ACCURACY OF THE QUAD4 THICK SHELL ELEMENT

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SUMMARY

The accuracy of the relatively new QUAD4 thick shell element is assessed via comparison with a theoretical solution for thick homogeneous and honeycomb flat simply supported plates under the action of a uniform pressure load. The theoretical thick plate solution is based on the theory developed by Reissner and includes the effects of transverse shear flexibility which are not included in the thin plate solutions based on Kirchoff plate theory. In addition, the QUAD4 is assessed using a set of finite element test problems developed by the MacNeal-Schwendler Corp. (MSC). Comparison of the COSMIC QUAD4 element as well as those from MSC and Universal Analytics, Inc. (UAI) for these test problems is presented. The current COSMIC QUAD4 element is shown to have excellent comparison with both the theoretical solutions and also those from the two commercial versions of NASTRAN that it was compared to.

INTRODUCTION

The QUAD4 thick shell element, added to NASTRAN in 1987, is one of the most important additions to the program since the original writing of the code. The deficiencies of the original QUAD1 and QUAD2 quadrilateral shell elements have been recognized for years and have been reported in the literature. At the Goddard Space Flight Center (GSFC), the quadrilateral shell element is in use in virtually all structural analyses of our spaceraft and related hardware. Typical applications are for the modelling of cylindrical shells and flat plates made of honeycomb or machined, lightweighted, metal that make up the structure of spacecraft and scientific instruments. In some cases these models require that the effects of transverse shear flexibility be included due to their thickness. The QUAD4 element includes these effects and, in addition, has an improved isoparametric membrane capability for in-plane loading.

The purpose of the study reported herein is to assess the accuracy of the QUAD4 element in modelling a variety of situations involving both solid cross-section plates as well as those constructed of honeycomb. Three goals of the study were to determine:

a) what is the rate of convergence to the theoretical solution as the mesh is refined;

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b) whether the element exhibits sensitivity to aspect ratios significantly different than 1.0;

c) how the element behaves in a wide variety of modelling situations, such as those included in the MSC element test library (discussed below).

The first two questions were addressed in the same manner as several other studies reported by one of the authors in prior NASTRAN colloquia (references 1 and 2). The procedure used in those studies, and followed here also, is to isolate the effects of mesh refinement and aspect ratio. That is, the mesh refinement study is done using elements with an aspect ratio of 1.0. Then, once a fine enough mesh has been reached such that the errors are small, the effects of aspect ratio can be investigated by keeping the mesh the same (i.e. same number of elements) and varying the overall dimensions of the problem, thus resulting in each element aspect ratio changing. Obviously, in order to accomplish this latter step there must be a theoretical solution (or some other equally acceptable comparison solution) to the problem with which to compare the finite element model results. This is needed since, at each step, a problem of different dimensions (and therefore different theoretical solution) is being modelled.

The above tests are important in that they show the rate of convergence toward the theoretical solution as the mesh is refined. Those tests, however, are not sufficient to completely test the accuracy of a finite element since they do not test irregular geometries, or a variety of loadings or material properties. The MSC has developed a comprehensive set of problems for testing finite elements in a variety of situations (reference 3). The library of problems consists of 15 test problems for the QUAD4 element that cover all of the parameters mentioned above. A test of the COSMIC QUAD4 using these elements was reported at the 17th NASTRAN Users Colloquium in 1987 by Victoria Tischler of the Air Force Wright Aeronautical Laboratories (AFSC) at Wright Patterson Air Force Base, Ohio, but was not included in the formal proceedings. Due to the fact that it was not included in the formal proceedings, and also due to the fact that errors in the QUAD4 code for nonhomogeneous plates (to be discussed later) have been corrected, the results of our testing of the latest version of the element with the MSC library are include herein.

RESULTS OF MESH AND ASPECT RATIO STUDY

For the mesh and aspect ratio study a theoretical comparison solution is highly desirable. Since the effects of transverse shear flexibility are included in the QUAD4 element formulation, a theoretical solution for moderately thick plates, based on Reissner (or Mindlin) thick plate theory is also desirable. Such a solution is given in references 4 and 5 for rectangular simply supported thick plates under the action of a pressure load. Thus, this problem was used for the mesh and aspect ratio portions of the study. Figure 1 defines the geometry, coordinate system, boundary conditions and loading for the rectangular plate. The thickness indicates a moderately thick plate of length to thickness ratio of .20. The effect of transverse shear flexibility is only approximately 1% on the maximum displacement but is important in discerning the quality of the convergence of the finite element results to the exact theoretical solution. By exact is meant the theoretical basis for the QUAD4 element, which is expressed in the Reissner thick plate theory. Figure 2 shows the finite element mesh geometry used in the mesh and aspect ratio studies. Due to symmetry only one quarter of the plate was modelled The 4 x 4 mesh shown on figure 2 is an example only; the mesh was varied during the mesh study.

Figures 3a - 3c show characteristics of the theoretical solution. As indicated in figure 3a the central displacement solution is represented as an infinite series of hyperbolic functions. A FORTRAN computer program was written to compute the theoretical solutions for displacements (using the series shown) as well as stresses (solution not shown). As m gets large, where m is the number of terms included in the series, the hyperbolic functions tend to overflow the exponent range of the computer. This does not indicate a problem with the series shown, as the hyperbolic functions appear in both the numerator and denominator and their ratio is numerically stable. However, in separately evaluating the numerator and denominator the overflow problem was encountered. In order to circumvent this problem, the hyperbolic functions were rewritten in terms of exponentials allowing the programmed equations, in terms of ratios of numerator and denominator terms, to be evaluated without overflow problems.

Figures 3b and 3c show the stiffness parameters needed in the theoretical solution for the homogeneous (i.e. solid) plate and the honeycomb plate. For the honeycomb plate, two different core stiffnesses were investigeted. The stiffer one is representative of aluminum honeycomb construction that has been used at the GSFC. The more flexible one was chosen because it represents a core flexibility that is quite low and was expected to be a more critical test of the QUAD4's shear flexibility formulation.

The results of the mesh study, showing the convergence of the QUAD4 solutions to the theoretical, are presented in tabular form in tables 1 - 2 and in graphical form in figures 4 - 7. Both formats show % error in displacement at the center of the plate as a function of mesh refinement. Results are included for COSMIC 88, MSC 65C and UAI 10.0 NASTRAN. In the tables results for COSMIC version 87 is also indicated as will be discussed below. The tables merely give exact numbers (along with the theoretical displacements) and the figures contain the same error information, but in graphic form.

Figures 4 and 5 and table 1 are the results for the homogeneous plate. The difference between the results in figures 4 and 5 (and that in the two parts of table 1) is that figure 4 (and the top half of table 1) is for a solution in which shear flexibility is included and figure 5 (and the bottom half of table 1) is without shear flexibility. These two situations were investigated to test the MID3 option on the PSHELL NASTRAN bulk data deck card which allows the effects of shear flexibility to be ignored if MID3 is left blank. As seen in figures 4 and 5 the NASTRAN results converge very rapidly with mesh refinement for COSMIC 88, MSC 65C and UAI 10.0. Table 1 contains the same information along with results for COSMIC 87, the first COSMIC version to contain the QUAD4 element. As seen, all versions converge to less than 0.5% error for a mesh size of 8 x 8 with the results without shear flexibility converging a little more rapidly.

