# VISUAL SLANT UNDERESTIMATION 

John A. Perrone<br>NASA Ames Research Center<br>Moffett Field, California<br>Peter Wenderoth<br>University of Sydney<br>Sydney, Australia

## SUMMARY

Observers frequently underestimate the in-depth slant of rectangles under reduction conditions. This also occurs for slanted rectangles depicted on a flat display medium. Perrone (1982) provides a model for judged slant based upon properties of the two-dimensional trapezoidal projection of the rectangle. Two important parameters of this model are the angle of convergence of the sides of the trapezoid and the projected length of the trapezoid. We tested this model using a range of stimulus rectangles and found that the model failed to predict some of the major trends in the data. However, when the projected width of the base of the trapezoidal projection was used in the model, instead of the projected length, excellent agreement between the theoretical and obtained slant judgments resulted. The good fit between the experimental data and the new model predictions indicates that perceived slant estimates are highly correlated with specifiable features in the stimulus display.

## INTRODUCTION

Attempts at depicting surfaces slanted in depth on a flat display medium are often hampered by a common perceptual illusion which results in underestimation of the true depth. Surfaces appear to lie closer to the fronto-parallel plane than the perspective projection dictates. This has been a common finding in a wide range of experiments involving slant perception, starting with Gibson's study (1950) on texture gradients (e.g., Clark, Smith and Rabe, 1955; Gruber and Clark, 1956; Smith, 1956; Flock, 1965; Freeman, 1965; Braunstein, 1968; Wenderoth, 1970).

The mode of viewing slanted surfaces under the conditions used in slant perception experiments differs from the way we normally encounter visual slant in our environment (Perrone, 1980). Cutting and Millard (1984) has also questioned the use of slant as a variable in the understanding of surface perception. However, slant underestimation remains an interesting phenomenon because the information is present in the stimulus display for the veridical perception of slant (Perrone, 1982), yet apparently the human visual system does not use that information correctly.

Theories attempting to explain the underestimation are rare. Gogel (1965) applied his "equidistance tendency" theory to slant underestimation effects and Lumsden (1980) speculated that truncation of the visual field by the use of an aperture may be a factor causing underestimation.

Perrone $(1980,1982)$ has proposed several models of slant perception which attempt to account for the slant underestimation. This paper tests and modifies one of these models. Our aim is to pinpoint the stimulus features used by observers when making visual slant estimates. This would provide useful insights into areas such as spatial orientation, picture perception, and pilot night-landing errors (Perrone, 1984).

## MODEL OF SLANT UNDERESTIMATION

The slant angle $v$, is obtainable from the two-dimensional projection of the surface onto the retina. (For a technique using perspective lines, see Freeman, 1966; Perrone, 1982.)

The slant angle is found from the two-dimensional variables given in figure 1 using:

$$
\begin{equation*}
\theta=\tan ^{-1}(\tan \pi / X) \mathrm{f} \tag{1}
\end{equation*}
$$

This equation states that the slant angle, $\theta$, can be derived from the angle of convergence ( $\pi$ ) of the perspective line in the projection, and the distance, $X$, from the center of the projection out to the perspective line. In equation $1, f$ is a known constant and it is the arbitrary distance from the eye to the theoretical projection plane used to analyze the array of light reaching the eye.

The convergence angle of perspective lines, $\pi$, can give the slant angle $\theta$ as long as the correct distance $X$ is used. Using a value of $X$ greater than the true value will result in a calculated slant angle less than the actual slant angle, i.e., slant underestimation. Perrone $(1980,1982)$ proposed a model which suggested that deviation of the perceived straight-ahead direction results in a judgment of slant based on an incorrect value of X .

Two versions of the model have been proposed:
Model A. Perrone (1982) suggested that because of the reduced viewing conditions and because of the unusual form of the presenting slant, the observer's perceived straight-ahead direction deviates from the true straight-ahead (fig. 2) and that the visual system uses the length $\mathrm{X}^{\prime}$ (equal to the projected length Y ) instead of X .

It is proposed that the visual system is attempting to measure the change in width over a square area of the projection plane, determined by Y , but because there are no perspective lines a distance $X^{\prime}$ out from $c^{\prime}$, the outside edge of the rectangle is used instead. When $X^{\prime}$ is substituted into equation (1) instead of $X$, the equation for perceived slant becomes $\beta=\tan ^{-1}\left(\tan \pi / X^{\prime}\right)$. However, in order to use this equation for predicting perceived slant, we need to replace the twodimensional variables ( $\pi$ and $X^{\prime}$ ) with the three-dimensional parameters of the stimulus situation. This gives the following equation for perceived slant:

$$
\begin{equation*}
\beta=\tan ^{-1}\left[\frac{\mathrm{~W} \sin \theta\left(\mathrm{D}^{2}-\mathrm{L}^{2} \sin ^{2} \theta\right)}{4 \mathrm{~L} \mathrm{D}^{2} \cos ^{2} \theta}\right] \tag{2}
\end{equation*}
$$

$\theta=$ actual slant
$\mathrm{W}=$ actual width of rectangle
$\mathrm{L}=$ half the total length of rectangle
$\mathrm{D}=$ distance from eye to center of rotation
To date, Perrone (1982) has shown how this sort of analysis provides acceptable fits to data collected by others (e.g., Clark, Smith, and Rabe, 1955; Smith, 1956), but these studies were designed to investigate other aspects of slant perception and so did not involve direct manipulation of the variables integral to the model.

