Experimental Determination of Material Damping Using

Vibration Analyzer

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ABSTRACT

Structural damping is an important dynamic characteristic of engineering materials that helps to damp vibrations by reducing their amplitudes. In this investigation, an experimental method is illustrated to determine the damping characteristics of engineering materials using a dual channel FFT (Fast Fourier Transform) analyzer. A portable Compaq III computer which houses the analyzer, is used to collect the dynamic responses of three metal rods. Time - domain information is analyzed to obtain the logarithmic decrement of their damping. The damping coefficients are then compared to determine the variation of damping from material to material. The variations of damping from one point to another of the same material, due to a fixed point excitation, and the variable damping at a fixed point due to excitation at different points, are also demonstrated.

INTRODUCTION

A body once set to vibrate freely, will not do so indefinitely. The amplitude of the oscillation gradually decreases to zero as a result of friction. The body is said to be damped. An undamped material, once excited, will oscillate indefinitely with a constant amplitude. Any physical system, however, possesses some degree of damping forces which cause energy to dissipate during a cycle of vibration. The rate and the amount of this dissipated energy depend upon the physical and geometric properties of the material.

Damping of a vibrating material may be of two types: external and internal damping. External damping refers to separate energy absorber units which are added to the system for reducing resonant vibration. The internal damping which is an inherent material property causes heat build-up in a material due to the absorption of energy during a cycle of vibration. In this investigation only internal damping of materials is considered.

Damping forces can be used in an analytical model to determine structural dynamic response (Reference 1). A viscous type of damping which assumes that the forces are proportional to the magnitude of the velocity and opposite to the direction of motion is generally used in a mathematical model to consider its effect on the structural response. The inclusion of such an effect in a model, however, does not make significant variation in the quantitative magnitude of the dynamic properties than that of an undamped case. For instance,

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the damped natural frequency only slightly differs from the undamped frequency of most real structures. This is partly because of the low range of damping co-efficients(less than 20%) present in a real structure.

The importance of damping characteristics, however, is more significant in studying or selecting new engineering materials. Current trends of producing sophisticated, high - performance material to replace traditional ones need more attention towards the dynamic performance specifications. An evaluation of the dynamic damping properties can be a critical factor in the material selection process. A material which damps off more quickly would obviously be a better choice than one which oscillates longer once it has started vibrating after the excitation force is removed. The large amplitude, at resonance, which decays faster can be identified as a good dynamic performance material.

As stated before a dual channel FFT (Fast Fourier Transform) analyzer is used in this investigation to determine the internal damping of different materials. The damping ratios of three rods, a copper, a steel, and an aluminium rod, are measured. The logarithmic decrement procedure of calculating damping is used as is found in literature (Reference 1), and is obtained from the time-domain information.

EXPERIMENTAL METHOD

Material and Test Set up:

This experiment consists of a testing rod, an input trigger source to excite the rod, an output device to measure the response signal, and an analyzer to collect and analyze the time - domain waveforms. A schematic diagram of the instrumental set-up is shown in Figure 1.

Three rods of identical length and diameter are selected in this experiment for studying and comparing the damping of each. The length and diameter are kept constant in order to see the effects of other material properties, such as mass and stiffness, upon the damping of the rods. The physical parameters of the rods are presented in Table I. SI unit conversion of each of the parameters in the table is listed inside a parenthesis. The moduli of elasticity of the rods are assumed to be equal to their respective typical values as obtained from the literature (Reference 2).

The rods are tested under identical support conditions as shown in Figure 1. Sponge foams are used to support the rods. This is done in order to avoid the bouncing effect which occurs due to use of rubber bands. Eleven points are marked off on each of the rods thus dividing them into 10 segments of approximately 2 inches each. Each of the points are numbered as shown. The numbering is required for identifying response or excitation location in the rods.

The trigger, a hammer, is connected to one channel of a dual channel analyzer and the other channel is connected to the output device, an accelerometer. The modally tuned hammer triggers the analyzer's mode of operation. This means that as soon as the hammer strikes the rod, imparting an impulsive signal, the analyzer starts collecting samples from the input and output sources. Here the impulsive signal refers to a single strike by the hammer instead of multiple strikes. Care is taken to make sure that this is the case because multiple strikes will generate multiple impulses which will interfere with the desired results. The free vibration generated by the rod after a single strike of the hammer is measured through the output device.

