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Experiments and Other Methods for Developing Expertise  
with Design of Experiments in a Classroom Setting

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INTRODUCTION

The only way to gain genuine expertise in "SPC" (Statistical Process Control) and "DOX" (the design of experiments) is with repeated practice, but not on canned problems with "dead" data sets. Rather, one must negotiate a wide variety of problems each with its own peculiarities and its own constantly changing data. The problems should not be of the type for which there is a single, well-defined answer that can be looked up in a fraternity file or in some text. The problems should match as closely as possible the open-ended types for which there is always an abundance of uncertainty. These are the only kinds that arise in real research, whether that be basic research in academe or engineering research in industry.

To gain this kind of experience, either as a professional consultant or as an industrial employee, takes years. Vast amounts of money, not to mention careers, must be put at risk. The purpose here is to outline some realistic simulation-type lab exercises that are so simple and inexpensive to run that the students can repeat them as often as desired at virtually no cost. Simulations also allow the instructor to design problems whose outcomes are as noisy as desired but still predictable within limits. Also the instructor and the students can learn a great deal more from the postmortem conducted after the exercise is completed. One never knows for sure what the "true data" should have been when dealing only with "real life" experiments. To add a bit more realism to the exercises it is sometimes desirable to make the students pay for each experimental result from a make-believe budget allocation for the problem.

Of course, the students find all this open-endedness and uncertainty very unsettling, but this is what most characterizes the kinds of investigations they will encounter later and it is important that these features be part of the students' lab experience. Better that the students' first encounters with these kinds of anxieties come in a college classroom under the tutelage of an experienced educator, than on their first job under the direction of a stressed out supervisor.

MECHANICAL SIMULATION

To be most effective, a lab exercise should be mechanical in nature and completely open to visual inspection while it is working. The reason is simple: All our experience is at the macroscopic level where mechanical phenomena dominate and it is largely by "seeing" that we have gained our experience. Hence, we take

data points more seriously when we can plainly see them being generated in a simple, easily understood fashion. Only later should one go to more exotic data production schemes, such as analog circuits or computers whose inner workings are usually much further removed from the world we experience.

One of the most effective teaching devices for introducing students to statistical methods is the so-called Quincunx, originally introduced by Sir Francis Galton back in the 1800's (see Figure 1). These simple machines allow one to generate endless sequences of samples, each drawn from a well defined population of known mean and standard deviation. Even easier to understand are the sampling boxes and sampling bowl schemes that are also in wide use today (see Figures 2-4). None of these are difficult to construct, but all are commercially available. For example, the Lightning Calculator Company of Troy, Michigan ([313]-649-4462) sells ready-to-use versions of the Quincunx, chip boxes, sampling boxes, and sampling bowls. They come completely assembled and are shipped with brief instructional brochures, an audio cassette, and ready-to-copy data forms for use in the lab. Statco Products, also of Troy, Michigan ([313]-879-7224) offers a similar array of such items.

Xbar-R charts for in-control processes are easily constructed using these devices and the students benefit from seeing the histograms take shape as the beads are dropped. Attributes charts (% defective, etc.) are best illustrated by sampling from a mixture of differently colored beads all housed in the same bowl. Specially constructed paddles make it easy to draw samples of various sizes. Sampling bowls can be used to construct and study p charts, u charts and c charts.

To control the attribute's mean frequency (% defective etc.), one simply adjusts the fraction of beads having a particular color in the sampling bowl. To examine the effects of different sample sizes one simply uses a different sampling paddle. With the quincunx, one controls the location of the bead drop by sliding it left or right. Different pegboards can be inserted to change the standard deviation of the Quincunx and the user decides on the number of beads dropped per sample. Again, the ease of operation of sampling bowls and the Quincunx makes it possible for the students to conduct as many runs as they care to and the data generation procedure is not nearly the time consuming distraction it is when actual experiments are used. Sometimes students dream up personal mini-research investigations to check various aspects of the recommended SPC and DOX procedures. Consider, for example, a scatter diagram study of the relation between the ranges and standard deviation of samples drawn from a Quincunx. Will the relation depend on the number of items per sample? Could this kind of information be used to convert R bar values into control limits on an Xbar-R chart? The possibilities are almost endless.