Figures 6 and 7 and table 2 are the results for the honeycomb plate. Figure 6 (and the top half of table 2) are for the honeycomb plate with the stiffer core and figure 7 (and the bottom half of table 2) are for the more flexible core. As seen in figures 6 and 7 the NASTRAN results for COSMIC 88 and the two commercial NASTRAN versions converge very rapidly for the two honeycomb plates as they did for the homogenous plate. Table 2 contains the same information along with the results for COSMIC 87. As indicated, the errors in the first version containing the QUAD4 were extremely large for the honeycomb plate but, as reported above, were quite good for the homogenous plate. When this was discovered it was immediately reported to COSMIC. They found the problem in a program controlled adjustable parameter (which is used to avoid the infamous shear locking phenomena in earlier thick shell finite elements based on Reissner plate theory) and sent us a fix within two days. After modifying the subroutine containing the error, the results became that which is reported under the COSMIC 88 heading (the same fix was included by COSMIC in the 88 release).

In order to test the QUAD4's sensitivity to aspect ratio, the model with a 12 x 12 mesh was run in which the plate side dimension in the x direction was varied. This causes the element aspect ratio to vary while maintaining a constant mesh in an attempt to remove mesh refinement errors from significantly affecting the results. As seen in tables 1 and 2, the QUAD4 results with a 12 x 12 mesh (and aspect ratio of 1.0) have very little error. The results of the aspect ratio study are presented in figures 8 - 10 and tables 3 - 5. Tables 3 - 5 give % error in the displacement at the center of the plate versus aspect ratio for a model with a mesh of 12 x 12 QUAD4 elements (over one quarter of the plate). As mentioned above, the aspect ratio was varied by changing the dimension of the plate along the x axis. Thus, the results for the aspect ratio of 10 are for a plate (and all QUAD4 elements) that is 10 times as long in the x direction as in the y direction. Due to this the theoretical solution changes with aspect ratio. Figure 8 and table 3 are for the homogenous plate (with transverse shear flexibility) while figure 9 and table 4 are for the stiff core honeycomb plate and figure 10 and table 5 are for the more flexible core honeycomb plate. Investigation of the % error in the tables, as well as in figures 8 - 10 show that the QUAD4 has essentially no aspect ratio sensitivity over the range investigated.

Based on the above results, the COSMIC QUAD4 element is seen to give very accurate results for the displacements in the problem investigated, both in comparison to the exact theory and in comparison to the two commercial versions of NASTRAN that we have at the GSFC. Although the results are not presented herein, similarily accurate results were obtained for the shear and moment stress resultants as well.

RESULTS OF TESTING USING THE MSC ELEMENT TEST LIBRARY

As mentioned earlier, the mesh and aspect ratio studies, while a very useful tool in the evaluation of an element, do not test all of the important variables that affect accuracy in a finite element solution. The MSC element test library mentioned above represents a rather exhaustive series of tests that include many of the element related parameters which affect the accuracy of a finite element solution. Reference 3 gives a detailed description of the test problems along with theoretical answers and the results of the testing on several MSC elements. The reader should consult reference 3 for a complete description of the various problems in the test series. The portion of this series of element tests that relate to the QUAD4 element was run by the authors on the QUAD4 elements contained in COSMIC 88, UAI 9.8+ (not version 10.0 as for the mesh and aspect ratio study) and MSC 65C. As the MSC does in their report, the results are presented in detail and also in a summary form in which the element is given a letter grade of A through F based on the magnitude of the error. Table 6 shows the summary results for the 15 tests in the series ranging from a simple patch test to modelling of beams (using the QUAD4 element through the depth) and various plates and shells. The meaning of the letter grades is given at the bottom of the table. As pointed out in reference 3, a failing grade for an element in one test is not a reason to dismiss the element. For one thing, the test scores would improve with mesh refinement; the mesh used in most of the problems was quite coarse. Of importance in this discussion is not the actual grades listed in table 6 but the comparison of the COSMIC grades with those from the other two programs. As seen in table 6, the COSMIC QUAD4 element is as good as, or better than, those of the commercial programs. Although not shown in table 6, the old QUAD2 element (included in reference 3) has a D or F grade in 9 of the 15 problems. This is the reason for the longstanding need for an improved shell element and the QUAD4 element added to COSMIC NASTRAN clearly fills that need. Detailed results for each of the problems in the test series are contained in tables 7 - 12 and are included for completeness.

CONCLUSIONS

The COSMIC QUAD4 general purpose flat shell element has been shown to be an excellent element and significantly enhances the usefulness of COSMIC NASTRAN. The element has been shown to compare excellently with those available in two commercial versions of NASTRAN that are currently being used at the GSFC. The addition of an improved triangular shell element, anticipated in the near future, is highly desireable as a companion element to the QUAD4 in general analyses of complicated shell like structures.

REFERENCES

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List of Symbols

a, b = plate dimensions

t = plate thickness

p = pressure load

D, C_n, C_s = plate rigidities (see Figures 3b,3c)

E = Young's Modulus

ν = Poisson's Ratio

G_c = honeycomb core shear modulus

AR, AR_e = aspect ratio (ratio of planar dimensions of plate or element)

w = plate displacement

N_x, N_y = number of elements in model of plate in x, y directions respectively

TABLE 1

MESH STUDY

THICK HOMOGENEOUS PLATE

ELEMENT ASPECT RATIO 1.0

Theoretical Displacements
With Transverse Shear Flexibility: 3.571×10^{-5} m
(1.406×10^{-3} in.)
Without Transverse Shear Flexibility: 3.529×10^{-5} m
(1.390×10^{-3} in.)

	% Error			
Mesh	Cosmic 87	Cosmic 88	UAI Ver. 10.0	MSC Ver. 65C
With Transverse Shear Flexibility				
1x1	12.03	12.03	12.03	21.76
2x2	4.35	4.34	4.35	2.54
4x4	1.67	1.67	1.67	1.39
8x8	0.59	0.60	0.59	0.53
12x12	0.39	0.41	0.39	0.36
Without Transverse Shear Flexibility				
1x1	16.90	16.83	16.90	26.31
2x2	1.12	1.10	1.12	1.67
4x4	0.19	0.18	0.19	0.50
8x8	0.03	0.00	0.03	0.30
12x12	0.00	0.03	0.00	0.18

TABLE 2

MESH STUDY

THICK HONEYCOMB PLATE

ELEMENT ASPECT RATIO 1.0

Theoretical Displacements

$G_z = 1.517 \times 10^8$ N/m^2: 2.422×10^{-3} m (9.535×10^{-2} in.)

$G_z = 1.379 \times 10^7$ N/m^2: 3.102×10^{-3} m (1.221×10^{-1} in.)

