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Multivariable Frequency Weighted Model Order Reduction For Control Synthesis

David K. Schmidt
Department of Mechanical and Aerospace Engineering
Arizona State University

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Abstract

Quantitative criteria are presented for model simplification, or order reduction, such that the reduced order model may be used to synthesize and evaluate a control law, and the stability and stability robustness obtained using the reduced-order model will be preserved when controlling the full-order system. The error introduced due to model simplification is treated as modeling uncertainty, and some of the results from multivariable robustness theory are brought to bear on the model simplification problem. A numerical procedure developed previously is shown to lead to results that meet the necessary criteria. The procedure is applied to reduce the model of a flexible aircraft. Also, the importance of the control law itself, in meeting the modeling criteria, is underscored. An example is included that demonstrates that an apparently robust control law actually amplifies modest modeling errors in the critical frequency region, and leads to undesirable results. The cause of this problem is identified to be associated with the canceling of lightly-damped transmission zeroes in the plant.

Whether the engineer is developing a system model for dynamic analysis, control law synthesis, or simulation, a simple low-order model with the requisite validity is desirable for a variety of practice reasons. The question arises, therefore, as to how to obtain such a simple yet valid model. Even more fundamental is the question of what model characteristic are important such that one may strive to retain them. Although the initial question has been addressed for some time, from the attention still paid to model and controller order reduction (c.f. Refs. 1,2), it appears that the issues still remain unresolved.

In Refs 3-6, some previous offerings on the subject are presented. In this paper, discussion will continue, in the attempt to expand on some of the earlier results, to further clarify the theoretical basis behind the proposed methodology, and to reveal some important aspects of not only model-simplification, but also control-law synthesis for elastic vehicles.

1. Criteria for Modeling

The objective in model simplification, as with all system modeling, is to develop a fundamental understanding of the system in question. For the model to be useful, it should predict to the required engineering accuracy the behavior of the actual system. Note that it does not have to predict with perfect accuracy, and that is not possible anyway. The required accuracy depends on the application for which the model is intended.

In this paper, as in Refs. 3 - 6, the intended application of the model is to predict the behavior of the system when it is subject to feedback action, as shown, for example, in Fig. 1. Clearly, then, the critical characteristics of the actual system that must be adequately captured by the model are those characteristics important in a feedback system. (Note that the feedback action could represent an automatic control system, as well as that of a human, or manual controller.) Finally, the existence of a sufficiently valid, although perhaps complex model for the system is assumed to be available - admittedly a big assumption. Further, if this model is infinite-dimensional and/or non-linear, it is assumed that a locally linearized, finite-dimensional model may be obtained. The original (complex) model will be denoted as $\underline{G}$, while the linear model will be denoted as G.

As a result of any simplification process, differences between the more-accurate model and the simple model arise. Or conceptually, if G_R is a simpler model for G, the model-simplification error may be considered to be $\Delta G = G - G_R$. These errors are key to the research presented here. In contrast, model-simplification errors arising due to the development of G, or $\Delta\underline{G} = \underline{G} - G$, will be considered only indirectly.

The critical question then is what errors ΔG are critical, or should be minimized, and what procedure will do so? The answer to the first part of the question could be that ΔG's critical in a feedback loop should be minimized. Further, if these ΔG's are interpreted more generally as model uncertainty rather that model-reduction error, the recent research on multi-variable robustness theory may be bought to bear on the model-simplification problem. This is the main idea in this research.

2. Robustness and Model Reduction

In this section, some key results from robust control theory will be noted, and they will be interpreted in the context of the model reduction problem.

With reference to the system shown in Fig. 2, G_R is the transfer-function-matrix representation of this simplified model, $\Delta G(s)$ in the analogous representation of the model-simplification error, and the full-order linear model is $G = G_R + \Delta G$. Likewise, K(s) is the matrix of control compensators, perhaps to be designed using G_R. Clearly in this context, one desires that the K(s) so obtained will control the "true" G(s) as predicted through the use of G_R. Attention is now turned to exposing the critical ΔG's via multi-variable Nyquist theory.[7]

Let $\Phi(s)$ be an analytic function of the complex variable s, and let the number of zeros of $\Phi(s)$ in the open right half of the complex plane be denoted as z. Then the Principle of the Argument states that

$$\underset{R\to\infty}{N} (O, \Phi(s), D_R) = z$$

or the number (N) of clockwise encirclements of the origin made by the image of the contour D_R, under the mapping of $\Phi(s)$, as s travels clockwise around D_R , equals z. Here D_R is the "Nyquist D contour" that encloses the entire right-half of the complex plane. Clearly, with regards to stability, the $\Phi(s)$ of interest is the closed-loop characteristic polynomial of the feedback system, denoted by $\Phi_{CL}(s)$.

Now, as shown in Ref. 8, and elsewhere, and referring to Fig. 1, for example,

$$\begin{aligned}\Phi_{CL}(s) &= \Phi_{OL}(s) \det [I + GK] \\ &= \Phi_{OL}(s) \det [I + KG] \qquad (1)\end{aligned}$$

where $\Phi_{OL}(s)$ is the characteristic polynomial of the open-loop system KG(s) or GK(s). That is, if either the transfer function matrix GK(s) or KG(s) has the state-space realization

$$\begin{aligned}\dot{x} &= A_{GK}x + B_{GK}e \\ y &= C_{GK}x\end{aligned}$$

then $\Phi_{OL}(s) = \det[sI - A_{GK}]$, and the zeros of $\Phi_{OL}(s)$ are the open-loop poles of the system. Note that Eqn. 1 may therefore be re-written as

$$\Phi_{OL}(s) = \det[sI - A_{GK}] \det [I + C_{GK} [sI - A_{GK}]^{-1} B_{GK}]$$

Now if the number of right-half-plane zeros of $\Phi_{OL}(s)$ is p, then the number of right-half-plane zeros of det $[I + C_{GK} [sI - A_{GK}]^{-1} B_{GK}]$ must be z-p. Furthermore, from the Principle of the Argument

$$\underset{R\to\infty}{N} (O, \det [I + C_{GK} (sI - A_{GK})^{-1} B_{GK}] ,D_R) = z\text{-}p$$

Consequently, if p is known, z may be deduced from

$$\begin{aligned}z &= p + (z - p) \\ &= p + [\underset{R\to\infty}{N} (O, \det [I + C_{GK} (sI - A_{GK})^{-1} B_{GK}] ,D_R)]\end{aligned}$$

or closed-loop stability is determined from knowledge of p and the examination of the Nyquist contour for det[I + GK] or det [I + KG]. Therefore, the closed-loop system is stable if and only if the Nyquist contour for det[I + GK](= det[I +KG]) encircles the origin counterclockwise exactly p times.