Also available from the Lightning Calculator people is a process simulation training kit called "prosim" (See Figure 5). It explains how the Quincunx can be used to demonstrate such statistical design strategies as One Way ANOVA,  $2^2$  and  $2^3$  factorial designs, two factor experiments with interactions, and Taguchi's  $L_8$  and  $L_9$  Orthogonal Arrays. After a little thought, however, it is easy to see how these instructions could be modified so as to simulate data for almost any kind of statistical design strategy in use today, including the so-called response surface methods.

## COMPUTER SIMULATION

Once you see how the mechanical simulators work, it is a fairly simple matter to write programs that do the same thing either on a PC or on a hand held programmable calculator. Two software packages, a statistics simulator and an SPC simulator, were described in pp 84-5 of the July 1988 Quality Progress. In essence, they are computer versions of the Quincunx and run on IBM/PC/XT/AT compatibles having 128K of memory and one or more disk drives.

In my view, computer simulation methods are most useful for teaching and learning the strategies for designing multi-factor experiments and then interpreting the results. Many people insist that only real life experiments carried out with actual laboratory equipment should be used when teaching the design of experiments; however, I disagree. Restricting oneself only to real life experiments of the simplest sort, there is no definitive way to critically check the inferences one is led to with the DOX strategies he or she is employing. And if one tries to solve this problem by going to a very simple experiment--such as studying the period of a simple pendulum as a function of the mass, length and starting angle of the pendulum--the students simply go to the closest physics text to see what dependences are predicted from theory. This eliminates the most important aspect of the exercise, namely the haunting feeling one has about investigating some response variable without knowing whether--much less how--it depends on the controls being studied. Knowing these things beforehand completely changes the mindset of the investigator and can severely undermine his or her ability to proceed objectively.

By using tailor-made simulation programs, the instructor can generate noisy data from a well understood law such as the gas law,  $V = nRT/P$ . But the law should then be disguised by replacing  $V$ ,  $n$ ,  $T$  and  $P$  with  $Y$ ,  $u$ ,  $v$  and  $w$  respectively. Also, by adding a superfluous control variable or two, say  $x$  and  $z$ , (that do not appear in the gas law formula), the instructor knows to look for these as being totally insignificant factors in the experiment. If they do register significant effects, either the student or the DOX software must be doing something wrong. Afterwards, the students can be told that  $Y$  was really the volume of a perfect gas,  $u$  was the number of moles,  $v$  the absolute temperature,  $w$  the pressure and that  $x$  and  $z$  were dummy variables all along. When all this is revealed afterwards, the students find "the scales falling from their eyes", so to speak, and can then review the decisions they made in a totally different light, namely that of informed hindsight.

Here in undisguised form is a DOX problem based on the gas law. Shown in Figure 7 is the computer code I used to simulate noisy  $V$  data on a Casio fx-8000G (or fx-7000 G) hand held programmable calculator.

### HANDS-ON, IN CLASS EXERCISE USING SIMULATED DATA

Design, execute and interpret an experiment to study the alleged dependence of gas volume ( $V:10\ 350$ )\* on mole number ( $n:0.5\ 2.0$ ), temperature ( $T:300\ 1000$ ),

\*The values over which the variables are to be ranged are bracketed by the numerical values following the colon. The same is true for the control variables  $n$ ,  $T$  and  $P$ .

pressure (P:0.5 2.0), and xylene content (x:0.01.0). Assume we require a resolution of 1.5 units for the response variable V and that its estimated standard deviation is about 0.7.

- Steps:
- 1) Execute the resolution preliminaries.
  - 2) Design an experiment using a linear, factorial, quadratic or cubic design for all four of the control variables listed above (n,T,P,x). Use five replications.
  - 3) Conduct experiments at the control settings given your experimental design and analyze the data.
  - 4) Produce an effects table and analyze it to see if any of the four control factors has little or no effect.
  - 5) Produce contour maps of P vs T for n = 0.5 and 1.25 and shade in the P-T domain(s) for which V is less than 50 but greater than 30.
  - 6) Now use the ideal gas law to sketch in the "true" contours for V = 50 (or any other choice that meets your fancy) and compare these to the same contours on the response surface map.