	% Error			
Mesh	Cosmic 87	Cosmic 88	UAI Ver. 10.0	MSC Ver. 65C
$G_z = 1.517 \times 10^8$ N/m^2 (22000. psi)				
1x1	747.3	-16.31	-7.21	-17.98
2x2	589.9	-1.17	4.87	3.26
4x4	311.4	-0.25	1.46	1.19
8x8	103.3	-0.06	0.37	0.31
12x12	47.9	-0.03	0.16	0.14
$G_z = 1.379 \times 10^7$ N/m^2 (2000. psi)				
1x1	-6550.4	-6.71	10.31	4.92
2x2	-5127.3	0.26	5.51	4.57
4x4	-2689.0	0.09	1.42	1.22
8x8	-888.5	0.02	0.36	0.31
12x12	-412.2	0.01	0.16	0.14

TABLE 3

ASPECT RATIO STUDY

THICK HOMOGENEOUS PLATE

WITH TRANSVERSE SHEAR FLEXIBILTY

12X12 MESH

AR	theoretical w, m (in.)	% Error Cosmic 88	% Error UAI Ver. 10.0	% Error MSC Ver. 65C
1	3.571×10^{-5} (1.406×10^{-3})	-0.38	0.39	0.36
2	8.865×10^{-5} (3.490×10^{-3})	0.28	0.26	0.27
5	11.34×10^{-5} (4.465×10^{-3})	-0.83	-0.01	0.05
10	11.38×10^{-5} (4.482×10^{-3})	-0.04	-0.06	-0.02

TABLE 4

ASPECT RATIO STUDY

THICK HONEYCOMB PLATE, Gz=1.517 x 10^8 N/m^2 (22000. psi)

12X12 MESH

		% Error		
AR	theoretical w, m (in.)	Cosmic 88	UAI Ver. 10.0	MSC Ver. 65C
1	2.422×10^{-3} (9.535×10^{-2})	0.02	-0.16	-0.14
2	5.974×10^{-3} (2.352×10^{-1})	0.05	-0.12	-0.13
5	7.631×10^{-3} (3.004×10^{-1})	0.24	0.13	0.07
10	7.660×10^{-3} (3.016×10^{-1})	0.27	0.17	0.14

TABLE 5

ASPECT RATIO STUDY

THICK HONEYCOMB PLATE, Gz=1.379 x 10^7 N/m^2 (2000. psi)

12X12 MESH

AR	theoretical w, m (in.)	Cosmic 88	% Error UAI Ver. 10.0	MSC Ver. 65C
1	3.102×10^{-3} (1.221×10^{-1})	-0.01	-0.16	-0.49
2	7.026×10^{-3} (2.766×10^{-1})	0.03	-0.12	0.23
5	8.785×10^{-3} (3.459×10^{-1})	0.20	0.01	0.06
10	8.815×10^{-3} (3.470×10^{-1})	0.24	0.41	0.14

TABLE 6
SUMMARY OF TEST RESULTS FOR QUAD4 SHELL ELEMENTS

Test	Elem. Loading In Plane	Elem. Loading Out of Plane	Element Shape	COSMIC 88	UAI 9.8+	MSC 65C
1. Patch Test	X		Irregular	A	A	A
2. Patch Test		X	Irregular	A	A	A
3. Straight Beam, Extension	X		All	A	A	A
4. Straight Beam, Bending	X		Regular	B	B	B
5. Straight Beam, Bending	X		Irregular	F	F	F
6. Straight Beam, Bending		X	Regular	A	A	A
7. Straight Beam, Bending		X	Irregular	A	A	B
8. Straight Beam, Twist			All	B	B	B
9. Curved Beam	X		Regular	C	C	C
10. Curved Beam		X	Regular	B	B	B
11. Twisted Beam	X	X	Regular	A	A	A
12. Rectangular Plate (N=4)		X	Regular	A	A	B
13. Scordelis-Lo Roof (N=4)	X	X	Regular	B	B	B
14. Spherical Shell (N=8)	X	X	Regular	A	A	A
15. Thick-Walled Cylinder (nu=.4999)	X		Regular	B	B	F
Number of Failed Tests (D's and F's)				1	1	2

Grading for Shell Element Test Results

Grade	Requirement
A	2% ≥ Error
B	10% ≥ Error > 2%
C	20% ≥ Error > 10%
D	50% ≥ Error > 20%
F	Error ≥ 50%

TABLE 7
PATCH TEST RESULTS

	Maxium % Error in Stress		
	Cosmic 88 Quad4	UAI Ver 9.8+ Quad4	MSC Ver. 65C Quad 4
Constant-Stress Loading	0.00	0.00	0.00
Constant-Curvature Loading	0.00	0.00	0.00

TABLE 8
RESULTS FOR STRAIGHT CANTILEVERED BEAM

Tip Loading Direction	Normalized Tip Displacement* in Direction of Loading Cosmic 88 Quad4	UAI Ver. 9.8+ Quad4	MSC Ver. 65C Quad 4
	(a) Rectangular Elements		
Extension	0.996	0.996	0.995
In-plane Shear	0.904	0.904	0.904
Out-of-plane Shear	0.985	0.985	0.986
Twist	0.958	0.957	0.941
	(b) Trapezoidal Elements		
Extension	1.00	0.992	0.996
In-plane Shear	0.071	0.071	0.071
Out-of-plane Shear	0.980	0.979	0.968
Twist	0.937	0.934	0.951
	(c) Parallelogram Elements		
Extension	0.992	0.992	0.996
In-plane Shear	0.080	0.080	0.080
Out-of-plane Shear	0.986	0.986	0.977
Twist	0.895	0.892	0.945

*: Normalizing displacement values listed in Ref. 3. It is usually a theoretical value.

TABLE 9
RESULTS FOR CURVED BEAM

Tip Loading Direction	Normalized Tip Displacement* in Direction of Loading Cosmic 88 Quad4	UAI Ver. 9.8+ Quad4	MSC Ver. 65C Quad 4
In-plane Shear	0.834	0.833	0.833
Out-of-plane Shear	0.971	0.971	0.951

RESULTS FOR TWISTED BEAM

Tip Loading Direction	Normalized Tip Displacement* in Direction of Loading Cosmic 88 Quad4	UAI Ver. 9.8+ Quad4	MSC Ver. 65C Quad 4
In-plane Shear	0.995	0.995	0.993
Out-of-plane Shear	0.984	0.984	0.985

*: Normalizing displacement values listed in Ref. 3. It is usually a theoretical value.

TABLE 10
RESULTS FOR RECTANGULAR PLATE

Normalized Lateral Deflection at Center*

	Uniform Load			Concentrated Load		
	Cosmic 88	UAI V. 9.8+	MSC V. 65C	Cosmic 88	UAI V. 9.8+	MSC 65C
Nodes/ Edge	Simple Supports					
	(a) Aspect Ratio = 1.0					
2	1.01	1.05	0.981	1.05	1.04	1.02
4	1.01	1.02	1.00	1.02	1.02	1.02
6	1.01	1.01	1.00	1.01	1.01	1.01
8	1.00	1.01	1.00	1.01	1.01	1.01
	(b) Aspect Ratio = 5.0					
2	0.986	0.983	1.05	0.999	0.989	0.811
4	0.988	0.984	0.991	1.02	1.01	0.932
6	0.995	0.995	0.997	1.03	1.02	0.973
8	0.997	0.997	0.998	1.03	1.02	0.989
	Clamped Supports					
	(a) Aspect Ratio = 1.0					
2	1.052	1.046	1.008	0.971	0.963	0.994
4	1.038	1.034	1.032	1.020	1.015	1.010
6	1.024	1.022	1.023	1.027	1.018	1.012
8	1.017	1.016	1.016	1.013	1.012	1.010
	(b) Aspect Ratio = 5.0					
2	1.121	1.112	1.314	0.689	0.663	0.519
4	1.023	1.019	1.016	0.987	0.974	0.863
6	1.013	1.010	1.017	1.028	1.019	0.940
8	1.014	1.013	1.017	1.034	1.027	0.972

*: Normalizing displacement values listed in Ref. 3. It is usually a theoretical value.

