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EIGENVALUE COMPUTATIONS WITH THE QUAD4 CONSISTENT-MASS MATRIX

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SUMMARY

The NASTRAN user has the option of using either a lumped-mass matrix or a consistent- (coupled-) mass matrix with the QUAD4 shell finite element. At the Sixteenth NASTRAN Users' Colloquium (1988), Melvyn Marcus and associates of the David Taylor Research Center summarized a study comparing the results of the QUAD4 element with results of other NASTRAN shell elements for a cylindrical-shell modal analysis. Results of this study, in which both the lumped- and consistent-mass matrix formulations were used, implied that the consistent-mass matrix yielded poor results. In an effort to further evaluate the consistent-mass matrix, a study was performed using both a cylindrical-shell geometry and a flat-plate geometry. Modal parameters were extracted for several modes for both geometries leading to some significant conclusions. First, there do not appear to be any fundamental errors associated with the consistent-mass matrix. However, its accuracy is quite different for the two different geometries studied. The consistent-mass matrix yields better results for the flat-plate geometry and the lumped-mass matrix seems to be the better choice for cylindrical-shell geometries.

INTRODUCTION

At the 1988 NASTRAN Users' Colloquium, results of a study using the QUAD4, four-node, shell finite element for shell vibrations was presented (ref. 1). This study indicated that using the QUAD4 element with a consistent-mass matrix results in poor prediction of natural frequencies of a cylindrical shell. The errors in predicted frequencies were small for lower circumferential harmonics ($n<4$) and grew from approximately 10 per cent for the fourth circumferential harmonic to over 20 per cent for the sixth circumferential harmonic. The errors seemed to be relatively independent of the longitudinal harmonics. The authors conclude that the poor performance is probably caused by either a bad formulation of the consistent-mass matrix or, more likely, a coding error in the program. The QUAD4 element is described in reference 2.

In an effort to determine whether either of the above reasons for the poor results is correct, a study was undertaken at Los Alamos to gain more insight into the problem. Earlier studies to evaluate the performance of the element for static problems indicate that the stiffness matrix formulation is correct. Also, results reported in reference 1 for the QUAD4 element with a lumped-mass matrix indicate that the problem is not with the stiffness matrix because the error in frequency prediction is quite low (less than 4 percent even for higher circumferential harmonics). Of course, some degradation in accuracy is expected for higher harmonics because the mesh density can become a limiting factor.

As a first step in our evaluation, an independent check of the formulation and coding was performed. No problems were found with either the formulation or the coding. As a final check, the mass matrix for a single element oriented at a skew angle to the global coordinate system was calculated by hand, and then results of the code were compared. There apparently are no errors in either the formulation or the coding.

A brief review of the literature on the subject of consistent-mass matrices does lend some insight into the problem. Clough and Wilson (ref. 3) state that, if the consistent-mass formulation is used with a displacement compatible element, resulting frequencies are always larger than the true frequencies. With a lumped-mass matrix, the frequencies may be above or below the true frequencies leading to the possibility that use of the lumped-mass matrix formulation <u>may</u> result in more accurate frequency predictions. The NASTRAN Theoretical Manual (ref. 4) describes errors associated with both consistent- and lumped-mass matrices for the rod elements. Fortunately, in this case, the errors turn out to be opposite in sign and of equal magnitude for lower-order terms. Thus, an accurate mass matrix can be generated simply by averaging the lumped- and consistent-mass matrices. The case does not appear to be the same for shell elements. Stavrinidis et al. (ref. 5) propose improved mass matrices for several elements, including the one-dimensional bar, two-dimensional membrane, and the pure bending beam element. Their method and other methods using so-called "higher-order" mass matrices depend on the predicted frequencies being consistently high or low. With significant effort, similar methods may be applicable to the current three-dimensional shell problem. However, as will be seen later in this paper, solutions with the consistent-mass matrix for the QUAD4 element can be either high or low, depending on the geometry of the structure.