In the MSE 341 course at Iowa State, we use a software package called EChip to teach expertise in experimental design. This will be described further below. The following problem is taken directly from the EChip text and the software contains a simulator program that can generate response data for whatever control variable settings the user wishes to specify. I include here the entire problem description but not the simulator program nor the extended text discussion of the problem. However, I think the reader will appreciate the realistic nature of the problem, which is based on an industrial problem that was successfully solved by an EChip user.

#### ECHIP PROBLEM 5: EXTRUSION OF A NEW THERMOPLASTIC

This is a wrap-up problem that ties together all you have learned so far. It should take about four hours to complete and everyone should be able to obtain a defensible solution using the methods discussed thus far.

#### Background

You work in R and D in the plastics division and have been asked to fully define the process for making a new plastic in one of the extruders on the compounding line. An extruder is like a giant meat grinder with a massive internal screw (auger) in the extended barrel. Raw materials are fed into the receiving funnel at one end; these are melted and mixed by a combination of heat added by elements on the barrel wall and by the shearing action of the auger. Everything eventually gets pumped out the extruder die (hole) at the exit end of the barrel.

## The Control Variables

On talking with the inventor of the new plastic, you find he has worked only with small, lab scale equipment and that he has not defined the precise viscosity of the base polymer of the new plastic. Consequently, you will have to determine the percentage of the two additives add 1 and add 2. He also says that the moisture content of the base polymer (which you can control) often seems to be important.

You can control the temperature in each of four zones along the extruder barrel ( $T_1, T_2, T_3$  and  $T_4$ ) by automatic controllers on the heater elements.

Adding all this to information obtained from the operators of the extruder, you find there are at least ten control variables that may or not be important. Here they are along with the ranges over which they can be adjusted.

Add 1.... 0.0% to 0.5%. This is supposed to induce uniform melting.  
Add 2... 2.0% to 4.0%. This is a filler that is said to improve strength.  
Viscosity... The viscosity can be set anywhere from 60 to 80.  
Moisture... The moisture content can be set from 0.1% to 0.25%.  
 $T_1$  through  $T_4$ ... Each can be controlled at 260C min to 320C max, independently.  
Rate... Feed rate of input is from 100 to 200 pounds/hr.  
RPM (auger).. Can be set anywhere from 150 to 300 rpm.

## The Response Variables

On talking to your marketing people, you conclude that tensile strength (TS) is the most important property but measuring it requires a destructive test. It would be nice if the TS correlated with some easily monitored manufacturing variables so that the TS for each batch could be estimated from the other measurable variable. Old hands tell you they can pretty well estimate the TS of thermoplastics from the temperature of the melt during production. This is almost too good to be true so you decide to verify their claim by studying both the TS and the melt T as response variables. So your responses are chosen as follows.

Melt T... Estimated range is from 280 to 325C  
TS... Ranges from 15000 psi to 30,000 psi with the minimum advertised specification being 25000 psi

## Standard Deviation and Resolution

Discussions with marketing have resulted in a desired resolution of 10%. The general argument is that products less than 22,500 psi will be totally unacceptable. Since this is 10% less than 25,000 psi, the resolution of the experiment must be capable of detecting at least this difference.

You are fortunate in having a data base of tensile testing data from which you have calculated a standard deviation estimate of 0.02 for log 10 tensile strength. The antilog of 0.02 is approximately 1.05 which means that the standard deviation is about half of the desired resolution.

You may use 0.04 (the common log of 1.10) for resolution and 0.02 for standard deviation in the number of trials calculations.

## Final Hints

Make sure you have the goals clear. Many students, proceeding in haste, think optimization is the only goal. If you think this, reread p 5.4 very carefully.

Check it out. Try your model out and see how well it predicts.

All of the indicators may be positive. There may be no lack of fit indicated in the Effects Table, and the contour plots may make sense, but the model may still be in error. There is little redundancy in the recommended designs and they can fit the particular set of data but not the response surface from which the data comes; recording errors are a common problem.