Of course the determination of z is possible from other means, and the real utility of the above fact is in defining the concept of relative stability, and in identifying factors that are critical to closed-loop system stability. These issues are of special import here.

Consider the model error, or uncertainty, to be ΔG (as in Fig. 2), and assume that K is such that KG_R leads to a stable closed-loop system with good stability margins. (Note this assumption should always be true as it involves a key objective in determining K(s) using G_R to begin with.) Then if (assumption 1) the number of right-half-plane poles of KG (= p) is identical to the number of right-half-plane poles of KG_R (= p_R), K will stabilize G if and only if (assumption 2)

$$\underset{R\to\infty}{N}(O, \det [I + GK], D_R) = \underset{R\to\infty}{N}(O, \det [I + G_RK], D_R)$$

or the number of encirclements of the origin made by the Nyquist contours associated with G and with G_R are identical.

Stability is guaranteed as follows:

Let z = no. of unstable closed-loop poles of the KG loop.
z_R = no. of unstable closed-loop poles of the KG_R loop
p, p_R = defined above

Then to show stability (or z = 0), note that if (assumption 1) $p_R = p$, then

$$z = z_R - (z_R - p_R) + (z - p)$$

By the assumption KG_R leads to a stable system, $z_R = 0$, and from assumption 2, $(z_R - p_R) = (z-p)$. Hence, z = 0.

This now establishes in a meaningful way, qualitative criteria for model simplification, the simplification must at least lead to ΔG's such that assumption 1 and 2 are satisfied. But the criteria goes further. Not only must stability of the KG loop be assured (i.e., z = 0) but the margins "designed" into KG_R should carry over to the closed-loop system associated with KG. Otherwise, the K so designed would not be satisfactory. It is for this reason that any model reduction technique that just assures stability of the full-order closed-loop system may not be good enough!

To satisfy assumption 2, or to assure that the number of encirclements of the origin is unchanged due to ΔG, requires that [9]

$$\det [I + G_RK + \varepsilon\Delta GK] \neq 0 \qquad \forall\ \omega>0,\ \varepsilon \in [0,1] \tag{2}$$

In other words, if as the Nyquist contour for det[I + G_RK] is continually warped to that for det [I + GK] the origin is never intersected, the number of encirclements of the origin cannot change. Furthermore, Eqn. 2 is assured if (c.f., Ref. 9)

$$\bar{\sigma}(\Delta GK) < \underline{\sigma}[I + G_RK] \quad \forall\ \omega>0 \tag{3}$$

Finally, it is known that an alternative to Eqn. 3 is

$$\bar{\sigma}(E_m) < \underline{\sigma}[I + (G_RK)^{-1}] = \underline{\sigma}\{[G_RK(I + G_RK)^{-1}]^{-1}\} \quad \forall\ \omega>0 \tag{4}$$

where $E_m = G_R^{-1}\Delta G$

The above expressions (Eqns. 2 - 4) may be extended by breaking the frequency domain ($0 \leq \omega < \infty$) into the domains ($0 \leq \omega \leq \omega^*$) and ($\omega^* < \omega < \infty$). Note that these domains are non-intersecting. Now it can be argued that Eqn. 2 will be satisfied if

$$\det [\, I + G_R K + \varepsilon \Delta G K] \neq 0 \qquad \begin{matrix} (0 \leq \omega \leq \omega^*) \\ (0 \leq \varepsilon \leq 1) \end{matrix} \qquad (5)$$

and

$$\det [\, I + G_R K + \varepsilon \Delta G K] \neq 0 \qquad \begin{matrix} (\omega^* < \omega < \infty) \\ (0 \leq \varepsilon \leq 1) \end{matrix} \qquad (6)$$

Further, Eqn. 5 is assured if Eqn. 3 is satisfied for $\omega \leq \omega^*$, while satisfying Eqn. 4 for $\omega > \omega^*$ assures that Eqn. 6 is satisfied. Hence, in such a situation, Eqn. 2 is satisfied.

By Eqns. 3 and 4, quantitative criteria on critical ΔG's are established. Further, the overall strategy for model simplification becomes apparent, and the interaction between model simplification and control law synthesis is underscored. Regarding the later, it should be clear that the allowable ΔG's (those that do not destroy closed-loop stability of the full-order system controlled by K(s)) depend on K itself. In other words, designing a "good" K(s) increases that allowable ΔG, while designing a bad one may put very strict limitations on the allowable ΔG, and hence model accuracy. The former K(s) is robust, the latter is not.

Regarding the model simplification strategy, then, first observe the right side of Eqn. 3. When $\underline{\sigma}\,(G_R K) >> 1$, $\underline{\sigma}\,[I + G_R K] \approx \underline{\sigma}\,(G_R K)$. Conversely, when $\bar{\sigma}(G_R K) << 1$, $\underline{\sigma}\,[I + G_R K] \approx 1$. Finally, the $\underline{\sigma}\,[I + G_R K\}$ will take on its minimum value in the frequency range where $\sigma_i\,(G_R K) \approx 1$. The frequency range where the latter occurs is of course the (multi-variable) gain crossover region. Consequently, it is this frequency range where the ΔG must be the smallest, and this can be assured if each element of the ΔG matrix is small in this frequency range.