TABLE 11
RESULTS FOR THICK-WALLED CYLINDER

Poisson's Ratio	Normalized Radial Displacement* at Inner Boundary Cosmic 88 Quad4	UAI Ver. 9.8+ Quad4	MSC Ver. 65C Quad4
0.4900	1.027	1.027	0.864
0.4990	1.032	1.032	0.359
0.4999	1.033	1.033	0.053

*: Normalizing displacement values listed in Ref. 3. It is usually a theoretical value.

TABLE 12
RESULTS FOR SCORDELIS-LO ROOF

No. of Spaces per Edge	Normalized Vertical Deflection* at Midpoint of Free Edge Cosmic 88 Quad4	UAI Ver. 9.8+ Quad4	MSC Ver. 65C Quad4
2	1.450	1.450	1.376
4	1.070	1.070	1.050
6	1.030	1.030	1.018
8	1.019	1.019	1.008
10	1.015	1.015	1.004

RESULTS FOR SPHERICAL SHELL

No. of Spaces per Edge	Normalized Vertical Deflection* at Midpoint of Free Edge Cosmic 88 Quad4	UAI Ver. 9.8+ Quad4	MSC Ver. 65C Quad4
2	1.020	1.011	0.972
4	1.043	1.040	1.024
6	1.023	1.020	1.013
8	1.010	1.009	1.005
10	1.004	1.003	1.001
12	1.000	0.999	0.998
16	0.998	0.997	0.997

*: Normalizing displacement values listed in Ref. 3. It is usually a theoretical value.

Fig. 1
Test Problem

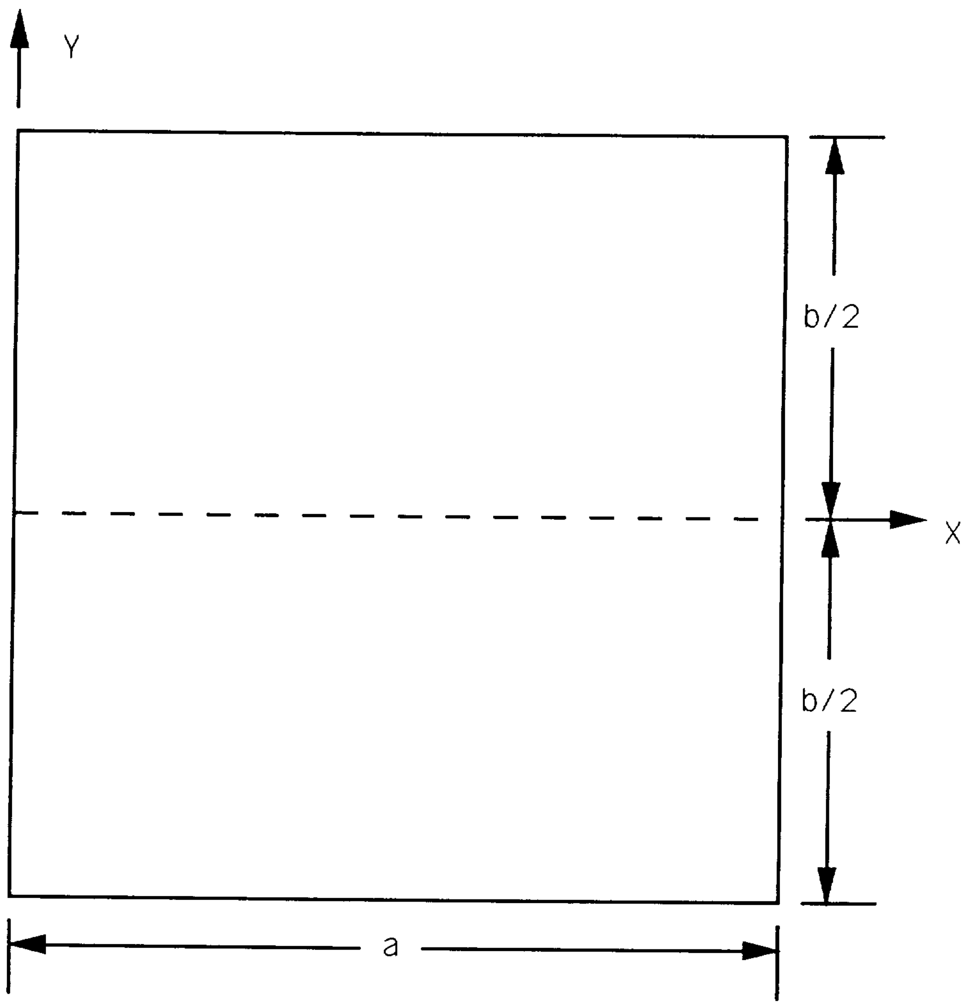
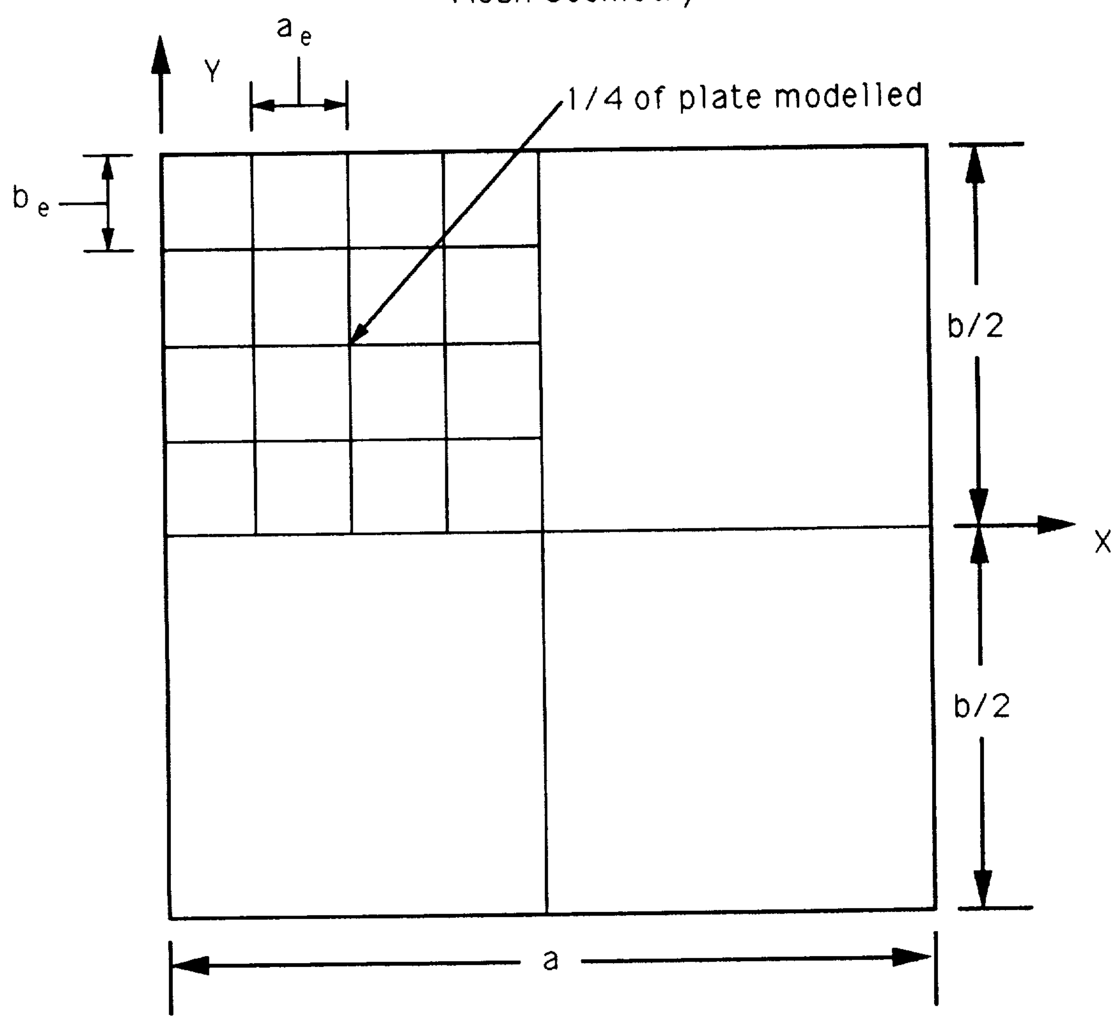