The comments about the analysis of linear designs in the last paragraph of section 10.2 are important.

This problem should be worked several times. The first time without using design augmentation. Get fresh data at each stage. Once you have "solved" it this way and read Appendix B, you can redo it with design augmentation. For a third trial, you might try blocking-- once without blocking is recommended.

## ECHIP'S DOX SOFTWARE

Many factors have conspired to keep undergraduate engineers from gaining knowledge of the DOX strategies they so badly need. For example, few if any engineering students have had the time or inclination to fulfill the prerequisites required for the DOX courses traditionally offered in statistics departments. This has tended to keep them out of formal DOX courses in college. Those engineers who have learned these methods have done so either as part of a graduate program or, more likely, by attending intensive short courses after graduation and at some employer's expense. Such short courses are marketed by numerous companies, consultants and continuing education units around the country. Interestingly, they learned to use DOX without taking all the traditional prerequisites. Also, more and more software packages with DOX capabilities are becoming available to the users of personal computers and these, too, can be used quite effectively by students who have not taken extensive prerequisite courses in statistics. In other words, engineers are bypassing departments of statistics and are learning DOX strategies on the job, either from short courses or through the use of software packages or both.

In an effort to better prepare our MSE students for what they will face upon graduation, we have purchased a license to use the EChip software package in our MSE 341 course. We feel its statistical methods are more than rigorous enough for our purposes and we especially appreciate EChip's emphasis on strategies rather than statistics or calculation procedures. The software focuses more on how to make the key decisions when designing and executing experiments and leaves the computational details and statistical analyses to the computer and keeps them "behind the scenes", as it were. After first deciding whether the proposed investigation will fit within the available budget, the user is directed on a swift but systematic course of action through the cycle shown in Fig. 8. An extensive array of response surfaces produced in the form of contour maps that can be scrolled, sorted through and

interacted with in the most useful fashion. Upon completion of the study, one decides on whether or not another battery of experiments should be undertaken. If so, the procedure is repeated, though in a modified form that accounts for what has been learned or what variables may be left out.

EChip provides a choice of several standard designs plus a powerful algorithmic design option for use when none of the standard ones can possibly suffice. This can occur when certain regions of the control variable space simply cannot be sampled due to unavoidable technical difficulties. EChip enables the user to disallow whole groups of troublesome control variable settings and then to algorithmically devise an optimal design in the subregion of settings that is left. Algorithmic designs are also used when mixture variables are to be included or when one wishes to augment the design executed in the previous iteration.

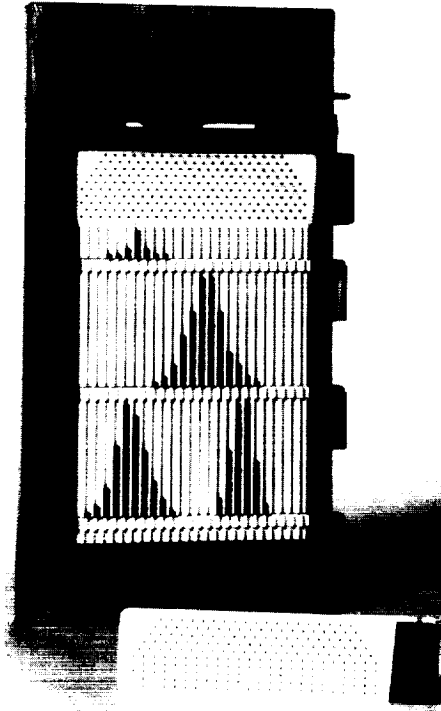
As EChip is response surface oriented, the response variables (several may be studied simultaneously) must be of the continuous type. Some but not all the controls may be of the categorical variety but the software is intended for studies involving continuous control variables for the most part. EChip's ability to handle both mixture and nonmixture controls simultaneously makes it especially useful for materials and chemical engineers.