Also, noting the above discussion, Eqn. 3 may be satisfied by rather large ΔG in any frequency range where $\underline{\sigma}\,[I + G_R K]$ is large, and this will occur when $\underline{\sigma}\,(G_R K)$ is large. If K is designed to give a good classical Bode loop shape, $\underline{\sigma}\,(G_R K)$ will be large for frequencies below crossover.[9]

Now consider Eqn. 4. When $\bar{\sigma}\,(G_R K) << 1$, $\underline{\sigma}\,(G_R K)^{-1} >> 1$, and $\underline{\sigma}\,\{I + (G_R K)^{-1}] \approx \underline{\sigma}\,(G_R K)^{-1} >> 1$. Hence the allowable ΔG may also be rather large in this case. Further, if K yields a good loop shape, or is well attenuated, at high frequencies, $\sigma\,(G_R K)$ will be small for frequencies above crossover. So clearly, the ΔG must be smallest in the region of multi-variable crossover, while if K yields a good bode loop shape, rather large ΔG elsewhere may be acceptable and Eqns. 3 and 4 may be satisfied. The above discussion is summarized in Fig. 3.

The final issue to be addressed is that of satisfying assumption 1, or the number of unstable poles of KG_R must be identical to the number of unstable poles of KG. First note that this is equivalent to requiring the number of unstable poles of G and G_R to be the same, since only one K is involved. Then observe that the poles of G are the poles of $G_R + \Delta G$, which consists of the poles of G_R plus the poles of ΔG. Hence to satisfy assumption 1, ΔG must be stable.

Attention will now turn to some additional criteria arising from performance considerations rather that from stability robustness. The system to be considered is that shown in Fig. 4. The vector of responses Y(s) is given by

$$\begin{aligned} Y = [I + (G_1 + \Delta G_1)K]^{-1}\,(G_1 + \Delta G_1)K(Y_C - N) \\ + [I + (G_1 + \Delta G_1)K]^{-1}\,(G_2 + \Delta G_2)\,D \end{aligned}$$

Here G_1 is the reduced-order model for the response of G to control inputs, where G_2 is the reduced-order model for the response of G to disturbances being considered. ΔG_1 and ΔG_2 are the analogous model-simplification errors.

The first observation to be made is that stability and stability robustness depends on G_1 and ΔG_1, not on G_2 and ΔG_2. Note that the poles of $(G_1 + \Delta G_1)$ are the poles of the "true" plant G, as are the poles of $G_2 + \Delta G_2$. Hence if K stabilizes G, which will be assured if G_1 and ΔG_1 satisfy the criteria developed previously, K must therefore stabilize $(G_2 + \Delta G_2)$. This is significant since some (stable) poles of G may be approximately cancelled by some zeroes for the transfer functions governing responses to control inputs, but not cancelled in those governing responses to disturbances. Cancelling these poles to obtain G_1 has raised questions by some as to whether those poles so cancelled could lead to problems later in analysis. The answer appears to be that they will not if G_2 is obtained such that those poles are retained. But from the above discussion on stability, the only reason to keep these poles in G_2 (that by assumption are not approximately cancelled) is such that the disturbance-rejection performance predicted using G_2 (when designing K, for example) will be reasonably accurate.

Finally, noting that the disturbance response due to ΔG_2 is

$$Y_{D_2} = [I + (G_1 + \Delta G_1)K]^{-1} \Delta G_2 D$$

for good performance prediction (Y_{D_2} small), ΔG_2 should tend to be small whenever D is large and $(G_1 + \Delta G_1)K$ is small. But here again, if K is designed to obtain a "good loop," it will be designed such that G_1K (and by implication $(G_1 + \Delta G_1)$ K) will be large over the frequency range where D is large. Consequently, this should not pose stringent requirements on ΔG_2.

In ending this section, it is worth noting that assuming K is designed properly has been critical. By doing so, one takes advantage of one of the basic advantages of a good feedback system, reduction in sensitivity to plant (or plant model) variations. This allows the development of a modeling procedure that focuses on the really critical problem of obtaining a good model in the crossover region.

3. Methodology and Sample Results

The procedure offered was discussed in detail in Ref. 5, and the computational technique is summarized again in Table 1. The technique is a frequency weighted internally-balanced approach, with stable factorization in the case of an unstable plant G. The stable factorization procedure sets the unstable subsystem of G aside via partial fraction expansion, leaving the remaining subsystem G_S stable. This stable subsystem is then reduced, such that a stable reduced order model G_{RS} is guaranteed. The unstable subsystem is then rejoined with G_{RS} to obtain the final reduced-order model G_R. By this procedure, the number of unstable poles of G are preserved. In fact the unstable poles in G are exactly retained in G_R.

The internally balanced technique [10] requires the frequency-weighting extension[5] since the basic technique leads to small model-simplification errors ΔG where the elements of G have large magnitude, which is not necessarily the crossover region. Further, a very poor model may be obtained where the elements of G have small magnitude. As will be shown later, this can be totally unacceptable.

In Ref. 11, a frequency-weighted approach was also suggested, but the weighting required the knowledge of the compensator K, obtained using the full-order plant. Since designing a simple K using the simpler plant G_R is the typical design objective, the above weighting is undesirable. In Ref. 5, it was noted that simply adding a weighting filter obtainable by inspection of the Bode plots of G and knowledge of the desired crossover frequency range let to excellent results. This filter is easily discarded after G_R is retained. In the example presented later, it will be shown that this approach again appears quite acceptable.

The key to the concept is the knowledge of the fact that the internally balanced approach yields a small ΔG where the elements of G have large magnitude. Heuristically, if a filter W(s) is used such that W(s)G(s) has large magnitude in the required frequency range, and if WG reduced such that WG_R is obtained, then G_R will have the desired properties.

Table 1. Frequency Weighted Internally Balanced Reduction

Given: System State space description A, B, C and weighting filter state space description A_w, B_w, C_w.