TEST PROBLEMS

Two test problems were chosen for this study. The first was a free-free flat plate for whose natural frequencies we have closed-form, analytical solutions. The second was a right, circular cylinder. Closed-form solutions do not exist for this geometry, so the finite-difference code BOSOR (ref. 6) was used with a fine mesh to establish the reference frequencies and mode shapes. BOSOR results compare favorably with approximate solutions presented by Blevins (ref. 7).

The flat plate was a 10 by 10, 0.1-thick square. Its elastic modulus was 1.0×10^5, Poisson's ratio was 0.3, and the density was 1.0. Figure 1 shows the first three vibration modes of the plate with the theoretical frequencies.

The cylindrical shell had a radius of 300 and a length of 600. The material thickness was 3.0. Its elastic modulus, Poisson's ratio, and density were 3.0×10^6, 0.3, and 2.588×10^{-4}. The cylinder ends were simply supported without axial constraint (rigid diaphragm). Table I gives the reference frequencies calculated with BOSOR (ref. 6) for the cylindrical shell, along with the approximate solutions given by Blevins (ref. 7).

FINITE ELEMENT MODELS

Three different finite-element codes were used to model each of the two test problems. The finite-element code SPAR (ref. 8) was used with its E43, four-node quadrilateral element. This element is based on a mixed formulation first proposed by Pian (ref. 9). For analyzing these problems both the lumped- and coupled-mass matrices in the SPAR code were used. Because the E43 element is based on an assumed-stress function, rather than an assumed-displacement function, its coupled-mass matrix is not "consistent." That is, it is not derived from the same displacement functions used in deriving the stiffness matrix. Two types of elements were used with the ABAQUS finite-element code (ref. 10). The S8R5 element is an eight-node element that has only a consistent-mass matrix option. The S4R5 element is a four-node element that offers only a lumped-mass matrix. Finally, NASTRAN was used with the QUAD4 element with both the lumped- and the consistent-mass matrix options. In addition, the problems were analyzed with NASTRAN using a matrix that is the average of the consistent- and lumped-mass matrices.

The flat plate was modeled with three mesh densities having three, five, or seven nodes along each edge of the plate. ABAQUS was not used with the coarsest mesh because that would have resulted in a one-element mesh for the eight-node S8R5. The cylindrical shell was also modeled with three different mesh densities. These meshes had 5, 9, or 17 nodes on each edge. For the ABAQUS eight-node element, fewer total nodes were present because of the lack of the middle node. For this study, only one eighth of the shell was modeled, and symmetry conditions were used on all boundaries. Thus, only the even circumferential and odd longitudinal harmonics were determined.

All the NASTRAN solutions were obtained using the FEER eigenvalue extraction method. The mass-orthogonality test parameter was 0.0001 for the analyses.

RESULTS

Results for the flat plate are shown in figures 2 through 4. In these figures, the horizontal axes show the number of nodes on each side of the square mesh and the vertical axis is the natural log of the absolute value of the error in predicted frequency. The error is simply the ratio of the calculated frequency to the theoretical frequency minus 1.0. For reference, a plotted ln(error) of -4.6 is approximately 1.0 per cent in error in absolute frequency determination. A plotted value of -8.0 is roughly equivalent to 0.03 per cent error.

The data points labeled "lumped," "consistent," and "average" are all for the NASTRAN QUAD4 element. Study of the results reveals some definite patterns. As might be expected, the consistent-mass matrix always outperforms the lumped-mass matrix. However, the rates of convergence seem to be approximately the same. The SPAR results that were obtained by using the coupled-mass matrix are consistently better than the NASTRAN results. However, the convergence pattern is not smooth and, for all cases, the SPAR E43 element with its coupled-mass matrix yields better answers with the intermediate, rather than the fine, mesh. This result is somewhat disturbing, although, in all cases, the errors were small. The ABAQUS S8R5 element also gives slightly better results than does the NASTRAN QUAD4 element. For the flat plate, the elements with the consistent-mass matrix formulation always overpredicted the frequencies and those with the lumped-mass matrices always underpredicted the frequencies.

