N90-23051

MODELING OF CONTROL FORCES FOR KINEMATICAL CONSTRAINTS IN THE DYNAMICS OF MULTIBODY SYSTEMS-A NEW APPROACH

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ABSTRACT

Conventionally kinematical constraints in multibody systems are treated similar to geometrical constraints and are modeled by constraint reaction forces which are perpendicular to constraint surfaces. However, in reality, one may want to achieve the desired kinematical conditions by control forces having different directions in relation to the constraint surfaces. In this paper the conventional equations of motion for multibody systems subject to kinematical constraints are generalized by introducing general direction control forces. Conditions for the selections of the control force directions are also discussed. A redundant robotic system subject to prescribed end-effector motion is analyzed to illustrate the methods proposed.

1. INTRODUCTION

In many applications of multibody systems certain points are desired to follow prescribed paths, such as the end-effector in a robotic system. Such kinematical conditions are treated constraint equations to determine the system motion and the control forces.

In this paper those constraints which arise from geometrical restrictions such as closed loops and physical guides are termed geometrical constraints. On the other hand, kinematical constraints are defined as those conditions which represent desired motions or desired paths of certain points or bodies.

In the conventional methods of analysis, regardless of the fundamental dynamic equations (Newton-Euler, Lagrange, Kane, etc.) used, the constraints are modeled by constraint reaction forces which are perpendicular to the constraint surfaces. (See Arnold [1], Hemami and Weimer [2], Kamman and Huston [3], Wehage and Haug [4], Nikravesh [5], Kim and Vanderploeg [13], Amirouche and Jia [6].)

However kinematical constraints do not have to be satisfied by constraint reaction forces, and usually have to be realized by control forces applied by the actuators in the system. Hence the conventional solution procedure imposes an arbitrary restriction to the directions of the control forces. Depending on the places of the actuators in the system one may want to achieve the desired kinematical conditions by control forces having different directions in relation to the constraint surfaces.

In this paper the conventional equations of motion are generalized by inroducing general direction control forces for kinematical constraints, that replaces the constraint force representation. And the dynamic equations for multibody systems subject to geometrical and kinematical constraints are developed. By the proposed method of solution the control forces and the system motion are solved simultaneously.

This paper is divided into five sections. After the introduction, the second section outlines the conventional equations of motion for constrained multibody systems. In the third section the general direction control forces for kinematical constraints are introduced and the conditions for the control force directions are discussed. In section four simulations of a redundant manipulator by the conventional and the proposed methods are presented.Conclusions form the last section.

2. CONVENTIONAL EQUATIONS OF MOTION

Consider a mechanical system where $q_1,\dots,q_n$ are a set of generalized coordinates chosen for convenience to specify the configuration of the system. Let the system be subject to c constraints. Kane's equations for an arbitrary system of particles and rigid bodies can be expressed as (Kamman and Huston [3], Baumgarte [7]),

$$F^* + F + F^c = 0 \tag{1}$$

where

$$F^c_p = \lambda_i \frac{\partial f_i}{\partial y_p} \qquad i=1,\dots,c\ , \quad p=1,\dots,n \tag{2}$$

F^*, F and F^c are the vectors of generalized inertia, external and constraint forces respectively. In equation (2) $f_i=0$, $i=1,\dots,c$ are the constraint equations in the acceleration level, λ_i are undetermined multipliers, and $y_1,\dots,y_n$ are the generalized speeds of the system chosen for convenience as independent linear combinations of $\dot{q}_p$. The tranformation between $\dot{q}_p$ and y_p, e.g. Euler angle derivatives and relative angular velocity components can be expressed as

$$\dot{q}_h = T_{hp}\, y_p \qquad h,p=1,\dots,n \tag{3}$$

where T_{hp} are functions of q_p (Kane and Levinson [8]).

The generalized inertia forces can be expressed in the following form (Huston and Passarello [9]),

$$F^* = M\,\dot{y} + Q \qquad (4)$$

where M is the generalized mass matrix of the unconstrained system being functions of q_p, and Q contains the quadratic velocity terms.

The holonomic and nonholonomic constraint equations can be expressed in the acceleration level as below

$$B_{ip}\,\dot{y}_p = h_i \qquad i=1,\dots,c \qquad (5)$$

In eq. (5) B is cxn constraint matrix, and B_{ip} and h_i are in general functions of q_p and y_p.

