N90-23021

Frequency Response Modeling and Control of Flexible Structures: Computational Methods[1]

William H. Bennett

TECHNO–SCIENCES, INC.
7833 Walker Dr. – Suite 620
Greenbelt, MD 20770

ABSTRACT

The dynamics of vibrations in flexible structures can be conveniently modeled in terms of frequency response models. For structural control such models capture the *distributed parameter* dynamics of the elastic structural response as an irrational transfer function. For most flexible structures arising in aerospace applications the irrational transfer functions which arise are of a special class of pseudo-meromorphic functions which have only a finite number of right half plane poles. In this paper, we demonstrate computational algorithms for design of multiloop control laws for such models based on optimal Wiener-Hopf control of the frequency responses. The algorithms employ a sampled-data representation of irrational transfer functions which is novel and particularly attractive for numerical computation. One key algorithm for the solution of the optimal control problem is the spectral factorization of an irrational transfer function. We highlight the basis for the spectral factorization algorithm together with associated computational issues arising in optimal regulator design. We also highlight options for implementation of wide band vibration control for flexible structures based on the sampled-data frequency response models. A simple flexible structure control example is considered to demonstrate the combined frequency response modeling and control algorithms.

1 Introduction

Frequency response methods offer several advantages for modeling the dynamics of small amplitude vibrations in flexible structures. Such models capture the *distributed parameter* dynamics of the elastic structural response as an irrational transfer function from localized actuation to localized deformation measurements. Interest in frequency response models can arise from a desire to predict modal frequencies with increased accuracy over that obtainable from finite element methods. The frequency domain approach is well suited to optimal control law synthesis with specific requirements for precision vibration supression and isolation. Most computational methods for optimal control synthesis available to design engineers focus on the manipulation of state space models. For flexible structure control, state space models are problematic since the question of model order required must be resolved as part of the optimal control computation. This paper reports progress in the development and testing of computational methods for design of precision control systems for mechanical structures with

[1]Work supported by SDIO and Air Force Wright Research and Development Center under contract F33615-88-C-3215.

$\mathbf{C}_+$	open right half complex s-plane, $\Re e\ s > 0$
$\mathbf{H}_\infty$	Hardy space of complex functions, analytic and essentially bounded in $\mathbf{C}_+$
$\mathbf{H}_2$	Hardy space of complex functions, $f(s)$, analytic in $\mathbf{C}_+$and such that; $\frac{1}{2\pi j}[\int_{-j\infty}^{j\infty} \|f(s)\|^2\, ds]^{1/2} < \infty$ for $s \in \mathbf{C}_+$.
$\mathbf{RH}_\infty$	rational functions in $\mathbf{H}_\infty$

Table 1.1: Notation

elastic effects based on direct frequency response models. The frequency response models can arise from finite element analysis, transfer function methods, wave propagation models, and/or empirical measurements. Moreover, the computational approach offers a framework for integration of frequency response data from various modeling approachs which offer varying precision in different frequency bands. The current paper extends the efforts reported in [1].

We will use the following notation and conventions in this paper. The transpose of a column vector will be denoted as x^T, $Tr\ X$ is the trace of the square matrix X, and $j = \sqrt{-1}$. A Laplace (resp. z) transform will normally be indicated by dependent variable; $x(s)$ (resp. $x(z)$), however, we often drop the explicit dependence where the meaning is clear from the context. The notation $u_*(s) = u^T(-s)$ will be frequently used. $E\{x(t)\}$ indicates the expectation of the random process $x(t)$. In this work all random processes are assumed wide sense stationary and ergodic so that expectation can be replaced with ensemble average where convenient. The notation contained in Table 1.1 specifies the classes of transfer function models considered at various points. A rational function has a (partial fraction) expansion $A(s) = \{A(s)\}_+ + \{A(s)\}_- + \{A(s)\}_\infty$ where $\{.\}_+$ (resp. $\{.\}_-$) is analytic in $\Re es > 0$–the causal part ($\Re es < 0$–the anti-causal part) and $\{.\}_\infty$ is the part associated with poles at infinity. Thus the operation $\{A(s)\}_+$ is *causal projection* of the frequency response model.

In section 2 we provide an overview of frequency domain models and modern Wiener-Hopf design of multiloop control systems. We motivate the role of frequency response modeling and optimal control and identify critical computational steps required for the method. Section 3 discusses a new approach to the required computations for Wiener-Hopf control which extend the algebraic constructions for rational transfer functions to certain irrational cases. We highlight the role of coprime factorization in design of distributed systems. Section 4 considers a simple, but nontrivial distributed parameter system design. Finally, in section 5 we discuss new options for real time control implementation suggested by the computational approach of section 3.

2 Optimal Control of Frequency Response Models: Wiener-Hopf Design

Frequency domain models have been used to articulate the full range of opportunities for feedback compensation for internal model stabilization. Algebraic constructions based on Laplace transform models of linear, time-invariant system dynamics have been used to describe alternatives for standard control computations and realizations for stabilizing controllers [2, 3]. This together with Wiener-Hopf optimization provides a general approach for resolving tradeoffs in regulator design where natural, frequency domain specifications for model-based, control performance are available. We remark that restricting attention to rational transfer function models in computational approachs to flexible structure control has been primarily motivated by convenience [1]. Specific results which extend the constructions of coprime factorization and internal stabilization to a certain class of irrational transfer functions have been obtained [4]. In the present effort we restrict attention to transfer functions in $\mathbf{H}_\infty$and meromorphic with the exception of a small number of right half plane poles.

Optimal regulator design via Wiener-Hopf methods. Techniques for the solution of $\mathbf{H}_2$ optimization problems in multiloop feedback systems have received considerable attention in the control theory literature for a number of years. A comprehensive approach to Wiener-Hopf design using transfer function models is given by Youla et al [5].

