm{E} \cdot 00$ | 1.1423E+00 | $1.1839 E+00$ | 1.228IE+00 | 1.2691E*00 | 1.2575E*00 | $1.0528 \mathrm{E}+00$ |
| 349 | 9.8425E-01 | $1.0195 E+00$ | $1.0605 \mathrm{E}+00$ | $1.0978 \mathrm{E}+00$ | $1.1430 E+00$ | 1.1840E+00 | 1.2279E*00 | 1.2697E+00 | $1.2662 E+00$ | $1.0576 \mathrm{E} \cdot 00$ |
| 33 | $9.8331 E-01$ | $1.0190 \mathrm{E}+00$ | $1.0596 E+00$ | $1.0965 \mathrm{E}+00$ | $1.1414 E+00$ | 1.1812E+00 | 1.2254E+00 | 1.2677E*00 | $1.2733 \mathrm{E}+00$ | $1.0552 \mathrm{E}+00$ |
| 32 | $0.8161 \mathrm{E}-01$ | $1.0177 \mathrm{E}+00$ | $1.0580 \mathrm{E}+00$ | 1.0945E +00 | $1.1396 E+00$ | $1.1780 \mathrm{E}+00$ | 1.2222E-00 | $1.2653 E+00$ | $1.2812 \mathrm{E}+00$ | $1.0524 \mathrm{E}+00$ |
| 31 | $9.7946 E-01$ | $1.0160 \mathrm{E}+00$ | $1.0560 E+00$ | 1.0921E+00 | $1.1374 E+00$ | 1.1743E+00 | $1.2188 \mathrm{E}+00$ | $1.2618 \mathrm{E}+00$ | $1.2889 E+00$ | $1.0508 E+00$ |
| 30 | $9.7707 \mathrm{E}-01$ | 1.0142E+00 | $1.0539 \mathrm{E}+00$ | 1.0897E+00 | 1.1348E*00 | $1.1708 \mathrm{E}+00$ | $1.2148 \varepsilon+00$ | $1.2578 \mathrm{E}+00$ | $1.2952 \mathrm{E}+00$ | 1.0503E+00 |
| 29 | 9. $7460 \mathrm{E}-01$ | $1.0123 E+00$ | 1.0520E*00 | $1.0873 E+00$ | 1.1323E +00 | $1.1676 E+00$ | $1.2112 E+00$ | $1.2534 E+00$ | $1.2986 \mathrm{E}+00$ | $1.0525 \mathrm{E}+00$ |
| 289 | 9.7226E-01 | $1.0107 \mathrm{E}+00$ | 1.0503E-00 | $1.0851 E+00$ | 1.1300E +00 | $1.1643 E+00$ | $1.2074 E+00$ | $1.2501 E+00$ | $1.2986 \mathrm{E}+00$ | $1.0508 \mathrm{BE}+00$ |
| 279 | $9.6995 \mathrm{E}-01$ | 1.0091E+00 | 1.04B7E 00 | $1.0831 E+00$ | 1.1277E +00 | $1.1613 \mathrm{E}+00$ | $1.2039 \mathrm{E}+00$ | $1.2471 E+00$ | $1.2952 \mathrm{E}+00$ | $1.0636 E+00$ |
| 269 | $9.6779 \mathrm{E}-01$ | $1.0076 E+00$ | 1.0471E+00 | 1.0813E+00 | 1.1252E+00 | 1.1587E+00 | $1.2007 E+00$ | $1.2446 E+00$ | $1.2901 E+00$ | 1.0707E +00 |
| 259 | $9.6584 \mathrm{E}-01$ | $1.0063 \mathrm{E}+00$ | $1.0456 \mathrm{E} \cdot 00$ | $1.0799 E+00$ | 1.1229E+00 | $1.1565 E+00$ | $1.1980 \mathrm{E}+00$ | 1.2420E +00 | $1.2846 \mathrm{E}+00$ | $1.0769 E+00$ |
| 249 | 9.6420E-01 | $1.0052 \mathrm{E}+00$ | 1.0443E-00 | $1.0788 E+00$ | 1. $1206 \mathrm{E}+00$ | 1.1546E+00 | $1.1962 \mathrm{E}+00$ | $1.2393 \mathrm{E}+00$ | $1.2793 \mathrm{E}+00$ | $1.0814 \mathrm{E}+00$ |
| 239 | $9.6290 E-01$ | 1.0042E+00 | $1.0432 E+00$ | 1.0781E+00 | 1.1186E +00 | $1.1534 E+00$ | $1.1948 \mathrm{E}+00$ | $1.2370 \mathrm{E}+00$ | $1.2736 \mathrm{E}+00$ | $1.0847 \mathrm{E}+00$ |
| 229 | 9.6197E-01 | 1.0035E*00 | 1.0424E+00 | $1.0776 \mathrm{E}+00$ | $1.1170 E+00$ | $1.1525 E+00$ | $1.1941 \mathrm{E}+00$ | $1.2349 E+00$ | 1.2673E 00 | $1.0873 \mathrm{E}+00$ |
| 219 | $9.6140 \mathrm{E}-01$ | $1.0031 \mathrm{E}+00$ | $1.0419 E+00$ | 1.0774E+00 | $1.1159 \mathrm{E}+00$ | $1.1522 \mathrm{E}+00$ | $1.1937 \mathrm{E} \cdot 00$ | 1.2332E*00 | $1.2606 E+00$ | $1.0895 \mathrm{E}+00$ |
| 209 | $9.6114 \mathrm{E}-01$ | $1.0028 \mathrm{E}+00$ | $1.0417 E+00$ | 1.0774E-00 | 1.1151E*00 | 1.1522E+00 | 1.1937E*00 | 1.2315E+00 | $1.2540 \mathrm{E}+00$ | 1.0917E+00 |
| 199 | $9.6115 E-01$ | $1.0029 E+00$ | $1.0418 \mathrm{C}+00$ | 1.0777E+00 | 1.1147E*00 | $1.1527 \mathrm{E}+00$ | $1.1939 \mathrm{E}+00$ | $1.2303 \mathrm{E}+00$ | $1.2479 \mathrm{E}+00$ | $1.0934 \mathrm{E}+00$ |
| 189 | $9.6142 \mathrm{E}-01$ | $1.0032 \mathrm{E}+00$ | $1.0422 \mathrm{E}+00$ | $1.0783 E+00$ | 1.1150E*00 | $1.1536 E+00$ | $1.19495+00$ | $1.2303 E+00$ | $1.2432 \mathrm{E}+00$ | $1.0954 \mathrm{E}+00$ |
| 179 | $9.6150 \mathrm{E}-01$ | $1.0033 \mathrm{E}+00$ | 1.0623E +00 | $1.0787 \mathrm{E}+00$ | 1.1152E-00 | $1.1541 \mathrm{E}+00$ | $1.1953 \varepsilon * 00$ | $1.2308 \mathrm{E}+00$ | $1.2405 \mathrm{E}+00$ | $1.0990 \mathrm{E}+00$ |