Note that for holonomic constraints $\Phi_i(q_p,t)=0$,

$$B_{ip} = \frac{\partial \Phi_i}{\partial q_h} T_{hp}$$

and for velocity level nonholonomic constraints $\Psi_i(q_p,y_p,t)=0$,

$$B_{ip} = \frac{\partial \Psi_i}{\partial y_p}$$

Then, using eq. (2), the generalized constraint forces are

$$F^c = B^T \lambda \qquad (6)$$

The undetermined multipliers λ_i represent the restraining constraint forces and moments generated by the constraints at the points of application (Ider and Amirouche [11]).

Equations (1) and (5) represent the governing dynamical equations. Combining these and making use of equations (4) and (6), we have

$$\begin{bmatrix} M & B^T \\ B & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \lambda \end{bmatrix} = \begin{bmatrix} F-Q \\ h \end{bmatrix} \qquad (7)$$

The accelerations obtained from eq. (7) are then used for numerical integration for the time history of y_p and, through the use of eq. (3), the generalized coordinates q_p.

Lagrange multipliers could be eliminated to reduce the equations for computational efficiency. To this end let C represent a nx(n-c) matrix which is orthogonal complement to B, obtained either by Singular value decomposition (Singh and Likins [12]), Zero eigenvalue method (Kamman and Huston [3]), Q-R decomposition (Kim and Vanderploeg [13], Amirouche and Jia [6]), or row equivalence transformation (Ider and Amirouche [10]). Premultiplying eq. (1) by C^T yields

$$C^T(F^*-F) = 0 \qquad (8)$$

Combining equations (8) and (5) and utilizing eq. (4), we obtain the reduced equations

$$\begin{bmatrix} C^T M \\ --- \\ B \end{bmatrix} \dot{y} = \begin{bmatrix} C^T(F-Q) \\ ------- \\ h \end{bmatrix} \tag{9}$$

Equations (9) and (3) form a set of 2n first order ordinary differential equations that can be numerically integrated to obtain the time history of y_p and q_p.

When relative joint coordinates are selected as the generalized coordinates and the corresponding partial velocity vectors are developed using recursive multibody kinematics (Huston and Passarello [9], Ider and Amirouche [10]), constraint equations for joint connections are automatically eliminated. Hence, in this paper an open tree-like system represents an unconstrained system where n is the total number ofthe free joint degrees of freedom.

3. CONTROL FORCES FOR KINEMATICAL CONSTRAINTS

Now consider that a tree-like multobody system is subject to geometrical and kinematical constraints. Kinematical constraints represent desired motions or desired paths of certain points or bodies. They are the conditions that have to be realized by the actuators in the system. The desired motions could be specified at position, velocity or acceleration levels and could be holonomic or nonholonomic.

Whether one uses Newton-Euler, Lagrange or Kane's equations or other variations of these, in the conventional approach the constraints in the system are modeled by constraint reaction forces which are perpendicular to the constraint surfaces. In the case of geometrical constraints perpendicular reaction forces at the application points are generated, hence the above approach is necessary. On the other hand kinematical constraints could be achieved by a number of alternative control forces whose directions in the generalized space can be be selected by physical considerations. Therefore the conventional equations of motion should be generalized by considering general direction control forces for kinematical constraints.

Consider c constraint equations (5), and let c_1 of the constraints in the system be geometrical and the remaining c_2 ($c_2=c-c_1$) be kinematical. The constraint matrix B and the vector of constraint force magnitudes λ can be partitioned such that

$$B = [B^{G^T} \quad B^{K^T}]^T \tag{10}$$

and

$$\lambda = [\lambda^{G^T} \quad \lambda^{K^T}]^T \tag{11}$$

where B^G is a $c_1$xn matrix, B^K is a c_2 matrix, λ^G is a c_1 dimensional vector and λ^K is a c_2 dimensional vector.