The signal generated by the impulse hammer and the response accelerometer are analogs in form and are digitized by the ADC (Analog to Digital Convertor), then displayed on the computer screen. A typical time -domain signal output response of the aluminium rod is shown in Figure 2. This signal shows the fluctuating movement of the rod as it decays with time. This decay of the amplitude of motion describes the damping of the system.

For a linear system of vibration the ratio of the amplitude for any given cycle of vibration to the amplitude of another cycle is a constant. This constant is called the logarithmic decrement δ, which is defined as proportional to the natural logarithm of the ratio of two amplitudes which are apart at a multiple of time period as in Equation 1. Here Z_0 is the height of the peak of one of the periods of motion, and Z_n is the height of the peak after n cycles of vibration.

$$\delta = -\frac{1}{2\pi n} \ln\left(\frac{Z_n}{Z_0}\right) \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots (1)$$

The determination of damping, therefore, requires the measurement of two peak amplitudes, Z_0 and Z_n and the number of periods in between them, as shown in Figure 2a. The illustrated method is based upon certain assumptions and has some limitations in measuring the damping of a material. These assumptions and limitations are listed below.

Assumptions and Limitations:

(1) The relationship used to determine the decay rate of a motion is adequate for a system having damping ratio less than 20% of the critical damping. Critical damping is that type of system damping which once excited will result in a non-oscillatory motion, the magnitude of which decays exponentially with time to zero,

(2) Peak amplitude of a period is measured from an acceleration response instead of a displacement waveform. This can be done because it is assumed that the decay rate of the acceleration response is identical to that of displacement response, since the displacement amplitude is a constant multiple of the acceleration amplitude. Actually the double integration of the acceleration signal results in a displacement signal with identical pattern but different amplitudes,

(3) Inspection of the periodic variation of time waveforms are done by naked eye, and

(4) The sample rate of the analyzer was adjusted according to the nature of the resulting waveform from the output signal. A lower sample rate filters out the high frequency response content resulting in a signal pattern in which the periods are visually identifiable. Sample rates are considered in such a way that the definite pattern of periods are recognized, which may restrict the collections of some unrecognized frequency contributions.

RESULTS AND DISCUSSIONS

The time-domain responses of the rods are analyzed to compute the damping ratios of the testing materials. The damping ratios for the aluminium, steel, and copper rods are compared in Table II. The data are taken under identical support and excitation conditions. The accelerometer is kept fixed at the mid point of the rod, marked #6 on Figure 1. The

responses at this point are collected by successively exciting each of the eleven points on the rods. A typical signal for each of the materials is shown in Figures 2 to 4. To measure the accurate peak values and periods of the signal a portion of the waveform is expanded, as shown in Figure 2a.

The data obtained from this experiment are statistically analyzed to determine the differences among the groups. The analysis of variance (ANOVA) for the damping data of Table II shows that the damping ratios for each of the materials are significantly different. The probability that the mean values of these materials are equal is 0.003 (0.3%). As expected, the test result also shows that the mean value of the damping ratios for copper is greater than steel which has a mean value greater than that of aluminium. This means that if these rods were to be displaced equally under identical conditions and made to vibrate freely then the copper rod would stop vibrating fastest, the steel second fastest, and the aluminium rod third. Lower damping ratio means longer decay time. The deviations of the damping ratios from the respective means are also computed in Table II. This coefficient of variation is calculated as the standard deviation divided by the mean and multiplied by 100 (Reference 3). The high degree of dispersion of damping ratios from point to point for each case supports the conclusion that the decay rate varies from point to point of the rods.