Find: r^{th} order system

Step 1: Solve for X and Y

$$\begin{bmatrix} A & BC_w \\ 0 & Z_w \end{bmatrix} \begin{bmatrix} X & X_{12} \\ X_{21} & X_{22} \end{bmatrix} + \begin{bmatrix} X & X_{12} \\ X_{21} & X_{22} \end{bmatrix} \begin{bmatrix} A^T & 0 \\ C_w^T B^T & A_w^T \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & B_w B_w^T \end{bmatrix} = 0$$

$$\begin{bmatrix} A^T & 0 \\ C_w^T B^T & A_w^T \end{bmatrix} \begin{bmatrix} Y & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} + \begin{bmatrix} Y & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} A & BC_w \\ 0 & A_w \end{bmatrix} + \begin{bmatrix} CC^T & 0 \\ 0 & 0 \end{bmatrix} = 0$$

Step 2: Find T and Σ where $XY = T\Sigma^2 T^{-1}$, $T = [T_r \; T_{n-r}]$, $T^{-T} = [U_r, U_{n-r}]$

$$\Sigma^2 = \begin{bmatrix} \Sigma^2 & 0 \\ 0 & \Sigma_{n-r} \end{bmatrix} \qquad \text{where}$$

$$\Sigma r = \text{diag}(v_{ci}\, v_{oi}) \quad i = 1, \ldots, r$$

$$\Sigma_{n-r} = \text{diag}(v_{ci}\, v_{oi}) \quad i = r+1, \ldots, n$$

$$v_{ci}\, v_{oi} \geq \ldots \geq v_{ci}\, v_{oi} \geq 0$$

Step 3: r^{th} order system is

$$A_r = U_r^T A T r$$

$$B_r = U_r^T B$$

$$C_r = CT_r$$

As the example, consider an elastic aircraft identical to the configuration investigated in Refs. 3 and 6. This configuration is of reasonably conventional geometry with a low-aspect ratio swept wing, conventional tail, and canard. A numerical model for the longitudinal dynamics is available from the above references. Both rigid-body modes and four elastic modes (resulting in a 11^{th} order model) are included. The in-vacuo vibration frequencies are 6.3, 7.0, 10.6, and 11.0 rad/s, and are representative for a supersonic/hypersonic cruise vehicle. These frequencies, furthermore, are all near the anticipated frequencies at crossover for the control systems to be designed.

Control inputs are elevator deflection δ_E and canard deflection δ_C, while the disturbance is the perturbation in angle of attack due to atmospheric turbulence α_g. Selected responses are vertical acceleration a_z' measured at the cockpit and pitch rate q measured at the antinode of the first bending mode. Therefore, the flight and structural mode control loops in the context of Figure 4, might correspond to the following, for example

$$Y = [a_z'\; q]^T$$
$$U = [\delta_E\; \delta_C]^T$$
$$D = \alpha_g$$

Obtaining the reduced order model G_1 was the subject of Ref. 6. An anticipated crossover frequency range (for G_1K) was assumed as 1 to 10 rad/s. In that reference, it was also noted that a fourth-order for G_1 was sought based on the observation that the full order model has two oscillatory models in this frequency range.

Attention is now turned to the requirements for G_2. As a realistic example, the Dryden gust spectrum for turbulence is used to describe the disturbance. A fourth-order model for G_2 is sought based on the observation that the full order model has two oscillatory models in the frequency range where the spectrum of D is largest. This frequency range is coincidentally also 1 to 10 rad/s.

The reduced order models for G_1 and G_2 were then obtained simultaneously from the frequency-weighted internally-balanced reduction technique [5] which was specifically developed to meet the criteria in Section 3. The frequency-weighting filter used was a band pass filter of unity magnitude in the 1 to 10 rad/s frequency range with 40 db/dec roll off on either side of this frequency range.

Table 2 contains the reduced order state space matrices A, B, C and D. Figures 5 through 10 show the reduced order and full order frequency response magnitudes for G_1 and G_2. Observe that the reduced order model accuracy approximates the full order model in the 1 to 10 rad/s frequency range as desired. To complete this example, a simple control law, consisting of three constant gains was synthesized using the model G_1. The synthesis objective was to augment the damping of the first aeroelastic mode with acceleration feedback to the canard, to augment the short period damping with pitch-rate feedback to the elevator, and to provide some response decoupling with a cross feed from the elevator to the canard. The resulting control law is of the following form

$$\begin{bmatrix} \delta_C \\ \delta_E \end{bmatrix} = \begin{bmatrix} K_1 & K_2K_3 \\ 0 & K_3 \end{bmatrix} \begin{bmatrix} a_z' \\ q \end{bmatrix} + \begin{bmatrix} 1 & K_2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \delta_{C_{com}} \\ \delta_{E_{com}} \end{bmatrix}$$

Actuation effects were modeled with simple first-order lags, with corner frequencies at 20 r/s for both the canard and the elevator.

Table 2 Reduced Order Model

$$\begin{array}{c|c} A & B \\ \hline C & D \end{array}$$

-.9932	.8294	-.0138	-.0507	-31.67	14.48	13.59
-2.013	-.0137	.0121	.0329	35.92	-21.42	-24.38
-5.593	-.6638	-.3175	-9.658	-593.7	-420.0	700.1
4.934	.2098	3.739	-.5171	-281.4	-175.2	342.5
.0665	-.03471	.0017	.0015	0	0	0
8.762	.7218	.9287	-2.038	52.01	-244.5	333.0

y = q (r/s), az' (ft/s) $\qquad$ u = δ_E (rad), δ_C (rad) $\qquad$ D = α_g (rad)

Shown in Fig. 11 is the plot of Eqn. 3, while Eqn. 4 is shown in Fig. 12. Note that although this control law did not result in high gain (large G, K) at low frequencies, Eqn. 3 was still satisfied below crossover region. Conversely, Eqn. 4 is satisfied, although barely, in the frequency range above crossover. Hence, from the argument in Section 2, if ω^* in Eqns. 5 and 6 is in the crossover region, stability is assured. For reference, the pitch-rate to elevator transfer function is

$$\frac{q(s)}{\delta_{E_C}(s)} = \frac{50\,(0.33)[.13,4.84][.01,10.6][.03,11.0][.21,13.](45.)}{[.53,1.81][.15,4.78][.02,10.8][.03,11.][.19,13.3](19.)(69.)}$$

4. An Additional Criteria

As noted in Section 3, the ΔG arising from the model simplification must satisfy stringent criteria in the crossover region, and if Eqn. 3 and/or 4 (or 5 and 6) is satisfied, closed-loop stability is assured. To be discussed here is the fact that the controller K should not be such that small ΔG is amplified such that $\bar{\sigma}(\Delta GK)$ becomes large. It will be shown by example that this can easily occur where the magnitudes of G (or of the g_{ij}'s) are small. Hence, the example will demonstrate why obtaining a good model in this situation is important (recall that unweighted balanced reduction has a problem here), and some implications regarding control-law synthesis will also arise.