Addition of control forces to the equations of motion yields

$$M \dot{y} + Q + B^{G^T}\lambda^G + B^{K^T}\lambda^K + A^T \mu = F \tag{12}$$

where A is a $c_2$xn control force matrix where each row represents the direction of the control force for each kinematical constraint in the generalized space, and μ is c_2 dimensional vector of control force magnitudes. Now assume that the control force directions and magnitudes are selected such that the restraining constraint forces λ^G become zero. This leads to

$$M \dot{y} + Q + B^{G^T} \lambda^G + A^T \mu = F \tag{13}$$

Eq. (13) can be written in the following form

$$M \dot{y} + Q + Z^T \nu = F \tag{14}$$

where Z is the augmented matrix of constraint and control force directions

$$Z = [B^{G^T} \quad A^T]^T \tag{15}$$

and is the vector of constraint and control force magnitudes,

$$\nu = [\lambda^{G^T} \quad \mu^T]^T \tag{16}$$

Once the control force directions A are selected by physical considerations eq. (14) needs to be solved together with eq. (5) to determine the control force magnitudes simultaneously with the generalized accelerations. Hence the augmented equations of motion are

$$\begin{bmatrix} M & Z^T \\ B & 0 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \nu \end{bmatrix} = \begin{bmatrix} F-Q \\ h \end{bmatrix} \tag{17}$$

Alternatively the equations could be reduced in a manner similar to Section 2. To this end, let $\bar{C}$ be a nx(n-c) matrix orthogonal complement to Z. Premultiplying eq. (14) by $\bar{C}^T$ and augmenting with eq. (5) leads to

$$\begin{bmatrix} \bar{C}^T M \\ \hline B \end{bmatrix} \dot{y} = \begin{bmatrix} \bar{C}^T (F-Q) \\ \hline h \end{bmatrix} \tag{18}$$

In the case when the reduced equations are used ν can be obtained from eq. (14) utilizing the computed accelerations, as

$$\nu = (Z\ Z^T)^{-1}\ Z\ (F-My-Q) \tag{19}$$

Note that A should be selected such that rank Z=c, because otherwise there will be less than c_2 control forces to control c_2 kinematical conditions.

The augmented equations have a solution if and only if the augmented mass matrix is nonsingular, in which case the prescribed conditions are achieved with the corresponding control forces. On the other hand if it is not physically possible to realize the kinematical constraints with the selected control force directions, this reveals itself as a singular (or near singular) augmented mass matrix. Therefore the condition for the existence of solution could be expressed as follows: Directions A should be chosen such that a linear combination of the rows of $\tilde{C}^T M$ should not be a linear combination of the rows of B. In other words the vector space spanned by the rows of $\tilde{C}^T M$ and the vector space spanned by the rows of B should be nonintersecting (except the zero vector). Since the dimension of the vector space spanned by the rows of B is c, and that of $\tilde{C}^T M$ is n-c, the possibilities for $\tilde{C}^T M$ are infinitely many (provided that n-c>0). Hence one can construct various vector spaces for $\tilde{C}^T M$ by different selections of the control force directions A. $C^T M$ that correspond to directions B is only one of them.

For redundant systems, i.e. when c<n, it has been observed that one usually has several physically meaningful control force directions to satisfy the given kinematical conditions.

In the special case when A is selected such that its rows are linear combinations of the rows of B^K, then since $\tilde{C}$ is the same as C in the conventional model, y becomes the same as the conventional case and $Z^T \nu$ becomes equal to $B^T \lambda$. However ν_i may be different than λ_i depending on A.

Similarly for nonredundant systems, i.e. n=c, B is nxn, and rows of Z are necessarily linear combinations of the rows of B. In this case $\tilde{C}$ is null matrix and the above procedure reduces to the conventional method where $Z^T \nu$ is equal to $B^T \lambda$.

It should be noted that $\tilde{C}^T M$ and B may form nonintersecting vector spaces even if C^T and B do not. Hence realization of the prescribed motions is possible even in the extreme case when a control force direction is tangent to the corresponding constraint surface. This is due to the inertia coupling between the generalized coordinates.

4. SIMULATIONS OF A REDUNDANT ROBOTIC SYSTEM

In the three link manipulator shown in Figure 1, the configuration of the system can be described by three generalized coordinates θ_1, θ_2, θ_3. The generalized speeds y_p are defined as

$$y_1 = \dot{\theta}_1 \ , \ y_2 = \dot{\theta}_2 \ , \ y_3 = \dot{\theta}_3$$

The data used are $L_1=L_2=L_3=1.0$m, $m_1=30$kg, $m_2=m_3=18$kg, $I_1=10$ kg.m^2, $I_2=I_3=8.64$ kg.m^2.