A general framework for resolution of tradeoffs in multiloop control design was recently outlined by Park and Bongiorno [6]. In general, control design involves the resolution of choices in the use of dynamic (feedback) compensation with respect to a nominal dynamic model of the system response to an n-vector control, u and an m-vector of exogenous system disturbances, e, as seen by p available sensors, y and ℓ (possibly nonmeasurable) regulated variables. A frequency domain model for the control design problem (shown in Figure 2.1) can be expressed using Laplace transforms as,

$$\begin{aligned} \begin{pmatrix} y(s) \\ z(s) \end{pmatrix} &= G(s) \begin{pmatrix} u(s) \\ e(s) \end{pmatrix} \\ &= \begin{bmatrix} -G_{yu} & G_{ye} \\ G_{zu} & G_{ze} \end{bmatrix} (s) \begin{pmatrix} u(s) \\ e(s) \end{pmatrix}. \end{aligned} \tag{2.1}$$

The control architecture is assumed to involve feedback,

$$u(s) = C(s)y(s). \tag{2.2}$$

Then the closed loop compensation will alter the response of the system to disturbances as seen in terms of the *regulated variables* as[2],

$$\begin{aligned} z &= [G_{zu}CS, I] \begin{bmatrix} G_{ye} \\ G_{ze} \end{bmatrix} e \\ &= G_{zu}CSG_{ye}e + G_{ze}e \end{aligned} \tag{2.3}$$

where the system closed loop *sensitivity operator* is $S = [I + G_{yu}C]^{-1}$.

A major consideration in the classical methods of frequency domain design is closed loop stability. In such methods stability considerations must be continually evaluated (using root locus or Nyquist plots) as performance tradeoffs are evaluated. For single loop designs of relatively low order systems, classical frequency domain methods focus attention on the tradeoff between stability margins and performance. The modern approach is to use optimization to

[2]Suppressing dependence on the Laplace variable s.

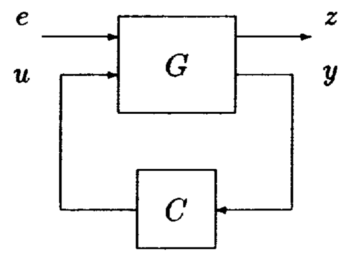

Figure 2.1: General Multiloop Control System Design Problem

resolve complicated engineering tradeoffs in multiloop design—subject to the constraint of system internal stability.

The well known Youla parametrization of all stabilizing feedback controllers C which stabilize a given plant model was orginally derived for rational matrix transfer functions which have *coprime factorizations* over the ring of polynomials in the Laplace s-variable [5]. It is now understood that the construction goes through without modification for the ring of stable rational functions, $\mathbf{RH}_\infty$which includes all rational transfer functions analytic in the closed right half plane including the point at infinity [7]. Thus under the assumption that the plant transfer function has right and left factorizations,

$$G_{yu} = ND^{-1} = D_\ell^{-1}N_\ell, \tag{2.4}$$

coprime over $\mathbf{RH}_\infty$, then there exist $X, Y, X_\ell, Y_\ell \in \mathbf{RH}_\infty$ such that

$$D_\ell X_\ell + N_\ell Y_\ell = I, \qquad XD + YN = I, \tag{2.5}$$

and each controller, C, which obtains $R = CS$ with $R \in \mathbf{RH}_\infty$ can be *parametrized* by the factorization formulae,

$$C = (X - KN_\ell)^{-1}(Y + KD_\ell) \tag{2.6}$$

$$= (Y_\ell + KD)(X_\ell - KN)^{-1} \tag{2.7}$$

for some $K \in \mathbf{RH}_\infty$. The importance of this construction is that optimization procedures can be applied directly to the choice of $K \in \mathbf{RH}_\infty$ without concern for closed loop stability. This fact was first exploited by Youla et al[5] in the description of Wiener-Hopf optimal control for frequency response models. More recently, the parametrization has been utilized by Desoer and his students [8].

Our concern here is with $\mathbf{H}_2$ optimization for a certain class of irrational transfer functions which are "pseudo-meromorphic" in the sense stable coprime factorizations exist; i.e., the constructions in (2.4)–(2.7) obtain closed loop stability with $N, D, N_\ell, D_\ell, X, Y, X_\ell, Y_\ell, K \in \mathbf{H}_\infty$[1, 4]. In the research reported herein we avoid algebraic constructions related to stable, coprime factorization of irrational transfer functions as considered by Desoer [7] and instead, focus on the development of numerical algorthims for approximating the frequency response of the required objects. Such transfer functions can be adequately approximated over finite frequency ranges by (rather high order) rational transfer functions. Models of this type arise in the study of vibrations of multibody systems with flexible interactions [9] and wave propagation in flexible mechanical structures [10, 11].

PSD modeling of control processes for performance specification. A wide class of standard control design problems including symultaneous requirements for tracking, disturbance rejection and accomodation, etc. can be represented in the form of the general linear, time-invariant regulator problem of Fig. 2.1 and (2.1)-(2.2) where the objective is to choose a controller which stabilizes the closed loop system and minimizes a performance criterion in the form,

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} Tr[Q(s)P_z(s)]\, ds, \tag{2.8}$$

where P_z is an effective Power Spectral Density (PSD) of the regulated variables, z.