Consider the simple scalar plant

$$g(s) = \frac{(s^2 + .04s + 1^2)}{s(s^2 + .032s + 0.8^2)}$$

The plant is stable and minimum phase, so a robust control law should be obtainable. Using LQG/LTR or H_∞, for example, the following compensator could be obtained.

$$k(s) = \frac{8(s^2 + .032s + 0.8^2)}{(s^2 + .04s + 1^2)(s + 8)}$$

It can be easily verified that the loop shape kg is very good, yielding infinite gain margin, 90 degree phase margin, and good roll off above 8 rad/s.

Now assume that the "true" plant is

$$g_{true} = \frac{0.69\,(s^2 + .048s + 1.2^2)}{s(s^2 + .032s + 0.8^2)}$$

or the numerator "frequency" is in error by 20% (1.0 → 1.2). Note that this could occur, for example, if a vibration mode shape was slightly off in the modeling. Shown in Fig. 13 is the plot of Eqn. 3 for this example, and clearly $\bar{\sigma}(\Delta gk) > \underline{\sigma}(1 + gk)$ at 1 r/s (the designed crossover frequency). Further, a quick check would show the kg_{true} loop to be unstable. But the $|\Delta g| = |g - g_{true}|$ (not shown) would be found to be rather modest at ω = 1r/s, with much larger $|\Delta g|$ at lower frequencies. The problem could be interpreted as one of the control law k amplifying the $|\Delta g|$ at ω = 1 r/s, and this is confirmed from the plot of $|k(j\omega)|$ in Fig. 14.

Stability of the kg_{true} loop would result, and Eqn. 3 satisfied, if the $|k(j\omega)|$ at ω = 1 rad/sec were simply reduced. This is accomplished with the following compensator

$$k_{mod}(s) = \frac{8(s^2 + .032s + 0.8^2)}{(s^2 + 1.2s + 1^2)(s + 8)}$$

or the damping of the complex compensator poles is increased, and the plant model zeroes close to the imaginary axis are not exactly cancelled. Clearly the loop shape with this compensator is not as "optimal" as the original, but this control law is more robust against this Δg.

Noting that the problem arose with a modeling error that is associated with lightly-damped zeroes, the critical Δg was at a frequency (ω = 1 r/s), where $|g(j\omega)|$ was relatively small as shown in Fig. 15. Hence, obtaining a good model at this frequency is important. Furthermore, by attempting to cancel those lightly-damped zeroes in the plant, the original controller was very sensitive to their location. Increasing the damping of the compensator poles, as in a classical notch filter, made the loop more robust against the uncertainty in the location of these plant zeroes. (Incidentally, this can be accomplished with a modified LTR procedure, as noted in Ref. 12 and in another paper in preparation.)

As a final remark, it is observed that lightly-damped zeroes in the compensator are different from similar zeroes in the plant since through the design and implementation of the compensator, the location of its zeroes may be more accurately defined.

5. Conclusions

Quantitative criteria are presented for model (or controller) simplification. The reduced order model (or controller) must well approximate the full-order system in the (multivariable) crossover region for stability, and stability robustness, to be assured. Bounds on the model-simplification error were noted, and if the bounds are satisfied, stability is assured. It was also noted that the model reduction criteria were functions of the control law, and by synthesizing a robust control law, the criteria could be easier to satisfy.

A numerical procedure, consisting of stable factorization with weighted balancing of coordinates has been shown, by example, to meet the above criteria. The example involved reducing an eleventh order linear model of an elastic aircraft to obtain a fourth-order model leading to the desired six transfer functions.

Finally, another example demonstrated the importance of obtaining good agreement between the full- and reduced-order model in the crossover region, even where the transfer function (or functions) have relatively small magnitude. Furthermore, the example demonstrated that an apparently robust controller could in fact amplify small errors, and lead to unstable results. The problem would occur with any control law that had the effect of cancelling lightly-damped transmission zeroes of the plant model.

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7. References

1. Liu, Y. and Anderson, B.D.O., "Controller Reduction Via Stable Factorization and Balancing," Int. Jrn. of Auto. Cont., Vol. 44, No. 2, 1986.

2. Anderson, B.D.O., "Controller Reduction," Plenary Session, 1987 ACC, Minneapolis.

3. Waszak, M. R. and Schmidt, D. K., "Flight Dynamics of Aeroelastic Vehicles," Journal of Aircraft, Vol. 25, No. 6, June, 1988.

4. Bacon, B. J. and Schmidt, D. K., " A Fundamental Approach to Equivalent Systems Analysis," Journal of Guidance, Control, and Dynamics, Vol. 11, No. 6, Nov. - Dec., 1988.

5. Bacon, B. J. and Schmidt, D. K., "Multivariable Frequency-Weighted Order Reduction," Journal of Guidance, Control, and Dynamics, Vol 12, No. 1, Jan. - Feb., 1989.

6. Schmidt, D. K. and Newman, B., "Modeling, Model Simplification, and Stability Robustness With Aeroelastic Vehicles," 1988 AIAA Guidance And Control Conference, Minneapolis.

7. Stein, G., Lecture Notes on Control System Design, MIT, 1979.

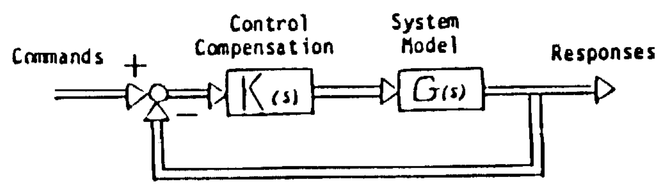
8. Kwakernaak, H. and Sivan, R., Linear Optimal Control Systems, New York: Wiley-Interscience, 1972.

9. Doyle, J. C. and Stein, G., "Multivariable Feedback Design: Concepts for a Classical/Modern Synthesis," IEEE Transactions Automatic Control, Vol. AC-26, Feb., 1981.

10. Moore, B.C., "Principal Component Analysis in Linear Systems: Controllability, Observability, and Model Reduction," IEEE Transactions On Automatic Control, Vol. AC-26, February 1981.