The end-effector (point A) is desired to move in the horizontal direction with a constant velocity v^A. Hence the constraint equations in the system are

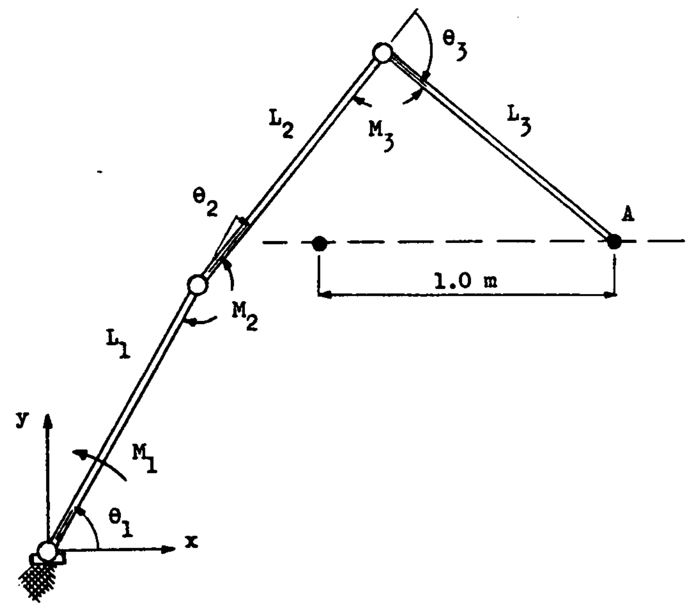

Figure 1. Three link manipulator

$$L_1c_1 + L_2c_{12} + L_3c_{13} = v^At + 1.9088 \qquad (20)$$

$$L_1s_1 + L_2s_{12} + L_3s_{13} = 0.9893$$

where $c_1=\cos\theta_1$, $c_{12}=\cos(\theta_1+\theta_2)$, $c_{13}=\cos(\theta_1+\theta_2+\theta_3)$, and similarly $s_1=\sin\theta_1$, $s_{12}=\sin(\theta_1+\theta_2)$, $s_{13}=\sin(\theta_1+\theta_2+\theta_3)$. At the acceleration level the constraint equations are given by eq. (5) where B and h are

$$B = \begin{bmatrix} L_1s_1+L_2s_{12}+L_3s_{13} & L_2s_{12}+L_3s_{13} & L_3s_{13} \\ L_1c_1+L_2c_{12}+L_3c_{13} & L_2c_{12}+L_3c_{13} & L_3c_{13} \end{bmatrix} \qquad (21)$$

and

$$h = \begin{bmatrix} -L_1s_1y_1{}^2-L_2s_{12}(y_1+y_2)^2-L_3s_{13}(y_1+y_2+y_3)^2 \\ -L_1c_1y_1{}^2-L_2c_{12}(y_1+y_2)^2-L_3c_{13}(y_1+y_2+y_3)^2 \end{bmatrix} \qquad (22)$$

Initially the system is at the configuration $\theta_1=60^o$, $\theta_2=-10^o$, $\theta_3=-90^o$. The initial generalized speeds are $y_1=0.386$ rad/sec, $y_2=0$, $y_3=-0.9618$ rad/sec, which correspond to $v^A= -1$ m/sec.

First the system is simulated using the conventional method. The generalized constraint forces can be expressed from equations (6) and (21) as,

$$\begin{bmatrix} F_1^c \\ F_2^c \\ F_3^c \end{bmatrix} = \begin{bmatrix} \lambda_1 \ (L_1 s_1 + L_2 s_{12} + L_3 s_{13}) + \lambda_2 \ (L_1 c_1 + L_2 c_{12} + L_3 c_{13}) \\ \lambda_1 \ (L_2 s_{12} + L_3 s_{13}) + \lambda_2 \ (L_2 c_{12} + L_3 c_{13}) \\ \lambda_1 \ L_3 s_{13} + \lambda_2 \ L_3 c_{13} \end{bmatrix} \tag{23}$$

In particular we wish to determine joint moments denoted as M_i, i=1,2,3 that would achieve the desired kinematical conditions. F_i^c, i=1,2,3 represent the required joint moments. It is seen from eq. (23) that all three joint moments are nonzero, i.e. motors are required at all three joints.