#### CONCLUDING REMARKS

In the foregoing sections, I have promoted the use of simulation schemes, first of the mechanical-visual sorts and later of the computer-numerical types. In my view, these are the best ways to generate response data for multifactor experiments. Since the instructor specifies everything (including the noise level!) in the model that generates the data, it is relatively easy to diagnose the mistakes that arise in the students' work because there is no possibility for poor lab methods or experimental equipment failures to cause problems. Discrepancies can only be due either to software bugs or to inadequacies in the DOX strategies being used or to mistakes or poor judgment by the user. I, for one, very much favor the simulation approach when trying to help students acquire expertise in the use of DOX methods for analyzing or optimizing complicated multifactor processes. On the other hand there is indeed a lot to be said for also including real experiments in such a course, despite all the difficulties they may entail. For this reason, I conclude by citing a number of candidate experiments we are hoping to incorporate in the MSE 341 course in future offerings. Unlike the examples discussed so far, these place much greater emphasis on mixture variables.

- 1) Study the dependence of AC conductance and capacitance for aqueous solutions ( $\text{H}_2\text{O} + \text{NaCl} + \text{sugar} + \text{H}_2\text{CO}_3 + \text{etc.}$ ) on temperature ( $^{\circ}\text{K}$ ) compositions (mixture variables), AC frequency ( $\text{H}_2$ ), geometric variables (distance between electrodes, their areas, etc.).
- 2) Study AC conductance, capacitance and density of moist soils ( $\text{H}_2\text{O} + \text{sand}$ , silt, etc.) and their dependences on compaction pressure, water contact, soil type, AC frequency, etc.
- 3) Study dependence of freezing point depression of aqueous (or other) solutions on composition variables.

- 4) Study dependence of density and permeability of sintered  $\text{TiO}_2$  (or  $\text{ZnO}$ ,  $\text{NaCl}$ , etc.) powders on binder type, compaction pressure sintering temperature, sintering time, particle size distribution, etc.
- 5) Study dependence of final density and porosity of hot pressed pellets of bakelite powder on hot pressing temperature, pressure and time.

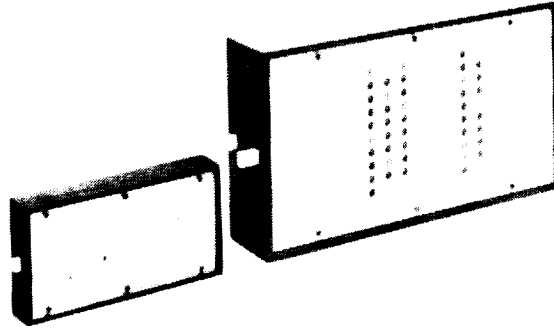


### THE QUINCUNX

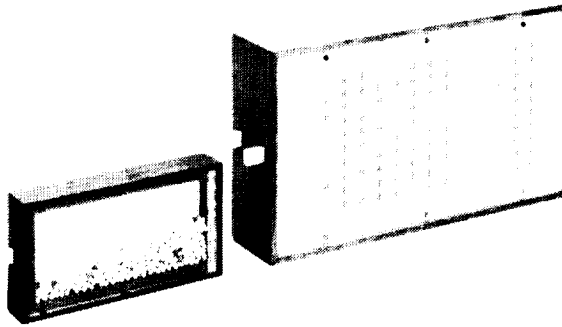
Figure 1. The Quincunx.

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Front



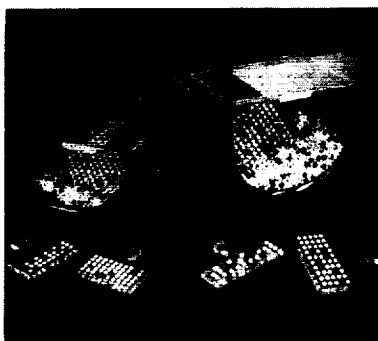
Back

## THE SAMPLING BOX

Figure 2. The sampling box.

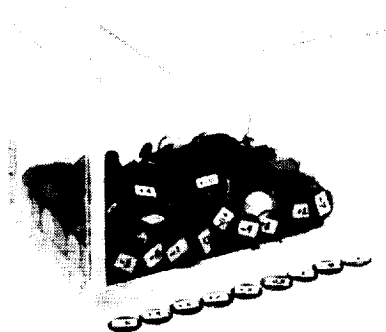
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## **THE SAMPLING BOWL**

Figure 3. The sampling bowl.