The representation of regulation performance in terms of PSD is quite practical for a variety of design problems arising in multiloop systems and provides a frequency dependent specification of control performance consistent with design requirements for vibration rejection. One can extend the significance of PSD modeling to include a wide range of practical design considerations. The regulation PSD, P_z, can be related to modeling assumptions on the exogenous inputs in terms of the closed loop transfer functions;

$$P_z = [G_{uz}R,\ I]\begin{pmatrix} G_{ye} \\ G_{ze} \end{pmatrix} P_e\,(G_{ye*},\ G_{ze*})\,. \tag{2.9}$$

PSD models of exogenous inputs may include deterministic transient effects together with steady state stochastic PSD, Φ_e; viz.,

$$P_e(s) = \lambda_1 E\{e(s)e_*(s)\} + \lambda_2\Phi_{ee}(s), \tag{2.10}$$

Formulation of the performance objective, J, may include real, positive, λ_i, $i = 1,2$ which permit scaling relative importance of steady state and transient considerations to the composite performance and $Q(s)$ is included to permit frequency weighting. PSD modeling has recently received increased emphasis in the study of vibration control in acoustic regimes [11].

Park and Bongiorno [6] also highlight the use of PSD models for minimizing closed loop system sensitivity to model uncertainty. Let the system model uncertainty be given as a frequency dependent, additive perturbation, $G := G+\Delta$, which can be expressed in partitioned form as,

$$\Delta = \begin{bmatrix} -\Delta_{yu} & \Delta_{ye} \\ \Delta_{zu} & \Delta_{ze} \end{bmatrix}.$$

Following [6] an effective model uncertainty PSD, $P_\Delta = E\{\Delta\Delta_*\}$, can be reflected to the system regulated outputs, and via superposition, a composite performance objective of the form,

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} Tr\left\{Q[G_{uz}R,\ I]\begin{bmatrix} \Phi_{yy} & \Phi_{yz} \\ \Phi_{zy} & \Phi_{zz} \end{bmatrix}\right\} ds$$

is obtained for optimal regulator design, where the system model uncertainty can be obtained commensurate with the performance specifications as an effective disturbance PSD,

$$\begin{bmatrix} \Phi_{yy} & \Phi_{yz} \\ \Phi_{zy} & \Phi_{zz} \end{bmatrix} = \begin{pmatrix} G_{ye} \\ G_{ze} \end{pmatrix} P_e \left(G_{ye*}, G_{ze*} \right) + \mu P_\Delta. \tag{2.11}$$

Here the partitioned terms can be expressed in terms of a priori modeling assumptions; [6]

$$\Phi_{yy} = G_{ye} P_e G_{ye*} + \mu(E\{\Delta_{yu}(\Delta_{yu})_*\} E\{\Delta_{ye}(\Delta_{ye})_*\}), \tag{2.12}$$

$$\Phi_{yz} = G_{ye} P_e G_{ze*} + \mu(E\{\Delta_{yu}(\Delta_{zu})_*\} E\{\Delta_{ye}(\Delta_{ze})_*\}) = \Phi_{zy*}, \tag{2.13}$$

$$\Phi_{zz} = G_{ze} P_e G_{ze*} + \mu(E\{\Delta_{zu}(\Delta_{zu})_*\} E\{\Delta_{ze}(\Delta_{ze})_*\}). \tag{2.14}$$

It is by now widely recognized that frequency domain response considerations are extremely important for robust control design and that performance objectives formulated in the frequency domain are important tools for resolving design tradeoffs of relevance to practical design problems. However, the common wisdom is that state space modeling offers the most reliable numerical framework for the computational problems which arise in optimal regulator design. The Wiener-Hopf approach identifies the solution for the optimal controller in an explicit form which highlights the role of the algebraic constructions generic to stabilization and the quantitative computations required for identifying an optimal controller. Thus given the system architecture (2.1)–(2.2), appropriately chosen stable coprime factors for the plant (2.4), a nominal stabilizing controller given in terms of its coprime factors as solutions of the Diophantine relations (2.5), and performance PSD's (2.12)–(2.14), then an optimal closed loop system response is obtained (assuming a solution exists) by the formula,

$$R = D\Lambda^{-1} \left(\{\Lambda D^{-1} Y \Omega\}_- - \{\Lambda_*^{-1} D_* G_{zu*} Q \Phi_{zy} D_{\ell*} \Omega_*^{-1}\}_+ \right) \Omega^{-1} D_\ell. \tag{2.15}$$

The explicit form given here depends on operations of causal projection and the solution of two causal, spectral factorizations;

$$D_* G_{zu*} Q G_{zu} D = \Lambda_* \Lambda \tag{2.16}$$

$$D_\ell \Phi_{yy} D_{\ell*} = \Omega \Omega_* \tag{2.17}$$

with $\Lambda, \Lambda^{-1}, \Omega, \Omega^{-1} \in \mathbf{H}_\infty$. The required controller can then be obtained in the explicit form, $C = (I - RG_{yu})^{-1} R$.

The computational steps required to identify candidate optimal control solutions for the regulator problem include: 1) stable coprime factorization (as in (2.4), 2) identify candidate solution to Diophantine relations (2.5), 3) causal spectral factorization, and 4) causal projection. We contend that such computations can be effectively supported (in finite precision arithmetic) by obtaining state space realizations [3] *only for relatively low order, rational transfer functions.* In the sequel, we specifically avoid such an approach since we are ultimately concerned with the approximate solution of large (or even infinite) dimensional models.

3 Frequency Response Computations for Optimal Control.

Our approach to optimal control computation is motivated by distributed parameter models which arise in flexible structure control. The approach we have in mind is based on sampling and interpolation of the frequency response models for the system. The choice of sampling and the resulting high order, rational approximations are obtained in the context of the optimal control problem as summarized above.