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Note that this calculation has not reached the level of convergence specified in the input $\left(\Delta Q_{\max } \leq 10^{-6}\right.$ ). However, close examination of several parameters near the end of the calculation indicates that the solution is no longer changing appreciably with time, but oscillates slightly about some mean steady level. This type of result appears to be fairly common, especially for flows with shock waves. The reason is not entirely clear, but may be related to inadequate mesh resolution, discontinuities in metric information, etc. For this particular case, the cause may also be inherent unsteadiness in the flow. The experimental data for this duct show a self-sustained oscillation of the normal shock at Mach numbers greater than about 1.3 (Bogar, Sajben, and Kroutil, 1983).

The three runs for this case required $709.2,679.3$, and 694.4 seconds of CPU time, respectively, for execution. About 142 additional seconds were required in each run to up-dimension and compile the code.

## Computed Results

The computed flow field is shown in Figure 9.6 in the form of constant Mach number contours. Contours are plotted at Mach numbers ranging from 0.0 to 1.2 in increments of 0.1 .


Figure 9.6-Computed Mach number contours for transonic diffuser flow.
The flow enters the duct at about $M=0.46$, accelerates to just under $M=1.3$ slightly downstream of the throat, shocks down to about $M=0.78$, then decelerates and leaves the duct at about $M=0.51$. The normal shock in the throat region and the growing boundary layers in the diverging section can be seen clearly. Because this is a shock capturing analysis, the normal shock is smeared in the streamwise direction.

The computed distribution of the static pressure ratio along the top and bottom walls is compared with experimental data (Hsieh, Wardlaw, Collins, and Coakley, 1987) in Figure 9.7. The static pressure ratio is here defined as $p /\left(p_{T}\right)_{0}$, where $\left(p_{T}\right)_{0}$ is the inlet core total pressure.