## **THE CHIP BOX**

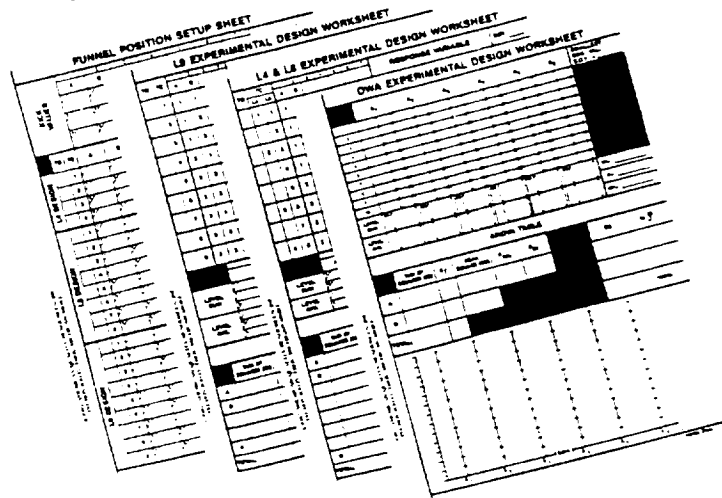
Figure 4. The chip box.

**PROSIM**, which stands for process simulator, is a training aid that uses a quincunx to teach designed experiments.

### The PROSIM kit includes :

- Master Forms to be used by the instructor and student.
- Overhead Masters for instruction and demonstrations.
- A "D" ring binder for the 140 plus page manual.

The training package includes step by step procedures for how to set up any quincunx with a variety of factors and degrees of significance. The handy setup forms and work sheets will help simplify any design experiment class. The data analysis forms included treat conventional designed experiments and Taguchi Orthogonal Arrays.



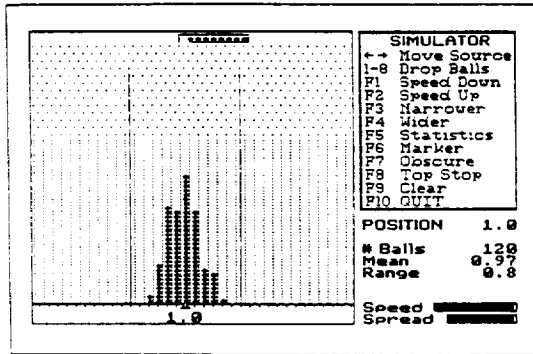
The detailed procedures included with PROSIM address the following topics:

1. How to set up and scale the quincunx.
2. One way ANOVA.
3.  $2^2$  Full Factorial Designs.
4.  $2^3$  Full Factorial Designs.
5. Two factor experiments with interactions.
6. Taguchi  $L_8$  Orthogonal Arrays.
7. Taguchi  $L_9$  Orthogonal Arrays.

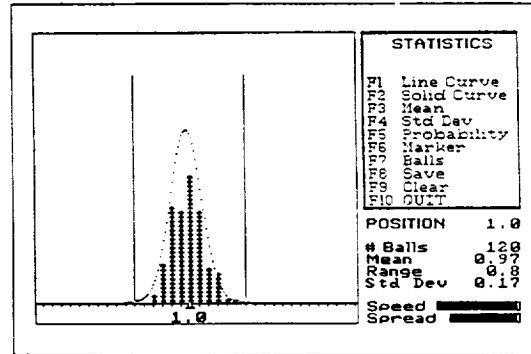
## PROSIM

Figure 5. Prosim.