A computational approach to spectral factorization. Recall that a transfer function $H(s) \in \mathbf{H}_2 \cap \mathbf{H}_\infty$ has a unique *spectral factorization* $H(s) = F(s)F_*(s)$ with $F \in \mathbf{H}_\infty$ if:

1. $\overline{H(s)} = H(\bar{s})$; i.e., $H(s)$ is the transform of a real-valued function $h(t)$.
2. $H(s) = H_*(s)$; i.e., $H(s)$ is "para-hermittian".
3. $H(s)$ is of normal rank; i.e., full rank almost everywhere in $\mathbf{C}$.
4. $H(i\omega)$ is positive, semi-definite and bounded for $\omega \in \mathbf{R}$.

To see that causal projection is a closely related problem consider the following. If $H(s)$ is scalar, then with $\Phi(s) = \ln H(s)$ we obtain

$$\begin{aligned} \Phi(s) &= \{\Phi(s)\}_+ + \{\Phi(s)\}_-, && (3.1) \\ &= \ln F(s) + \ln F_*(s), && (3.2) \end{aligned}$$

so that the causal, spectral factorization is related to causal projection via the logarithmic transformation; $F(s) = \exp\{\ln H(s)\}_+$.

Our goal is to obtain numerically stable approximations to these related problems for transfer functions in $\mathbf{H}_\infty$. For application to precision control of flexible structures we require wide band frequency domain models so that even rational approximations will be of relatively high order. An approach to model order reduction which has recently received attention in the literature is based on Fourier series approximation of irrational frequency responses [12] in $\mathbf{H}_\infty$. Our approach to computations for such models is also based on sampling and interpolation of the spectrum, but is motivated by computational requirements for Wiener-Hopf optimization. From the above discussion of causal projection we motivate a class of algorithms of interest from basic properties of the Hilbert transforms applied to the frequency response $\Phi(j\omega)$. Recall that the Hilbert transform of a time signal $f(t)$ is defined as a convolution; $\check{f}(t) = \int_{-\infty}^{\infty} \frac{f(\tau)}{\pi(t-\tau)}\, d\tau$ and it's Fourier transform has the property,

$$\check{f}(\omega) = \begin{cases} -jf(\omega), & \omega > 0 \\ \;\; jf(\omega), & \omega > 0 \end{cases}.$$

The inverse Fourier transform of $\check{\Phi}$ is $-j\mathrm{sgn}(t)\phi(t)$ where $\phi(t)$ is the inverse Fourier transform of $\Phi(\omega)$. A consequence is that the casual projection can be obtained as

$$\{\Phi(\omega)\}_+ = \frac{1}{2}[\Phi(\omega) + j\check{\Phi}(\omega)].$$

In previous studies we reported computational algorithms for causal projection and scalar spectral factorization by numerical evaluation of the Hilbert transform integral. Computational cost was high due to the fact that the Hilbert transform integral is convergent only in the Cauchy principal value sense [13]. An alternate method for causal projection and spectral factorization was considered in [14] based on sampling and interpolation of the system frequency response. The algorithm developed in [14] employs results of Stenger [15] on numerical solution of Wiener-Hopf integrals by sampling and interpolation. Details of the algorithm used for the current studies and computer implementation are given in [14].

In the multiloop, optimal regulator design problem we require the solution of two matrix spectral factorization problems (analogous to the solution of control and filtering Riccati equations for time domain models). The computational approach exploited in the current study is based on a Newton-Raphson iteration for the matrix causal spectral factor;

$$F_{n+1}(i\omega) := \left\{[F_n^*(i\omega)]^{-1}H(i\omega)[F_n(i\omega)]^{-1}\right\}_+ F_n(i\omega). \tag{3.3}$$

The recursion (3.3) can be replaced with a numerically well conditioned problem by iteration on the inverse spectral factor;

$$[F_{n+1}]^{-1} := [F_n]^{-1}\left(I + \left\{[F_n^*]^{-1}H\,[F_n]^{-1} - I\right\}_+\right)^{-1}. \tag{3.4}$$

By initializing with F_0 (an $m \times m$ diagonal matrix) with diagonal elements equal to the spectral factors of the diagonal elements of H the second term of (3.4) remains a perturbation of the identity (since $[F_n^*]^{-1}H[F_n]^{-1} - I \to 0$) which regularizes the computations. The algorithm used in this work is based on that reported in [14] and is a modified form of the method reported in [16].

Computation of stable coprime factorizations for flexible structure models. Simple models of structural components with elastic effects typically lead to transfer functions in $\mathbf{H}_\infty$once realistic damping models are included. Linear vibration models of more complex structures arising in aerospace applications usually will have transfer function models with only a finite number of poles in the closed right half plane. Restricting attention to such transfer functions we indicate a simple procedure for coprime factorization over $\mathbf{H}_\infty$.

Let $\mathcal{S} \subseteq \mathbf{H}_\infty$ be a set of transfer functions analytic in a half plane including $\mathbf{C}_+$. Under the above assumption any such transfer function $P(s)$ can be expressed in the form,

$$P(s) = P_{\mathcal{S}}(s) + P_{\bar{\mathcal{S}}}(s) \tag{3.5}$$

where $P_{\mathcal{S}} \in \mathcal{S} \subseteq \mathbf{H}_\infty$ and $P_{\bar{\mathcal{S}}}$ is rational and analytic in the complement of $\mathcal{S}$ with (a finite number of) poles outside $\mathcal{S}$. A stable coprime factorization can be readily obtained for the (typically low order) transfer function as, $P_{\bar{\mathcal{S}}} = \tilde{N}_r\tilde{D}_r^{-1}$, by well known state space constructions [3]. Then P has stable coprime factorization,

$$P = N_rD_r^{-1} = [\tilde{N}_r - P_{\mathcal{S}}\tilde{D}_r]\tilde{D}_r^{-1}, \tag{3.6}$$

where N_r, D_r are $\mathcal{S}$-stable. The separation of terms in (3.5) is readily carried out given $P(s)$ by computing the residues of the finite number of unstable poles contributing to $P_{\bar{\mathcal{S}}}$.