Plate Size: a=1.016 m (40. in.)* b=1.016m (40.in.)
Boundary Conditions: simply supported on all edges
Loading: pressure load, p=6895. N/m^{-2} (1.0 psi) +Z direction
Thickness: t=0.0508 m (2.0 in.)

*: Variable in aspect ratio studies

Fig. 2
Mesh Geometry



$AR_e = a_e/b_e$ = element aspect ratio

$N_x = a/2a_e$ = number of elements in X direction in 1/4 of plate

$N_y = b/2b_e$ = number of elements in Y direction in 1/4 of plate

Fig. 3a

Theoretical Solution - Central Displacement

Central Displacement

$$w\left(x=\frac{a}{2}, y=0\right) = \frac{4p}{aD} \sum_{m=1,3,5,\ldots} \left[1 + C_5 \cosh(\mu y) + \mu y C_6 \sinh(\mu y) + \mu^2 D\left(\frac{1}{C_s} - \frac{1+\nu}{C_n}\right) \right] \frac{\sin \mu x}{\mu^5}$$

where,

$$C_5 = -\frac{1}{\cosh \alpha_m}\left[1 + \mu^2 D\left(\frac{1}{C_s} - \frac{1+\nu}{C_n}\right) + \frac{1}{2}\alpha_m \tanh(\alpha_m) \right]$$

$$C_6 = \frac{1}{2 \cosh \alpha_m}$$

$$\alpha_m = \frac{m\pi}{2}\frac{b}{a}, \quad \mu = \frac{m\pi}{a}$$

Fig. 3b

Theoretical Solution - Homogeneous Plate Parameters

Homogeneous Plate

$$D = \frac{Et^3}{12(1-\nu^2)}$$

$$C_n = \frac{5}{6}\frac{Et}{\nu}$$

$$C_s = \frac{5}{6}Gt, \quad G = \frac{E}{2(1+\nu)}$$

$E = 6.89 \times 10^{10}$ N/m^2 (10.0×10^6 lb/in^2)

$\nu = 0.33$

Fig. 3c

Theoretical Solution - Honeycomb Plate Parameters

Honeycomb Plate

$$D = \frac{E_f t_f (t_c + t_{f/2})^2}{4(1-\nu^2)}$$

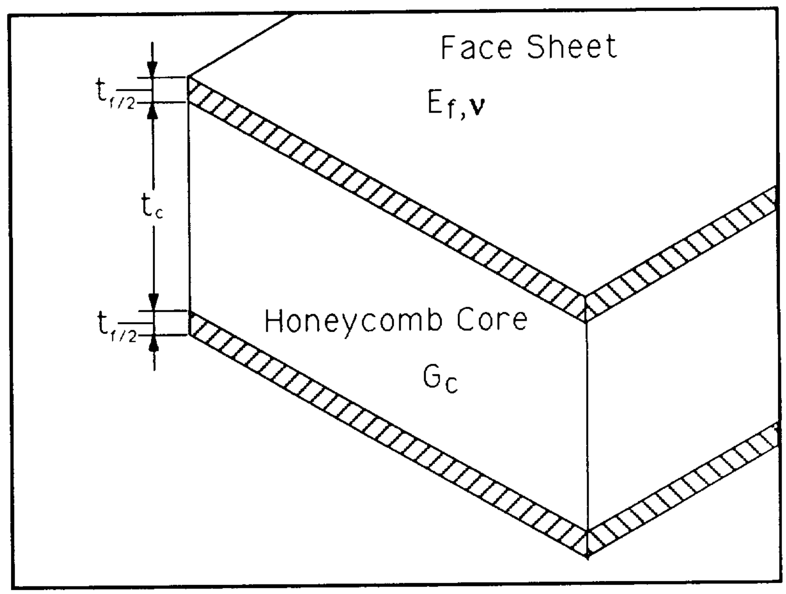
$C_n = \infty$

$C_s = t_c G_c$

$E_f = 6.89 \times 10^{10}\ \mathrm{N/m^2}$
$(10 \times 10^6\ \mathrm{lb/in^2})$

$\nu = 0.33$

$G_c = 1.379 \times 10^7\ \mathrm{N/m^2}\ (2000.\ \mathrm{lb/in^2})$
or
$1.517 \times 10^8\ \mathrm{N/m^2}\ (22000.\ \mathrm{lb/in^2})$



Core Detail

Fig. 4
Error in Displacement at Center of Plate
Mesh Size Study
Homogeneous Plate with Transverse Shear Flexibility
Element Aspect Ratio 1.0