Statistical analysis is performed to find out how much of the variation in the damping is due to differences between a fixed point trigger and a fixed point response. The damping ratios for these two cases are determined under the following conditions: (i) the triggering is at a fixed position (#6) while response is measured at different points, and (ii) the accelerometer collects the response at a fixed point (#6) while excitation is made at various positions. The data for this comparison are tabulated in Table III. Analysis of variance of this data is made by excluding the damping ratios of the points where the rod rests on the supports. This exclusion is done because of the greater variation of results at these two points, which may be due to the external influence of the sponge foam. It is found from the analysis that the damping ratios obtained from the two cases (i) and (ii) are not significantly different. The level of significance for this analysis is 0.2412 (24.12%). The coefficient of variation among the points of a single rod, however, is greater for fixed point triggering than for fixed point response. From the data it can be concluded that the response at one point due to excitation at another point is the same when the positions of the input and output devices are reversed. Moreover, it can also be concluded that the reciprocity between the response and trigger sources can be used to measure the damping of a linear system.

To understand the causes of variations of damping ratios among the different points of a single material, further analysis is performed. The sample rate which dictates the number of samples to be collected by the analyzer controls the frequency content of the output signal. For a lower sampling rate value, only the low frequency oscillation patterns can be measured while the contributions of the higher frequencies are truncated. Figure 5 shows a low frequency oscillation of the steel rod when the samples are collected at a rate of 250 per seconds. For a sample rate of 1200 per sec. the response collected for the same rod, as shown in Figure 3, differs significantly from the response with lower rate. This is because the higher frequency responses are also collected by the analyzer, as a result of which an unclear or fuzzy graphical reading is observed . In other words, in the second case, the total response includes the contributions of many modes of vibration. Modes are uncoupled dynamic parameters which describe the vibration of a physical structure (Reference 4). The variation of damping ratios when sample rates are changed is shown in Table IV. The clearer the response, the easier it is to identify periods. This in turn increases the accuracy of computing the damping ratio. The clarity of the response depends not only on the frequency content of the signal, but also, as shown in the Figures 2 and 3, on the material. For example, with a

low rate of sampling the steel rod gives a clear waveform pattern while, on the other hand, the aluminium rod shows a good waveform with a higher rate.

The modes of a structure are defined as the definite regular waveforms corresponding to each of its resonant frequencies. Modal analysis can isolate these resonant frequencies and the associated deflection pattern of Mode Shapes (Reference 4). The damping and frequencies for each of the modes of vibration of each of the rods are tabulated in Table V. The results of the modal analysis as presented in this table show that the damping ratios vary with the modes. Also it is seen that higher frequencies have lower damping ratio and therefore take longer to damp off. Again in Table IV it is found that for a sampling ratio of 1500 which includes higher frequencies determines smaller damping ratio than when sampling ratio equals 250. This variation can be used to explain why the damping among different points of a single material is not a constant. At a particular point certain modes will affect the response while others may have no affect at all. This latter case depends on the location of the nodes. For example, when the response is taken from the midspan, the second mode has no influence since at this point a node exists. A relationship can be found between the material damping and the weighted contributions of the modal damping in order to determine real life material damping.

CONCLUSIONS

The logarithmic decrement procedure measures the damping ratios in a reasonably accurate way. The results are consistent to the expected physical behavior of the materials. The damping ratios for copper, steel, and aluminium are in a decreasing order. The significant variations of damping ratios among the materials as found in this experiment may be due to the differences in their physical properties, while the variation among the points of an individual material is due to the type of signal output of the system which eventually depends upon the modal behavior of the system. Statistical confidence interval of the damping ratios can be determined from the experimental results which then can be estimated as a damping ratio of the material. A weighted average of the modal damping can be used to determine the damping of a system. The damping ratio is a variable entity which will depend upon material type, support conditions, and the frequency content of the output signal.

REFERENCES

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3.   Schlotzhauer, Sandra and Littell, Ramon: SAS System for Elementary Statistical Analysis, SAS Institute, Inc., Cary, NC, 1987.