4 Control Computations for an Elastic Structure

To illustrate the computational approach for a simple elastic structure we consider the simply supported Euler beam with torque control at one end. The beam lateral deformation is given by $y(t,z)$ with $0 \le z \le L$ and has dynamics described by the dimensionless PDE;

$$\frac{\partial y^2}{\partial t^2} - 2\zeta\frac{\partial^3 y}{\partial t \partial z^2} + \frac{\partial^4 y}{\partial z^4} = 0. \tag{4.1}$$

with boundary conditions at $z = 0$,

$$y(t,0) = 0, \qquad \left.\frac{\partial^2 y}{\partial z^2}\right|_{z=0} = 0,$$

and at $z = L$,

$$y(t,L) = 0, \qquad \left.\frac{\partial^2 y}{\partial z^2}\right|_{z=L} = \tau,$$

with the control moment, τ, applied at the right hand end of the beam. The transfer function $(\tau \to y)$ for beam control is

$$G_{yu}(s,z) = L^2\,\frac{\sin\lambda_1 \sinh\lambda_2\frac{z}{L} - \sin\lambda_1\frac{z}{L}\sinh\lambda_2}{(\lambda_1^2+\lambda_2^2)\sin(\lambda_1)\sinh(\lambda_2)}, \tag{4.2}$$

where $\lambda_1^2 = (-\zeta + i\sqrt{1-\zeta^2})sL^2$, $\lambda_2^2 = (\zeta + i\sqrt{1-\zeta^2})sL^2$, L is the beam length, ζ is the damping factor, and z is the observation point to be regulated on the beam. The transfer function is meromorphic, and $G_{yu}(s,z) \in \mathbf{H}_\infty$ for any $0 \le z \le L$.

The regulator problem considered arises from a requirement for asymptotic rejection of constant load disturbances at a point $\frac{z}{L} = 0.7$. For the current numerical studies we take $L = 10.$, and the effective damping ratio, $\zeta = 0.01$. Stable coprime factorization is trivial and we take $N_r = N_\ell = G_{yu}$, $D_r = D_\ell = 1$. Exogenous inputs here include the output load disturbance d and measurement noise model n and are described by their effective PSD models representing constant (step) load disturbance and narrowband sensor noise as shown in Figure 4.2. The frequency response of G_{yu} is shown in Figure 4.1 with 1024 uniform frequency samples over a bandwidth of $0 < \omega < 100$. Clearly, the frequency response is irrational and no obvious rational approximation is evident.

The optimal control design is regulation of the beam deflection at $z/L = .7$ and the performance objective is given as,

$$J = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} Tr\left\{\Phi_y + \mu\Phi_u\right\}\, ds$$

where the tracking cost is modeled by PSD, Φ_y, and the control saturation PSD is $\Phi_u = E\{uu_*\}$. Then given a constraint on the control power the scalar $\mu > 0$ plays the role of a Lagrange multiplier for the optimal design. In this case the required spectral factors;

$$(G_{yu*}G_{yu} + \mu) = \Lambda_*\Lambda \tag{4.3}$$

$$(G_{yd}\Phi_d P_{yd*} + \Phi_n) = \Omega\Omega_* \tag{4.4}$$

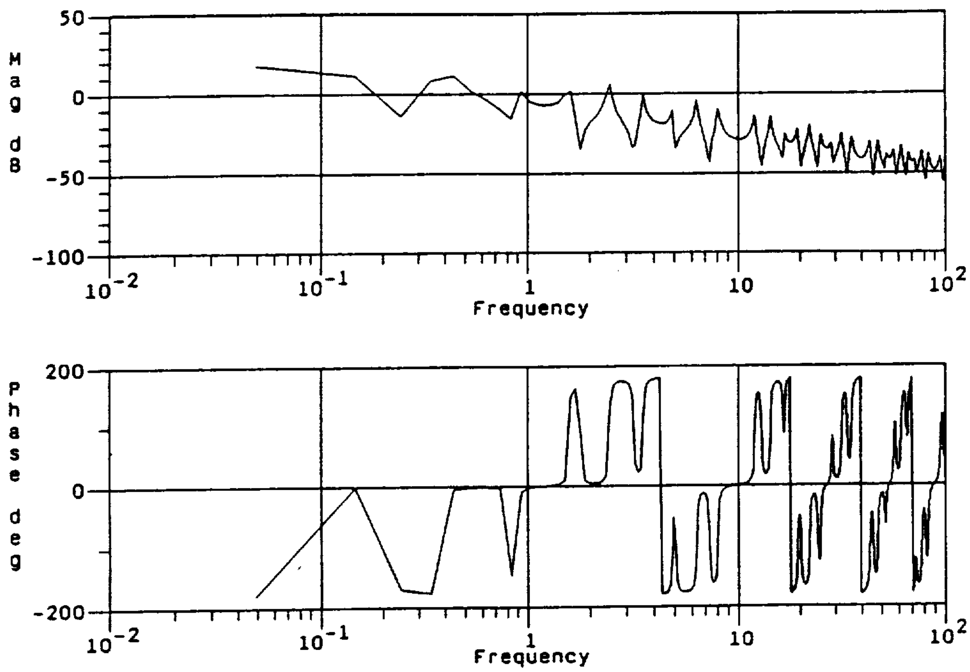

Figure 4.1: Frequency response for pinned-pinned beam control.

were obtained for $\mu = .1$ by the frequency sampled procedure and are displayed in Figure 4.3. We remark that the computations of the indicated spectral factors effectively replace the computational step of solving a pair Riccati matrix equation for the control (resp. filter) problem typically encountered in state-space methods for control design. For distributed parameter systems, solution of the Riccati equation (a PDE) requires discretization which is accomplished using the current algorithms by sampling and interpolation of the frequency response. Thus numerical precision is concentrated over frequency bands significant for the given control problem and with sampling under direct control of the designer. The solution obtained is effectively a high order rational approximation of the optimal solution with frequency response interpolation points chosen by the design engineer. The optimal controller frequency response thus obtained is shown for $\mu = .1$ in Figure 4.4.