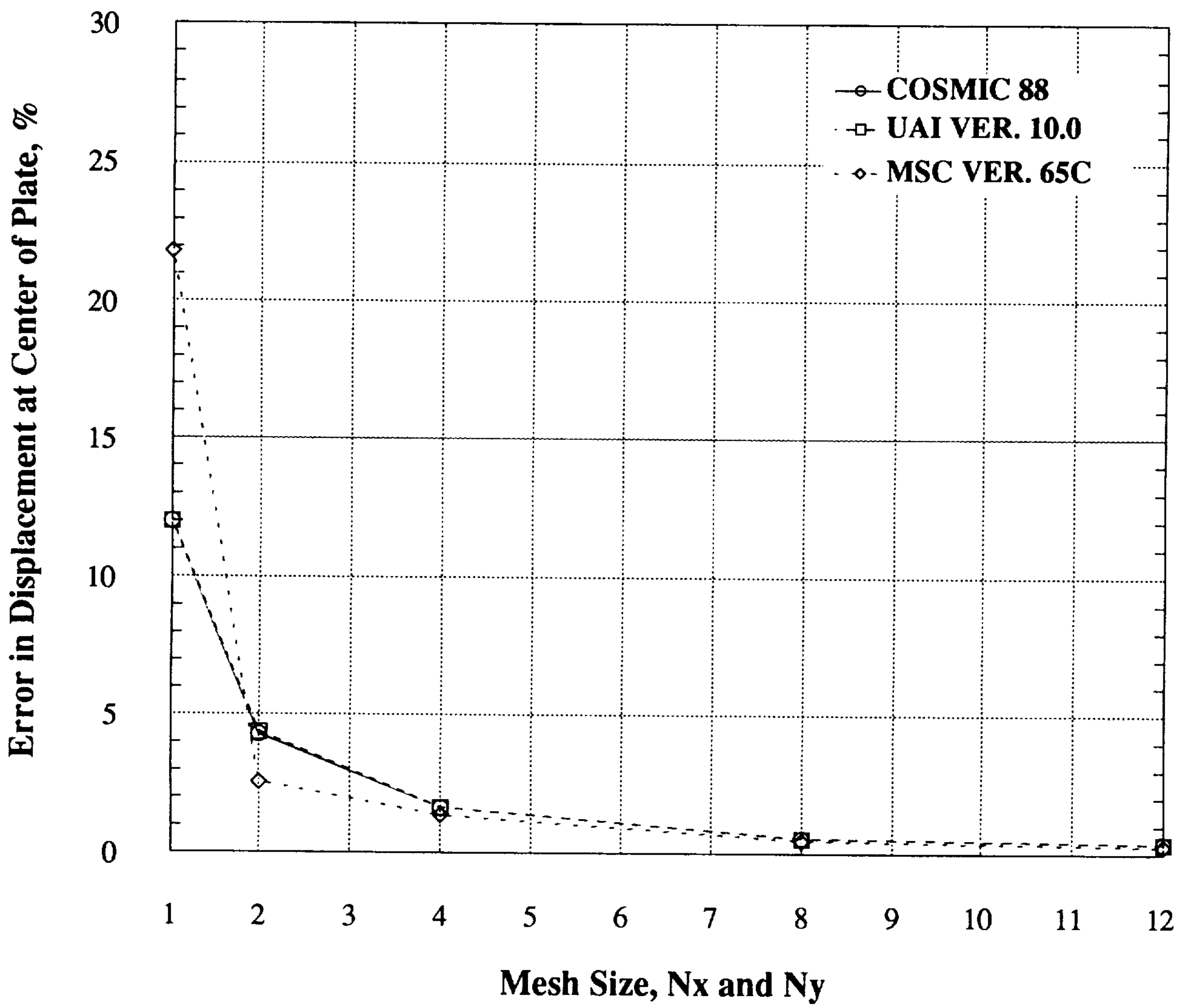


Fig. 5
Error in Displacement at Center of Plate
Mesh Size Study
Homogeneous Plate without Transverse Shear Stiffness
Element Aspect Ratio 1.0

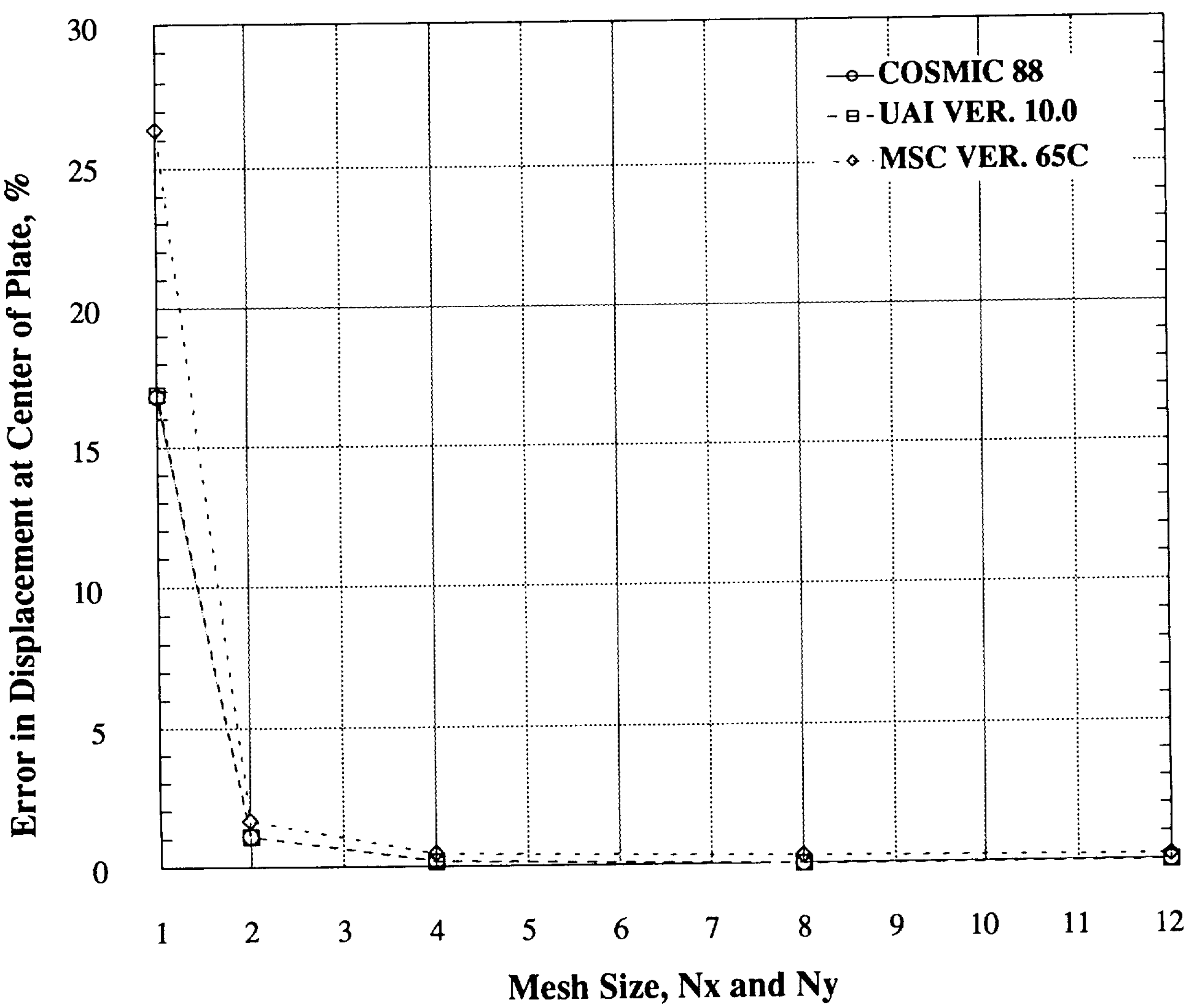


Fig. 6
Error in Displacement at Center of Plate
Mesh Size Study
Stiff Honeycomb Plate
Element Aspect Ratio 1.0