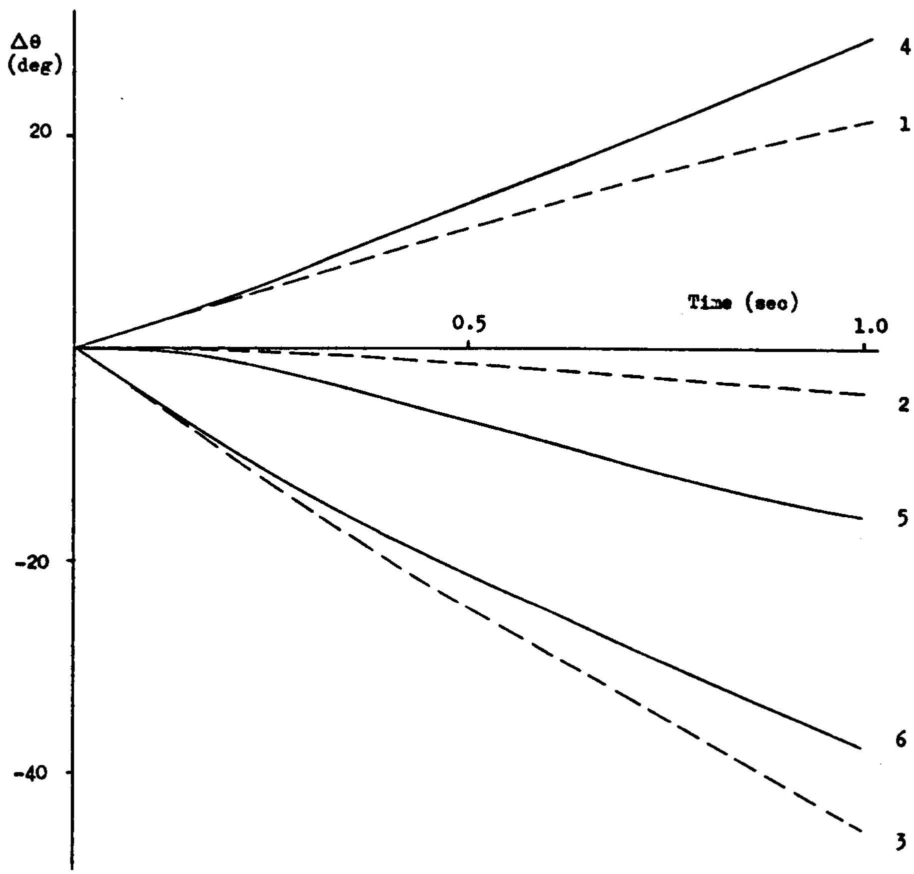

Figure 2. Displacements.
Conventional method : 1. $\Delta\theta_1$, 2. $\Delta\theta_2$, 3. $\Delta\theta_3$
Control force method: 4. $\Delta\theta_1$, 5. $\Delta\theta_2$, 6. $\Delta\theta_3$