5 Frequency Sampling Filters for Real Time Control Implementation

The frequency response computations for Wiener-Hopf control outlined and illustrated in the previous sections identify various frequency sampled approximations to the ideal, possible irrational frequency response for the desired optimal controller. Bandwidth and sampling can be chosen by the design engineer to represent specific concerns based on models and/or control performance. The frequency sampled computations obtain a *specification* for the frequency response of the ideal (optimal) controller via its sampled representation. The design engineer now has several options for implementing the controller depending on available hardware. In contrast to the state space approach for finite dimensional systems, several new realization opportunities are suggested by the frequency sampling approach.

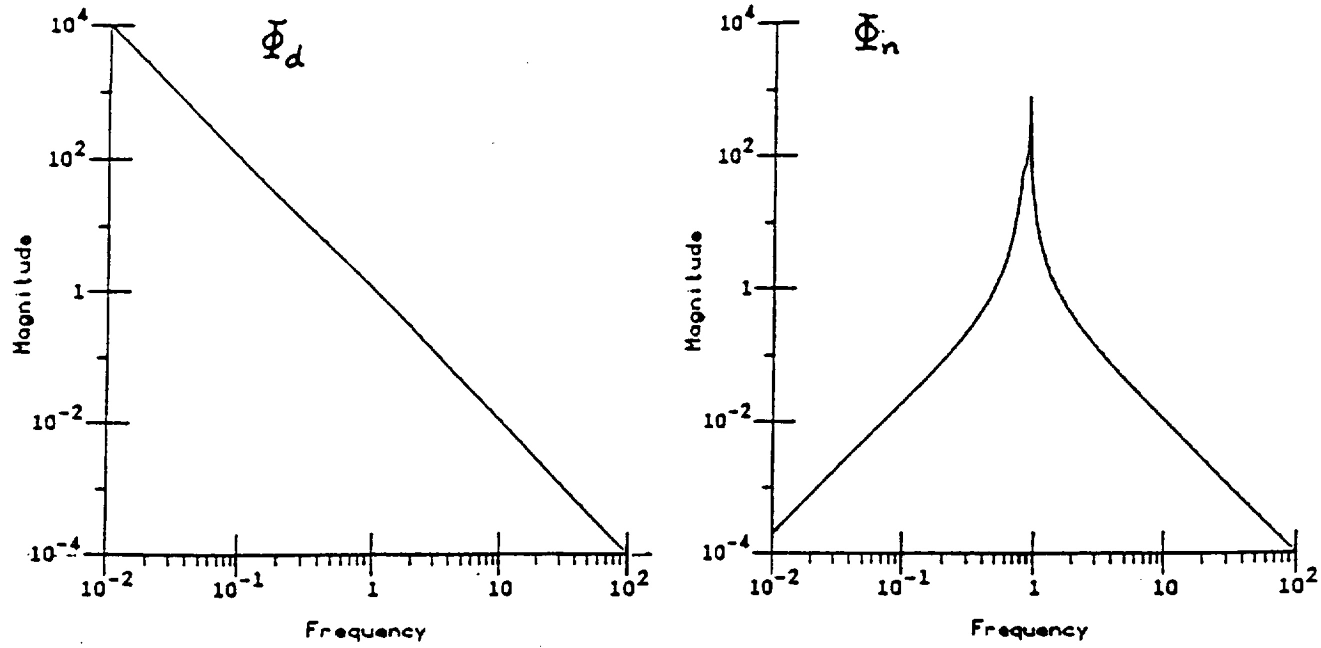

Figure 4.2: PSD for disturbance and sensor noise inputs for beam control.

A principal concern in implementation of high precision control laws for flexible structure control is the order of the realization required for the online controller. The controller order is usually taken to mean the dimension of the state variable realization of the transfer function $C(s)$ which will be implemented for realtime control. Implementation using analog components of high order models is limited by circuit complexity, reliability and cost. As a result considerable effort has been expended in methods for model order reduction. One approach to controller realization which follows from the frequency sampled computations of the previous section is to compute reduced order, continuous time, state space realizations for the controller by techniques such as in [12].

Digital computer implementations are primarily limited by computational speed and algorithm complexity effecting the ultimate obtainable sampling rate and considerations for reduced order realization of the controller may be required. However, the emergence of specialized computer hardware implemented in VLSI single chip circuits for digital signal processing opens new opportunities for realization of realtime control for flexible structures. We prefer to consider realization options for the optimal controller in discrete time for implemenation on a digital computer. Realization of the controller specified by its frequency samples can be obtained using a FIR digital filter implementation.

Given the specified frequency samples obtained for the optimal controller,

$$C_k = C(j\omega_k),$$

at frequencies, $\omega_k = k\omega_{BW}/N$, where ω_{BW} is bandwidth and N the number of uniformly spaced frequency samples we describe the digital filter realization using z-transforms. With discrete time sampling rate $\omega_s > 2\omega_{BW}$ the frequency samples correspond to interpolation points in the z-plane given by[3], $z_k = e^{jk2\pi/N_1}$, for $k = 0, \ldots, N-1$. The z transform which

[3]Bandwidth and sampling requirements would typically require padding the sequence of frequency samples of length N with $N_1 - N$ zero values to avoid aliasing.