Results for the cylinder are not as clear as for the flat plate. Figures 5 through 7 show the frequency-convergence characteristics for the elements that are being considered for three different modes. These involve the second, fourth, and sixth circumferential harmonics (n=2,4,and 6) and the first longitudinal harmonic (m=1). The most striking observation is that, for the QUAD4 element, the lumped-mass matrix is now outperforming the consistent-mass matrix. This observation seems to confirm the result of Marcus (ref. 1). To illustrate the point, data from reference 1 have been added to the figures. Here, the definition of the ordinate axis has to be qualified. In reference 1, a 13 node by 37-node mesh was used in modeling one half of a cylinder. This becomes a 7-node by 19-node mesh when an eighth of the cylinder is considered, as is the case for this study. Because, for the modes presented, only the first longitudinal harmonic is present, we can loosely define this as a 7 by 7 mesh and plot it as such on our figures.

Only for the lower, second harmonic (fig. 5) does the ABAQUS S8R5 element outperform the NASTRAN lumped-mass element. For all three modes, the QUAD4 with the lumped-mass matrix yields the best results. Its deviation from the QUAD4 with the consistent-mass matrix increases with higher circumferential harmonics. The SPAR, E43 element with a coupled-mass matrix tends to follow the QUAD4 element closely for these modes.

Except for a few cases, the frequencies were overpredicted for consistent-, lumped-, and coupled-mass matrices for the cylindrical-shell problem. The exceptions were the QUAD4 lumped-mass matrix and the E43 coupled-mass matrix for the finest mesh for the second and third modes considered here.

Frequency is not the only parameter that should be considered for modal analyses. The other is, of course, the mode shape. One method of comparing mode shapes is to compare calculated generalized masses for the solutions using the different elements being considered. Another is to use a parameter frequently calculated when comparing calculated mode shapes with experimentally measured mode shapes. This parameter is called the mode shape correlation coefficient (MSCC) and is described in reference 11. It essentially provides a measure of the least-squares deviation of the points being compared from a straight-line correlation. Both these measures were used in comparing solutions for the n=8, m=5 mode for the cylinder being considered here. Results of these comparisons are summarized in table II, along with comparisons of the predicted frequencies. The predicted frequencies are normalized using the BOSOR code results as the baseline. The generalized masses were normalized using the theoretical value obtained by direct integration of the square of the analytically perfect mode shape multiplied by the material density. For the MSCC comparisons, mode shapes predicted by BOSOR were used as the "correct" shape.

A study of the results summarized in table II shows again that the lumped-mass matrix provides better frequency predictions than does the consistent-mass matrix for the NASTRAN QUAD4 shell element. Note that for the 9-node by 9-node mesh, the error for the consistent-mass matrix is over 30 per cent. A finer mesh (17 by 17) with the consistent-mass matrix provides better frequency approximations, but the prediction is still not as good as for the lumped-mass matrix with a coarser mesh. The generalized mass is in considerable error for both QUAD4 cases in which the consistent-mass matrix is used.

The generalized mass is a much more sensitive measure of mode shape error than the MSCC, as evidenced by data for the ABAQUS results that used the S8R5 element. Here the MSCC is quite close to 1.0 for the 9-node by 9-node mesh, but the generalized mass is over 30 per cent in error. As the mesh is refined, the generalized mass improves, but it is still not as accurate as for the QUAD4 when a lumped-mass matrix is used. Note that the performance of the ABAQUS S4R5 element compares favorably with the NASTRAN QUAD4 element.

The SPAR E43 element, which performed nearly as well as the QUAD4 in predicting frequencies for all the shell modes considered in this study, apparently predicts both the frequency and generalized mass accurately if the coupled-mass matrix is used. However, somewhat unexpectedly, this element does not perform quite so well with a lumped-mass matrix. In this case, the frequency is predicted accurately but the mode shape has considerable error associated with it, as evidenced by the underprediction of the generalized mass.

CONCLUSIONS

Among the elements considered in this study, the NASTRAN QUAD4 element with a lumped-mass matrix seems to be the best choice when the geometry is a cylindrical shell. A general rule seems to be that, for any element considered here, consistent-mass matrices should be avoided for this particular geometry. On the other hand, for flat-plate geometries, the consistent-mass matrix outperforms the lumped-mass matrix.