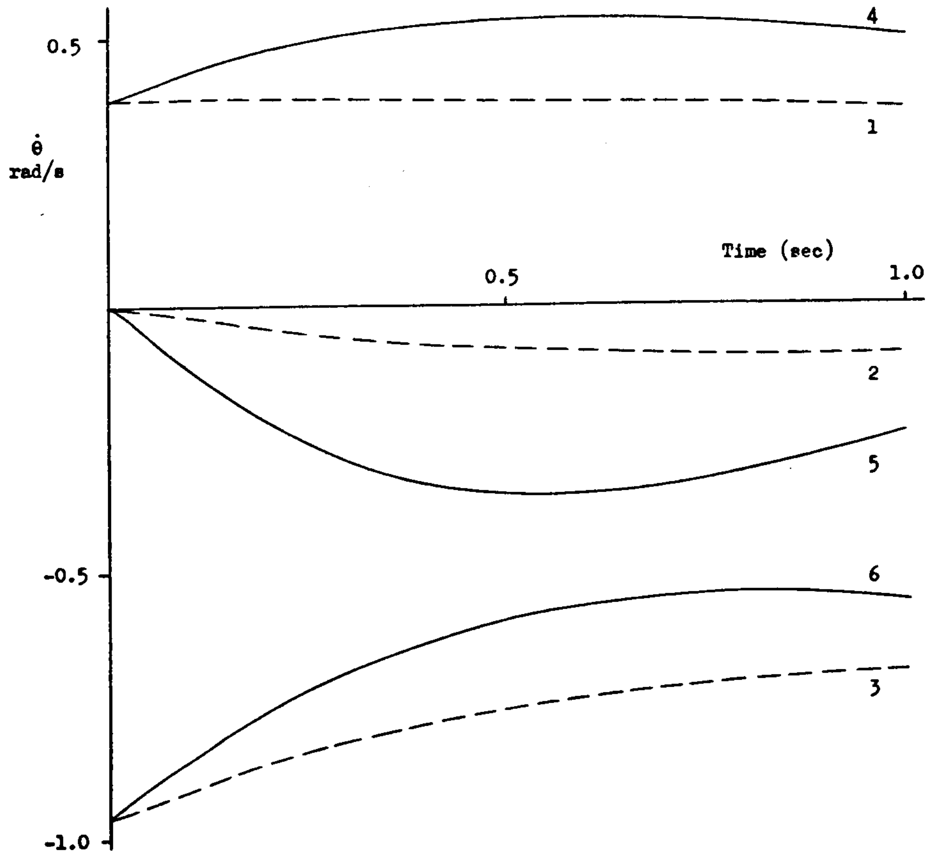

Figure 3. Velocities.
Conventional method : 1. $\dot{\theta}_1$, 2. $\dot{\theta}_2$, 3. $\dot{\theta}_3$
Control force method: 4. $\dot{\theta}_1$, 5. $\dot{\theta}_2$, 6. $\dot{\theta}_3$

The simulation is performed for 1 sec., until the end effector moves 1m in -x direction. $\Delta\theta_i$ and $\dot{\theta}_i$, i=1,2,3 are plotted in Figures 2 and 3 respectively. The joint moments M_i required to obtain the desired motion of the end effector as obtained by the conventional method are shown in Figure 4.

Second the system is resimulated by the proposed control force approach. As an illustration the control force directions are selected such that no moments are needed at the lower joint of link 1. The corresponding control force directions are