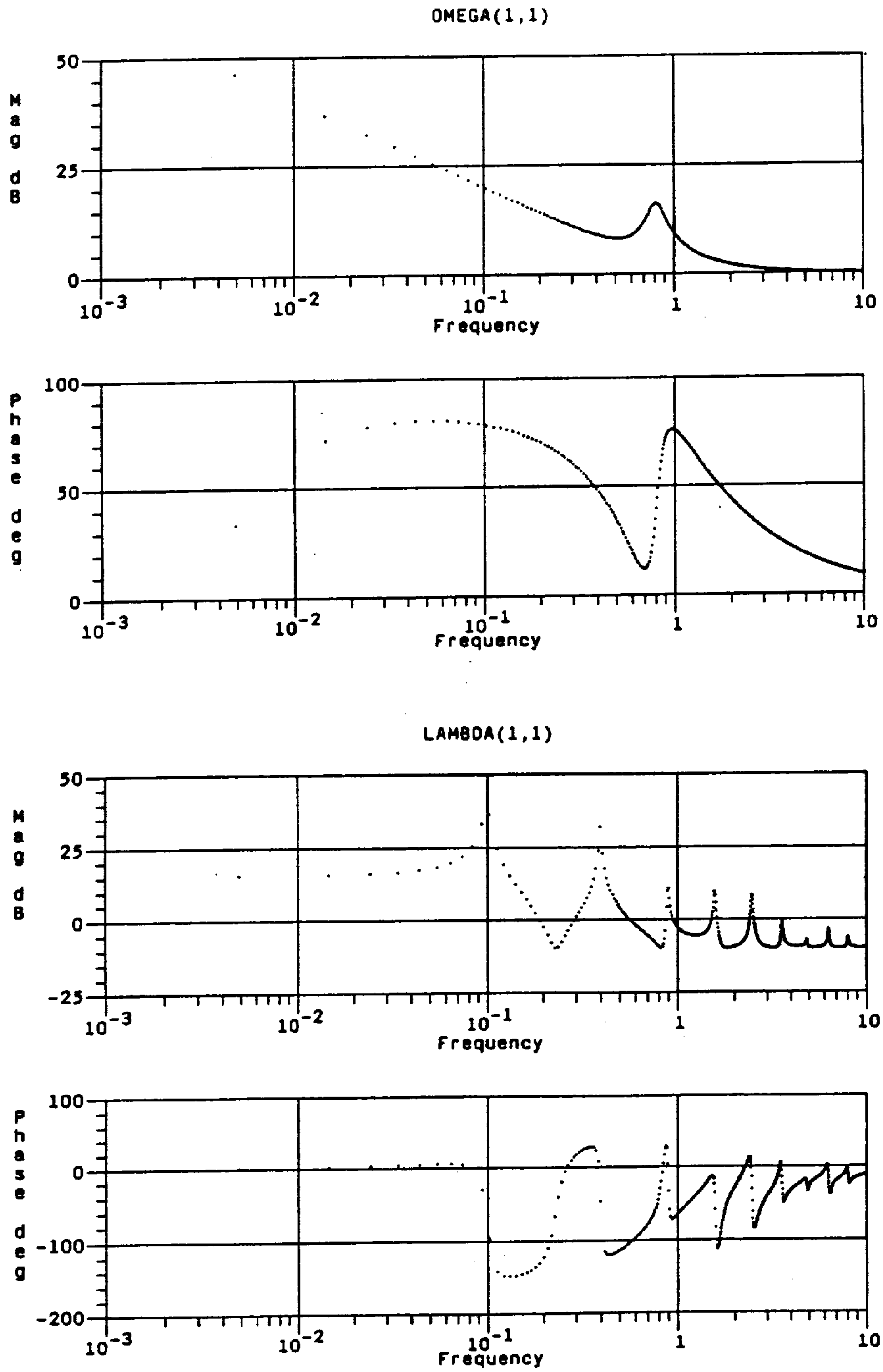

Figure 4.3: Spectral Factors for pinned-pinned beam control.

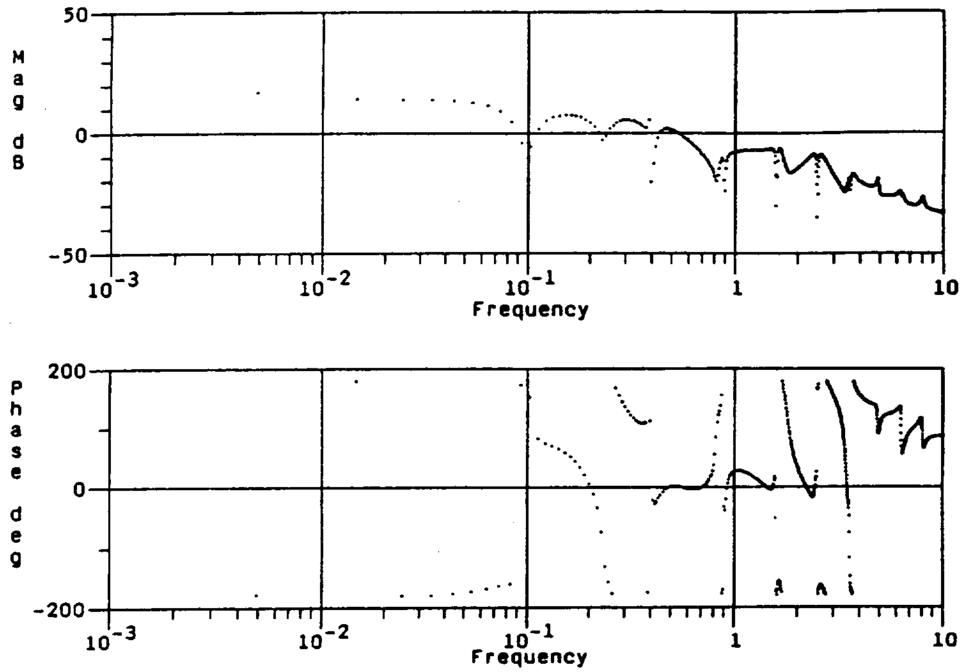

Figure 4.4: Optimal Controller Frequency Response for $\mu = .1$.