Figure 9.7 - Computed and experimental static pressure distribution for transonic diffuser flow.

The computed results generally agree well with the experimental data, including the jump conditions across the normal shock. The predicted shock position, however, is slightly downstream of the experimentally measured position. The pressure change, of course, is also smeared over a finite distance. There is also some disagreement between analysis and experiment along the top wall near the inlet. This may be due to rapid changes in the wall contour in this region without sufficient mesh resolution.

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| 7. Author(s) <br> Charles E. Towne, John R. Schwab. Thomas J. Benson, and Ambady Suresh |  |  |  | 8. Pertorming Organization Report No. E-5367 |  |
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| 16. Abstract <br> A new computer code has been developed to solve the two-dimensional or axisymmetric, Reynolds-averaged, unsteady compressible Navier-Stokes equations in strong conservation law form. The thin-layer or Euler equations may also be solved. Turbulence is modeled using an algebraic eddy viscosity model. The objective in this effort has been to develop a code for aerospace applications that is easy to use and easy to modify. Code readability, modularity, and documentation have been emphasized. The equations are written in nonorthogonal body-fitted coordinates, and solved by marching in time using a fully-coupled alternating-direction-implicit procedure with generalized first- or second-order time differencing. All terms are linearized using second-order Taylor series. The boundary conditions are treated implicitly, and may be steady, unsteady, or spatially periodic. Simple Cartesian or polar grids may be generated internally by the program. More complex geometries require an externally generated computational coordinate system. The documentation is divided into three volumes. Volume 2 is the User's Guide, and describes the program's general features, the input and output, the procedure for setting up initial conditions, the computer resource requirements, the diagnostic messages that may be generated, the job control language used to run the program, and several test cases. |  |  |  |  |  |
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[^0]:    1 It should be noted that namelist input is not part of ANSI standard Fortran 77, but is nevertheless available with most Fortran compilers. See Section 2.3.1 of Volume 3 for a discussion of possible computer-dependent features in the PROTECS code.

[^1]:    3 The definitions of $k_{l}$ and $k_{r}$ in Table 3.3 (IVOUT $=92$ and 102) assume a constant turbulent Prandu number is being specified in namelist TURB.

[^2]:    4 The IPLOT $=-1$ option is the better one to use for CONTOLR plot files. The IPLOT $=1$ option is included only to be consistent with the various PLOT3D and PLOT2D options.
    5 The current version of PLOT3D does not work for multiple time levels, although future versions might. You can,
    however, fake it out using the IPLOT $=-3$ option.
    6 Note that with IDTAU $=5$ or $6, \tau$ will vary in space, and therefore $\tau_{i, j} \neq \tau_{1,1}$.

[^3]:    7 There may be some confusion between the axisymmetric flow option and the polar coordinate system option, or between the axisymmetric radius $r$ and the polar coordinate $r^{\prime}$. They are not the same thing. The governing flow equations were developed by originally writing them in Cartesian $(x-y)$ coordinates, then transforming them into generalized $(\xi-\eta)$ coordinates. Therefore, any computational coordinate system that is used, including the polar coordinate system, must be related to the original Cartesian system through the transformation metrics and Jacobian. The parameters $r^{\prime}$ and $\theta^{\prime}$ are used only to initially define the coordinates in the $\mathrm{XGEOM}=2$ option. Now, if the $(x-y)$ coordinates, no matter how they are obtained, are rotated about the Cartesian $x$ axis, the result is a cylindrical coordinate coordinate system with $y$ representing the radius $r$. Thus, the axisymmetric flow option can be used with any of the coordinate system options. The polar coordinate option would be useful, for instance, for flow over a sphere.

[^4]:    B However, note that a specified point-by-point distribution of a function value is most easily set using the "no change from initial conditions" option with the JBC parameters.

[^5]:    ${ }^{9}$ In $\Delta \mathbf{Q}_{\max }$, the total energy $\bar{E}_{T}$ has been divided by $E_{T_{r}}=\rho_{r} \bar{R} T_{r} /\left(y_{r}-1\right)+\rho_{r} u_{r}^{2} / 2$ so that it is the same order of magnitude as the other conservation variables.