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{24}$$

Note that since there are no geometrical constraints in this system, the

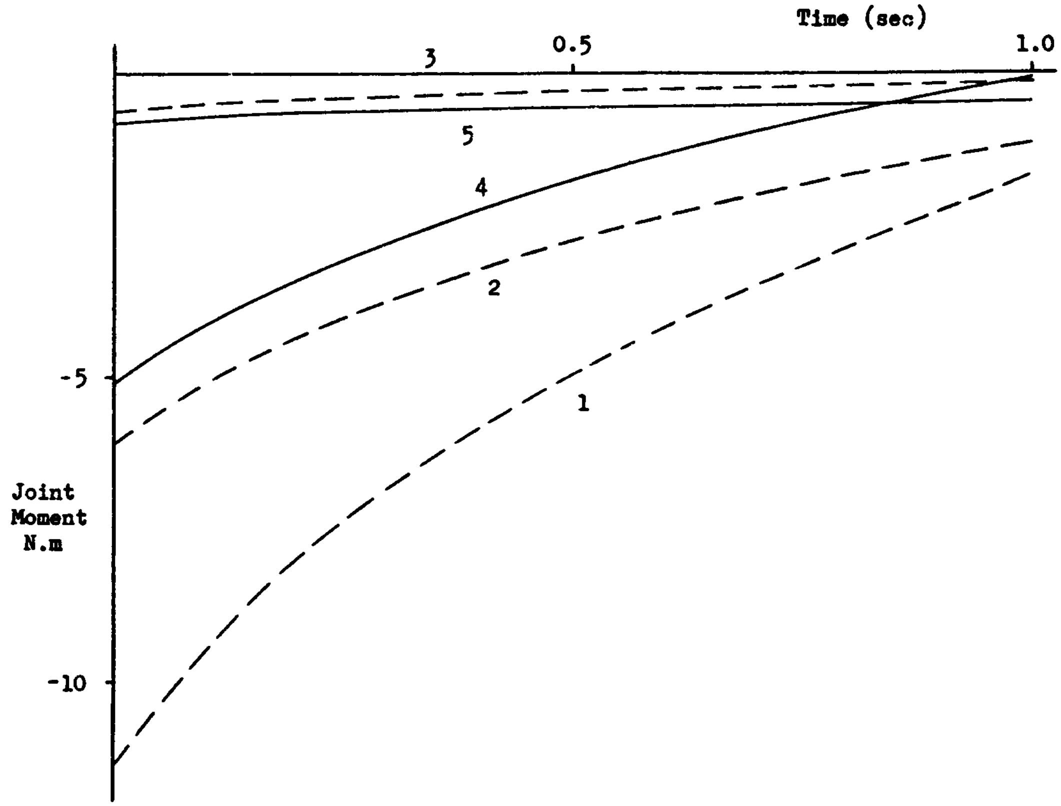

Figure 4. Joint moments.
Conventional method : 1. M_1, 2. M_2, 3. M_3
Control force method: 4. M_2, 5. M_3

matrix Z is identical to A, and the vector ν is identical to μ. The control forces $Z^T\nu$ are

$$Z^T\nu = \nu_1\begin{bmatrix}0\\1\\0\end{bmatrix} + \nu_2\begin{bmatrix}0\\0\\1\end{bmatrix} = \begin{bmatrix}0\\\nu_1\\\nu_2\end{bmatrix} \tag{25}$$

Hence, in this case, the required joint moments for the prescribed end effector motion are $M_1=0$, $M_2=\nu_1$, $M_3=\nu_2$.

The augmented mass matrix was observed to be full rank as expected. θ_i and $\dot{\theta}_i$, i=1,2,3 for a simulation of 1 sec. are plotted in Figures 2 and 3. The joint moments are plotted in Figure 4. Note that in this case a motor is not needed at the lower joint of link 1.

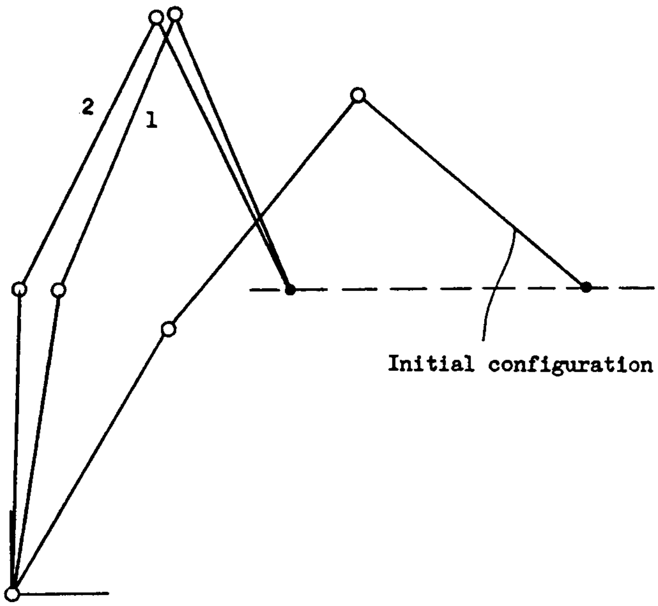

Figure 5. Final configurations: 1. Conventional method; 2. Control force method.

The system's motion differ in the above two approaches although in both cases the end effector performs the same motion. The configurations at t=1 sec. corresponding to the conventional model and the control force model are shown in Figure 5.

5. CONCLUSIONS

This paper presented a general procedure for the dynamic modeling of multibody systems subject to kinematical constraints. General direction control forces have been introduced that replace the conventional constraint reaction forces, hence increasing the ways of realization of the prescribed motions. It is shown that the possible conrol force directions are more than one, and the criteria for the existence of solution have been presented.

The method proposed in this paper involves selecting the control force directions in the generalized space by physical considerations, and then solving their magnitudes simultaneously with the corresponding system motion. As a result one can design alternative control forces that can be applied by the actuators in the system. The method is expected to be especially useful to control the extra degrees of freedom in systems that have joint flexibility or joint clearance.

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