These conclusions imply that choices are difficult when modeling geometries that deviate from the simple geometries considered here. It is possible that an alternate method of deriving the mass matrix, such as the SPAR coupled-mass matrix, would generate a result that would be more generally applicable. Note that it seems to perform well for both geometries. However, for the present, if the geometry is predominantly cylindrical, the lumped-mass matrix should always be used with the NASTRAN QUAD4 element.

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TABLE I

PREDICTED FREQUENCIES (Hz) FOR CYLINDRICAL SHELL USING BOSOR (BLEVINS) FOR EVEN CIRCUMFERENTIAL HARMONICS (n) AND ODD LONGITUDINAL HARMONICS (m).

n \ m	1	3	5
2	19.61 (21.82)	47.97 (48.61)	54.69 (54.83)
4	7.92 (8.27)	33.27 (33.85)	47.05 (47.30)
6	7.32 (7.59)	23.64 (24.00)	39.56 (39.83)
8	10.76 (11.68)	20.63 (20.94)	35.18 (35.47)

TABLE II

COMPARISON OF FREQUENCY, GENERALIZED MASS, AND MODE SHAPE PREDICTED BY VARIOUS FINITE ELEMENT MODELS FOR CYLINDRICAL SHELL MODE n=8, m=5.

Nodes/side	Computer code/ element	Mass matrix	Normalized frequency	Mode shape correlation coef.	Normalized generalized mass
9	SPAR/E43	coupled	1.032	0.9995	1.011
9	SPAR/E43	lumped	1.046	0.9853	0.810
9	ABAQUS/S8R5	consistent	0.982	0.9998	0.677
17	ABAQUS/S8R5	consistent	1.010		0.971
9	ABAQUS/S4R5	lumped	0.987		0.995
9	NASTRAN/QUAD4	lumped	1.014	0.9991	1.010
9	NASTRAN/QUAD4	consistent	1.336		0.603
17	NASTRAN/QUAD4	consistent	1.066		0.876

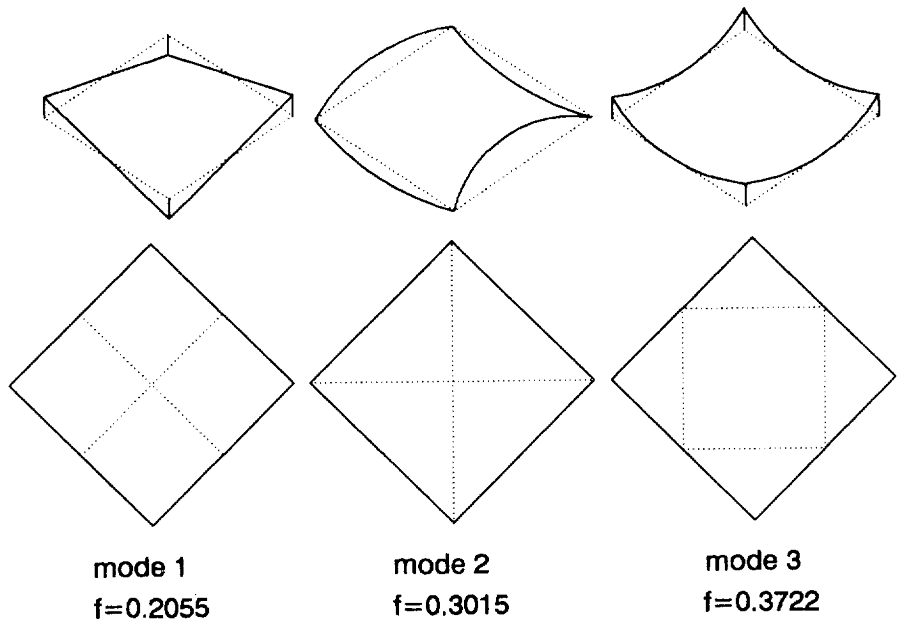

Figure 1. Mode shapes and frequencies (Hz) for flat plate.