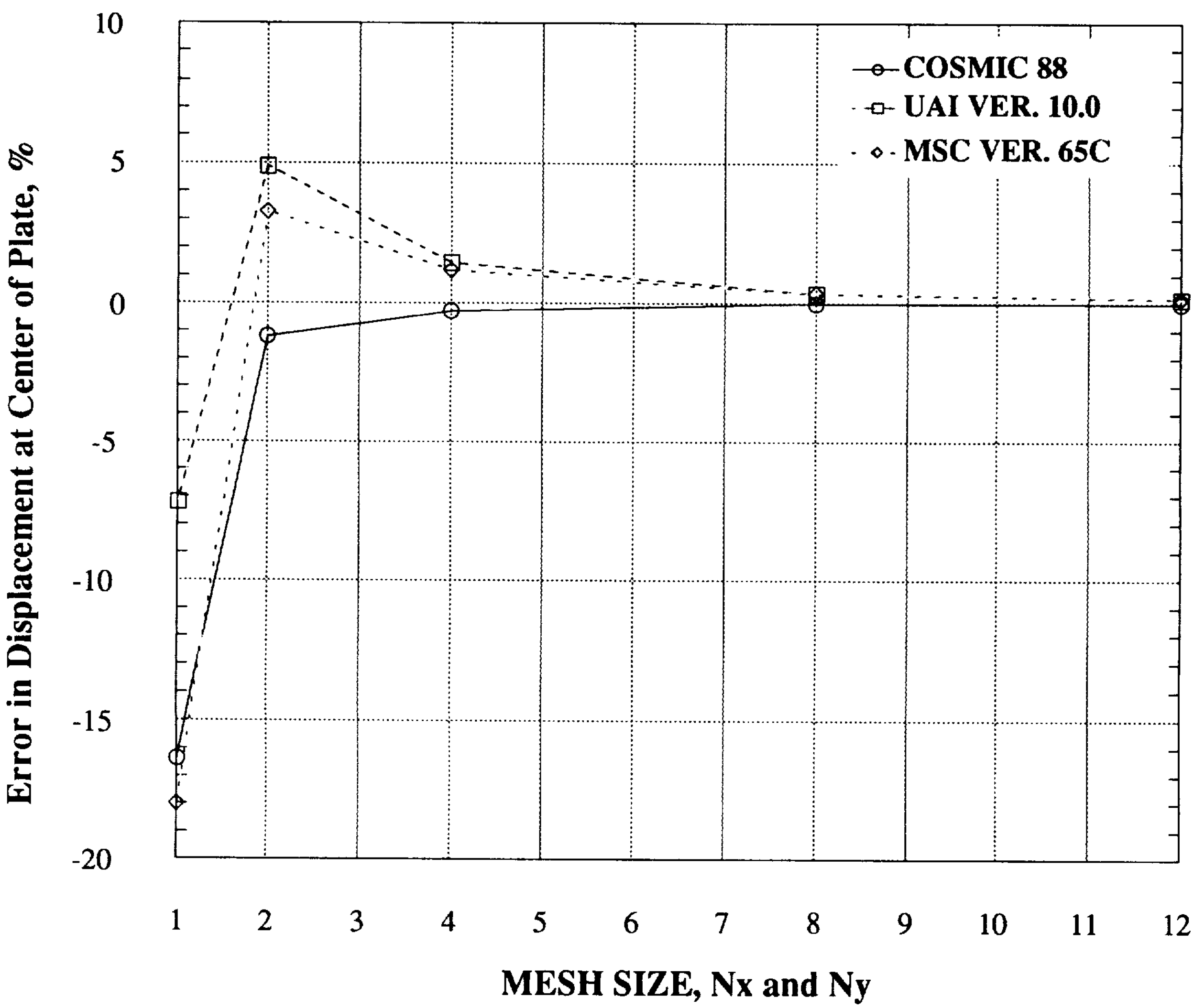


Fig. 7
Error in Displacement at Center of Plate
Mesh Size Study
Flexible Honeycomb Plate
Element Aspect Ratio 1.0

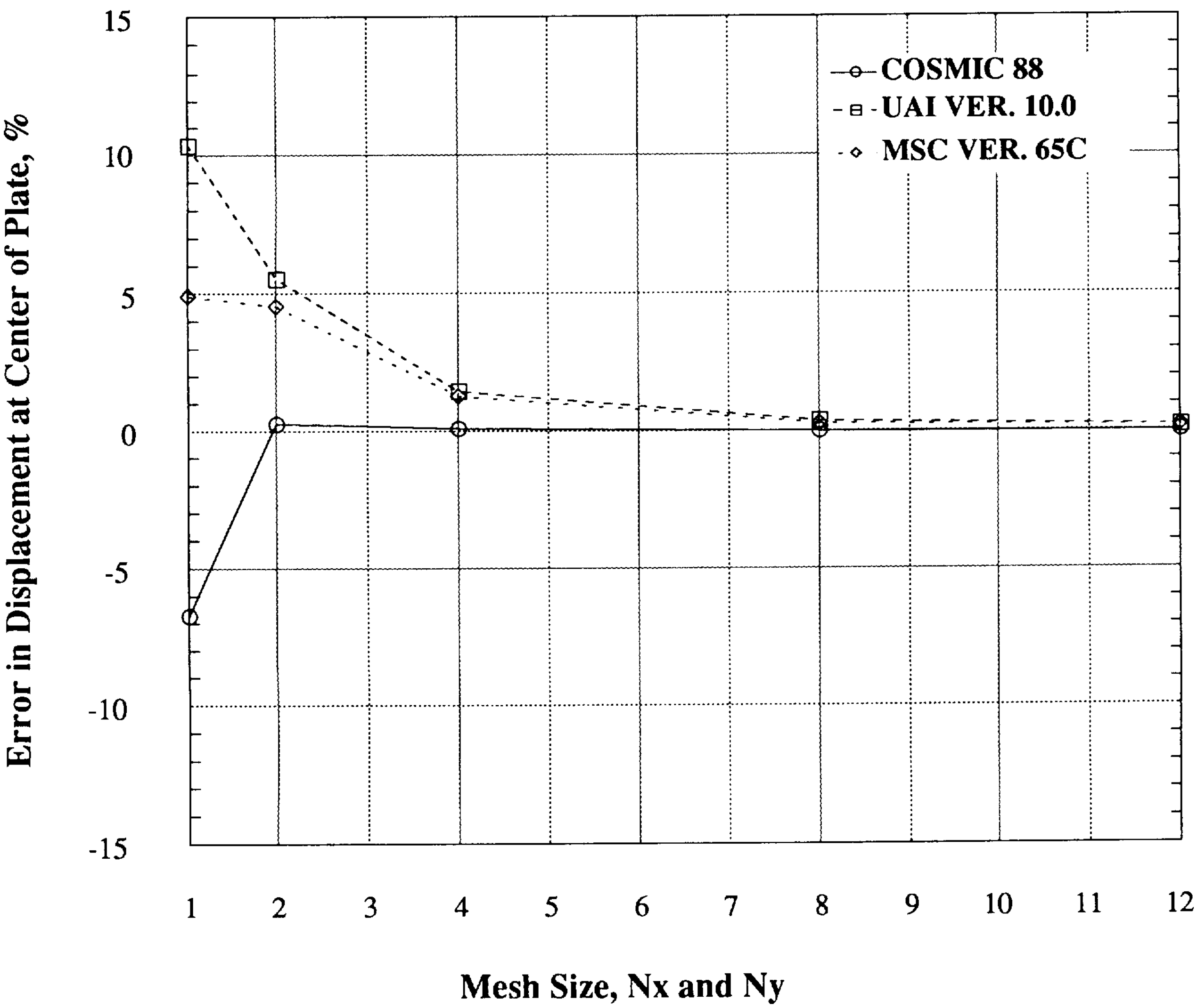


Fig. 8
Error in Displacement at Center of Plate
Aspect Ratio Study
Homogeneous Plate
12 x 12 Mesh