11. Enns, D.F., "Model Reduction for Control System Design," Ph.D. dissertation, Department of Aeronautics and Astronautics, Stanford University, June, 1984.

12. Schmidt, D.K., "Research on Control System Design to Meet Aircraft Handling Qualities Requirements," Final Report for MCAIR, July, 1989.



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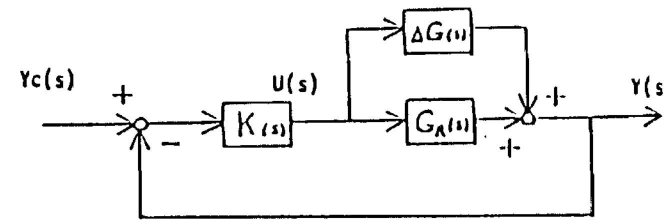

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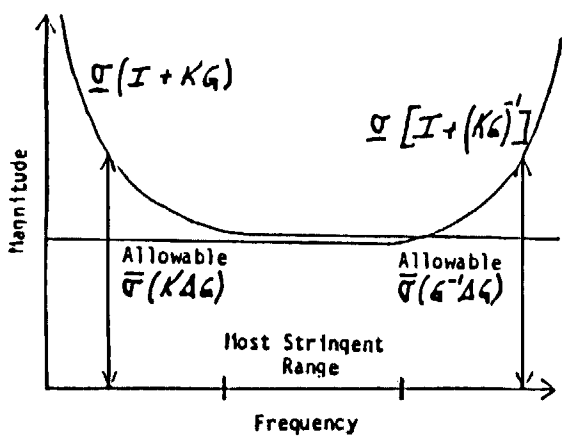

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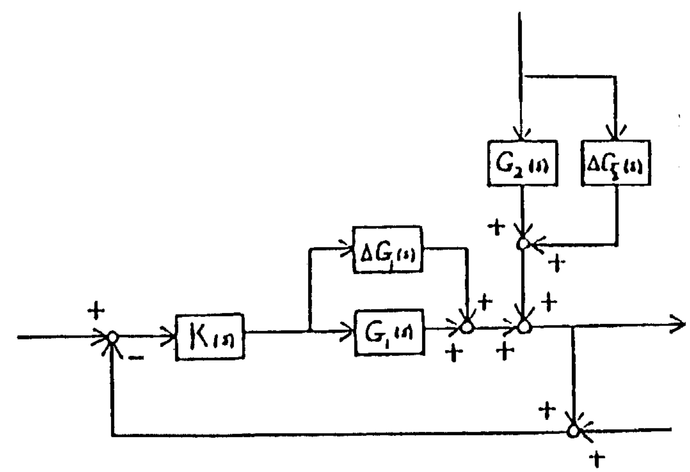

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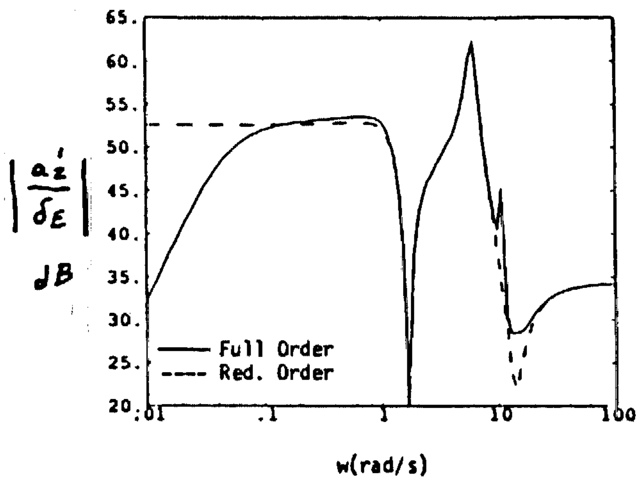

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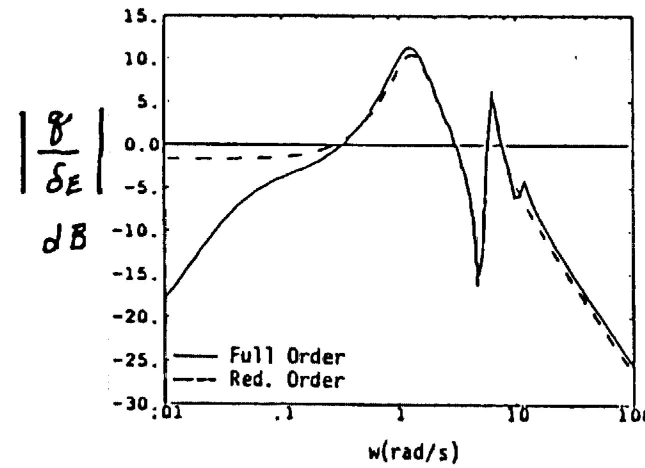

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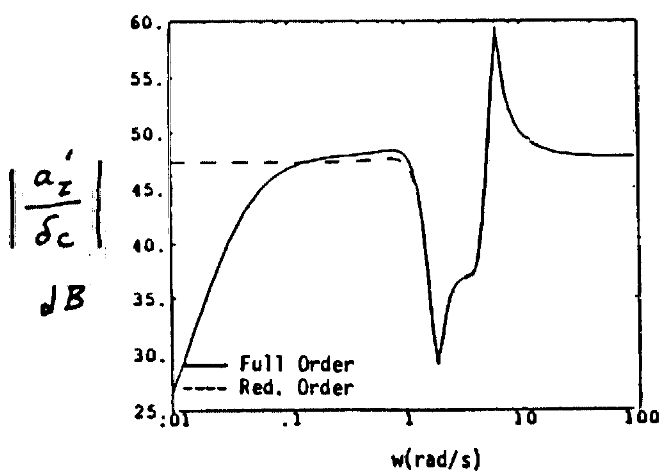