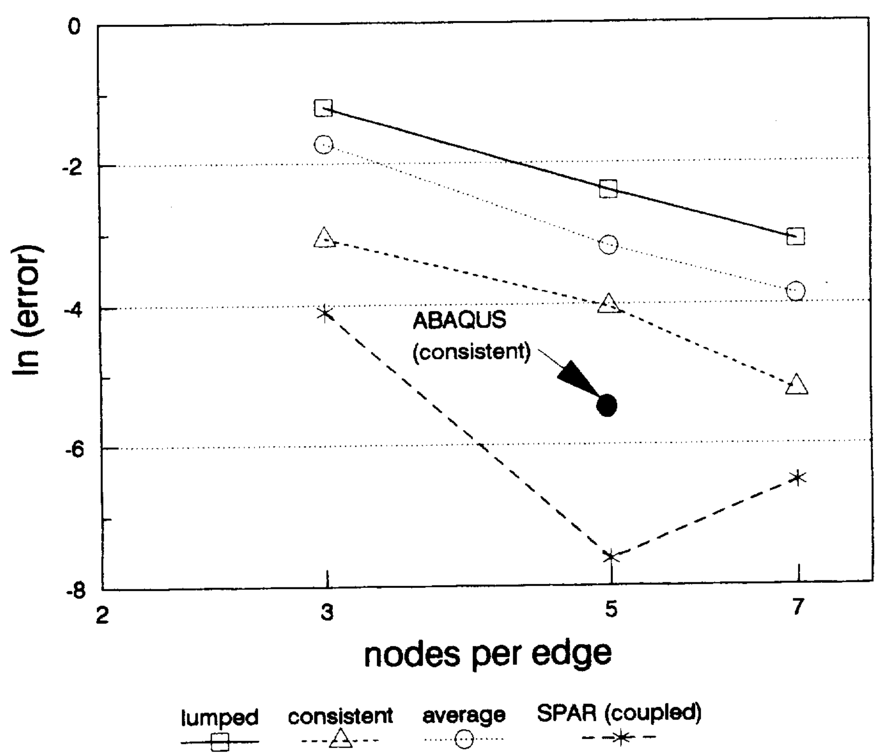

Figure 2. Frequency convergence for mode 1 of a flat plate as a function of mesh density for different finite elements.

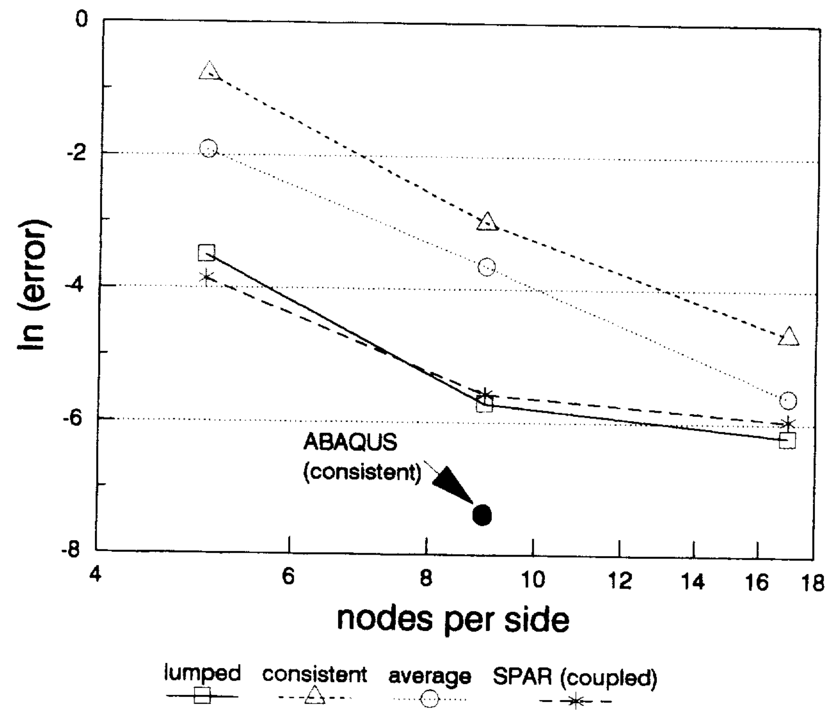

Figure 3. Frequency convergence for mode 2 of a flat plate as a function of mesh density for different finite elements.