One problem with this version of the model is that it predicts that slant overestimation will occur when the projected height of the test rectangle (Y) becomes less than the projected half-width at the axis of rotation (X). However, there have been no published accounts of slant overestimation occurring, but this may simply be because nobody has used test rectangles with the appropriate length-to-width ratio.

Model B. (Modified version of Model A). This version proposes that the total base width of the rectangle $\left(\mathrm{X}_{\mathrm{b}}\right)$ is used in the evaluation of the slant angle instead of X . This new form of the model can be interpreted as saying that the observers are basing their slant estimates on the convergence angle, $\pi$, of perspective lines which they believe to be twice the true distance out from the center. It may be that it is a difficult and unnatural task for the observer to judge the slant of a surface which is centered on the median plane of the eye. It is easier if we have a side view or at least a more oblique view of the slanted surface. The observers may resort to making their judgments on the basis that they have a more extreme or displaced viewpoint than is in fact the case. Their interpretation of the slant of the rectangle may be based on an assumed view of the rectangle which is displaced or rotated relative to its true position.

When this error is combined with the proposed deviation of the perceived straight-ahead (Perrone 1982), the result may be the erroneous use of the total base width of the projected trapezoid rather than the correct half-width at the axis of rotation. When the total projected base width of a slanted rectangle is used to estimate theta from equation 1 , the predicted perceived slant angle is found using

$$
\begin{equation*}
\beta=\tan ^{-1}\left[\frac{\tan \theta(\mathrm{D}-\mathrm{L} \sin \theta)}{2 \mathrm{D}}\right] \tag{3}
\end{equation*}
$$

## $\theta=$ actual slant

$\mathrm{L}=$ half the total length of rectangle
$\mathrm{D}=$ distance from eye to center of rotation

## TESTING THE MODEL

An experiment was designed to verify which of the two cases (equation 2 or equation 3 ) best models the data from human observers in the slant perception task. If it can be established that specific features of the stimulus display are being used in the slant estimation process, then the more difficult task of discovering why these particular variables are being used can be attempted. The model provides a means of narrowing down the choice of possible variables and the combination in which they are used.

## Experiment

The stimuli were computer-generated two-dimensional perspective representations of rectangular outline figures, presented on a CRT and viewed monocularly through an aperture. These figures represented rectangles measuring 25 cm wide with the following lengths: 50 cm (condition 1), 25 cm (condition 2), and 15 cm (condition 3). These were depicted to be at a distance of 57 cm from the subject's eye and slanted backwards away from the observer by varying angles of slant. The actual slant angles used were $20^{\circ} 40^{\circ}, 60^{\circ}$ and $80^{\circ}$ measured from the vertical.

The subject reproduced the judged slant of the rectangle on a response device which was located $90^{\circ}$ to the right and positioned at eye level. The response device consisted of a thin black line inscribed on a clear plexiglass strip which was mounted on a circular white metal disk 23 cm in diameter. Vertical and horizontal black lines were drawn on the disk to provide anchor points (Wenderoth, 1970). Subjects were 10 paid volunteers, naive as to the aims of the experiment.

## Predictions

If Model A is correct, then the slant estimates for the three different conditions should lie along three distinct curves given in figure 3a. For some of the stimulus conditions, the subjects should judge the rectangle to be slanted farther back from the fronto-parallel plane than the true position (slant overestimation). This corresponds to any region of the curves which lies above the dotted line in figure 3a. If a Model B is correct, the slant estimates for all three conditions should all lie on approximately the same curve of the shape shown in figure 3b. No slant overestimation should occur.

## Results

The data from the 10 subjects have been plotted in figure 4 along with the predictions from Model B. For the case in which a tall narrow rectangle was used (Condition 1), the results are similar to those obtained in past slant perception experiments which used rectangles with a length-to-width ratio greater than one, (e.g., Smith, 1956). For this condition, both Model A and B give reasonable predictions for the smaller test angles (see C1 predictions in fig. 3a). However, for the remaining conditions, the data depart greatly from the Model A predictions and none of the predicted overestimation of slant occurred.

The mean absolute error between the Model A predictions and the data over the three conditions was $13.9^{\circ}$, (sd = 8.1). For Model B, on the other hand, the mean absolute error was only $2.6^{\circ}$, ( $\mathrm{sd}=1.9$ ). The mean absolute errors from Model A are significantly greater than those from Model B, $(t=4.5, p<0.05,22 d f)$ and represent a worse fit between the model predictions and data.