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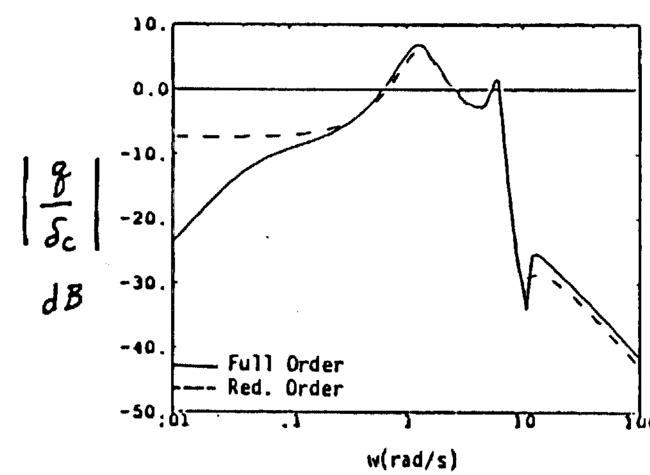

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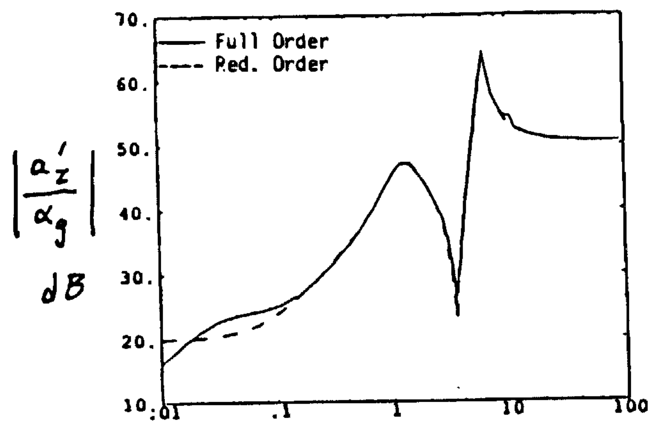

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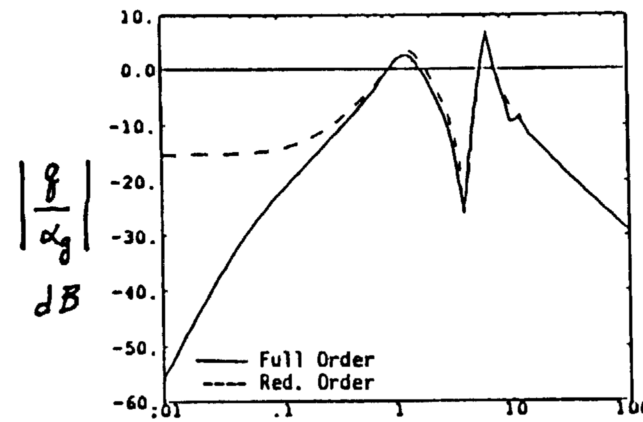

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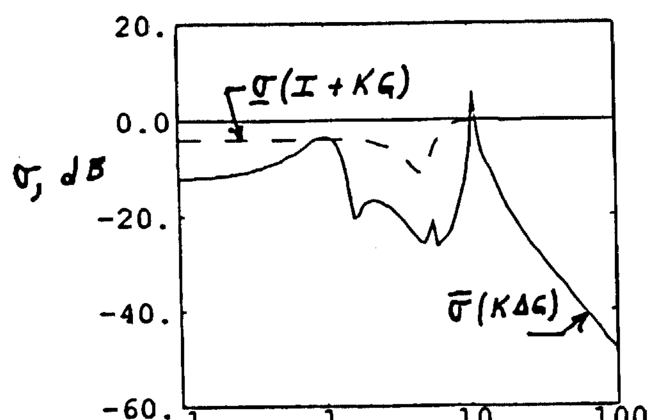

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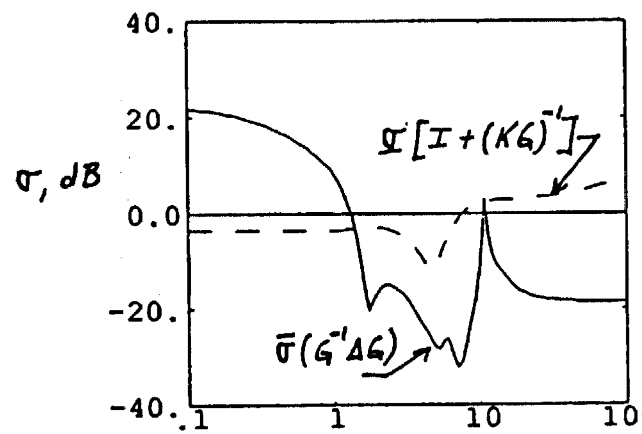

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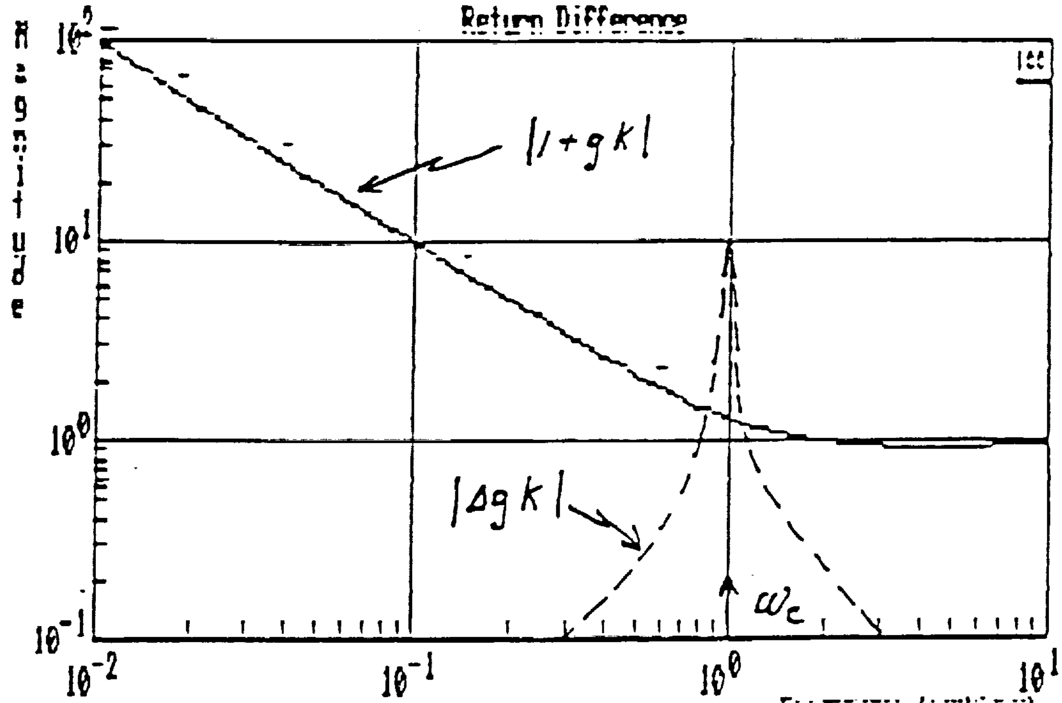