realizes the frequency sampling filter is

$$C(z) = \sum_{k=0}^{N-1} C_k F_k(z)$$

where the interpolating functions are,

$$F_k(z) = \frac{1 - z^{-N}}{N(1 - e^{jk2\pi/N} z^{-1})},$$

with $k = 0, \ldots, N-1$. A standard computation shows that the frequency sampling filter has transfer function

$$C(z) = \frac{1 - z^{-N}}{N} \sum_{k=0}^{N-1} \frac{C_k}{1 - e^{jk2\pi/N} z^{-1}} = \sum_{\ell=0}^{N-1} c_\ell z^{-\ell} \tag{5.1}$$

where the coefficients,

$$c_\ell = \frac{1}{N} \sum_{k=0}^{N-1} C_k e^{j(2\pi/N)\ell k} \tag{5.2}$$

for $\ell = 0, \ldots, N-1$, are the *Inverse Discrete Fourier Transform (IDFT)* of the sequence $C_0, \ldots, C_{N-1}$. The final form in (5.1) shows that the realization is a FIR realization. Such realizations are nonrecursive and are efficiently implemented using high speed, single chip DSP processors which utilize highly pipelined architectures to achieve high throughput.

6 Conclusions and Directions

Wiener-Hopf optimization of frequency domain models has been shown to offer significant advantages for computation of precision controllers for irrational transfer functions arising in control of flexible structures. Computational algorithms for causal spectral factorization and causal projection can be implemented based on frequency sampled representation of the model response. Such models can be obtained from transfer function models or from frequency response measurements of the controlled structure. Computations based on frequency response sampling have been demonstrated for irrational transfer function models arising in the control of flexible structures.

Requirements for precision control will involve frequency response models which are characterized by a large number of flexible modes within the control bandwidth. However, for control of relatively large, flexible space structures control bandwidth and resulting sampling requirements for discrete time control implementations are well within the state-of-the-art for high speed digital computers. Frequency sampling filters based on nonrecursive implementations can be efficiently implemented in modern DSP single chip processors for realtime control of such systems.

Application of FIR realizations for realtime, closed loop control have not received much consideration in the literature primarily due to increased phase lag by comparison with a recursive realization. However, the rapidly developing technology for realtime DSP using special purpose architectures offers throughput capabilities which may reduce the achievable computational delay to within acceptable limits for certain applications. In such cases, high order realizations may be feasible using nonrecursive implemenations which cannot be realized by recursive methods.

References

[1] W. H. Bennett, "Computation and implementation of precision control for flexible structures," in *Proc. Com Con 88*, Oct. 1988.

[2] C. Desoer, R.-W. Liu, J. Murray, and R. Saeks, "Feedback systems design: The fractional representation approach to analysis and synthesis," *IEEE Trans. on Auto. Cntrl.*, vol. AC-25, no. 3, pp. 399–412, 1980.

[3] M. Vidyasagar, *Control System Synthesis: A Factorization Approach.* MIT Press, 1985.

[4] J. S. Baras, "Complex variable methods in the control of distributed systems," 1985. Presented at NSF, SIAM, AFOSR Workshop on Control of Distributed Systems.

[5] D. Youla, H. Jabr, and J. J.J. Bongiorno, "Modern Wiener-Hopf design of optimal controllers–parts I and II," *IEEE Trans. on Auto. Cntrl.*, vol. AC-30, no. 7, pp. 3–13 and 319–338, 1976.

[6] K. Park and J. J.J. Bongiorno, "A general theory for the Wiener-Hopf design of multivariable control systems," *IEEE Trans. on Auto. Cntrl.*, vol. AC-34, no. 6, pp. 619–626, 1989.

[7] C. Desoer, R.-W. Liu, J. Murray, and R. Saeks, "Feedback system design: The fractional representation approach," *IEEE Trans. on Auto. Control*, vol. AC–25, pp. 399–412, 1980.

[8] C. Desoer and C. L. Gustafson, "Algebraic theory of linear multivariable feedback systems," *IEEE Trans. Auto. Cntrl.*, vol. AC–29, no. 10, pp. 909–917, 1984.

[9] W. Bennett and H. Kwatny, "Continuum modeling of flexible structures with application to vibration control," *AIAA J.*, September 1989. to appear.

[10] A. von Flotow, "Traveling wave control for large spacecraft structures," *AIAA J. Guidance*, vol. 9, no. 4, pp. 462–468, 1986.

[11] D. Miller, A. von Flotow, and S. R. Hall, "Active modification of wave reflection and transmission in flexible structures," in *Proc. 1987 Amer. Cntrl. Conf.*, pp. 13128–1324, 1987.

[12] G. Gu and P. Khargonekar, "Approximation of infinite-dimensional systems," *IEEE Tran. on Auto. Cntrl.*, vol. AC-34, no. 6, pp. 610–618, 1989.

[13] W. Bennett and N. Barkakati, "FlexCAD: Prototype software for modeling and control of flexible structures," in *Proc. IEEE CACSD Symp.*, Sept. 1986.

[14] W. Bennett and I. Yan, "A computer algorithm for causal spectral factorization," in *Proc. 1988 NAECON*, May 1988.

[15] F. Stenger, "The approximate solution of Wiener-Hopf integral equations," *J. Math. Analysis and Applic.*, vol. 37, pp. 687–724, 1979.

[16] J. Davis and R. Dickinson, "Spectral factorization by optimal gain iteration," *J. Appl. Math.*, vol. 43, pp. 389–301, 1983.