4.   Practical Aspects of Modal Analysis. Reference Materials, Structural Measurement Systems, Inc., San Jose, CA 1988.

TABLE I. - PHYSICAL CHARACTERISTICS OF RODS

Material Type	Length inches (cm)	Area of Cross-Section Sq. in, (cm^2)	Weight Lbs. (N)	Modulus of Elasticity lbs./ in ^{2} (Mpa)
ALUMINIUM	25.47 (64.7)	0.20258 (1.307)	0.51609 (2.29)	10×10^6 (0.6895×10^5)
STEEL	25.35 (64.4)	0.1995 (1.2868)	1.416 (6.299)	30×10^6 (2.0685×10^5)
COPPER	25.43 (64.6)	0.2186 (1.4103)	1.753 (7.797)	17×10^6 (1.172×10^5)

TABLE II. - COMPARISON OF DAMPING RATIO FOR DIFFERENT MATERIALS: FIXED POINT RESPONSE DUE TO EXCITATION AT VARIABLE POSITIONS.

Position	Damping Ratio		
(Triggering point, Accelerometer point)	Aluminium Rod (Sampling Rate = 1500)	Steel Rod (Sampling Rate = 1200)	Copper Rod (Sampling Rate = 1500)
(1,6)	0.00400	0.00138	0.00900
(2,6)	0.00560	0.01140	0.00400
(3,6)	0.00354	0.00051	0.00268
(4,6)	0.00410	0.00820	0.01140
(5,6)	0.00306	0.01080	0.02200
(6,6)	0.00400	0.00052	0.01134
(7,6)	0.00491	0.01100	0.01625
(8,6)	0.00550	0.00910	0.01209
(9,6)	0.00250	0.00080	0.01409
(10,6)	0.00180	0.01060	0.00380
(11,6)	0.00390	0.00110	0.00868
Mean	0.003901	0.005946	0.0105
Coefficient of Variation (%)	28.65	79.42	52.72

TABLE III. - COMPARISON OF DAMPING RATIOS FOR ALUMINIUM ROD: FIXED POINT TRIGGERING VS. FIXED POINT RESPONSE (Sampling Rate equals 1500)

Fixed Point Trigger			Fixed Point Response		
Position (Trgr, Accl)	Damping Ratio (a)		Position (Trgr, Accl)	Damping Ratio (b)	Ratio (a/b)
(6,1)	0.00400		(1,6)	0.00400	1.00
(6,2)**	0.01700		(2,6)	0.00560	3.03
(6,3)	0.00490		(3,6)	0.00354	1.38
(6,4)	0.00218		(4,6)	0.00410	0.53
(6,5)	0.00508		(5,6)	0.00306	1.66
(6,6)	0.00400		(6,6)	0.00400	1.00
(6,7)	0.00499		(7,6)	0.00491	1.01
(6,8)	0.00670		(8,6)	0.00550	1.22
(6,9)	0.00420		(9,6)	0.00250	1.68
(6,10)**	0.01398		(10,6)	0.00180	7.76
(6,11)	0.00496		(11,6)	0.00390	1.27
Mean	0.0065445			0.003901	
Coefficient of Variation(%)	67.06			28.65	28.65*

**Rods are supprted at these points.
* Excluding support ratios.

TABLE IV. - COMPARISON OF DAMPING RATIO FOR STEEL AND ALUMINIUM ROD WITH DIFFERENT SAMPLING RATES (SR): FIXED POINT RESPONSE

Position (Trgr., Accl.)	Aluminium Rod		Steel Rod	
	Damping Ratio when SR = 250 (Samples/sec)	Damping Ratio when SR = 1500 (Samples/sec)	Damping Ratio when SR= 250 (Samples/sec)	Damping Ratio when SR = 1200 (Samples/sec)
(1,6)	0.00552	.004000	0.10280	0.00138
(2,6)	0.01194	0.00560	0.07600	0.01140
(3,6)	0.00231	0.00354	0.0930	0.00051
(4,6)	0.00766	0.00410	0.09600	0.00820
(5,6)	0.00088	0.00306	0.08100	0.01080
(6,6)	0.00192	0.00400	0.08030	0.00052
(7,6)	0.00597	0.00491	0.09300	0.01100
(8,6)	0.00640	0.00550	0.09030	0.00910
(9,6)	0.00825	0.00250	0.09500	0.00080
(10,6)	0.02378	0.00180	0.07900	0.01060
(11,6)	0.00416	0.00390	0.09800	0.00110
Mean	0.00716	0.00390	0.08949	0.00595
Coefficient of Variation(%)	84.65	28.65	9.5	79.42