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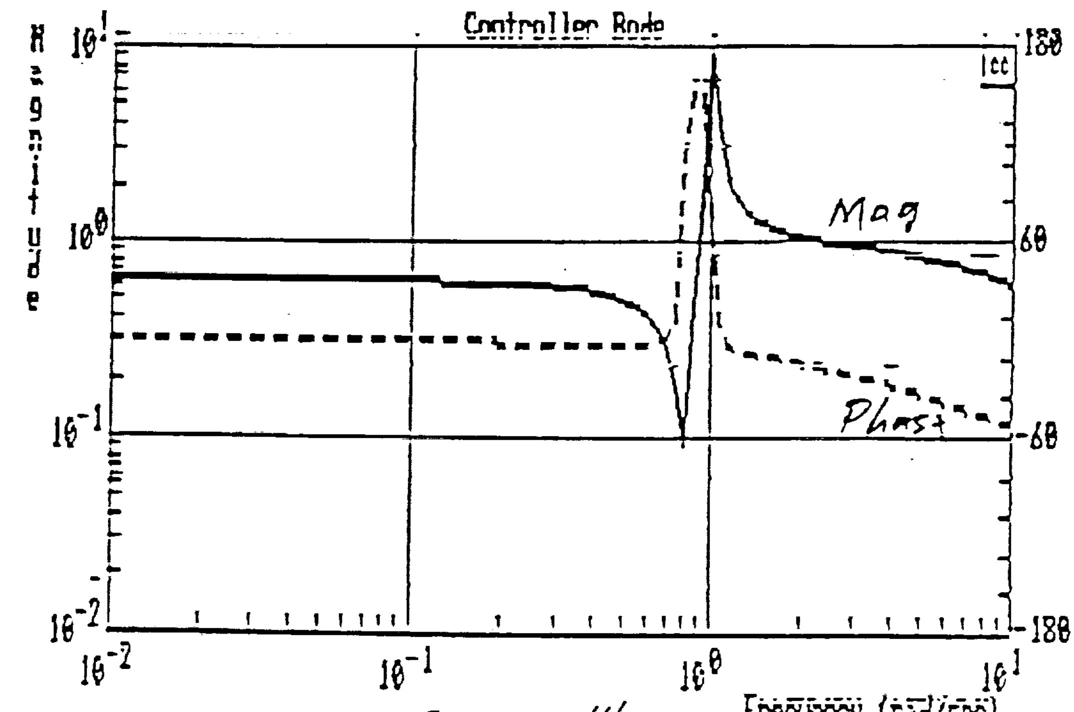

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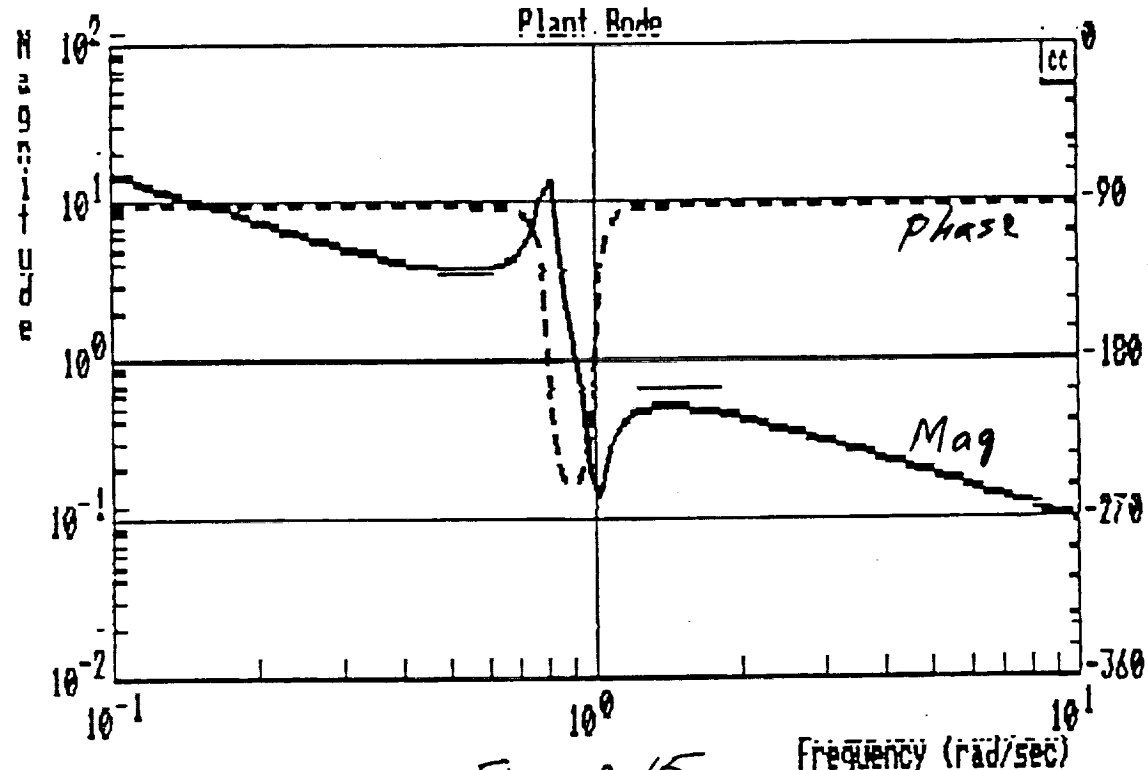

Figure 15.