[^6]:    ${ }^{10}$ Note that if IDTAU $=2$ or $6, \mathrm{Cl}$ (1) only sets $\Delta \tau$ for the first time step, and that the time step sequencing option does not apply.
    ${ }^{11}$ Note that if IDTAU $=4$, DT(1) only sets $\Delta \tau$ for the first time step, and that the time step sequencing option does not apply.

[^7]:    12 The total energy $\bar{E}_{T}$ is divided by $E_{T_{r}}=\rho_{r} \bar{R} T_{r l} /\left(y_{r}-1\right)+\rho_{r} u_{r}^{2} / 2$ before testing for convergence, so that it is the same order of magnitude as the other conservation variables.

[^8]:    ${ }^{\text {a }}$ Use the ${ }^{*}+^{*}$ sign for 2 -point one-sided differencing of first derivatives, and the " - " sign for 3 -point differencing of first derivatives.

[^9]:    ${ }^{\text {a }}$ Use the " $+^{*}$ sign for 2 -point one-sided differencing, and the " - " sign for 3 -point one-sided differencing.

[^10]:    ${ }^{13}$ In this discussion, when "pages" of output are referred to it is assumed that the file is printed with Fortran carriage control in effect.

[^11]:    is Second order time differencing should be used only for unsteady problems, for which "convergence" has no meaning. It should also be noted that the computation of the residuals in the code is correct only for first-order time differencing.

    For the encrgy equation, the change in $\bar{E}_{T}$ is divided by $E_{T_{r}}=\rho_{r} \bar{R} T_{l} /\left(y_{r}-1\right)+\rho_{r} u_{r}^{2} / 2$, so that it is the same order
    of magnitude as the other conservation variables.

[^12]:    16 If only the last computed time level is of interest, the restart files may also be used for ploting with PLOT3D. See Section 4.4.

[^13]:    1 The IPLOT $=-1$ option, discussed in the next section, is the better one to use for CONTOUR plot files. The IPLOT = 1 option is included only to be consistent with the various PLOT3D and PLOT2D options.

[^14]:    18 The current version of PLOT3D does not work for multiple time levels, although future versions might. You can, however, fake it out using the IPLOT $=-3$ option described in Section 4.2.5.

[^15]:    19 Note that with IDTAU $=5$ or $6, \tau$ will vary in space, and therefore $\tau_{i, j} \neq \tau_{1,1}$.
    ${ }^{20}$ Note that the number of time levels that end up being written into the files is not known until the end of the

[^16]:    21 Actually, if the input parameters IPLT and IPLTA are such that only the final time level is written into the Q file, the XYZ and Q files may be used for an "approximate" restart. In this case, PROTEUS will set $\mathrm{QL}=\mathrm{Q}$.

[^17]:    22 Fewer variables may actually be needed, depending on the values of the input parameters IHSTAG and ISWIRL.
    ${ }^{23}$ Note that some input variables, like the Mach number $M$, are not normally saved as separate Fortran variables. To save storage they are to be loaded into existing Fortran variables in INIT. These Fortran variables will later be loaded with their normal values.

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[^18]:    24 See Volume 3 for definitions of all the common block variables.

[^19]:    2. The diagnostic messages described in this section are generated by the PROTELS code itself, and appear as part of the standard output. Any computer system error messages due to floating point errors, etc., are, of course, sy stem-dependent. On $\mathbf{U} \backslash 1 X$-based systems, system errors will normally appear in the standard error file.
[^20]:    ${ }^{27}$ See Section 9.0 for specific examples of actual cases.
    28 In many LXIX environments, all files that are created become permanent. As implemented at NASA Lewis, however, in the Bourne shell batch jobs are executed from a temporary working directory that is removed when the job terminates. All files are thus temporary, unless the full path name of a permanent file is used, or unless they are explicitly saved by copying them into a permanent directory.

[^21]:    ${ }^{29}$ It should be noted, however, that the incompressible governing equation for this flow is linear. It turns out that even first-order Euler-implicit time differencing gives accurate unsteady results.
    ${ }_{30}$ Smaller time steps were originally used, but it was found that equally good results could be obtained for this case with the input value shown here.

[^22]:    31 The solution presented by Schlichting is actually for a stationary top wall and moving bottom wall. The solution presented here, for a stationary bottom wall and moving top wall, was derived from it by replacing $y$ with $h-y$.

[^23]:    32 The distinction between the computational coordinate system and the computational mesh is explained in Section 2.2.

[^24]:    - For sale by the National Technical Information Service. Springfield, Virginia 22161