TABLE V. - COMPARISON OF MODAL DAMPING RATIOS AND FREQUENCIES FOR DIFFERENT MATERIALS

| Mode of Vibration	Types of Rods					
	Aluminium		Steel		Copper	
	Frequency (Hz)	Damping Ratio	Frequency (Hz)	Damping Ratio	Frequency (Hz)	Damping Ratio
1	135.15	0.003990	115.57	.224100	106.39	0.000869
2	371.88	0.000466	385.52	0.002636	292.50	0.000472
3	729.25	0.000093	751.49	0.000207	571.89	0.0001327
4	1201.00	0.000309	1234.00	0.000055	1399.00	0.0000794
5	1780.00	0.000477	1842.00	0.000050	1939.00	0.0003735
Mean		0.001067		— —		0.00039
Coefficient of Varition		137.0%				73.3%

__ __ Not Computed

60
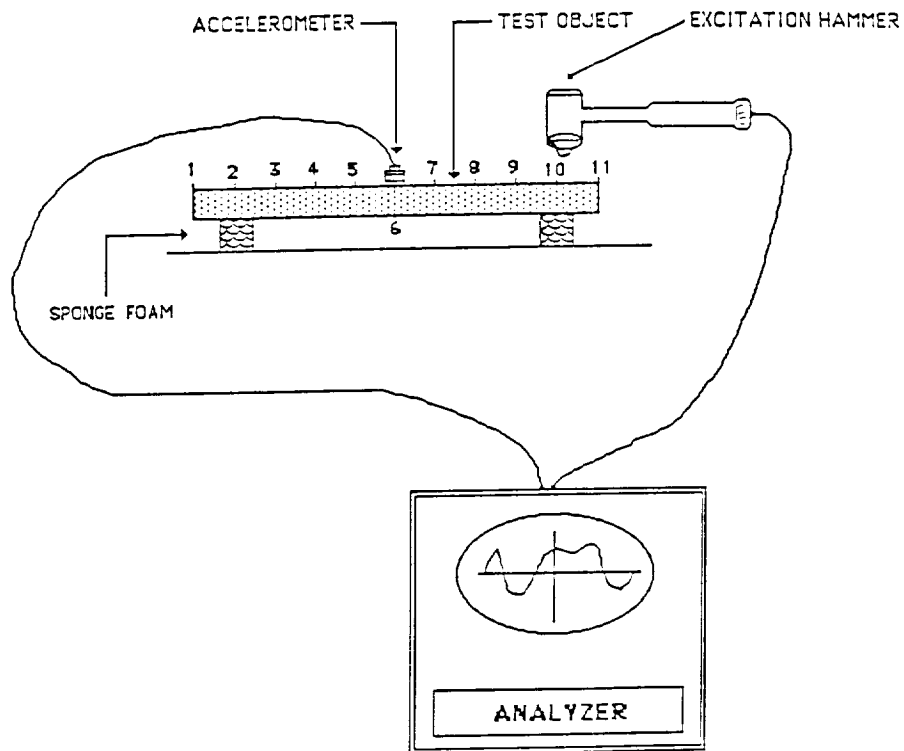
Figure 1: Schematic Diagram of the Test Set-up.
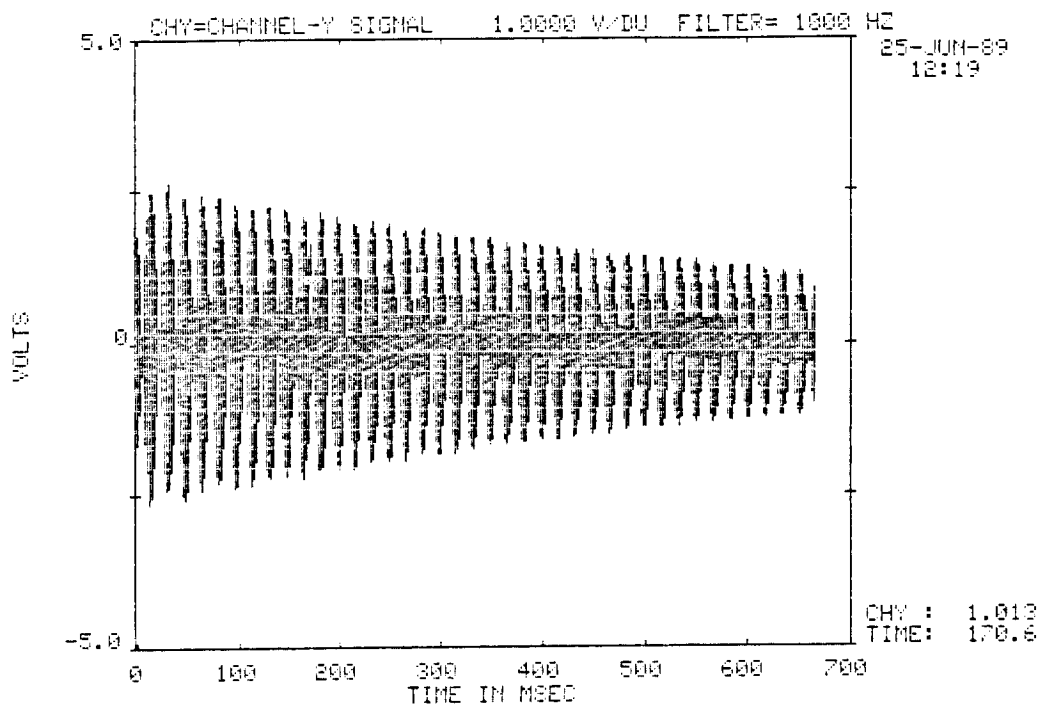
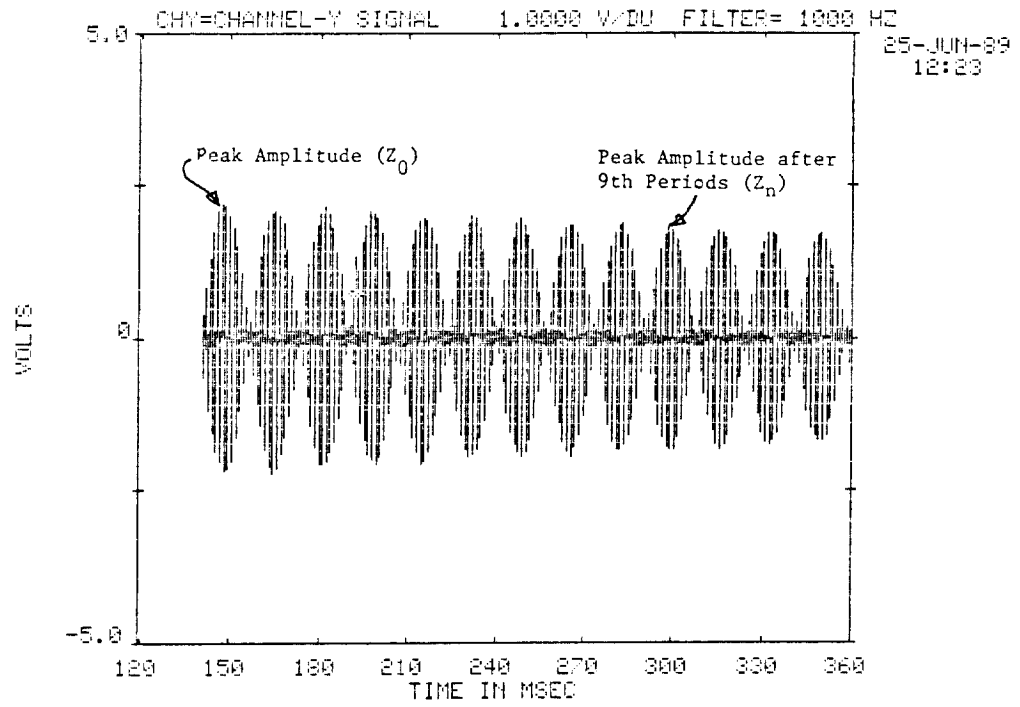Figure 2: Free Vibration Response for Aluminium Rod (Accelerometer at Point #6 and Trigger at Point #3).
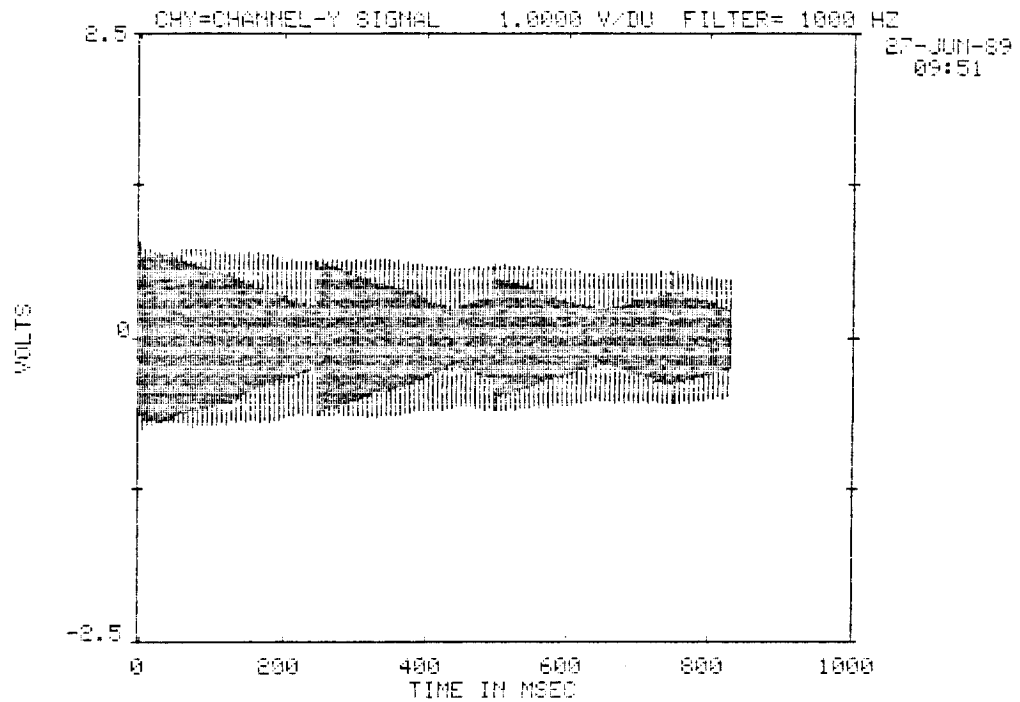
5.0