## CONCLUSIONS

Slant underestimation Model A (Perrone 1982) incorrectly predicts overestimation to occur for rectangles which have a projected length less than half of the base width. In fact, the influence of
the projected length of the rectangle on slant judgments is minimal. However, Model B provides an excellent fit between the experimental data and the predictions. These predictions are based on measurable features of the experimental configuration. There are no free parameters. Model B states that the total projected base width of the rectangle is used instead of half the projected width at the axis of rotation. Two parameters of the two-dimensional projection are important in the slant estimation process: (1) the angle of convergence of perspective lines and (2) the distance of the perspective lines from the center of the projection. The success of Model B suggests the human observers make errors in slant estimates because they misperceive this second parameter.

The question remains as to why human observers use "incorrect" features of the stimulus in their assessment of the slant angle. It has been shown that the correct slant angle is obtainable from the appropriate use of the variables given in equation 1. These variables are known to be present in the two-dimensional stimulus reaching the observer's eye. The experimental data are consistent with the proposal that the total base width of the trapezoidal projection is used instead of half the projected width at the axis of rotation. However, it does not shed any light as to why this should be the case.

Further research is required before we can conclude the actual mechanisms used by the human visual system in making slant estimates. In the meantime, sufficient evidence exists to conclude that slant judgments by an observer are highly correlated with specific measurable features in the two-dimensional array of light reaching the observer's eye. The slant estimates exhibit a large amount of error and often greatly underestimate the true slant angle. This paper shows that such errors cannot be attributed to the fact that insufficient information exists in the stimulus for veridical slant judgments. The information is available, but is incorrectly used.

## REFERENCES

Braunstein, M. L.: Motion and texture as sources of slant information. J Exper. Psychol., vol. 8, 1968, pp. 584-590.

Clark, W. C.; Smith, A. H.; and Rabe, A.: Retinal gradients of outline as a stimulus for slant. Canadian J. Psychol., vol. 9, 1955, pp. 247-253.

Cutting, J. C.; and Millard, R. T.: Three gradients and the perception of flat and curved surfaces. J. Exper. Psychol.: General, vol. 113, 1984, pp. 198-216.

Flock, H. R.: Optical texture and linear perspective as stimuli for slant perception. Psychol. Rev., vol. 72, 1965, pp. 505-514.

Freeman, R. B., Jr.: Ecological optics and visual slant. Pyschol. Rev., vol. 72, 1965, pp. 501504.

Freeman, R. B., Jr.: Function of cues in the perceptual learning of visual slant. Psychol. Monogr.: Gen. Appl., vol. 80, 1966, (2) whole no. 610.
Gibson, J. J.: The perception of surfaces. Amer. J. Psychol., vol. 63, 1950, pp. 367-384.
Gogel, W. C.: Equidistance tendency and its consequences. Psychol. Bull., vol. 64, 1965, pp. 153-163.

Gruber, H. E.; and Clark, W. C.: Perception of slanted surfaces. Percept. Motor Skills, vol. 16, 1956, pp. 97-106.

Lumsden, E. A.: Problems of magnification and minification: An explanation of the distortions of distance, slant, shape and velocity. In M. A. Hagen (Ed.) The Perception of Pictures, vol. 1, 1980, (pp. 91-134). New York: Academic Press.

Perrone, J. A.: Slant underestimation: A model based on the size of the viewing aperture. Perception, vol. 9, 1980, pp. 285-302.

Perrone, J. A.: Visual slant underestimation: A general model. Perception, vol. 11, 1982, pp.
$641-654$.
Perrone, J. A.: Visual slant misperception and the 'black-hole' landing situation. Av. Space Environ. Medicine, vol. 55, 1984, pp. 1020-5.

Smith, A. H. Gradients of outline convergence and distortion as stimuli for slant. Canadian J. Psychol., vol. 10, 1956, pp. 211-218.

Wenderoth, P. M.: A visual spatial after-effect of surface slant. Amer. J. Pyschol., vol. 83, 1970, pp. 576-590.


Figure 1.- The two-dimensional information reaching the eye is analyzed on a theoretical projection plane an arbitrary distance $f$ from the eye. All measurements on the projection plane are made within the plane of the page.


Figure 2.- Deviation of the perceived straight-ahead results in the analysis being carried out about $\mathrm{c}^{\prime}$ instead of c . Model A states that the length $\mathrm{X}^{\prime}$ (equal to Y ) is used instead of X . Model B proposes that $X_{b}$ is used instead of $\mathbf{X}$.


Figure 3.- Plots showing (a) predictions from Model A for each of the three experimental conditions and (b) predicted slant versus actual slant for Model B. No slant overestimation is predicted to occur.


Figure 4.- Data are plotted from conditions 1, 2, and 3 along with predictions from Model B. Error bars have been omitted for clarity, but the largest standard error was $4.5^{\circ}$ for the $80^{\circ}$ slant angle.