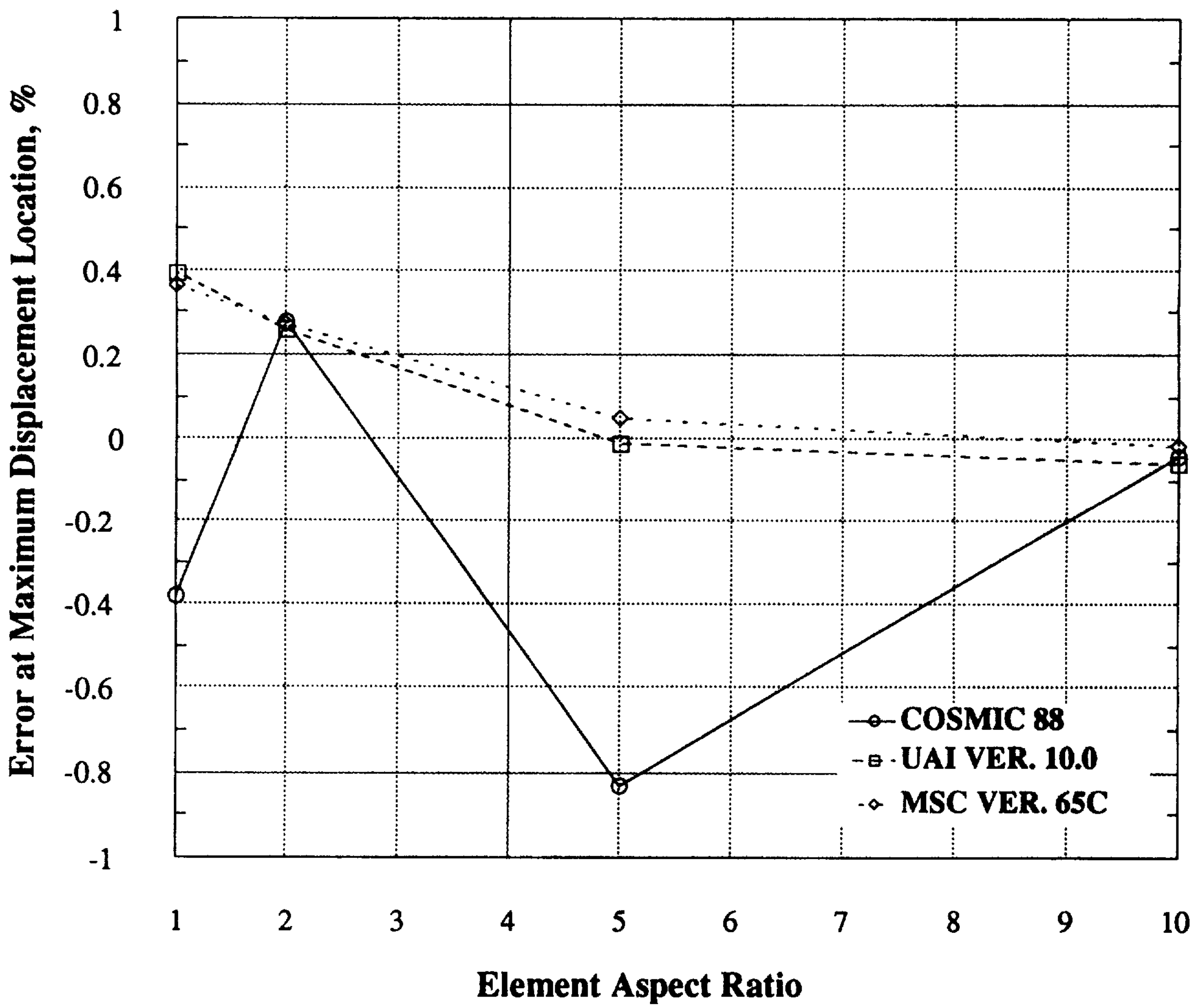


Fig. 9
Error in Displacement at Center of Plate
Aspect Ratio Study
Stiff Honeycomb Plate
12 x 12 Mesh

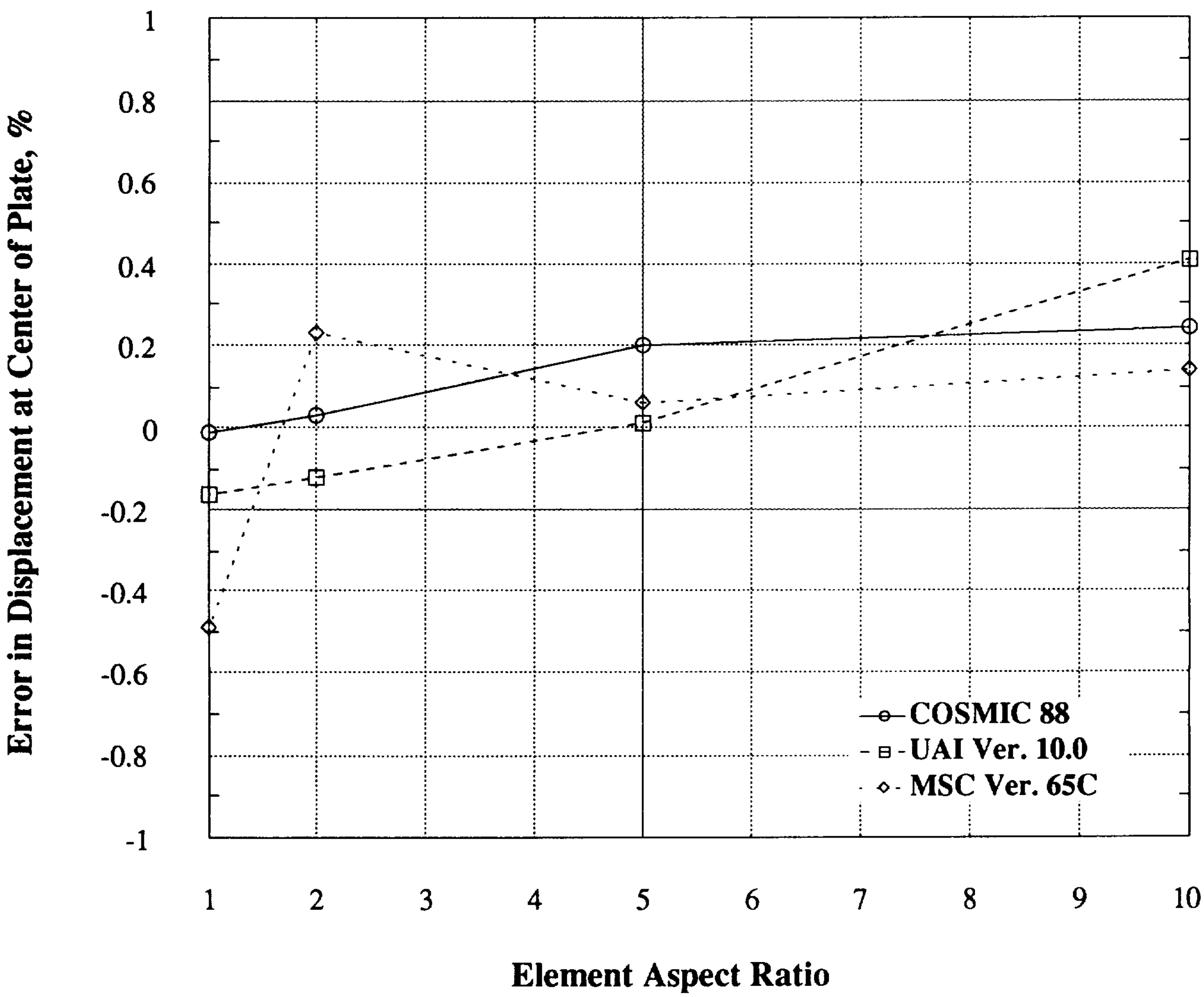


Fig. 10
Error in Displacement at Center of Plate
Aspect Ratio Study
Flexible Honeycomb Plate
12 x 12 Mesh