Peak Amplitude (Z_0)

Peak Amplitude after
9th Periods (Z_n)

VOLTS

0

-5.0

120   150   180   210   240   270   300   330   360

TIME IN MSEC

Figure 2a: Expansion of Figure 2.

2.5

VOLTS

0

-2.5

0   200   400   600   800   1000

TIME IN MSEC

Figure 3: Free Vibration Response for Steel Rod (Accelerometer at Point #6
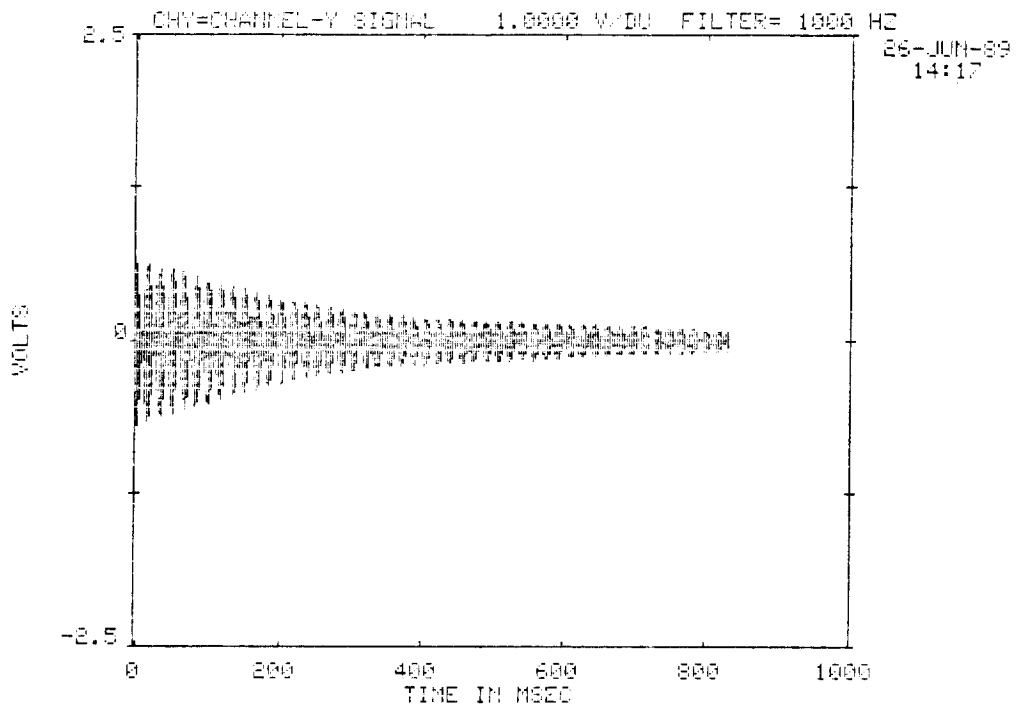and Trigger at Point #3).

Figure 4: Free Vibration Response for Copper Rod (Accelerometer at Point #6 and Trigger at Point #3).
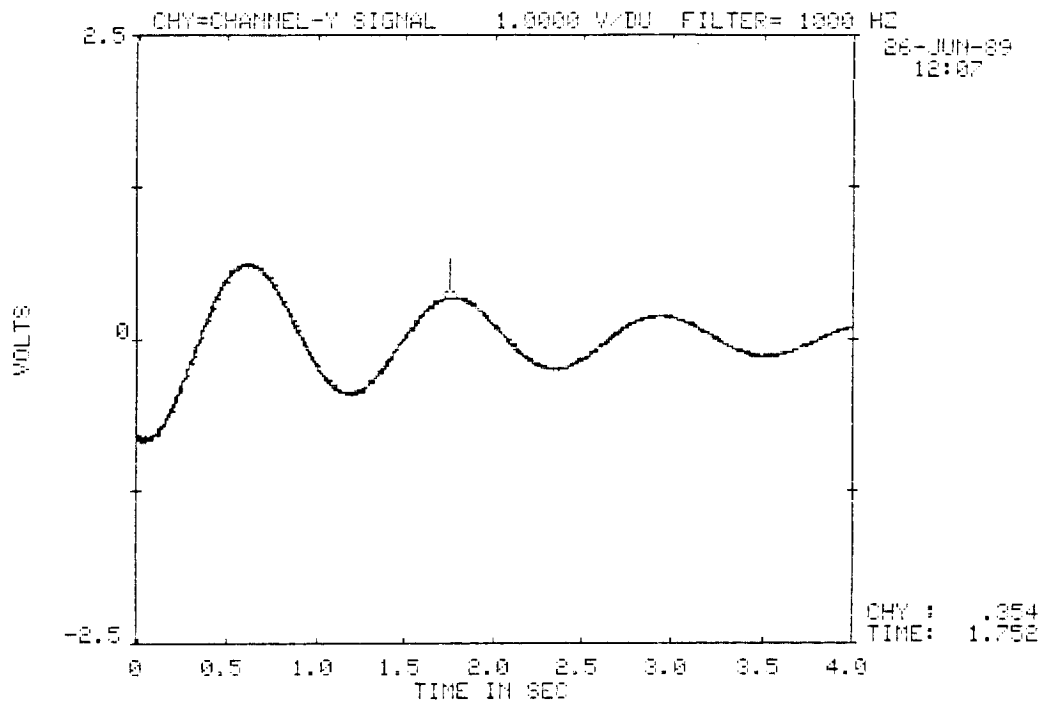

Figure 5: Damped Vibration of Steel Rod when Sample rate equals 250 (Accelerometer at Point #6 and Trigger at Point #3).