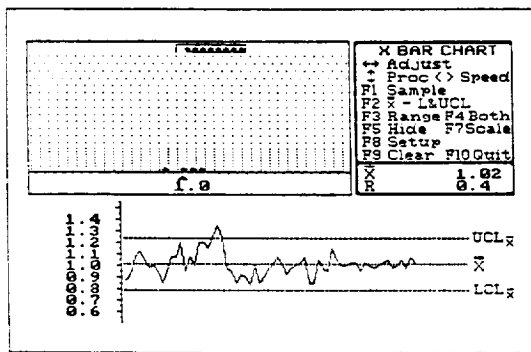
# Two Quincunxes for the PC



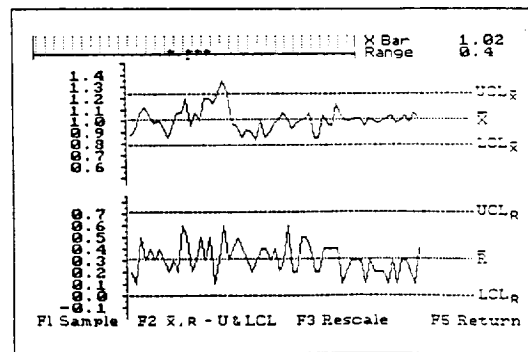
This is the bead frame in the Statistics Simulator. It shows a sample of 120 centered on 1. The standard deviation can be varied by using the F3 and F4 keys. Upper and lower spec limits were drawn in using the F6 key.



This is the Statistics Simulator analysis screen based on the information in Figure 1. Plus and minus standard deviation lines also can be drawn in, and the proportion between any two limits can be calculated.



The Statistical Process Control Simulator produces an X-bar chart with limits that can be updated. The effect of adjusting the mean, spread, or both can be demonstrated easily.



Pressing the F5 key displays the X-bar and range charts. Menu selections are always visible on the screen in the Statistical Process Control Simulator.

Figure 6. Two Quincunxes for the PC

Figure 6 reprinted with permission, *Quality Progress*, July 1988

PROG 5	Comments and Remarks
Mcl "SD"?→S Lbl 1 PROG 3  "n"?→N "T"?→T "P"?→P "x"?→x  .082NT ÷ P→V V + S × A→V  "V=" :V Δ GOTO 1 (CASIO fx-7000G)	Clears all memory registers Prompts for STD. DEV. "(TRUE)" and stores it in S  PROG 3 is a subroutine that returns a number "A" drawn from a normal (0.1) population  Prompts for a value of n and stores it in reg. N " " " " " T " " " " " T " " " " " P " " " " " P " " " " " x " " " " " x  Calculates the "TRUE" value of V and stores it in V Adds a normal residual noise contribution (STD DEV = S) onto V(TRUE) and stores <u>that</u> in V  Displays V = and the value just simulated cycles back to Lbl 1 for the next simulated experimental result
PROG 3	Comments and Remarks
φ→A:48→F:Lbl2 A + RAN#→A DS2 F:GOTO 2 (A-24) ÷ 2→A  (CASIO fx-7000G)	Sets A to φ and counter, F, to 48 Adds RAN# (uniform in int. 0-1) to register A Decreases counter By 1.0 when F > φ, GOTO Lbl2 When F drops to φ, convert the sum in A to a normal (0,1) value and store it in A for use in the main program.

Figure 7. A program with subroutine for simulating volume data with a known variability. Assumes  $V = \pi RT/P$ .

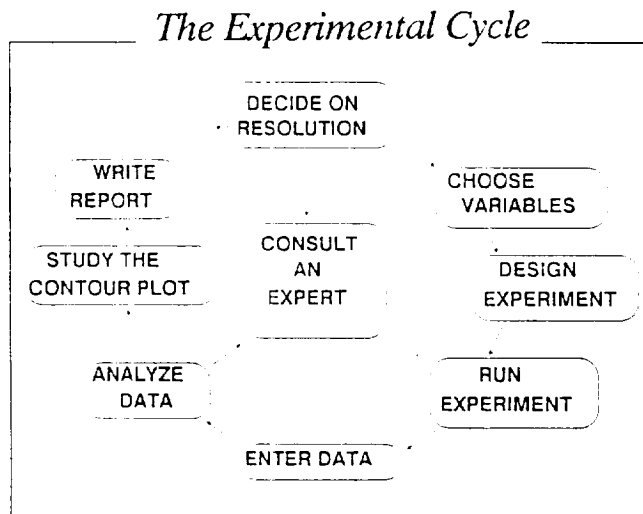


Figure 8.