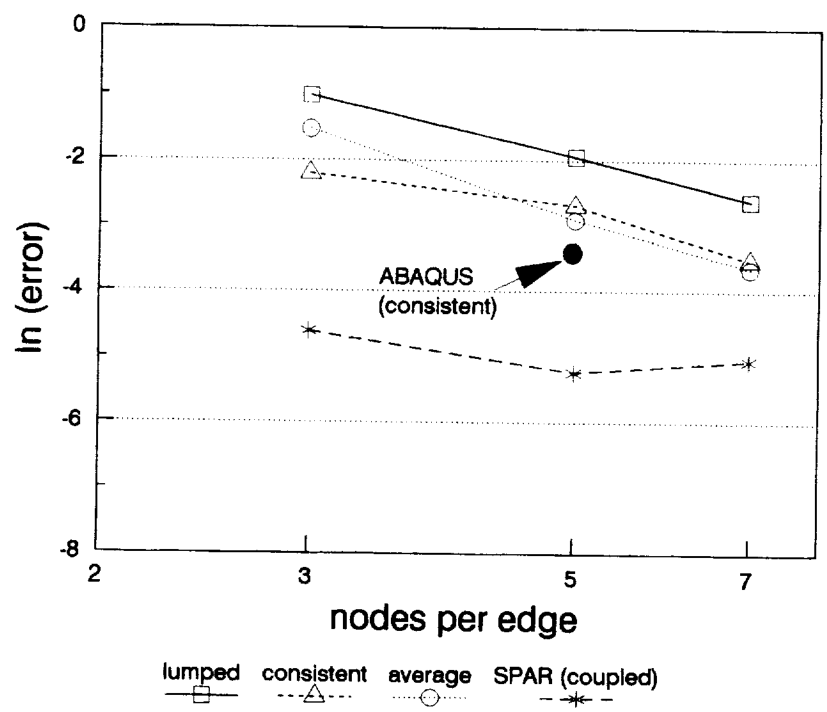

Figure 4. Frequency convergence for mode 3 of a flat plate as a function of mesh density for different finite elements.

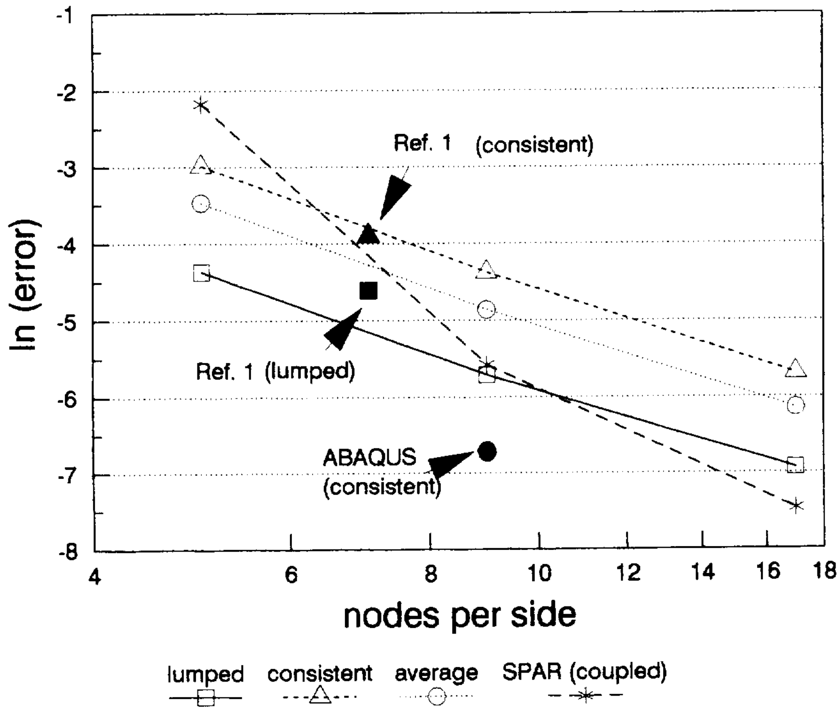

Figure 5. Frequency convergence for mode n=2, m=1 of cylindrical shell as a function of mesh density for different finite elements.

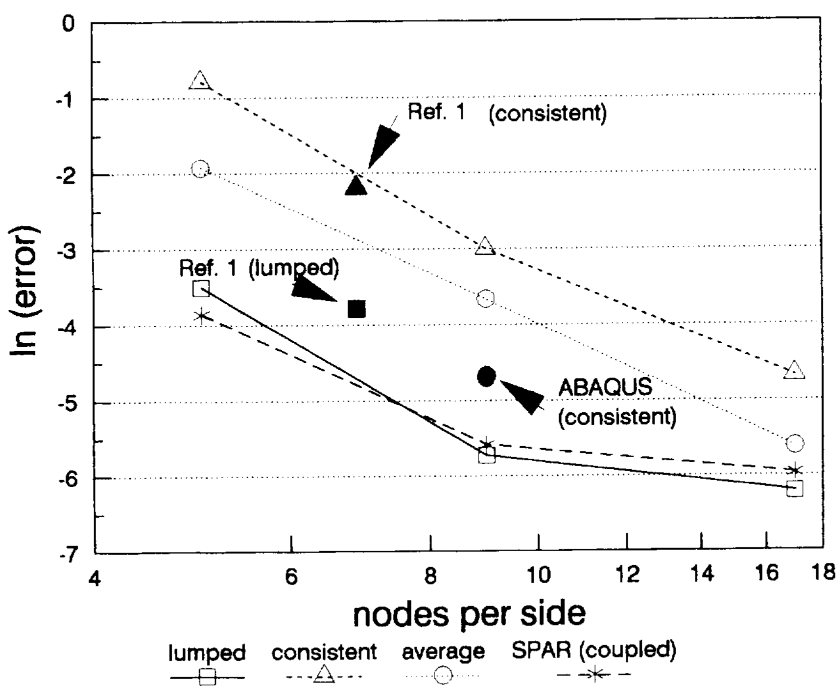

Figure 6. Frequency convergence for mode n=4, m=1 of a cylindrical shell as a function of mesh density for different finite elements.

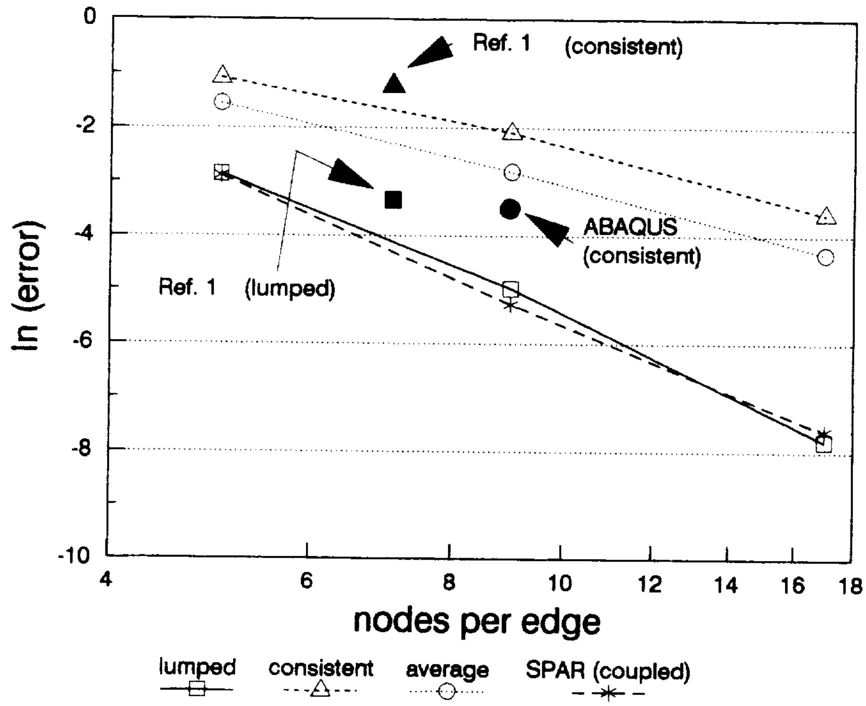

Figure 7. Frequency convergence for mode n=6, n=1 of a cylindrical shell as a function of mesh density for different finite elements.